Minimize using VEM (Variable Entered Map)

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- VEM is similar to the **K-map**
- By putting several input variables into the map (as the output);
- This will reduce the number of the input variables as much as those we put into the map;
- So, this method is so-called **Variable Entered Map (VEM).**
- Typically use to reduce several inputs for the system with more than 4 variables.

VEM Plot and Minimization

Example 1 : Simplify the Boolean Equation below, from 3 variables to 2 variables.

$$F(A, B, C) = \Sigma m(0, 4, 6, 7)$$

Solution: Write the Boolean Equation in the SOP form:

$$F(A, B, C) = \overline{A}\,\overline{B}\,\overline{C} + A\,B\,\overline{C} + A\,\overline{B}\,\overline{C} + A\,B\,C$$

Change the function from 3 variables to 2 variables of A and B:

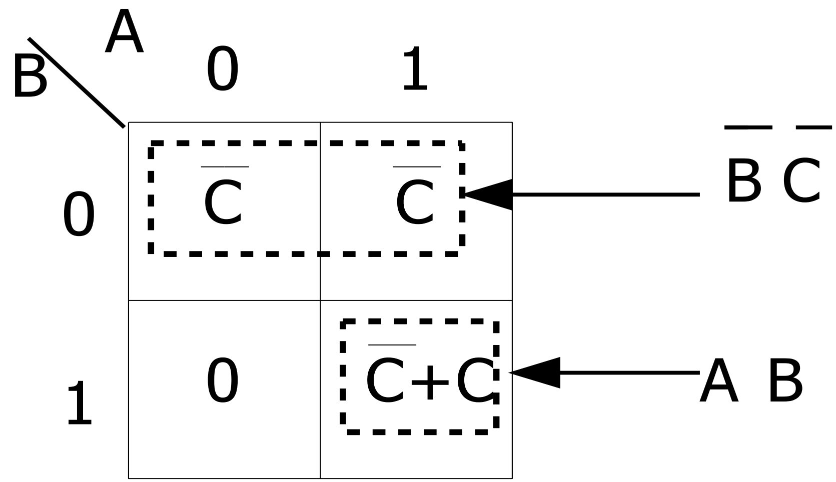
$$F(A, B, C) = G(A, B) = \underbrace{\overline{A}\,\overline{B}}_{0}(\overline{C}) + \underbrace{A\,B}_{3}(\overline{C}) + \underbrace{A\,\overline{B}}_{2}(\overline{C}) + \underbrace{A\,B}_{3}(C)$$

$$G(A, B) = \sum m(A, B) = \sum m(0\cdot\overline{C}, 2\cdot\overline{C}, 3\cdot(C+\overline{C}))$$

$$G(A, B) = \sum m(A, B) = \sum m(0\cdot C, 2\cdot C, 3)$$

B \ A	0	1
0		
1		

we can write VEM by Putting *C* into the map as:



Then we can minimize as we do in the Karnaugh Map by

- finding the terms with same variable or 1,
- and grouping it by the number of 2, 4, 8 or 16.

So, we get $F(A, B, C) = \overline{B}\,\overline{C} + A\,B$

Example 2 : Simplify the Boolean Equation below from 4 variables to 3 variables.

$$F(A,B,C,D) = A\bar{B}CD + \bar{A}B\bar{C}D + A\bar{B}C\bar{D} + AB\bar{C}D + \bar{A}\,\bar{B}\,\bar{C}D$$

Solution: Need to write form:

$$F(A,B,C,{\color{red}D}) = G(A,B,C)$$

$$F(A,B,C,{\color{red}D}) = A\bar{B}C(D) + \bar{A}B\bar{C}(D) + A\bar{B}C(\bar{D}) + AB\bar{C}(D) + \bar{A}\,\bar{B}\,\bar{C}(D)$$

$$= A\bar{B}C(D+\bar{D}) + \bar{A}B\bar{C}(D) + AB\bar{C}(D) + \bar{A}\,\bar{B}\,\bar{C}(D)$$

So, we can put D into the map as:

C \ A B	00	01	11	10
0	D	D	D	0
1	0	0	0	$D+\bar{D}$

$$F(A,B,C,D) = G(A,B,C)$$

$$= \bar{A}\,\bar{C}\,D + B\,\bar{C}\,D + A\,\bar{B}\,C$$

Example 3: Simplify the Boolean Equation below from 5 variables to 4 variables.

$$F(A,B,C,D,E) = \bar{A}B\bar{C}\bar{D}\bar{E} + AB\bar{C}\bar{D}\bar{E} + \bar{A}B\bar{C}D\bar{E} + \bar{A}BC\bar{D} + \bar{A}\bar{B}C\bar{D}E$$
$$+ ABC\bar{D}\bar{E} + ABC\bar{D}E + \bar{A}B\bar{C}DE$$

Solution: To convert $F(A,B,C,D,E) = G(A,B,C,D)$

by putting E in the map, then

$$G(A,B,C,D) = \bar{A}B\bar{C}\bar{D}\left(\bar{E}\right) + AB\bar{C}\bar{D}\left(\bar{E}\right) + \bar{A}B\bar{C}D\left(\bar{E}\right) + \bar{A}BC\bar{D} + \bar{A}\bar{B}C\bar{D}(E)$$
$$+ ABC\bar{D}\left(\bar{E}\right) + ABC\bar{D}(E) + \bar{A}B\bar{C}D(E)$$

then will get the VEM :

CD \ AB	00	01	11	10
00	0	$\bar{E}$	$\bar{E}$	0
01	0	1	0	0
11	0	0	0	0
10	E	1	1	0

$$1 = E + \overline{E}$$

$$F(A,B,C,D,E) = B\,\bar{D}\,\bar{E} + B\,C\,\bar{D} + \bar{A}\,B\,\bar{C}\,D + \bar{A}\bar{B}C\bar{D}E$$

VEM Plot with Several Variables in The Map

Minimization VEM by putting several variables into the map is quite difficult. It is still easier than the actual K-Map though.

However, this method is easily feasible in practice if associated with the Multiplexer IC .

Therefore, VEM is the potentially used in the circuit design with the MUX IC [In Chapter 8] .

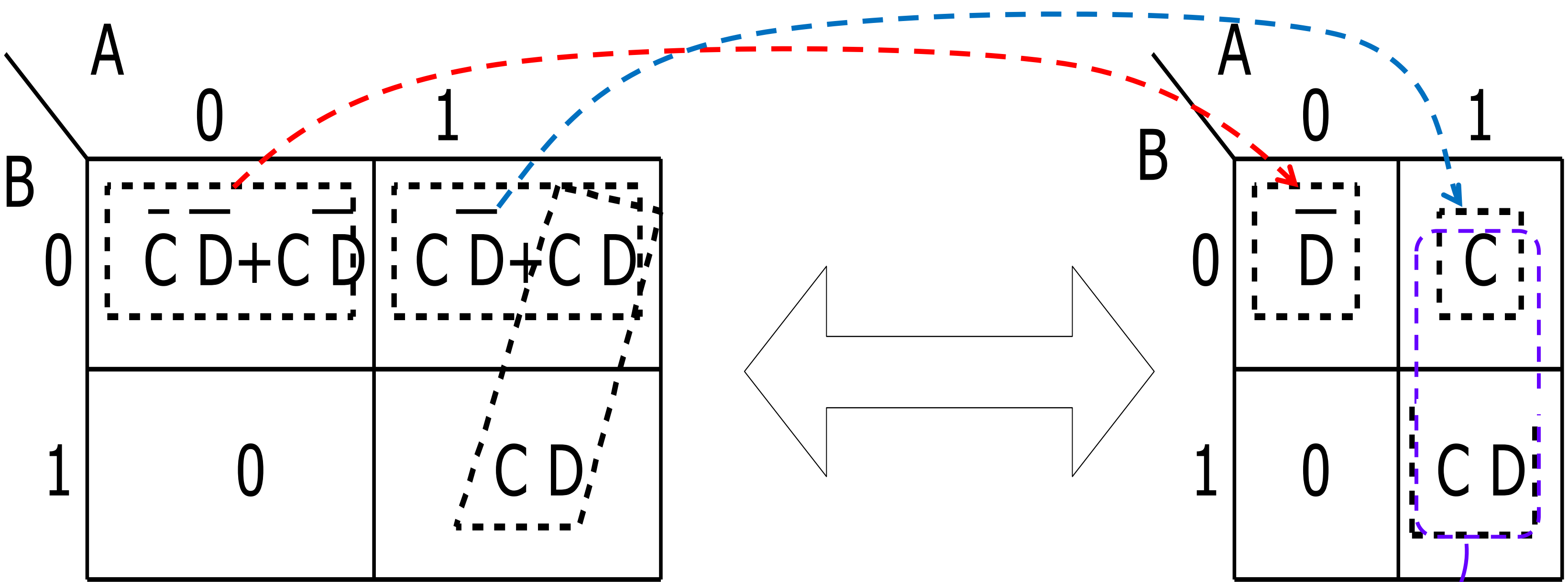
Example 4: Simplify the Boolean Equation below from 4 variables to 2 variables.

Solution: To convert $F(A,B,C,D) = G(A, B)$

by putting C and D into the map, then

$$F(A, B, C, D) = \bar{A}\,\bar{B}\,\bar{C}\,\bar{D} + \bar{A}\,\bar{B}\,C\,\bar{D} + A\,B\,C\,D + A\,\bar{B}\,C\,D + A\,\bar{B}\,C\,\bar{D}$$

then will get : $G(A, B) = \bar{A}\,\bar{B}(\bar{C}\bar{D} + C\bar{D}) + A\bar{B}(C\bar{D} + CD) + AB(CD)$



$$F(A,B,C,D) = \bar{A}\,\bar{B}\,\bar{D} + A\,\bar{B}\,C + A\,C\,D$$

VEM Plot with Don't Care

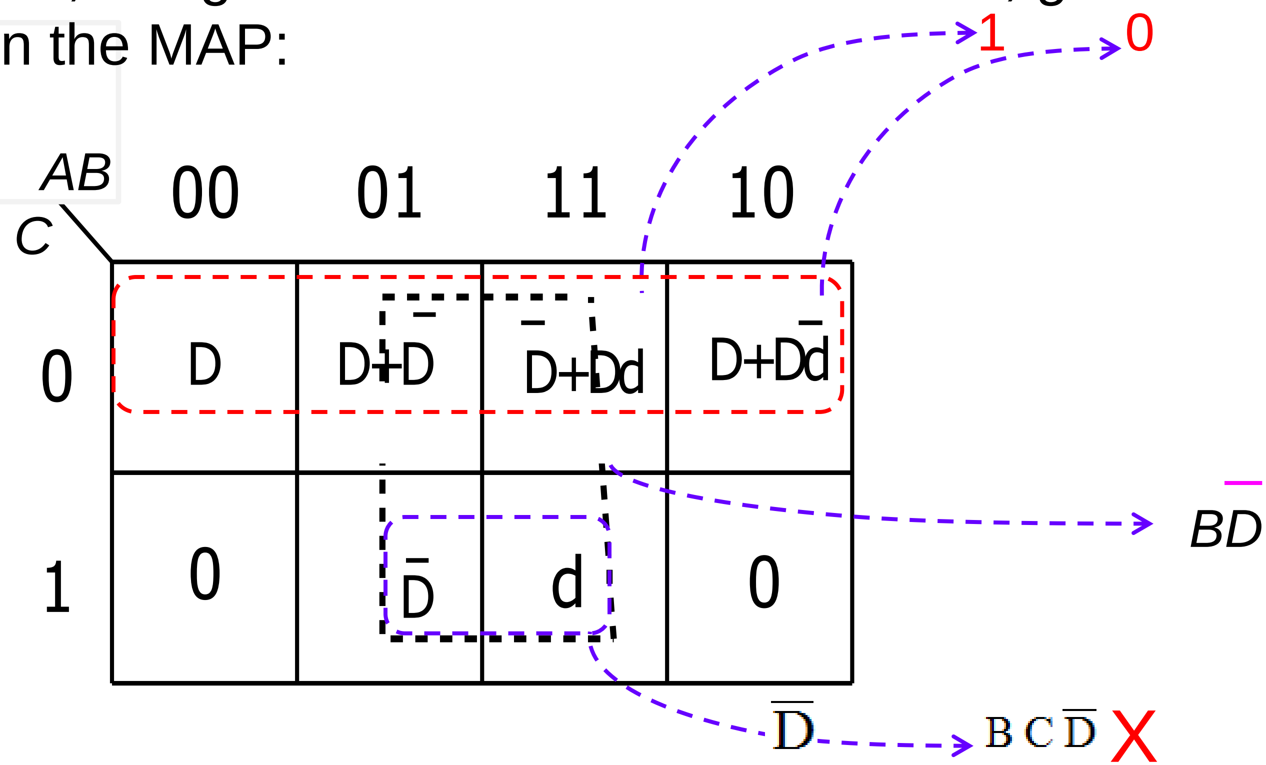
The VEM with Don't Care can be produced by:

- The Truth Table with Don't Care,
- then merge/group the conditions/minterns with the same INPUTs;
- Don't Care may be used to help for minimizing the terms by replacing it with either
  - the appropriate variable or
  - Logic 1.

Example 5: Find the VEM function $G(A, B, C)$ for the truth table below.

Decimal	Binary				Output	
N	A	B	C	D	Y	
0	0	0	0	0	0	
1	0	0	0	1	1	$Y = D$
2	0	0	1	0	0	
3	0	0	1	1	0	
4	0	1	0	0	1	$Y = \overline{D} + D = 1$
5	0	1	0	1	1	
6	0	1	1	0	1	$Y = \overline{D}$
7	0	1	1	1	0	
8	1	0	0	0	d	$Y = D + \overline{D}d$
9	1	0	0	1	1	
10	1	0	1	0	0	
11	1	0	1	1	0	
12	1	1	0	0	1	$Y = \overline{D} + Dd$
13	1	1	0	1	d	
14	1	1	1	0	d	$Y = d$
15	1	1	1	1	d	

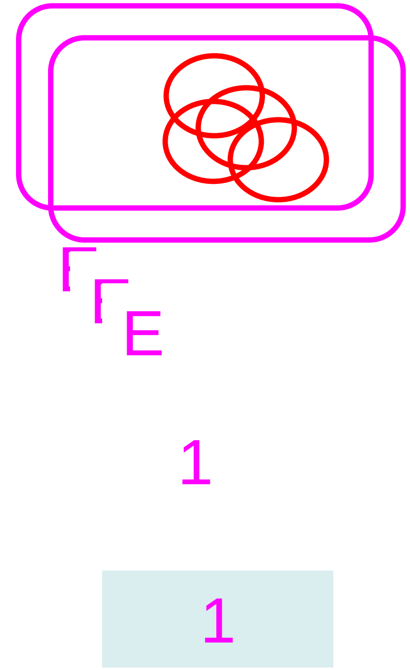
Solution: From the table, merge the same conditions of *ABC, get D and Don't Care,* and put it in the MAP:



- Remember that ALL entered variables must be used;
- Don't cares make the minimization possibly occurred:
  - d(111) = $\overline{D}$ ➔ Group 4 for B$\overline{D}$;
  - d(110) = D/1 ➔ Group 4 for $\overline{C}$D;
  - d(110) = 0 ➔ Group 4 for $\overline{C}$D;

Therefore, we get the minimal Y = B$\overline{D}$ + $\overline{C}$D

Example 6: Find the output function for the VEM below.



AB / CD	00	01	11	10
00	0	d	E	0
01	0	dE	d	$E + \bar{E}d$
11	0	0	d	$E + \bar{E}d$
10	0	0	0	0

- Remember that ALL entered variables must be used;
- Don’t cares make the minimization possibly occurred:
  - Group 4 for $B\bar{C}E$: ➔ d(0100), d(0101), d(1101) = E
  - Group 4 for AD: ➔ d(1101), d(1111), d(1001), d(1011) = 1

Therefore, we get the minimal $Y = AD + B\bar{C}E$