1 Magnetic Dipole

A magnetic dipole is a type of a dipole. A dipole is an idealized system that is used for an approximate description of a field created by more complex system of charges. The types of dipoles include:

- Electric
- Magnetic

A magnetic dipole is equivalent to the flow of current around a loop. It is the simplest structure in magnetism. If you take a bar magnet and cut off the south pole, it will still show to have one polar to the north pole. This can continue upto atomic level and even then magnetism is only occurent in the form of a dipole. The smallest system being that of an electron revolving around the nucleus, which acts analogically to current flowing in a circular wire. This would be an example of a microscopic magnetic dipole.
Many millions of iron atoms spontaneously locked into the same alignment to form a ferromagnetic domain also constitute a magnetic dipole. Magnetic compass needles and bar magnets are examples of macroscopic magnetic dipoles.

2 Magnetic Dipole Moment

Magnetic dipole moment is an inherent property of magnetic substances. Whenever a magnetic object is placed in a magnetic field, the field will interact with that object or we can say that it applies some $force$ on it. This exists in the form of torque $\vec{\tau}$. Hence for some magnetic with a magnetic dipole moment μ present in a magnetic field of intensity $\vec{B}$, the value of the torque applied on it by the field is given by:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

The magnetic dipole moment is also a vector quantity and it has the units $A\,m^2$.
Now before we go on the mathemtically describe it, we need to describe the Biot-Savart law briefly which will be important in deriving the value of μ later.

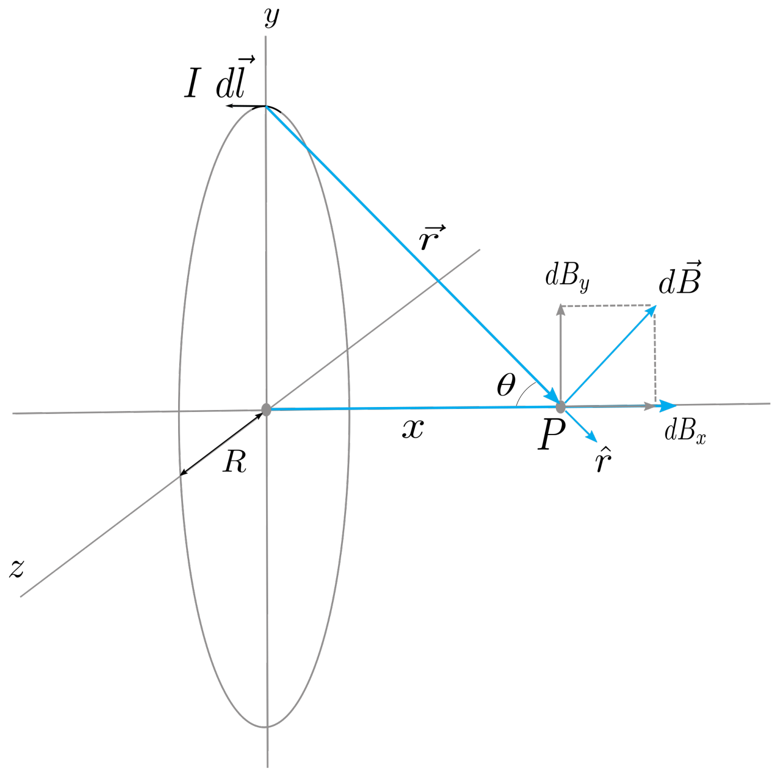
2.1 Biot-Savart Law

The magnetic field $d\vec{B}$ produced by a current element $I\vec{l}$ is given by:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} \tag{1}$$

This is known as the Biot-Savart law and it was deduced by Ampere. It is analogous to Coulomb's law for the electric field of a point charge.

2.2 Derivation of Magnetic Dipole Moment



(Figure 1)

For some charge $\vec{q}$ moving with velocity $\vec{v}$, the magnetic field $\vec{B}$ generated by it is given by:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2} \tag{2}$$

Here μ_0 is the permeability of free space.

Now Figure 1 shows a point P at a distance x on the axis of a circular current loop's center. At the top of the loop, we consider a current element Idl which at all times on the loop, remains tangent to the loop and perpendicular to the vector $\vec{r}$ from the current element to the field point P. The magnetic field due to this current element $d\vec{B}$ is given by:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I|d\vec{l} \times \hat{r}|}{r^2}$$

Here from the equation by pythagorean theorem, we know that $r^2 = x^2 + R^2$ and $|d\vec{l} \times \hat{r}| = dl$. So the above equation becomes:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Idl}{(x^2 + R^2)}$$

In order to find the magnetic field generated by the whole loop, we will need to integrate over the all the current elements in it. But based on symmetry, the current elements dB_y end up cancelling themselves and the net result from them is 0. Hence we only need to integrate over the x components of the field. The x component of the current element is given as follows:

$$dB_x = dB \sin\theta$$

From figure 1, $\sin\theta = \frac{R}{\sqrt{x^2+R^2}}$ which we put in the equation:

$$dB_x = \frac{\mu_0}{4\pi}\frac{Idl}{(x^2+R^2)}\ \frac{R}{\sqrt{x^2+R^2}}$$

$$dB_x = \frac{\mu_0}{4\pi}\frac{Idl\ R}{(x^2+R^2)^{\frac{3}{2}}}$$

Now for the field generated by the entire loop, we integrate it:

$$B_x = \oint\ dB_x = \oint\ \frac{\mu_0}{4\pi}\frac{I\ R}{(x^2+R^2)^{\frac{3}{2}}}dl$$

Now x and R remain the same throughout the loop so the above integral simplifies to:

$$B_x = \oint\ dB_x = \frac{\mu_0}{4\pi}\frac{I\ R}{(x^2+R^2)^{\frac{3}{2}}}\oint\ dl$$

Integrating over dl gives us the circumference of the loop, or $2\pi R$, hence we have:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi\ R^2 I}{(x^2+R^2)^{\frac{3}{2}}}$$

If our point P is at a distance much greater than R or $x >> R$, then the above equation simplifies further as follows:

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi\ R^2 I}{(x^2(1+\frac{R^2}{x^2}))^{\frac{3}{2}}}$$

$$B_x = \frac{\mu_0}{4\pi}\frac{2\pi\ R^2 I}{x^3}$$

$$\boxed{B_x = \frac{\mu_0}{4\pi}\frac{2\mu}{x^3}} \tag{3}$$

Here μ is the magnetic dipole moment for the loop and it is equal to $I\pi R^2$. Hence we have derived an expression that would allow us to calculate the magnetic dipole moment of any loop present in some magnetic field $\vec{B}$.

3 Applications

3.1 Nuclear magnetic resonance

All isotopes that contain an odd number of protons or neutrons have an intrinsic nuclear magnetic moment and angular momentum. When placed in a strong magnetic field, these weak oscillating magnetic fields cause a perturbation which can be detected. This results due to the magnetic properties of certain nuclei. NMR is used widely to determine the structure of organic molecules. The technique has a vast amount of applications in many fields. MRI is also an important medical application of NMR.

3.2 Magnetic Resonance Imaging

Before the 21st century, internal medical imaging was nearly impossible. The modern ways included the use of X rays which is non invasive enough but it is also a potential biohazard and can only be used on a patient for a limited amount of times due to it. MRI on the other hand, does not use any type of ionizing radiation.

It uses strong magnetic fields to form images of the internal organs of the body non invasively. In most medical applications, hydrogen nuclei, which consist solely of a proton, that are in tissues create a signal that is processed to form an image of the body in terms of the density of those nuclei in a specific region. Given that the protons are affected by fields from other atoms to which they are bonded, it is possible to separate responses from hydrogen in specific compounds.

The major components of an MRI scanner are the main magnet, which polarizes the sample, the shim coils for correcting shifts in the homogeneity of the main magnetic field, the gradient system which is used to localize the region to be scanned and the RF system, which excites the sample and detects the resulting NMR signal. The whole system is controlled by one or more computers.