Numerical Methods I
Quiz #1 Tips

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Exam #1

0.1 Absolute Error

$$e_{abs} = |e_{exact} - e_{approx}| \tag{1}$$

0.2 Relative Error

$$e_{rel} = \frac{|e_{exact} - e_{approx}|}{|e_{exact}|} \tag{2}$$

0.3 Bisection Method

$$m = \frac{b+a}{2} \tag{3}$$

If $y < 0$, $a = m$;
Else if $y > 0$, $b = m$;
Else $y = 0$.

0.4 Regula Falsi

$$x = b - y_b \frac{b-a}{y_b - y_a} \tag{4}$$

If $y < 0$, $a = x$;
Else if $y > 0$, $b = x$;
Else $y = 0$.

0.5 Secant Method

$$x_{n+1} = x_n - \frac{f(x_n) * (x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} \tag{5}$$

0.6 Newton‘s Method

$$x_{n+1} = x_n - \frac{y(x_n)}{y'(x_n)} \tag{6}$$

0.7 Fixed-Point

$$f(p) = p \tag{7}$$

$$x = f(x) = 0 \tag{8}$$

Find a convenient $x = g(x)$, and use:

$$x_{n+1} = g(x_n) \tag{9}$$

0.8 Existence (Intermediate Value Theorem)

If y changes signal in $[a, b]$ and is continuous, exists at least one root in $[a, b]$.

0.9 Uniqueness (Monotone)

If y' does not change signal in $[a, b]$ and it is continuous, exists only one root in $[a, b]$.

0.10 Newton‘s First Modification Method

$$x_{n+1} = x_n - \frac{my(x_n)}{y'(x_n)} \tag{10}$$

Used when multiplicity of the root is known to be $m > 1$. Converges Quadratically.

0.11 Newton‘s Second Modification Method

$$x_{n+1} = x_n - \frac{y(x_n)y'(x_n)}{[y'(x_n)]^2 - y(x_n)y''(x_n)} \tag{11}$$

Used when multiplicty of the root is not know. Converges Quadratically.

0.12 Discovering Multiplicity

If $x = \alpha$ is root of the function. Do continuous derivatives until:

$$\begin{aligned} f(a) &= 0 \\ f'(a) &= 0 \\ f^{(n-1)}(a) &= 0 \\ f^{(n)}(a) &\neq 0 \qquad (12) \\ & \qquad (13) \end{aligned}$$

The function in root $x = \alpha$ have multiplicity n.

0.13 Order Convergence

$$|e_{n+1}| \approx \beta |e_n|^R \tag{14}$$

β is the asymptotic error constant. R is the order of convergence.

0.14 Fixed-Point Theorem

If $g(x)$ is continuous on $[a, b]$ and $g(x) \in [a, b] \forall x \in [a, b]$. Then, g is guaranted to have at least one fixed-point in $[a, b]$.

Moreover, if $|g'(x)| \leq k < 1 \forall x \in [a, b]$, then the fixed-point is unique.

Step one: Verify the boundaries. If positive, g have at least one fixed-point. If negative, g does not have fixed points. Don't forget to check if g is monotone.

$$\begin{aligned} g(a) &\in [a,b]? && (15) \\ g(b) &\in [a,b]? && (16) \\ g'(x) &\geq 0 \forall x \in [a,b]?, \ \textit{or} \\ g'(x) &\leq 0 \forall x \in [a,b]? && (17) \end{aligned}$$

Step two: Verify if the modular derivated function have maximum value less than 1. If $|g'(x)| > 1$, the sequence will divere. Otherwise, the fixed point is unique and can be discovered by the iterative scheme.

$$x_{n+1} = g(x_n), \forall x_0 \in [a,b] \tag{18}$$

0.15 Bisection Error Bound

$$|E| \leq \frac{b-a}{2^n} \tag{19}$$

To discover the minimum number of iterations needed to ensure a required tolerance (T):

$$\lceil n \rceil \geq \frac{\ln \frac{b-a}{T}}{\ln 2} \tag{20}$$

0.16 Fixed-Point Error Bound

$$|E| \leq \frac{k^n}{1-k}|x_0 - x_1| \tag{21}$$

To discover the minimum number of iterations needed to ensure a required tolerance (T):

$$\lceil n \rceil \geq \frac{\ln \frac{T(1-k)}{|x_0 - x_1|}}{\ln k} \tag{22}$$

To discover the order of convergence, for an $x = \alpha$ as fixed point of g.

$$\lim_{n \to \inf} \frac{|e_{n+1}|}{|e_n|} = \frac{g^{(R)}(\alpha)}{R!} \tag{23}$$