

The complete graph is a graph with all possible edges. Use induction to show that an n -vertex complete graph has $\frac{n(n-1)}{2}$ edges.

Proof

Let P_n be " n -vertex complete graph has $\frac{n(n-1)}{2}$ edges "

Base Case : $P(1)$: • 1-vertex complete graph has $\frac{1(1-1)}{2} = 0$ edge which is true.

IH : Assume $P(k)$: " k -vertex complete graph has $\frac{k(k-1)}{2}$ edges " is true.

$$\begin{aligned}
 \text{IS : } P(k) \rightarrow P(k+1) : & 0 + 1 + 2 + \dots + (k-1) + k = \frac{k(k-1)}{2} + k \\
 &= \frac{k^2 - k + 2k}{2} \\
 &= \frac{k^2 + k}{2} \\
 &= \frac{(k+1)(k)}{2} = \frac{(k+1)(k+1-1)}{2}
 \end{aligned}$$

Note: Everytime the vertex increases the number of edge increases by $n-1$ ($n-1$ is from $n-1$ more edges created). \therefore If increase to $k+1$ nodes, number of edge will be increased by k

\therefore " n -vertex complete graph has $\frac{n(n-1)}{2}$ edges " is true by Mathematical Induction.