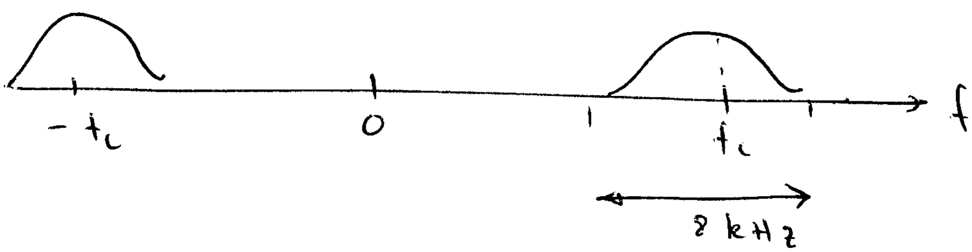
(1) (a) $z(t) = v(t)\cos 2\pi f_c t + v^2(t)\sin 2\pi f_c t$.

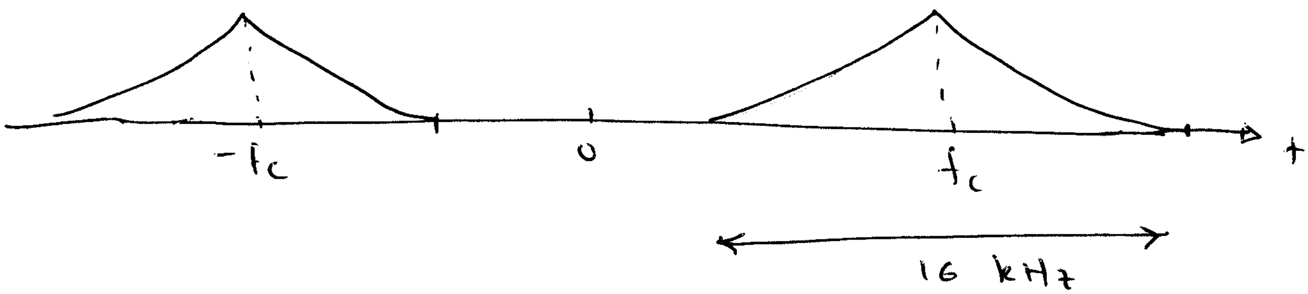
$v(t)$ has a bandwidth of 4 kHz

$\therefore$ $v^2(t)$ " " " 8 kHz.

Spectrum of $v(t)\cos 2\pi f_c t$:



Spectrum of $v^2(t)\cos 2\pi f_c t$:



$\therefore$ Bandwidth of $z(t)$ = $\underline{\underline{16 \text{ kHz}}}$.

(b) Clearly $g(t) = \hat{v}(t)$

$\therefore$ $z(t) = v(t)\cos 2\pi f_c t + \hat{v}(t)\sin 2\pi f_c t$.

This is an SSB signal (LSB signal).

$\therefore$ Bandwidth of $z(t) = \underline{\underline{4\text{ kHz}}}$

(2) (a) $u(t) = [A + m(t)]\cos 2\pi f_c t$

$= A\cos 2\pi f_c t + m(t)\cos 2\pi f_c t$

$$\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} u^2\,dt = \frac{A^2}{2} + \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\frac{m^2(t)}{2}\,dt.$$

$$= \underset{\substack{\uparrow \\ \text{carrier power}}}{\frac{A^2}{2}} + \underset{\substack{\uparrow \\ \text{side-band power}}}{\frac{P_m}{2}}.$$

$$\therefore \eta = \frac{P_m/2}{\frac{A^2}{2} + \frac{P_m}{2}} = \frac{P_m}{A^2 + P_m}.$$

modulation index $\mu = \dfrac{\text{negative peak of } m(t)}{A}$

$= 1$

$\therefore \dfrac{1}{A} = 1 \implies A = 1$

$P_m = \dfrac{(4 \times 0.5 + 1 \times 1)}{2} = 1.5$ (area under $m^2(t)$ over a period of 2 divided by 2)

$\therefore \eta = \dfrac{1.5}{1 + 1.5} = 0.6$ or 60%

(b) If $m(t)$ is replace by $-m(t)$:

$\dfrac{2}{A} = 1 \quad \therefore A = 2.$

Since P_m is unchanged, η is now decreased.

3 Using given data

unmodulated carrier $c(t) = \overset{A_c}{5} \cos(\overbrace{1000 \times 10^3 \pi}^{f_c/2} t)$

In FM, instantaneous frequency $f(t) = f_c + k_f m(t)$

modulator sensitivity

$\therefore$ int. angle $= \int_{-\infty}^{t} 2\pi f(\tau)\, d\tau = \theta(t)$

$\therefore \quad \theta(t) = 2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau.$

$\therefore$ FM signal $u(t) = 5 \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right]$

with $m(t) = A_m \cos 2\pi f_m t$

$$\int_{-\infty}^{t} m(\tau)\, d\tau = \frac{A_m}{2\pi f_m} \sin 2\pi f_m t \quad \text{(ignoring initial values)}.$$

$$\therefore \quad u(t) = 5 \cos\left[2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right]$$

with $A_m \cos 2\pi f_m t = 4 \cos(20 \times 10^3 \pi t)$

$$\frac{4 k_f}{10 \times 10^3} = 2 \quad \Rightarrow \quad k_f = 5 \text{ kHz/V}.$$

Now, with $m(t) = 2.5 \cos(50 \times 10^3 \pi t)$

$$\beta = \frac{5 \times 10^3 \times 2.5}{25 \times 10^3} = 0.5$$

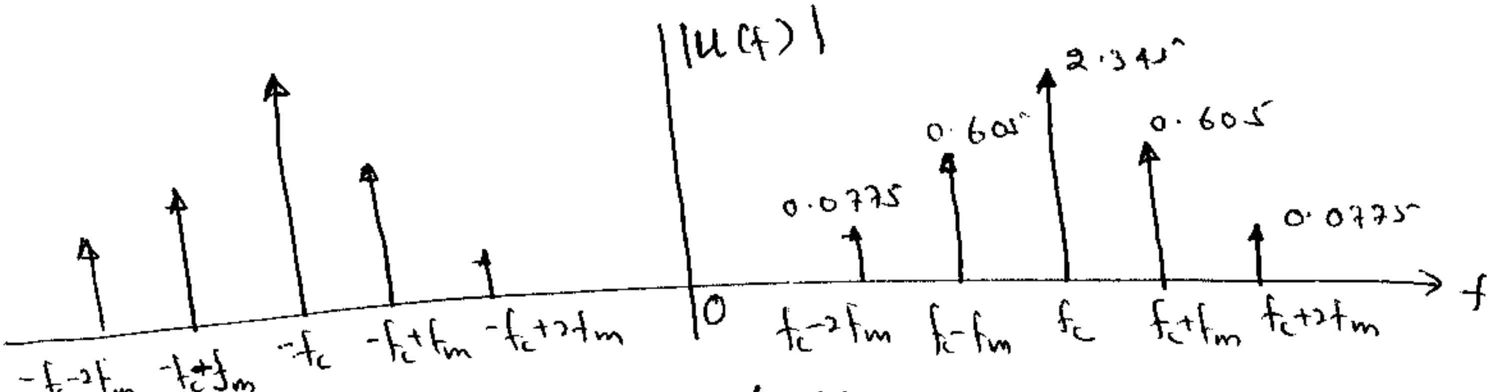
Hence the modulator output is

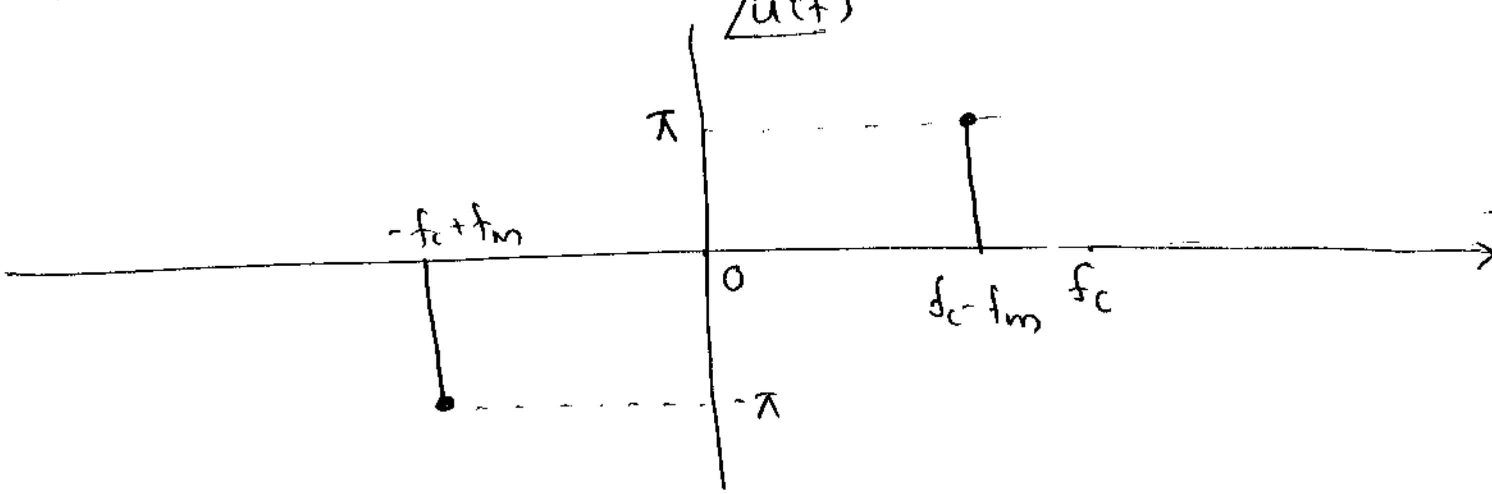
$$u(t) = 5 \sum_{n=-\infty}^{\infty} J_n(0.5) \cos 2\pi (f_c + f_m) t$$

$f_c = 500$ kHz
$f_m = 25$ kHz.

Using the Bessel function table:

$$u(t) = 5\Big[0.938 \cos 2\pi f_c t + 0.242 \cos 2\pi (f_c + f_m) t - 0.242 \cos 2\pi (f_c - f_m) t + 0.031 \cos 2\pi (f_c + 2f_m) t + 0.031 \cos 2\pi (f_c - 2f_m) t\Big]$$





4 FM Modulator output is

$$u(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right]$$

Since a PM demodulator detects the phase variation, demodulator output

$$x(t) = k_f \int_{0}^{t} m(\tau)\,d\tau, \quad \text{when } k_f = 1.$$