Homework 4 – Algebraic Structures

Ch 7: 3.1, 3.3, 3.4, 4.7, 4.9, 5.3, 5.6, 5,7, 5.12, 6.4, 6.5
Pre-lect: 5.1

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Exercise (7.3.1)**.** *Prove the Fixed Point Theorem (7.3.2).*

Proof. □

Exercise (7.3.3)**.** *A nonabelian group G has order p^3, where p is prime.*

1. *What are the possible orders of the center Z?*
2. *Let x be an element of G that isn't in Z. What is the order of its centralizer $Z(x)$?*
3. *What are the possible class equations for G?*

Proof. □

Exercise (7.3.4)**.** *Classify groups of order 8.*

Proof. □

Exercise (7.4.7)**.** *Let G be a group of order n that operates nontrivially on a set of order r. Prove that if $n > r!$, then G has a proper normal subgroup.*

Proof. □

Exercise (7.4.9)**.** *Let x be an element of a group G, not the identity, whose centralizer $Z(x)$ has order pq, where p and q are primes. Prove that $Z(x)$ is abelian.*

Proof. □

Exercise (7.5.3)**.** *Determine the orders of the elements of the symmetric group S_7.*

Proof. □

Exercise (7.5.6)**.** *Find all subgroups of S_4 of order 4, and decide which ones are normal.*

Proof. □

Exercise (7.5.7). *Prove that A_n is the only subgroup of S_n of index 2.*

Proof. □

Exercise (7.5.12). *Determine the class equations of S_6 and A_6.*

Proof. □

Exercise (7.6.4). *Let H be a normal subgroup of prime order p in a finite group G. Suppose that p is the smallest prime that divides the order of G. Prove that H is in the center $Z(G)$.*

Proof. □

Exercise (7.6.5). *Let p be a prime integer and let G be a p-group. Let H be a proper subgroup of G. Prove that the normalizer $N(H)$ of H is strictly larger than H, and that H is contained in a normal subgroup of index p.*

Proof. □

Pre-Lecture Problems

Exercise (7.5.1).

1. *Prove that the transpositions $(\mathbf{12}), (\mathbf{23}), \ldots (\mathbf{n-1}, \mathbf{n})$ generate the symmetric group S_n.*
2. *How many transpositions are needed to write the cycle $(\mathbf{123}\ldots\mathbf{n})$?*
3. *Prove that the cycles $(\mathbf{12}\ldots\mathbf{n})$ and $(\mathbf{12})$ generate the symmetric group S_n.*

Proof. □