Problem Set 2: Discrete Mathematics Theepakorn Phayonrat 67011352

1. Prove that (n+1)^3>=3^n if n∈N with n=<4.

2. For any real number x the absolute value of x is denoted by |x| and is defined as

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Show that for every real x, if |x+7|> 8, then |x|>1.

3. Show that the product of any even integer and any other integer is even.

4. Show that product of 2 consecutive integers is even.

5. Show that the square of an odd integer equals 8k+1 for some integer k.

6. Show that if n is a multiple of three, $n^2$ is a multiple of three.

7. Show that product of 3 consecutive integers is even.

8. Show that for any sets A and B, if A⊆B then (A∪B)=B.

9. Show by contraposition that for any sets A and B, if A⊆B then (A∩B)=A.

10. Let A,B,C,D be arbitrary sets. Prove that (A∩C)∪(B∩D) ⊆ (A∪B)∩(C ∪D).

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## 1. Prove that (n+1)^3>=3^n if nЄN with n=<4.

Proof

Proof by exhaustion.

Case I $n=0$ : $(0+1)^3 \geq 3^0$

$1^3 \geq 3^0$

$1 \geq 1$

$\therefore n=0 \rightarrow (n+3)^3 \geq 3^n$

Case II $n=1$ : $(1+1)^3 \geq 3^1$

$2^3 \geq 3^1$

$8 \geq 3$

$\therefore n=1 \rightarrow (n+1)^3 \geq 3^n$

Case III $n=2$ : $(2+1)^3 \geq 3^2$

$3^3 \geq 3^2$

$27 \geq 9$

$\therefore n=2 \rightarrow (n+1)^3 \geq 3^n$

Case IV $n=3$ : $(3+1)^3 \geq 3^3$

$4^3 \geq 3^3$

$64 \geq 27$

$\therefore n=3 \rightarrow (n+1)^3 \geq 3^n$

Case V $n=4$ : $(4+1)^3 \geq 3^4$

$5^3 \geq 3^4$

$125 \geq 81$

$\therefore n=4 \rightarrow (n+1)^3 \geq 3^n$

$\therefore$ From every case, we can conclude that $(n+1)^3 \geq 3^n$ if $n \in \mathbb{N}$ with $n \leq 4$ //

2. For any real number x the absolute value of x is denoted by |x| and is defined as

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Show that for every real x, if |x+7|> 8, then |x|>1.

Proof

Proof by contrapositive

$\therefore$ We must show that if $|x| \leq 1$, then $|x+7| \leq 8$, $\forall x \in \mathbb{R}$.

From $|x| \leq 1$, we can conclude that $-1 \leq x \leq 1$

From $|x+7| \leq 8$, we can conclude that $-8 \leq x+7 \leq 8$

$\therefore$ We must show that $-1 \leq x \leq 1 \rightarrow -8 \leq x \leq 8$

$-1 \leq x \leq 1 \rightarrow 6 \leq x+7 \leq 8$

$\rightarrow -8 \leq x+7 \leq 8$

$\rightarrow |x+7| \leq 8$

$\therefore |x| \leq 1 \rightarrow |x+7| \leq 8$, $\forall x \in \mathbb{R}$ //

3. Show that the product of any even integer and any other integer is even.

Proof

Proof by Cases.

Case I a is even and b is even : $a = 2x, \exists x \in \mathbb{Z}$, $b = 2y, \exists y \in \mathbb{Z}$ $\}$ $ab = (2x)(2y) = 4xy = 2z$ ; $\exists z = 2xy \in \mathbb{Z}$

$\therefore ab$ is even

Case II a is even and b is odd: $a = 2x, \exists x \in \mathbb{Z}$, $b = 2y+1, \exists x \in \mathbb{Z}$ $\}$ $ab = (2x)(2y+1) = 4xy + 2x = 2z$

$\exists z = (2xy+x) \in \mathbb{Z}$

$\therefore ab$ is even

$\therefore$ From every case we can conclude that "The product of any even integer and any other integer is even" //

Theepatorn Phayarat 67011352

4. Show that product of 2 consecutive integers is even.

Proof

Proof by cases.

Let the first integer be $n$.

Case I: If $n$ is even: $n = 2k$

$\therefore n(n+1) = (2k)(2k+1) = 4k^2 + 2k = 2(2k^2+k) = 2x \; ; \exists x = 2k^2+k \in \mathbb{Z}$

Case II: If $n$ is odd: $n = 2k+1$

$\therefore n(n+1) = (2k+1)[(2k+1)+1] = (2k+1)^2 + (2k+1) = (4k^2+4k+1) + (2k+1) = 4k^2+6k+2$

$= 2(2k^2+3k+1)$

$= 2x \; ; \exists x = 2k^2+3k+1 \in \mathbb{Z}$

$\therefore$ From every case we can conclude that

"The product of 2 consecutive integers is even"

5. Show that the square of an odd integer equals 8k+1 for some integer k.

Proof

Proof by direct proof

We know that $k$ is odd number. $\therefore k = 2x+1$

$\therefore (2x+1)^2 = (2m)(2m) + 4m + 1 = 4(m)(m+1) + 1$

From PS 2 part 4 : $m(m+1) = 2n \; ; \exists n \in \mathbb{Z}$ $\quad \therefore 4(m)(m+1)+1 = 4(2n)+1 = 8n+1$

$\therefore$ We can conclude that "The square of an odd integer equals $8k+1$ for some integer $k$"

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6. Show that if n is a multiple of three, $n^2$ is a multiple of three.

Proof

If n is a multiple of three $\therefore n = 3m \ ; \exists m \in \mathbb{Z}$

$\therefore n^2 = (3m)^2 = 9m^2 = 3(3m^2) = 3k \ ; \exists k \in \mathbb{Z}$

$\therefore n^2$ is a multiple of three.

$\therefore$ We can conclude that "If n is a multiple of three, $n^2$ is a multiple of three" #

7. Show that product of 3 consecutive integers is even.

Proof

Proof by cases.

Let n be the first integer

Case I If n is even: $n = 2m \ ; \exists m \in \mathbb{Z}$

$\therefore$ The product of the 3 consecutive integer is $n(n+1)(n+2) = (2m)(2m+1)(2m+2)$

From PS2 part 4 $2m(2m+1)(2m+2) = 2x$ ; x is even number

From PS2 part 3 $2x$ is even

$\therefore$ If the first consecutive integer (n) is even, then the product of 3 consecutive number is even. #

Case II If n is odd : $n = 2m+1 \ ; \exists m \in \mathbb{Z}$

$\therefore$ The product of 3 consecutive integer is $(2m+1)(2m+2)(2m+3)$

From PS2 part 4 $(2m+1)(2m+2)(2m+3) = (2m+1)x$ ; x is even

From PS2 part 3 $(2m+1)x$ is even.

$\therefore$ If the first consecutive integer (n) is odd, then the product of 3 consecutive integer is even. #

$\therefore$ From every case, we can conclude that "The product of 3 consecutive integers is even". #

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8. Show that for any sets A and B, if $A \subseteq B$ then $(A \cup B) = B$.

Proof We must show that $A \cup B \subseteq B$ and $B \subseteq A \cup B$

For any set $A$ and $B$

Lemma I $A \cup B \subseteq B$ : We must show that "For and $x \in A \cup B \rightarrow x \in B$"

For any $x \in A \cup B$ : $A \cup B$ is the set of all $x$ where $x \in A$ or $x \in B$

Case I $x \in A$ : We must show that "$x \in A \rightarrow x \in B$"

We know that $A \subseteq B$ $\therefore x \in A \rightarrow x \in B$

$\therefore x \in B$

Case II $x \in B$ : We must show that "$x \in B \rightarrow x \in B$"

We all know that $x \in B \rightarrow x \in B$

$\therefore x \in B$

$\therefore A \cup B \subseteq B$ *

Lemma II $B \subseteq A \cup B$ : We must show that "For any $x \in B \rightarrow x \in A \cup B$"

We know that $x \in B \rightarrow x \in A \cup B$

$\therefore B \subseteq A \cup B$ *

From every lemmas, we con-conclude that

"For any sets $A$ and $B$, if $A \subseteq B$ then $A \cup B = B$ *

Theepakorn Phayonrat 67011352

9. Show by contraposition that for any sets A and B, if A⊆B then (A∩B)=A.

Proof

Proof by contraposition

To show that $A \subseteq B \to A \cap B = A$, we must show that $A \cap B \neq A \to A \not\subseteq B$

From " $A \cap B \neq A$ " we can conclude that " There exists some $x \in A$ that $x \notin A \cap B$ "

which mean " $x \notin A$ or $x \notin B$ "

Case I $x \in A \to x \notin A$ : We know it is False.

Case II $x \in A \to x \notin B$ : We know that we can conclude that

$$\boxed{A \not\subseteq B}$$

∴ We can conclude that " $A \cap B \neq A \to A \not\subseteq B$ "

∴ " $A \subseteq B \to A \cap B = A$ " From contraposition

Theepakorn Phayonrat 67011352

10. Let A,B,C,D be arbitrary sets. Prove that (A∩C)∪(B∩D) ⊆ (A∪B)∩(C ∪D).

Proof

We must show that $x \in (A\cap C)\cup(B\cap D) \rightarrow x \in (A\cup B)\cap(C\cup D)$

From "$x \in (A\cap C)\cup(B\cap D)$" we know that "$(x\in A$ and $x\in C)$ or $(x\in B$ and $x\in D)$"

Case I "$x\in A$ and $x\in C$":

"$x\in A$ and $x\in C$" $\rightarrow$ "$x\in A$"
$\rightarrow$ "$x\in A$ or $x\in B$"
$\rightarrow$ "$x\in A\cup B$" ①

"$x\in A$ and $x\in C$" $\rightarrow$ "$x\in C$"
$\rightarrow$ "$x\in C$ or $x\in D$"
$\rightarrow$ "$x\in C\cup D$" ②

From ① and ② "$x\in A\cup B$" and "$x\in C\cup D$" $\rightarrow$ "$x\in (A\cup B)\cap(C\cup D)$"

$\therefore$ "$x\in A\cap C$" $\rightarrow$ $x\in$ "$(A\cup B)\cap(C\cup D)$"

Case II "$x\in B$ and $x\in D$":

"$x\in B$ and $x\in D$" $\rightarrow$ "$x\in B$"
$\rightarrow$ "$x\in A$ or $x\in B$"
$\rightarrow$ "$x\in A\cup B$" ③

"$x\in B$ and $x\in D$" $\rightarrow$ "$x\in D$"
$\rightarrow$ "$x\in C$ or $x\in D$"
$\rightarrow$ "$x\in C\cup D$" ④

From ③ and ④ "$x\in A\cup B$" and "$x\in C\cup D$" $\rightarrow$ $x\in$ "$(A\cup B)\cap(C\cup D)$"

$\therefore$ "$x\in B\cap D$" $\rightarrow$ $x\in (A\cup B)\cap(C\cup D)$

$\therefore$ From every case we can conclude that

"For any sets A, B, C, D $(A\cap C)\cup(B\cap D)\subseteq(A\cup B)\cap(C\cup D)$"