1. See lecture slides. The most important fundamental advantages of digital communication are

   (i) *Coding can be used for error correction.* This allows near error free communication over long distances and is perhaps the single most important advantage (the Internet and digital wireless communication would not have been possible without error correction coding). The possibility of digital instead of analog communication using error control coding and data compression was first described by Claude Shannon in his landmark 1948 paper "A mathematical theory of communication" which appeared in Bell Systems Technical Journal.

   (ii) *Digital signal processing and VLSI circuits allow very complex systems to be realized.* Analog systems used in practice, such as AM and FM, are not able to match the performance of their more complex digital counterparts.

   (iii) *Digital transmission allows packet-switching, routing, etc.,.* In comparison, the best known analog network was the circuit-switched telephone network (which is no longer in use).

   However, digital communication is not without weaknesses:

   (i) more complex and is not cost effective for simple applications.

   (ii) usually requires more channel bandwidth. This issue is also related to the previous point- bandwidth efficient digital transmission requires very complex coding and modulation techniques.

   (iii) suffers from a severe *threshold effect*- when error control coding fails, data cannot be decoded at all. In contrast, analog systems exhibit "graceful degradation" and can tolerate much more noise before completely failing (think about analog TV and digital TV). Analog systems also suffer from a less severe threshold effect- for example FM radio stations abruptly disappear at some point, as one drives away from the city (transmitter). The threshold effect in AM is negligible.

2. This problem is based on problem 14 of set 2 (textbook problem 3.16).

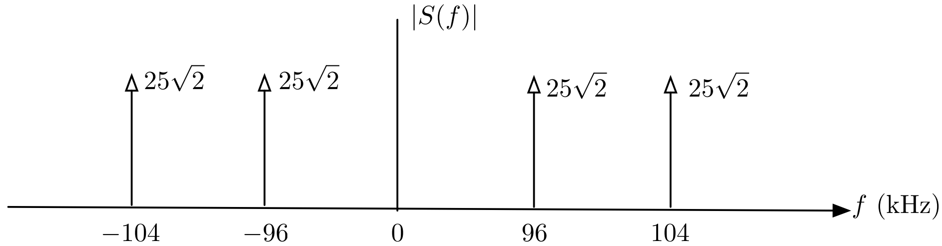
   (a) DSB-SC signal is
   $$s(t) = 100m(t)\cos(20 \times 10^4 \pi t)$$

   (b) There are several ways to solve this problem. Perhaps the simplest is the one shown in problem 14 of set 2. That is, using the last trig. identity in the test appendix
   $$\begin{aligned} m(t) &= \cos(8000\pi t) + \sin(8000\pi t) \\ &= \sqrt{2}\cos(8000\pi t - \frac{\pi}{4}) \end{aligned}$$

Then it is easy to show that

$$
\begin{aligned}
S(f) &= 25\sqrt{2}[\delta(f-104\times 10^3)e^{-j\pi/4} + \delta(f+104\times 10^3)e^{j\pi/4} \\
&\quad +\delta(f-96\times 10^3)e^{j\pi/4} + \delta(f+96\times 10^3)e^{-j\pi/4}]
\end{aligned}
$$

(c) The magnitude spectrum is



3. (a) See slide 12 under angle modulation.

   (b) See Example on slide 26.
   Let $m'(t) = \frac{d}{dt}m(t)$. For the FM modulator, the instantaneous frequency and angle respectively of the modulated carrier are given by

$$
\begin{aligned}
f(t) &= f_c + k_f m'(t) \text{ Hz} \\
\theta(t) &= 2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m'(\tau)d\tau \\
&= 2\pi f_c t + \underbrace{2\pi k_f m(t)}_{\phi(t)} \text{ rad.}
\end{aligned}
$$

   where $\phi(t)$ is the instantaneous phase deviation. Therefore the maximum phase deviation is $2\pi k_f \max\{m(t)\}$. From the given plot of $\max\{m(t)\} \approx 1.75$ V, and hence $k_f \approx 0.136$ Hz/V.

4. According to the spectrum shown, $\frac{A_c}{2}J_0(\beta) = \frac{A_c}{2}J_1(\beta) = 0.55$, $\frac{A_c}{2}J_2(\beta) = 0.2$, $\frac{A_c}{2}J_3(\beta) = 0.05$ and $J_n(\beta) \approx 0$ for $n > 3$. From the Bessel function plot, we find that this corresponds to $\beta \approx 1.5$. Note that A_c need not be known to determine β from the spectrum, as $\frac{A_c}{2}$ is a common factor.

5. This problem is similar to the problem 1 of set 3 (textbook 4.16).

   (a) Instantaneous frequency is $f(t) = f_c + k_f m(t)$ (Hz), where $f_c = 10^6$ Hz and $k_f = 1.2$ kHz/V. Hence frequency deviation $\Delta f(t) = k_f m(t)$ as a function of t is simply

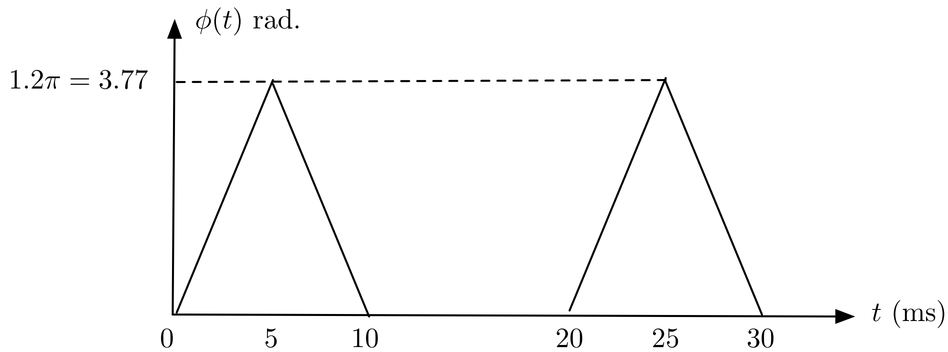
$$\Delta f(t) = 1200m(t) \text{ Hz.}$$

   The plot of $\Delta f(t)$ is identical to that of $m(t)$ in shape, except that the maximum and minimum values respectively are 120 Hz and -120 Hz.

(b) Instantaneous phase angle is

$$
\begin{aligned}
\theta(t) &= 2\pi \int_{\infty}^{t} f(\tau) d\tau \\
&= 2\pi f_c t + \underbrace{2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau}_{\phi(t)}
\end{aligned}
$$

where $\phi(t)$ is the phase deviation. Since $\int m(\tau)d\tau$ is the area under $m(t)$, $\phi(t)$ as a function of t is as follows.



Note that the maximum value is the area of $m(t)$ from 0 to 5 ms (a rectangle): $2\pi \times 1.2 \times 10^3 \times 0.1 \times 5 \times 10^{-3} = 1.2\pi$ rad.

(c) To estimate the bandwidth of the modulated signal, we have to use the Carson's rule. Since this is FM

$$B = 2\Delta f + 2W$$

where $\Delta f = 120$ Hz is the maximum frequency deviation (from part a), and W is the maximum frequency in the message signal. Since most of the total power of the message signal is contained in it's first 30 harmonics

$$W = 30 \frac{1}{20 \times 10^{-3}} = 1500 \text{ Hz.}$$

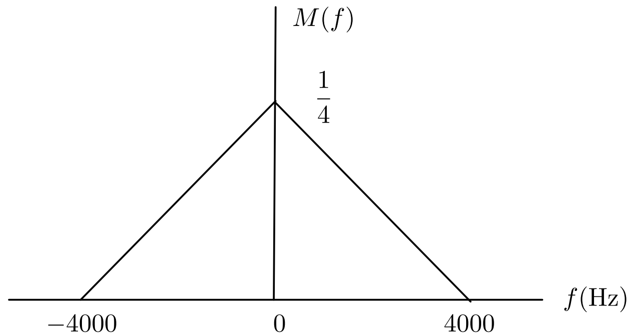
Hence the bandwidth of the modulated signal is $B \approx 3.4$ kHz.

(d) An angle modulated signal has the form $A_c \cos(2\pi f_c t + \phi(t))$ Hence, the average power is $P = \frac{1}{2} A_c^2$ where $A_c = 100$ mV is given ($P = 5$ mW.)

6. This problem is based on the assignment 2 and the problems 8 and 9 in set 2.

(a)

$$M(f) = \frac{1}{4} \Delta \left(\frac{f}{4000} \right)$$



(b) Using the Fourier series of $x(t)$, we find that

$$m(t)x(t) = \frac{8}{\pi}m(t) + \sum_{k=1}^{\infty} \frac{16m(t)}{\pi(1-k^2)} \cos(2\pi k f_0 t)$$

where $f_0 = \frac{1}{20\times 10^{-6}} = 200$ kHz. This signal has the message itself and a sum of DSB-SC signals at carrier frequencies $f_0, 2f_0, 3f_0, \ldots$. Since the BPF has a center frequency 198 kHz and a bandwidth of 4 kHz, it only passes the signal components in the frequency range 198 ± 2 kHz, that is $196 - 200$ kHz. Therefore, the BPF output $y(t)$ is the bandpass filtered version of

$$y_1(t) = -\frac{16}{15\pi} m(t) \cos(400 \times 10^3 \pi t)$$

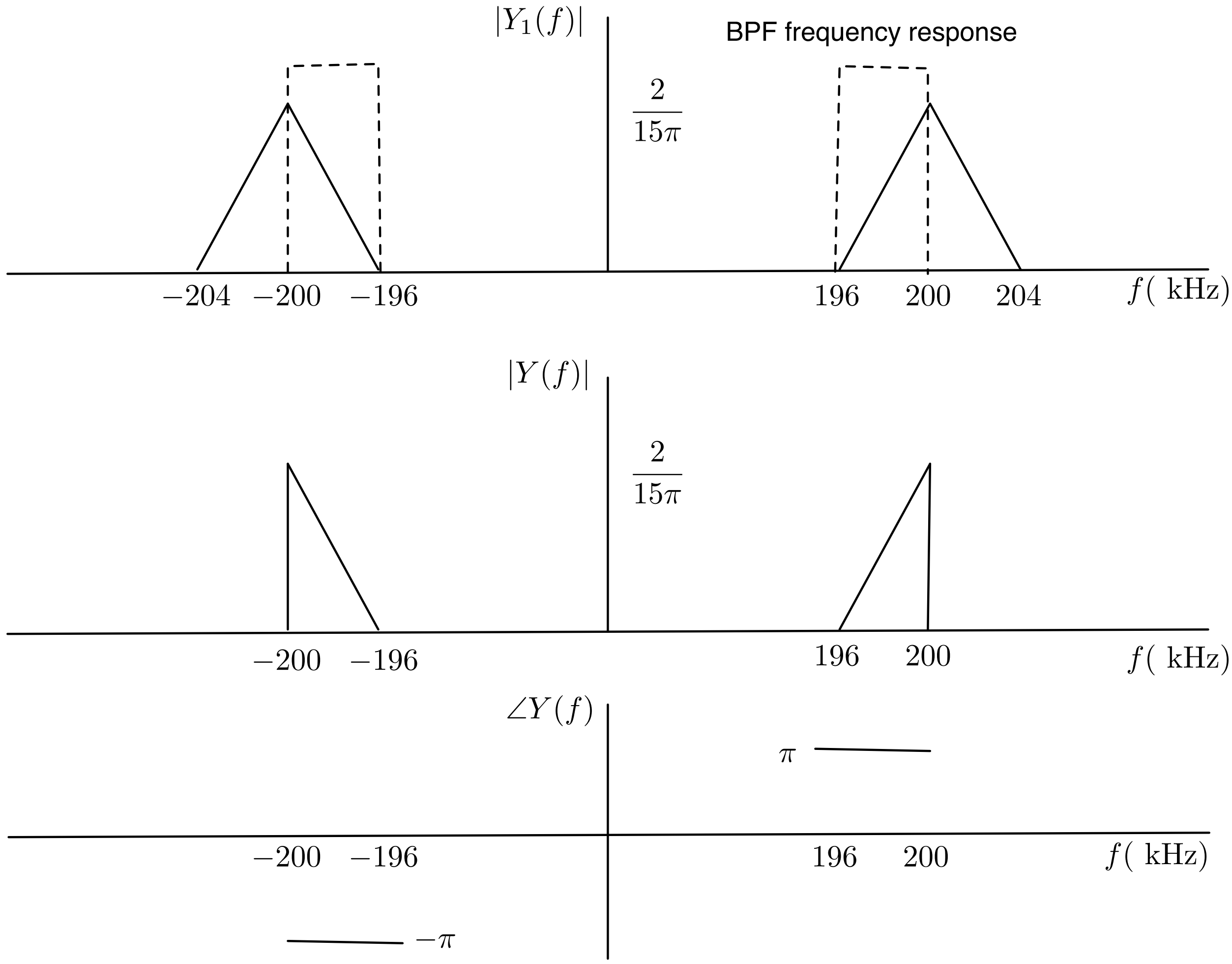
where

$$Y_1(f) = -\frac{8}{15\pi} \left[M(f - 2f_0) + M(f + 2f_0)\right]$$

$Y(f)$ is therefore as shown in the figure below.

(c) Since only the lower sideband appears at the output, this is LSSB.

(d) For LSSB, the upper edge of the sideband is the carrier frequency, i.e., $f_c = 2f_0 =$ 200 kHz.


$|Y_1(f)|$
BPF frequency response
$\frac{2}{15\pi}$
-204 -200 -196
196 200 204
$f($ kHz$)$
$|Y(f)|$
$\frac{2}{15\pi}$
-200 -196
196 200
$f($ kHz$)$
$\angle Y(f)$
π
-200 -196
196 200
$f($ kHz$)$
$-\pi$