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1. Show that $|\mathbb{E}| = |\mathbb{Z}|$

We have to find injection from $\mathbb{E}$ to $\mathbb{Z}$ Let $f : \mathbb{E} \to \mathbb{Z}$ be defined by $f(e) = e/2$, for $e \in \mathbb{E}$

We prove that f is a bijection.

    1. Suppose $f(e_1) = f(e_2)$ for $e_1, e_2 \in \mathbb{E}$ then $e_1/2 = e_2/2$ and $e_1 = e_2$. Hence, f is injective.
    2. Suppose $x \in \mathbb{Z}$, $2x$ is even, so $2x \in \mathbb{E}$ and $f(2x) = x$. Hence, f is surjective.

$\therefore$ We now have that is a bijection

2. Show that $(x^3 + 2x)/(2x+1)$ is $O(x^2)$

$$\frac{x^3 + 2x}{2x+1} \leqslant \frac{4x^3 + 2x^2}{2x+1} \leqslant 2x\left(\frac{2x+1}{2x+1}\right) = 2x^2 \leqslant c \cdot x^2 \qquad C = 2$$
$$k = 1$$

3. $\displaystyle\sum_{i=1}^{100} (4 + 3i) = \sum_{i=1}^{100} 4 + \sum_{i=1}^{100} 3i$

$$= 4(100) + 3\left(\frac{(100)(101)}{2}\right)$$

$$= 400 + 3(50)(101)$$

$$= 400 + 15150$$

$$= 15550$$