Problem Set 11: Discrete Mathematics Name Theepakorn Phayarrat ID 6701135Z

1. How many integer solutions are there to the equation

A + B + C + D = 17

A, B, C, D ≥ 0,

$\left(\sum_{i=0}^{17} x^i\right)^4$

Find coefficient of x^{17}

OR $\binom{20}{3} = \binom{20}{17} = 1140$

2. How many different ways are there to choose 8 bottles of beer in the Bar, when they have Leffe, Chimay, Budvar and Hoeggarden?

$A+B+C+D=8$

$A,B,C,D \geq 0$

$(x^0+x^1+x^2+x^3+x^4+x^5+x^6+x^7+x^8)^4 = \left(\sum_{i=0}^{8} x^n\right)^4$

OR $\binom{11}{3} = \binom{11}{8} = 165$

Find coefficient of x^8

3. Use generating functions to find the number of k-combinations of a set with n elements.

$(1+x)^n$

Find coefficient of x^k

4. Set up a generating function and use it to find the number of ways in which 12 oranges can be allocated for three children, Grace, Mary, and Frank. Grace gets at least four, and Mary and Frank gets at least two, but Frank gets no more than five.

$\left(\sum_{i=4}^{8} x^i\right)\left(\sum_{i=2}^{6} x^i\right)\left(\sum_{i=2}^{5} x^i\right)$

Find coefficient of x^{12}

5. Four kinds of jelly beans, Red, Green, White, Black
In how many ways can we select 24 jelly beans so that we have an even number of white beans and at least six black ones? Set up a generating function and use it to find a solution.

Black $\left(\sum_{i=6}^{24} x^i\right)$ White $\left(\sum_{i=0}^{9} x^{2i}\right)$ Red $\left(\sum_{i=0}^{18} x^i\right)$ Green $\left(\sum_{i=0}^{18} x^i\right)$

Find coefficient of x^{24}

6. Set up a generating function and use it to find how many nonnegative integer solutions are there for $c_1+c_2+c_3+c_4=25$?

$\left(\sum_{i=0}^{25} x^i\right)^4$

Find coefficient of x^{25}