Lecture 2

These notes cover:

- Graphing in polar coordinates
- Symmetry
- A review of slope in polar coordinates

Graphing in polar coordinates:

The easiest way to graph in polar coordinates is to plot the values on a cartesian graph after creating a table and then to translate these values on a polar graph. A polar graph is distinct from a cartesian one in the sense that it can be thought of as a graph of circles ever incrementing. The way you vary in them is via the angle. Or you could consider θ as a function of r or $\theta = f(r)$.

The following figure shows a polar graph. Notice that you can take a vector of magnitude 1 and rotate it through an angle of 2π to achieve a full circle on it. Changing the magnitude will result in a spiral.

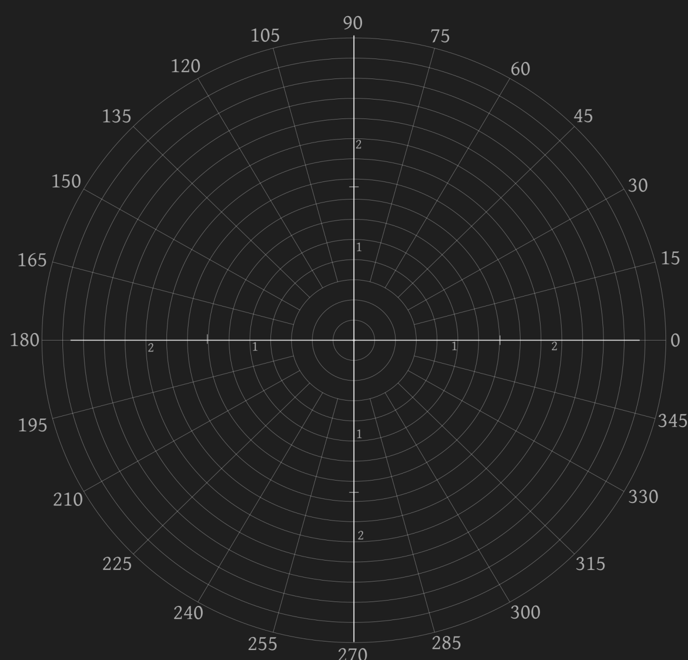


Figure 1: Polar Graph

Before discussing the symmetries of the graphs, it is important to understand that the graph itself would reveal moreabout it's symmetry than algebraic analysis.
However the functions can be analyzed in alongside the cartesian plotting in order to figure out if any symmetries exist which often aid in plotting the graph. For example if a function is symmetric about the y axis then we can determine the values of r w.r.t θ from 0 to $\frac{\pi}{2}$ and then just reflect that on the other side of the axis.
It is helpful to draw a unit circle before attempting any of these questions.
But before studying symmetries we will look at the general process of plotting a polar graph.

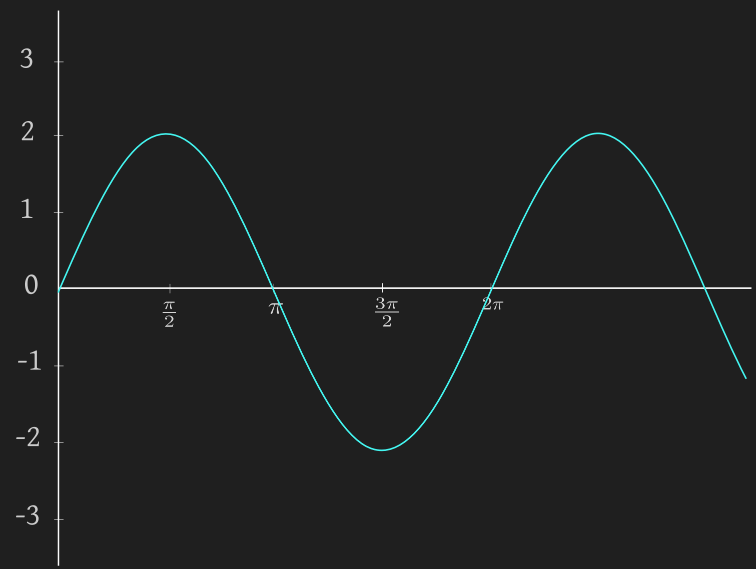
Example: Plot the graph of $2\sin\theta$ in polar coordinates.

Now we first write our function as:

$$r(\theta) = 2\sin\theta$$

Now we choose some increment value for θ and then make a table of corresponding values of r for that θ. Then we plot that table on a cartesian graph which gives us a general idea about how that graph behaves.
Keep the calculator in degree mode while creating the values for cartesian table.
Then we take that cartesian graph and plot the on a polar graph using the angle to rotate around a circle. As angle goes from 0 to $\frac{\pi}{2}$ then value of r goes from 0 to 2.

θ	r
0	0
$\frac{\pi}{2}$	2
π	0
$\frac{3\pi}{2}$	-2
2π	0



Using this graph as an aid, we can plot the polar graph. It is visible here that as θ goes from 0 to 90, the value goes to 2, then θ going from 90 to 180, the value drops to 0. On a polar graph this will create a circle. Then the value changes to negative so as the angle goes from 180 to 270 then value goes to 2 but in the opposite direction. Hence this value will again be plotted in the first quadrant.

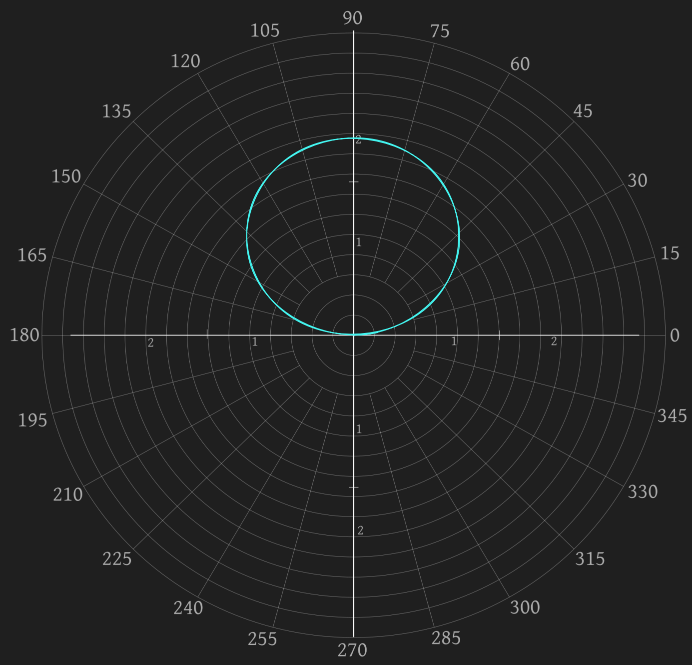


Figure 2: Example 1

Symmetry:

About X axis: If the point (r, θ) lies on the graph, then $(r, -\theta)$ or $(-r, \pi - \theta)$ will also lie on the graph.

Note here that $(\pi - \theta)$ and $(\theta - \pi)$ are not associative i.e. $(\pi - \theta) \neq (\theta - \pi)$.

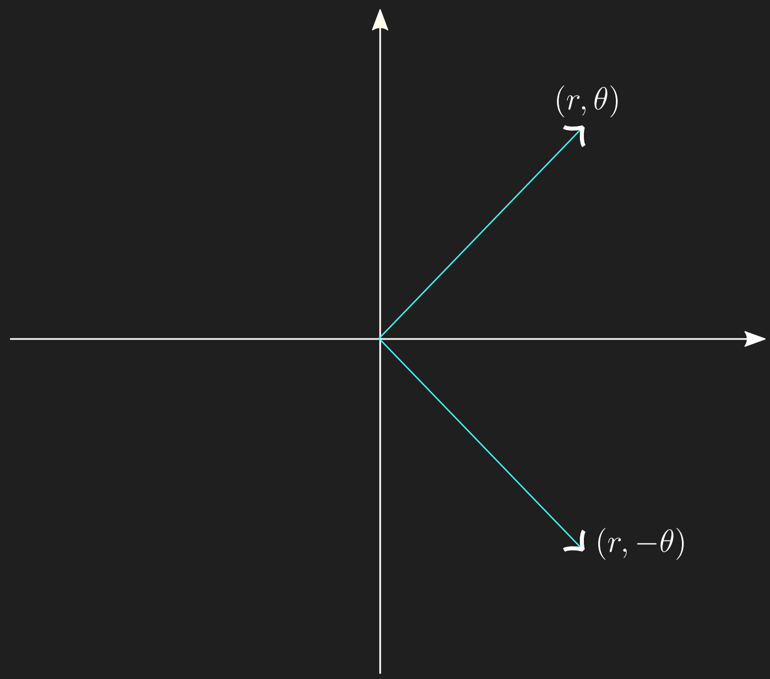


Figure 3

And for the second one we take r with the angle $\pi - \theta$ and then we put r negative to get the symmetry:

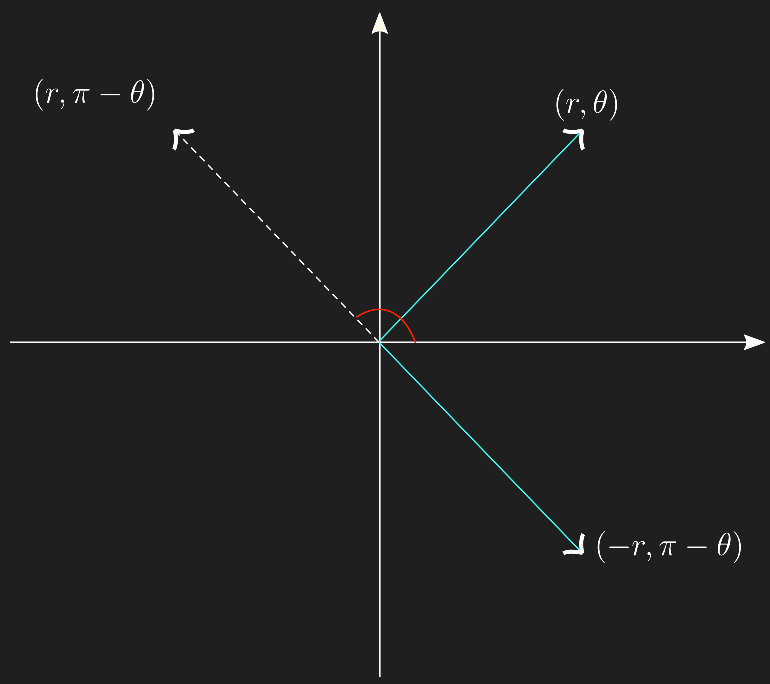


Figure 4

About y axis:
If the point (r, θ) lies on the graph, then $(r, \theta - \pi)$ or $(-r, -\theta)$ will also lie on the graph.

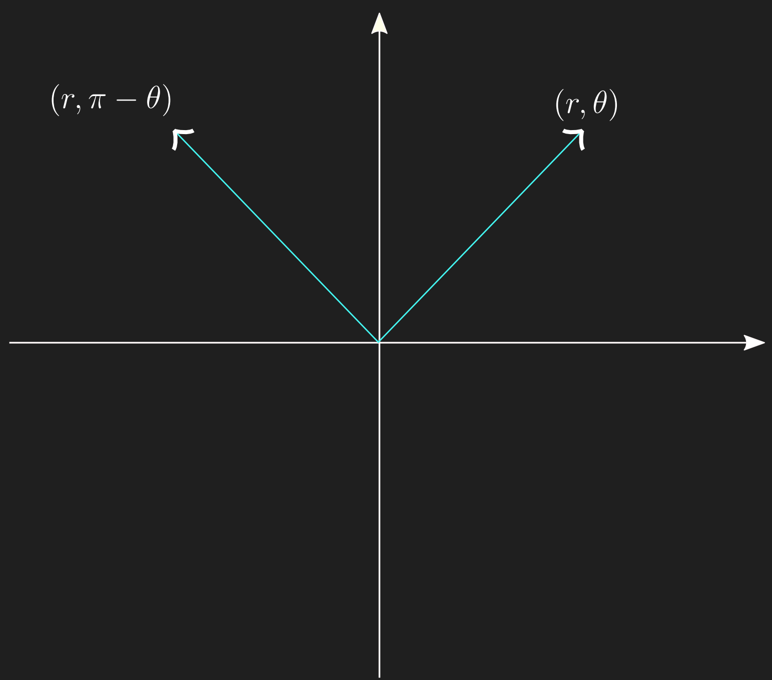


Figure 5

And for the second one, we take the angle to be $-\theta$ and then putting r to be $-r$ gives us the symmetry about y axis:

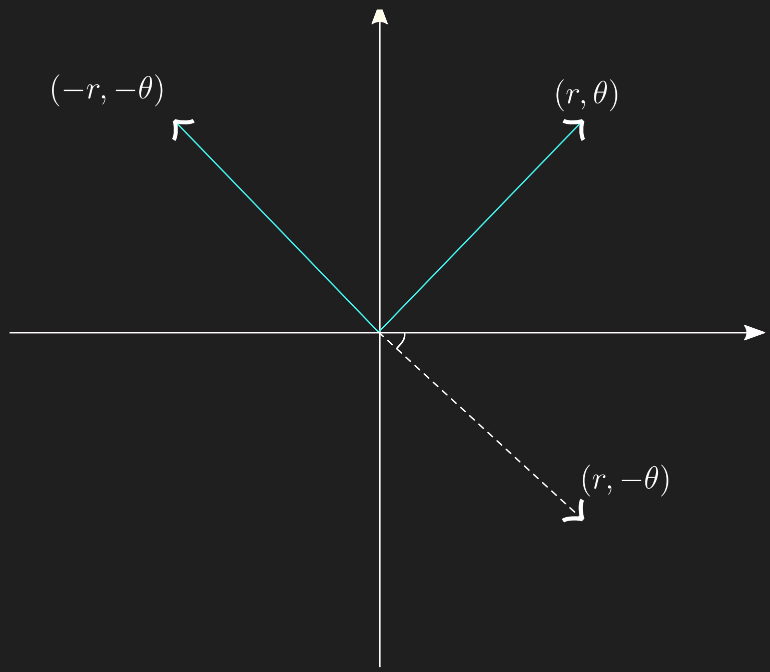


Figure 6

About the origin:
If the point (r, θ) lies on the graph, then $(-r, \theta)$ or $(r, \theta + \pi)$ will also lie on the graph.

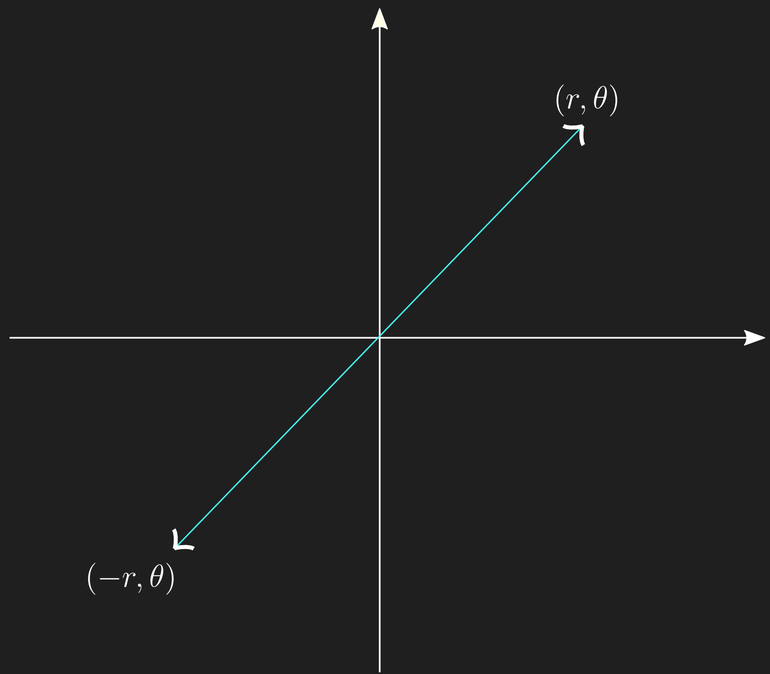


Figure 7

And for the second one, we take the same r and just add π to the angle which gives us the same line:

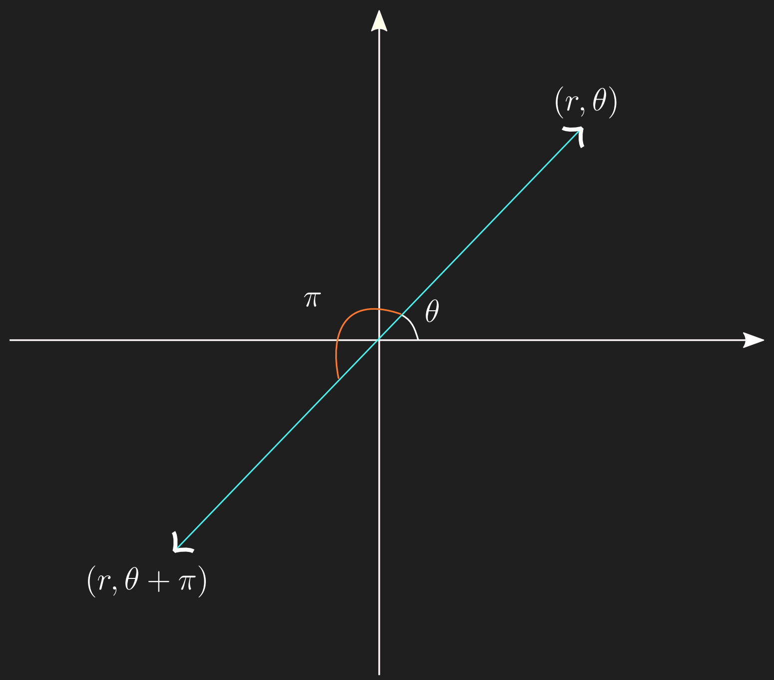


Figure 8

PS 10.6

1. $r = 1 + \cos\theta$

Symmetry about the x axis:

$$r = 1 + \cos\theta = 1 + \cos(-\theta) = 1 + \cos\theta \quad \checkmark$$

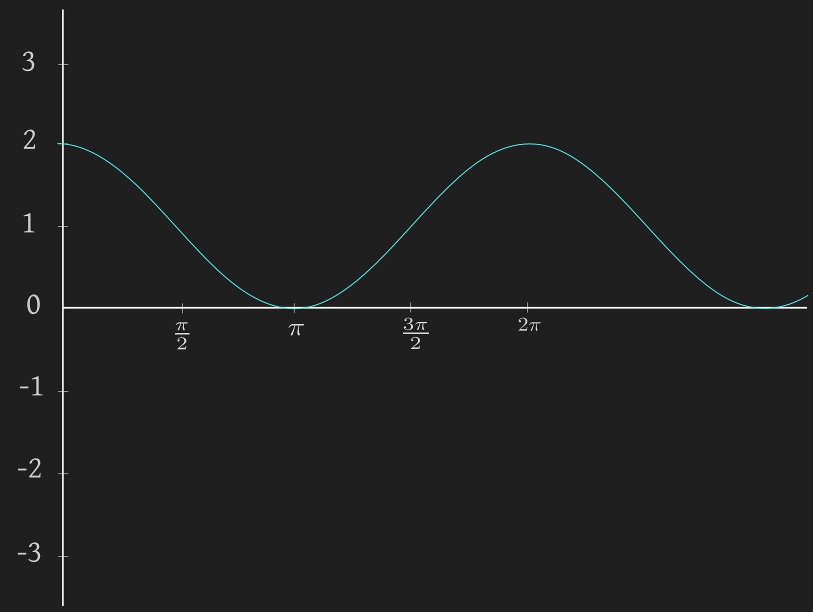
Symmetry about the y axis:

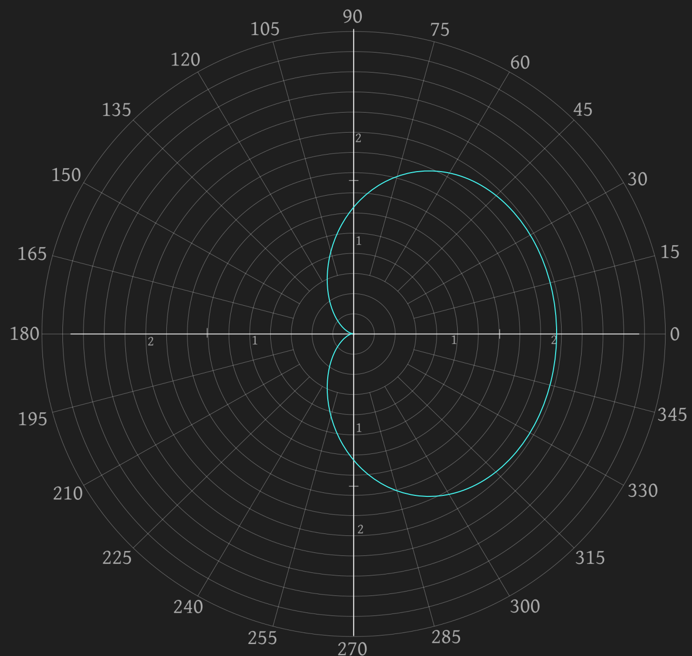
$$r = 1 + \cos(-\theta) \implies r = -1 - \cos\theta \quad ✗$$

Symmetry about the origin:

$$-r = 1 + \cos\theta \implies r = -1 - \cos\theta \quad ✗$$

θ	r
0	2
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	1
2π	2


3
2
1
0
-1
-2
-3
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π


90
75
60
45
30
15
0
345
330
315
300
285
270
255
240
225
210
195
180
165
150
135
120
105

2. $r = 2 - 2\cos\theta$

θ	r
0	0
$\frac{\pi}{2}$	2
π	4
$\frac{3\pi}{2}$	2
2π	0

Symmetry about the x axis:

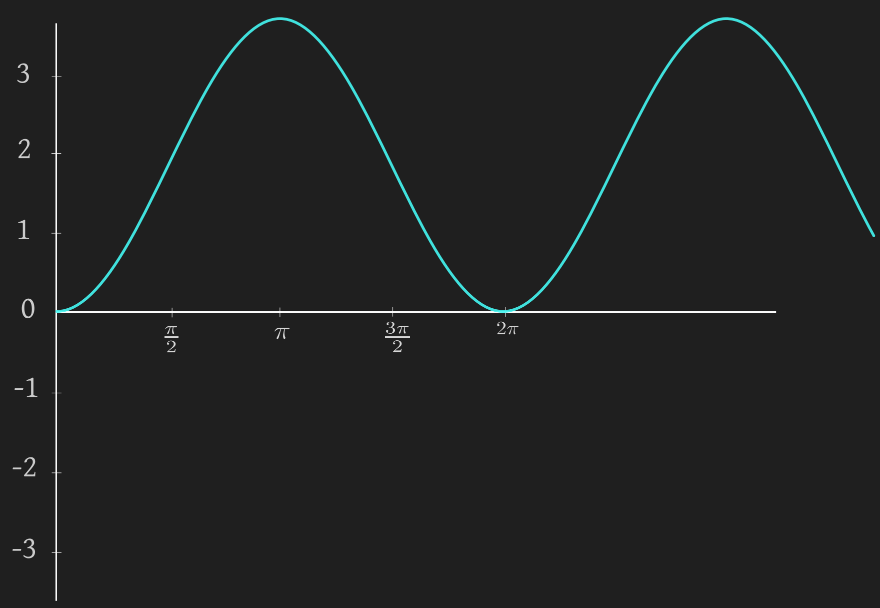
$$r = 2 - 2\cos(-\theta) = 2 - 2\cos\theta \quad \checkmark$$

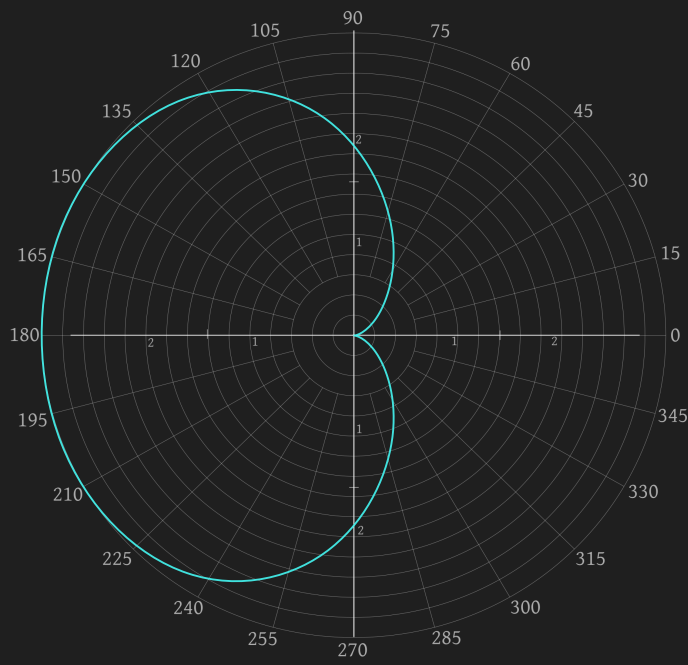
Symmetry about the y axis:

$$-r = 2 - 2\cos(-\theta) \implies r = -2 + 2\cos\theta \quad ✗$$

Symmetry about the origin:

$$-r = 2 - 2\cos\theta \implies r = -2 + 2\cos\theta \quad ✗$$





3. $r = 1 + \sin\theta$

Symmetry about the x axis:

$$r = 1 - \sin(-\theta) = 1 + \sin\theta \quad ✗$$

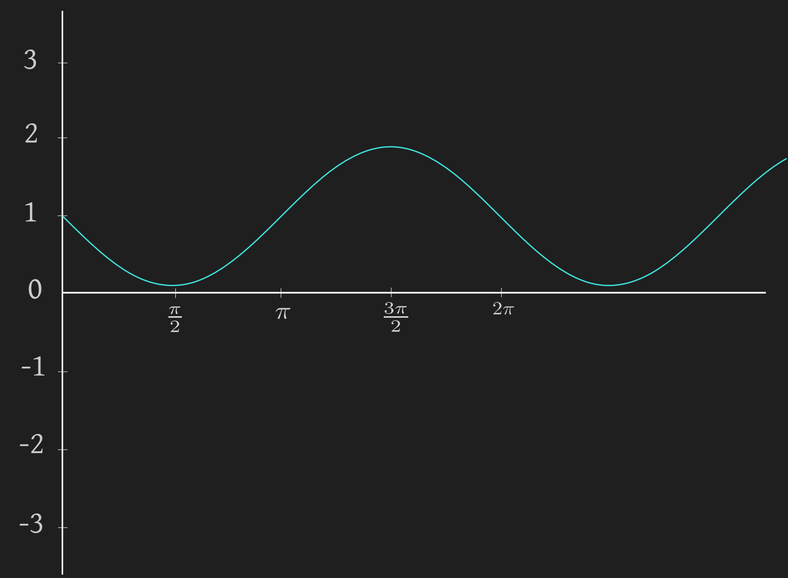
Symmetry about the y axis:

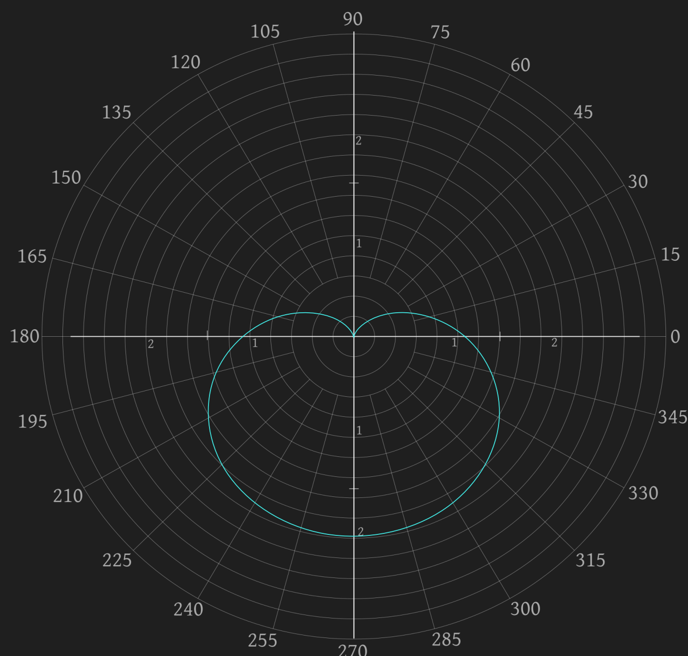
$$r = 1 - \sin(\pi - \theta) = 1 - \sin\theta \quad ✓$$

Symmetry about the origin:

$$-r = 1 - \sin\theta \implies r = -1 + \sin\theta \quad ✗$$

θ	r
0	1
$\frac{\pi}{2}$	0
π	1
$\frac{3\pi}{2}$	2
2π	1





Question 4 and 5 are similar to the ones above.

6. $r = 1 + 2\sin\theta$

Symmetry about the x axis:

$$r = 1 + 2\sin(-\theta) = 1 - 2\sin\theta \quad ✗$$

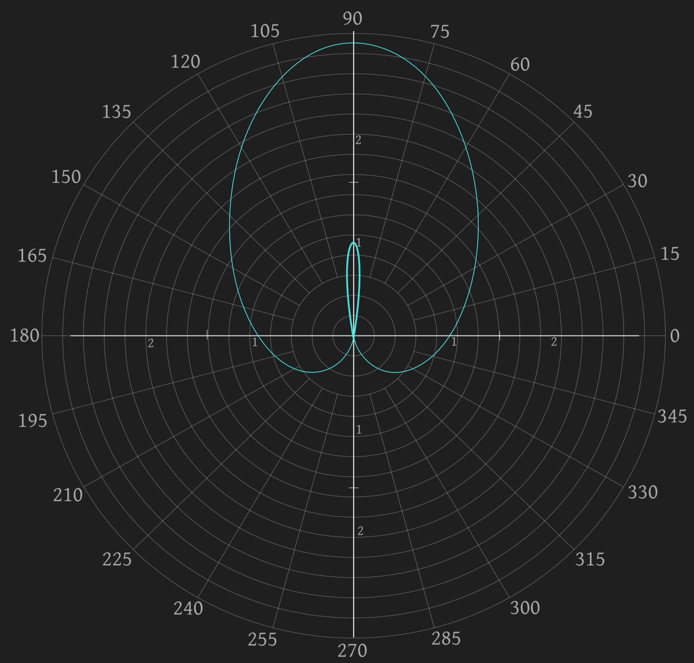
Symmetry about the y axis:

$$r = 1 + 2\sin(\pi - \theta) = 1 + 2\sin\theta \quad ✓$$

Symmetry about the origin:

$$-r = 1 + 2\sin\theta \implies r = -1 - 2\sin\theta \quad ✗$$

θ	r
0	1
$\frac{\pi}{2}$	3
π	1
$\frac{3\pi}{2}$	-1
2π	1



Here after angle π it is supposed to go to 1 on the mid of the 3 and 4 quadrant but since value is negative it goes into 1 on the 1 and 2 quadrant.

7. $r = \sin\frac{\theta}{2}$

Symmetry about the x axis:

$$r = \sin\frac{-\theta}{2} = -\sin\frac{\theta}{2} \quad ✗$$

Symmetry about the y axis:

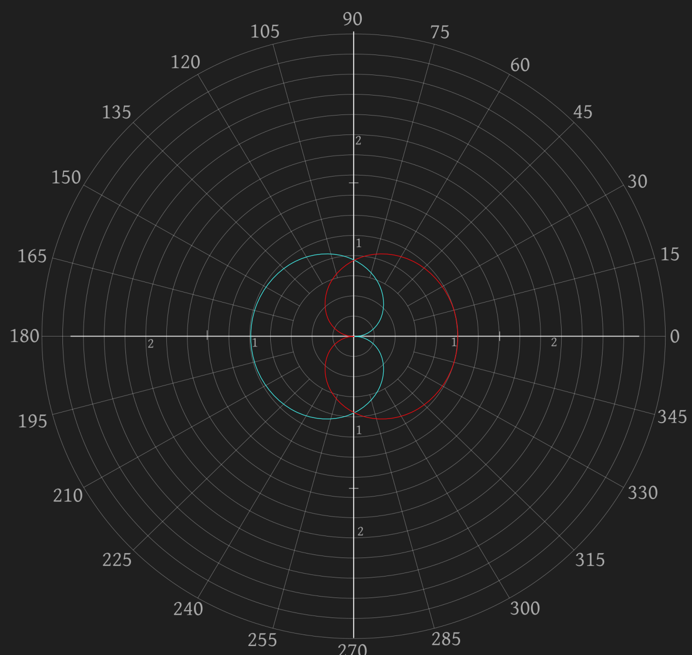
$$-r = \sin\frac{-\theta}{2} \implies r = \sin\frac{\theta}{2} \quad \checkmark$$

Symmetry about the origin:

$$-r = \sin\frac{\theta}{2} \quad ✗$$

θ	r
0	0
$\frac{\pi}{2}$	0.7071
π	1
$\frac{3\pi}{2}$	0.7071
2π	0
$\frac{5\pi}{2}$	-0.7071
3π	-1
$\frac{7\pi}{2}$	-0.7071
4π	0

Now in this graph, we plot only from 0 to 2π and then make sure of symmetry around the y axis to reflect the graph. The reflected graph is in red.



In fractional functions, it is preferred to not use the $\pi - \theta$ value and to instead use e.g $-r = -\theta$ for y axis. Question 8 is similar to this one.

9. $r^2 = \cos\theta$

The way to solve this one is to split the function into two, because to graph it, we need r to be linear.

$$r^2 = \cos\theta$$

$$r = \pm\sqrt{\cos\theta}$$
$$r = \sqrt{\cos\theta} \quad \text{and} \quad r = -\sqrt{\cos\theta}$$

For $r = \sqrt{\cos\theta}$

Symmetry about the x axis:

$$r = \sqrt{\cos(-\theta)} = \sqrt{\cos\theta} \quad \checkmark$$

Symmetry about the y axis:

$$-r = \sqrt{\cos(-\theta)} \implies r = -\sqrt{\cos\theta} \quad ✗$$

Symmetry about the origin:

$$-r = \sqrt{\cos\theta} \implies r = -\sqrt{\cos\theta} \quad ✗$$

For $r = -\sqrt{\cos\theta}$

Symmetry about the x axis:

$$r = -\sqrt{\cos(-\theta)} = -\sqrt{\cos\theta} \quad \checkmark$$

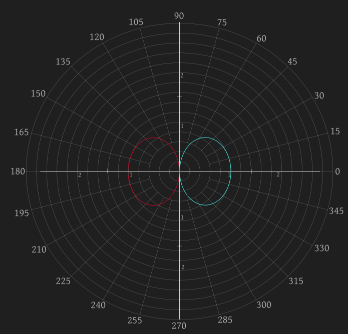
Symmetry about the y axis:

$$-r = -\sqrt{\cos(-\theta)} \implies r = \sqrt{\cos\theta} \quad ✗$$

Symmetry about the origin:

$$-r = -\sqrt{\cos\theta} \implies r = \sqrt{\cos\theta} \quad ✗$$

But for the equation $r^2 = \cos\theta$ we have symmetry about x, y and origin. This happens when we combine the graphs.



θ	$r = \sqrt{cos\theta}$
0	1
$\frac{\pi}{2}$	0
π	Undefined
$\frac{3\pi}{2}$	0
2π	1

Here the entire second quadrant is undefined.

Here the red circle shows the graph of $r = -\sqrt{\cos\theta}$. In the combined graph, symmetry exists about x, y and origin.