CS 3500 AA – Numerical Methods I
Fall 2015
Homework Problem Set #1
Due: 09/03/2015

Be sure to do all your work on separate paper, and include all steps where appropriate. All homework must follow the formatting rules posted on Blackboard. For the optional MATLAB exercises, print the output from the command window and attach to the rest of your solutions.

1. Let $f(x) = 2x^2 - x - 1$.
   (a) Find $f(1.234)$.
   (b) Approximate $f(1.234)$ by rounding to one decimal place at **every** arithmetic operation. What is the absolute error of the approximation?

2. The Maclaurin series for $f(x) = \sin(x)$ is given by
$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$
   (a) Use the first three non-zero terms of the Maclaurin series to approximate $\sin(\pi/4)$.
   (b) What is the absolute error of the approximation in part (a)?
   (c) What is the percentage error of the approximation in part (a)?
   (d) To how significant digits is the approximation in part (a) accurate?

3. Prove analytically that each of the following functions possess a unique root on the indicated interval.
   (a) $f(x) = e^{-2x} - x$ on $[-1, 1]$
   (b) $f(x) = \cos(x) - x$ on $[0, \pi/2]$

4. Explain or demonstrate how each of the following problems can be reformulated as finding a root of a non-linear function.
   (a) What is an approximate value of $\sqrt{3/5}$?
   (b) Where do the curves $y = \sin(x)$ and $y = x^3 - 1$ intersect?

5. Use four iterations of the bisection method to approximate the root of the function on the given interval.
   (a) $f(x) = x^3 - 4x - 2$ on $[1, 3]$
   (b) $f(x) = x^4 + \ln(3x - 1)$ on $[0.4, 0.7]$

6. **[MATLAB]** Use MATLAB to solve the equation $2x + 9 = 5$.

7. **[MATLAB]** The largest real root of $f(x) = x^3 - 6x^2 + 11x - 6.1$ is known to be in the interval $[2.5, 3.5]$. Approximate this root with a tolerance of 10^{-8} using the bisection method. Your answer should be a table similar to that on page 52. You may use the same stopping criteria used in the m-file given in the text.