

Tower of Hanoi Problem

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The **Tower of Hanoi** (also called the **Tower of Brahma** or **Lucas' Tower** and sometimes pluralized) is a mathematical game or puzzle. It consists of three rods and a number of disks of different sizes, which can slide onto any rod. The puzzle starts with the disks in a neat stack in ascending order of size on one rod, the smallest at the top, thus making a conical shape.

The objective of the puzzle is to move the entire stack to another rod, obeying the following simple rules:

1. Only one disk can be moved at a time.
2. Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty rod.
3. No disk may be placed on top of a smaller disk.

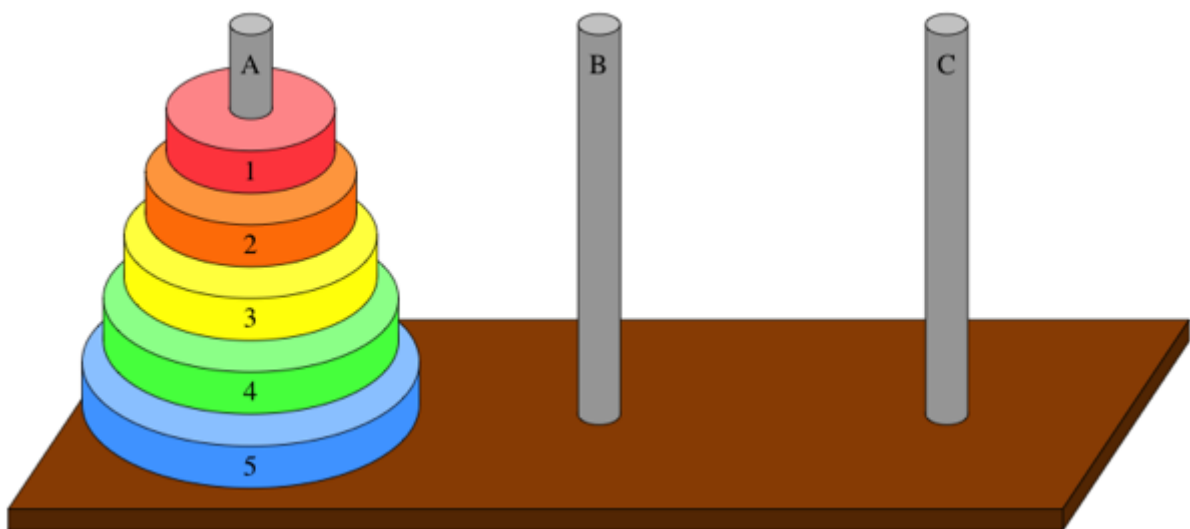


Fig.-1

Approach

Illustrated in the Fig. 2.

Step #1: Recursively move top (N-1) disk from source to auxiliary peg.

Step #2: Move the last disk (largest disk at the bottom) from **source** to **destination** peg.

Step #3: Recursively move (N-1) disk from **auxiliary** to **destination** peg.

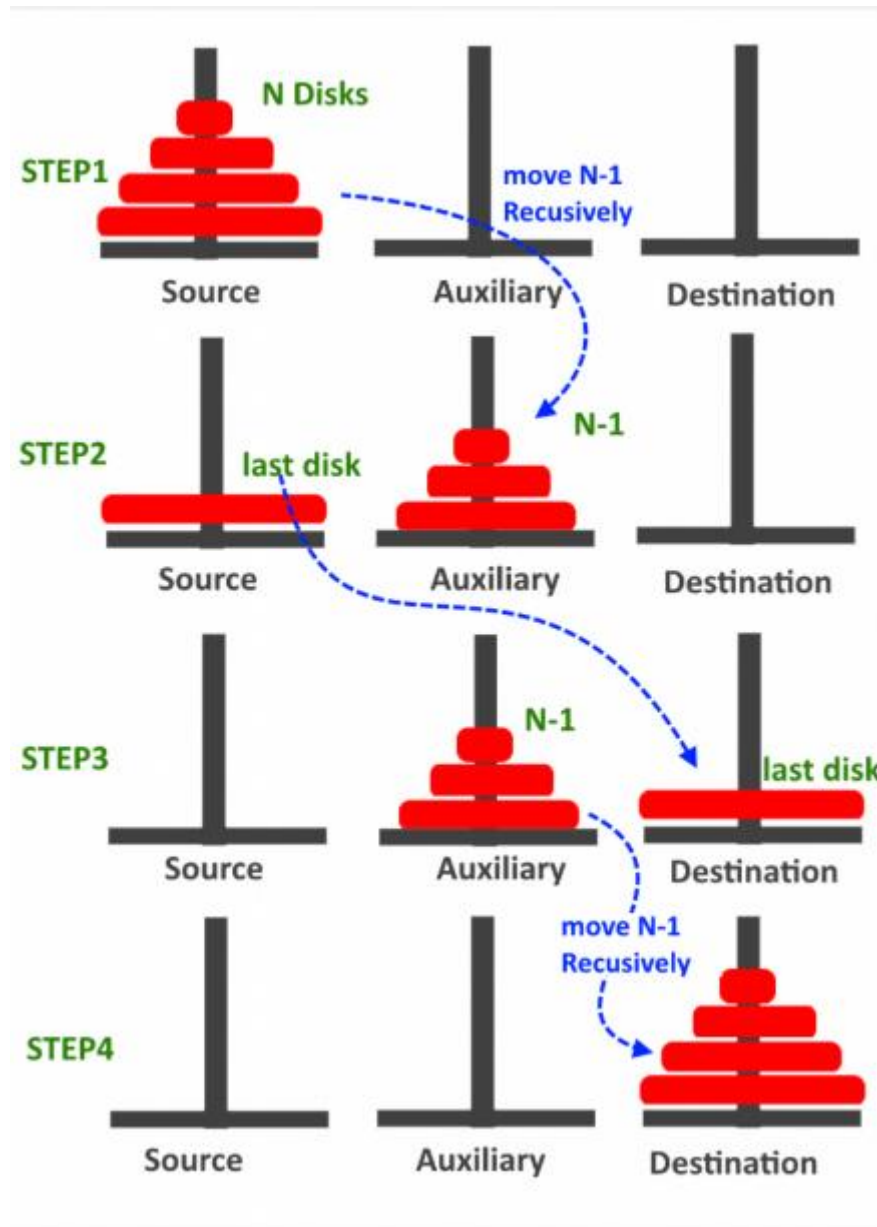


Fig.-2

Terminating Condition for Recursion:

If $N=0$ then No disk transfer is required.

Recursive C function to solve Tower of Hanoi Problem

```
# include <stdio.h>
```

```
void initialisation (int *);
```

```
void Tower_of_Hanoi ( int *, int , char, char, char);
```

```
void initialisation (int * step_count)
```

```
{
```

```
    *step_count=0;
```

```
}
```

```
void Tower_of_Hanoi ( int *step_count, int N, char Source, char Aux, char Target)
```

```
{
```

```
    if(N>0)
```

```
    {
```

```
        Tower_of_Hanoi (step_count, (N-1), Source, Target, Aux);
```

```
        printf("\n Step No. %d : Transfer Disk %d from %c to  
                %c", ++(*step_count), N, Source, Target);
```

```
        Tower_of_Hanoi (step_count, (N-1), Aux, Source, Target);
```

```
    }
```

```
}
```

```

void main( )
{
    int n, step_no;
    printf("\n Enter the no. of disks:");
    scanf("%d", &n);
    initialisation(&step_no);
    Tower_of_Hanoi ( &step_no, n, 'S', 'A', 'T');
}

```

No. of Steps Required to Transfer N no. Disks

Let, $T(N)$ = No. of steps required to transfer N disks

$$T(N) = 2T(N-1) + 1 \quad \text{if } N > 0$$

$$T(0) = 0$$

$$T(N) = 2T(N-1) + 1$$

$$= 2[2T(N-2) + 1] + 1$$

$$= 2^2 T(N-2) + (2+1)$$

$$= 2^2 [2T(N-3) + 1] + (2+1)$$

$$= 2^3 T(N-3) + (2^2 + 2 + 1)$$

$$\dots \quad \dots \quad \dots$$

$$= 2^r T(N-r) + (2^{r-1} + \dots + 2^2 + 2 + 1)$$

(after r no. of steps)

$$= 2^N T(0) + (2^{N-1} + \dots + 2^2 + 2 + 1)$$

(after N no. of steps)

$$= 0 + (2^{N-1} + \dots + 2^2 + 2 + 1) \quad [\because T(0) = 0]$$

$$= 2^N - 1$$