**Homework 6 – Algebraic Structures**

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Ch 7: 7.2, 7.4, 7.5, 7.9, 7.10, 8.3, 8.4, 8.5, M.5, M.6
Pre-lect: 5.1

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**Exercise** (7.7.2)**.** *Let $G_1 \subset G_2$ be groups whose orders are divisible by $p$, and let $H_1$ be a Sylow $p$-subgroup of $G_1$. Prove that there is a Sylow $p$-subgroup $H_2$ of $G_2$ such that $H_1 = H_2 \cap G_1$.*

*Proof.* Put a tautology here. □

**Exercise** (7.7.4)**.**

1. *Prove that no simple group has order $pq$, where $p$ and $q$ are prime.*
2. *Prove that no simple group has order $p^2q$, where $p$ and $q$ are prime.*

*Proof.* □

**Exercise** (7.7.5)**.** *Find Sylow 2-subgroups of $D_{10}$.*

*Proof.* □

**Exercise** (7.7.9)**.** *Classify groups of order* ***(1)*** *33* ***(2)*** *18.*

*Proof.* □

**Exercise** (7.7.10)**.** *Prove that the only simple groups of order ¡60 are the groups of prime order.*

*Proof.* □

**Exercise** (7.8.3)**.** *Determine the class equations of the groups of order 12.*

*Proof.* □

**Exercise** (7.8.4)**.** *Prove that a group of order $n = 2p$, where $p$ is a prime, is either cyclic or dihedral.*

*Proof.* □

**Exercise** (7.8.5)**.** *Let $G$ be a nonabelian group of order 28 whose sylow 2 subgroups are cyclic.*

1. *Determine the numbers of sylow 2 subgroups and sylow 7 subgroups.*
2. *Prove that there is at most one isomorphism class of such groups.*
3. *Determine the numbers of elements of each order, and the class equation of $G$.*

*Proof.* □

**Exercise** (7.M.5)**.** *Let $H$ and $N$ be subgroups of a group $G$, and assume that $N$ is a normal subgroup.*

1. *Determine the kernels of the restrictions of the canonical homomorphism $\pi : G \to G/N$ ot the subgroups $H$ and $HN$.*
2. *Applying First Isomorphism Theorem to these restrictions, prove the Second Isomorphism Theorem: $H/(H \cap N)$ is isomorphic to $(HN)/N$.*

*Proof.* □

**Exercise** (7.M.6)**.** *Let $H$ and $N$ be normal subgroups of a group $G$ such that $H \supset N$. Let $\overline{H} = H/N$ and $\overline{G} = G/N$.*

1. *Prove that $\overline{H}$ is a normal subgroup of $\overline{G}$.*
2. *Use the composed homomorphism $G \to \overline{G} \to \overline{G}/\overline{H}$ to prove the Third Isomorphism Theorem: $G/H$ is isomorphic to $\overline{G}/\overline{H}$.*

*Proof.* □