Test 5 Discrete Mathematics Name________________________________ ID_________________

Let S be given by

Basis: $2 \in S$; $5 \in S$;

Recursive: if $x \in S$ then $x+2 \in S$.

Show that "n ∈ S for all integers n ≥ 4".

Answer:

Prove by complete induction that for all n ≥ 4, n ∈ S

Let P(n) be "for all $n \geq 4$, $n \in S$"

Base Cases:

P(4): $2 \in S \Rightarrow 2+2 \in S \Rightarrow 4 \in S$

P(5): $5 \in S$

Inductive Hypothesis:

$P(4), P(5), \ldots, P(k)$ for $k \geq 5$

Inductive Step:

We want to show that P(k+1), that is k+1 ∈ S.

Using the inductive hypothesis, $P(k-1)$ holds since $k-1 \geq 4$.

Thus, $k-1 \in S$ (because $k-1 \geq 4$, since $k \geq 5$).

Then, by definition we have $(k-1)+2 \in S \Rightarrow k+1 \in S$.

So $P(k+1)$ holds.

By the principle of complete induction, $P(n)$ holds for all $n \in Z$, $n \geq 4$.