

Problem Set 2: Discrete Mathematics

1. Prove that $(n+1)^3 \geq 3^n$ if $n \in \mathbb{N}$ with $n \leq 4$.
2. For any real number x the absolute value of x is denoted by $|x|$ and is defined as
$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$
Show that for every real x , if $|x+7| > 8$, then $|x| > 1$.
3. Show that the product of any even integer and any other integer is even.
4. Show that product of 2 consecutive integers is even.
5. Show that the square of an odd integer equals $8k+1$ for some integer k .
6. Show that if n is a multiple of three, n^2 is a multiple of three.
7. Show that product of 3 consecutive integers is even.
8. Show that for any sets A and B , if $A \subseteq B$ then $(A \cup B) = B$.
9. Show by contraposition that for any sets A and B , if $A \subseteq B$ then $(A \cap B) = A$.
10. Let A, B, C, D be arbitrary sets. Prove that $(A \cap C) \cup (B \cap D) \subseteq (A \cup B) \cap (C \cup D)$.