

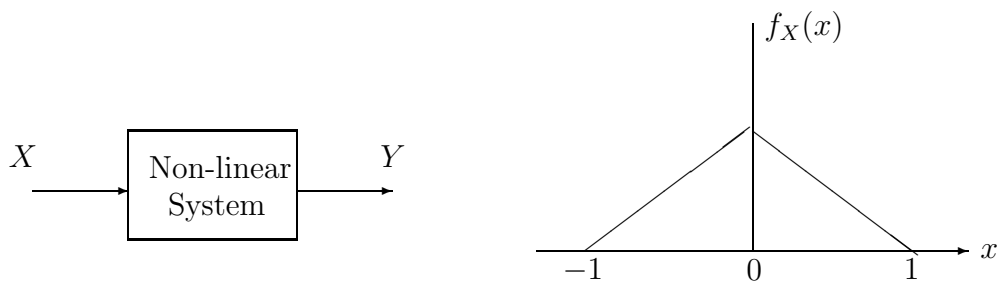
Closed-book exam: No printed or hand-written material allowed.

A calculator may be used.

If you expect part marks, steps of your calculations must be shown clearly.

8 Marks

- The input to a non-linear system is a random variable X with the probability density function $f_X(x)$ as shown below.



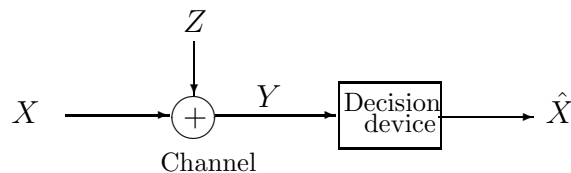
The input-output relationship of this system is given by

$$Y = X^2$$

- Find the mean value of Y
- Find $P(Y < 0.25)$
- Find $P(X > 0 | Y < 0.25)$

8 Marks

- In the communication system shown below, a discrete random variable X is transmitted across a noisy channel whose output is Y . The input X can assume two values 0 and 1, with $P(X = 0) = 0.8$. Channel noise is a Gaussian random variable Z with mean 0 and variance 0.04 (the formula for Gaussian pdf is given at the end of this problem).



The decision device attempts to guess the transmitted value X , based on the observed channel output Y . It operates as follows:

$$\hat{X} = \begin{cases} 0 & \text{if } Y \leq 0.7 \\ 1 & \text{otherwise} \end{cases}$$

The output is correct if $\hat{X} = X$.

- (a) What is the probability that $\hat{X} = 1$ if we transmit $X = 0$?
- (b) What is the probability that $\hat{X} = 0$ if we transmit $X = 1$?
- (c) What is the probability that the system produces a correct output ?

Note: Gaussian pdf with mean μ and variance σ^2 is given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x - \mu)^2}{2\sigma^2}\right]$$

2 Marks

3. Let Θ be a continuous random variable, uniformly distributed in $(-\pi, \pi)$. Suppose we generate two other random variables X and Y as follows:

$$\begin{aligned} X &= \cos(\Theta) \\ Y &= X^2 \end{aligned}$$

- (a) Determine if X and Y are statistically independent.
- (b) Determine if X and Y are correlated.

Supplementary Information

- Trigonometric identities

TABLE A6.4 *Trigonometric Identities*

$$\begin{aligned} \exp(\pm j\theta) &= \cos \theta \pm j \sin \theta \\ \cos \theta &= \frac{1}{2}[\exp(j\theta) + \exp(-j\theta)] \\ \sin \theta &= \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)] \\ \sin^2 \theta + \cos^2 \theta &= 1 \\ \cos^2 \theta - \sin^2 \theta &= \cos(2\theta) \\ \cos^2 \theta &= \frac{1}{2}[1 + \cos(2\theta)] \\ \sin^2 \theta &= \frac{1}{2}[1 - \cos(2\theta)] \\ 2 \sin \theta \cos \theta &= \sin(2\theta) \\ \sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\ \tan(\alpha \pm \beta) &= \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \\ \sin \alpha \sin \beta &= \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\ \cos \alpha \cos \beta &= \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)] \\ \sin \alpha \cos \beta &= \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)] \end{aligned}$$

THE UNIVERSITY OF MANITOBA

TERM TEST 2

OCTOBER 16 2006, 6:00-7:30 PM

TIME ALLOWED: 1.5 HOURS

DEPARTMENT: Electrical and Computer Engineering

COURSE: ECE 4260 Communication Systems

EXAMINER: P. Yahampath

PAGE NO: 3 OF 3

- Q-function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-z^2/2) dz$$

TABLE 5.1 TABLE OF THE Q FUNCTION

0	5.000000e-01	2.4	8.197534e-03	4.8	7.933274e-07
0.1	4.601722e-01	2.5	6.209665e-03	4.9	4.791830e-07
0.2	4.207403e-01	2.6	4.661189e-03	5.0	2.866516e-07
0.3	3.820886e-01	2.7	3.466973e-03	5.1	1.698268e-07
0.4	3.445783e-01	2.8	2.555131e-03	5.2	9.964437e-08
0.5	3.085375e-01	2.9	1.865812e-03	5.3	5.790128e-08
0.6	2.742531e-01	3.0	1.349898e-03	5.4	3.332043e-08
0.7	2.419637e-01	3.1	9.676035e-04	5.5	1.898956e-08
0.8	2.118554e-01	3.2	6.871378e-04	5.6	1.071760e-08
0.9	1.840601e-01	3.3	4.834242e-04	5.7	5.990378e-09
1.0	1.586553e-01	3.4	3.369291e-04	5.8	3.315742e-09
1.1	1.356661e-01	3.5	2.326291e-04	5.9	1.817507e-09
1.2	1.150697e-01	3.6	1.591086e-04	6.0	9.865876e-10
1.3	9.680049e-02	3.7	1.077997e-04	6.1	5.303426e-10
1.4	8.075666e-02	3.8	7.234806e-05	6.2	2.823161e-10
1.5	6.680720e-02	3.9	4.809633e-05	6.3	1.488226e-10
1.6	5.479929e-02	4.0	3.167124e-05	6.4	7.768843e-11
1.7	4.456546e-02	4.1	2.065752e-05	6.5	4.016001e-11
1.8	3.593032e-02	4.2	1.334576e-05	6.6	2.055790e-11
1.9	2.871656e-02	4.3	8.539898e-06	6.7	1.042099e-11
2.0	2.275013e-02	4.4	5.412542e-06	6.8	5.230951e-12
2.1	1.786442e-02	4.5	3.397673e-06	6.9	2.600125e-12
2.2	1.390345e-02	4.6	2.112456e-06	7.0	1.279813e-12
2.3	1.072411e-02	4.7	1.300809e-06		