Solutions to H.W. #9

1. (a) $I = \int_1^2 \ln(x+1)\,dx$ $\qquad u = \ln(x+1) \qquad dv = dx$

$\qquad du = \frac{1}{x+1}dx \qquad v = x$

$$\therefore I = x\ln(x+1)\Big|_1^2 - \int_1^2 \frac{x}{x+1}\,dx$$

Using long division:

$$I = x\ln(x+1)\Big|_1^2 - \int_1^2 1 - \frac{1}{x+1}\,dx$$

$$= x\ln(x+1)\Big|_1^2 - \Big[x - \ln(x+1)\Big]_1^2$$

$$= \Big[x\ln(x+1) + \ln(x+1) - x\Big]_1^2$$

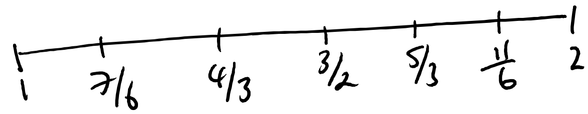
$$= \big(2\ln(3) + \ln(3) - 2\big) - \big(\ln(2) + \ln(2) - 1\big)$$

$$= 3\ln(3) - 2\ln(2) - 1$$

$$= \ln(27) - \ln(4) - 1$$

$$= \ln\left(\frac{27}{4}\right) - 1$$

(b) Let $f(x) = \ln(x+1)$, $n=6 \Rightarrow h = \frac{2-1}{6} = \frac{1}{6}$



$$\int_1^2 f(x)\,dx \approx \frac{1/6}{2}\Big[f(1) + 2f(7/6) + 2f(4/3) + 2f(3/2) + 2f(5/3) + 2f(11/6) + f(2)\Big]$$

$$= 0.9091568907$$

$$\text{Abs. error} = \left|0.9091568907 - I\right|$$

$$= 0.00038561532$$

(c) $$\int_1^2 f(x)\,dx \approx \frac{1/6}{3}\Big[f(1) + 4f(7/6) + 2f(4/3) + 4f(3/2) + 2f(5/3) + 4f(11/6) + f(2)\Big]$$

$$= 0.9095417598$$

$$\text{Abs. error} = \left|0.9095417598 - I\right|$$

$$= 0.0000074624$$

(d) $$\int_1^2 f(x)\,dx \approx \frac{3}{8}\cdot\frac{1}{6}\Big[f(1) + 3f(7/6) + 3f(4/3) + 2f(3/2) + 3f(5/3) + 3f(11/6) + f(2)\Big]$$

$$= 0.909540847 9$$

$$\text{Abs. error} = |0.9095408479 - I|$$

$$= 0.0000165814$$

2. Let $f(x) = e^{-x^2}$, $n = 8 \Rightarrow h = \frac{1-0}{8} = \frac{1}{8}$

$$\therefore \int_0^1 f(x)\,dx \approx \frac{1/8}{3}\Big[f(0) + 4f(1/8) + 2f(1/4) + 4f(3/8) + 2f(1/2)$$
$$+ 4f(5/8) + 2f(3/4) + 4f(7/8) + f(1)\Big]$$

$$= 0.7468261206$$

3. Distance traveled $= \displaystyle\int_0^{10} |v(t)|\,dt$

Since $v(t) \geq 0$ at each $t = 0, 1, \ldots, 10$,

$$= \int_0^{10} v(t)\,dt$$

$$\approx \frac{1}{3}\Big[v(0) + 4v(1) + 2v(2) + 4v(3) + 2v(4)$$
$$+ 4v(5) + 2v(6) + 4v(7) + 2v(8) +$$
$$4v(9) + v(10)\Big]$$

$$= 40.1 \text{ cm}$$

4. (a) For the composite trapezoidal method, the truncation error term is $\frac{h^2}{12}(b-a)f''(c)$, $c \in (a,b)$

Since $\max_{0 \le x \le 5} |f''(x)| = 8$, we require the maximum h such that

$$\frac{h^2}{12} \cdot (5-0) \cdot 8 \le 10^{-6}$$

$$\frac{10}{3} h^2 \le 10^{-6}$$

$$h^2 \le 3 \cdot 10^{-7}$$

Since $h \ge 0$,

$$h \le \sqrt{3 \cdot 10^{-7}} = 0.005477226$$

Moreover, $\frac{5}{n} \le 0.005477226$

$$\therefore n \ge \frac{5}{0.005477226} = 9128.709292$$

Take $n = 9129$.

(b) Since the error term for Simpson's 1/3 method is $\frac{h^4}{180}(b-a)f^{(4)}(c)$, $c \in (a,b)$, we require the maximum h such that

$$\frac{h^4}{180} \cdot 5 \cdot 18 \leq 10^{-6}$$

$$\frac{h^4}{2} \leq 10^{-6}$$

$$h^4 \leq 2 \cdot 10^{-6}$$

Since $h \geq 0$, $h \leq \sqrt[4]{2 \cdot 10^{-6}} = 0.03760603$

Moreover, $\frac{5}{n} \leq 0.03760603$

$$n \geq \frac{5}{0.03760603} = 132.9573974$$

In the Simpson's 1/3 method, n must be even, so, take $n = 134$.

5. Let $f(x) = \sqrt[3]{x+1} = (x+1)^{1/3}$

$\therefore f'(x) = \frac{1}{3}(x+1)^{-2/3}$, $f''(x) = -\frac{2}{9}(x+1)^{-5/3}$

So, $|f''(x)| = \frac{2}{9(x+1)^{5/3}}$. Since $|f''(x)|$ is decreasing on $0 \le x \le 2$, the maximum of $|f''(x)|$ on $0 \le x \le 2$ occurs at $x = 0$:

$$\max_{0 \le x \le 2} |f''(x)| = \frac{2}{9}$$

We require the maximum h such that

$$\frac{h^2}{12} \cdot (2) \cdot \frac{2}{9} \le 10^{-6}$$

$$\frac{h^2}{27} \le 10^{-6}$$

$$h^2 \le 27 \cdot 10^{-6}$$

Since $h \ge 0$, $h \le \sqrt{27 \cdot 10^{-6}} = 0.005196152$

Moreover, $\frac{2}{n} \le 0.005196152$

$$\therefore n \ge \frac{2}{0.005196152} = 384.9001794$$

Take $n = 385$.