1. (a) See slide 21 of Introduction slide set.
   (b) See slide 48 of Amplitude Modulation slide set.
   (c) See slide 25 of Amplitude Modulation slide set.

2. This signal has period of $T = 2$ or the fundamental frequency 0.5 Hz.. The total power is
$$P_T = \frac{1}{T}\int_0^2 (6\sin \pi t)^2 dt = 18$$
Power in the dc component is
$$P_{dc} = \left(\frac{2X_o}{\pi}\right)^2 = 14.59$$
where $X_o = 6$.
Power in the k-th harmonic ($k \geq 1$) is
$$P_k = \frac{1}{2}\left(\frac{4X_o}{\pi(1-4k^2)}\right)^2$$
We find that $P_{dc} + P_1 + P_2 = 17.96$ and $P_{dc} + P_1 + P_2 + P_3 = 17.99$. Therefore 99.9% of the total power P_T is in the dc component and the first 3 harmonics. The bandwidth is thus 1.5 Hz.

3. To obtain the USSB spectrum, first find the DSB-SC spectrum. The DSB-SC signal is
$$\begin{aligned} u_{DSB-SC}(t) &= 50m(t)\left(\frac{-je^{j2\pi f_c t} + je^{-j2\pi f_c t}}{2}\right) \\ &= 25m(t)(e^{j(2\pi f_c t - \frac{\pi}{2})} + e^{-j(2\pi f_c t - \frac{\pi}{2})}), \end{aligned}$$
where $f_c = 5000$ Hz. The Fourier transform is
$$U_{DSB-SC}(f) = 25e^{-j\frac{\pi}{2}}M(f - f_c) + 25e^{j\frac{\pi}{2}}M(f + f_c)$$
We find that
$$M(f) = \frac{4}{200}\prod\left(\frac{f}{200}\right).$$
$U_{DSB-SC}(f)$ is shown in Fig. 1.
But we need the USSB spectrum, which only includes the upper sideband (Fig. 2).

4. (a) AM signal is $u(t) = 10[A + m(t)]\cos 2\pi f_c t$ where $f_c = 2500$ Hz and A is to be determined.
   The modulation index is
$$a = \frac{|\text{Lowest negative value of } m(t)|}{A} \approx \frac{3.5}{A} = 0.7$$
   which gives $A = 5$.

(b) The highest frequency in the message is 300 Hz. Hence the AM bandwidth is 600 Hz.

(c) First, using trigonometry

$$u(t) = 50\cos(5000\pi t) + \underbrace{15\cos(5200\pi t) + 15\cos(4800\pi t) + 5\sin(5600\pi t) - 5\sin(4400\pi t)}_{\text{sidebands}}.$$

The total power is 1500 W. Power in sidebands is 250 W. The fraction of power in sidebands is therefore $\frac{1}{6}$.

5. (a) $\frac{-750}{15}\sin 500\pi t + \frac{800}{3}\cos 200\pi t$

(b) $\frac{9}{\pi}\cos 500\pi t + \frac{12}{\pi}\sin 200\pi t$

6. (a) The instantaneous frequency deviation is $k_f a\cos 2000\pi t$, where k_f is the modulator sensitivity. At $t = 125\ \mu s$

$$k_f a\cos(2000\pi \times 125 \times 10^{-6}) = 1414 \text{ Hz}$$

From this, we find that the maximum frequency deviation is $\Delta f = k_f a = 1414\sqrt{2}$ Hz. Now the instantaneous phase deviation is

$$\phi(t) = 2\pi\int_{-\infty}^{t}\Delta f\cos(2000\pi u)du = 2\sin 2000\pi t \text{ rad.}$$

Therefore, the maximum phase deviation is 2 rad.

(b) The FM signal is given by

$$u(t) = \cos(2\pi 100 \times 10^3 t + 2\sin 2000\pi t) = \sum_{n=-\infty}^{\infty} J_n(2)\cos[2\pi 10^3(100+n)t]$$

The frequency spectrum is given by the Fourier transform

$$U(f) = \sum_{n=-\infty}^{\infty}\frac{J_n(2)}{2}[\delta\left(f - 10^3(100+n)\right) + \delta\left(f + 10^3(100+n)\right)].$$

Using the Bessel function plots, we find that

$$\begin{aligned} J_0(2) &\approx 0.21 \\ J_1(2) = -J_{-1}(2) &\approx 0.58 \\ J_2(2) = J_{-2}(2) &\approx 0.35 \\ J_3(2) = -J_{-3}(2) &\approx 0.11 \\ J_4(2) = J_{-4}(2) &\approx 0.04 \end{aligned}$$

and $J_n(2)$ for $|n| > 4$ is negligible. The two-sided magnitude spectrum is shown in Fig. 3.

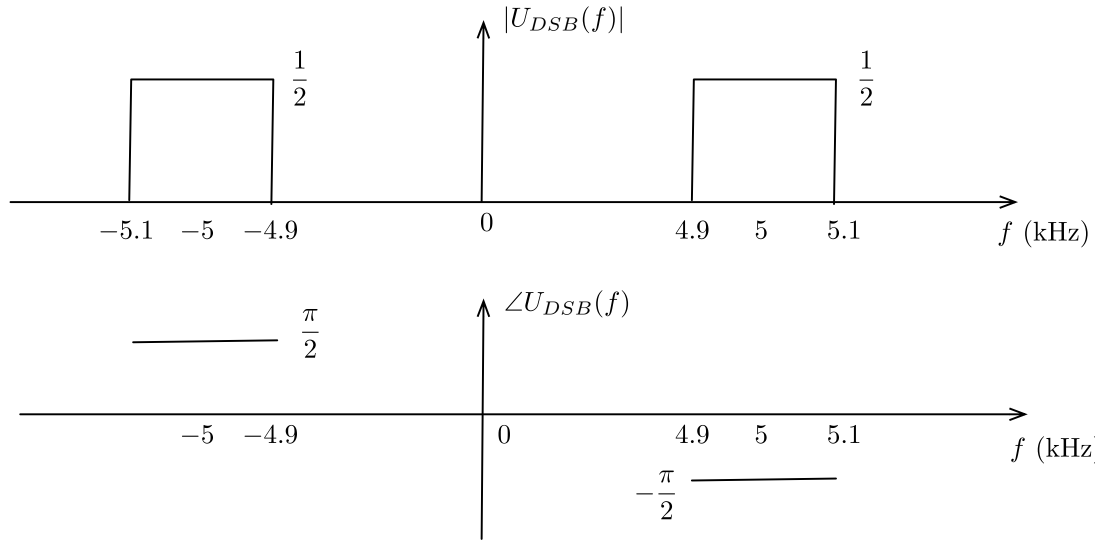

FIg. 1

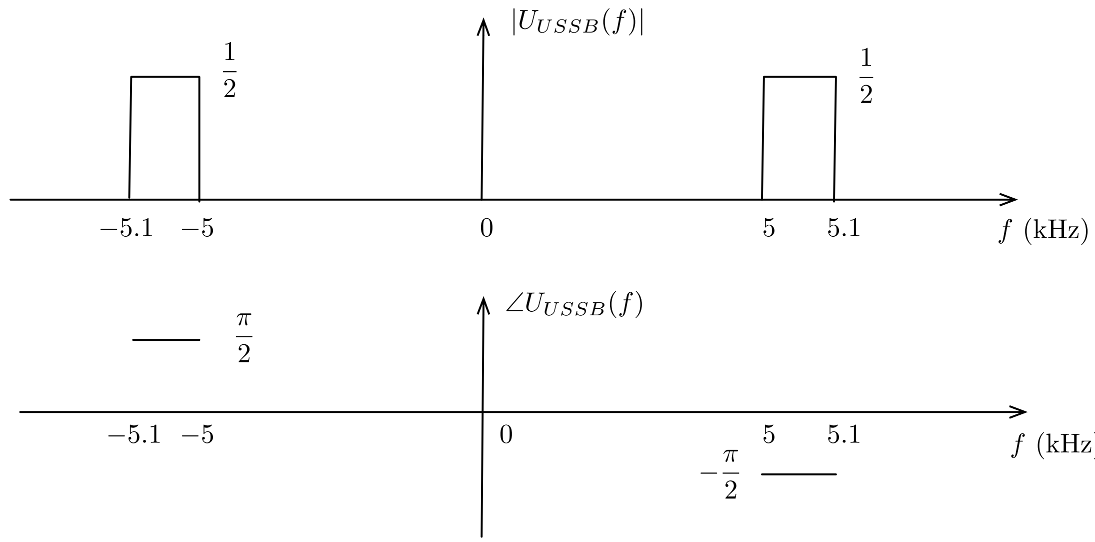

Fig. 2

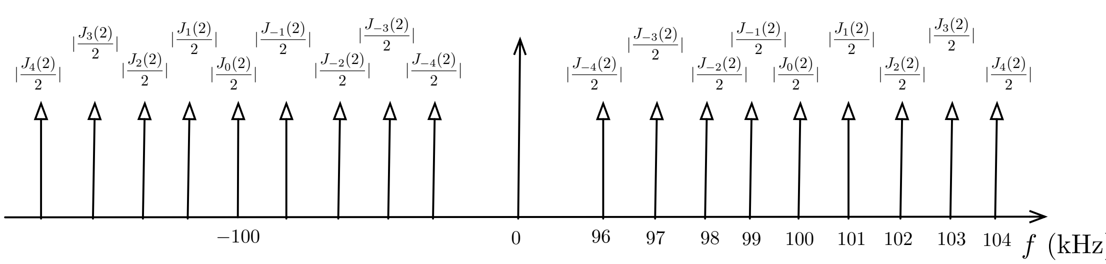

Fig. 3