LAB-7

Q1)

- when $x = [0 \quad 1 \quad 3 \quad 4 \quad 8]$
  $y = [8 \quad 12 \quad 2 \quad 6 \quad 0]$

$$p(x) = -0.2607x^4 + 3.58x^3 - 13.95x^2 + 14.628x + 8$$

- when $x = [0 \quad 1 \quad 3 \quad 8]$
  $y = [8 \quad 12 \quad 2 \quad 0]$

$$p(x) = 0.457x^3 - 4.828x^2 + 8.371x + 8$$

- when $x = [0 \quad 1 \quad 8]$
  $y = [8 \quad 12 \quad 0]$

$$p(x) = -0.714x^2 + 4.714x + 8$$

- when $x = [0 \quad 8]$
  $y = [8 \quad 0]$

$$p(x) = -x + 8$$

- For two points we obtain a linear line. on including more points, we obtain more higher degree terms. The method followed is Lagrange's Interpolation.

- On increasing the no. of points, we obtain a better approximation to the existing polynomial function. Hence we can also expect lower error for higher number of points.

- All graphs are attached at the end.

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Q2) • Via Lagrange's Method:

$p(x) = -0.00024x^{10} + 0.0023x^9 - 0.012x^8 + 0.0481x^7 -$
$0.155x^6 + 0.426x^5 - 0.971x^4 + 1.767x^3 -$
$2.413x^2 + 3.197x - 1$

• Via Newton's Interpolation:

$p(x) = \cancel{2.97x - 1.94}$
$-1 + 2.97x - 1.945x(x-0.1) + 1.27x(x-0.1)(x-0.2) +$
$-0.6308x(x-0.1)(x-0.2)(x-0.3) + 0.248x(x \ldots)\ldots(x-0.4)$
$-0.818(x)\ldots(x-0.5) + 0.23x\ldots(x-0.6)$
$-0.0056(x)(\ldots)(x-0.7) + 0.0012x\ldots(x-0.8)$
$-0.00024(x)\ldots(x-0.9)$

On expanding, both the polynomial turn out to be the same.

This can be seen from the Graph as well

• On using Bisection Method for 30 iterations, we obtain a root $x = 0.408$

• Another interesting observation is that, all 3 graphs almost coincide. This can be attributed to the fact that since we have 11 points between 0 to 1 (relatively high) hence we have a greater accuracy while approximating. (Although the above fact is not always necessary - Runge Phenomenon)

~~Graph attached at the end.~~

• Graph attached at the end.

Q3) Runge Phenomenon is the problem of oscillation at the edges when using polynomial interpolation with polynomials of high degree. The above phenomenon can be clearly seen from the graphs.

Observations:

A) Error Plots:

- as we increase n from 5 to 15, the error at the edge points increase significantly, but the error at the midpoints decreases to a considerable extent. Hence Accuracy need not always increase when number of nodes are increased. This can be clearly seen from the error plots.

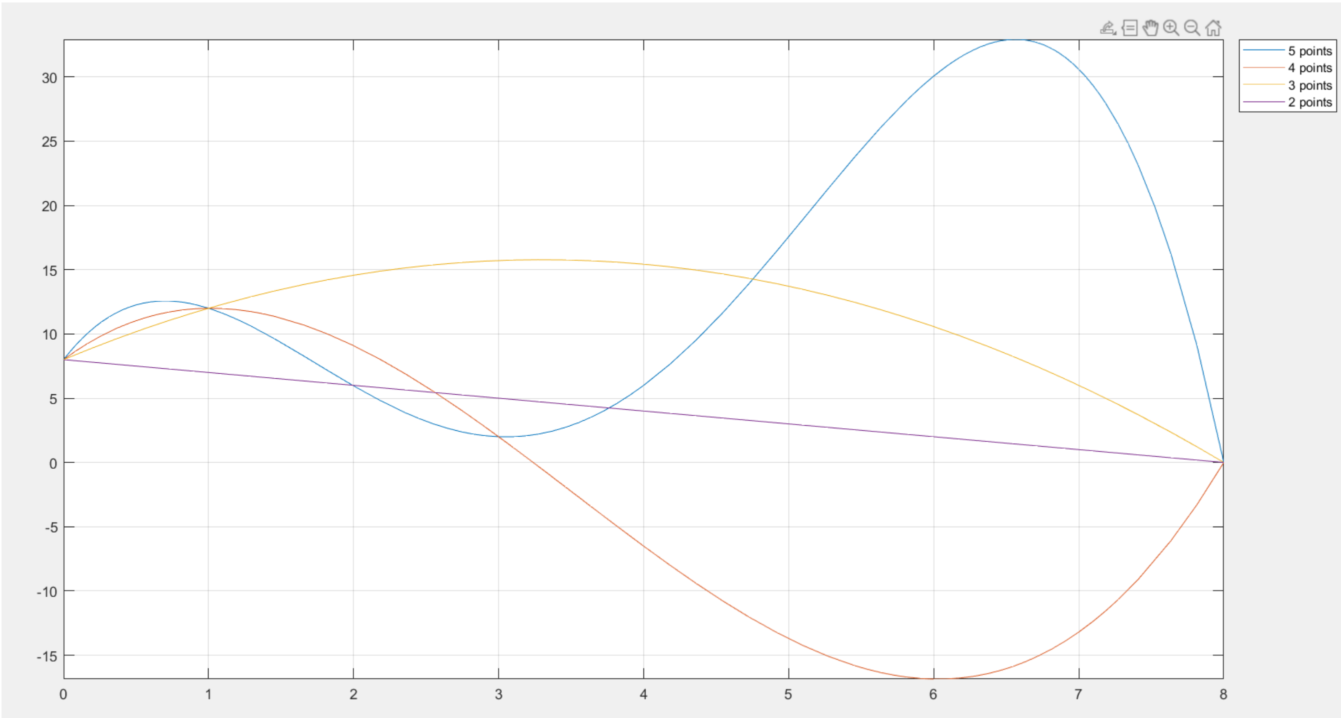
B) Polynomials:

- The interpolated polynomials (via Newton's Methods), also display a similar property like the error plots. Higher 'n' polynomials oscillate wildly at the edges, but have a better estimation in the mid part.

  In contrast, lower 'n' values, have low error at the edges, but have a poor estimation in the midpoints. This can also be seen from the polynomial plot.

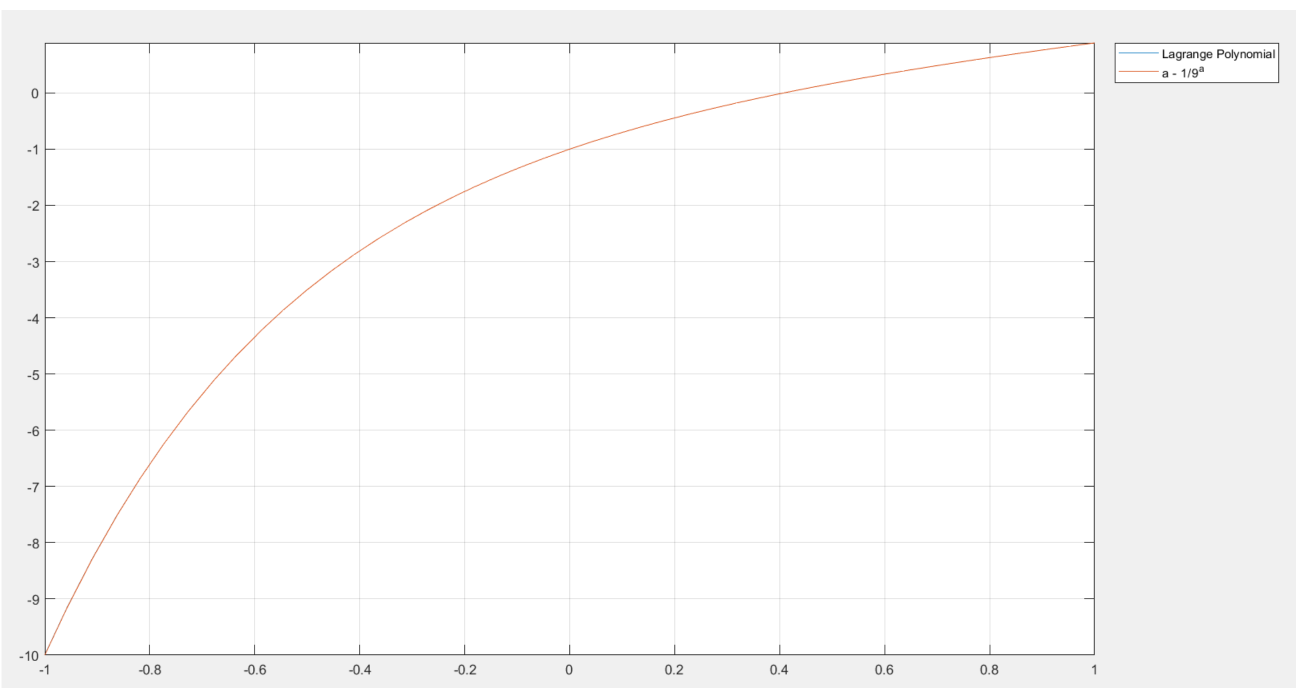
- For $n = 5$ & $n = 15$, the interpolated & actual polynomial are technically also supposed to meet at $x = 0$, and have only one intersection point there, but due to MATLAB's accuracy/precision limitation, the interpolated polynomial graph shifts slightly down and it now intersects with the actual graph at $x = -\delta$ & $x = +\delta$, leading to an extra intersection point (which should technically not be there).

Q1)

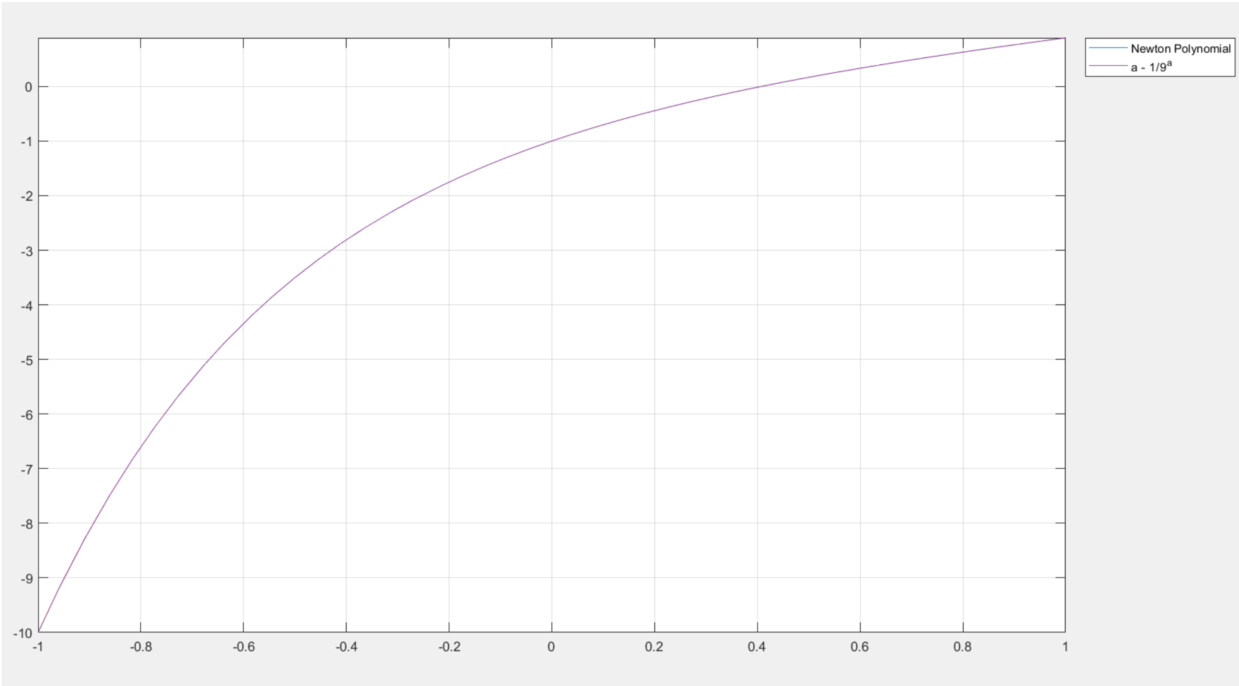


Q2)a)

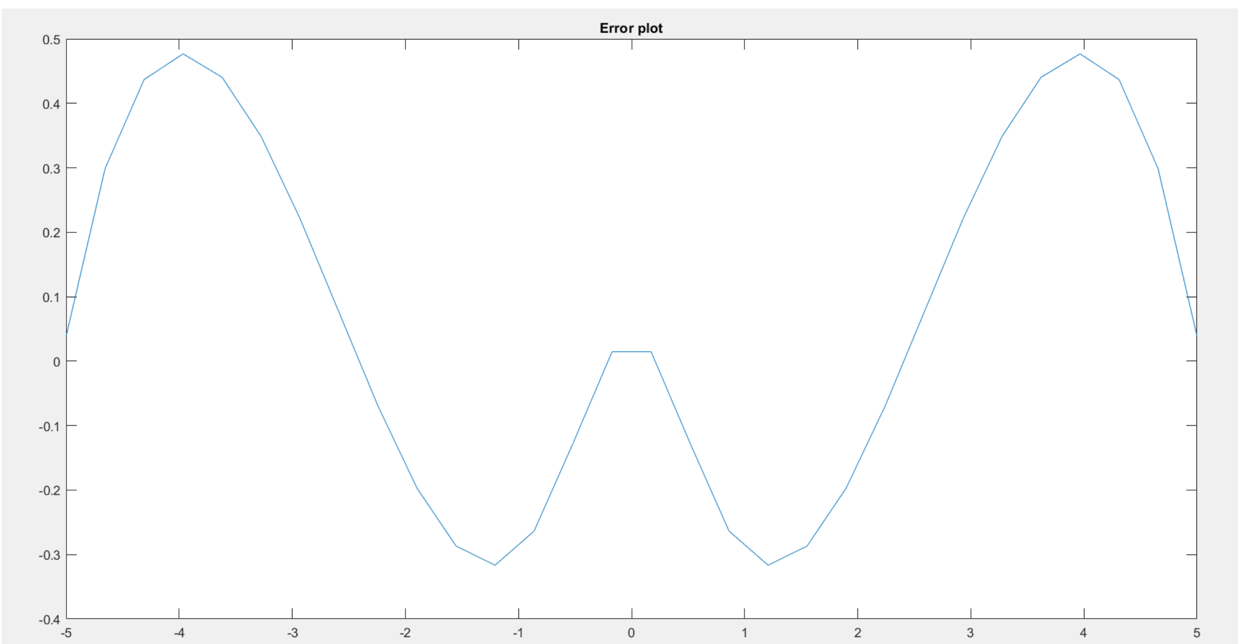
Lagrange's form of Interpolating Polynomial



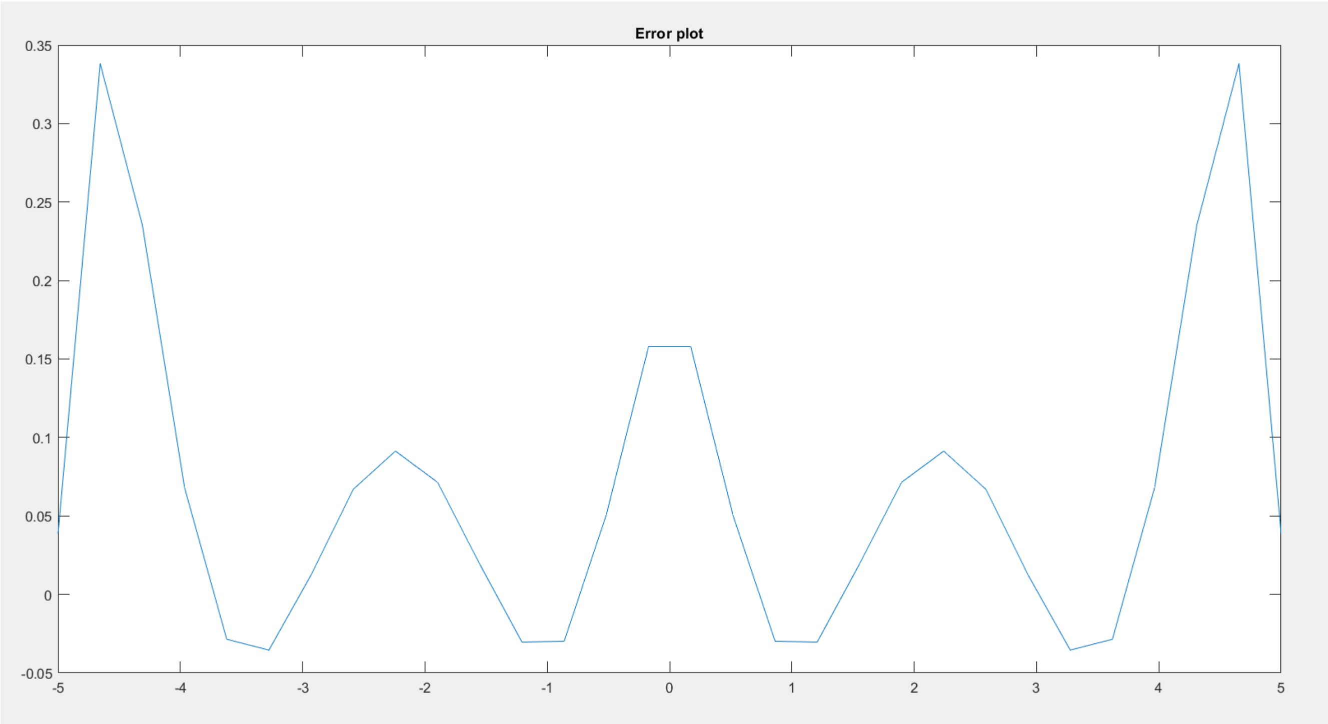
Newton's form of Interpolating Polynomial



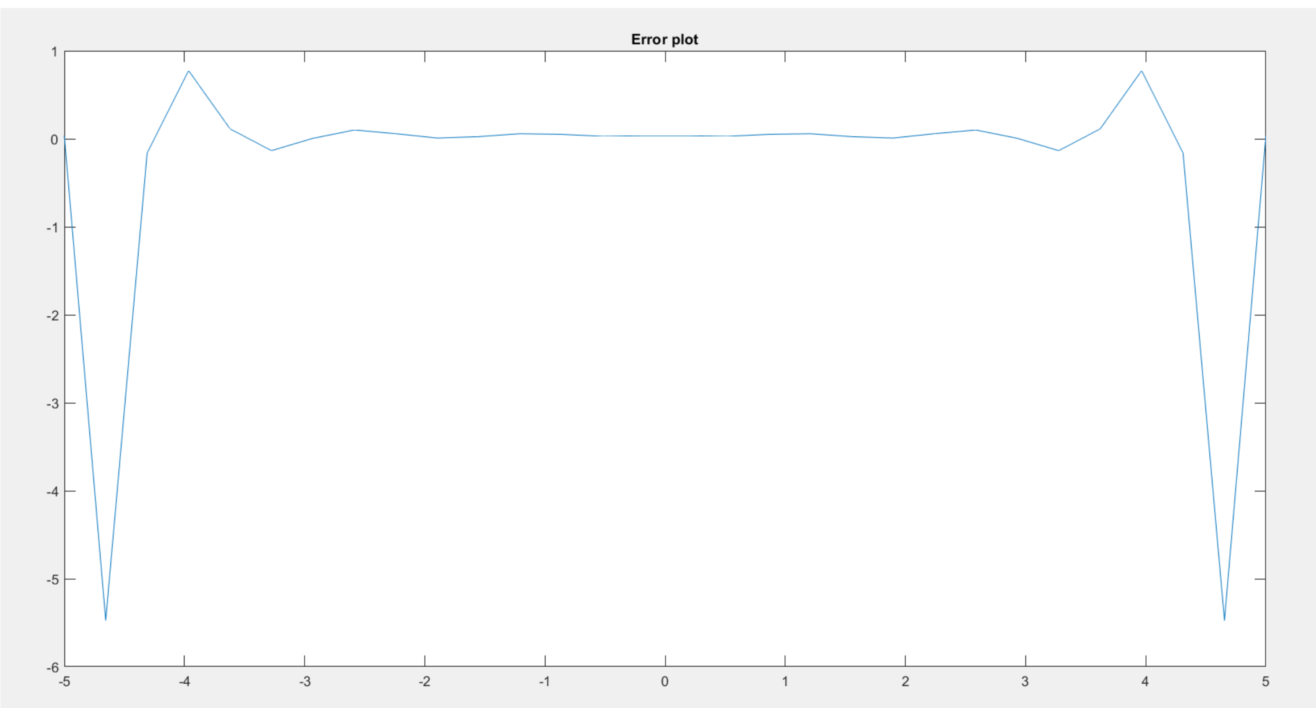
Q3) error plot for n=5



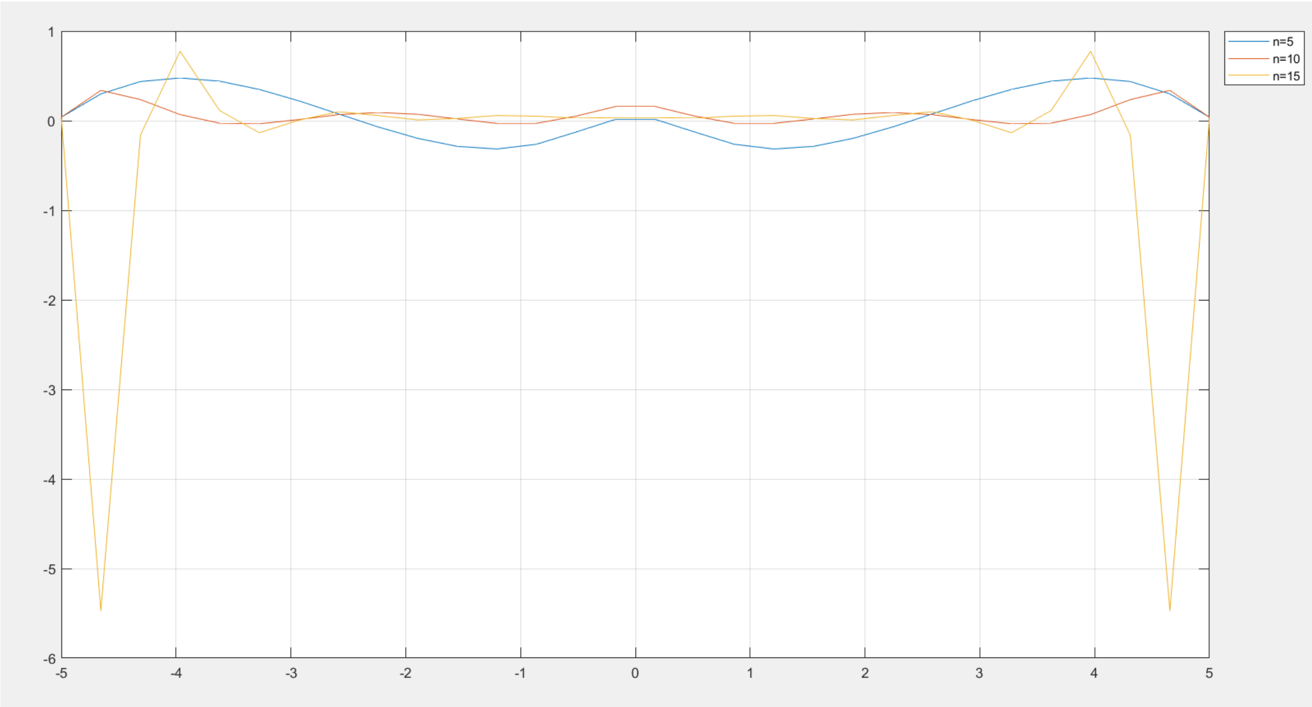
error plot for n=10



error plot for n=15

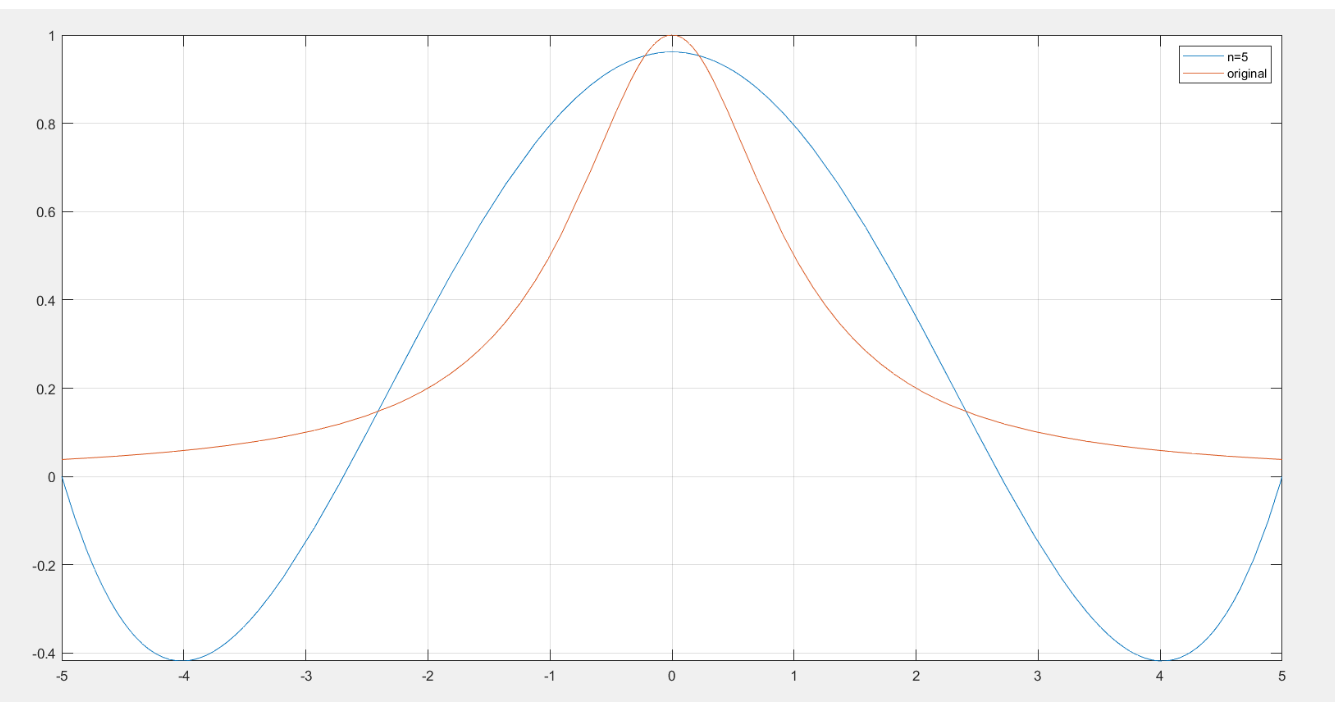


Combined Error Plot

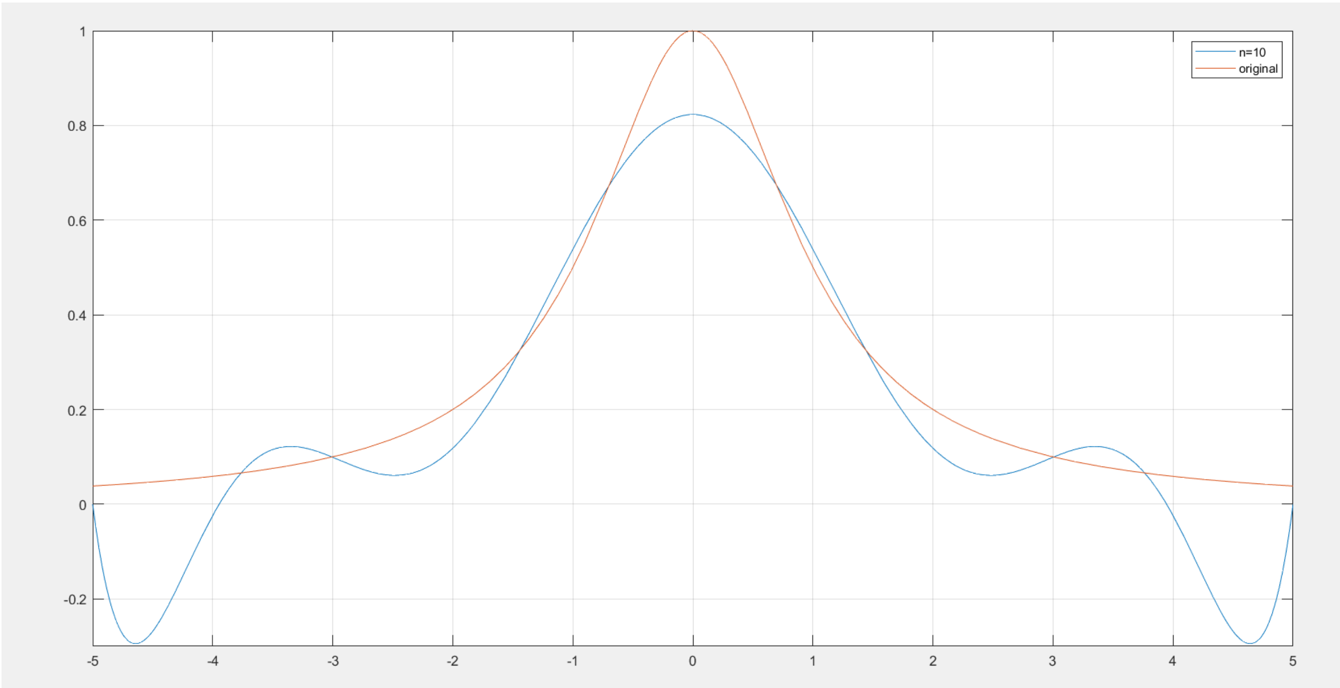


Polynomial plots

N=5

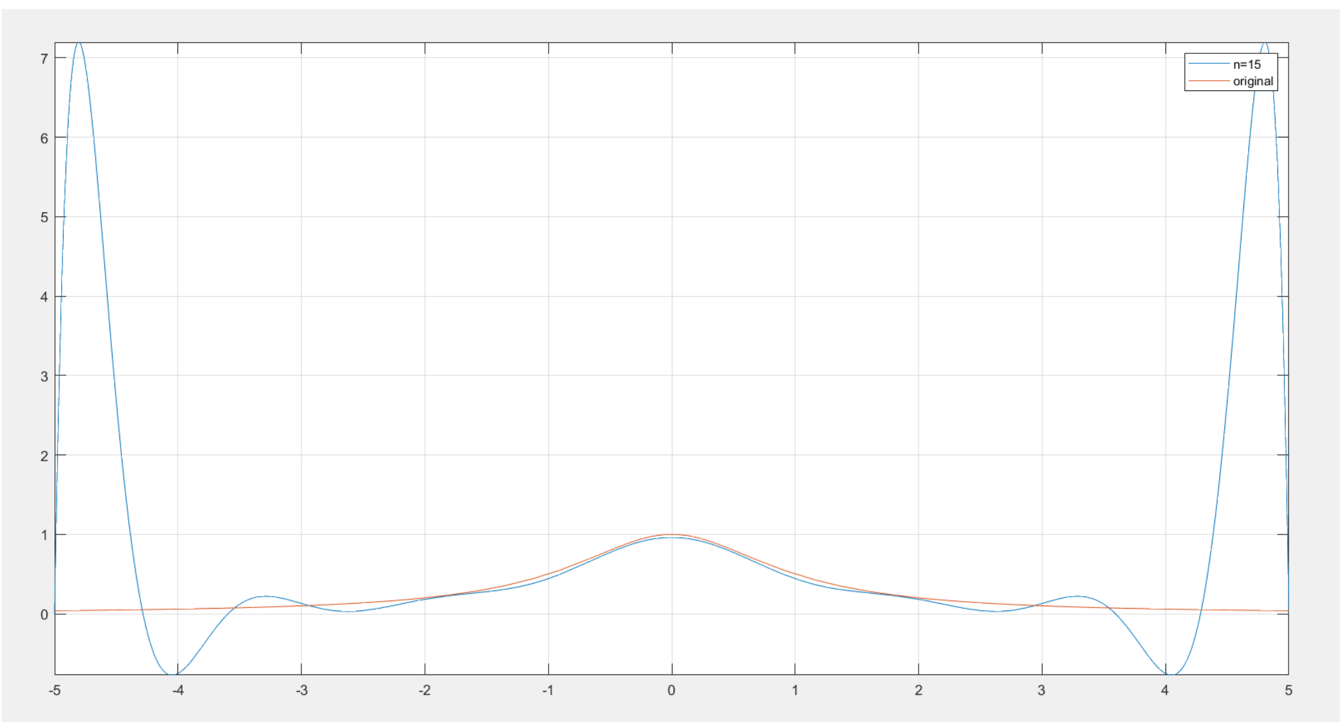


N=10



n=10
original
1
0.8
0.6
0.4
0.2
0
-0.2
-5
-4
-3
-2
-1
0
1
2
3
4
5

N=15



n=15
original
7
6
5
4
3
2
1
0
-5
-4
-3
-2
-1
0
1
2
3
4
5

Combined Plot