separable equations

1. $dy/dx = 4x\sqrt{y}$, with $y(0) = 1$.

$\frac{dy}{dx} = 4x\sqrt{y}$

$\frac{dy}{\sqrt{y}} = 4x\,dx$

$\int y^{-\frac{1}{2}} dy = \int 4x\,dx$

$2y^{\frac{1}{2}} = 2x^2 + C$

$y^{\frac{1}{2}} = x^2 + C$

$y = (x^2 + C)^2$

Since $y(0) = 1$

$y(0) = (0)^4 + C = 1$

$C = 1$

$\boxed{y = (x^2+1)^2}$

2. Solve $y' + y^2 \sin x = 0, \quad y(0) = 1.$

$\frac{dy}{dx} + y^2 \sin x = 0$

$\frac{dy}{dx} = -y^2 \sin x$

$\int \frac{dy}{y^2} = \int -\sin x\,dx$

$\int y^{-2}\,dy = \cos x + C$

$-y^{-1} = \cos x + C$

$\frac{1}{y} = -\cos x - C$

$y = -\frac{1}{\cos x + C}$

Since $y(0) = 1$

$1 = -\frac{1}{\cos 0 + C}$

$= -\frac{1}{1 + C}$

$C = -2$

$\boxed{y = -\frac{1}{\cos x - 2}}$

31. $2y^2\,dx = 3x^2\,dy$ when $x = 2, y = -1$

$\int \frac{dx}{3x^2} = \int \frac{dy}{2y^2}$

$\frac{1}{3}\int x^{-2}dx = \frac{1}{2}\int y^{-2}dy$

$\frac{1}{3}\cdot -x^{-1} + C = \frac{1}{2}\cdot -y^{-1}$

when $x = 2$, $y = -1$

$\frac{1}{3}\cdot -\frac{1}{2} + C = \frac{1}{2}\cdot -\frac{1}{-1}$

$-\frac{1}{6} + C = \frac{1}{2}$

$C = \frac{1}{2} + \frac{1}{6}$

$= \frac{4}{6}$

$-\frac{1}{3x} + \frac{4}{6} = -\frac{1}{2y}$

$\frac{1}{3x} - \frac{4}{6} = \frac{1}{2y}$

$\frac{6xy}{3x} - \frac{24xy}{6} = \frac{6xy}{2y}$

$\boxed{2y - 4xy = 3x}$

33. $x^2e^{2y}\,dy = (x^3 + 1)\,dx$ when $x = 1, y = 0$

$u = 2y$
$du = 2dy$
$\frac{du}{2} = dy$

$\int e^{2y}\,dy = \int \frac{x^3+1}{x^2}\,dx$

$\frac{1}{2}\int e^u\,du = \int x + x^{-2}\,dx$

$\frac{1}{2}e^u = \frac{1}{2}x^2 - x^{-1} + C$

$\frac{1}{2}\cdot e^{2y} = \frac{1}{2}x^2 - x^{-1} + C$

when $x = 1$, $y = 0$

$\frac{1}{2}\cdot e^{2(0)} = \frac{1}{2}(1) - (1)^{-1} + C$

$\frac{1}{2} = \frac{1}{2} - 1 + C$

$\frac{1}{2} = -\frac{1}{2} + C$

$\frac{1}{2} + \frac{1}{2} = C$

$1 = C$

$\frac{1}{2}e^{2y} = \frac{1}{2}x^2 - \frac{1}{x} + 1$

$\boxed{e^{2y} = x^2 - \frac{2}{x} + 2}$

19. $dx = (x^2y^2 + x^2)\,dy$

$\frac{dx}{dy} = x^2y^2 + x^2$

$\frac{dx}{dy} = x^2(y^2+1)$

$\int \frac{dx}{x^2} = \int (y^2+1)\,dy$

$\int x^{-2}dx = \frac{1}{3}y^3 + y$

$\boxed{-x^{-1} + C = \frac{1}{3}y^3 + y}$

20. $dy = (x^2y^3 + xy^3)\,dx$

$\frac{dy}{dx} = x^2y^3 + xy^3$

$\frac{dy}{dx} = y^3(x^2 + x)$

$\int \frac{dy}{y^3} = \int (x^2+x)\,dx$

$\int y^{-3}dy = \frac{1}{3}x^3 + \frac{1}{2}x^2 + C$

$\boxed{-\frac{1}{2}y^{-2} = \frac{1}{3}x^3 + \frac{1}{2}x^2 + C}$

21. $y^2\,dx = x\,dy$

$\int \frac{dx}{x} = \int \frac{dy}{y^2}$

$\ln x + C = \int y^{-2}\,dy$

$\boxed{\ln x + C = -y^{-1}}$

22. $y\,dx = x\,dy$

① separable

$\int \frac{dx}{x} = \int \frac{dy}{y}$

$\ln x + C = \ln y$

$\ln y - \ln x = C$

$\ln\left(\frac{y}{x}\right) = C$

$e^{\ln\left(\frac{y}{x}\right)} = C$

$\boxed{\frac{y}{x} = C}$

② Linear

$\frac{y}{x} = \frac{dy}{dx}$

$\frac{dy}{dx} - \frac{y}{x} = 0$

Integrating Factor $= e^{-\int \frac{1}{x}dx}$

$= e^{-\ln x}$

$= x^{-1}$

Multiply both sides

$x^{-1}\frac{dy}{dx} - \frac{1}{x^2}y = 0$

$(x^{-1}y)' = C$

$\boxed{\frac{y}{x} = C}$

* Keep track of the c if you need to find an equation

41. ***Investing*** When the interest on an investment is compounded continuously, the investment grows at a rate that is proportional to the amount in the account, so that if the amount present is P, then

$$\frac{dP}{dt} = kP$$

where P is in dollars, t is in years, and k is a constant. If \$100,000 is invested (when $t = 0$) and the amount in the account after 15 years is \$211,700, find the function that gives the value of the investment as a function of t. What is the interest rate on this investment?

$P(0) = 100{,}000$

$P(15) = 211{,}700$

$P(t) = ?$

$$\left.\begin{aligned} \frac{dP}{dt} &= kP \\ \int \frac{dP}{P} &= \int k\,dt \\ \ln P &= kt + C \\ e^{\ln P} &= e^{kt} \cdot e^{C} \\ P &= e^{kt} \cdot e^{C} \end{aligned}\right\} \text{Separable method}$$

$\longrightarrow P(t) = e^{kt} \cdot 100{,}000$

$= e^{0.05t} \cdot 100{,}000$

The interest rate on the account is 5%

$$(0,\ 100000) \left\{ \begin{aligned} P(0) &= 1 \cdot e^{C} \\ 100{,}000 &= e^{C} \end{aligned} \right.$$

$$(15,\ 211700 \left\{ \begin{aligned} P(15) &= e^{k(15)} \cdot 100{,}000 \\ 211700 &= e^{15k} \cdot 100{,}000 \\ 2.117 &= e^{15k} \\ k &= 0.05 \end{aligned} \right.$$

$$\left.\begin{aligned} \frac{dP}{dt} - kP &= 0 \\ e^{-\int k\,dt} &= e^{-kt} \\ e^{-kt}\frac{dP}{dt} - kP &= 0 \\ \int \left[(e^{-kt})(P)\right]' &= \int 0\,dt \\ e^{-kt} P &= C \end{aligned}\right\} \text{Linear method}$$

$$(0,\ 100000) \left\{ \; 1 \cdot 100{,}000 = C \right.$$

$$(15,\ 211700) \left\{ \begin{aligned} e^{-k(15)}\, 211700 &= 100{,}000 \\ e^{-15k} &= 0.4723 \\ -15k &= -0.749 \\ k &= 0.05 \end{aligned} \right.$$