Problem Set 13: Discrete Mathematics Name Theepakorn Phayomrat ID 67011352

1. Solve: $a_n = 10a_{n-1} - 25a_{n-2}$, $a_0 = 3$, $a_1 = 4$.

$a_n - 10a_{n-1} + 25a_{n-2} = 0$

$r^2 - 10r + 25 = 0$

$(r-5)(r-5) = 0$

$r = 5, 5$

$\therefore a_n = C_1 5^n + C_2 n 5^n$

$\therefore 3(5^n) - \frac{11}{5}(n)(5^n)$ ✱

$a_0 = 3 = C_1$

$a_1 = 4 = 5C_1 + 5C_2$

$4 = 15 + 5C_2$

$C_2 = -\frac{11}{5}$

2. Solve the recurrence relation $a_n = 3a_{n-1} + 2^n$, with initial condition $a_0 = 2$.

$a_n - 3a_{n-1} = 0$

$r - 3 = 0$

$r = 3$

$a_n^h = C\,3^n$

$k\,2^n = 3k\,2^{n-1} + 2^n$

$k\,2^n = \frac{3}{2}k\,2^n + 2^n$

$k2^n = \left(\frac{3}{2}k + 1\right)2^n$

$k = \frac{3}{2}k + 1$

$k = -2$

$a_n^p = -2(2^n) = -2^{n+1}$

$a_n = C\,3^n + 2(2)^n = C3^n - 2^{n+1}$

$a_0 = 2 = C - 2$

$C = 4$

$\therefore a_n = 4(3^n) - 2^{n+1}$ ✱

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3. Solve the recurrence relation $a_n = 8a_{n-1} - 12a_{n-2} + 3n$, with initial conditions $a_0 = 1$ and $a_1 = 5$.

4. What is the form of the solution of the nonlinear recurrence relation $a_n = 6a_{n-1} - 9a_{n-2} + F(n)$ when $F(n) = n^2 2^n$?

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3. Solve the recurrence relation $a_n = 8a_{n-1} - 12a_{n-2} + 3n$, with initial conditions $a_0 = 1$ and $a_1 = 5$.

$r^2 - 8r + 12 = 0$

$r = 2, 6$

$\therefore a_n^h = C_1 2^n + C_2 6^n$ #

$a_n = 8a_{n-1} - 12a_{n-2} + 3n$

$a_n^p = (b_1 n + b_0) s^n \; ; s = 1$

$a_n^p = b_1 n + b_0$

$(b_1 n + b_0) = 8(b_1(n-1) + b_0) - 12(b_1(n-2) + b_0) + 3n$

$b_1 n + b_0 = (-4b_1 + 3)n + 16b_1 - 4b_0$

$b_1 = \frac{3}{5}$

$b_0 = \frac{48}{25}$

$\therefore a_n^p = \frac{3}{5}n + \frac{48}{25}$ #

$\therefore a_n = C_1 2^n + C_2 6^n + \frac{3}{5}n + \frac{48}{25}$

$a_0 = 1 = C_1 + C_2 + \frac{48}{25}$

$-\frac{23}{25} = C_1 + C_2$

$C_1 = -2$

$C_2 = \frac{29}{25}$

$\boxed{\therefore a_n = -2(2^n) + \frac{29}{25}6^n + \frac{3}{5}n + \frac{48}{25}}$ #

$a_1 = 5 = 2C_1 + 6C_2 + \frac{3}{5} + \frac{48}{25}$

$5 = 2C_1 + 6C_2 + \frac{63}{25}$

$= 2C_1 + 6C_2$

$\frac{58}{25} = -\frac{46}{25} + 4C_2$

$=$

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4. What is the form of the solution of the nonlinear recurrence relation $a_n = 6a_{n-1}$ - $9a_{n-2}$ + F(n) when F(n) = $n^2 2^n$?

$$a_n - 6a_{n-1} + 9a_{n-2} = 0$$

$$r^2 - 6r + 9r = 0$$

$$(r-3)^2 = 0$$

$$r = 3, 3$$

$$a_n^h = (b_1 n + b_o)\, 3^n$$

$$F(n) = (b_2 n^2 + b_1 n + b_o)\, s^n \; ; \; s = 2$$

$$a_n^p = (b_2 n^2 + b_1 n + b_o)\, 2^n$$

$$a_n^p = 6a_{n-1}^p - 9a_{n-2}^p + n^2 2^n$$

$$(b_2 n^2 + b_1 n + b_0) 2^n = 6(b_2(n-1)^2 + b_1(n-1) + b_o) 2^{n-1} - 9(b_2(n-2)^2 + b_1(n-2) + b_o) 2^{n-2} + n^2 2^n$$

$b_2 = 4$

$b_1 = 48$

$b_o = 192$

$$\therefore a_n^p = (4n^2 + 48n + 192)\, 2^n$$

$$a_n = 3(c_1 + c_2 n) + 4n^2 + 48n + 192 \; \#$$