MATH3202
Assignment 1
Practical Group: P02 (Wednesday 10am)

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Section A: Supervisor Report

Sets

N	Nodes	{Node #, X, Y, Demand}
G	Generator Nodes	{Node #, X, Y, 0} ((Demand $= \mathbf{0}$) $\implies$ Generator)
A	Arcs (i.e. Transmission Lines)	{Number: (Node 1, Node 2)}
U	Unrestricted Arcs	($U \subseteq A$)
T	Time Period	$\{0, 1, 2, 3, 4, 5\}$ ($0 \implies$ 12am to 4am, $1 \implies$ 4am to 8am, etc.)

Data

d_{nt}	Demand of node $n \in N$ at time period $t \in T$ (MW)
c_g	Capacity of generator $g \in G$ (MW)
cost_g	Cost of generator $g \in G$ ($/MWh)
$l = 106$ MW	Power limit for all restricted arcs (i.e. $\forall a \in (A \setminus U)$)
(x_n, y_n)	Co-ordinates of node $n \in N$
length_a	Length of arc $a \in A$ (Calculated via Euclidean Distance - assumes arcs are straight lines)
powerChange = 170 MW	Maximum change in generator power allowed between each time period t (MW)
powerLoss = 0.001	0.1% Loss in power through arc per km
timePeriodDuration = 4	Duration of each time period (i.e. 4 hours)
E_a	efficiency of arc $a \in A$ (i.e. $E_a = (\text{powerLoss})(\text{length}_a)$)
(f_a, t_a)	From and to node for arc $a \in A$

Variables

x_{gt}	amount of power (in MW/hr) to produce from each generator $g \in G$ at the beginning of time period $t \in T$
y_{at}	amount of power (in MW/hr) in each arc $a \in A$ at the beginning of time period $t \in T$

Objective

$$\min \left(\sum_{t \in T} \sum_{g \in G} \left((\text{timePeriodDuration}) \left(\text{cost}_g\right) \left(x_{gt}\right) \right) \right)$$

Constraints

$$
\begin{aligned}
x_{gt} &\leq c_g & \forall g \in G, \forall t \in T \quad (1)\\
\sum_{\substack{a \in A \\ \text{s.t.} t_a = n}} y_{at} E_a &= d_{nt} + \sum_{\substack{a \in A \\ \text{s.t.} f_a = n}} y_{at} & \forall n \in (N \setminus G), \forall t \in T \quad (2)\\
x_{gt} + \sum_{\substack{a \in A \\ \text{s.t.} t_a = g}} y_{at} E_a &= \sum_{\substack{a \in A \\ \text{s.t.} f_a = g}} y_{at} & \forall g \in G, \forall t \in T \quad (3)\\
y_{at} &\leq l & \forall a \in (A \setminus U), \forall t \in T \quad (4)\\
x_{gt} - x_{g(t+1)} &\leq \text{powerChange} & \forall g \in G, \forall t \in T \setminus \{5\} \quad (5)\\
x_{g(t+1)} - x_{gt} &\leq \text{powerChange} & \forall g \in G, \forall t \in T \setminus \{5\} \quad (6)\\
x_{g5} - x_{g0} &\leq \text{powerChange} & \forall g \in G \quad (7)\\
x_{g0} - x_{g5} &\leq \text{powerChange} & \forall g \in G \quad (8)\\
x_{gt} &\geq 0 & \forall g \in G, \forall t \in T \quad (9)\\
y_{at} &\geq 0 & \forall a \in A, \forall t \in T \quad (10)
\end{aligned}
$$

- Constraint (1) ensures no generator exceeds its power capacity at any time.
- Constraint (2) ensures the total amount of power transmitted into each substation node via each connected transmission line, scaled down to account for loss in power across the transmission lines, is equal to the total amount of power out of each substation node (via each connected transmission line), along with the demand of each substation node, at any given time. This is to ensure that just enough electricity is produced so that each substation's demand is met, and enough power is provided for each node to transmit electricity to other nodes. In other words, the net flow of electricity through each substation should be 0.
- Constraint (3) ensures the total power received by all substation nodes directly connected to a generator, is equal to the amount of power said generator produced at any given time, along with the the amount of power received by a generator, again, scaled down to account for loss in power across the transmission lines. This is to ensure we consider the situation when demands are low and the optimal solution is for a generator to produce little to no electricity. At this point, the generator is essentially equivalent to a substation node, and must therefore be treated like one.
- Constraint (4) ensures that all transmission lines with a power limit do not exceed said limit.
- Constraints (5) and (6) ensure changes in the amount of power produced by any generator between time periods does not exceed the imposed power change limits.
- Constraints (7) and (8) ensure power change constraints are met overnight. We assume the data provided to us, is of a 'typical day', and hence, is accurate to estimate demands in time periods of different days (i.e. constraining the first time period of the day by its final time period, is equivalent to constraining the first time period of the day, by the final time period of the previous day).
- Constraints (9) and (10) provide non-negativity constraints to the amount of power produced by each generator, and the amount of power in each arc at any given time period, preventing nonsensical solutions.

Please refer to `ElectrigridSolution.py` for a `Gurobi` model that implements the above formulation.

Section B: Client Report

Communication 1

Dear Client,

The overall demand of the network is 1864 MW/hr (i.e. 44 736 MW/day).

The optimal cost for meeting the current demand over a whole day from your generators is \$3 159 144.00.

To best achieve this, we recommend:

- Generator 1 (Node 12) produce 382.0 MW per hour
- Generator 2 (Node 18) produce 761.0 MW per hour
- Generator 3 (Node 37) produce 177.0 MW per hour
- Generator 4 (Node 41) produce 544.0 MW per hour

As per the provided constraints, all generators are below, or at the specified capacities, and the minimum amount of power was produced as to only meet the required demands of all substation nodes.

As Generator 3 (Node 37) is the most expensive, it should produce the least amount of power. As the total demand is quite high, it is required that most generators are at capacity.

Communication 2

Dear Client,

Although the total demand remains unchanged, to account for this power loss in the transmission lines, generators must consequently produce more electricity to meet the substation demands.

This has led to an increase in the optimal cost, which is now \$3 407 848.59.

This is best achieved by the following generator production values (rounded to 2 decimal places):

- Generator 1 (Node 12) produces 331.35 MW per hour
- Generator 2 (Node 18) produces 761.0 MW per hour
- Generator 3 (Node 37) produces 347.77 MW per hour
- Generator 4 (Node 41) produces 544.0 MW per hour

It may be noted that the distribution of power between generators has slightly varied. This is due to the fact that more expensive generators that are closer to certain substations, are now more optimal options to provide power to said substations, than cheaper generators which are much farther away, due to this new loss constraint. This is because cheaper generators that are much farther away from certain substations, would be required to produce significantly more power to meet the required demands, as a greater amount of electricity would be lost in transmission lines.

Communication 3

Dear Client,

The total demand has once again remained unchanged.

However, as many transmissions lines have restricted power loads, to ensure all substations still met the required demands, longer paths between generators and substations had to be taken.

With the per km loss in power across transmission lines, it follows that longer paths would lead to a greater power loss. As a result, slightly more power had to be produced, leading to a higher optimal cost. This cost is now $3 409 280.33.

This is best achieved by the following generator production values:

- Generator 1 (Node 12) produces 315.93 MW per hour
- Generator 2 (Node 18) produces 761.0 MW per hour
- Generator 3 (Node 37) produces 363.35 MW per hour
- Generator 4 (Node 41) produces 544.0 MW per hour

Communication 4

Dear Client,

As the substation demands vary throughout the day, the total demand over a whole day is significantly lower (See Table 1).

Time Period	Hourly Demand (MW/hr)
Midnight to 4am	629.0
4am to 8am	985.0
8am to 12pm	1424.0
12pm to 4pm	1580.0
4pm to 8pm	1926.0
8pm to Midnight	1559.0

Table 1: Hourly Demands at Different Time Periods - Communication 4

From these hourly demands, we obtain a total demand of 32 412 MW/day, which is much lower than the previous total demand of 44 736 MW/day.

As a result of these changing demands, generators are now able to produce varying power at different time periods. This is much more efficient that providing constant power output throughout the day, as generators may now produce less power when it is not required.

This has led to lower electricity production requirements from the generators, and as a result, a lower optimal cost. The optimal cost is now $2 425 479.97.

This is best achieved by the following generator production values, for the given time periods (See Table 2):

Time Period	Generator 1 Power (MW/hr)	Generator 2 Power (MW/hr)	Generator 3 Power (MW/hr)	Generator 4 Power (MW/hr)
Midnight to 4am	0.0	684.66	0.0	31.07
4am to 8am	0.0	761.0	0.0	338.53
8am to 12pm	148.22	761.0	96.51	544.0
12pm to 4pm	191.88	761.0	200.96	544.0
4pm to 8pm	341.42	761.0	401.26	544.0
8pm to Midnight	219.30	761.0	194.83	544.0

Table 2: Optimal Generator Power Output at Different Time Periods - Communication 4

Communication 5

Dear Client,

Although demands remain unchanged, the optimal cost has increased. This cost is now \$2 524 145.32.

Ideally, the optimised solution would be to rely much more on the cheaper generators to power the network (within reason, based on the previously introduced constraints). However, to ensure changes in generator outputs between time periods are within the provided limit, these generators are limited in how much they can change according to varying demands. As a result, when these generators meet power change limits, more expensive generators must produce more power to meet remaining demands.

Furthermore, all generators must sometimes produce more power than required during certain time periods, as to again, not exceed this power change limit. This is especially seen before/after peak periods, where the changes in demands are quite high.

These aforementioned factors are the cause for this increased optimal cost.

This cost is best achieved with the following generator production values, for the given time periods (See Table 3):

Time Period	Generator 1 Power (MW/hr)	Generator 2 Power (MW/hr)	Generator 3 Power (MW/hr)	Generator 4 Power (MW/hr)
Midnight to 4am	12.06	591.0	61.26	374.0
4am to 8am	0.0	741.79	0.0	356.48
8am to 12pm	154.35	761.0	105.92	526.48
12pm to 4pm	171.42	761.0	231.26	532.46
4pm to 8pm	341.42	761.0	401.26	544.0
8pm to Midnight	182.06	761.0	231.26	544.0

Table 3: Optimal Generator Power Output at Different Time Periods - Communication 5