

Problem Set 3: Discrete Mathematics

1. Using proof by contrapositive to show that “If  $m - n$  is even, where  $m$  and  $n$  are integers; then, either  $m$  and  $n$  are both odd, or  $m$  and  $n$  are both even.”
2. Using proof by contrapositive to show that “for all integers  $m$  and  $n$ , if the product of  $m$  and  $n$  is even, then  $m$  is even or  $n$  is even.”
3. Use Proof by Contrapositive to show that “If  $(x + 3)^2 \cdot (x - 3) + 12$  where  $x$  is natural number is an odd integer, then  $x$  must be an even integer.”
4. Prove that “If a number added to itself gives itself, then the number is 0.”
5. Prove that “For all real numbers  $x$  and  $y$ , if  $x + y \geq 2$ , then either  $x \geq 1$  or  $y \geq 1$ .”
6. Using proof by contradictions to show that “The sum of even integers is even.”
7. Prove that for all positive integers  $a$ ,  $b$ , and  $c$ ; if  $a^2 + b^2 = c^2$ , then at least one of  $a$ ,  $b$ , or  $c$  must be even.
8. Prove that for some positive integer  $x$ ,  $x + 1/x \geq 2$ .
9. Prove that there are no integers  $a$  and  $b$  for which  $18a + 6b = 1$ .
10. Use proof by contradiction to show that  $(A \cap B) = \emptyset \Rightarrow A \subseteq B^c$