Numerical Methods I
Exam #2 Tips

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Quiz #2

0.1 LU Factorizations

Purpose:

$$A = LU \tag{1}$$

$$A\vec{x} = (LU)\vec{x} = L(U\vec{x}) = b \qquad U\vec{x} = \vec{y}$$

$$L\vec{y} = b \tag{2}$$

Procedure (3×3 matrix):

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \tag{3}$$

$$= \begin{bmatrix} l_{11}u_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + l_{22}u_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + l_{33}u_{33} \end{bmatrix} \tag{4}$$

Types of equations:

$$i < j \qquad l_{i1} + u_{1j} + l_{i2}u_{2j} + \cdots + l_{ii}u_{ij} = a_{ij} \tag{5}$$

$$i = j \qquad l_{i1} + u_{1j} + l_{i2}u_{2j} + \cdots + l_{ii}u_{jj} = a_{ij} \tag{6}$$

$$i > j \qquad l_{i1} + u_{1j} + l_{i2}u_{2j} + \cdots + l_{ij}u_{jj} = a_{ij} \tag{7}$$

0.1.1 Doolittle

$A = 3 \times 3$ matrix:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \tag{8}$$

$$= \begin{bmatrix} (1)u_{11} & (1)u_{12} & (1)u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + (1)u_{22} & l_{21}u_{13} + (1)u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + (1)u_{33} \end{bmatrix} \tag{9}$$

0.1.2 Crout

$A = 3 \times 3$ matrix:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

$$= \begin{bmatrix} l_{11}(1) & l_{11}u_{12} & l_{11}u_{13} \\ l_{21}(1) & l_{21}u_{12} + l_{22}(1) & l_{21}u_{13} + l_{22}u_{23} \\ l_{31}(1) & l_{31}u_{12} + l_{32}(1) & l_{31}u_{13} + l_{32}u_{23} + l_{33}(1) \end{bmatrix} \tag{11}$$

0.1.3 Cholesky

$A = 3 \times 3$ matrix:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} x_{11} & 0 & 0 \\ l_{21} & x_{22} & 0 \\ l_{31} & l_{32} & x_{33} \end{bmatrix} \begin{bmatrix} x_{11} & u_{12} & u_{13} \\ 0 & x_{22} & u_{23} \\ 0 & 0 & x_{33} \end{bmatrix} \tag{12}$$

$$= \begin{bmatrix} x_{11}x_{11} & x_{11}u_{12} & x_{11}u_{13} \\ l_{21}x_{11} & l_{21}u_{12} + x_{22}x_{22} & l_{21}u_{13} + x_{22}u_{23} \\ l_{31}x_{11} & l_{31}u_{12} + l_{32}x_{22} & l_{31}u_{13} + l_{32}u_{23} + x_{33}x_{33} \end{bmatrix}$$

$$= \begin{bmatrix} x_{11}^2 & x_{11}u_{12} & x_{11}u_{13} \\ l_{21}x_{11} & l_{21}u_{12} + x_{22}^2 & l_{21}u_{13} + x_{22}u_{23} \\ l_{31}x_{11} & l_{31}u_{12} + l_{32}x_{22} & l_{31}u_{13} + l_{32}u_{23} + x_{33}^2 \end{bmatrix} \tag{13}$$

0.2 Identifying Matrices

0.2.1 Strictly Diagonally Dominant (SDD)

A matrix $A_{n\times n}$ is strictly diagonally dominant if:

$$|a_{ii}| > \sum_{j=1, j\neq i}^{n} |a_{ij}|, \qquad \forall i \tag{14}$$

0.2.2 Symmetric Positive Definite (SPD)

A symmetric matrix $A_{n\times n}$ is positive definite if:

$$\vec{x}^T A \vec{x} > 0 \tag{15}$$

$$\vec{x_{ij}} \in \mathbb{R}_{\neq 0} \tag{16}$$

0.2.3 Coefficient Matrix (A) Significance

0.3 Matrix Norms

0.3.1 1- Norm

0.3.2 2- Norm

0.3.3 ∞- Norm

0.4 Linear System Root Approximations

0.4.1 Jacobi Method

Criteria for Convergence

Error-Bound for Absolute Error

0.4.2 Gauss-Seidel

Criteria for Convergence

Error-Bound for Absolute Error

0.5 Matrix Condition

0.5.1 Ill-Conditioned Matrices

0.5.2 Condition Number of a Matrix