**CS 3500 AA – Numerical Methods I**
**Fall 2015**
**Homework Problem Set #5**
**Due: 10/01/2015**

**Be sure to do all your work on separate paper, and include all steps where appropriate. All homework must follow the formatting rules posted on Blackboard.**

1. Find the Doolittle factorization of the following matrix.

$$A = \begin{bmatrix} 8 & 5 & 1 \\ 3 & 7 & 4 \\ 2 & 3 & 9 \end{bmatrix}$$

   Then use the factorization to solve

$$Ax = \begin{bmatrix} 6 \\ 1 \\ -2 \end{bmatrix}.$$

2. Use the Crout factorization to solve the following system.

$$\begin{aligned} x_1 - x_2 &= 0 \\ -2x_1 + 4x_2 - 2x_3 &= -1 \\ -x_2 + 2x_3 &= 4 \end{aligned}$$

3. Find the Cholesky factorization of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

4. Why does $A = \begin{bmatrix} 4 & 0 & -3 \\ 0 & 6 & 2 \\ -3 & -2 & 7 \end{bmatrix}$ not have a Cholesky factorization?

5. Classify each matrix as SDD, SPD, both, or neither.

   (a) $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{bmatrix}$ (b) $B = \begin{bmatrix} 1 & 2 & 0 \\ 4 & 6 & -1 \\ -3 & 2 & 0 \end{bmatrix}$ (c) $C = \begin{bmatrix} 5 & -3 & 2 \\ -3 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}$

6. Consider the matrix

$$\begin{bmatrix} 5 & -2 & 2 \\ -2 & 6 & a \\ 2 & a & 7 \end{bmatrix}$$

   (a) For what value(s) of $a$ will this matrix be strictly diagonally dominant?

   (b) For what value(s) of $a$ will this matrix be symmetric positive definite?