UNIVERSITY OF MANITOBA

TERM TEST, FALL 2016 — October 19, 6:00-8:00 PM
COURSE: ECE 4260 Communication Systems
DEPARTMENT: Electrical and Computer Engineering
TIME ALLOWED: 2 HOURS
EXAMINER: P. Yahampath —

- **Closed-book exam**:
  - No printed or handwritten material of any form allowed.
  - No Internet access or the use of any other form of electronic media permitted.
  - A non-programmable calculator may be used.
- **Answer all 6 questions**. For full credit, you must clearly show how you arrive at the solution, including all relevant calculations. Solutions without intermediate steps will not receive any marks.

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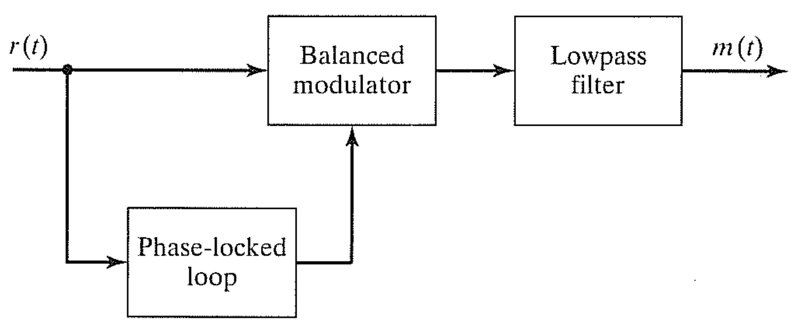
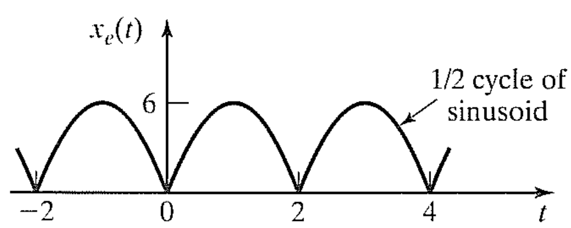
3 Marks

1. Write brief (but clear) answers to the following questions.
   (a) Name two key factors that limit the information capacity of a communication channel.
   (b) What is the main advantage of SSB AM over DSB-SC AM?
   What is the main disadvantage?
   (c) A demodulator is shown below. What is the purpose of the block labeled *phase-locked loop*?



3 Marks

2. Determine the 99.9%-power bandwidth of the following full-wave rectified sinusoidal signal.



4 Marks

3. The message signal $m(t) = 4\text{sinc}(200t)$ is the input to an upper-sideband SSB modulator whose carrier is $c(t) = 50\sin(10000\pi t)$.

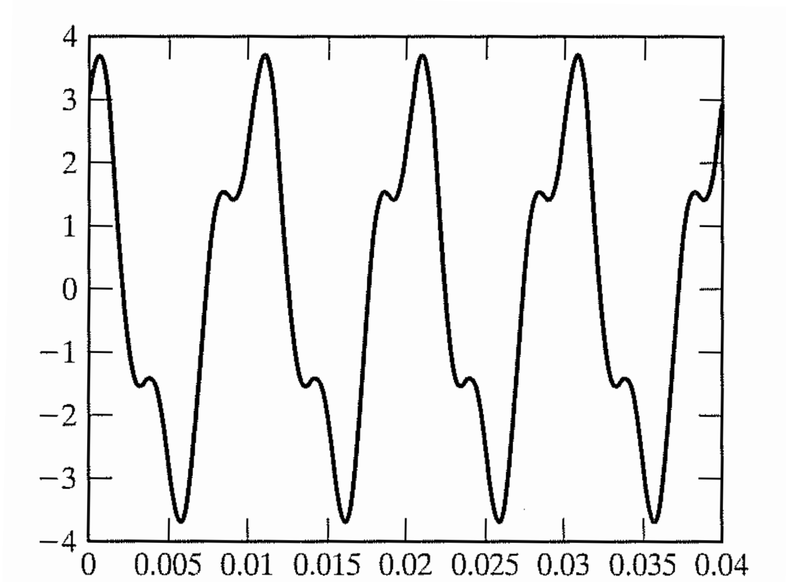
   Plot and label the *two-sided* magnitude and phase spectra of the modulator output.

4 Marks

4. Consider the message signal

$$m(t) = 3\cos(200\pi t) + \sin(600\pi t)$$

a plot of which is shown below.



This message is used in a conventional AM modulator whose unmodulated carrier is $10\cos(5000\pi t)$. The modulation index of the AM signal is found to be 70%.

(a) Find an expression for the AM signal.

(b) What is the bandwidth of the AM signal?

(c) Determine the fraction of power in the sidebands.

3 Marks

5. Consider the following angle modulated signal.

$$u(t) = 100\cos(2\pi 10^4 t + 3\cos 500\pi t + 4\sin 2000\pi t)$$

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(a) The signal has been generated by a frequency modulator with sensitivity 15 Hz/V. Determine the message signal.

(b) The signal has been generated by a phase modulator with sensitivity $\pi/3$ rad/V. Determine the message signal.

3 Marks

6. A sinusoidal message $m(t) = a\cos 2000\pi t$ (V) is used to frequency modulate the carrier $\cos 2\times 10^5\pi t$, where the time is in seconds. The instantaneous frequency deviation of the modulated carrier at time $t = 125\ \mu s$ is observed to be 1414 Hz.

(a) Determine the maximum phase deviation of the modulated carrier.

(b) Plot and label the two-sided *magnitude* spectrum of the FM signal.

——— END OF TEST ———

Appendix

- Carson's rule: $2(\beta+1)W$
- **Trigonometric identities**:

$$\cos[2\pi f_c t + a\sin 2\pi f_m t] = \sum_{n=-\infty}^{\infty} J_n(a)\cos\left[2\pi(f_c+nf_m)t\right]$$

$$\cos[2\pi f_c t + a\cos 2\pi f_m t] = \sum_{n=-\infty}^{\infty} J_n(a)\cos\left[2\pi(f_c+nf_m)t+\frac{n\pi}{2}\right]$$

$$e^{\pm jx} = \cos x \pm j\sin x$$

$$\cos x = \frac{1}{2}(e^{jx}+e^{-jx})$$

$$\sin x = \frac{1}{2j}(e^{jx}-e^{-jx})$$

$$\cos\left(x\pm\frac{\pi}{2}\right) = \mp\sin x$$

$$\sin\left(x\pm\frac{\pi}{2}\right) = \pm\cos x$$

$$2\sin x\cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1-\cos 2x)$$

$$\cos^3 x = \frac{1}{4}(3\cos x + \cos 3x)$$

$$\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$$

$$\sin(x\pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x\pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\tan(x\pm y) = \frac{\tan x \pm \tan y}{1\mp\tan x\tan y}$$

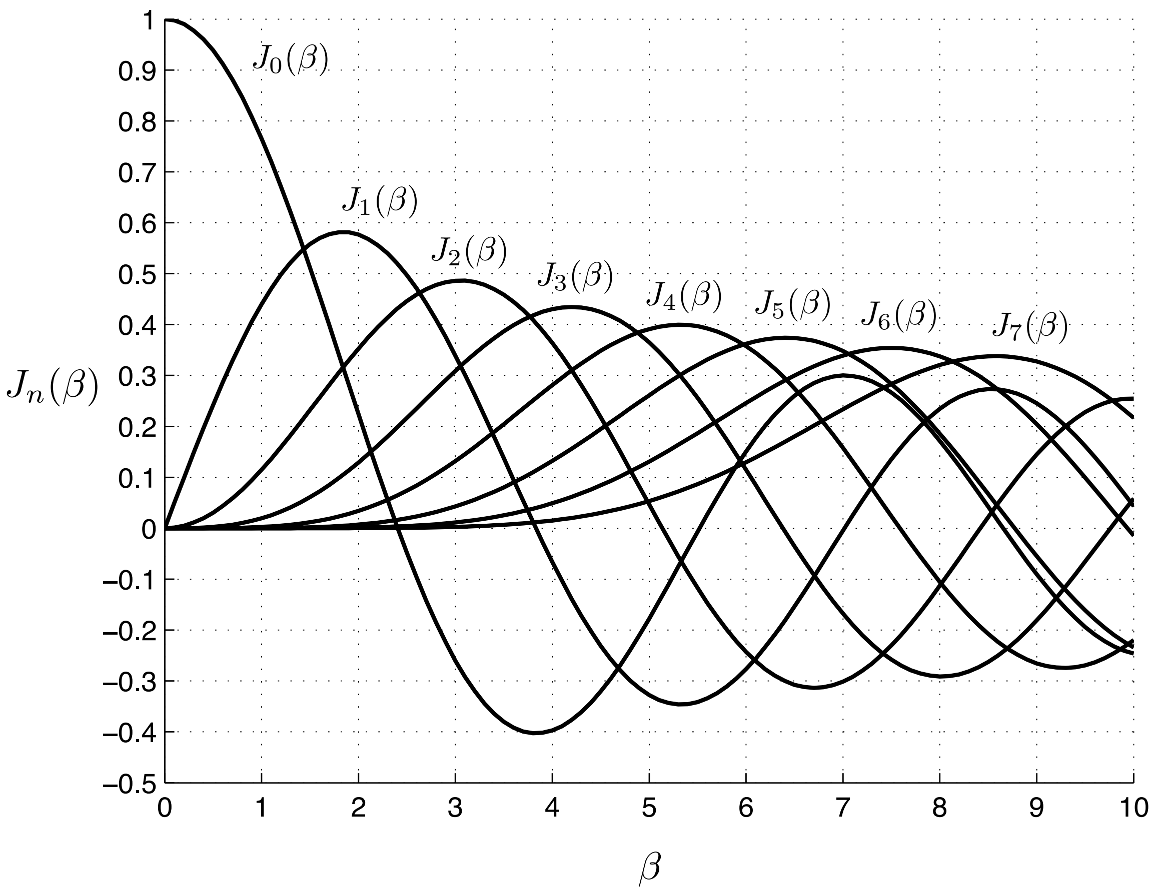
$$\sin x\sin y = \frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

$$\cos x\cos y = \frac{1}{2}[\cos(x-y)+\cos(x+y)]$$

$$\sin x\cos y = \frac{1}{2}[\sin(x-y)+\sin(x+y)]$$

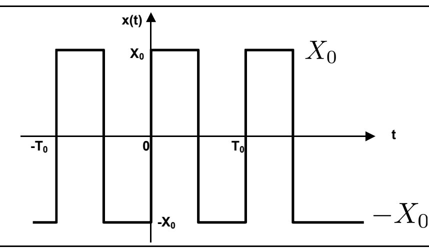
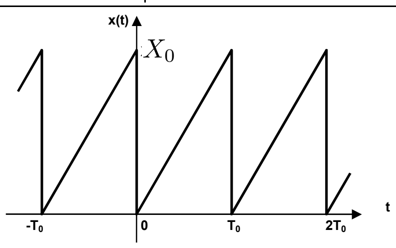
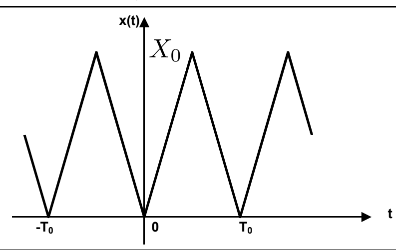
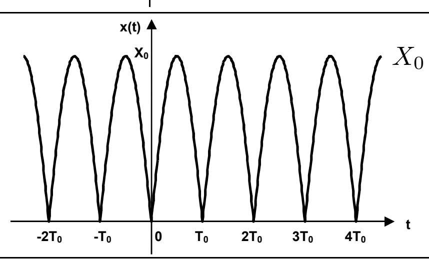
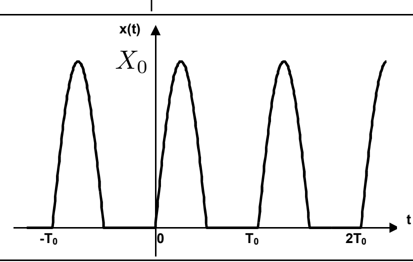
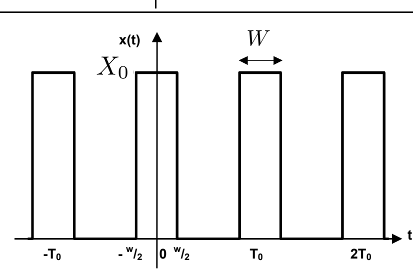
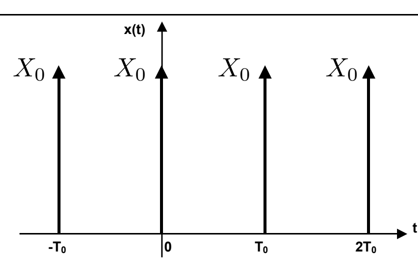
$$a\cos x + b\sin x = C\cos(x+\theta)$$

in which $C=\sqrt{a^2+b^2}$ and $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$



Note: $J_{-n}(\beta) = (-1)^n J_n(\beta)$

Table of Common Fourier Series

Name	Waveform	a_0	a_k	Comments
1. Square wave		0	$-j\frac{2X_0}{\pi k}$ when k is odd	$a_k = 0$ when k is even
2. Sawtooth		$\frac{X_0}{2}$	$j\frac{X_0}{2\pi k}$	
3. Triangular wave		$\frac{X_0}{2}$	$\frac{-2X_0}{(\pi k)^2}$ when k is odd	$a_k = 0$ when k is even
4. Full-wave rectified		$\frac{2X_0}{\pi}$	$\frac{2X_0}{\pi(1-4k^2)}$	
5. Half-wave rectified		$\frac{X_0}{\pi}$	$\frac{X_0}{\pi(1-k^2)}$ when k is even	$a_k = 0$ when k is odd, except $a_1 = -j\frac{X_0}{4}$
6. Rectangular Wave		$\frac{wX_0}{T_0}$	$\frac{wX_0}{T_0} sinc\frac{wk\omega_0}{2}$ $\frac{wX_0}{T_0}\frac{\sin\frac{wk\omega_0}{2}}{\frac{wk\omega_0}{2}}$ $\frac{X_0}{k\pi}\sin\frac{wk\omega_0}{2}$	$\omega_0 = \frac{2\pi}{T_0}$ $sinc(x) = \frac{\sin x}{x}$
7. Impulse train		$\frac{X_0}{T_0}$	$\frac{X_0}{T_0}$	

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = a_0 + \sum_{k=1}^{\infty} 2|a_k|\cos(k\omega_0 t + \angle a_k)$$

TABLE OF FOURIER TRANSFORMS

Time Domain $(x(t))$	Frequency Domain $(X(f))$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t-t_0)$	$e^{-j2\pi f t_0}$
$e^{j2\pi f_0 t}$	$\delta(f-f_0)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\delta(f-f_0)+\frac{1}{2}\delta(f+f_0)$
$\sin(2\pi f_0 t)$	$-\frac{1}{2j}\delta(f+f_0)+\frac{1}{2j}\delta(f-f_0)$
$\Pi(t)=\begin{cases}1, & \|t\|<\frac{1}{2}\\ \frac{1}{2}, & t=\pm\frac{1}{2}\\ 0, & \text{otherwise}\end{cases}$	$\text{sinc}(f)=\frac{\sin(\pi f)}{\pi f}$
$\text{sinc}(t)$	$\Pi(f)$
$\Lambda(t)=\begin{cases}t+1, & -1\le t<0\\ -t+1, & 0\le t<1\\ 0, & \text{otherwise}\end{cases}$	$\text{sinc}^2(f)$
$\text{sinc}^2(t)$	$\Lambda(f)$
$e^{-\alpha t}u_{-1}(t), \alpha>0$	$\frac{1}{\alpha+j2\pi f}$
$te^{-\alpha t}u_{-1}(t), \alpha>0$	$\frac{1}{(\alpha+j2\pi f)^2}$
$e^{-\alpha\|t\|}$	$\frac{2\alpha}{\alpha^2+(2\pi f)^2}$
$e^{-\pi t^2}$	$e^{-\pi f^2}$
$\text{sgn}(t)=\begin{cases}1, & t>0\\ -1, & t<0\\ 0, & t=0\end{cases}$	$1/(j\pi f)$
$u_{-1}(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
$\delta'(t)$	$j2\pi f$
$\delta^{(n)}(t)$	$(j2\pi f)^n$
$\frac{1}{t}$	$-j\pi\,\text{sgn}(f)$
$\sum_{n=-\infty}^{n=+\infty}\delta(t-nT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{n=+\infty}\delta\left(f-\frac{n}{T_0}\right)$

TABLE 2.2 TABLE OF FOURIER-TRANSFORM PROPERTIES

Signal	Fourier Transform
$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(f) + \beta X_2(f)$
$X(t)$	$x(-f)$
$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right)$
$x(t - t_0)$	$e^{-j2\pi f t_0} X(f)$
$e^{j2\pi f_0 t} x(t)$	$X(f - f_0)$
$x(t) \star y(t)$	$X(f)Y(f)$
$x(t)y(t)$	$X(f) \star Y(f)$
$\frac{d}{dt} x(t)$	$j2\pi f X(f)$
$\frac{d^n}{dt^n} x(t)$	$(j2\pi f)^n X(f)$
$tx(t)$	$\left(\frac{j}{2\pi}\right) \frac{d}{df} X(f)$
$t^n x(t)$	$\left(\frac{j}{2\pi}\right)^n \frac{d^n}{df^n} X(f)$
$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{X(f)}{j2\pi f} + \frac{1}{2} X(0)\delta(f)$