MATH3202
Assignment 2
Practical Group: P02 (Wednesday 10am)

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Section A: Supervisor Report

Sets

N	Nodes	{Node #, X, Y, {D0, D1, ..., D5}}
G	Main Generator Nodes	{Number} ((Demand **=0**) $\implies$ Generator)
A	Arcs (i.e. Transmission Lines)	{Number: (Node 1, Node 2)}
U	Unrestricted Arcs	{Number}
R	Restricted Arcs	{Number}
T	Time Periods	$\{0, 1, 2, 3, 4, 5\}$ (0 $\implies$ 12am to 4am, 1 $\implies$ 4am to 8am, etc.)
B	MainGeneratorsAndBannedNode	{Number}

Data

d_{nt}	Standard demand of node $n \in N$ at time period $t \in T$ (MW)
c_n	Total capacity of main generator node $n \in G$ (MW)
cost_n	Standard cost of main generator node $n \in G$ (\$/MWh)
$l = 83$ MW	Base capacity for all restricted arcs (i.e. $\forall a \in R$)
(x_n, y_n)	Co-ordinates of node $n \in N$
length_a	Length of arc $a \in A$ (Calculated via Euclidean norm - assumes arcs are straight lines)
$\text{powerLoss} = 0.001$	0.1% Loss in power through arc per km
$\text{timePeriodDuration} = 4$	Duration of each time period (i.e. 4 hours)
(f_a, t_a)	From and to node for arc $a \in A$
$\text{numIncreasedArcLimit} = 3$	The number of arcs $a \in R$ with the increased capacity
$\text{numGasGenerators} = 1$	The number of gas generator nodes $n \in N$
$\text{arcCapacityIncrease} = 50$	The amount to increase the capacity of a restricted arc $a \in R$ by (MW)
$\text{gasGeneratorCost} = 77$	The cost of the gas generator node $n \in B^C$ (\$/MWh)
$\text{gasGeneratorCapacity} = 200$	Capacity of the gas generator node $n \in B^C$ (MW)
$\text{bannedNode} = 3$	The substation $n \in N$ which the gas generator is not allowed to be installed at
$\text{gasGeneratorNumTimePeriods} = 4$	Number of time periods which the gas generator can be run in
$\text{numSolarFarms} = 1$	The number of solar farms to be built
$\text{solarFarmCost} = 42$	The cost of the solar farm (in MW) built at node $n \in N$
$\text{solarFarmCapacities}_t$	The capacity of the solar farm during time period $t \in T$
$\text{efficientCapacityScale} = 0.6$	Percentage of capacity in which a main generator is running efficiently
$\text{costScale} = 0.3$	Scale factor to increase main generator cost by when producing more than (100 * efficientCapacityScale)% of its capacity
$\text{demandScale} = 0.1$	Scale factor of reduced demand
$\text{numDemandReduceTime} = 1$	Maximum number of time periods in which customers will strive to reduce their demand at any given node
$\text{numDemandReduceNode} = 9$	Maximum number of nodes in which customers will strive reduce their demand, during any given time period

Variables

x_{at}	amount of power (in MW/hr) in arc $a \in$ A at the beginning of time period $t \in T$
y_{nt}	amount of power produced efficiently (in MW/hr - i.e. up to [efficientCapacityScale $*$ 100]% generator capacity) by main generator node $n \in N$ at the beginning of time period $t \in T$
g_{nt}	amount of power generated (in MW/hr) by the gas generator node $n \in N$ at the beginning of time period $t \in T$
s_{nt}	amount of power generated (in MW/hr) by the solar farm at node $n \in N$ at the beginning of time period $t \in T$
z_{nt}	amount of power produced inefficiently (in MW/hr - i.e. once a generator is above 60% of its capacity) by main generator node $n \in N$ at the beginning of time period $t \in T$

$$p_a = \begin{cases} 1 & \text{if capacity of restricted arc } a \in R \text{ should be increased by arcCapacityIncrease} \\ 0 & \text{otherwise} \end{cases}$$

Binary variable which determines if a given restricted arc $a \in R$ has an increased capacity

$$k_n = \begin{cases} 1 & \text{if building the gas generator at node } n \in B^C \\ 0 & \text{otherwise} \end{cases}$$

Binary variable which determines if a given substation node $n \in B^C$ is the gas generator

$$o_t = \begin{cases} 1 & \text{if the gas generator should be running in time period } t \in T \\ 0 & \text{otherwise} \end{cases}$$

Binary variable which determines if the gas generator node should be running in time period $t \in T$

$$j_n = \begin{cases} 1 & \text{if we should build the solar farm at node } n \in N \\ 0 & \text{otherwise} \end{cases}$$

Binary variable which determines if we should build the solar farm at node $n \in N$

$$e_{nt} = \begin{cases} 1 & \text{If demand at node } n \in N \text{ during time period t in T is reduced by [demandScale * 100]\%} \\ 0 & \text{otherwise} \end{cases}$$

Binary variable which determines if a given node $n \in N$ should have its overall demand reduced in a given time period $t \in T$

Objective

$$\min \left(\text{timePeriodDuration} \left(\sum_{t \in T} \left(\sum_{n \in G} ((\text{cost}_n)(y_{nt}) + (1 + \text{costScale})(\text{cost}_n)(z_{nt})) + \sum_{n \in G^C} ((\text{gasGeneratorCost})(g_{nt})) + \sum_{n \in N} ((\text{solarFarmCost})(s_{nt})) \right) \right) \right)$$

Constraints

$$
\begin{aligned}
y_{nt} &\le \text{efficientCapacityScale} * c_n && \forall n \in G, \forall t \in T && (1)\\
y_{nt} + z_{nt} &\le c_n && \forall n \in G, \forall t \in T && (2)\\
y_{nt} &= 0 && \forall n \in G^C, \forall t \in T && (3)\\
z_{nt} &= 0 && \forall n \in G^C, \forall t \in T && (4)
\end{aligned}
$$

$$
\begin{aligned}
&\left(\sum_{\substack{a\in A\\ \text{s.t.}\, t_a = n}} x_{at} * (1 - \text{powerLoss} * \text{length}_a)\right) + y_{nt} + g_{nt} + \\
&s_{nt} + z_{nt} = d_{nt} - \text{demandScale} * d_{nt} * e_{nt} + \sum_{\substack{a\in A\\ \text{s.t.}\, f_a = n}} x_{at} \qquad \forall n \in N, \forall t \in T \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
x_{at} &\le l + \text{arcCapacityIncrease} * p_a && \forall a \in R, \forall t \in T && (6)\\
\sum_{a\in R} p_a &= \text{numIncreasedArcLimit} && && (7)\\
g_{nt} &\le \text{gasGeneratorCapacity} * k_n && \forall n \in B^C, \forall t \in T && (8)\\
g_{nt} &= 0 && \forall n \in B, \forall t \in T && (9)\\
\sum_{n\in N} k_n &= \text{numGasGenerators} && && (10)\\
g_{nt} &\le \text{gasGeneratorCapacity} * o_t && \forall n \in B^C, \forall t \in T && (11)\\
\sum_{t\in T} o_t &= \text{gasGeneratorNumTimePeriods} && && (12)\\
s_{nt} &\le \text{solarFarmCapacities}_t * j_n && \forall n \in N, \forall t \in T && (13)\\
\sum_{n\in N} j_n &= \text{numSolarFarms} && && (14)\\
\sum_{t\in T} e_{nt} &\le \text{numDemandReduceTime} && \forall n \in N && (15)\\
\sum_{n\in N} e_{nt} &\le \text{numDemandReduceNode} && \forall t \in T && (16)\\
x_{at} \ge 0,\ y_{nt} \ge 0, g_{nt} &\ge 0, s_{nt} \ge 0, z_{nt} \ge 0 && \forall a \in A, \forall n \in N, \forall t \in T && (17)
\end{aligned}
$$

- Constraint (1) ensures the power (efficiently) produced by a main generator is bounded by 60% of its capacity at all times.
- Constraint (2) ensures the total amount of power produced by a main generator does not exceed its capacity at any given time.
- Constraints (3) and (4) ensure substation nodes cannot 'produce power' from a main generator (i.e. main generators only exist at the main generator nodes)
- Constraint (5) ensures a balanced power flow across each node, taking power loss into account on inflow arcs, the power generated at each node (from the main generators, the gas generator, and the solar farm), and the (potentially reduced) demand of each node.
- Constraint (6) ensures the power flowing through a given restricted arc is no greater than its capacity. If a restricted arc is to be upgraded, its capacity is increased suitably.
- Constraint (7) ensures that the number of restricted arcs which will be upgraded is equal to the specified amount.

- Constraint (8) ensures that if the gas generator exists at a given node, it does not exceed its capacity.
- Constraint (9) ensures that the gas generator cannot be built at a main generator node, or the node at which it is banned.
- Constraint (10) ensures that only one gas generator is built.
- Constraint (11) ensures the gas generator is only running in the optimal time periods.
- Constraint (12) ensures the gas generator only runs in four time periods each day.
- Constraint (13) ensures that if a solar farm is present at a given substation, it does not exceed its capacity at any given time.
- Constraint (14) ensures only one solar farm is built.
- Constraint (15) ensures the number of time periods in which consumers strive to lessen demand on the grid does exceed the defined amount.
- Constraint (16) ensures the number of nodes that have a reduced demand during any given time period does not exceed the defined amount.
- Constraint (17) provides non-negativity constraints to:
  The amount of power produced efficiently by each main generator,
  The amount of power in each arc,
  The amount of power produced by the gas generator,
  The amount of power produced by the solar farm and,
  The amount of power produced inefficiently by each main generator, at any given time period, preventing nonsensical solutions.

Please refer to `ElectrigridSolution.py` for a `Gurobi` model that implements the above formulation.

Section B: Client Report

Communication Solutions

Communication 6

Dear Client,

As requested, we have ignored your previous instructions to limit the changes to generator output across a given day, and have resumed work based on our solution to the fourth communication.

With the new transmission line capacity upgrades, the optimal cost for meeting this demand over a whole day has reduced. It is now $2 691 719.06.

To achieve this reduced cost, we recommend upgrading transmission lines 88, 111 and 113, and setting the initial generator production values to those specified in Table 1.

As three of the restricted transmission lines are able to receive a capacity increase of 50MW, we may transmit higher amounts of power through said lines. These specific lines have been chosen by our analysts as they form the most optimal routes for power distribution, leading to lower power losses as a result.

This is what results in the reduced generator production values that we recommend in Table 1, which thereby reduce the optimal cost.

Time Period	Generator 1 Power (MW/hr)	Generator 2 Power (MW/hr)	Generator 3 Power (MW/hr)	Generator 4 Power (MW/hr)
Midnight to 4am	377.0	158.53	111.24	161.95
4am to 8am	377.0	270.2	253.68	269.14
8am to 12pm	377.0	388.87	462.23	349.59
12pm to 4pm	377.0	419.45	541.53	399.76
4pm to 8pm	377.0	554.1	652.41	559.94
8pm to Midnight	377.0	448.72	524.53	417.24

Table 1: Optimal Generator Production Values for Each Time Period - Communication 6

Communication 7

Dear Client,

Once again, the optimal cost has reduced. It is now $2 671 896.38.

To achieve this reduced cost, we recommend that the gas generator is built at Node 3. As requested, we have ensured that the existing demand at Node 3 is still met.

It should be noted that the specific transmission lines to be upgraded in capacity are the same as our previous communication.

The initial gas generator production values we recommend are given in Table 2, and the initial main generator production values we recommend are given in Table 3.

Constructing a gas generator at an existing substation allows us to reduce the impact of power loss over long distances. This is done by supplying power to nodes where it is inefficient for main generators to do so. As a result, the gas generator provides a cheaper alternative to the main generators.

In doing this, we are able to reduce the strain on the main generators, consequently reducing the overall cost.

Time Period	Gas Generator Output (MW/hr)
Midnight to 4am	98.21
4am to 8am	170.46
8am to 12pm	200.0
12pm to 4pm	200.0
4pm to 8pm	200.0
8pm to Midnight	200.0

Table 2: Optimal Gas Production Values for Each Time Period - Communication 7

Time Period	Generator 1 Power (MW/hr)	Generator 2 Power (MW/hr)	Generator 3 Power (MW/hr)	Generator 4 Power (MW/hr)
Midnight to 4am	377.0	158.53	57.01	113.43
4am to 8am	377.0	254.4	144.64	216.69
8am to 12pm	377.0	388.87	310.11	291.68
12pm to 4pm	377.0	419.45	408.05	322.18
4pm to 8pm	377.0	554.1	532.99	467.17
8pm to Midnight	377.0	448.72	382.19	348.71

Table 3: Optimal Main Generator Production Values for Each Time Period - Communication 7

Communication 8

Dear Client,

As expected, the optimal cost has now increased. It is now \$2 673 089.37.

To achieve this cost, we now recommend that the gas generator is built at Node 25. Again, we have ensured that the existing demand at Node 25 is still met.

It should be noted that once again, the specific transmission lines to be upgraded in capacity are the same as our previous communication.

The initial gas generator production values we recommend are given in Table 4, and the initial main generator production values we recommend are given in Table 5.

With the local government declining your application to build a gas generator at Node 3, the gas generator can no longer supplement the main generators where it is most efficient. This is what has caused this increased cost.

Time Period	Gas Generator Output (MW/hr)
Midnight to 4am	77.29
4am to 8am	121.48
8am to 12pm	159.47
12pm to 4pm	184.61
4pm to 8pm	200.0
8pm to Midnight	176.15

Table 4: Optimal Gas Generator Production Values for Each Time Period - Communication 8

Time Period	Generator 1 Power (MW/hr)	Generator 2 Power (MW/hr)	Generator 3 Power (MW/hr)	Generator 4 Power (MW/hr)
Midnight to 4am	377.0	158.53	65.48	125.89
4am to 8am	377.0	270.2	178.28	216.69
8am to 12pm	377.0	388.87	351.94	291.68
12pm to 4pm	377.0	419.45	423.93	322.18
4pm to 8pm	377.0	554.1	532.99	467.17
8pm to Midnight	377.0	448.72	406.92	348.71

Table 5: Optimal Main Generator Production Values for Each Time Period - Communication 8

Communication 9

Dear Client,

As expected, the optimal cost has increased once again. It is now \$2 676 929.23.

To achieve this cost, we recommend that the gas generator, which should still be constructed at Node 25, operate in time periods 2, 3, 4, and 5 (i.e. 8am - Midnight).

Again, the specific transmission lines to be upgraded in capacity are the same as our previous communication.

Furthermore, the initial gas generator production values we recommend are given in Table 6, and the initial main generator production values we recommend are given in Table 7.

As we can see from the previous communication, the optimal solution was to have the gas generator run during all time periods. The addition of this supplementary power was seen to be more cost-efficient than running the main generators alone.

Constraining the gas generator to only run in four time periods requires the main generators to produce more power in compensation, when the gas generator is not running. This is what results in the increased optimal cost.

Time Period	Gas Generator Output (MW/hr)
Midnight to 4am	0.0
4am to 8am	0.0
8am to 12pm	159.47
12pm to 4pm	184.61
4pm to 8pm	200.0
8pm to Midnight	176.15

Table 6: Optimal Gas Generator Production Values for Each Time Period - Communication 9

Time Period	Generator 1 Power (MW/hr)	Generator 2 Power (MW/hr)	Generator 3 Power (MW/hr)	Generator 4 Power (MW/hr)
Midnight to 4am	377.0	158.53	111.24	161.95
4am to 8am	377.0	270.2	253.68	269.14
8am to 12pm	377.0	388.87	351.94	291.68
12pm to 4pm	377.0	419.45	423.93	322.18
4pm to 8pm	377.0	554.1	532.99	467.17
8pm to Midnight	377.0	448.72	406.92	348.71

Table 7: Optimal Main Generator Power Output for Each Time Period - Communication 9

Communication 10

Dear Client,

The introduction of the solar farm has greatly reduced the optimal cost, which is now $2 633 644.98.

To achieve this cost, we recommend constructing the solar farm at Node 49.

We also still recommend that the gas generator be constructed at Node 25, and operate in time periods 2, 3, 4, and 5 (i.e. 8am - Midnight).

Once again, the specific transmission lines to be upgraded in capacity are the same as our previous communication.

The initial gas generator production values we recommend are given in Table 8, and the initial main generator production values we recommend are given in Table 9. Furthermore, the initial solar farm production values we recommend are given in Table 10.

Similar to the gas generator, the solar farm will supplement the generators throughout the day. As the farm is significantly cheaper, the optimal solution is to run it as much as possible. For the farm capacities which you have given us, this means using all 1080MW of power produced. This leads to the generators producing 1080MW less power each day, and so, as the generators are far more expensive, this greatly reduces the optimal cost.

Time Period	Gas Generator Output (MW/hr)
Midnight to 4am	0.0
4am to 8am	0.0
8am to 12pm	159.47
12pm to 4pm	184.61
4pm to 8pm	200.0
8pm to Midnight	176.15

Table 8: Optimal Gas Generator Production Values for Each Time Period - Communication 10

Time Period	Generator 1 Power (MW/hr)	Generator 2 Power (MW/hr)	Generator 3 Power (MW/hr)	Generator 4 Power (MW/hr)
Midnight to 4am	377.0	158.53	111.24	161.95
4am to 8am	377.0	266.94	253.68	250.81
8am to 12pm	377.0	388.87	301.21	215.88
12pm to 4pm	377.0	410.05	386.4	251.82
4pm to 8pm	377.0	554.1	532.5	445.6
8pm to Midnight	377.0	448.72	406.92	348.71

Table 9: Optimal Main Generator Power Output for Each Time Period - Communication 10

Time Period	Solar Generator Output (MW/hr)
Midnight to 4am	0.0
4am to 8am	20.0
8am to 12pm	120.0
12pm to 4pm	110.0
4pm to 8pm	20.0
8pm to Midnight	0.0

Table 10: Optimal Solar Farm Production Values for Each Time Period - Communication 10

Communication 11

Dear Client,

This new information has led to a significant increase in the optimal cost, which is now \$2 709 415.59.

First, we now recommend that the solar farm be built at Node 32 instead. Also, we recommend that transmission lines 15, 111, and 113 be upgraded in capacity instead.

However, it should be noted that we still recommend that the gas generator be constructed at Node 25, and operate in time periods 2, 3, 4, and 5 (i.e. 8am - Midnight).

The initial gas generator production values we recommend are given in Table 11, and the initial main generator production values we recommend are given in Table 12. The initial solar farm production values we recommend have remain unchanged, i.e., the solar farm should be running at capacity at all times.

As the total demand of the system is simply too high for all main generators to run below 60% of their capacity, we are guaranteed an increase in the optimal cost.

Naturally, the optimal solution then is to minimise the duration in which the main generators exceed 60% of their capacity.

However, a consequence of this solution is that other main generators must compensate for this reduction in power production. This new combination of power production values leads to an overall increase in the optimal cost.

This can be especially seen when Generator 1, which is the cheapest main generator, is limited to produce 60% of its capacity. The other, more expensive generators must produce more power to compensate, thereby leading to a higher optimal cost.

Time Period	Gas Generator Output (MW/hr)
Midnight to 4am	0.0
4am to 8am	0.0
8am to 12pm	159.47
12pm to 4pm	184.61
4pm to 8pm	200.0
8pm to Midnight	200.0

Table 11: Optimal Gas Generator Production Values for Each Time Period - Communication 11

Time Period	Generator 1 Power (MW/hr)	Generator 2 Power (MW/hr)	Generator 3 Power (MW/hr)	Generator 4 Power (MW/hr)
Midnight to 4am	226.2	189.14	204.78	175.97
4am to 8am	226.2	309.36	338.31	267.03
8am to 12pm	226.2	452.4	352.39	250.31
12pm to 4pm	248.18	452.4	420.87	303.05
4pm to 8pm	377.0	606.05	510.8	415.29
8pm to Midnight	308.51	452.4	447.0	348.0

Table 12: Optimal Main Generator Production Values for Each Time Period - Communication 11

Communication 12

Dear Client,

We can confirm that the scheme to reduce demands will lead to an optimal cost of $2 643 590.38.

No changes to the transmission lines to upgrade, the gas generator location and optimal time periods, or the solar farm location are required.

As a reminder, we recommend upgrading transmission lines 15, 111, and 113, we recommend building the gas generator at Node 25, we recommend running said generator in time periods $2-5$ (i.e. 8am-Midnight), and we recommend building the solar farm at Node 32, running at capacity, at all times.

The initial gas generator production values we recommend are given in Table 14, and the initial main generator production values we recommend are given in Table 15.

Trivially, it follows that a reduced demand would result in less power required to be produced, thereby reducing the optimal cost.

Table 13 presents our recommendations for the nodes during each time period which should ideally reduce in demand, to best implement this new scheme.

Time Period	Nodes to Reduce Demand
Midnight to 4am	8
4am to 8am	4, 22, 23, 30, 34, 36, 45, 48, 49
8am to 12pm	1, 3, 5, 9, 13, 24, 26, 39, 42
12pm to 4pm	0, 11, 27, 32, 33, 38, 43, 46, 47
4pm to 8pm	10, 12, 14, 15, 17, 21, 25, 35, 37
8pm to Midnight	6, 7, 16, 18, 28, 29, 31, 41, 44

Table 13: Optimal Nodes to Reduce Demand at Each Time Period - Communication 12

Time Period	Gas Generator Output (MW/hr)
Midnight to 4am	0.0
4am to 8am	0.0
8am to 12pm	159.47
12pm to 4pm	181.55
4pm to 8pm	200.0
8pm to Midnight	200.0

Table 14: Optimal Gas Generator Production Values for Each Time Period - Communication 11

Time Period	Generator 1 Power (MW/hr)	Generator 2 Power (MW/hr)	Generator 3 Power (MW/hr)	Generator 4 Power (MW/hr)
Midnight to 4am	226.2	187.42	204.78	175.97
4am to 8am	226.2	301.66	334.48	260.16
8am to 12pm	226.2	452.4	339.75	232.25
12pm to 4pm	245.65	452.4	409.31	283.58
4pm to 8pm	377.0	584.24	476.78	400.41
8pm to Midnight	275.75	452.4	447.0	348.0

Table 15: Optimal Main Generator Production Values for Each Time Period - Communication 11

Further Insights

In this section of the report, we provide some additional insights on the final solution that may be of use to you. These changes have been constrained to what we deemed as realistically implementable.

Gas vs Solar

We begin by investigating the effect of adding another solar farm on the optimal cost, and compare it to the the effect of adding another gas generator. Note, we consider the case in which the costs and capacities of the added solar farm/gas generator are the same as those already in the network.

Building a second solar farm, at Node 44 specifically, reduces the optimal cost to \$2 598 709.01, saving approximately \$44 881.37 each day.

This solar farm would also be running at capacity, at all times. It should be noted that adding this solar farm would change our recommendations for the transmission lines to be upgraded to lines 15, 41, and 111. No other recommendations would change.

Next, building a second gas generator, at Node 34 specifically, reduces the optimal cost to \$2 611 810.19, saving approximately \$31 780.19 each day.

It should be noted that adding this gas generator would change our recommendations for the transmission lines to be upgraded to lines 15, 41, and 125 instead. Once again, no other recommendations would change.

As we can see, both options decrease the optimal cost, but, as expected, the addition of the solar farm results in the lower optimal cost, if you were to decide between one or the other. Trivially, adding both a second solar farm and gas generator would optimise the solution even further. In fact, it would lead to an optimal cost of \$2 568 622.91.

Main Generator Efficiency

The main generator efficiency threshold introduced in Communication 11 caused a substantial increase in the optimal cost.

In the scenario that a main generator can be efficient to a higher threshold, say 70% of its capacity, the optimal cost will clearly decrease.

If this is possible, this would reduce the optimal cost to \$2 610 501.16, i.e., a reduction of \$33 089.22.

Node Demands

Here we will examine whether your implementation of the customer demand reduction scheme can be improved on, within reason.

We have considered three possible alternatives to the current scheme:

1. 9 nodes reduced per time period with a 15% node demand reduction
   - Optimal Cost : \$2 610 812.64
2. 5 nodes reduced per time period with a 20% node demand reduction
   - Optimal Cost : \$2 615 386.68
3. 9 nodes reduced per time period with a 10% node demand reduction, but up to 2 time periods per node
   - Optimal Cost : \$2 625 802.21

This is only a small example of how adjusting the demand reduction scheme parameters can change the optimal cost. If these parameters are able to be changed within reason, it may be interesting to investigate this further, as all of the samples above have led to a reduced optimal cost.