CS 3500 AA – Numerical Methods I
Fall 2015
Homework Problem Set #7
Due: 10/22/2015

Be sure to do all your work on separate paper, and include all steps where appropriate. All homework must follow the formatting rules posted on Blackboard. To compute any needed inverses, you may use your graphing utility, Maple, or MATLAB.

1. Will the sequence $\{\mathbf{x}^{(k)}\}$ generated by the following converge for some $\mathbf{x}^{(0)} \in \mathbb{R}^2$? Justify your answer.

$$\mathbf{x}^{(k+1)} = \begin{bmatrix} \frac{2}{3} & 0 \\ 1 & -\frac{4}{5} \end{bmatrix} \mathbf{x}^{(k)} + \begin{bmatrix} 6 \\ -5 \end{bmatrix}$$

2. Consider the following linear system of equations:

$$\begin{aligned} 4x_1 - 2x_2 + x_3 &= 0 \\ -x_1 + 3x_2 + x_3 &= 1 \\ x_1 + x_2 + 5x_3 &= -1 \end{aligned}$$

   (a) Show that the Jacobi method will converge by establishing the fact that $\|T\|_\infty < 1$.
   (b) Find the third iterate, $\mathbf{x}^{(3)}$, when the initial approximation is $\mathbf{x}^{(0)} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$.
   (c) Compute the error bound for your approximation in part (b).
   (d) Determine the minimum iterations are needed to get an absolute error no greater than 10^{-8}?

3. Suppose the solution of some linear system is approximated using the Gauss-Seidel method with $\mathbf{x}^{(0)} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ and

$$\mathbf{x}^{(k+1)} = \begin{bmatrix} 0 & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{3} & 0 & -\frac{1}{3} \\ -\frac{1}{5} & -\frac{2}{5} & 0 \end{bmatrix} \mathbf{x}^{(k)} + \begin{bmatrix} \frac{1}{2} \\ 1 \\ -\frac{1}{3} \end{bmatrix}$$

   Using the l_∞-norm, determine the minimum number of iterations need to ensure an approximation accurate to within 10^{-8}.

4. Consider the linear system

$$\begin{bmatrix} 1 & 2 \\ 0.9999 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{b}$$

   (a) Solve the system when $\mathbf{b} = \begin{bmatrix} 3 & 3.0001 \end{bmatrix}^T$.
   (b) Solve the system when $\mathbf{b} = \begin{bmatrix} 3.1 & 3.0001 \end{bmatrix}^T$.
   (c) Comment on the effect perturbing $\mathbf{b}$ has on the solution, $\mathbf{x}$. Is A ill-conditioned? Explain. If needed, use the l_1-norm for any calculations.

5. The $n \times n$ Hilbert matrix, H_n, is defined as

$$[h_{ij}] = \frac{1}{i+j-1}, \quad 1 \le i, j \le n$$

   Find $K(H_3)$ using the l_1-norm.

6. Find the condition number of the following matrix.

$$\begin{bmatrix} 1 & 1 \\ 1 & 1+\frac{1}{n} \end{bmatrix}, \text{ for } n > 1$$

   What happens to the condition number as $n \to \infty$?