CS 3500 AA – Numerical Methods I
Fall 2015
Homework Problem Set #2
Due: 09/10/2015

Be sure to do all your work on separate paper, and include all steps where appropriate. All homework must follow the formatting rules posted on Blackboard. For the optional MATLAB exercises, print the output from the command window and attach to the rest of your solutions.

1. The following equations has a solution in the interval $[0,1]$. Using the endpoints of the interval as initial values and three iterations of false position, approximate the solution. You may give your answer in the form of a table.
$$\ln(x+1)-\cos(x)=0$$

2. Repeat question 1 using the secant method and the same initial values.

3. The given equation has one real solution. Approximate it using Newton's method with the closest integer as the initial approximation. Give your approximation to as many digits as your calculator will allow.

   (a) $x^3+x-1=0$

   (b) $x^5-3x+3=0$

4. The **mechanic's rule** for approximating square roots states that $\sqrt{a}\approx x_{n+1}$, where
$$x_{n+1}=\frac{1}{2}\left(x_n+\frac{a}{x_n}\right),\quad n=0,1,2,\ldots$$
and x_0 is any positive approximation to $\sqrt{a}$.

   (a) Apply Newton's method to $f(x)=x^2-a$ to derive the mechanic's rule.

   (b) Use 3 iterations of the mechanic's rule method with $x_0=3$ to approximate $\sqrt{10}$. What is the absolute and percentage error of the third iterate?

5. Find all real fixed-points of the following functions algebraically.

   (a) $f(x)=x\ln(x+1)$

   (b) $f(x)=\sqrt{x+1}$

6. Show the algebraic steps needed to write the equation $x^3-3x-20=0$ in each of the following forms.
$$x=\underbrace{\frac{x^3-20}{3}}_{g_1(x)},\qquad x=\underbrace{\frac{20}{x^2-3}}_{g_2(x)},\qquad x=\underbrace{(3x+20)^{1/3}}_{g_3(x)}$$
Using $x_{n+1}=g(x_n)$ with $x_0=3.5$, compute x_7. Does it appear the each sequence is converging? If so, how quickly?

7. **[MATLAB]** It can be shown that if a spherical object of radius R and density ρ_o is placed on the surface of a fluid of density ρ_f, the object would sink to a depth h which is a solution of the equation
$$\frac{\rho_f}{3}h^3-R\rho_f h^2+\frac{4}{3}R^3\rho_o=0$$

Suppose we place a spherical ball of cork with a radius of $R = 5$ cm and a density of $\rho_o = 0.120$ g/cm^3 into motor oil with a density of $\rho_f = 0.890$ g/cm^3. Approximate the height to which the cork will sink with an absolute tolerance of 10^{-8} using each of the following methods and initial condition(s) of your choice. It may be wise to look at the graph of the function to determine appropriate initial conditions. Comment on the relative speed of all the methods used.

(a) bisection (b) false position (c) secant (d) Newton's