

Assignment on Random Variable

ASSIGNMENT 1

1. A die is tossed two times. Let X be the sum of face values on the two tosses and Y be the absolute value of the difference in face values. Note that both X and Y are random variables. Find out range of X that is $\mathcal{R}(X)$ and range of Y that is $\mathcal{R}(Y)$. Find out the events $(X = 2)$ and $(Y = 2)$.
2. Let X be a random variable. Is $|X|$ also an random variable? If $\mathcal{R}(X) = \{-2, -1, 0, 1, 2\}$ then find out $\mathcal{R}(|X|)$.
3. A die is rolled five times. Let X be the sum of the face values. Find out (a) $P(X = 6)$ and (b) $P(X \geq 29)$.
4. Let X be a discrete random variable with the following probability mass function (pmf): $P(X = 0) = p$ and $P(X = 1) = 1 - p$. Find out corresponding distribution function $F(x)$.
5. Let X be a discrete random variable with $\mathcal{R}(X) = \{1, 2, 3, \dots\}$. Find the value of c so that the function $f(k) = \frac{c}{k^2}$, for all $k \in \mathcal{R}(X)$ can be a probability mass function of X . Hence find out corresponding distribution function $F(X)$.
6. Show that the function

$$f(x) = \frac{1}{2}e^{-|x|}$$

is a probability density function of some random variable X . Find out the distribution function $F(x)$.

7. Let X be a continuous random variable with the following triangular probability density function (pdf):

$$f(x) = \begin{cases} x, & \text{if } 0 < x \leq 1 \\ 2 - x, & \text{if } 1 < x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find out the following:

- (a) The distribution function $F(X)$ of the random variable X ,
(b) $P(0.3 < X \leq 1.5)$.

Why this is called a triangular pdf?

8. Is the following function a distribution function of some continuous random variable? If so, find the corresponding probability density function.

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1 - e^{-x^2}, & \text{if } x \geq 0. \end{cases}$$

9. The kms X in thousands of kms which a car owner get with certain kind of tyre is a random variable with a probability density function

$$f(x) = \begin{cases} \frac{1}{20}e^{-\frac{x}{20}}, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0. \end{cases}$$

Find out the probability that one of these tyres will last anywhere from 16,000 to 24,000 kms.

10. Let $f(x)$ be the probability mass function (pmf) of a discrete random variable X which assumes the values x_1, x_2, x_3, x_4 such that

$$2f(x_1) = 3f(x_2) = f(x_3) = 5f(x_4).$$

Find the pmf and corresponding distribution function $F(x)$.