Problem Set 6: Discrete Mathematics

1. We define the degree of a node in an undirected graph without loops as the number of edges incident with it. The degree of the node v is denoted by deg(v). Prove the following theorems.

1.1 In any graph G = (V, E), the sum of degrees of all its nodes equals twice the number of edges.

1.2 In any graph G = (V, E), the number of odd nodes is even. (an even number of nodes has an odd degree).

2. A graph with v vertices and n edges has at least v−n connected components.

3. A graph G = (V, E) is a forest if it doesn't contain any cycles.
Let G = (V, E) be a forest. There must exist a vertex $v \in V$ with $\deg(v) \leq 1$.
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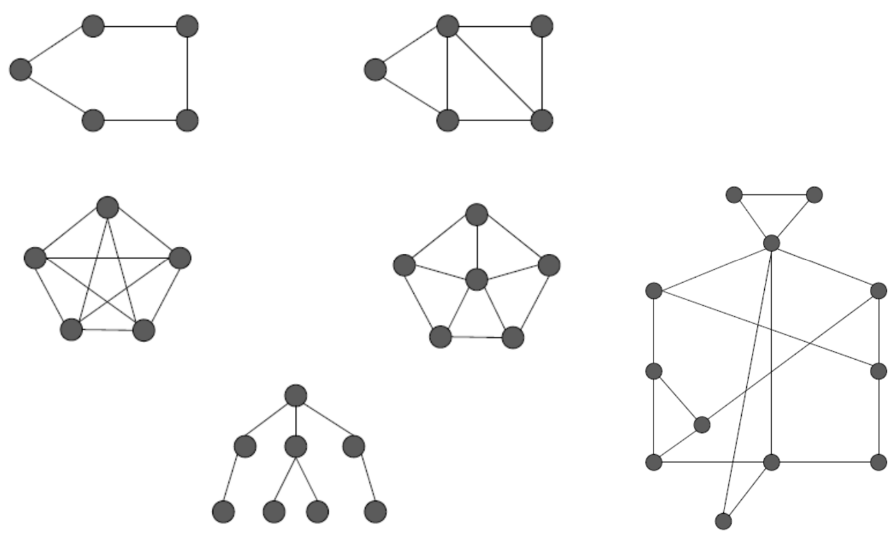
4. Let T = (V, E) be a tree. Show that if |V| ≥ 2, T has at least two leaves.

5. The chromatic number of a graph is the least number of colors required to do a vertex-coloring of a graph. Calculate the chromatic number of the following graphs.



6. A computer network (a connected graph) consists of 6 computers. Show that there are at least two computers in the network that are directly connected to the same number of computers.

7. What is the minimum number of students needed in a class to guarantee that there are at least 6 students whose birthdays fall in the same month?