1. Use the Principle of Mathematical Induction to prove that
$1-2+2^2-2^3+\ldots+(-1)^n 2^n = \dfrac{2^{n+1}(-1)^n+1}{3}$ for all positive integers n.

2. Use the Principle of Mathematical Induction to prove that $1 + 2^n \leq 3^n$ for all $n \geq 1$.

3. Use the Principle of Mathematical Induction to prove that $1+3+9+27+...+3^n = \dfrac{3^{n+1}-1}{2}$ for all $n \geq 0$.

4. Prove that $\sum_{j=n}^{2n-1}(2j+1) = 3n^2$ for all positive integers *n*.

5. Prove that for any positive integer n, the number 2^{2n}-1 is divisible by 3.

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6. Prove that $x^0 + x^1 + ... + x^n = (x^{n+1} - 1)/(x - 1)$ for all integers $n \geq 0$, using induction.

7. For each natural number $n \geq 1$, the n^{th} Fibonacci number, F_n, is defined inductively by

$F_1=1$, $F_2 = 1$, and $F_{n+2} = F_{n+1} + F_n$ for $n \geq 1$.

Prove that $F_{n+6} = 4F_{n+3} + F_n$ for $n > 0$.

8. Prove that for any $n \geq 1$, $\Sigma_{i=1}^{n} (i^2) = n(n+1)(2n+1)/6$

9. Prove that $n! > 2^n$ for all $n \geq 4$

10. Show that $\Sigma_{i=1}^{n} (i^3) = (\Sigma_{i=1}^{n} i)^2$ for all $n \geq 1$