General Physics I
Homework Chapter 8

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Homework: Chapter 8

Problem (1)

You drop an 11.2 lb book to a friend who stands on the ground at distance $D = 39.0\ ft$ below. If your friend's outstretched hands are at distance $d = 5.3\ ft$ above the ground:

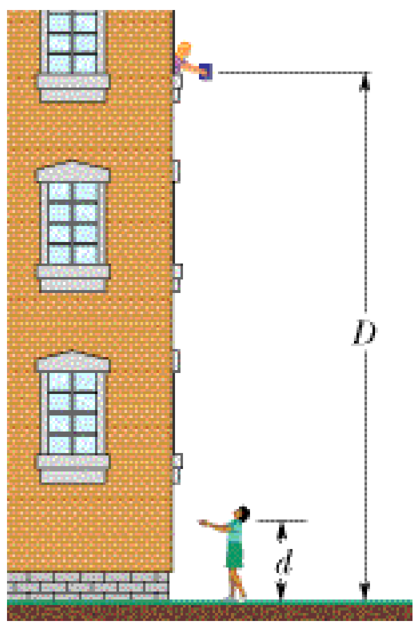

Figure 1: Illustration of Problem 1

Question (a)

How much work W_g does the gravitational force do on the book as it drops to her hands?

R:

$$\begin{aligned} W_g &= F_g \Delta y \\ &= (11.2\ lb)[(5.3\ ft) - (39.0\ ft)] \\ &= -377.44\ ft \times lb \end{aligned} \tag{1}$$

Question (b)

What is the change ΔU in the gravitational potential energy of the book-Earth system during the drop?

R:

$$\begin{aligned} \Delta U_g &= U_{g_f} - U_{g_i} \\ &= mgy_f - mgy_i \\ &= [(11.2\ lb)(5.3\ ft)] - [(11.2\ lb)(39.0\ ft)] \\ &= (59.36\ ft \times lb) - (436.8\ ft \times lb) \\ &= -377.44\ ft \times lb \end{aligned} \tag{2}$$

Problem (2)

What is the spring constant of a spring that stores 34 J of elastic potential energy when compressed by 7.6 cm from its relaxed length?

R:

$$
\begin{aligned}
U_s &= \frac{1}{2}kx^2 \\
k &= 2\frac{U_s}{x^2} \\
&= 2\frac{34\ J}{(0.076m)^2} \\
&= \frac{68\ J}{0.0058m^2} \\
&= 11\ 772.8532\ N/m
\end{aligned} \tag{3}
$$

Problem (3)

In the fig. 2, a frictionless roller coaster car of mass $m = 16\ sl$ tops the first hill with speed $v_0 = 16\ ft/s$ at height $h = 36\ ft$.

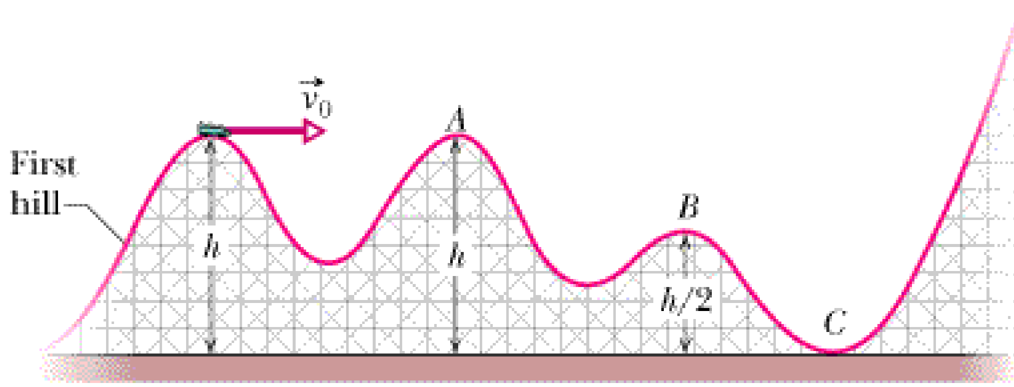

Figure 2: Illustration of Problem 3

Question (a)

What is the speed of the car at point A?

R:

$$
\begin{aligned}
E_{tot,0} &= K_0 + U_{g_0} \\
&= \frac{1}{2}mv_0^2 + mgy_0 \\
&= \left[\frac{1}{2}(16\ sl)(16\ ft/s)^2\right] + \left[(16\ sl)\left(32.2\ ft/s^2\right)(36\ ft)\right] \\
&= (2048\ ft \times lb) + (18\ 547.2\ ft \times lb) \\
&= 20\ 595.2\ ft \times lb
\end{aligned}
$$

$$
\begin{aligned}
E_{tot,1} &= K_1 + U_{g_1} \\
E_{tot,1} &= \frac{1}{2}mv_1^2 + mgy_1 \\
v_1 &= \sqrt{\frac{E_{tot,1} - mgy_1}{\frac{1}{2}m}} \\
&= \sqrt{\frac{(20\ 595.2\ ft \times lb) - (18\ 547.2\ ft \times lb)}{8\ sl}} \\
&= \sqrt{\frac{2048\ ft \times lb}{8\ sl}} \\
&= \sqrt{256 ft^2/s^2} \\
&= 16\ ft/s
\end{aligned}
\tag{4}
$$

Question (b)

What is the speed of the car at point B?

R:

$$\begin{aligned}
E_{tot,2} &= K_2 + U_{g_2} \\
E_{tot,2} &= \frac{1}{2}mv_2^2 + mgy_2 \\
v_2 &= \sqrt{\frac{E_{tot,2} - mgy_2}{\frac{1}{2}m}} \\
&= \sqrt{\frac{(20\ 595.2\ ft \times lb) - (9273.6\ ft \times lb)}{8\ sl}} \\
&= \sqrt{\frac{11\ 321.6\ ft \times lb}{8\ sl}} \\
&= \sqrt{1415.2 ft^2/s^2} \\
&= 37.62\ ft/s \qquad (5)
\end{aligned}$$

Question (c)

What is the speed of the car at point C?
R:

$$\begin{aligned}
E_{tot,3} &= K_3 + U_{g_3} \\
E_{tot,3} &= \frac{1}{2}mv_3^2 + mg(0) \\
v_3 &= \sqrt{\frac{E_{tot,3}}{\frac{1}{2}m}} \\
&= \sqrt{\frac{20\ 595.2\ ft \times lb}{8\ sl}} \\
&= \sqrt{2574.4 ft^2/s^2} \\
&= 50.74\ ft/s \qquad (6)
\end{aligned}$$

Question (d)

How high will the car go on the last hill, which is too high for it to cross?
R:

$$
\begin{aligned}
E_{tot,4} &= K_4 + U_{g4} \\
E_{tot,4} &= \frac{1}{2}m(0)^2 + mgy_4 \\
y_4 &= \frac{E_{tot,4}}{mg} \\
&= \frac{20\ 595.2\ ft \times lb}{(16\ sl)\,(32.2\ ft/s^2)} \\
&= \frac{20\ 595.2\ ft \times lb}{515.2\ lb} \\
&= 39.98\ ft
\end{aligned} \tag{7}
$$

Problem (4)

A 0.35 kg rock is fired from a cliff 14 m high with an initial velocity of 17 m/s, directed 49^o above the horizontal.

Question (a)

Using energy techniques, find the speed of the rock as it reaches the ground below the cliff.

R:

$$
\begin{aligned}
E_{tot,0} &= K_0 + U_{g_0} \\
&= \frac{1}{2}mv_0^2 + mgy_0 \\
&= \left[\frac{1}{2}(0.35\ kg)(17\ m/s)^2\right] + \left[(0.35\ kg)\left(9.8m/s^2\right)(14\ m)\right] \\
&= (50.575\ J) + (48.02\ J) \\
&= 98.595\ J
\end{aligned}
$$

$$\begin{aligned} E_{tot,1} &= K_1 + U_{g_1} \\ E_{tot,1} &= \frac{1}{2}mv_1^2 + mg(0) \\ v_1 &= \sqrt{\frac{E_{tot,1}}{\frac{1}{2}m}} \\ &= \sqrt{\frac{98.595\ J}{0.175\ kg}} \\ &= \sqrt{563.4\ m^2/s^2} \\ &= 23.736\ m/s \end{aligned} \tag{8}$$

Question (b)

What is that speed if the launch angle is changed to 49^o below the horizontal?

R: Since the absolute value of the initial speed $|\vec{v_0}|(= v_0)$ remains the same and there are no non-conservative forces applied, the final speed will be the same: $23.736\ m/s$. The angle will change the total time elapsed. (In this case, will be shorter than if launched with the original angle).

Question (c)

What is that speed if the mass is changed to $1.5\ kg$?

R: It will be the same.

$$\begin{aligned} E_{tot,0} &= K_0 + U_{g_0} \\ &= \frac{1}{2}mv_0^2 + mgy_0 \\ &= \left[\frac{1}{2}(1.5\ kg)(17\ m/s)^2\right] + \left[(1.5\ kg)\left(9.8m/s^2\right)(14\ m)\right] \\ &= (216.75\ J) + (205.8\ J) \\ &= 422.55\ J \end{aligned}$$

$$
\begin{aligned}
E_{tot,1} &= K_1 + U_{g_1} \\
E_{tot,1} &= \frac{1}{2} m v_1^2 + mg(0) \\
v_1 &= \sqrt{\frac{E_{tot,1}}{\frac{1}{2} m}} \\
&= \sqrt{\frac{422.55\ J}{0.75\ kg}} \\
&= \sqrt{563.4\ m^2/s^2} \\
&= 23.736\ m/s
\end{aligned} \tag{9}
$$

Problem (5)

A 5.1 g marble is fired vertically upward using a spring gun. The spring must be compressed 3.8 cm if the marble is to just reach a target 9.6 m above the marble's position on the compressed spring.

Question (a)

What is the change ΔU_g in the gravitational potential energy of the marble-Earth system during the 9.6 m ascent?

R:

$$
\begin{aligned}
\Delta U_g &= U_{g_f} - U_{g_i} \\
\Delta U_g &= mgy_f - mg(0) \\
\Delta U_g &= (0.0051\ kg)\left(9.8 m/s^2\right)(9.6\ m) \\
\Delta U_g &= 0.4798\ J
\end{aligned} \tag{10}
$$

Question (b)

What is the change ΔU_s in the elastic potential energy of the spring during its launch of the marble?

R:

$$\Delta U_s = U_{s_f} - U_{s_i}$$
$$\Delta U_s = \frac{1}{2}k(0)^2 - \frac{1}{2}kx_i^2$$
$$\Delta U_s = -\frac{1}{2}kx_i^2$$

Since the change in the elastic potential energy is based only on its initial condition and the change in the gravitational potential energy is based only on its final condition:

$$\begin{aligned} E_{tot,f} = U_{g_f} = E_{tot,i} = U_{s_i} = -\Delta U_s \\ \Delta U_s = -0.4798\ J \end{aligned} \tag{11}$$

Question (c)

What is the spring constant of the spring?

R:

$$\begin{aligned} U_{s_i} &= \frac{1}{2}kx_i^2 \\ k &= 2\frac{U_{s_i}}{x_i^2} \\ &= 2\frac{0.4798\ J}{(0.038\ m)^2} \\ &= 664.5429\ N/m \end{aligned} \tag{12}$$

Problem (6)

A collie drags its bed box across a floor by applying a horizontal force of 3.1 lb. The kinetic frictional force acting on the box has magnitude 1.8 lb. As the box is dragged at constant speed through 4.2 ft along the way:

Question (a)

What is the work done by the collie's applied force?

R:

$$\begin{aligned} W_{\text{collie}} &= \vec{F}_{\text{collie}}\ \Delta\vec{r} \\ &= (3.1\ lb)(4.2\ ft) \\ &= 13.02\ ft \times lb \end{aligned}$$

Question (b)

What is the work done by the friction force?

R:

$$\begin{aligned} W_{F_s} &= \vec{F}_s \Delta\vec{r} \\ &= -(1.8\ lb)(4.2\ ft) \\ &= -7.56\ ft \times lb \end{aligned}$$

Problem (7)

A 62 kg skier leaves the end of a ski-jump ramp with a velocity of 22 m/s directed 32^o above the horizontal. Suppose that as a result of air drag the skier returns to the ground with a speed of 17 m/s, landing 19 m vertically below the end of the ramp. From the launch to the return to the ground, how much work has been done by air drag?

R:

$$\begin{aligned} E_{tot,i} &= K_i + U_{g_i} \\ &= \frac{1}{2}mv_i^2 + mgy_i \\ &= \left[\frac{1}{2}(62\ kg)(22\ m/s)^2\right] + \left[(62\ kg)\left(9.8\ m/s^2\right)(19\ m)\right] \\ &= (15\ 004\ J) + (11\ 544.4\ J) \\ &= 26\ 548.4\ J \end{aligned}$$

$$
\begin{aligned}
E_{tot,f} &= K_f + U_{g_f} \\
&= \frac{1}{2}mv_f^2 + mg(0) \\
&= \frac{1}{2}(62\ kg)(17\ m/s)^2 \\
&= 8959\ J
\end{aligned}
$$

$$
\begin{aligned}
W_{nc} &= \Delta E_{tot} \\
&= (8959\ J) - (26\ 548.4\ J) \\
&= -17\ 589.4\ J
\end{aligned} \tag{13}
$$