- **Closed-book exam**:
  - No printed or handwritten material of any form allowed.
  - No Internet access or any other form of electronic media permitted.
  - A non-programmable calculator may be used.
- **Answer all 4 questions**. For full credit, you must clearly show how you arrive at the solution, including all relevant calculations. Solutions without intermediate steps will not receive any marks.

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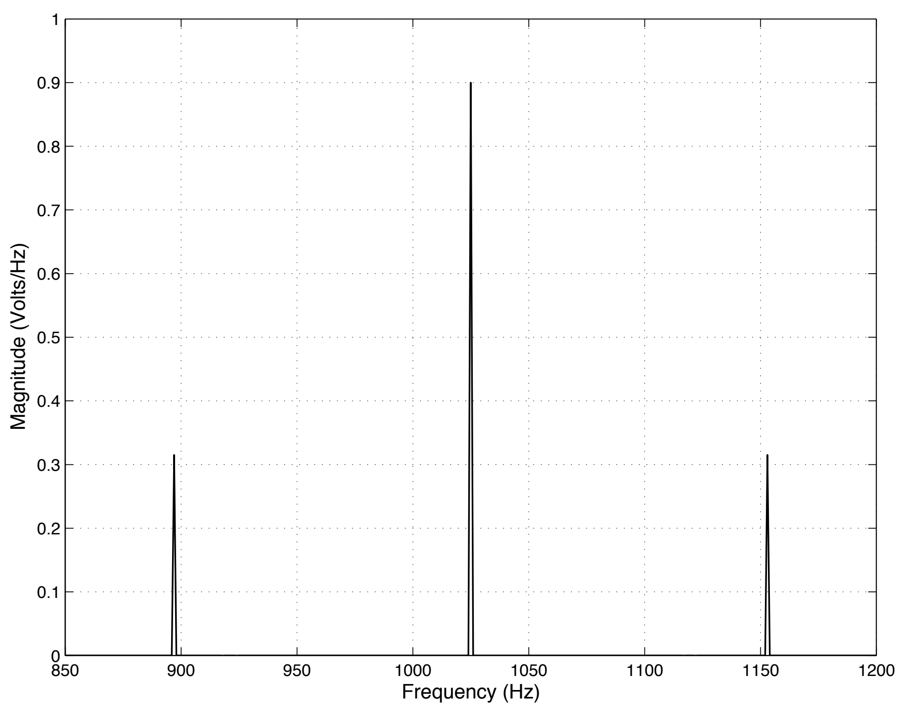
3 Marks

1. Provide short (but clear) answers.
   (a) What are the most important advantages of digital communication over analog communication?
   (b) What is the significance of the modulation index in conventional AM?
   (c) What is the main advantage of FM compared to AM?

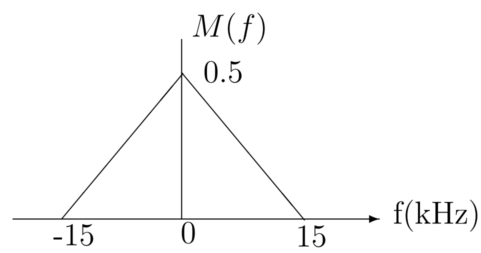
2. (a) The following frequency spectrum was recorded (plotted to scale) on a spectrum analyzer, when a sinusoidal message signal having a dc off set is applied to a DSB-SC modulator.



2 Marks i. Estimate the modulation index.

1 Mark ii. By how much should the dc off set be changed to achieve a 100 % modulation index? Explain why.

4 Marks (b) A message signal $m(t)$, whose frequency spectrum is shown below, is transmitted using lower sideband suppressed carrier amplitude modulation. The unmodulated carrier is $50\cos(2\pi f_c t)$, where $f_c = 500$ kHz.

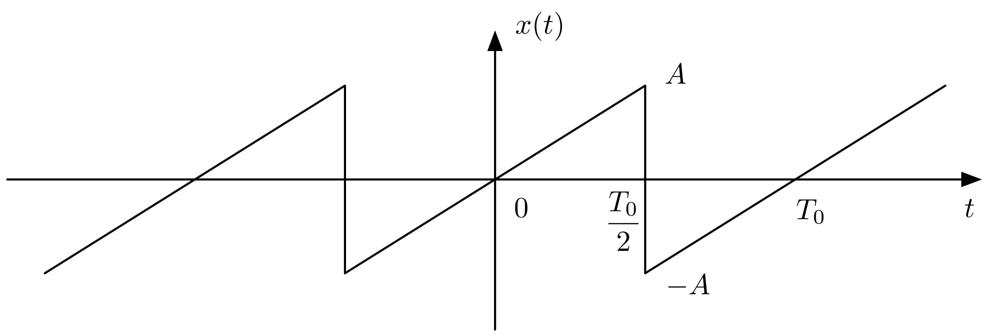


The transmitted signal is demodulated by a receiver with an imperfect synchronous demodulator whose local oscillator output is $\cos(1100\pi \times 10^3 t)$, where t is in seconds. Assume that the demodulator employs an ideal low-pass filter which has a cut-off frequency equal to the message bandwidth.

i. Plot the two-sided magnitude spectrum of the input to the LPF in the demodulator. Indicate all important values on your plots.

ii. Plot the spectrum of the demodulated signal.

3. (a) The periodic signal $x(t)$ shown below (assume that amplitude is in Volts) is used as the input to a frequency modulator whose sensitivity is 3 Hz/volt.



The Fourier series expansion of $x(t)$ is

$$x(t) = \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin\left(\frac{2\pi n}{T_o} t - n\pi\right)$$

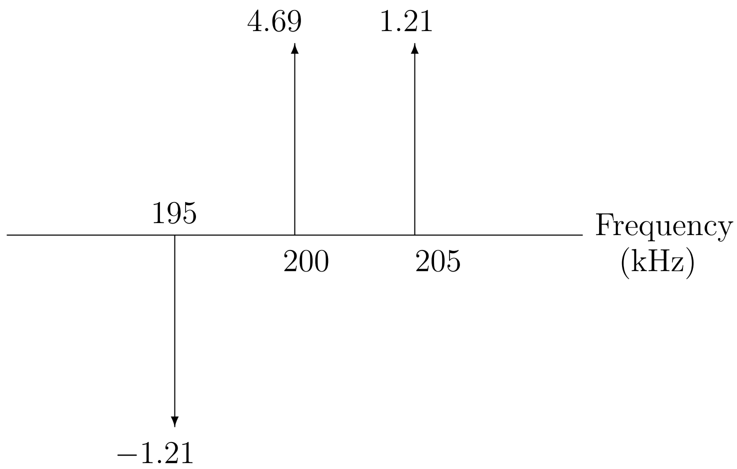
Assume $A = 2$ Volts and $T_0 = 0.25$ sec. Also consider the modulated carrier phase to be 0 rad. at $t = 0$ (i.e., the phase reference)

2 Marks i. Calculate the peak phase deviation (in radians) of the modulator output.

4 Marks ii. By considering 90%-power bandwidth as the bandwidth of $x(t)$, find an estimate for the bandwidth of the modulator output (clearly show how you obtain your estimate).

1 Marks (b) The signal $x(t)$ is applied to a phase modulator with a sensitivity of 0.6 radians/volt. Find an estimate for the bandwidth of the modulator output.

3 Marks 4. (a) Suppose a *cosine wave* is used as the message signal in a frequency modulator whose sensitivity is 10 kHz per volt. The significant components of the frequency spectrum of the resulting FM signal are shown below (only the positive half of the frequency axis is shown here and spectral components with phase shifts of π are shown as negative impulse functions).



Calculate the amplitude and the frequency of the message signal.

——— END OF TEST ———

Appendix

- Carson's rule: $2(\beta+1)W$
- **Trigonometric identities**:

$$\cos[2\pi f_c t + a\sin 2\pi f_m t] = \sum_{n=-\infty}^{\infty} J_n(a)\cos\left[2\pi(f_c+nf_m)t\right]$$

$$\cos[2\pi f_c t + a\cos 2\pi f_m t] = \sum_{n=-\infty}^{\infty} J_n(a)\cos\left[2\pi(f_c+nf_m)t + \frac{n\pi}{2}\right]$$

$$e^{\pm jx} = \cos x \pm j\sin x$$

$$\cos x = \frac{1}{2}(e^{jx}+e^{-jx})$$

$$\sin x = \frac{1}{2j}(e^{jx}-e^{-jx})$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin x$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos x$$

$$2\sin x\cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1-\cos 2x)$$

$$\cos^3 x = \frac{1}{4}(3\cos x + \cos 3x)$$

$$\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$$

$$\sin(x\pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x\pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\tan(x\pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$

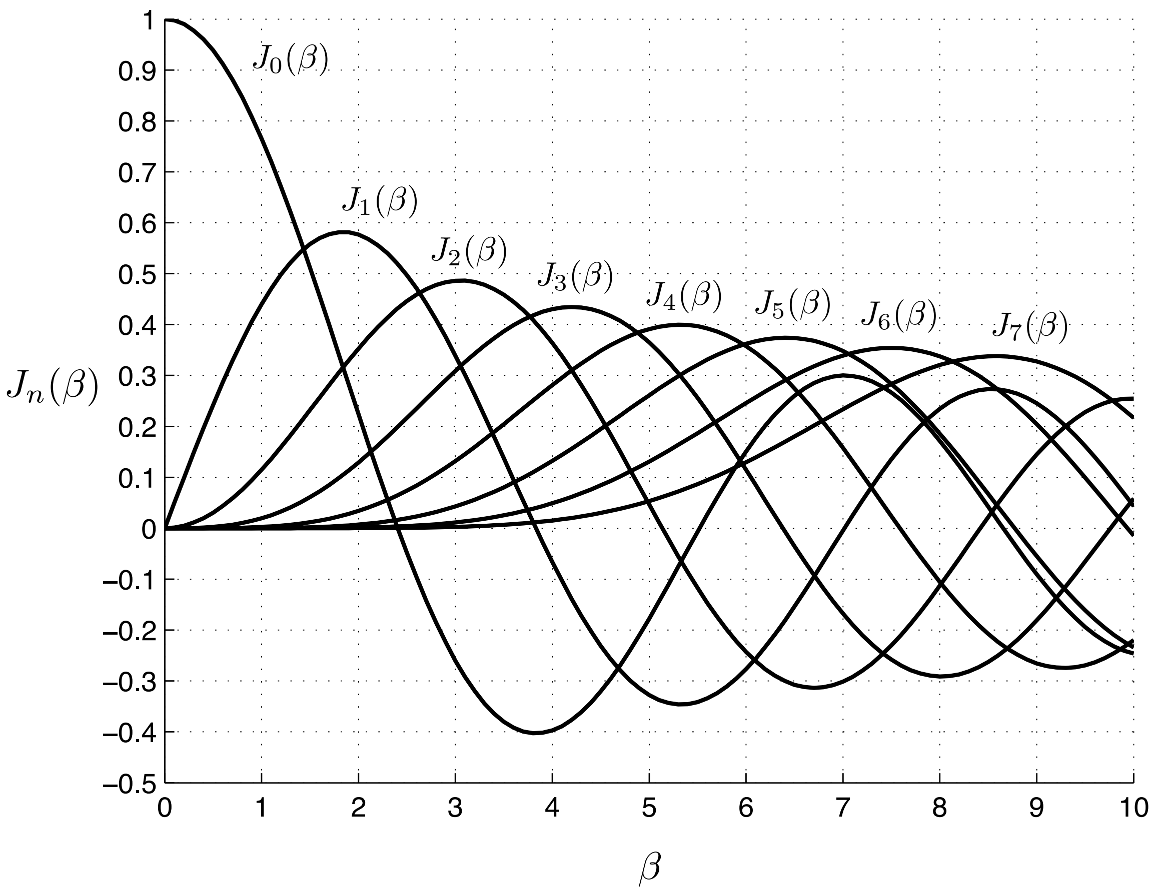
$$\sin x\sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x\cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$$

$$\sin x\cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)]$$

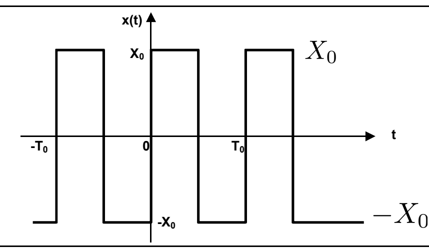
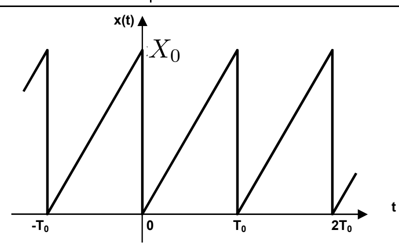
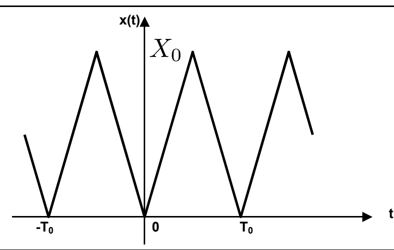
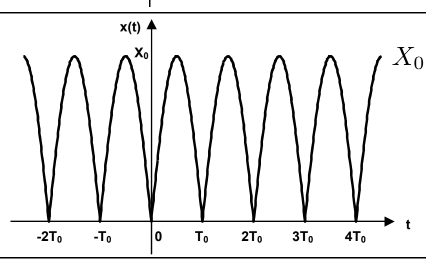
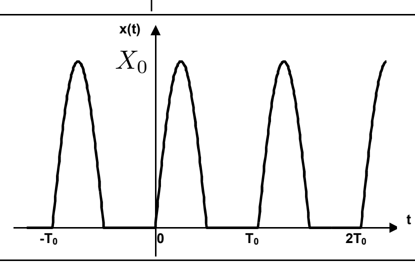
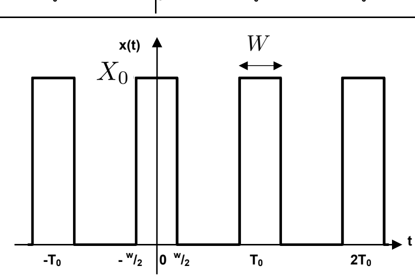
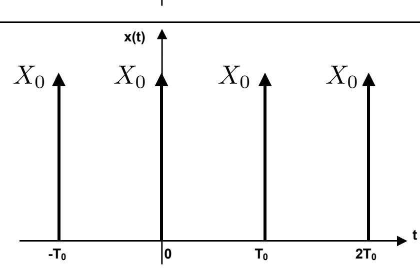
$$a\cos x + b\sin x = C\cos(x+\theta)$$

in which $C = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$



Note: $J_{-n}(\beta) = (-1)^n J_n(\beta)$

Table of Common Fourier Series

Name	Waveform	a_0	a_k	Comments
1. Square wave		0	$-j\dfrac{2X_0}{\pi k}$ when k is odd	$a_k = 0$ when k is even
2. Sawtooth		$\dfrac{X_0}{2}$	$j\dfrac{X_0}{2\pi k}$	
3. Triangular wave		$\dfrac{X_0}{2}$	$\dfrac{-2X_0}{(\pi k)^2}$ when k is odd	$a_k = 0$ when k is even
4. Full-wave rectified		$\dfrac{2X_0}{\pi}$	$\dfrac{2X_0}{\pi(1-4k^2)}$	
5. Half-wave rectified		$\dfrac{X_0}{\pi}$	$\dfrac{X_0}{\pi(1-k^2)}$ when k is even	$a_k = 0$ when k is odd, except $a_1 = -j\dfrac{X_0}{4}$
6. Rectangular Wave		$\dfrac{wX_0}{T_0}$	$\dfrac{wX_0}{T_0} sinc \dfrac{wk\omega_0}{2}$ $\dfrac{wX_0}{T_0}\dfrac{\sin\frac{wk\omega_0}{2}}{\frac{wk\omega_0}{2}}$ $\dfrac{X_0}{k\pi}\sin\dfrac{wk\omega_0}{2}$	$\omega_0 = \dfrac{2\pi}{T_0}$ $sinc(x) = \dfrac{\sin x}{x}$
7. Impulse train		$\dfrac{X_0}{T_0}$	$\dfrac{X_0}{T_0}$	

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = a_0 + \sum_{k=1}^{\infty} 2|a_k|\cos(k\omega_0 t + \angle a_k)$$

TABLE OF FOURIER TRANSFORMS

Time Domain ($x(t)$)	Frequency Domain ($X(f)$)
$\delta(t)$	1
1	$\delta(f)$
$\delta(t-t_0)$	$e^{-j2\pi f t_0}$
$e^{j2\pi f_0 t}$	$\delta(f-f_0)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}\delta(f-f_0)+\frac{1}{2}\delta(f+f_0)$
$\sin(2\pi f_0 t)$	$-\frac{1}{2j}\delta(f+f_0)+\frac{1}{2j}\delta(f-f_0)$
$\Pi(t)=\begin{cases}1, & \lvert t\rvert<\frac{1}{2}\\ \frac{1}{2}, & t=\pm\frac{1}{2}\\ 0, & \text{otherwise}\end{cases}$	$\text{sinc}(f)=\frac{\sin(\pi f)}{\pi f}$
$\text{sinc}(t)$	$\Pi(f)$
$\Lambda(t)=\begin{cases}t+1, & -1\le t<0\\ -t+1, & 0\le t<1\\ 0, & \text{otherwise}\end{cases}$	$\text{sinc}^2(f)$
$\text{sinc}^2(t)$	$\Lambda(f)$
$e^{-\alpha t}u_{-1}(t), \alpha>0$	$\frac{1}{\alpha+j2\pi f}$
$te^{-\alpha t}u_{-1}(t), \alpha>0$	$\frac{1}{(\alpha+j2\pi f)^2}$
$e^{-\alpha\lvert t\rvert}$	$\frac{2\alpha}{\alpha^2+(2\pi f)^2}$
$e^{-\pi t^2}$	$e^{-\pi f^2}$
$\text{sgn}(t)=\begin{cases}1, & t>0\\ -1, & t<0\\ 0, & t=0\end{cases}$	$1/(j\pi f)$
$u_{-1}(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
$\delta'(t)$	$j2\pi f$
$\delta^{(n)}(t)$	$(j2\pi f)^n$
$\frac{1}{t}$	$-j\pi\,\text{sgn}(f)$
$\sum_{n=-\infty}^{n=+\infty}\delta(t-nT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{n=+\infty}\delta\left(f-\frac{n}{T_0}\right)$

TABLE 2.2 TABLE OF FOURIER-TRANSFORM PROPERTIES

Signal	Fourier Transform
$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(f) + \beta X_2(f)$
$X(t)$	$x(-f)$
$x(at)$	$\frac{1}{\lvert a \rvert} X\left(\frac{f}{a}\right)$
$x(t - t_0)$	$e^{-j2\pi f t_0} X(f)$
$e^{j2\pi f_0 t} x(t)$	$X(f - f_0)$
$x(t) \star y(t)$	$X(f)Y(f)$
$x(t)y(t)$	$X(f) \star Y(f)$
$\frac{d}{dt} x(t)$	$j2\pi f X(f)$
$\frac{d^n}{dt^n} x(t)$	$(j2\pi f)^n X(f)$
$tx(t)$	$\left(\frac{j}{2\pi}\right) \frac{d}{df} X(f)$
$t^n x(t)$	$\left(\frac{j}{2\pi}\right)^n \frac{d^n}{df^n} X(f)$
$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{X(f)}{j2\pi f} + \frac{1}{2} X(0)\delta(f)$