Problem Set 7: Discrete Mathematics

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1. List all binary relations on the set {0, 1}.

16 Relations

1) $\emptyset$
2) $\{(0,0)\}$
3) $\{(0,1)\}$
4) $\{(1,0)\}$
5) $\{(1,1)\}$
6) $\{(0,0),(0,1)\}$
7) $\{(0,0),(1,0)\}$
8) $\{(0,0),(1,1)\}$
9) $\{(0,1),(1,0)\}$
10) $\{(0,1),(1,1)\}$
11) $\{(1,0),(1,1)\}$
12) $\{(0,0),(0,1),(1,0)\}$
13) $\{(0,0),(0,1),(1,1)\}$
14) $\{(0,0),(1,0),(1,1)\}$
15) $\{(0,1),(1,0),(1,1)\}$
16) $\{(0,0),(0,1),(1,0),(1,1)\}$ #

2. List all binary relations on the set {0, 1} that are equivalence relations.

8) $\{(0,0),(1,1)\}$

16) $\{(0,0),(0,1),(1,0),(1,1)\}$ #

3. Suppose relation R is defined on set Z where aRb means ab<0. Determine whether R is an equivalence relation on Z.

consider R in reflexive property $\rightarrow a \not R a$ because $\forall a \in \mathbb{Z}$ if $a<0$ $a(a) \not< 0$

$\therefore$ No, R is not an equivalence relation on $\mathbb{Z}$ #

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4. The relation R on Z defined by x R y if x + 3y is even, is an equivalence relation.

Reflexive: $a + 3a = 4a = 2(2a) = 2(x)$; $\exists x = 2a \in \mathbb{Z}$ $\therefore$ $a + 3a$ is even.

$\therefore$ Reflexive

Symmetric: From $\overset{aRb}{a+3b}$ is even, $a+3b = 2m$; $\exists m \in \mathbb{Z}$. $\therefore$ $a = 2m - 3b$ which mean $\overset{bRa}{b + 3a} = b + 3(2m - 3b)$

$= b - 9b + 6m$

$= -8b + 6m$

$= 2(3m - 4b)$

$= 2x$; $\exists x = (3m - 4b) \in \mathbb{Z}$

$\therefore b + 3a$ is even. $\therefore$ Symmetric

Transitive: Assume aRb and bRc.

$\therefore a + 3b = 2m$; $\exists n \in \mathbb{Z}$ —①

and $b + 3c = 2n$ $\exists n \in \mathbb{Z}$ —②

①+②: $a + 3b + b + 3c = 2m + 2n$

$a + 3c = 2m + 2n - 4b$

$a + 3c = 2(m + n - 2b)$

$a + 3c = 2x$; $x = (m + n - 2b) \in \mathbb{Z}$

$\therefore$ aRb and $bRc \rightarrow aRc$

$\therefore$ Transitive

$\therefore$ R is an equivalence relation.

5. Suppose A is the set composed of all ordered paired of positive integers. Let R be the relation defined on set A where (a,b)R(c,d) means that a+d = b+c.
   - Prove that R is an equivalence relation.
   - Find $[(2, 4)]_R$.

Proof Reflexive: $a + b = b + a \rightarrow \therefore (a,b)R(a,b)$ $\therefore$ Reflexive.

Symmetric: If $a + d = b + c$, then $c + b = d + a$; $\therefore$ $(a,b)R(c,d) \rightarrow (c,d)R(a,b)$ $\therefore$ Symmetric

Transitive: If $a + d = b + c$ and $c + f = d + e$, then $(a+d) - (d+e) = (b+c) - (c+f)$

$a - e = b - f$

$a + f = b + e$ $\therefore (a,b)R(c,d) \rightarrow (c,d)R(e,f)$ $\therefore$ Transitive

$\therefore$ R is an equivalence relation

$[(2,4)]_R = \{(a,b) \mid a + 4 = b + 2\}$

$= \{(a,b) \mid b = a + 2\}$

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6. Suppose the relation R is defined on the set of all subsets of {1,2,3,4} where SRT means S is a proper subset of T. Determine whether R is a strict order relation on these subsets.

Show that

Irreflexive : $S \not R S$

S cannot be a proper subset of itself by definition of proper subset.

$\therefore S \not R S$ $\therefore$ R is irreflexive

$\therefore$ R is a strict relation on these subsets.

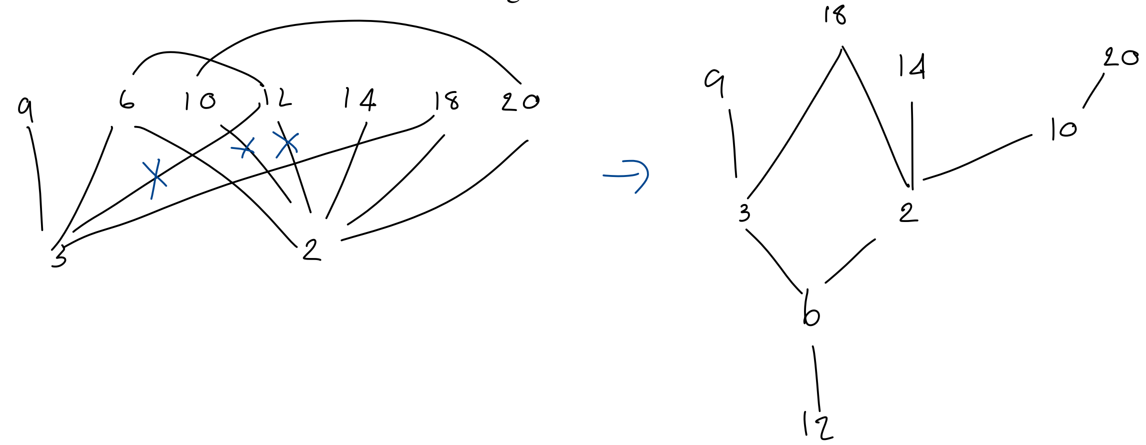
Transitive $SRT \wedge TRU \rightarrow SRU$

By PS 1 part 3 we have prove that $\forall S,T,U;\ S \subseteq \{1,2,3,4\}$ and $T \subseteq \{1,2,3,4\}$ and $U \subseteq \{1,2,3,4\}$

If $S \subset T$ and $T \subset U \Rightarrow S \subset U$

$\therefore SRT \wedge TRU \rightarrow SRU$ $\therefore$ R is transitive

7. Suppose A={2,3,6,9,10,12,14,18,20} and R is a strict order relation defined on A where aRb means a is a divisor of b. Draw the Hasse diagram for R.



8. Let R be the relation on the set of people such that xRy if x and y are people and x is older than y. Show that R is not a partial ordering.

R is antisymmetric because if x is older than y, then y must not be older than x.

$\therefore xRy \Rightarrow y \not R x$ $\therefore$ R is antisymmetric.

R is transitive because if x is older than y and y is older than z, then for sure x is older than z.

$\therefore xRy \wedge yRz \rightarrow xRz$ $\therefore$ R is transitive

BUT R is not reflexive because x cannot be older than himself/herself.

$\therefore x \not R x$ $\therefore$ R is irreflexive.

$\therefore$ R is not a partial ordering