Problem Set 1: Discrete Mathematics

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1. For all set S, show that $\emptyset \subseteq S$.

Let S be any set: To show that $\emptyset \subseteq$ S, we must show that "$\forall x \in \emptyset \to x \in S$" is True. Because the empty set has no element, it follows that $x \in \emptyset$ is always False. It follows that "$\forall\ x \in \emptyset \to x \in S$ is True. By Vacuous Proof #

2. Prove or give a counterexample: A ∩ P(A) = A.

$A = \{1\}$

$P(A) = \{\emptyset, \{1\}\}$

$\to A \cap P(A) = \emptyset \neq A$

$\therefore A \cap P(A) = A$ is False #

3. Show that "For all sets A, B, and C, if A ⊆ B and B ⊆ C, then A ⊆ C".

Direct Proof

Assume : $a \in A$

We know that $A \subseteq B \therefore a \in B$

we know that $B \subseteq C \therefore a \in C$

$\therefore A \subseteq C$ #

4. Prove that if A ⊆ B then ℘(A) ⊆ ℘(B).

Direct Proof Assume $A \subseteq B$. We need to show that $P(A) \subseteq P(B)$

Let $x \in P(A)$, we need show that $x \in P(B)$

Since $x \in P(A) \therefore x \subseteq A$

But $A \subseteq B$, then $x \subseteq B$

$\therefore x \subseteq B \to x \in P(B)$ #

5. Prove that ℘(A) ∩ ℘(B) = ℘(A∩B).

We need to show that

$P(A) \cap P(B) \subseteq P(A \cap B)$

and

$P(A \cap B) \subseteq P(A) \cap P(B)$

$P(A) \cap P(B) \subseteq P(A \cap B)$

Let $c \in P(A) \cap P(B)$. There $c \in P(A)$ and $c \in P(B)$. So, $c \subseteq A$ and $c \subseteq B$ $\therefore c \in P(A \cap B)$

$\therefore P(A) \cap P(B) \subseteq P(A \cap B)$

6. Prove that ℘(A) ∪ ℘(B) ⊆ ℘(A∪B).

$P(A \cap B) \subseteq P(A) \cap P(B)$

Let $c \in P(A \cap B)$. There $c \subseteq A \cap B$. So, $c \subseteq A$ and $c \subseteq B$ $\therefore c \in P(A) \cap P(B)$

$\therefore P(A) \cap P(B) = P(A \cap B)$ #

Assume $x \in P(A)$ or $x \in P(B)$. There $x \subseteq A$ or $x \subseteq B$

So, $x \subseteq A \cup B \therefore x \in P(A \cup B)$

$\therefore P(A) \cup P(B) \subseteq P(A \cup B)$ #

7. Give a counterexample to disprove "For all sets A, B, and C, if A ∈ B and B ∈ C, then A ∈ C".

$A = \{1\}$

$B = \{\{1\}, 2\}$

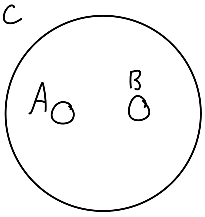
$C = \{\{\{1\}, 2\}, 3\}$ #

8. Give a counterexample to disprove "For all sets A, B, and C, if A U C = B U C, then A = B".

$A = \{1\}$ $C = \{1, 2\}$

$B = \{2\}$ #

C



9. Show that there exists a set A where ℘(A) = {A}.

If $A = \emptyset$ $P(A) = \{\emptyset\}$

$\therefore$ There exists $A = \emptyset$ that $P(A) = \{A\}$. #

10. Prove or give a counterexample: If A ⊂ B and A ⊂ C, then A ⊂ (B ∩ C).

To prove that $A \subset B \cap C$ we must show that

we must show that $x \in A \to (x \in B \wedge x \in C)$

Proof Let $x \in A$

Since $A \subseteq B \therefore x \in B$

Since $A \subseteq C \therefore x \in C$

$\therefore x \in A \to (x \in B \wedge x \in C)$

$\therefore A \subseteq (B \cap C)$ #

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1 Let $A = \emptyset$, $B = \emptyset$, show that $A = B$

From PS1 Part 1: $\emptyset \subseteq S$ for all set S

To proof that $A = B$
we must show that $A \subseteq B$ and $B \subseteq A$.

Show that $A \subseteq B$
Let $A = \emptyset$
From PS1 Part 1, $A \subseteq B$
$\therefore A \subseteq B$

Show that $B \subseteq A$
Let $B = \emptyset$
From PS1 Part 1, $B \subseteq A$
$\therefore B \subseteq A$

$\therefore A \subseteq B \wedge B \subseteq A$

$\therefore A = B$

$\therefore$ If $A = \emptyset$ and $B = \emptyset$ then $A = B$ #

2 Show that $P(A) = P(B) \rightarrow A = B$

Proof To proof that $A = B$
we must show that $A \subseteq B$ and $B \subseteq A$

Show that $A \subseteq B$
Let $a \in A$
$\{a\} \subseteq A$ $\therefore \{a\} \in P(A)$
We know that $P(A) = P(B)$
$\therefore \{a\} \in P(B)$
$\therefore a \in B$
$\therefore A \subseteq B$

Show that $B \subseteq A$
Let $b \in B$
$\{b\} \subseteq B$ $\therefore \{b\} \in P(B)$
We know that $P(A) = P(B)$
$\therefore \{b\} \in P(A)$
$\therefore b \in A$
$\therefore B \subseteq A$

$\therefore A \subseteq B \wedge B \subseteq A$

$A = B$

$\therefore P(A) = P(B) \rightarrow A = B$ #