Test 8: Name____ ID__________________________

1. Let E denote the set of even integers. Show that |E| = |Z|.

Proof:

We have to find a bijection from E to Z. Let f : ___E=>Z_____ be defined by f(e) = e/2 , for e ∈ E.

We prove that f is a bijection.

1. Suppose $f(e_1) = f(e_2)$ for $e_1, e_2 \in E$, then $e_1/2 = e_2/2$ and e_1 = __e_2_. Hence, f is _injective____.
2. Suppose x ∈ Z, 2x is even, so 2x ∈ E and f(2x) = __x______. Hence, f is __surjective___.

We now have that f is a bijection. ■

2. Show that $(x^3 + 2x)/(2x+1)$ is $O(x^2)$

$(x^3 + 2x)/(2x + 1) < (x^3 + 2x)/2x = (½)x^2 + 1 \leq 2x^2$
Therefore, let c=2 and k=1

3. Evaluate

$$\sum_{i=1}^{100}(4+3i)$$

$$\sum_{i=1}^{100}(4+3i) = \sum_{i=1}^{100} 4 + \sum_{i=1}^{100} 3i$$

$$= \sum_{i=1}^{100} 4 + 3\left(\sum_{i=1}^{100} i\right)$$

$$= 4(100) + 3\left\{\frac{100(100+1)}{2}\right\}$$

$$= 400 + 15,150$$

$$= 15,550 .$$