ECE2460 Communication Systems
Solutions to Term-test

1. (a) The main advantages of digital communication over analog communication (from a communication point of view):
   i. Ability to apply error-correction coding and hence transmit information (nearly) error-free. Error correction is not possible with analog.
   ii. Possibility of realizing complex (hence more capable) systems using DSP, VLSI, and programming.
   iii. Easy to network (packet-switching, routing, ...)

   (b) The significance of the modulation index is that the wrong value (> 1) will make it impossible to demodulate the AM signal using a (simple) envelope detector (if modulation index greater than 1, then synchronous demodulation is needed which of course completely defeats the purpose of using conventional AM).

   (c) With FM, the receiver SNR can be increased by either increasing the transmitter power or the transmitted signal bandwidth (*power-bandwidth trade-off*). With AM, the receiver SNR can be increased only by increasing the transmitter power. As a result AM transmitters require much more power (> 10 kW) than FM transmitters (< 5 kW) for same performance.

2. (a) i. The AM signal has the form

$$s(t) = [A + m(t)] \cos 2\pi f_c t,$$

where A is a constant (a dc value), $m(t)$ is the message signal, and f_c is the carrier frequency. In this case, the message signal has the form $m(t) = A_m \cos 2\pi f_m t$
We can expand the AM signal as

$$s(t) = A \cos 2\pi f_c t + \frac{A_m}{2} \cos 2\pi(f_c + f_m)t + \frac{A_m}{2} \cos 2\pi(f_c - f_m)t.$$

Taking the Fourier transform we get the following equation for the spectrum shown in the problem.

$$S(f) = \frac{A}{2}[\delta(f-f_c)+\delta(f+f_c)]+\frac{A_m}{4}[\delta(f-f_c-f_m)+\delta(f+f_c+f_m)+\delta(f+f_c-f_m)+\delta(f-f_c+f_m)].$$

From the figure, we conclude that

$$\begin{aligned} \frac{A}{2} &\approx 0.9 \\ \frac{A_m}{4} &\approx 0.32 \end{aligned}$$

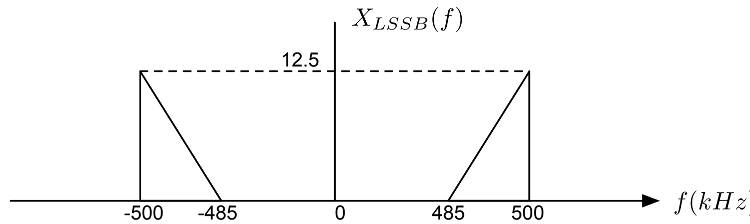
Hence, $A = 1.8$, $A_m = 1.28$, and the modulation index is $\frac{A_m}{A} = 0.71$.

   ii. For 100% modulation index we need $A = A_m$. Hence the dc level has to be reduced by 0.52 or 29%.

(b) To find the LSSB signal, we first find the DSB-SC signal and then delete the upper-sideband.
The DSB-SC signal and its spectrum are

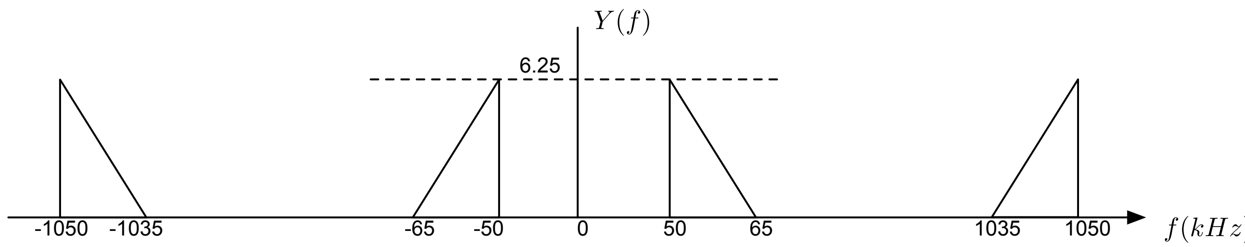
$$\begin{aligned} x_{DSB}(t) &= 50m(t)\cos 2\pi f_c t \\ X_{DSB}(f) &= 25M(f-f_c) + 25M(f+f_c) \end{aligned}$$

Hence the transmitted signal spectrum is as follows.



i. The signal at the input of the LPF and its spectrum are

$$\begin{aligned} y(t) &= x_{LSSB}(t)\cos(2\pi 550\times 10^2 t) \\ Y(f) &= \frac{1}{2}[X_{LSSB}(f-550\times 10^3) + X_{LSSB}(f+550\times 10^3)] \end{aligned}$$



ii. Since the LPF bandwidth is 15 kHz, the demodualtor output is zero.

3. (a) i. The instantaneous frequency deviation and angle are

$$\begin{aligned} f(t) &= f_c + 3x(t) \text{ Hz} \\ \theta(t) &= 2\pi \int_0^t f(u)du \\ &= 2\pi f_c t + \underbrace{6\pi \int x(t)dt}_{\text{phase deviation}} \quad \text{rad.} \end{aligned}$$

The peak phase deviation occurs when the integral reaches its maximum, which is the area of a triangle, $AT_0/4 = 0.125$. Hence the max. phase deviation is $3\pi/4$ radians.

ii. The total power of $x(t)$ is $\frac{1}{T_0}\int_{-T_0/2}^{T_0/2}(2A/T_0)^2 t^2 dt = 4/3$. Using the Fourier series, we then find that, to include at least 90% of the total power, we need 6 harmonics. Hence the bandwidth of $x(t)$ is $W = 24$ Hz.
Maximum frequency deviation is 6 Hz. From the Carson's rule, the bandwidth of the FM signal is $2\times 6 + 2\times 24 = 60$ Hz.

(b) The maximum phase deviation is 0.6×2 radians. Hence the PM signal bandwidth is $2(1.2 + 1)24 \approx 106$ Hz.

4. The message frequency is 5 kHz and the message has the form $A_m \cos 10^4 \pi t$.

   According to the spectrum shown, this FM signal should have the form

$$s(t) \approx A_c \sum_{n=-1}^{1} J_n(\beta) \cos[2\pi(f_c + nf_m)t]$$

   From the spectrum we see that $J_0(\beta)/J_1(\beta) \approx 3.88$ (and $J_n(\beta) \approx 0$ for $n > 1$). From the Bessel function plots, we note that this happens when $\beta \approx 0.5$.

   The instantaneous frequency deviation is $k_f A_m \cos 10^4 \pi t$ Hz, and the phase deviation is

$$\begin{aligned} \Delta\theta(t) &= 2\pi k_f A_m \int_{-\infty}^{t} \cos(10^4 \pi u) du \\ &= \frac{2k_f A_m}{10^4} \sin 10^4 \pi t + \phi_0 \text{ rad.} \end{aligned}$$

   Since ϕ_0 is a constant, the maximum phase deviation is $\beta = \max|\Delta\theta(t)| = 2A_m$, and hence $A_m = 0.25$. The message signal is $0.25 \cos 10^4 \pi t$.