CS 3500 AA – Numerical Methods I
Fall 2015
Homework Problem Set #4
Due: 09/24/2015

Be sure to do all your work on separate paper, and include all steps where appropriate. All homework must follow the formatting rules posted on Blackboard.

1. For some convergent fixed-point iterative scheme, $x_{n+1} = g(x_n)$, on $[0,1]$, we have $x_0 = 1$ and $x_1 = \frac{1}{2}$. Suppose further that for all $x \in [0,1]$, $|g'(x)| \le \frac{1}{3}$.
   (a) Compute the error bound for x_7.
   (b) Determine the minimum number of iterations needed to approximate the fixed-point to within 10^{-6} using and $x_0 = 1$.
2. Let $g(x) = \frac{1}{5}(x+1)^{3/2}$.
   (a) Prove that g has a unique-fixed point in $[0,1]$.
   (b) For $x_{n+1} = g(x_n)$, compute the error bound for x_5 starting with $x_0 = 0$.
   (c) Determine the minimum number of iterations needed to approximate the fixed-point to within 10^{-8} using $x_0 = 0$
3. Verify that $x = \sqrt{a}$ is a fixed point of the function
$$g(x) = \frac{1}{2}\left(x + \frac{a}{x}\right).$$
   Determine the order of convergence of the sequence $x_{n+1} = g(x_n)$ as it converges to $x = \sqrt{a}$.
4. Consider the iterative scheme
$$x_{n+1} = 0.4 + x_n - 0.1x_n^2, \quad n \ge 0$$
   Will this scheme converge to the fixed point $x = 2$? If yes, find its rate of convergence.
5. Both of the following sequences will converge to the fixed-point $\sqrt{5}$. Determine analytically which one will do so at a faster rate.
   (a) $x_{n+1} = x_n + 1 - \dfrac{x_n^2}{5}$
   (b) $x_{n+1} = \dfrac{x_n^2 + 5}{2x_n}$