Next

- Chapter 2:
  - Pushdown Automata

More examples of CFLs

- $L(G) = \{0^n 1^{2n} \mid n = 1,2,\ldots\}$
- $L(G) = \{xx^R \mid x \text{ is a string over } \{a,b\}\}$
- $L(G) = \{x \mid x$ is a string over $\{1,0\}$ with an equal number of 1's and 0's$\}$

Next: Pushdown automata (PDA)

Add a stack to a Finite Automaton

- Can serve as type of memory or counter
- More powerful than Finite Automata
- Accepts Context-Free Languages (CFLs)
- Unlike FAs, nondeterminism **makes a difference** for PDAs. We will only study non-deterministic PDAs and omit treatment of Det-PDAs.

Pushdown Automata

Pushdown automata are for context-free languages what finite automata are for regular languages.

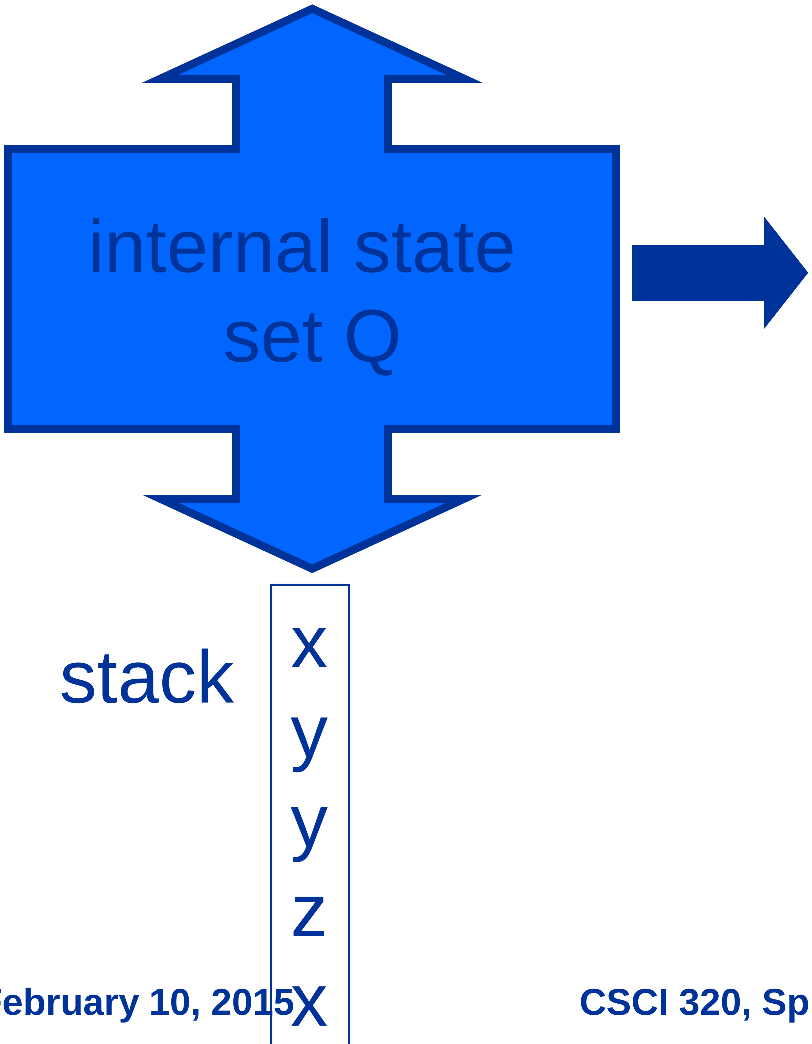
PDAs are *recognizing automata* that have a single stack (= memory):

Last-In First-Out *pushing* and *popping*

Non-deterministic PDAs can make non-deterministic choices (like NFA) to find accepting paths of computation.

Informal Description PDA (1)

input w = 00100100111100101



The PDA M reads w and stack element.

<u>Depending on</u>

- input $w_i \in \Sigma_\varepsilon$,
- stack $s_j \in \Gamma_\varepsilon$, and
- state $q_k \in Q$

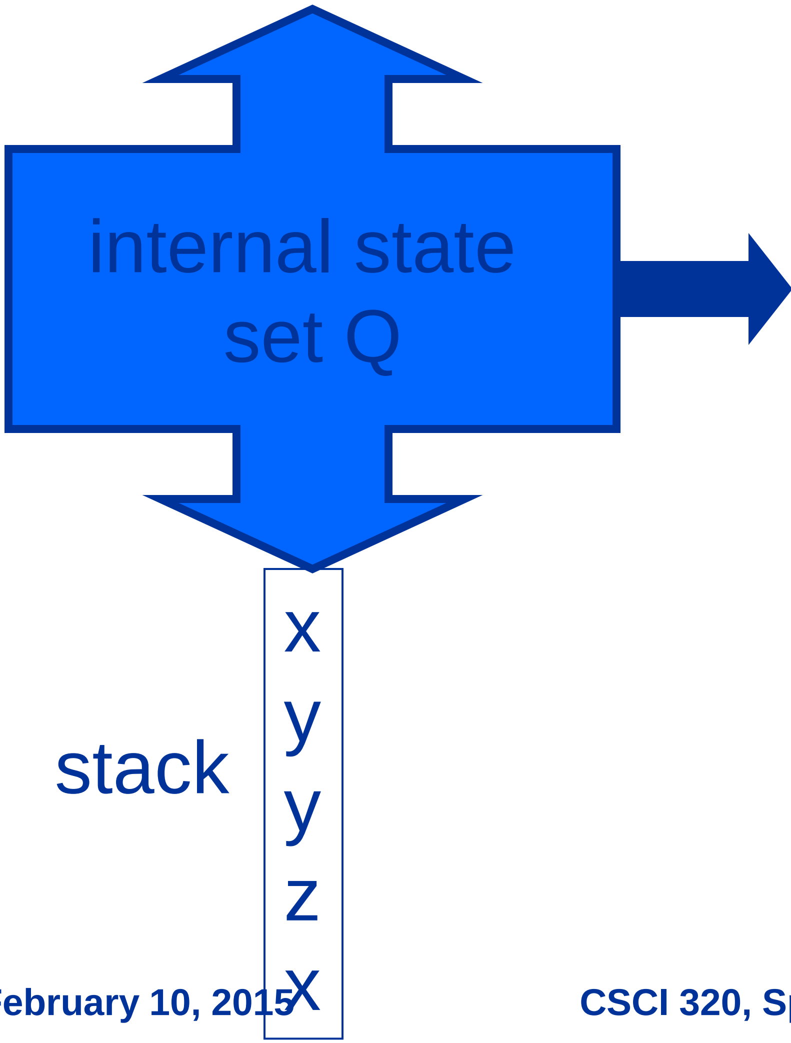
<u>the PDA M</u>:

- jumps to a new state,
- pushes an element Γ_ε

(nondeterministically)

Informal Description PDA (2)

input w = 00100100111100101



After the PDA has read complete input, M will be in state $\in$ Q

If possible to end in accepting state $\in F \subseteq Q$, with an empty stack then M accepts w

Formal Description of a PDA

A Pushdown Automata M is defined by a six tuple $(Q,\Sigma,\Gamma,\delta,q_0,F)$, with

- Q finite set of states
- Σ finite input alphabet
- Γ finite stack alphabet
- q_0 start state $\in$ Q
- F accepting states $\subseteq$Q
- Δ transition relation

$$\Delta: (Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon) \text{ x } (Q \times \Gamma_\varepsilon)$$

- where Σ_ε is Σ U {ε} (and Γ_ε is Γ U {ε})

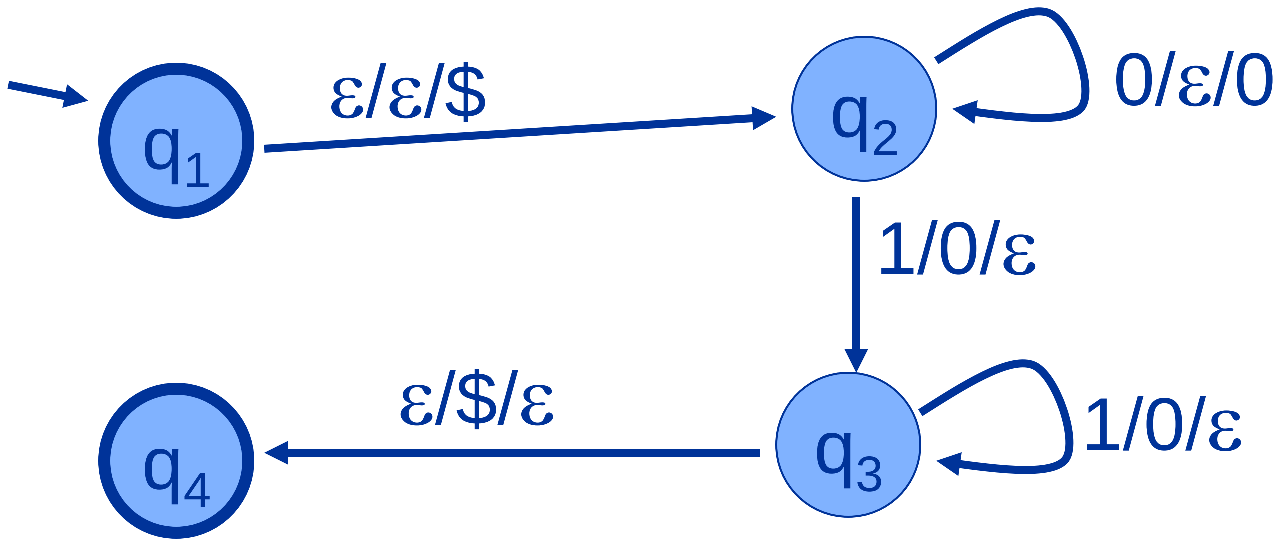
PDA for L = { $0^n1^n \mid n \geq 0$ }

<u>Example 2.9</u>:

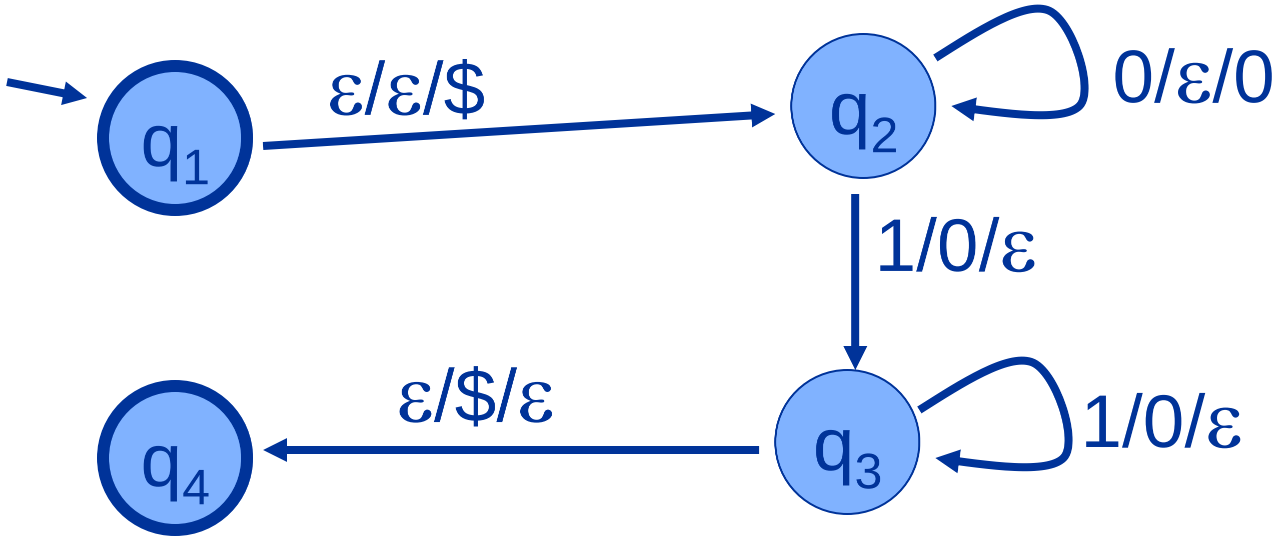
The PDA first pushes “ \$ 0^n ” on stack.

Then, while reading the 1^n string, the zeros are popped again.

If, in the end, \$ is left on stack, then “accept”



Machine Diagram for 0^n1^n

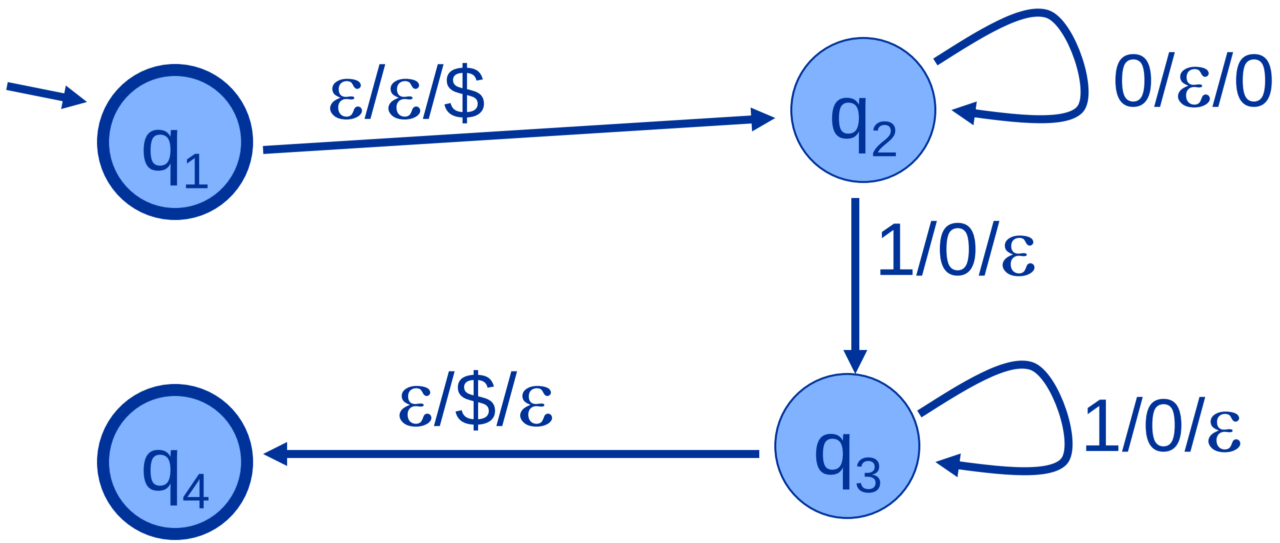


On w = 000111 computation:

$(q_1,000111,\varepsilon) \vdash (q_2,000111\$) \vdash (q_2,00111,0\$)$
$\vdash (q_2,0111,00\$) \vdash (q_2,111,000\$) \vdash (q_3,11,00\$)$
$\vdash (q_3,1,0\$) \vdash (q_3,\varepsilon,\$) \vdash (q_4,\varepsilon,\varepsilon)$

This final q_4 is an accepting state

Machine Diagram for 0^n1^n



On w = 0101 (state; stack) evolution:

$(q_1, 0101, \varepsilon) \vdash (q_2, 0101, \$) \vdash (q_2, 101, 0\$) \vdash (q_3, 01, \$) \vdash (q_4, 01, \varepsilon)$ …

But we still have part of input "01".

There is no accepting path.

An important example

- $L = \{a^i b^j a^k \mid i=j \text{ or } i=k\}$
- (Example 2.16, p 115. 3rd ed Sipser)

- Try $L = \{ww^R \mid w \text{ is any binary string}\}$

PDAs and CFL

Theorem 2.20 (2.12 in 2nd Ed Sipser):
A language L is context-free if and only if there is a pushdown automata M that recognizes L.

Two step proof:
1) Given a CFG G, construct a PDA M_G
2) Given a PDA M, make a CFG G_M

Converting a CFL to a PDA

- Lemma 2.21 in 3rd Ed
- The PDA should simulate the derivation of a word in the CFG and accept if there is a derivation.
- Need to store intermediate strings of terminals and variables. How?

Idea

- Store only a suffix of the string of terminals and variables derived at the moment starting with the first variable.
- The prefix of terminals up to but not including the first variable is checked against the input.
- A 3 state PDA is enough p 120 3rd Ed.

Converting a PDA to a CFG

- Lemma 2.27 in 3rd Ed
- Design a grammar equivalent to a PDA
- Idea: For each pair of states p,q we have a variable A_{pq} that generates all strings that take the automaton from p to q (empty stack to empty stack).

Some details

Assume

- Single accept state
- Stack emptied before accepting
- Each transition either pops or pushes a symbol

- Can create rules for all the possible cases (p 122 in 3rd Ed)