LAB-4

Q1) a) Note: All graphs are attached at the end.

For iterations ≥ 10, it more or less converges towards the root with any $x \in (0,1)$ as $|\phi'(x)| < 1$ for $\forall x \in (0,1)$, as seen in the graph, and is continuous and differentiable as well.
It converges to the root, $x = 0.2016$

$$x_{k+1} = \frac{1}{5}(x_k^3 + 1)$$

b) ~~The~~ $x_{k+1} = (5x_k - 1)^{1/3} = \phi(x)$

This method doesn't converge as ~~the~~ $\phi'(x)$ is not continuous or differentiable in $(0,1)$ and also $|\phi'(x)| \not< 1$ $\forall x \in (0,1)$.

This can be clearly seen in the graph attached.

c) $x_{k+1} = x_k^3 - 4x_k + 1$

This method also doesn't converge to the root as $|\phi'(x)| \not< 1$ $\forall x \in (0,1)$. Instead it $\phi'(x) \in (-4,-1)$ $\forall x \in (0,1)$. Hence the root can't converge to $x = 0.2016$

d) For computational purposes, $f(x) = x^2 - 3x + 2 = 0$. In this part $\alpha = 2$, $\beta = 1$

P.T.O

②

As seen in the graph if we select x_0 close to 0 as ~~...~~ $\phi'(x) > 1$ there. For x_0 near to α_1, the method always converges as $\phi'(x)$ is continuous, $\phi(x)$ is differentiable and $|\phi'(x)| < 1$ in that interval.

e) Here $\beta = 2$, $\alpha = 1$, $a = -3$, $b = 2$

$$x_{K+1} = \frac{-b}{x_K + a} = \frac{-2}{x_K - 3}$$

If we select a close enough value to 1, the root always converges to 1. This is because it satisfies all the constraints mentioned in the previous questions. Only at $x \to 3$, $\phi'(x) \to \infty$, hence selecting a value there will lead to divergence.

f) Here $\alpha = 1$, $\beta = 2$, $a = -3$, $b = 2$

$$x_{K+1} = \frac{x^2 + 2}{3} = \frac{(x_K^2 + b)(-1)}{a}$$

For values sufficiently ~~...~~ close to 1, the root converges to 1. For $x > 1.5$, $|\phi'(x)| > 1$, hence after that point root diverges.

g) It converges to $\sqrt{a}$.

For $a = 4$, it converged to root(2) in 10 iterations. For values sufficiently close to $\sqrt{a}$, $|\phi'(x)|$ is less than 1 and continuous & differentiable as well.

Refer Graph Attached at the end.

Q2) $\alpha = 2$

For values close to 2, it always converges to the root (2). for $x < 1$, $|\phi'(x)| > 1$, hence it doesn't converge then.

$$\phi(x) = x - \frac{2 * f(x)}{f'(x)}$$

Refer Graph for supporting Arguments.

Q3) (i) **Observations:** It converged to the root by both methods, with root
= ~~7.5446~~ 7.113197

(ii) for $x^5 + x^3 + 3 = 0$ & $x_0 = -1$ & $x_1 = 1$. (Actual root ≈ -1.2)

It didn't converge via the methods mentioned (for Regula falsi)

It was observed : • for secant methods, roots may/may not converge depending on the type of equation and constraints.

• for Regula-falsi Method, if the root is within the interval of (x_0, x_1), then it will always converge and is often faster and more efficient than the secant method (as seen from the order of convergence)

Iterative observation:

P.T.O.

(Part A) Accuracy 10^{-2} $x_1 = 7.2$
($x_0 = 7$, ~~$x_1 = 7.1$~~)

Secant → ~~7.51140~~ 7.113197 (5 iterations)

Regula → ~~7.51140~~ 7.11145 (4 iterations)

Part (B) $x_0 = -1$, $x_1 = 1$, $f(x_0) \cdot f(x_1) > 0$ so regula falsi doesn't converge ~~via either method~~, as it doesn't satisfy initial condition.

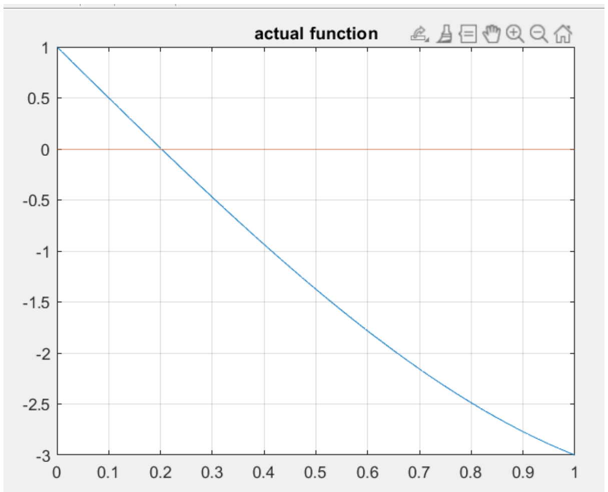
Accuracy 10^{-2}

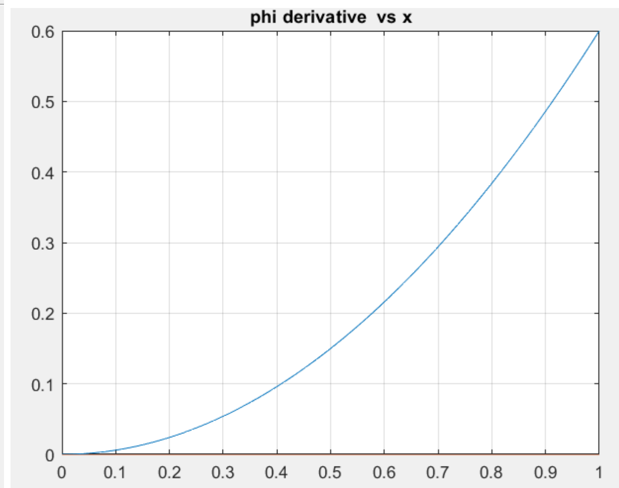
Secant → -0.38404 (6 iterations)

Accuracy 10^{-4}

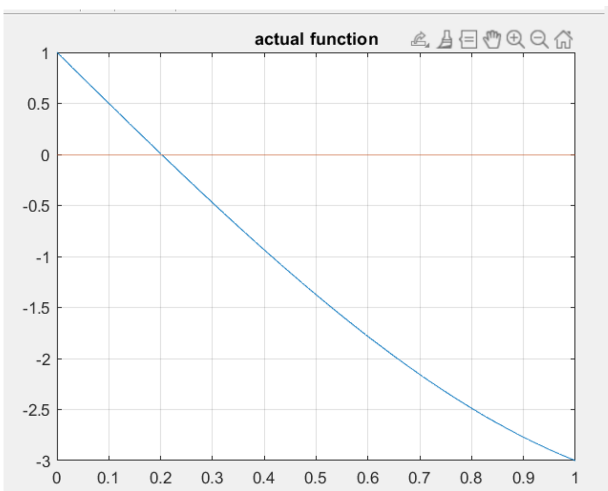
Secant → -1.10 5298 (5 iterations)

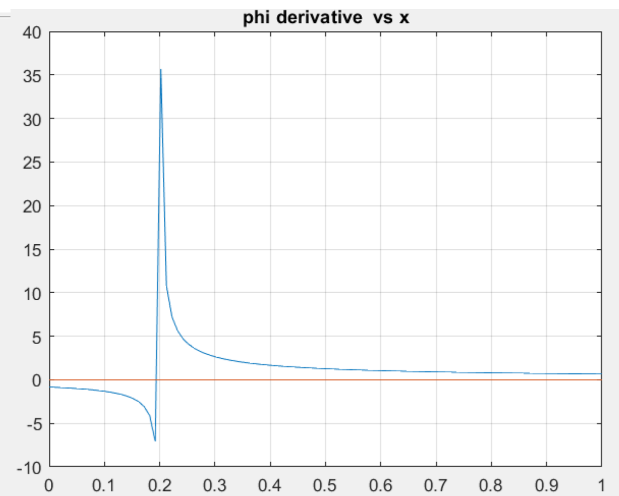
Q1)a)


actual function

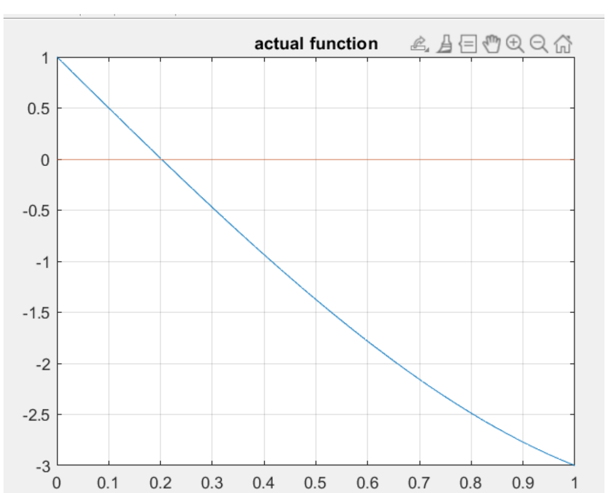

phi derivative vs x

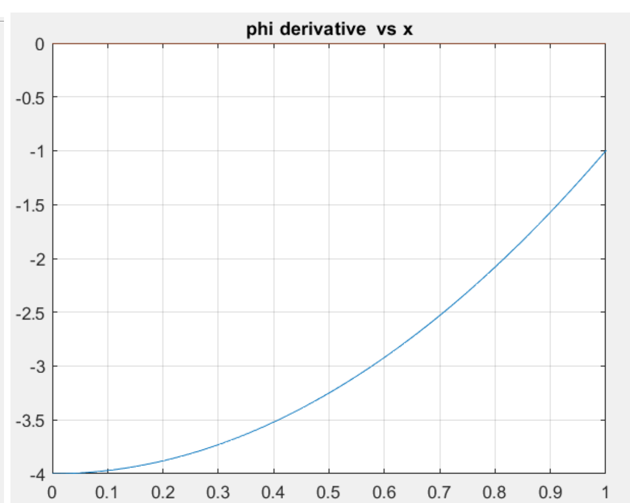
1)b)


actual function

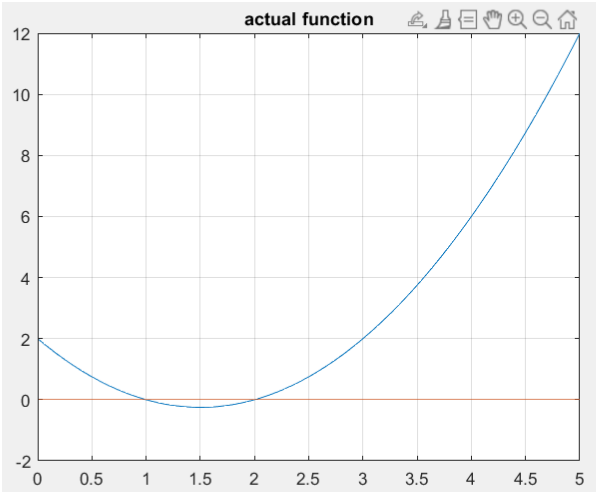

phi derivative vs x

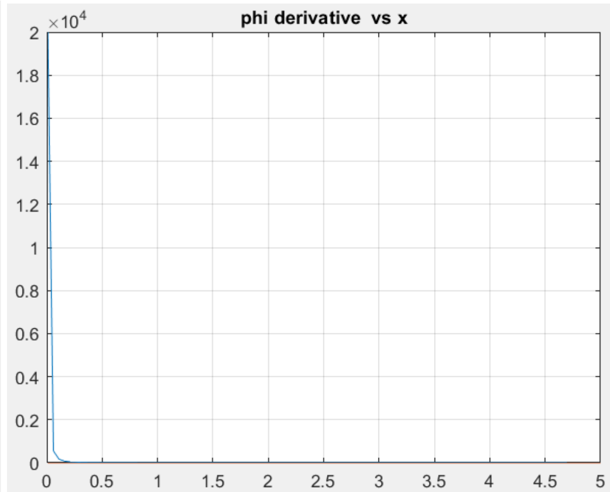
1)c)


actual function

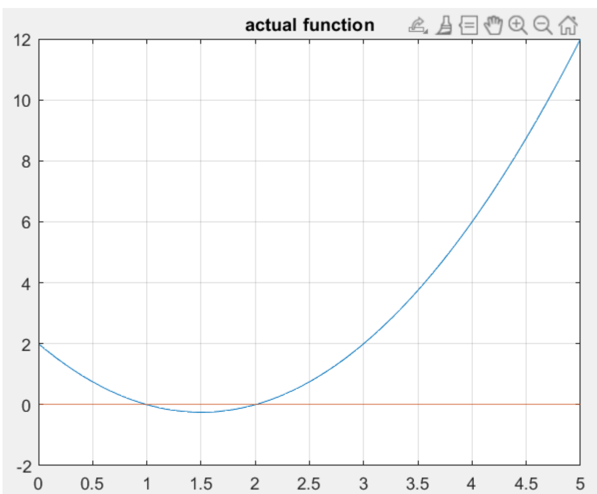

phi derivative vs x

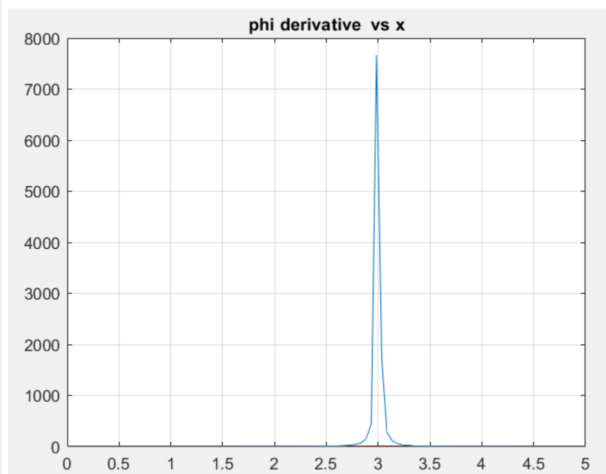
1)d)


actual function
12
10
8
6
4
2
0
-2
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5

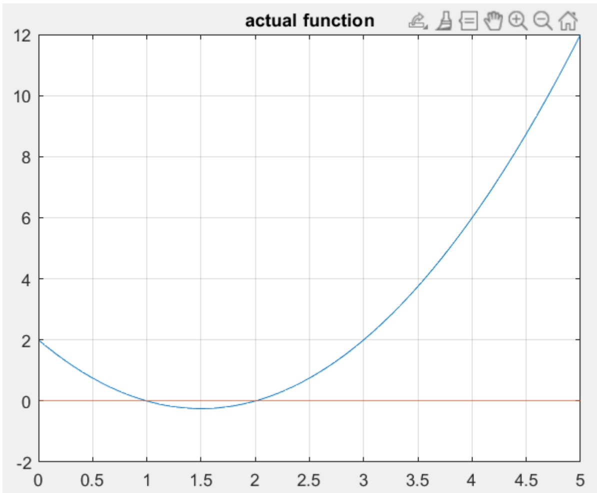

phi derivative vs x
$\times 10^4$
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5

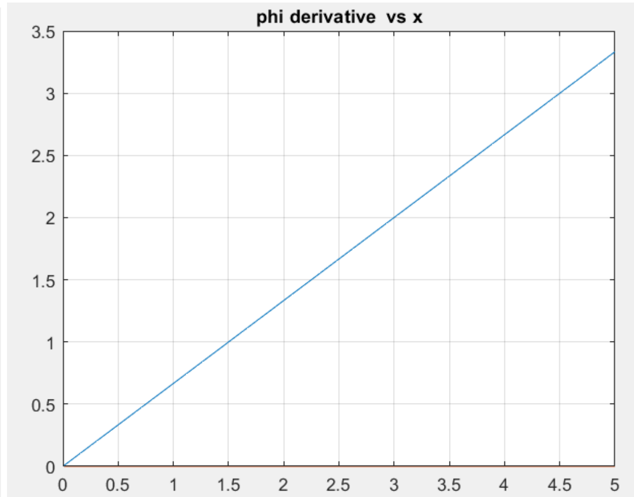
1)e)


actual function
12
10
8
6
4
2
0
-2
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5

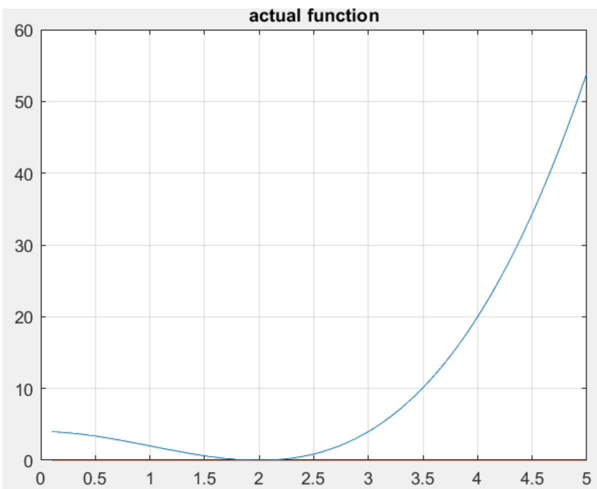

phi derivative vs x
8000
7000
6000
5000
4000
3000
2000
1000
0
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5

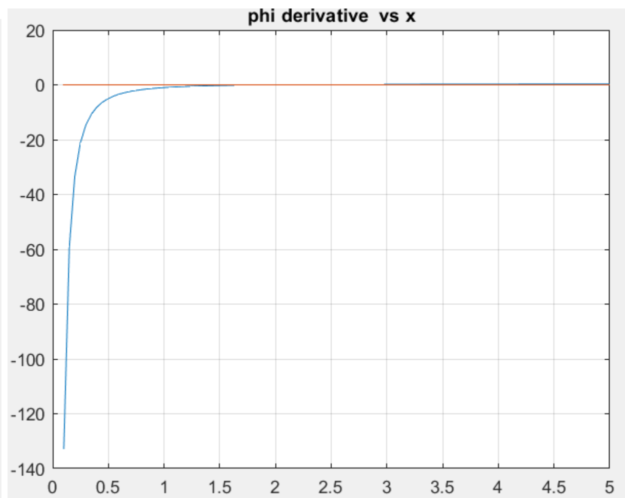
1)f)


actual function
12
10
8
6
4
2
0
-2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


phi derivative vs x
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

2)


actual function
60
50
40
30
20
10
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


phi derivative vs x
20
0
-20
-40
-60
-80
-100
-120
-140
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5