

1. R named Cyclic : $aRb \wedge bRa \rightarrow cRa$. Show that if R is equivalence relation in A, then R is reflexive and cyclic.

Proof

Since R is equivalence over A then it is already true that R is reflexive.

And also if R is equivalence over A then R is transitive and symmetric.

From above, to be cyclic $aRb \wedge bRc \rightarrow cRa$, and from transitive $aRb \wedge bRa \rightarrow aRc$ and $aRc \rightarrow cRa$

$$\therefore aRb \wedge bRc \rightarrow cRa$$

\therefore If R is equivalence over A then R is cyclic and reflexive ~~X~~

2. Let R be relation on set ordered pairs of \mathbb{Z} such that $\{(ab), (cd)\} \in R$ iff $ad = bc$. Show that R is an equivalence relation.

Proof

Reflexive : R is reflexive because $\{(a,b), (a,b)\} \in R$ because $ab = ba$.

Symmetric : R is symmetric because if $\{(ab), (cd)\} \in R$, then $ad = bc$, which means $cb = da$ which means $\{(cd), (ab)\} \in R$.

Transitive : If $\{(ab), (cd)\} \in R$ and $\{(cd), (ef)\} \in R$, then $ad = bc$ and $cf = de$

$$(1) \times (2) \text{ we get } adcf = bade$$

$$ad = be \text{ which means } \{(ab), (ef)\} \in R \therefore R \text{ is transitive.}$$

$\therefore R$ is an equivalence relation ~~X~~