① First consider DSB-SC system

$$S_1(t) = A_1 A_m \cos 2\pi f_m t \cos 2\pi f_c t$$

$$= \frac{A_1 A_m}{2}\left[\cos 2\pi (f_c - f_m)t + \cos 2\pi (f_c + f_m)t\right]$$

average power (transmitted) $P_1 = \frac{1}{2}\left(\frac{A_1 A_m}{2}\right)^2 + \frac{1}{2}\left(\frac{A_1 A_m}{2}\right)^2$

$$= \frac{A_1^2 A_m^2}{4} = 10$$

$$\therefore A_1 A_m = \sqrt{40} \quad \text{—— ①}$$

average message power $P_m = \frac{1}{2} A_m^2$

$$= 0.1$$

$$\therefore A_m = \sqrt{0.2}$$

from ① $\Rightarrow$ $A_1 = \sqrt{200}$

Next consider broadcast AM :

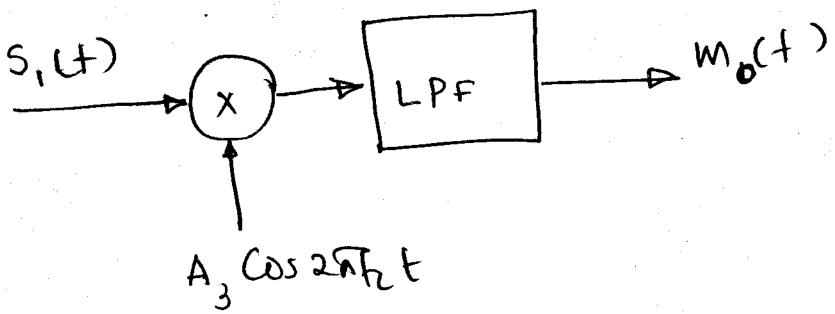
$$S_2(t) = A_2\left[1 + K_a A_m \cos 2\pi f_m t\right] \cos 2\pi f_c t$$

$$= A_2 \cos 2\pi f_c t + \frac{K_a A_2 A_m}{2}\left[\cos 2\pi (f_c - f_m)t + \cos 2\pi (f_c + f_m)t\right]$$

average power $P_2 = \frac{A_2^2}{2} + \frac{K_a^2 A_2^2 A_m^2}{4} = 400 \quad \text{—— ②}$

$=$

demodulated signal in DSB-SC system :



at the input of LPF, we have

$$S_1(t) \cdot A_3 \cos 2\pi f_c t = A_1 A_3 A_m \cos 2\pi f_m t \cos^2 2\pi f_c t$$

$$= \frac{A_1 A_3 A_m}{2}\left[1 + \cos 4\pi f_c t\right] \cos 2\pi f_m t.$$

after LPF $\quad m_0(t) = \dfrac{A_1 A_3 A_m}{2} \cos 2\pi f_m t \quad$ 1

also the demodulated signal with broadcast AM is

$$m_0(t) = \frac{K_a A_2 A_3 A_m}{2} \cos 2\pi f_m t.$$

$$\therefore \quad K_a A_2 = A_1 \quad 1$$

From (2) we have

$$\frac{A_2^2}{2} + \frac{A_1^2 A_m^2}{4} = 400$$

$$\frac{A_2^2}{2} + 10 = 400$$

$$\therefore \quad A_2 = \sqrt{780}$$

$$\therefore K_a = \frac{A_1}{A_2} = \frac{\sqrt{200}}{\sqrt{780}} = \sqrt{0.256} \approx 0.5 \quad 1$$

modulation index $= K_a A_m$

(2)

$$x(t) = \operatorname{sinc}^2(1000t)$$
$$y(t) = \sin(2\pi 10^4 t)$$

$$\therefore \hat{y}(t) = \sin(2\pi 10^4 t - \pi/2) = -\cos(2\pi 10^4 t)$$

Hence
$$z(t) = -x(t)\cos(2\pi 10^4 t) + \hat{x}(t)\sin(2\pi 10^4 t)$$
$$= -[x(t)\cos(2\pi 10^4 t) - \hat{x}(t)\sin(2\pi 10^4 t)]$$

This is an SSB signal, namely the upper-sideband signal of
$$u(t) = -2x(t)\cos(2\pi 10^4 t)$$

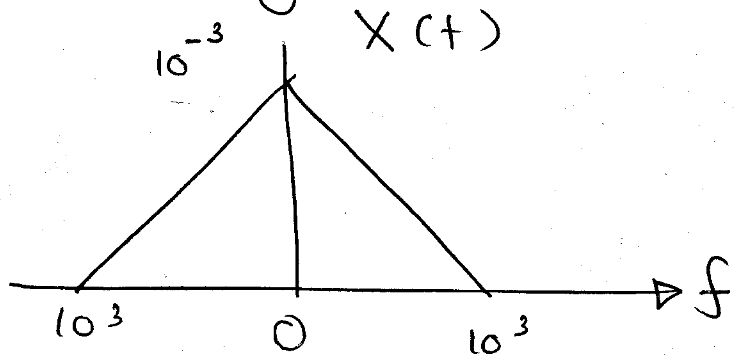
Hence, first plot $U(f)$:

$$U(f) = -[X(f - f_c) + X(f + f_c)]$$

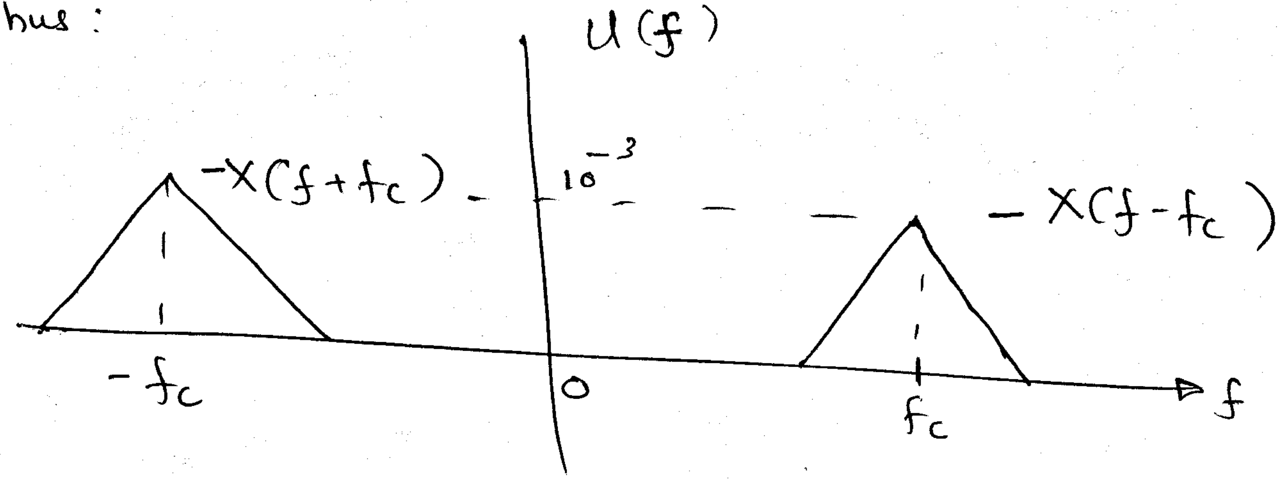
where $f_c = 10^4$

Since $x(t) = \operatorname{sinc}^2(1000t)$

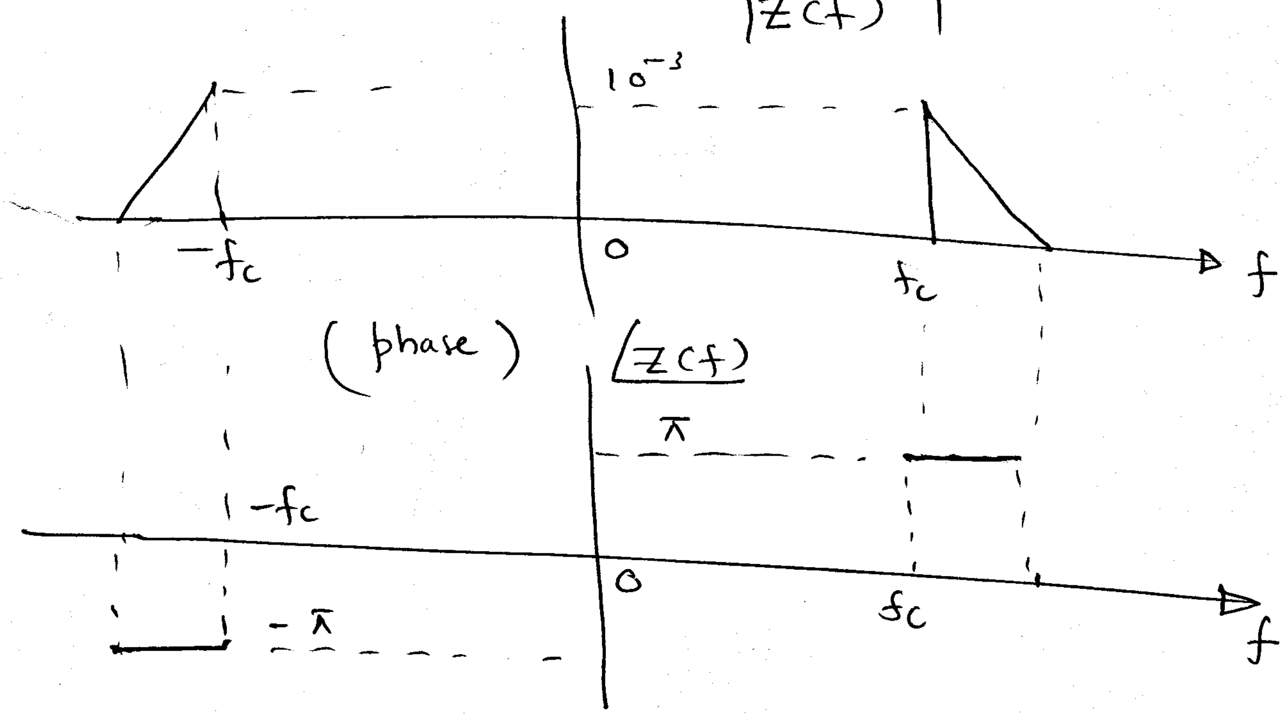
$X(f)$ is a triangular function as follows:



Thus:



Now, upper-sideband spectrum (magnitude)

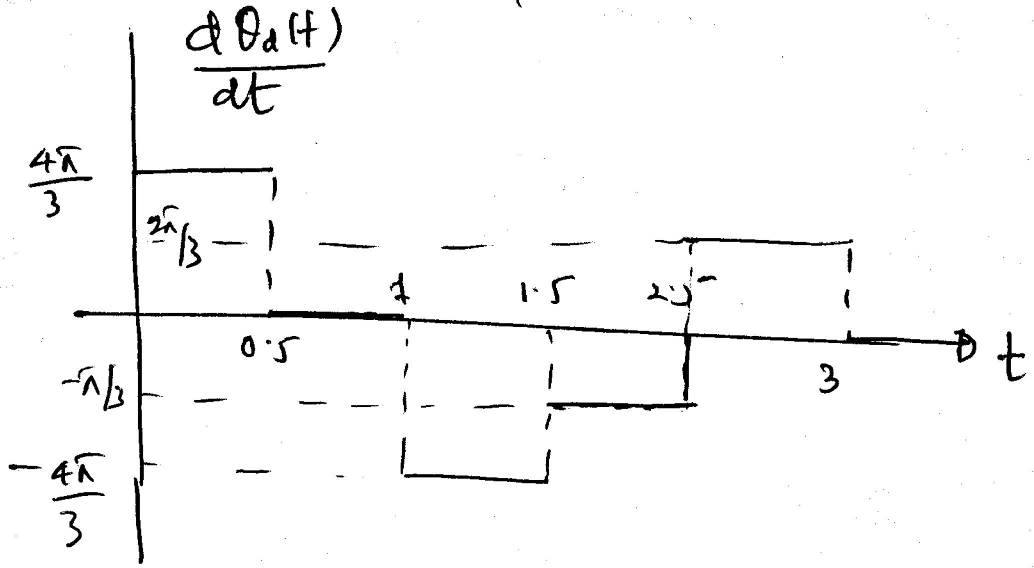


③ Frequency deviation $f_d(t) = k_f\, m(t)$

$\therefore$ phase deviation $\theta_d(t) = 2\pi k_f \int m(\tau)\,d\tau$.

Thus, what is shown in the graph is the $2\pi k_f$ (integral of the message)

$$\therefore \quad \cancel{m(t) = \frac{1}{2\pi k_f}}\ \frac{d\,\theta_d(t)}{dt} = 2\pi k_f\, m(t)$$



$\frac{2\pi/3}{0.5}$ $\quad 4\pi/3$

$-\pi/3$

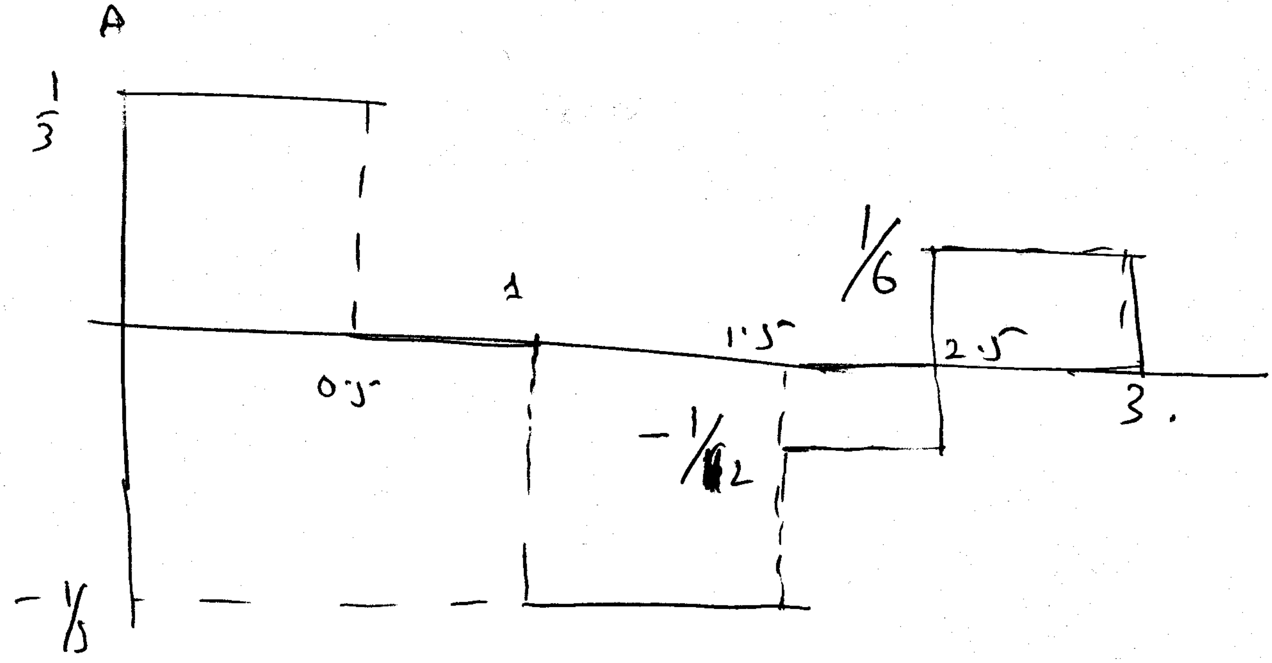
$\frac{\pi/3}{0.5}$

Since $|m(t)|_{max} = 1/3$, we see that

$$\cancel{2\pi}\left(\frac{1}{\cancel{3}}\right)k_f = \frac{4\cancel{\pi}}{\cancel{3}}\ 2$$

$$\underline{\underline{k_f = 2}}.$$

$m(t)$



Hence, phase deviation of PM signal $= k_p m(t)$

$k_p = \pi/10$