

1.  $R$  named cyclic :  $aRb \wedge bRa \rightarrow cRa$ . Show that if  $R$  is equivalence relation in  $A$ , then  $R$  is reflexive and cyclic.

Proof Since  $R$  is equivalence over  $A$  then it is already true that  $R$  is reflexive.

And also if  $R$  is equivalence over  $A$  then  $R$  is transitive and symmetric.

From above, to be cyclic  $aRb \wedge bRc \rightarrow cRa$ , and from transitive  $aRb \wedge bRa \rightarrow aRc$  and  $aRc \rightarrow cRa$

$$\therefore aRb \wedge bRc \rightarrow cRa$$

$\therefore$  If  $R$  is equivalence over  $A$  then  $R$  is cyclic and reflexive

2. Let  $R$  be relation on set ordered pairs of  $\mathbb{Z}^+$  such that  $\{(a,b), (c,d)\} \in R$  iff  $ad = bc$ . Show that  $R$  is an equivalence relation.

Proof Reflexive :  $R$  is reflexive because  $\{(a,b), (a,b)\} \in R$  because  $ab = ba$

Symmetric :  $R$  is symmetric since  $\{(a,b), (c,d)\} \in R$ , then  $ad = bc$ , which means  $cb = da$  which means  $\{(c,d), (a,b)\} \in R$

Transitive : If  $\{(a,b), (c,d)\} \in R$  and  $\{(c,d), (e,f)\} \in R$ , then  $ad = bc$  and  $cf = de$

$$\textcircled{1} \times \textcircled{2} \text{ we get } \overset{1}{ad}f = b\overset{1}{c}de$$

$$af = be \text{ which means } \{(a,b), (e,f)\} \in R \therefore R \text{ is transitive}$$

$\therefore R$  is an equivalence relation