

Model assisted (1+1)ES

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1 Introduction

In my experiments I followed the model-assisted algorithm suggested in [1] with some changes in the evolutionary part and approximation approach.

1/5 rule Instead of the median selection with CMA-ES that is used in [1], my algorithm evolves using the 1/5 rule explained in [2].

Local training The surrogate algorithm in [1] only learns from the last 30 training data available for training, because of this the model error decrease as the evolutionary algorithm converges to the optimum point. However, my experiments show that increasing the number of training data makes the model error decrease faster for more complex problems.

RBF Network In this paper we are using the RBF network suggested in [1] which contains 20 hidden nodes, and a k-means core.

Experiments Case studies are evaluated by three factors:

1.Original fitness First case study is a simple Sphere function $Y = X^2$. In this case study the minimum fitness that the algorithms can show in python is $10^{-323} * 10^{-323} = 10^{-646}$.

Second case study is a sphere function shifted for one unit to the left or right side $Y = (X - 1)^2$. In this case study the minimum fitness that the algorithms can show in python is $10^{-15} * 10^{-15} = 10^{-30}$.

2.Model fitness error In all experiments I measured the absolute fitness error of the new generated population to evaluate if the model is accurate enough to select the best individual in the population:

$$Error = |Fitness(ind) - Model(ind)|$$

3.Original fitness slope Finding the absolute slope between the starting point and the best evaluated fitness can also be beneficial to describe the behaviour of the algorithms.

$$Slope_{generation=N} = (\log(Bestfitness_{generation=0}) - \log(Bestfitness_{generation=N})) / N$$

2 Algorithms

2.1 Algorithm 1

In the first two lines of the algorithm, a RBF network with a K-means core is initialized as was suggested in[1].In lines 5-12,the algorithm uses the σ calculated in the previous cycle to discover random points in sigma-neighbourhood of the current point, and then selects the best point via the best available approximated model of the fitness function.

In lines 14-18, the algorithm adds a new individual into the training population of the RBF model only if its real fitness is better than any other fitness available.(the training population only stores the last 30 individuals)

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1:  $N = \text{numberofdimensions}$ 
2:  $\text{initialize}(\text{individual}, \sigma)$ 
3:  $\text{Model} \leftarrow (\text{RBFNetwork}(\text{Inputs} = N, \text{Hidden} = 10, \text{Output} = 1))$ 
4: for  $i \leq \text{Originalgeneration}$  do
5:    $\text{individual2} \leftarrow \text{individual}$ 
6:   for  $j \leq \text{Modelgeneration}$  do
7:      $\text{individual3} \leftarrow \text{individual2} + \sigma * I$ 
8:      $\text{fit2} \leftarrow \text{Modelfitness}(\text{individual2})$ 
9:      $\text{fit3} \leftarrow \text{Modelfitness}(\text{individual3})$ 
10:    if  $\text{fit3} \leq \text{fit2}$  then
11:       $\text{individual2} \leftarrow \text{individual3}$ 
12:    end if
13:  end for
14:   $\text{Newfitness} = \text{originalfitness}(\text{individual2})$ 
15:   $\text{printerror} \leftarrow \frac{(\text{Newfitness} - \text{Model}(\text{individual2}))}{\text{Originalgeneration}}$ 
16:   $\sigma \leftarrow \sigma * \exp^{\frac{1}{N}}((\text{Newfitness} \leq \text{Bestfitness}) - \frac{1}{5})$ 
17:  if  $\text{Newfitness} \leq \text{Bestfitness}$  then
18:     $\text{individual2} \leftarrow \text{individual}$ 
19:     $\text{Bestfitness} \leftarrow \text{Newfitness}$ 
20:     $\text{trainingpop.append}(\text{individual}, \text{Bestfitness})$ 
21:     $\text{Model.train}(\text{trainingpop})$ 
22:  end if
23: end for
24:  $\text{printfitness} \leftarrow \text{Bestfitness}$ 

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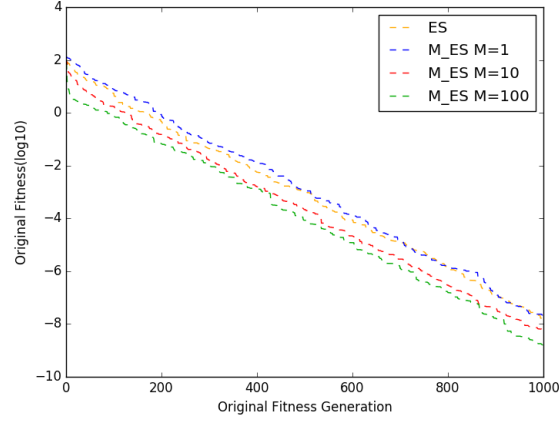


Figure 1: simple(1+1) ES fitness compared to M-ES(model assisted ES) using three different number of model generation(M) per original fitness evaluation on 10D $Y = (X)^2$ (5 trials)

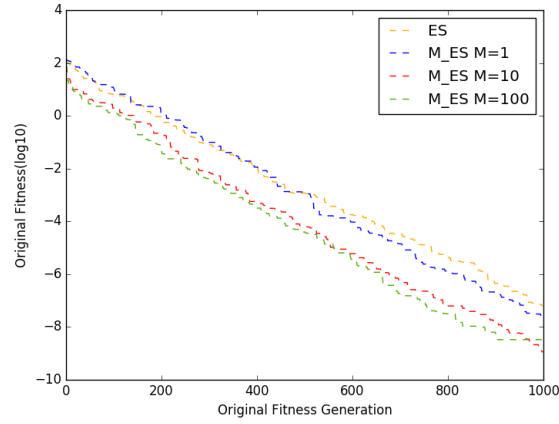


Figure 2: simple ES fitness compared to M-ES(model assisted ES) using three different number of model generation(M) per original fitness evaluation on 10D $Y = (X - 1)^2$ (5 trials)

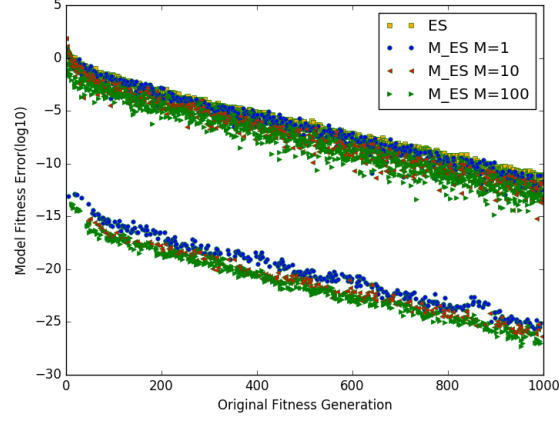


Figure 3: simple ES error compared to M-ES(model assisted ES) using three different number of model generation(M) per original fitness evaluation on 10D $Y = (X)^2$ (5 trials)

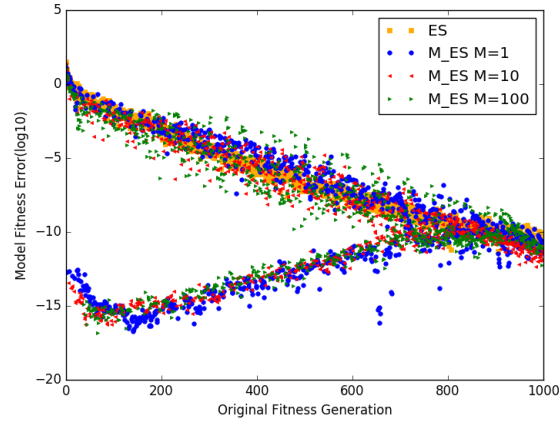


Figure 4: simple ES error compared to M-ES(model assisted ES) using three different number of model generation(M) per original fitness evaluation on 10D $Y = (X - 1)^2$ (5 trials)

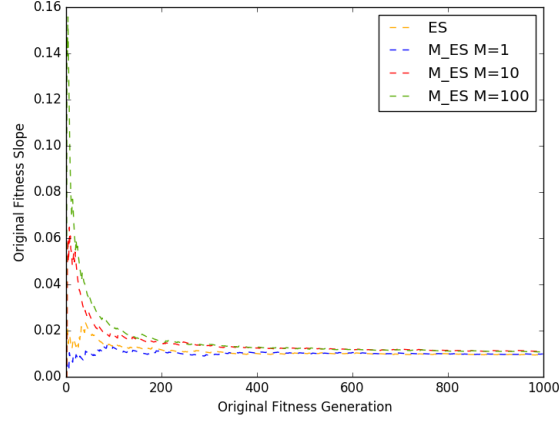


Figure 5: simple ES slope compared to M-ES(model assisted ES) using three different number of model generation(M) per original fitness evaluation on 10D $Y = (X)^2$ (5 trials)

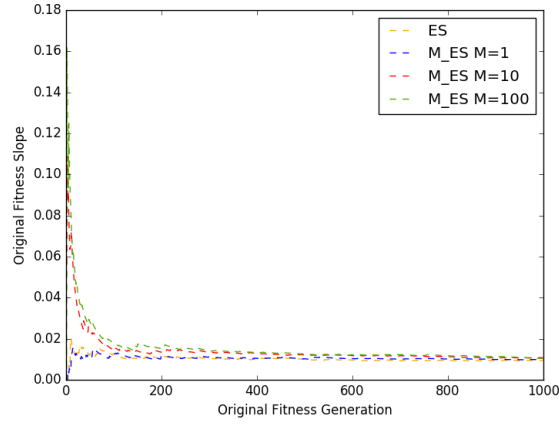


Figure 6: simple ES slope compared to M-ES(model assisted ES) using three different number of model generation(M) per original fitness evaluation on 10D $Y = (X - 1)^2$ (5 trials)

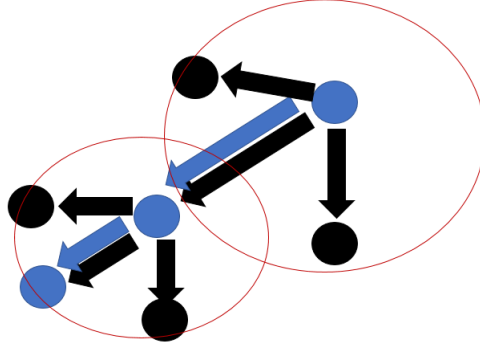


Figure 7: Shows how A1 algorithm works. Black arrow are the model evaluations in the sigma neighbourhood and the blue arrows are the real fitness evaluations on the best points selected by the model evolution

Looking at the results generated at figures 1 -6 we can say that the model-assisted algorithms helps the evolutionary algorithm by choosing the best point according to the approximated function. In Figure 4 however it can be seen that the fitness error fails to decrease the error after 10^{-15} probably because that's the minimum value that python can evaluate.

2.2 Algorithm 2

In algorithm 1 Model only finds the best point in the sigma neighbourhood which increases the convergence speed of the ES in early iterations where sigma is big (figure 5 and 6). However, after 200 iteration sigma is so small that having more approximated points can not make ES much faster than what it already is. Therefore, in the second algorithm in line 13 I added a sigma update function. In A2 algorithms ES evolves for a number of iteration on the approximated model before original fitness evaluation.

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1:  $N = \text{number of dimensions}$ 
2:  $\text{initialize}(\text{individual}, \sigma)$ 
3:  $\text{Model} \leftarrow (\text{RBFNetwork}(\text{Inputs} = N, \text{Hidden} = 10, \text{Output} = 1))$ 
4: for  $i \leq \text{Original generation}$  do
5:    $\text{individual2} \leftarrow \text{individual}$ 
6:   for  $j \leq \text{Model generation}$  do
7:      $\text{individual3} \leftarrow \text{individual2} + \sigma * I$ 
8:      $\text{fit2} \leftarrow \text{Model fitness}(\text{individual2})$ 
9:      $\text{fit3} \leftarrow \text{Model fitness}(\text{individual3})$ 
10:    if  $\text{fit3} \leq \text{fit2}$  then
11:       $\text{individual2} \leftarrow \text{individual3}$ 
12:    end if
13:     $\sigma \leftarrow \sigma * \exp^{\frac{1}{N}}((\text{New fitness} \leq \text{Best fitness}) - \frac{1}{5})$ 
14:  end for
15:   $\text{New fitness} = \text{original fitness}(\text{individual2})$ 
16:   $\text{print error} \leftarrow \frac{(\text{New fitness} - \text{Model}(\text{individual2}))}{\text{Original generation}}$ 
17:  if  $\text{New fitness} \leq \text{Best fitness}$  then
18:     $\text{individual2} \leftarrow \text{individual}$ 
19:     $\text{Best fitness} \leftarrow \text{New fitness}$ 
20:     $\text{trainingpop.append}(\text{individual}, \text{Best fitness})$ 
21:     $\text{Model.train}(\text{trainingpop})$ 
22:  end if
23: end for
24:  $\text{print fitness} \leftarrow \text{Best fitness}$ 

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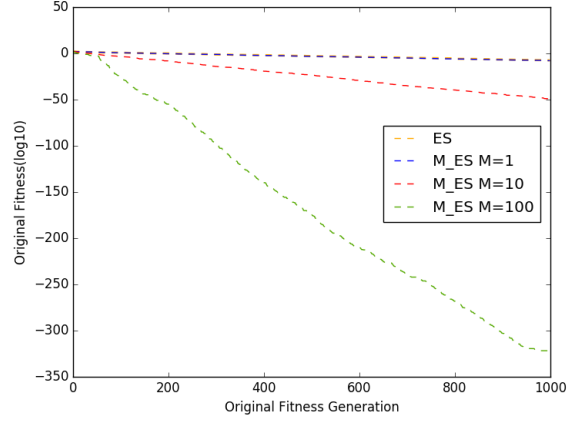


Figure 8: simple(1+1) ES fitness compared to M-ES(model assisted ES) using three different number of model generation(M) per original fitness evaluation on 10D $Y = (X)^2$ (5 trials)

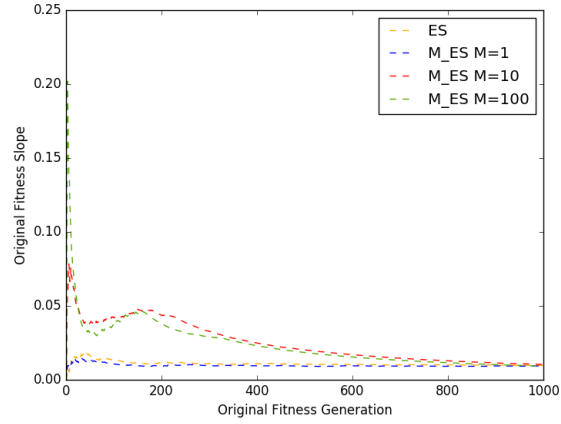


Figure 9: simple ES fitness compared to M-ES(model assisted ES) using three different number of model generation(M) per original fitness evaluation on 10D $Y = (X - 1)^2$ (5 trials)

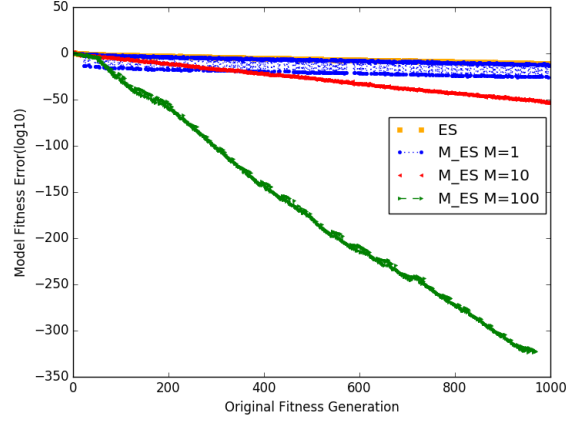


Figure 10: simple ES error compared to M-ES(model assisted ES) using three different number of model generation(M) per original fitness evaluation on 10D $Y = (X)^2$ (5 trials)

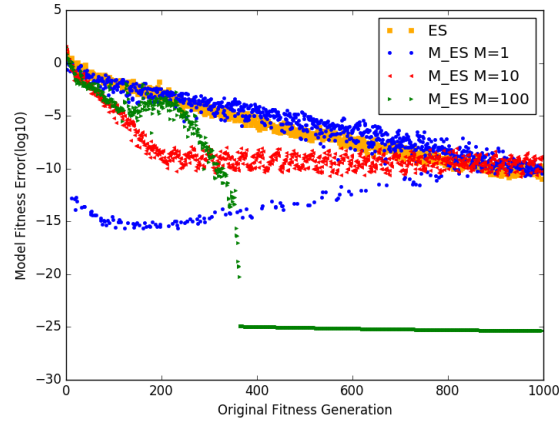


Figure 11: simple ES error compared to M-ES(model assisted ES) using three different number of model generation(M) per original fitness evaluation on 10D $Y = (X - 1)^2$ (5 trials)

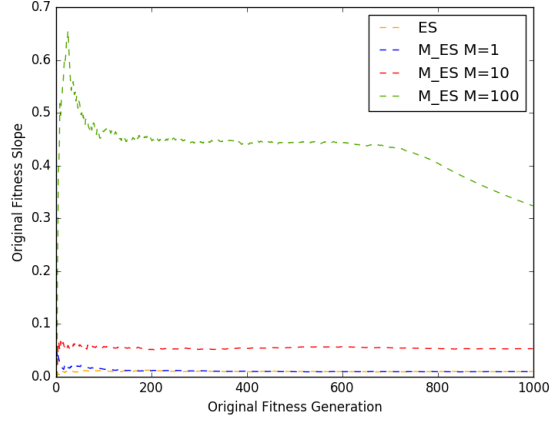


Figure 12: simple ES slope compared to M-ES(model assisted ES) using three different number of model generation(M) per original fitness evaluation on 10D $Y = (X)^2$ (5 trials)

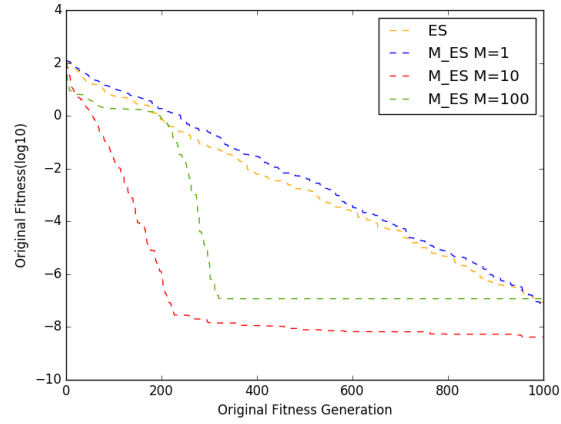


Figure 13: simple ES slope compared to M-ES(model assisted ES) using three different number of model generation(M) per original fitness evaluation on 10D $Y = (X - 1)^2$ (5 trials)

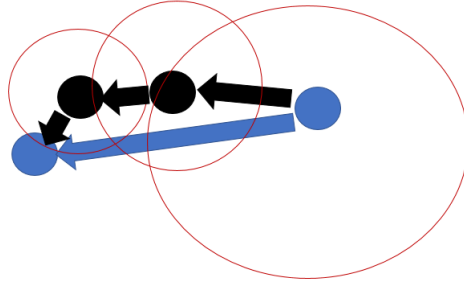


Figure 14: Shows how A2 algorithm works. Black arrow are the model evaluations in the sigma neighbourhood and the blue arrows are the real fitness evaluations on the best points selected by the model evolution

References

- [1] Ulmer, H., Streichert, F. and Zell, A., 2003, July. Model-assisted steady-state evolution strategies. In Genetic and Evolutionary Computation Conference (pp. 610-621). Springer Berlin Heidelberg.
- [2] Hansen, N., Arnold, D.V. and Auger, A., 2015. Evolution strategies. In Springer handbook of computational intelligence (pp. 871-898). Springer Berlin Heidelberg.