Solutions to H.W. # 8

1. $f(x) = e^{-x} \cdot \ln(x+2)$, $f'(x) = -e^{-x}\ln(x+2) + \dfrac{e^{x}}{x+2}$

$$f'(2) = -0.1537807192$$

Using the two-point backward difference approximation

$$f'(2) \approx \frac{f(2) - f(2-h)}{h}$$

h	Approximation	Absolute Error
0.1	-0.159448438	0.00566771880
0.05	-0.156591372	0.00281065280
0.025	-0.155180197	0.00139947780

2. $f(x) = 2\sin(x) - \sqrt{2x+3}$, $f'(x) = 2\cos(x) - \dfrac{1}{\sqrt{2x+3}}$

$$f'(0) = 1.422649731$$

Using the three-point central difference approximation:

$$f'(0) \approx \frac{f(h) - f(-h)}{2h}$$

$$a(1.56) \approx \frac{-185 + 16\cdot(208) - 30\cdot(249) + 16\cdot(261) - 271}{12\cdot(0.52)^2}$$

$$= -130.0542 \ m/s^2$$

5. Since the error is of $O(h^2)$:

$$E = 4.15831 + K\cdot(0.05)^2 \quad , \ K \in \mathbb{R}$$
$$E = 4.16361 + K\cdot(0.025)^2$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & -0.05^2 & 4.15831 \\ 1 & -0.025^2 & 4.16361 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & 4.16537 \\ 0 & 1 & 2.82617 \end{array}\right]$$

$$\therefore E = 4.16537$$

6. Since the error is of $O(h^4)$:

$$E = -3.2213 + K\cdot(0.01)^2$$

$$E = -3.3245 + K\cdot(0.005)^2$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & -0.01^2 & -3.2213 \\ 1 & -0.005^2 & -3.3245 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & -3.3314 \\ 0 & 1 & K = -1.1 \times 10^7 \end{array}\right]$$

$$\therefore E = -3.3314$$