Problem Set 5: Name____ID_____________________

The definition of a Fibonacci number is as follows:
Basis: $f_0 = 0$ and $f_1 = 1$
Recursive: $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$
Use this definition for question 1 and 2.

1. Show that $f_1^2 + f_2^2 + f_3^2 + \ldots + f_n^2 = f_n f_{n+1}$ for $n \geq 1$.

2. Show that $f_1 + f_2 + f_3 + \ldots + f_{2n-1} = f_{2n}$ for $n \geq 1$.

3. Show that the set S defined by
Basis step: $1 \in S$
Recursive step: $s + t \in S$ when $s \in S$ and $t \in S$
is the set of positive integers:
$Z^+ = \{ 1, 2, 3, \dots \}$

4. Let N be given by
Basis: $1 \in N$
Recursive: if $x \in N$ then $x+1 \in N$.
Show that "$5^n - 1$ is divisible by 4".

5. Use structural induction, to prove that the number of nodes in any full rooted binary tree is odd.