CS 3500 AA – Numerical Methods I
Fall 2015
Homework Problem Set #3
Due: 09/17/2015

Be sure to do all your work on separate paper, and include all steps where appropriate. All homework must follow the formatting rules posted on Blackboard.

1. Determine if the function is guaranteed to have at least one fixed-point on the indicated interval.
   (a) $g(x) = \frac{1}{2}e^{x/2}$, $[4,5]$
   (b) $g(x) = \frac{1}{5}\cos(x)$, $[0,\pi/2]$
2. Show that there exists a unique fixed-point of $g(x) = \frac{1}{2}e^{0.5x}$ on $[0,1]$.
3. Show that $x = 1$ is a root of $f(x) = (x-1)^2\ln(x)$. What is the multiplicity of $\alpha = 1$?
4. Show that $x = 1$ is a root of $f(x) = x^4 - x^3 - 3x^2 + 5x - 2$ and determine its multiplicity. Find x_3 starting with $x_0 = 0.5$, using
   (a) Newton's method.
   (b) the first modification of Newton's method.
5. Suppose an iterative scheme is known to converge with order $R = 2$ and asymptotic error constant $\beta = 0.5$. If e_0 is known to be 0.25, estimate e_1, e_2, and e_3.
6. The bisection method is used to generate a sequence of approximations, $\{x_n\}$, using a starting interval $[1,4]$. Give an error bound for the tenth iterate, x_{10}.
7. Determine the minimum number of iterations of the bisection method needed to approximate the unique zero of a continuous function, $f(x)$, known to exist in the interval $[-3,-2]$ to within
   (a) 10^{-5} (b) 10^{-8}