Computer Science 320 Midterm Test 2 NAME:____________________
Feb19 2015 Out of 48 marks

Recall that for a string w, $\#_\sigma(w)$ is the number of occurences of σ in w.

1. (2 mark) Either find, if they exist, a regular language L_1 and a context-free language L_2 whose intersection is regular, or explain why no such two languages exist.

$\exists$ many examples where the intersection is $\emptyset$; one example that results in an infinite but regular language $\neq L_1$ and $\neq L_2$:

$L_1 = L(ab^*c^*)$, $L_2 = \{a^i b^j c^k : i=j \text{ or } i=k\}$.

$L_1 \cap L_2 = L(abc^* + ab^*c)$

2. (4 marks) True or False:

   (a) T The union of a (possibly infinite) number of regular languages can be non-regular
   (b) T If L_1 is regular and L_2 is context-free, then $L_1 \cup L_2$ is also context-free.
   (c) T Given a regular grammar G, each string in $L(G)$ has a unique derivation in G.
   (d) T If L is context-free, then so is L^*. we did not cover this

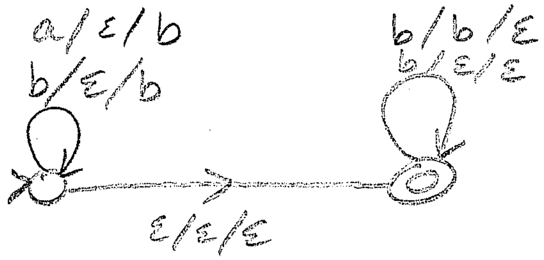
3. (6 marks) Give an algorithm to determine if a given Context-Free Grammar G is *usable*. Recall that a grammar is usable if there exists at least one string of terminals that can be derived from the start symbol of the grammar. In your answer, let the grammer be $G = (V, \Sigma, R, S)$, where
S is the start symbol,
Σ is the alphabet of terminals,
V is set of variables, and
R is the sets of production rules.
You may refer to the *left side* of a rule, which consists of a single variable, and the *right side* of a rule, which consists of a string from $(V \cup \Sigma)^*$.

```
Algorithm Usable (G)
{
  UsableVariables = ∅;   grows = True;
  while grows {
     grows = False;
     for each rule in R of form X -> α
            if X ∉ UsableVariables
                if α ∈ (Σ ∪ UsableVariables)*
                  { add X to UsableVariables;
                    grows = TRUE
                  }
  }
  if S ∈ UsableVariables return TRUE else return False.
```

4. (a) (0 marks) [Excercise: Prove using the Pumping Lemma that the language L is not regular, where]
$L = \{w \in \{a, b\}^* :$ the latter half of the string contains only b's$\}$. An example of a string in the language is *abaababbbbbbb*. In other words, the last a appears in the first half of the string. In odd strings, the middle symbol must be b.

(b) (4 marks) Give a PDA for the language given in part (a).

- push a "b" for every symbol in first half
- non-det. decide when at (or before) the middle.
- pop-and-match b's till end of string.



(c) (4 marks) Give a Context-Free Grammar for the language given above.

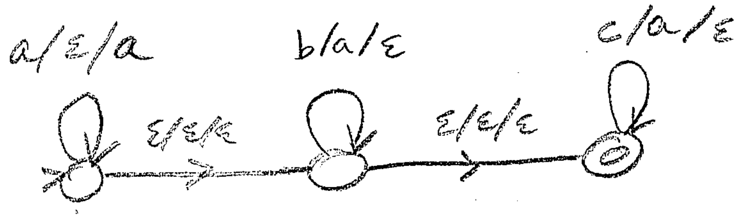
$S \rightarrow aSb \mid bSb \mid b \mid \varepsilon$

5. (4 marks) Give a Context-Free Grammar for the following: $\{a^{i+j}b^ic^j : i, j \geq 0\}$

$S \rightarrow aSc \mid X$

$X \rightarrow aXb \mid \varepsilon$

6. (4 marks) Give a PDA for the above language.



7. (4 marks) Give a Context-Free Grammar for the following: $\{a^n b^m : 2n = 5m - 4$, and $n, m \geq 0\}$

$\boxed{S \to aaaaaSbb \mid aaabb}$

m is even.

$2n \equiv 1 \bmod 5$

$\therefore n \equiv 3 \bmod 5$

base case

Subsequent rows grow by 5 a's, 2 b's.

n	$m = \frac{2n+4}{5}$
3	2
8	4
13	6
18	8

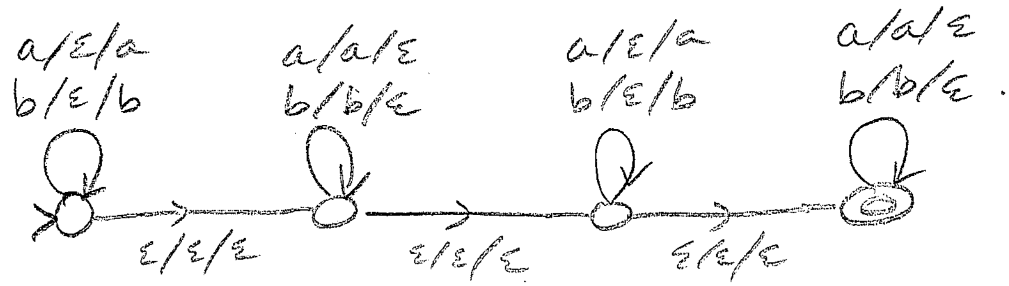
5, 2

8. (4 marks) Give a Context-Free Grammar for the following: $\{w \in \{a, b\}^* : w = w_1 w_2$, where w_1 and w_2 are palindromes$\}$

$S \to XX$

$X \to aXa \mid bXb \mid a \mid b \mid \varepsilon$

9. (4 marks) Give a PDA for the above language.



10. (4 marks) Give a Context-Free Grammar for the following: $\{w \in \{(,)\}^* : \#_{(}(w) = \#_{)}(w)$ and every suffix of w has at least as many ('s as)'s$\}$

$$S \to SS \mid)S(\mid \varepsilon$$

[observe: in a "normal" balanced string of parens, every suffix must have at least as many ")" as "(". Hence in this question, "(" and ")" have been swapped]

11. (4 marks) Give a PDA for the above language.