LAB - 6

Q1) • Two Test cases were taken :

① $A = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$ $\|A\|_\infty = 0.5 < 1$ [Satisfies conditions]

After 20 iterations

$$B = \sum_{j=0}^{20} A^j \Rightarrow (I-A)B \rightarrow I$$

$(I-A)B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

② $A = \begin{bmatrix} 0.3 & 0.2 & 0.1 \\ 0.02 & 0.7 & 0.01 \\ 0.4 & 0.07 & 0.06 \end{bmatrix}$ $\|A\|_\infty = \max\{0.6, 0.73, 0.53\}$

$= 0.73 < 1$

↳ Satisfies conditions

After 20 iterations :

$(I-A)B = \begin{bmatrix} 1.00 & -0.0005 & 0 \\ -0.0001 & 0.991 & 0 \\ 0 & -0.004 & 1 \end{bmatrix} \rightarrow I$ (tends to identity matrix)

This happens primarily because $B = (I-A)^{-1} = \sum_{k=0}^{\infty} A^k \therefore (I-A) \cdot B \rightarrow I$

On increasing N, we get better results

↳ Neumann Series

Q2)

$$\begin{bmatrix} 60 & 30 & 20 \\ 30 & 20 & 15 \\ 20 & 15 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 110 \\ 65 \\ 47 \end{bmatrix}$$

A

b

$$x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Iteration 1:

$$r_0 \rightarrow \begin{bmatrix} 110 \\ 65 \\ 47 \end{bmatrix}, \quad e_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Residual Vector

Error Vector

Iteration 2:

$$r_2 \rightarrow \begin{bmatrix} 2.13 \times 10^{-13} \\ 1.1 \times 10^{-13} \\ 0.7 \times 10^{-13} \end{bmatrix}, \quad e_2 = \begin{bmatrix} -0.0071 \times 10^{-13} \\ 0.2274 \times 10^{-13} \\ -0.2132 \times 10^{-13} \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

residual Vector

Error Vector

Iteration 3:

$$r_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad e \rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Residual Vector

Error Vector

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03) $A_1 = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/3 & 1 & 1/2 \\ 1/2 & 1/3 & 1 \end{bmatrix}$, $b_1 = \begin{bmatrix} 11/18 \\ 11/18 \\ 11/18 \end{bmatrix}$

Consider $x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

- Richardson:

$x = \begin{bmatrix} 0.33 \\ 0.33 \\ 0.33 \end{bmatrix}$ 73 iterations required to acheive an accuracy of 10^{-6}

- Jacobi:

$x = \begin{bmatrix} 0.33 \\ 0.33 \\ 0.33 \end{bmatrix}$ 73 iterations are required to acheive an accuracy of 10^{-6}

- Gauss-Siedel Count

$x = \begin{bmatrix} 0.33 \\ 0.33 \\ 0.33 \end{bmatrix}$ 12 iterations required to acheive an accuracy of 10^{-6}.

$A_2 = \begin{bmatrix} 10 & 2 \\ 4 & 12 \end{bmatrix}$, $b_2 = \begin{bmatrix} 12 \\ 16 \end{bmatrix}$

P.T.O

- Richardson

The answer diverged from the actual answer.
This is because it didn't satisfy the condition.

$\|I - A\|_\infty < 1$

$x = \begin{bmatrix} -4.08 \times 10^{222} \\ -8.19 \times 10^{222} \end{bmatrix}_{2\times 1}$ [More than 200 iterations]

- Jacobi

$x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_{3\times 1}$ 11 iterations required to achieve accuracy of 10^{-6}.

- Gauss-Siedel

$x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_{3\times 1}$ 6 iterations required to achieve accuracy of 10^{-6}

Jacobi and Gauss-Siedel converged because they satisfied $\|I - D^{-1}A\|_\infty < 1$ ~~and also~~ or

$\rho(A) \leq \inf \|A\|$

↳ holds higher precedence

In General, Gauss-Siedel has better efficiency than Richardson's & Jacobi's Method.

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Q4) ω is initialized as 0.05 and incremented in steps of 0.2

$$A = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/3 & 1 & 1/2 \\ 1/2 & 1/3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 11/18 \\ 11/18 \\ 11/18 \end{bmatrix}$$

(Max of 50 iterations are taken)

- $W = 0.05$

$$x = \begin{bmatrix} 0.3319 \\ 0.3313 \\ 0.3278 \end{bmatrix}$$

iterations → 50

Less Accuracy than 10^{-6} [Less Accuracy]

- $W = 0.25$

$$x = \begin{bmatrix} 0.3319 \\ 0.3313 \\ 0.3278 \end{bmatrix}$$

iterations → 50

Accuracy less than 10^{-6} [Less Accuracy]

- ~~0.45~~ $0.45 = W$

$$x = \begin{bmatrix} 0.3333 \\ 0.3333 \\ 0.3333 \end{bmatrix}$$

iterations → ~~36~~ 36

Accuracy → 10^{-6}

- ~~0.65~~ $0.65 = W$

$$x = \begin{bmatrix} 0.3333 \\ 0.3333 \\ 0.3333 \end{bmatrix}$$

iterations → ~~36~~ 23

Accuracy → 10^{-6}

- $W =$ ~~0.85~~ 0.85

$$x = \begin{bmatrix} 0.3333 \\ 0.3333 \\ 0.3333 \end{bmatrix}$$

iterations → 16

Accuracy → 10^{-6}

(w ∈ (0,1))

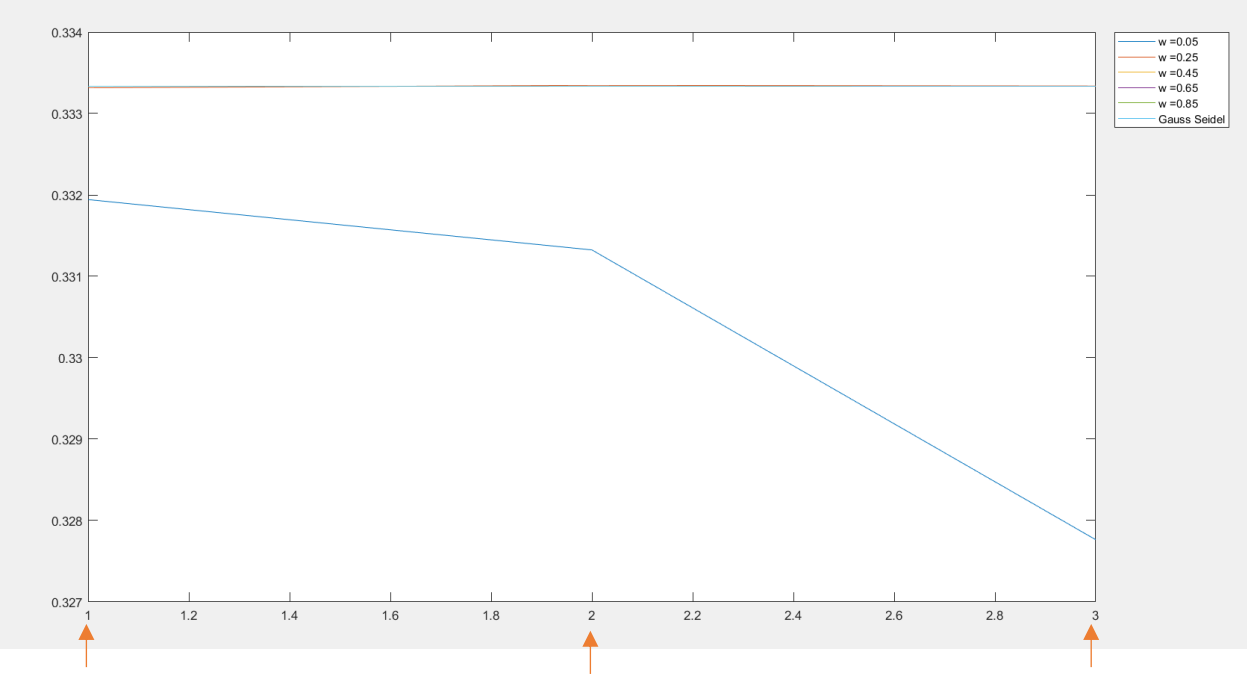
On increasing w, efficiency increases as it was previously underfitted. Also as $w \to 1$, we can see from the graph it tends to Gauss-Siedel solution.

- For 50 iterations, all the graphs tend to the same point although for higher w, it is more accurate
- For 20 iterations, the distinction can be seen better.
- Underfitted solutions are less efficient.

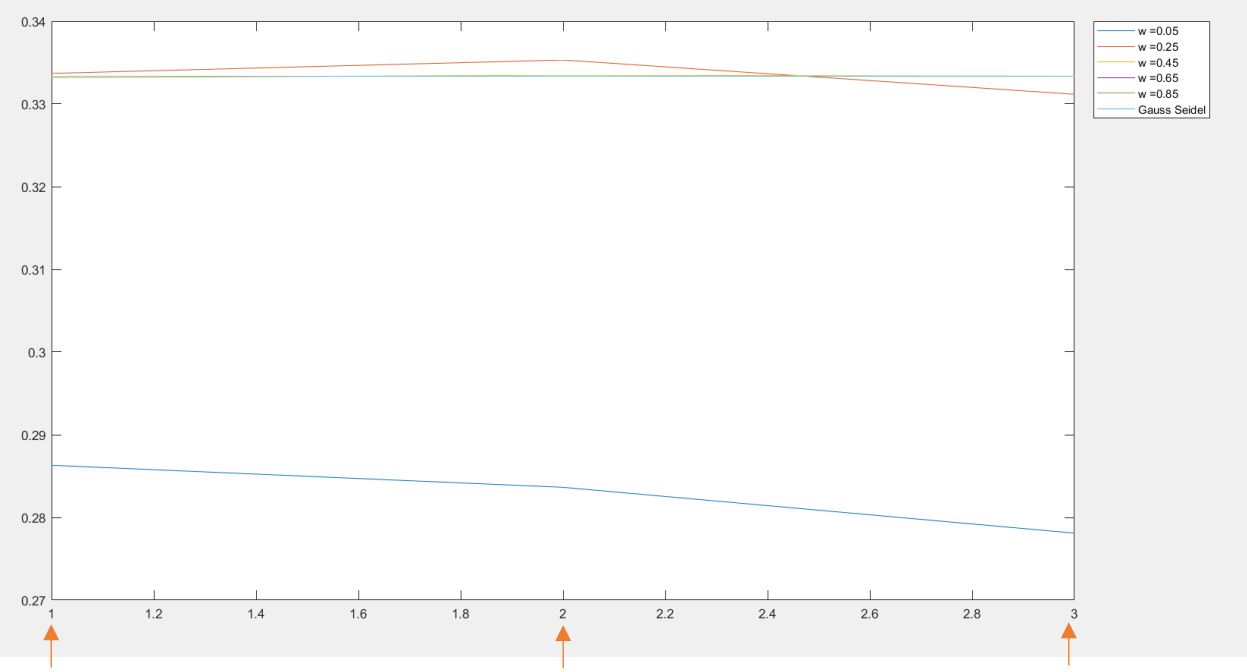
Two types of plots have been made:

- X-value plot: values of x_i after the final iteration for various w and Gauss-Seidel method, have been plotted. The arrows represent x_1, x_2 and x_3.
- Error plots: After the final iteration for each method the error vector (i.e. actual answer - x_{final}) has been calculated. The error corresponding to x_1, x_2, x_3 have been plotted. The x_i 's have been marked with arrows.
- We need to observe values only at x=1,2,3 for all the plots (where arrows are made).

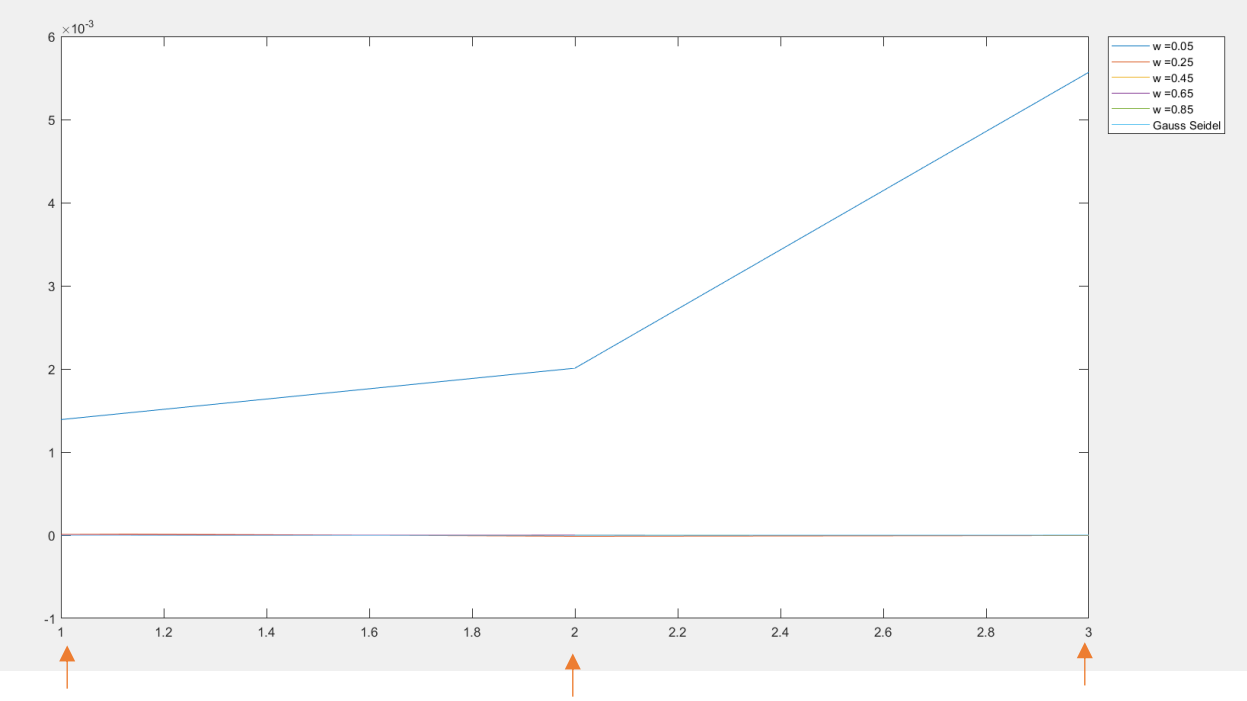
X-value Plot (max 50 iterations)



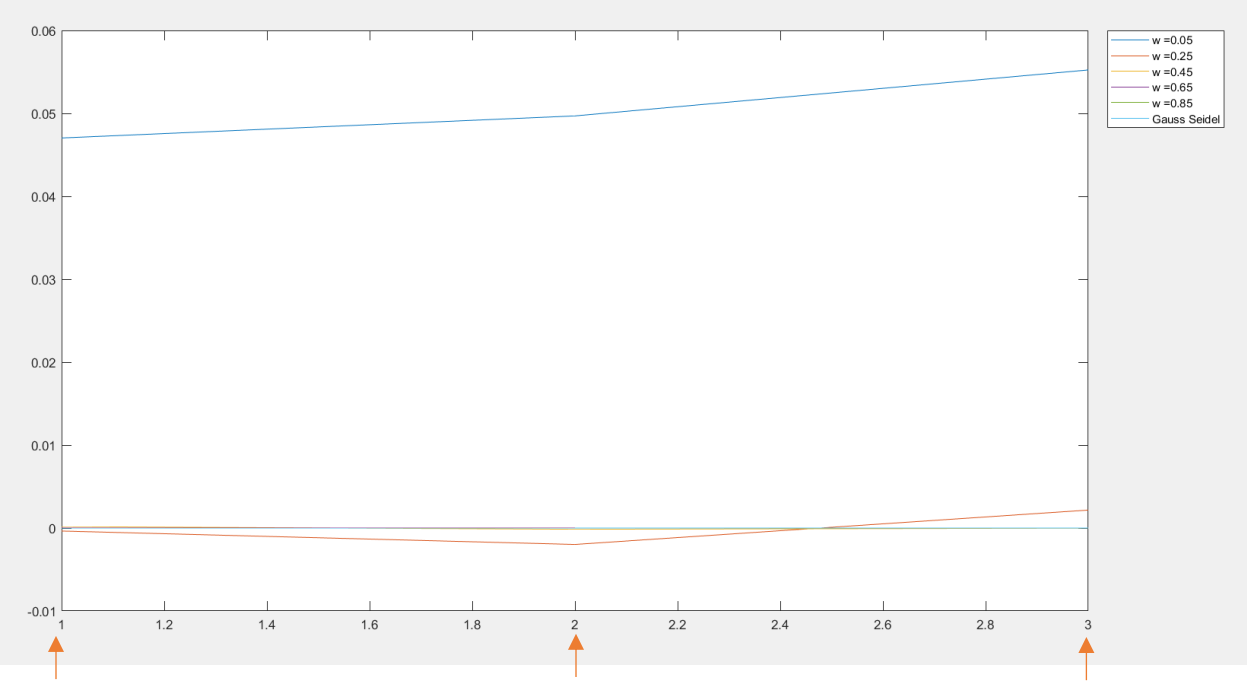
X-value Plot (max 20 iterations)



Error Plot (max 50 iterations)



Error Plot (max 20 iterations)



- For the X-plot's, as w tends to 1, SOR solutions also tends to the gauss-seidel solution for all x_i.
- For error plots as w tends to 1 the error value for every x_i also coincides with that of the gauss-seidel error and almost tends to 0.
- On increasing no. of iterations, even underfitted algorithms give a fairly decent approximation.
- The difference between the plots is more evident for 20 iterations.