Solutions to H.W. #1

1. $f(x) = 2x^2 - x - 1$

(a) $f(1.234) = 2\cdot(1.234)^2 - (1.234) - 1 = 0.811512$

(b) $f(1.234) \approx f(1.2) = 2\cdot(1.2)^2 - (1.2) - 1$

$\approx 2\cdot(1.4) - 1.2 - 1.0$

$= 2.8 - 1.2 - 1$

$= 0.6$

Absolute error $= |0.811512 - 0.6| = 0.211512$

2. $\sin(x) = \sum_{k=0}^{\infty} (-1)^k \cdot \frac{x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots$

(a) $\sin(\pi/4) \approx \pi/4 - \frac{(\pi/4)^3}{6} + \frac{(\pi/4)^5}{120} = 0.7071430459$

(b) Absoluter error $= |\sin(\pi/4) - 0.7071430459|$

$= \left|\frac{\sqrt{2}}{2} - 0.7071430459\right|$

$= 0.0000362649 = 3.62649 \times 10^{-5}$

(c) Percentage error $= \frac{|\sin(\pi/4) - 0.7071430459|}{|\sin(\pi/4)|}$

$= 0.0000512863134 0 = 5.128631340 \times 10^{-5}$

(d) 4 significant digits

3. (a) $f(x) = e^{-2x} - x$, $[-1,1]$

Since $f(x)$ is continuous on $[-1,1]$, $f(-1) > 0$ and $f(1) < 0$, by the Intermediate Value Theorem, $f(x)$ has a least one root in $[-1,1]$.

Moreover, since $f'(x) = -2e^{-2x} - 1 < 0 \quad \forall x \in [-1,1]$, $f(x)$ is monotone decreasing. Therefore, the root in $[-1,1]$ is unique! Q.E.D.

(b) $f(x) = \cos(x) - x$, $[0, \pi/2]$

Since $f(x)$ is continuous on $[0, \pi/2]$, $f(0) > 0$ and $f(\pi/2) < 0$, by the Intermediate Value Theorem, $f(x)$ has at least one root in $[0, \pi/2]$.

Moreover, since $f'(x) = -\sin(x) - 1 < 0 \quad \forall x \in [0, \frac{\pi}{2}]$, $f(x)$ is monotone decreasing. Therefore, $f(x)$ has a unique root in $[0, \pi/2]$! Q.E.D.

4. (a) Let $x = \sqrt{\frac{3}{5}}$

$$\therefore \quad x^2 = \frac{3}{5}$$

$$5x^2 - 3 = 0$$

So, approximating the positive root of $f(x) = 5x^2 - 3$ is equivalent to approximating $\sqrt{\frac{3}{5}}$.

(b) To find the desired intersection point(s), we must solve

$$\sin(x) = x^3 - 1$$

$$\therefore \quad \sin(x) - x^3 + 1 = 0$$

So, approximating the root(s) of $f(x) = \sin(x) - x^3 + 1$ is equivalent to finding said intersection point(s).

5. (a) $f(x) = x^3 - 4x - 2$, $[1, 3]$

a_n	$m_n = \frac{a_n + b_n}{2}$	b_n
1	2	3
2	2.5	3
2	2.25	2.5
2	2.125	2.25

The 4th iterate is 2.125

(b) $f(x) = x^4 + \ln(3x-1)$, $[0.4, 0.7]$

a_n	$m_n = \frac{a_n + b_n}{2}$	b_n
0.4	0.55	0.7
0.55	0.625	0.7
0.55	0.5875	0.625
0.5875	0.60625	0.625

The 4th iterate is 0.60625.

```
>> % Question 6
>> solve('2*x+9=5')

ans =

-2

>> % Question 7
>> bisection
    step        a          b        m          ym      bound

    1.0000    2.5000    3.5000    3.0000   -0.1000    0.5000

    2.0000    3.0000    3.5000    3.2500    0.6031    0.2500

    3.0000    3.0000    3.2500    3.1250    0.1988    0.1250

    4.0000    3.0000    3.1250    3.0625    0.0370    0.0625

    5.0000    3.0000    3.0625    3.0312   -0.0345    0.0312

    6.0000    3.0312    3.0625    3.0469    0.0004    0.0156

    7.0000    3.0312    3.0469    3.0391   -0.0172    0.0078

    8.0000    3.0391    3.0469    3.0430   -0.0084    0.0039

    9.0000    3.0430    3.0469    3.0449   -0.0040    0.0020

   10.0000    3.0449    3.0469    3.0459   -0.0018    0.0010

   11.0000    3.0459    3.0469    3.0464   -0.0007    0.0005

   12.0000    3.0464    3.0469    3.0466   -0.0001    0.0002

   13.0000    3.0466    3.0469    3.0468    0.0002    0.0001

   14.0000    3.0466    3.0468    3.0467    0.0000    0.0001

   15.0000    3.0466    3.0467    3.0467   -0.0000    0.0000

   16.0000    3.0467    3.0467    3.0467   -0.0000    0.0000

   17.0000    3.0467    3.0467    3.0467    0.0000    0.0000

   18.0000    3.0467    3.0467    3.0467   -0.0000    0.0000

   19.0000    3.0467    3.0467    3.0467    0.0000    0.0000

   20.0000    3.0467    3.0467    3.0467    0.0000    0.0000

   21.0000    3.0467    3.0467    3.0467    0.0000    0.0000

   22.0000    3.0467    3.0467    3.0467    0.0000    0.0000

   23.0000    3.0467    3.0467    3.0467    0.0000    0.0000

   24.0000    3.0467    3.0467    3.0467   -0.0000    0.0000
```

```
   25.0000    3.0467    3.0467    3.0467    0.0000    0.0000

   26.0000    3.0467    3.0467    3.0467   -0.0000    0.0000

   27.0000    3.0467    3.0467    3.0467    0.0000    0.0000

   28.0000    3.0467    3.0467    3.0467   -0.0000    0.0000

The bisection method has converged in 28 iterations.
The approximate solution is 3.046680528671.
The function value at the approximation is -7.166379489831e-09.
```