UNIVERSITY OF MANITOBA

TERM TEST, WINTER 2015 **February 12, 6:00-8:00 PM**
COURSE: ECE 4260 Communication Systems
DEPARTMENT: Electrical and Computer Engineering
TIME ALLOWED: 2 HOURS
EXAMINER: P. Yahampath

- **Closed-book exam**:
  - No printed or handwritten material of any form allowed.
  - No Internet access or the use of any other form of electronic media permitted.
  - A non-programmable calculator may be used.
- **Answer all 6 questions**. For full credit, you must clearly show how you arrive at the solution, including all relevant calculations. Solutions without intermediate steps will not receive any marks.

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3 Marks

1. Write brief (but clear) answers to the following questions.
   (a) What are the important advantages of digital communication compared to analog communication? List at least two.
   (b) What is the main advantage of conventional AM over DSB-SC AM?
   (c) What is the main advantage of FM over conventional AM?

4 Marks

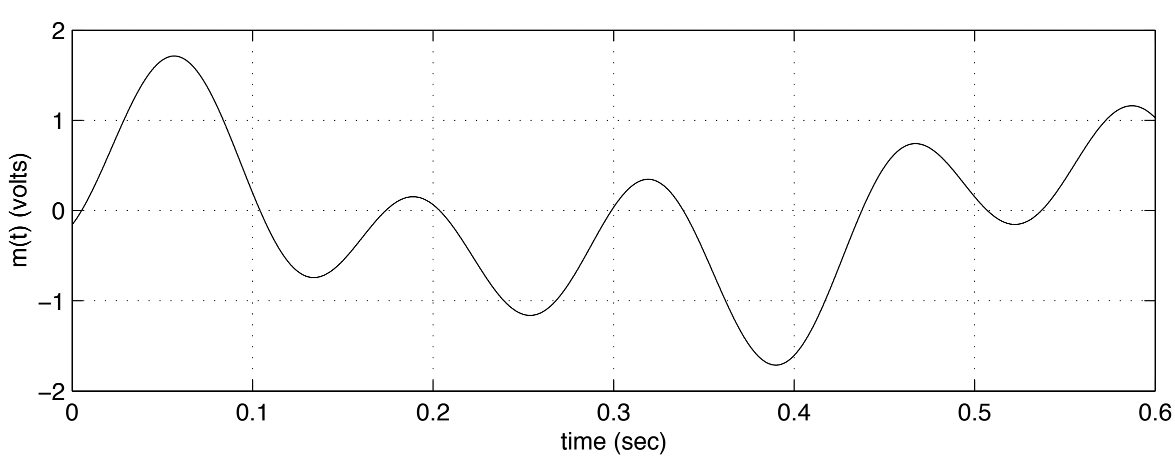
2. The carrier signal $c(t) = 100\cos(20 \times 10^4 \pi t)$ DSB-SC modulated using the following message.

$$m(t) = \cos 8000\pi t + \sin 8000\pi t$$

   (a) Write down a time-domain expression for the modulated signal.
   (b) Determine the Fourier transform the modulated signal.
   (c) Plot and label the magnitude spectrum of the modulated signal.

3 Marks

3. A message signal is shown below (plotted to scale).

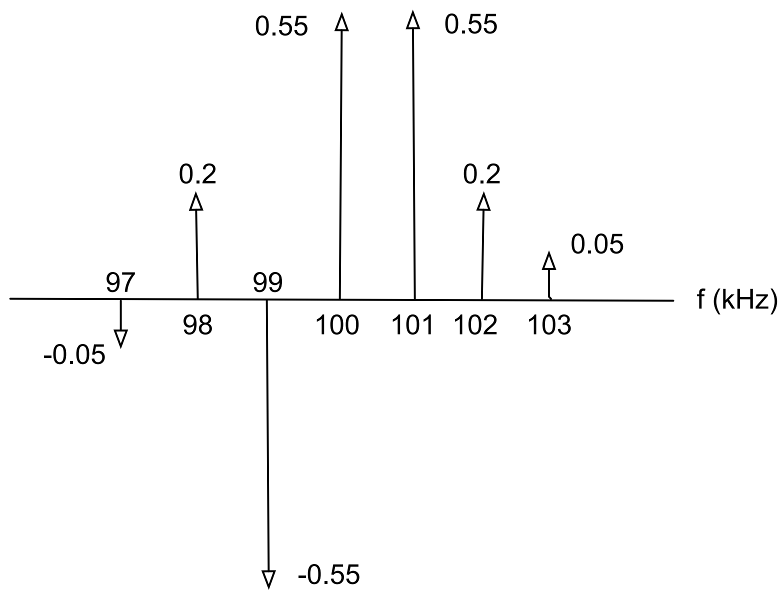


A frequency modulator (FM) is available to you.

   (a) You are supposed to generate a *phase modulated* (PM) signal using the message $m(t)$ shown above. How would you do this using the FM modulator available to you? Explain with the aid of a simple block diagram.
   (b) Suppose you can adjust the sensitivity (in Hz/volts) of the FM modulator. If the PM signal you generate in part (a) is required to have a *maximum phase deviation* of 1.5 radians, to what value would you set the FM modulator sensitivity? Why?

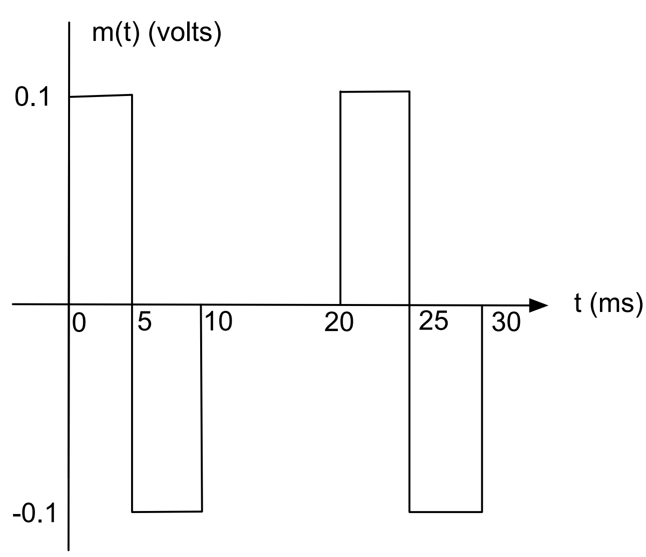
1 Mark

4. A sinusoidal message signal is used to angle modulate a carrier of 100 kHz. The frequency spectrum of the modulated signal as displayed on a spectrum analyzer is shown below (only the positive frequency axis shown). Assuming that the spectrum has no other significant components, estimate the maximum phase deviation of the modulated carrier.



4 Marks

5. The *periodic* message signal $m(t)$ shown below has most of its total power contained in the first 30 harmonics. This message is applied to a frequency modulator whose carrier frequency and amplitude are 1 MHz and 100 mV respectively, and the sensitivity is 1.2 kHz/v.



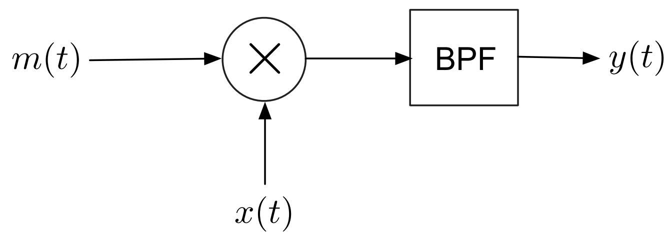
(a) Plot and label the frequency deviation (in kHz) of the modulated carrier as a function of time.

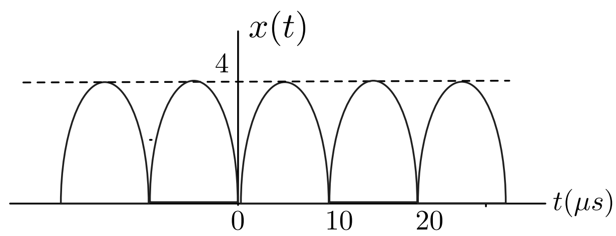
(b) Plot and label the phase deviation (in radians) of the modulated carrier as a function of time.

(c) Find an estimate for the bandwidth of the modulated signal.

(d) Find the average power of the modulated signal.

5 Marks 6. Shown below is a block diagram of a modulator, where $m(t)$ is the message signal.





The signal $x(t)$ is a full-wave rectified sinusoidal as shown (note that time is in μs). The center frequency and the bandwidth of the BPF are 198 kHz and 4 kHz respectively.

(a) Suppose $m(t) = 1000\text{sinc}^2(4000t)$. Determine the Fourier transform of $m(t)$ and plot its frequency spectrum.

(b) Plot the frequency spectrum of the modulated signal $y(t)$.

(c) What kind of a modulator is this (DSB-SC, conv. AM, SSB, FM, PM)?

(d) What is the carrier frequency?

——— END OF TEST ———

Appendix

- Carson's rule: $2(\beta + 1)W$
- **Trigonometric identities**:

$$\cos[2\pi f_c t + a\sin 2\pi f_m t] = \sum_{n=-\infty}^{\infty} J_n(a)\cos\left[2\pi(f_c + nf_m)t\right]$$

$$\cos[2\pi f_c t + a\cos 2\pi f_m t] = \sum_{n=-\infty}^{\infty} J_n(a)\cos\left[2\pi(f_c + nf_m)t + \frac{n\pi}{2}\right]$$

$$e^{\pm jx} = \cos x \pm j\sin x$$

$$\cos x = \frac{1}{2}(e^{jx} + e^{-jx})$$

$$\sin x = \frac{1}{2j}(e^{jx} - e^{-jx})$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin x$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos x$$

$$2\sin x\cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \frac{1}{4}(3\cos x + \cos 3x)$$

$$\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$

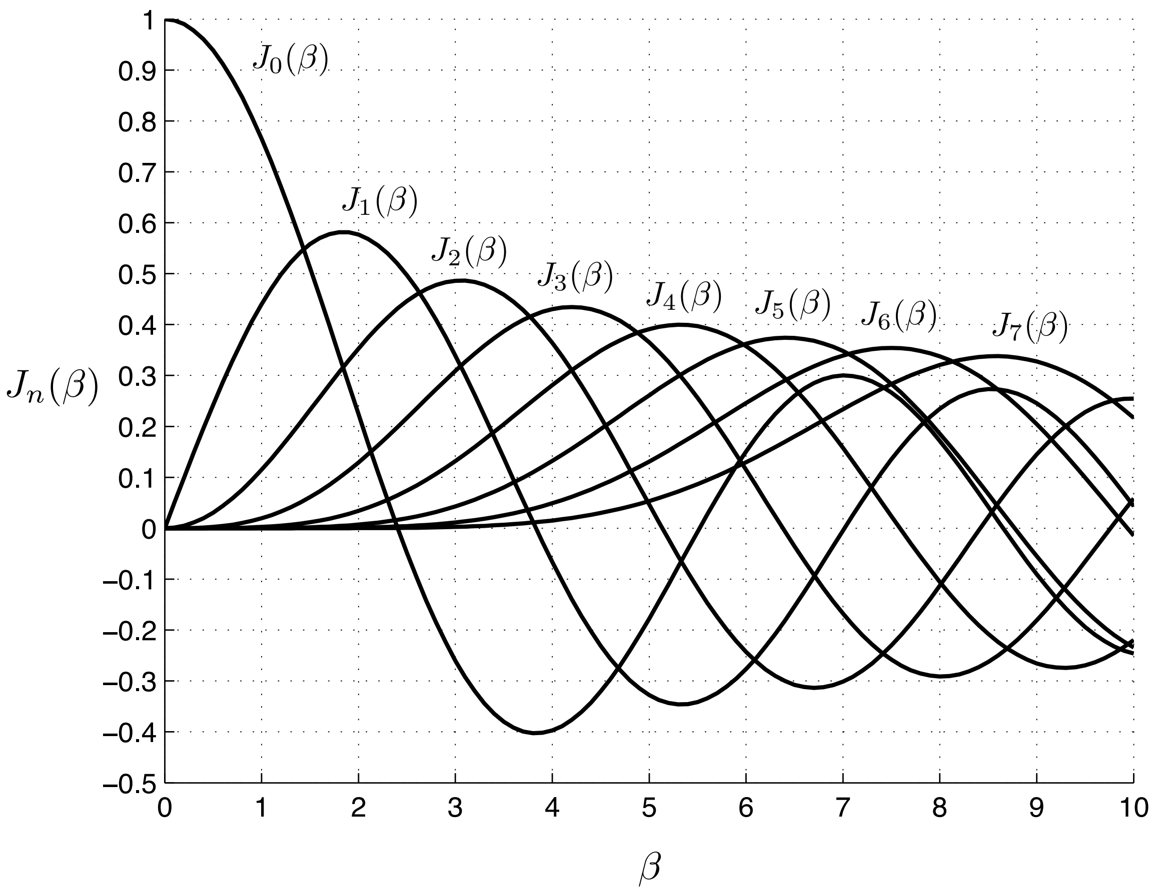
$$\sin x\sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x\cos y = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x\cos y = \frac{1}{2}[\sin(x - y) + \sin(x + y)]$$

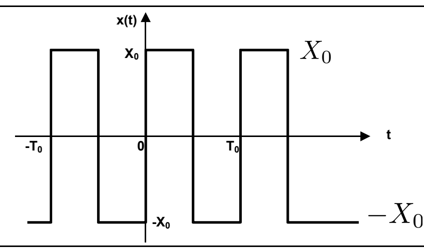
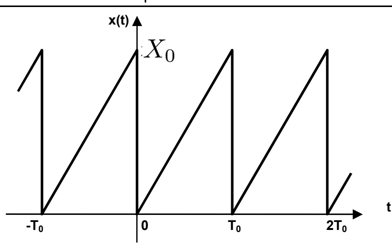
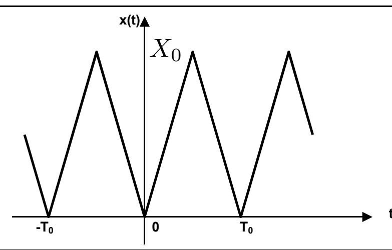
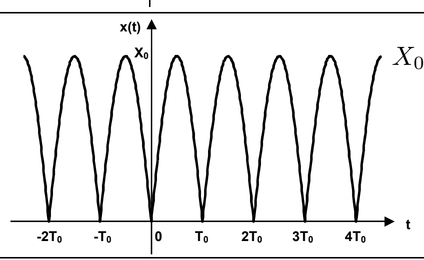
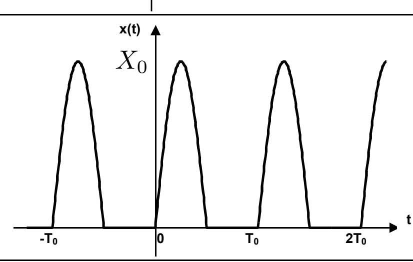
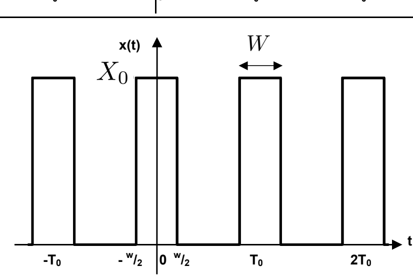
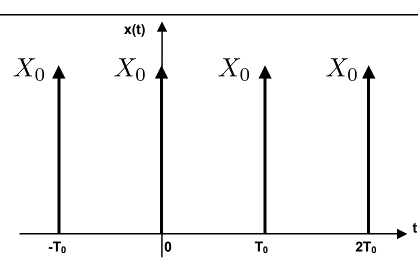
$$a\cos x + b\sin x = C\cos(x + \theta)$$

in which $C = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$



Note: $J_{-n}(\beta) = (-1)^n J_n(\beta)$

Table of Common Fourier Series

Name	Waveform	a_0	a_k	Comments
1. Square wave		0	$-j\dfrac{2X_0}{\pi k}$ when k is odd	$a_k = 0$ when k is even
2. Sawtooth		$\dfrac{X_0}{2}$	$j\dfrac{X_0}{2\pi k}$	
3. Triangular wave		$\dfrac{X_0}{2}$	$\dfrac{-2X_0}{(\pi k)^2}$ when k is odd	$a_k = 0$ when k is even
4. Full-wave rectified		$\dfrac{2X_0}{\pi}$	$\dfrac{2X_0}{\pi(1-4k^2)}$	
5. Half-wave rectified		$\dfrac{X_0}{\pi}$	$\dfrac{X_0}{\pi(1-k^2)}$ when k is even	$a_k = 0$ when k is odd, except $a_1 = -j\dfrac{X_0}{4}$
6. Rectangular Wave		$\dfrac{wX_0}{T_0}$	$\dfrac{wX_0}{T_0} sinc \dfrac{wk\omega_0}{2}$ $\dfrac{wX_0}{T_0}\dfrac{\sin\frac{wk\omega_0}{2}}{\frac{wk\omega_0}{2}}$ $\dfrac{X_0}{k\pi}\sin\dfrac{wk\omega_0}{2}$	$\omega_0 = \dfrac{2\pi}{T_0}$ $sinc(x) = \dfrac{\sin x}{x}$
7. Impulse train		$\dfrac{X_0}{T_0}$	$\dfrac{X_0}{T_0}$	

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = a_0 + \sum_{k=1}^{\infty} 2|a_k|\cos(k\omega_0 t + \angle a_k)$$

TABLE OF FOURIER TRANSFORMS

Time Domain ($x(t)$)	Frequency Domain ($X(f)$)
$\delta(t)$	1
1	$\delta(f)$
$\delta(t-t_0)$	$e^{-j2\pi f t_0}$
$e^{j2\pi f_0 t}$	$\delta(f-f_0)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\delta(f-f_0)+\frac{1}{2}\delta(f+f_0)$
$\sin(2\pi f_0 t)$	$-\frac{1}{2j}\delta(f+f_0)+\frac{1}{2j}\delta(f-f_0)$
$\Pi(t)=\begin{cases}1, & \lvert t\rvert<\frac{1}{2}\\ \frac{1}{2}, & t=\pm\frac{1}{2}\\ 0, & \text{otherwise}\end{cases}$	$\text{sinc}(f)=\frac{\sin(\pi f)}{\pi f}$
$\text{sinc}(t)$	$\Pi(f)$
$\Lambda(t)=\begin{cases}t+1, & -1\le t<0\\ -t+1, & 0\le t<1\\ 0, & \text{otherwise}\end{cases}$	$\text{sinc}^2(f)$
$\text{sinc}^2(t)$	$\Lambda(f)$
$e^{-\alpha t}u_{-1}(t), \alpha>0$	$\frac{1}{\alpha+j2\pi f}$
$te^{-\alpha t}u_{-1}(t), \alpha>0$	$\frac{1}{(\alpha+j2\pi f)^2}$
$e^{-\alpha\lvert t\rvert}$	$\frac{2\alpha}{\alpha^2+(2\pi f)^2}$
$e^{-\pi t^2}$	$e^{-\pi f^2}$
$\text{sgn}(t)=\begin{cases}1, & t>0\\ -1, & t<0\\ 0, & t=0\end{cases}$	$1/(j\pi f)$
$u_{-1}(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
$\delta'(t)$	$j2\pi f$
$\delta^{(n)}(t)$	$(j2\pi f)^n$
$\frac{1}{t}$	$-j\pi\,\text{sgn}(f)$
$\sum_{n=-\infty}^{n=+\infty}\delta(t-nT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{n=+\infty}\delta\left(f-\frac{n}{T_0}\right)$

TABLE 2.2 TABLE OF FOURIER-TRANSFORM PROPERTIES

Signal	Fourier Transform
$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(f) + \beta X_2(f)$
$X(t)$	$x(-f)$
$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right)$
$x(t - t_0)$	$e^{-j2\pi f t_0} X(f)$
$e^{j2\pi f_0 t} x(t)$	$X(f - f_0)$
$x(t) \star y(t)$	$X(f)Y(f)$
$x(t)y(t)$	$X(f) \star Y(f)$
$\frac{d}{dt} x(t)$	$j2\pi f X(f)$
$\frac{d^n}{dt^n} x(t)$	$(j2\pi f)^n X(f)$
$t x(t)$	$\left(\frac{j}{2\pi}\right) \frac{d}{df} X(f)$
$t^n x(t)$	$\left(\frac{j}{2\pi}\right)^n \frac{d^n}{df^n} X(f)$
$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{X(f)}{j2\pi f} + \frac{1}{2} X(0)\delta(f)$