THE UNIVERSITY OF MANITOBA

OCTOBER 13, 6:00-7:30 PM 2005 MID-TERM EXAMINATION
TIME: 1.5 HOURS PAGE NO: 1 OF 6
DEPARTMENT & COURSE NO.: 24.426
EXAMINATION: Communication Systems EXAMINER: P. Yahampath

Closed-book exam: No printed or hand-written material allowed.
A calculator may be used.
Answer all 4 questions.

5 Marks 1. Let $v(t)$ be a voice signal with a bandwidth of 4 kHz.

Consider the modulator shown below. Assume that $f_c = 100$ MHz.

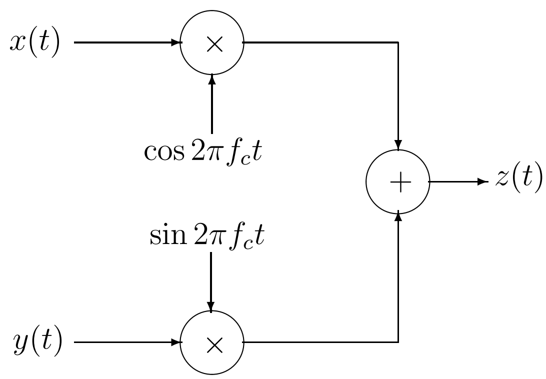

Figure 1:

(a) If you set the two modulator inputs as $x(t) = v(t)$ and $y(t) = v^2(t)$, what is the bandwidth of the modulator output $z(t)$?

(b) Let $g(t)$ be the signal obtained by filtering the voice signal $v(t)$ using the filter shown below.

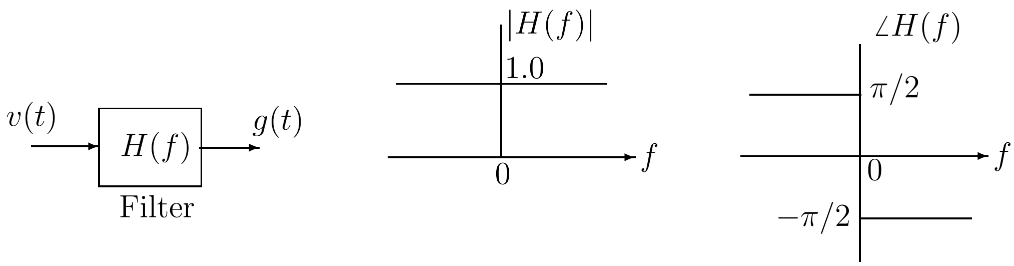

Figure 2:

If you set the two modulator inputs as $x(t) = v(t)$ and $y(t) = g(t)$, what is the bandwidth of the modulator output ?

5.5 Marks

2. A periodic signal $m(t)$ is shown below.

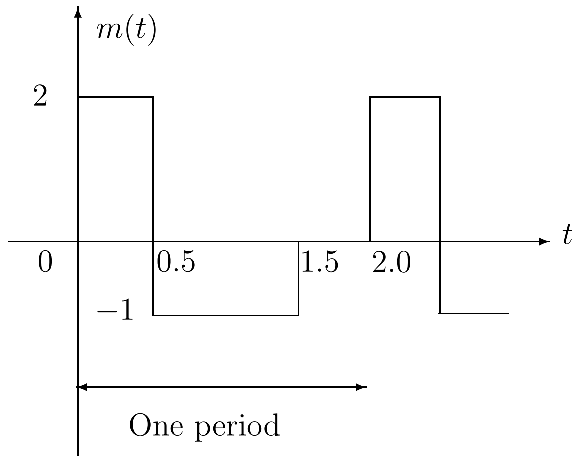

Figure 3:

(a) Suppose $m(t)$ is used as the message signal in a conventional amplitude modulator. If the modulation index is set to 100%, find the power efficiency of the modulator.

(b) If $-m(t)$ used as the message, instead of $m(t)$, will the power efficiency be lower or higher ? Explain very briefly, without any new calculations.

Note: The following relation may be useful:

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)\cos(2\pi f_c t)dt \simeq 0$$

when frequencies in $x(t)$ are much smaller compared to f_c.

6 Marks

3. Here is an experiment that a student did with a frequency modulator:

   - The output signal of the modulator with the message input grounded (0 V applied) was observed to be $5\cos(1000 \times 10^3 \pi t)$ V.
   - When the signal $4\cos(20 \times 10^3 \pi t)$ V is applied to the message input, the modulator output was observed to be $5\cos[1000 \times 10^3 \pi t + 2\sin(20 \times 10^3 \pi t)]$ V.

   Give a reasonably accurate plot of the modulator output frequency spectrum (both magnitude and phase) when the message input is $2.5\cos(50 \times 10^3 \pi t)$ V.

1 Mark

4. The following signal is applied to the message input of a frequency modulator having a sensitivity of 1 kHz/V.

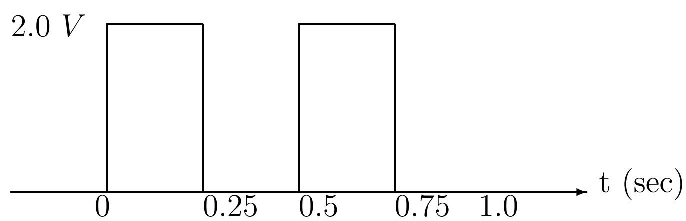

Figure 4:

The modulator output is then passed through a circuit that has been designed to demodulate phase modulated signals. This circuit has a sensitivity of $\frac{1}{2\pi}$ volts per radian of angle deviation in the input signal.

Plot the output waveform of the demodulator circuit for the interval 0-1 sec.

Useful Formulas

- $s_{DSB}(t) = A_c m(t) \cos 2\pi f_c t$
- $s_{USB}(t) = \frac{A_c}{2}[m(t) \cos 2\pi f_c t - \hat{m}(t) \sin 2\pi f_c t]$ where $\hat{m}(t)$ is the Hilbert transform of $m(t)$
- Tone modulated FM signal ($m(t) = A_m \cos 2\pi f_m t$)

$$s_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t]$$

  where β is the maximum angle deviation (modulation index) of the carrier.

- Fourier series of a periodic unipolar square wave

$$s(t) = \frac{1}{2} + \frac{2}{\pi}\Big(\cos 2\pi f_0 t - \frac{1}{3} \cos 2\pi 3 f_0 t + \frac{1}{5} \cos 2\pi 5 f_0 t - \frac{1}{7} \cos 2\pi 7 f_0 t + \dots \Big)$$

- Trigonometric relations

TABLE A6.8 *Trigonometric identities*

$\exp(\pm j\theta) = \cos\theta \pm j \sin\theta$

$\cos\theta = \frac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$

$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta - \sin^2\theta = \cos(2\theta)$

$\cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)]$

$\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)]$

$2 \sin\theta \cos\theta = \sin(2\theta)$

$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$

$\sin\alpha \sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\sin\alpha \cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$

Fourier Transform

TABLE 2.1 TABLE OF FOURIER-TRANSFORM PAIRS

Time Domain	Frequency Domain
$\delta(t)$	1
1	$\delta(f)$
$\delta(t-t_0)$	$e^{-j2\pi f t_0}$
$e^{j2\pi f_0 t}$	$\delta(f-f_0)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\delta(f-f_0)+\frac{1}{2}\delta(f+f_0)$
$\sin(2\pi f_0 t)$	$-\frac{1}{2j}\delta(f+f_0)+\frac{1}{2j}\delta(f-f_0)$
$\Pi(t)$	$\mathrm{sinc}(f)$
$\mathrm{sinc}(t)$	$\Pi(f)$
$\Lambda(t)$	$\mathrm{sinc}^2(f)$
$\mathrm{sinc}^2(t)$	$\Lambda(f)$
$e^{-\alpha t}u_{-1}(t), \alpha>0$	$\frac{1}{\alpha+j2\pi f}$
$te^{-\alpha t}u_{-1}(t), \alpha>0$	$\frac{1}{(\alpha+j2\pi f)^2}$
$e^{-\alpha\|t\|}$	$\frac{2\alpha}{\alpha^2+(2\pi f)^2}$
$e^{-\pi t^2}$	$e^{-\pi f^2}$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$u_{-1}(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
$\delta'(t)$	$j2\pi f$
$\delta^{(n)}(t)$	$(j2\pi f)^n$
$\frac{1}{t}$	$-j\pi\,\mathrm{sgn}(f)$
$\sum_{n=-\infty}^{n=+\infty}\delta(t-nT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{n=+\infty}\delta\left(f-\frac{n}{T_0}\right)$

TABLE 2.2 TABLE OF FOURIER-TRANSFORM PROPERTIES

Signal	Fourier Transform
$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
$X(t)$	$x(-f)$
$x(at)$	$\frac{1}{\|a\|}X\left(\frac{f}{a}\right)$
$x(t-t_0)$	$e^{-j2\pi f t_0}X(f)$
$e^{j2\pi f_0 t}x(t)$	$X(f-f_0)$
$x(t)\star y(t)$	$X(f)Y(f)$
$x(t)y(t)$	$X(f)\star Y(f)$
$\frac{d}{dt}x(t)$	$j2\pi f X(f)$
$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
$tx(t)$	$\left(\frac{j}{2\pi}\right)\frac{d}{df}X(f)$
$t^n x(t)$	$\left(\frac{j}{2\pi}\right)^n\frac{d^n}{df^n}X(f)$
$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{X(f)}{j2\pi f}+\frac{1}{2}X(0)\delta(f)$

Bessel functions

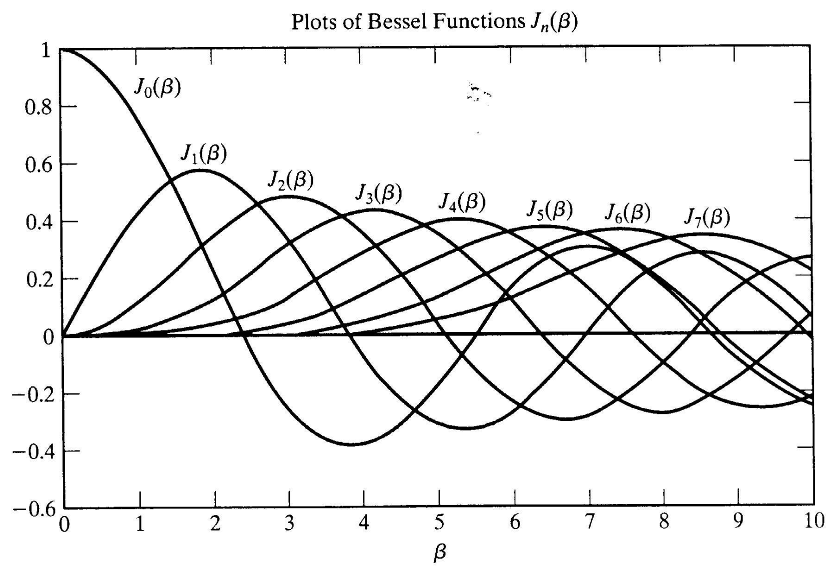

Figure 4.4 Bessel functions for various values of n.

TABLE 4.1 TABLE OF BESSEL FUNCTION VALUES

n	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.5$	$\beta = 1$	$\beta = 2$	$\beta = 5$	$\beta = 8$	$\beta = 10$
0	0.997	0.990	0.938	0.765	0.224	−0.178	0.172	−0.246
1	0.050	0.100	0.242	$\underline{0.440}$	$\underline{0.577}$	−0.328	0.235	0.043
2	0.001	0.005	0.031	$\underline{\underline{0.115}}$	0.353	0.047	−0.113	0.255
3				0.020	$\underline{\underline{0.129}}$	0.365	−0.291	0.058
4				0.002	0.034	$\underline{0.391}$	−0.105	−0.220
5					0.007	0.261	0.186	−0.234
6					0.001	$\underline{\underline{0.131}}$	0.338	−0.014
7						0.053	$\underline{0.321}$	0.217
8						0.018	0.223	$\underline{0.318}$
9						0.006	$\underline{\underline{0.126}}$	0.292
10						0.001	0.061	0.207
11							0.026	$\underline{\underline{0.123}}$
12							0.010	0.063
13							0.003	0.029
14							0.001	0.012
15								0.004
16								0.001

Single and double underlines indicate the number of harmonics containing 70% and 98% of total power, respectively.