Problem Set 12: Discrete Mathematics Name Theepakorn Phayonrat ID 67011352

1. Is the sequence $\{a_n\}$ a solution of the $a_n = -3a_{n-1} + 6a_{n-2}$ if $a_n = (-3)^n$.

$a_n = (-3)^n$

$a_{n-1} = (-3)^{n-1}$

$a_{n-2} = (-3)^{n-2}$

$(-3)^n \neq -3(-3)^{n-1} + 6(-3)^{n-2}$

$(-3)^n \neq (-3^n) + 6(-3)^{n-2}$

$(-3)^n \neq (-3^n) - 2(-3)^{n-1}$

$(-3)^n \neq (-3)^n - \frac{2}{3}(-3)^n$

$(-3)^n \neq \frac{1}{3}(-3)^n$ #

2. Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = 2a_{n-1} - a_{n-2} + 2$ if $a_n = n^2-3$.

$(n^2-3) = 2[(n-1)^2 - 3] - [(n-2)^2 - 3] + 2$

$n^2 - 3 = 2n^2 \cancel{-4n} \cancel{+2} \cancel{-6} - n^2 \cancel{+4n} \cancel{-4} \cancel{+3} \cancel{+2}$

$n^2 - 3 = n^2 - 3$ #

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3. Describe each sequence recursively. Include initial conditions and assume that sequences begin with a_1.

a. 1, 101, 10101, 1010101, ….

$a_1 = 1$

$a_n = 100a_{n-1} + 1$ #

b. $a_n = 1 + 2 + 3 + ... + n$.

$a_1 = 1$

$a_n = a_{n-1} + n$ #

c. *a_n = the number of subsets of a set of size n.*

$a_1 = 2$

$a_n = 2a_{n-1}$ #

4. Find the solution for the following recurrence relation:

$T(1) = 1$

$T(n) = T(n-1) + 3$ for $n \geq 2$

$T(n) = T(1) + 3(n-1)$

$T(n) = 3n - 2$ #

5. Find the solution for the following recurrence relation:

$a_n = 7a_{n-1} - 6a_{n-2}$, $a_0 = -1$, $a_1 = 4$.

$r^2 = 7r - 6$

$r^2 - 7r + 6 = 0$

$r = 1, 6$

$\alpha_n = \alpha_1 1^n + \alpha_2 6^n$

$\alpha_0 = \alpha_1 + \alpha_2 = -1 \rightarrow \alpha_1 = -1 - \alpha_2$

$\alpha_1 = \alpha_1 + 6\alpha_2 = 4$

$= -1 - \alpha_2 + 6\alpha_2 = 4$

$5\alpha_2 = 5$

$\alpha_2 = 1$

$\alpha_1 = -2$

$\therefore a_n = -2(1)^n + 1 \cdot 6^n$

$a_n = 6^n - 2$ #