4 Quine McCluskey Method

Quine McCluskey Method

We have learned how to reduce the equation using

- The Boolean Algebra
- The Karnaugh Map.

If the equation has **more** variables than 4,

The minimization will be more complicated and difficult to solve.

Here comes the method to handle more INPUTS:

The **Prime Implicant** method or The **Quine-McCluskey** method.

Quine McCluskey Method

- The principle is to compare between the similar minterms;
- and merge a pair of them together;
- This would reduce number of the marked (merged) minterms;
- Keep doing this until we cannot merge any pair of minterms;
- So, this might be a large number of pairs in comparison;
- Need to classify the groups of the minterms by

  the number of 1's existing in the minterms.

Quine McCluskey Method

Grouping Principle:

Arrange minterms, with the same amount of 1, in the same group as:

- Group 0: All variables in the terms is 0, or minterm 0.
- Group 1: There are just only one '1' in the minterms, such as minterm 1, 2, 4, 8, etc.
- Group 2: There are just only two '1's in the minterms, such as minterm 3, 5, 6, etc.
- and so on for Group 3, 4, …

The reason of grouping by the number of '1' is followings:

I. The terms with the **same** amount of 1 are impossible to be merged together e.g. minterm 1 (0001) , 2 (0010), 4 (0100) and 8 (1000).

II. The terms in the group with n number of 1's can be compared with that in the group with $n + 1$ number of 1 once.

e.g. term (0001) of group 1, can be compared with term (0011) of group 2.

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Prime Implicant Finding Procedure:

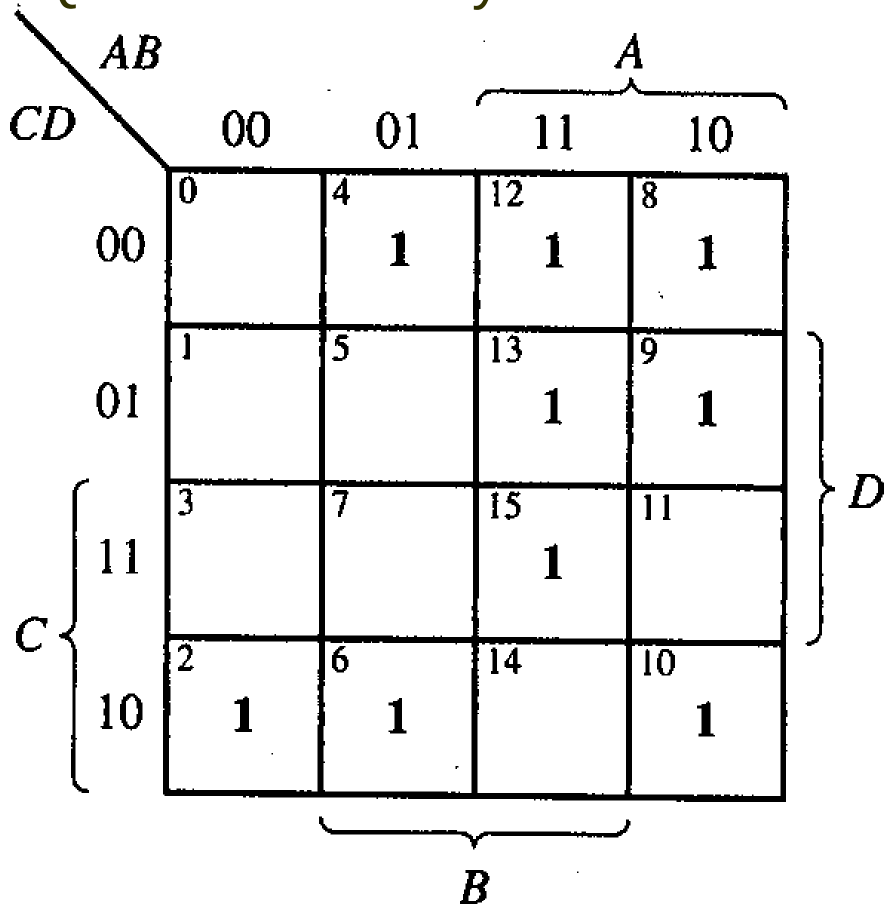
- Find the different bit and replace it with '-' , keep doing this until reaching the last group.

  e.g. 0010 and 0110 will be merge to 0-10.

- Mark the pair of minterms that can be merged, in the grouping table;
- Then the number of '1' in the minterms will be decreased.
- Then we get the new grouping list, which has less groups than the previous list.
- Then do comparing again until there is no pair for comparison.
- Entitle the **unmarked** minterms as the **Prime Implicant (PI).**

Quine McCluskey Method

Example: Use the Quine-McCluskey to minimize the K-Map below.



Solution: The Karnaugh Map is composed with 9 minterms:

m2 m4 m6 m8 m9 m10 m12 m13 and m15.

Quine McCluskey Method

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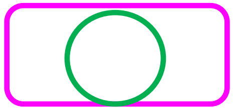
m2 m4 m6 m8 m9 m10 m12 m13 and m15.

Write all terms in binary codes as a function *F*(*A, B, C, D*), get:

m2 = 0010		m4 = 0100	
m6 = 0110		m8 = 1000	
m9 = 1001		m10 = 1010	
m12 = 1100		m13 = 1101	
m15 = 1111			

Having grouped them, we have :



- Group 1 (a single 1): m2, m4, m8
- Group 2 (two 1’s): m6, m9, m10, m12
- Group 3 (three 1’s): m13,
- Group 4 (four 1’s): m15

Quine McCluskey Method

Group them, we have

Minterms	ABCD	
2	0010	Group 1 (a single 1)
4	0100	
8	1000	
6	0110	Group 2 (two 1's)
9	1001	
10	1010	
12	1100	
13	1101	Group 3 (three 1's)
15	1111	Group 4 (four 1's)

Quine McCluskey Method

Compare them, we have

List 1			List 2			List 3		
Minterm	$ABCD$		Minterms	$ABCD$		Minterms	$ABCD$	
2	0010	✓	2, 6	0-10	PI_2	8, 9, 12, 13	1-0-	PI_1
4	0100	✓	2, 10	-010	PI_3			
8	1000	✓	4, 6	01-0	PI_4			
6	0110	✓	4, 12	-100	PI_5			
9	1001	✓	8, 9	100-	✓			
10	1010	✓	8, 10	10-0	PI_6			
12	1100	✓	8, 12	1-00	✓			
13	1101	✓	9, 13	1-01	✓			
15	1111	✓	12, 13	110-	✓			
			13, 15	11-1	PI_7			

Quine McCluskey Method

- The first grouping is illustrated as List 1;
- The first comparison is illustrated as List 2;
- The minterms used in comparison must be marked (✓), too.
- The second comparison is illustrated as List 3.

  It is found that no minterms cannot be compared further.
- Name the minterms cannot be compared as

  Prime Implicant (PI *n*) from the last List back to the prior groups.

Quine McCluskey Method

Plot the PI-Minterms Table:

Mark x to any minterms that are composed in the individual PI's.

	2	4	6	8	9	10	12	13	15
				√	√		√	√	√
* * PI_1				×	⊗		×	×	
PI_2	×		×						
PI_3	×					×			
PI_4		×	×						
PI_5		×					×		
PI_6				×		×			
* * PI_7								×	⊗

Find the Minterm(s) that occurs ONCE in through all PI's, by drawing a line vertically.

Call the PI of this mintern as the Essential PI (EPI), i.e. PI_1 and PI_7.

Quine McCluskey Method

The Essential PI's are composed with the minterms used for the answer.

These minterms would be taken away the table, leading to the PI table with less minterms (by taking off all above minterms).

Then cover the rest minterms by employing the number of PI's as LEAST as possible.

	√ 2	√ 4	√ 6	√ 10
PI_2	×		×	
$*PI_3$	×			×
$*PI_4$		×	×	
PI_5		×		
PI_6				×

PI Covering:

PI_3 and PI_4 can cover the rest minterms, and call them as Covering PI's (CPI).

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We have EPI: PI_1 and PI_7

And CPI: PI_3 and PI_4

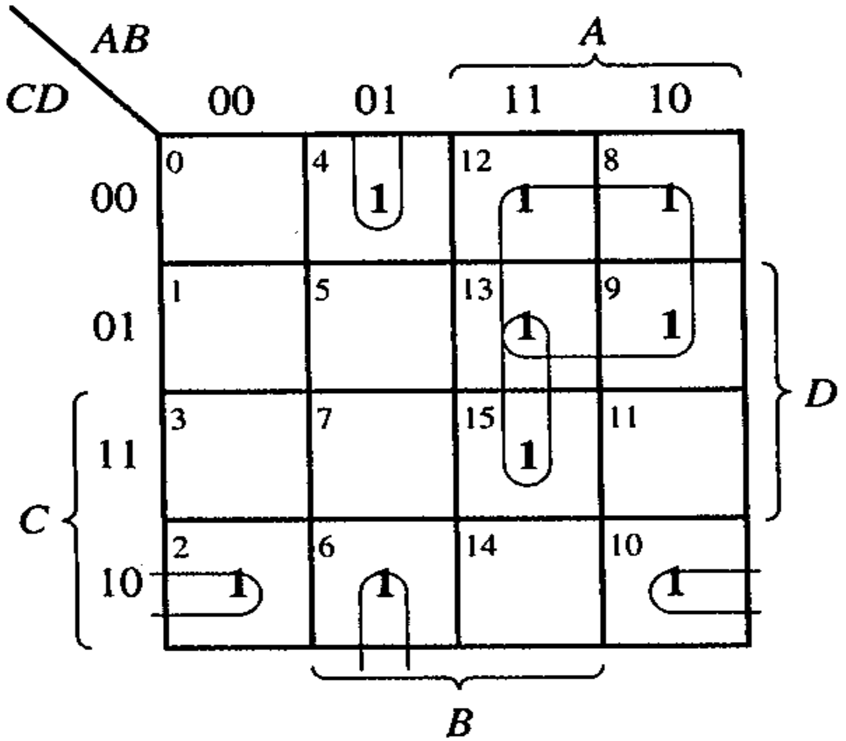
Therefore, the minimal output is sum of EPI and CPI:

Y = EPI + CPI

$$
\begin{aligned}
f(A,B,C,D) &= PI_1 + PI_3 + PI_4 + PI_7 \\
&= 1\text{-}0\text{-} + \text{-}010 + 01\text{-}0 + 11\text{-}1 \\
&= A\bar{C} + \bar{B}C\bar{D} + \bar{A}B\bar{D} + ABD
\end{aligned}
$$

Quine McCluskey Method

By K-Map, we can obtain the answer as:



The answers from both method are the same.

Quine McCluskey Method

Example: More practice in PI Covering, for a PI-Minterm table:

	√ 0	√ 1	5	√ 6	√ 7	√ 8	√ 9	10	11	13	√ 14	√ 15
* * PI_1	⊗	×				×	×					
PI_2		×	×				×			×		
PI_3			×		×					×		×
PI_4						×	×	×	×			
PI_5							×		×	×		×
PI_6								×	×		×	×
* * PI_7				⊗	×						×	×

From the table, we get EPI: PI_1 and PI_7.

Quine McCluskey Method

After cutting the minterms in the EPI’s (PI_1 and PI_7), the table reduces to:

	5	10	11	13
PI_2	×			×
PI_3	×			×
PI_4		×	×	
PI_5			×	×
PI_6		×	×	

PI Covering:

Find that PI_2 (or PI_3) and PI_4 (or PI_6) can cover the rest minterms.

Therefore, the minimal output is:

$Y = PI_1 + PI_2 + PI_4 + PI_7$

Y = -00- + --01 + 10-- + -11-

$Y = \bar{B}\bar{C} + \bar{C}D + A\bar{B} + BC$

Quine McCluskey Method

Example: Cyclic PI chart. Suppose the output function is

$$f(A,B,C) = \sum m(1,2,3,4,5,6)$$

	√		√			
	1	2	3	4	5	6
*PI_1	×		×			
PI_2		×	×			
PI_3		×				×
PI_4				×		×
PI_5				×	×	
PI_6	×				×	

Solution:

- There is NO EPI.
- Skip to the covering process.
- Find that it is a Cyclic PI chart.
- Need to define an EPI = PI_1.
- Redo the covering process.

Quine McCluskey Method

After cutting the minterms in PI_1, the table reduces to:

	2	4	5	6
PI_2	×			
PI_3	×			×
PI_4		×		×
PI_5		×	×	
PI_6			×	

PI Covering:

Find that PI_3 and PI_5 can cover the rest minterms.

Therefore the minimal output is:

$Y = PI_1 + PI_3 + PI_5$

$Y = 0\text{-}1 + \text{-}10 + 10\text{-}$

$Y = \overline{A}C + B\overline{C} + A\overline{B}$

It is just an answer !

There might be another solutions.