Test 12: Name Theepakorn Phayprat ID 67011352

1. Describe the sequence $1^2, 2^2, 3^3, 4^2, \ldots$ recursively. Include initial conditions and assume that sequences begin with a_1.

$a_1 = 1$

$a_n = (\sqrt{a_{n-1}} + 1)^2$ #

OR

$a_n = a_{n-1} + 2n - 1$ #

2. Find the solution of the recurrence relation $a_n = 2a_{n-1} + a_{n-2}$ where $a_0 = 1$, $a_1 = 2$.

$r^2 = 2r + 1$

$r^2 - 2r - 1 = 0$

$r = \frac{2 \pm \sqrt{4+4}}{2}$

$r = 1 \pm \sqrt{2}$

$a_n = \alpha_1 (1+\sqrt{2})^n + \alpha_2 (1-\sqrt{2})^n$

$a_0 = \alpha_1 + \alpha_2 \rightarrow \alpha_1 = 1 - \alpha_2$

$a_1 = \alpha_1 (1+\sqrt{2}) + \alpha_2 (1-\sqrt{2})$

$a_1 = (1-\alpha_2)(1+\sqrt{2}) + \alpha_2 (1-\sqrt{2}) = 2$

$2 = (1+\sqrt{2}) + 2\alpha_2$

$\alpha_2 = \frac{1-\sqrt{2}}{2}$

$\alpha_1 = \frac{1+\sqrt{2}}{2}$

$\therefore a_n = \left(\frac{1+\sqrt{2}}{2}\right)(1+\sqrt{2})^n + \left(\frac{1-\sqrt{2}}{2}\right)(1-\sqrt{2})^n$ #

$= \frac{1}{2}(1+\sqrt{2}^{\,n+1}) + \frac{1}{2}\left(\frac{1-\sqrt{2}}{2}\right)^{n+1}$