Problem Set 3: Discrete Mathematics

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1. Using proof by contrapositive to show that "If $m - n$ is even, where m and n are integers; then, either m and n are both odd, or m and n are both even."

2. Using proof by contrapositive to show that "for all integers m and n, if the product of m and n is even, then m is even or n is even."

3. Use Proof by Contrapositive to show that "If $(x + 3)^2.(x - 3) + 12$ where x is natural number is an odd integer, then x must be an even integer."

4. Prove that "If a number added to itself gives itself, then the number is 0."

5. Prove that "For all real numbers x and y, if $x + y \geq 2$, then either $x \geq 1$ or $y \geq 1$."

6. Using proof by contradictions to show that "The sum of even integers is even."

7. Prove that for all positive integers a, b, and c; if $a^2 + b^2 = c^2$, then at least one of a, b, or c must be even.

8. Prove that for some positive integer x, $x + 1/x \geq 2$.

9. Prove that there are no integers a and b for which $18a + 6b = 1$.

10. Use proof by contradiction to show that $(A \cap B) = \emptyset \Rightarrow A \subseteq B^c$

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1. Using proof by contrapositive to show that "If $m - n$ is even, where m and n are integers; then, either m and n are both odd, or m and n are both even."

Proof

Proof by contrapositive :

From contrapositive, we can assume that "If m and n neither both odd, nor both even, then m-n is odd".

Let m be even and n is odd : $m = 2a ; a \in \mathbb{Z}$

$n = 2b + 1 ; b \in \mathbb{Z}$

$\therefore m - n = 2a - (2b+1) = 2a - 2b - 1 = 2a - 2b - 1 + (1 - 1) = 2(a - b - 1) + 1$

$= 2c + 1 ; c = (a - b - 1) \in \mathbb{Z}$

$\therefore m - n$ is odd.

$\therefore$ If $m - n$ is even, where m and n are integers; then, either m and n are both even.

2. Using proof by contrapositive to show that "for all integers m and n, if the product of m and n is even, then m is even or n is even."

Proof

Proof by contrapositive:

From Contrapositive, we can assume that "If m and n are odd, then mn is odd"

$\therefore m = 2a + 1 ; a \in \mathbb{Z}$

$\therefore n = 2b + 1 ; b \in \mathbb{Z}$

$mn = (2a+1)(2b+1)$

$= 4ab + 2a + 2b + 1$

$= 2(2ab + a + b) + 1$

$= 2C + 1 ; C = (2ab + a + b) \in \mathbb{Z}.$

$\therefore$ If mn is odd, then m and n are both odd

$\therefore$ For all integer m and n, if the product of m and n is even, the m is even or n is even.

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3. Use Proof by Contrapositive to show that "If $(x + 3)^2.(x - 3) + 12$ where x is natural number is an odd integer, then x must be an even integer."

Proof

Proof by contrapositive:

From Ccontrapositive, we can assume that " If x is an odd integer, $(x+3)^2(x-1)+12$ is even"

$\therefore x = 2a+1;\ a \in \mathbb{Z}$

$$
\begin{aligned}
\therefore (x+3)^2(x-3)+12 &= [(2a+1)+3]^2[(2a+1)-3]+12 \\
&= (2a+4)^2(2a-2)+12 \\
&= 8(a+2)^2(a-1)+12 \\
&= 2[4(a+2)^2(a-1)+6] \\
&= 2b;\ b = [4(a+2)^2(a-1)+6] \in \mathbb{Z}
\end{aligned}
$$

$\therefore (x+3)^2(x-3)+12$ is even.

$\therefore$ If $(x+3)^2(x-3)+12$ where x is natural number is an odd integer, then x must be an even integer. $\#$

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4. Prove that "If a number added to itself gives itself, then the number is 0."

Proof

Proof by contradiction:

Assume that " $x+x=x \rightarrow x \neq 0$ "

From $x+x=x$

$2x = x$ divide by 0 since $x \neq 0$

$2 = 1$

which is false (We've reached Contradiction.)

$\therefore$ If a number added to itself then the number is 0. #

5. Prove that "For all real numbers x and y, if x + y $\geq$ 2, then either x $\geq$1 or y $\geq$1."

Proof

Proof by contradiction:

$\forall x \forall y \in \mathbb{R}$

Assume that " $x+y \geq 2$ and $(x<1$ and $y<1)$ "

From $(x<1$ and $y<1)$, then $x+y<2$ which is false because $x+y \geq 2$ from Assumption (We've reached Contradiction)

$\therefore \forall x \forall y \in \mathbb{R}$ if $x+y \geq 2$ then either $x \geq 1$ or $y \geq 1$ #

6. Using proof by contradictions to show that "The sum of even integers is even."

Proof

Proof by contradiction

Assume " The sum of even integers is not even" which means; "The sum of even numbers is odd"

which means, Assumptions : 1. x is even.

2. y is even.

3. $x+y$ is odd

4. $x=2a$; $a \in \mathbb{Z}$

5. $y=2b$; $b \in \mathbb{Z}$

6. $x+y=2(a+b)$

7. $x+y=2c$; $c=(a+b) \in \mathbb{Z}$

8. $x+y$ is even

From (3) and (8), we've reached contradiction.

$\therefore$ The sum of even integers is even #

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7. Prove that for all positive integers a, b, and c; if $a^2 + b^2 = c^2$, then at least one of a, b, or c must be even.

Proof

Proof by contradiction

For $\forall a \forall b \forall c \in \mathbb{Z}^+$

Assume " $a^2+b^2=c^2$ and (none of a, b and c is even)"
which mean " $a^2+b^2=c^2$ and a, b, c are all odd "

$\therefore a^2 + b^2 = c^2$

$(2x+1)^2 + (2y+1)^2 = (2z+1)^2$; $\exists x \exists y \exists z \in \mathbb{Z}$

$(4x^2+4x+1)+(4y^2+4y+1)=(4z^2+4z+1)$

$4x^2+4x+4y^2+4y+4z^2+4z+1=0$) +3 on both sides

$4(x^2+x+y^2+y+z^2+z+1)=3$

$4\alpha = 3$; $\alpha=(x^2+x+y^2+y+z^2+z) \in \mathbb{Z}$

$\therefore$ 4 is divisor of 3 which is false (We've reached Contradiction).

$\therefore \forall a \forall b \forall c \in \mathbb{Z}^+$ If $a^2+b^2=c^2$, then at least one of a, b or c must be even.

8. Prove that for some positive integer x, x + 1/x ≥ 2.

Proof

Proof by contradiction

Assume that, " $\exists x \in \mathbb{Z}^+$ but $x+\frac{1}{x} < 2$ "

From $x+\frac{1}{x} < 2$) multiply by x on both sides.

$x^2+1 < 2x$

$x^2-2x+1 < 0$

$(x-1)^2 < 0$ which is false because $\forall a \in \mathbb{R}$; $a^2 \nless 0$

(We've reached Contradiction).

$\therefore \forall x \in \mathbb{Z}^+$; $x+\frac{1}{x} \geq 2$ ※

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9. Prove that there are no integers a and b for which 18a + 6b = 1.

Proof

Proof by contradiction

Assume " $\exists a \exists b \in \mathbb{Z}$ which $18a + 6b = 1$ "

By algebra $18a + 6b = 1$

$2(9a + 6b) = 1$

$2c = 1$; $c = (9a + 6b) \in \mathbb{Z}$

$\therefore$ 1 is even which is false (We've reached Contradiction).

$\therefore$ There are no integers a and b for which $18a + 6b = 1$ #

10. Use proof by contradiction to show that (A ∩B)= Ø ⇒ A ⊆ B^c

Proof

Proof by Contradiction :

Assume that " $(A \cap B) = \phi$ and $A \not\subseteq B'$ "

From $A \not\subseteq B'$ we know that $\neg \forall x [x \in A \rightarrow x \in B']$

$\rightarrow \exists x \; x \in A$ and $x \notin B'$

$\rightarrow \exists x \; x \in A$ and $x \in B$

$\rightarrow \exists x \; x \in (A \cap B)$

$\rightarrow (A \cap B) \neq \phi$ which is against $A \cap B = \phi$ in Assumption (We've reached Contradiction)

$\therefore (A \cap B) = \phi \rightarrow A \subseteq B'$ #