

Numerical Methods I
Homework Problem Set #10

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Problem Set #10

1 Picard Iterate

$$y_{n+1} = y_0 + \int_{x_0}^x f(t, y_n(t)) dt \quad (1)$$

1.1 Question (a)

$$y' = x^2 - 2y^2 + 1 \quad y(0) = 0 \quad y_2(x) \quad (2)$$

$$y_1 = y_0 + \int_{x_0}^x t^2 - 2y^2 + 1 dt \quad (3)$$

1.2 Question (b)

2 Taylor Approximating Polynomial

2.1 Question (a)

$$y' = \cos(x) - y \quad y(0) = -1 \quad p_3(x) \quad (4)$$

$$p_3(x) = y(x_0) + y'(x_0)(x - x_0) + \frac{y''(x_0)}{2!}(x - x_0)^2 + \frac{y'''(x_0)}{3!}(x - x_0)^3 \quad (5)$$

$$y' = \cos(x) - y \quad (6)$$

$$y'(0) = \cos(0) - y(0) = 1 - (-1) = 2 \quad (7)$$

$$y'' = -\sin(x) - y' = -\sin(x) - \cos(x) + y \quad (8)$$

$$y''(0) = -\sin(0) - \cos(0) + y(0) = -1 - 1 = -2 \quad (9)$$

$$y''' = -\cos(x) - y'' = \sin(x) - y \quad (10)$$

$$y'''(0) = \sin(0) - y(0) = -(-1) = 1 \quad (11)$$

$$p_3(x) = y(0) + y'(0)x + \frac{y''(0)}{2}x^2 + \frac{y'''(0)}{6}x^3 \quad (12)$$

$$p_3(x) = -1 + 2x + \frac{-2x^2}{2} + \frac{x^3}{6} = \frac{x^3}{6} - x^2 + 2x - 1 \quad (13)$$

2.2 Question (b)

$$y' = e^x y \quad y(0) = 2 \quad p_2(x) \quad (14)$$

$$p_2(x) = y(x_0) + y'(x_0)(x - x_0) + \frac{y''(x_0)}{2!}(x - x_0)^2 \quad (15)$$

$$y' = e^x y \quad (16)$$

$$y'(0) = e^0 y(0) = 1(2) = 2 \quad (17)$$

$$y'' = e^x y + e^x y' = e^x (y + e^x y) \quad (18)$$

$$y''(0) = e^0 (y(0) + e^0 y(0)) = 1(2 + 1(2)) = 4 \quad (19)$$

$$p_2(x) = y(0) + y'(0)x + \frac{y''(0)}{2}x^2 \quad (20)$$

$$p_2(x) = 2 + 2x + 2x^2 = 2(x^2 + x + 1) \quad (21)$$

3 Euler's Method

3.1 Question (a)

3.2 Question (b)

4 Modified Euler's Method