General Physics I
Homework Chapter 11

Jonathan Henrique Maia de Moraes (ID: 1620855)

04/22/2016

Homework: Chapter 11

Problem (1)

A 5.6 sl hoop rolls along a horizontal floor so that its center of mass has a speed of 0.75 ft/s. How much work must be done on the hoop to stop it?

R:

$$\begin{aligned} W &= K_f - K_i \\ &= (0) - \frac{1}{2}mv_{cm}^2 \\ &= -\frac{1}{2}(5.6\ sl)(0.75\ ft/s)^2 \\ &= -(2.8\ sl)\left(0.5625\ ft^2/s^2\right) \\ &= -1.5750\ sl \times ft^2/s^2 \\ |W| &= 1.5750\ lb \times ft \end{aligned} \tag{1}$$

Problem (2)

A hollow ball rolls along the floor. What is the ratio of its translational kinetic energy to its rotational kinetic energy about the central axis parallel to its length?

R:

$$
\begin{aligned}
I_{cm} &= \frac{2}{3}MR^2 \\
K_{trans} &= \frac{1}{2}MV_{cm}^2 \\
&= \frac{1}{2}MR^2\omega^2 \\
K_{rot} &= \frac{1}{2}I_{cm}\omega^2 \\
&= \frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega^2 \\
&= \frac{1}{3}MR^2\omega^2 \\
\text{Ratio} &= \frac{K_{trans}}{K_{rot}} \\
&= \frac{\frac{1}{2}MR^2\omega^2}{\frac{1}{3}MR^2\omega^2} \\
&= \frac{3}{2} = 1.5 \qquad (2)
\end{aligned}
$$

Problem (3)

In the instant of the fig. 1, two particles move in an xy plane. Particle P_1 has mass 1.8 sl and speed $v_1 =$ 5.2 ft/s, and it is at distance $d_1 =$ 4.0 ft from point O. Particle P_2 has mass 1.2 sl and speed $v_2 =$ 7.6 ft/s, and it is at distance $d_2 =$ 7.5 ft from point O. What is the net angular momentum of the two particles about O? Take counterclockwise as the positive direction.

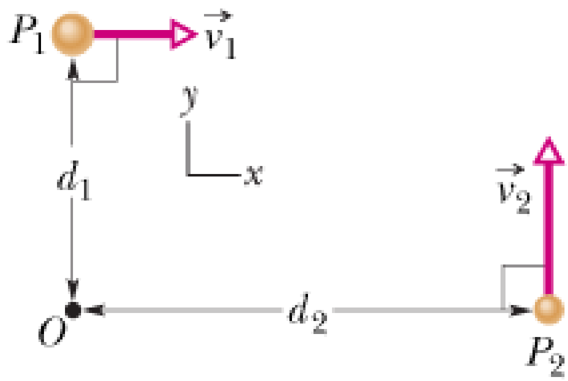

Figure 1: Illustration of Problem 3

R:

$$
\begin{aligned}
L_{P_1} &= d_{1\perp} m_1 v_1 \\
&= -(4.0\ ft)(1.8\ sl)(5.2\ ft/s) \\
&= -37.44\ sl \times ft^2/s \\
L_{P_2} &= d_{2\perp} m_2 v_2 \\
&= (7.5\ ft)(1.2\ sl)(7.6\ ft/s) \\
&= 68.40\ sl \times ft^2/s \\
L_{net} &= L_{P_1} + L_{P_2} \\
&= (-37.44\ sl \times ft^2/s) + (68.40\ sl \times ft^2/s) \\
&= 30.96\ sl \times ft^2/s
\end{aligned} \tag{3}
$$

Problem (4)

In the fig. 2, three particles of mass $m = 3.2\ kg$ are fastened to three rods of length $d = 0.45\ m$ and negligible mass. The rigid assembly rotates about point O at angular speeed $\omega =\ 9.0\ rad/s$. About O,

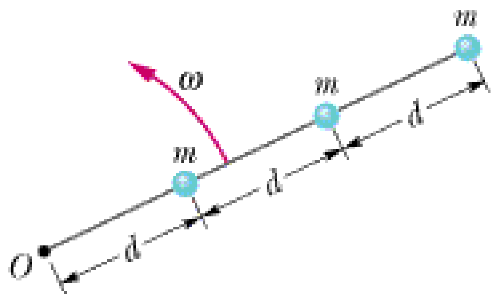

Figure 2: Illustration of Problem 4

Question (a)

What is the rotational inertia of the assembly?
R:

$$
\begin{aligned}
m_1 &= m_2 = m_3 = m \\
r_1 &= d \\
r_2 &= 2d \\
r_3 &= 3d \\
I &= \sum_i m_i r_i^2 \\
&= \left(m_1 r_1^2\right) + \left(m_2 r_2^2\right) + \left(m_3 r_3^2\right) \\
&= m\left[(d)^2 + (2d)^2 + (3d)^2\right] \\
&= m\left[14(d)^2\right] \\
&= (14)(3.2\ kg)(0.45\ m)^2 \\
&= (44.8\ kg)\left(0.2025\ m^2\right) \\
&= 9.0720\ kg \times m^2
\end{aligned} \tag{4}
$$

Question (b)

What is the magnitude of the angular momentum of the asembly?
R:

$$\begin{aligned} L &= I\omega \\ &= \left(9.0720\ kg \times m^2\right)(9.0\ rad/s) \\ &= 81.6480\ kg \times m^2/s \end{aligned} \tag{5}$$

Problem (5)

A man stands on a platform that is rotating (without friction) with an angular speed of 0.54 rev/s; his arms are outstretched and he holds a brick in each hand. The rotational inertia of the system consisting of the man, bricks, and platform about the central axis is 4.4 $kg \times m^2$. If by moving the bricks the man decreases the rotational inertia of the system to 2.8 $kg \times m^2$,

Question (a)

What is the resulting angular speed of the platform?

R:

$$\begin{aligned} \omega &= 0.54\ rev/s \times \left(\frac{2\pi\ rad}{1\ rev}\right) \\ &= 3.39292\ rad/s \\ \Delta L &= 0 \\ L_f &= L_i \\ I_f\omega_f &= I_i\omega_i \\ \omega_f &= \frac{I_i\omega_i}{I_f} \\ &= \frac{(4.4\ kg \times m^2)(3.39292\ rad/s)}{2.8\ kg \times m^2} \\ &= 5.33173\ rad/s \end{aligned} \tag{6}$$

Question (b)

What is the ratio of the new kinetic energy of the system to the original kinetic energy?

R:

$$
\begin{aligned}
\text{Ratio} &= \frac{K_f}{K_i} \\
&= \frac{\frac{1}{2} I_f \omega_f^2}{\frac{1}{2} I_i \omega_i^2} \\
&= \frac{(2.8\ kg \times m^2)\,(5.33173\ rad/s)^2}{(4.4\ kg \times m^2)\,(3.39292\ rad/s)^2} \\
&= \frac{(2.8\ kg \times m^2)\,(28.42734\ s^{-2})}{(4.4\ kg \times m^2)\,(11.51191\ s^{-2})} \\
&= \frac{79.59657\ kg \times m^2/s^2}{50.65239\ kg \times m^2/s^2} \\
&= 1.57143
\end{aligned} \tag{7}
$$

Problem (6)

A wheel is rotating freely at angular speed 960 rev/min on a shaft whose rotational inertia is negligible. A second wheel, initially at rest and with 4.0 times the rotational inertia of the first, is suddenly coupled to the same shaft.

Question (a)

What is the angular speed of the resultant combination of the shaft and two wheels?

R:

$$
\begin{aligned}
\omega_{1i} &= 960\ rev/min \times \left(\frac{2\pi\ rad}{1\ rev}\right) \times \left(\frac{1\ min}{60\ s}\right) \\
&= 100.5310\ rad/s \\
\omega_{2i} &= 0\ rad/s \\
I_{2i} &= 4 I_{1i} \\
I_{Rf} &= 5 I_{1i}
\end{aligned}
$$

$$\begin{aligned}
\Delta L &= 0 \\
L_f &= L_i \\
I_{Rf}\omega_{Rf} &= I_{1i}\omega_{1i} + I_{2i}\omega_{2i} \\
5I_{1i}\omega_{Rf} &= I_{1i}(100.531\ rad/s) + 4I_{1i}(0) \\
\omega_{Rf} &= \frac{I_{1i}(100.531\ rad/s)}{5I_{1i}} \\
&= 20.1062\ rad/s \qquad (8)
\end{aligned}$$

Question (b)

What fraction of the original rotational kinetic energy is lost?

R:

$$\begin{aligned}
\text{fraction}\quad \text{lost} &= 1 - \frac{K_{rot,f}}{K_{rot,i}} \\
&= 1 - \frac{5I_{1i}\omega_{Rf}^2}{\frac{1}{2}\left(I_{1i}\omega_{1i}^2 + I_{2i}\omega_{2i}^2\right)} \\
&= 1 - \frac{5\omega_{Rf}^2}{\omega_{1i}^2 + (0)} \\
&= 1 - \frac{5(20.1062\ rad/s)^2}{(100.5310\ rad/s)^2} \\
&= 1 - \frac{5(404.2593)}{10\ 106.4820} \\
&= 1 - \frac{2021.2965}{10\ 106.4820} \\
&= 1 - 0.2000 \\
&= 0.8000 = 80\% \qquad (9)
\end{aligned}$$

Problem (7)

Two disks are mounted (like a merry-go-round) on low friction bearings on the same axle and can be brought together so that they couple and rotate as one unit. The first disk, with rotational inertia 4.8 $sl \times ft^2$ about its central axis, is set spinning counterclockwise (which may be taken as the positive direction) at 420 rev/min. The second disk, with rotational inertia 3.6 $sl \times ft^2$ about its central axis, is set spinning counterclockwise at 640 rev/min. They then couple together.

Question (a)

What is the angular speed (rev/min) after coupling?
R:

$$\begin{aligned}
I_{Rf} &= I_{1i} + I_{2i} \\
I_{Rf} &= \left(4.8\ sl \times ft^2\right) + \left(3.6\ sl \times ft^2\right) \\
I_{Rf} &= 8.4\ sl \times ft^2 \\
\Delta L &= 0 \\
L_f &= L_i \\
I_{Rf}\omega_{Rf} &= I_{1i}\omega_{1i} + I_{2i}\omega_{2i} \\
\left(8.4\ sl \times ft^2\right)\omega_{Rf} &= \left(4.8\ sl \times ft^2\right)(420\ rev/min) + \left(3.6\ sl \times ft^2\right)(640\ rev/min) \\
\omega_{Rf} &= \frac{(2016\ sl \times ft^2 \times rev/min) + (2304\ sl \times ft^2 \times rev/min)}{8.4\ sl \times ft^2} \\
&= \frac{4320\ sl \times ft^2 \times rev/min}{8.4\ sl \times ft^2} \\
&= 514.2857\ rev/min \qquad (10)
\end{aligned}$$

Question (b)

If instead the second disk is set spinning clockwise at 640 rev/min, what is their angular velocity (in rev/min, using the correct sign for direction) after they couple together?
R:

$$\begin{aligned}
\omega_{Rf} &= \frac{(2016\ sl \times ft^2 \times rev/min) + (-2304\ sl \times ft^2 \times rev/min)}{8.4\ sl \times ft^2} \\
&= \frac{-288\ sl \times ft^2 \times rev/min}{8.4\ sl \times ft^2} \\
&= -34.2857\ rev/min \qquad (11)
\end{aligned}$$

Problem (8)

A horizontal vinyl record of radius 0.089 m rotates freely about a vertical axis through its center with an angular speed of 4.7 rad/s. The rotational inertia of the record about its axis of rotation is $3.0 \times 10^{-4} kg \times m^2$. A wad of wet putty of mass 24 g drops vertically onto the record from above and sticks to the edge of the record. What is the angular speed of the record immediately after the putty sticks to it?

R:

$$
\begin{aligned}
r_p &= r = 0.089\ m \\
v_f &= v_{vf} = v_{pf} = r\omega_{vf} \\
\Delta L &= 0 \\
L_f &= L_i \\
\left(m_p r^2 + I_v\right)\omega_{vf} &= I_v \omega_{vi} \\
\omega_{vf} &= \frac{I_v \omega_{vi}}{m_p r^2 + I_v} \\
\omega_{vf} &= \frac{(0.0003\ kg \times m^2)\,(4.7\ rad/s)}{[(0.024\ kg)(0.089\ m)^2] + (0.0003\ kg \times m^2)} \\
\omega_{vf} &= \frac{0.00141\ kg \times m^2 \times rad/s}{(1.9010 \times 10^{-4}\ kg \times m^2) + (3.0 \times 10^{-4}\ kg \times m^2)} \\
\omega_{vf} &= \frac{0.00141\ kg \times m^2 \times rad/s}{4.9010 \times 10^{-4}\ kg \times m^2} \\
\omega_{vf} &= 2.8769\ rad/s
\end{aligned} \tag{12}
$$