General Physics I
Homework Chapter 3

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Homework: Chapter 3

Problem (1)

The x component of vector $\vec{A}$ is $-27\ ft$ and the y component is $+44\ ft$.

Question (a)

What is the magnitude of $\vec{A}$?

R:

$$\begin{aligned} A = \left|\vec{A}\right| &= \sqrt{A_x^2 + A_y^2} \\ A &= \sqrt{(-27\ ft)^2 + (44\ ft)^2} \\ &= \sqrt{(729\ ft^2) + (1936\ ft^2)} \\ &= \sqrt{2665\ ft^2} \\ &= 51.624\ ft \end{aligned} \tag{1}$$

Question (a)

What is the angle between the direction of $\vec{A}$ and the positive direction of x?

R:

$$\begin{aligned}
\theta_A &= \tan^{-1}\left(\frac{A_y}{A_x}\right) \\
\theta_A &= \tan^{-1}\left(\frac{44\ ft}{-27\ ft}\right) \\
&= \tan^{-1}(-1.630) \\
&= -58.471^o\ (+\ 180^o) \\
&= 121.529^o
\end{aligned} \tag{2}$$

Problem (2)

Question (a)

What is the x component of a vector $\vec{a}$ in the xy plane if its direction is 71^o counterclockwise from the positive direction of the x axis and its magnitude is 8.1 m?

R:

$$\begin{aligned}
A_x &= A\cos\theta_A \\
A_x &= (8.1\ m)\cos 71^o \\
&= (8.1\ m)\times(0.326) \\
&= 2.641\ m
\end{aligned} \tag{3}$$

Question (b)

What is the y component of vector $\vec{a}$?

R:

$$\begin{aligned}
A_y &= A\sin\theta_A \\
A_y &= (8.1\ m)\sin 71^o \\
&= (8.1\ m)\times(0.946) \\
&= 7.663\ m
\end{aligned} \tag{4}$$

Problem (3)

A car is driven east for a distance of 71 mi, then north for 42 mi, and the in a direction 33^o east of north for 47 mi.

Question (a)

Determine the magnitude (in mi) of the car's total displacement from its starting point.

R:

$$
\begin{aligned}
\vec{C}_1 &= (71\ mi)\ @\ 0^o \\
\vec{C}_{1_x} &= 71\ mi \\
\vec{C}_{1_y} &= 0\ mi \\
\vec{C}_2 &= (42\ mi)\ @\ 90^o \\
\vec{C}_{2_x} &= 0\ mi \\
\vec{C}_{2_y} &= 42\ mi \\
\vec{C}_3 &= (47\ mi)\ @\ 57^o \\
\vec{C}_{3_x} &= (47\ mi)\cos 57^o \\
&= (47\ mi) \times (0.545) \\
&= 25.615\ mi \\
\vec{C}_{3_y} &= (47\ mi)\sin 57^o \\
&= (47\ mi) \times (0.839) \\
&= 39.433\ mi
\end{aligned}
$$

$$
\begin{aligned}
\vec{C} &= \vec{C}_1 + \vec{C}_2 + \vec{C}_3 \\
\vec{C} &= [(71\ mi) + (0\ mi) + (25.615\ mi)]\,\hat{i} + [(0\ mi) + (42\ mi) + (39.433\ mi)]\,\hat{j} \\
\vec{C} &= (96.615\ mi)\,\hat{i} + (81.433\ mi)\,\hat{j}
\end{aligned}
$$

$$\begin{aligned} C = \left|\vec{C}\right| &= \sqrt{(96.615\ mi)^2 + (81.433\ mi)^2} \\ C &= \sqrt{(9334.458\ mi^2) + (6631.333\ mi^2)} \\ &= \sqrt{15965.791\ mi^2} \\ &= 126.356\ mi \end{aligned} \tag{5}$$

Question (b)

Determine the angle (from east) of the car's total displacement measured from its starting direction.

R:

$$\begin{aligned} \theta_C &= \tan^{-1}\left(\frac{C_y}{C_x}\right) \\ \theta_C &= \tan^{-1}\left(\frac{81.433\ mi}{96.615\ mi}\right) \\ &= \tan^{-1}(0.843) \\ &= 40.131^o \end{aligned} \tag{6}$$

Problem (4)

$$\begin{aligned} \vec{a} &= (4.0\ m)\hat{i} + (3.0\ m)\hat{j} \\ \vec{b} &= (-13.0\ m)\hat{i} + (7.0\ m)\hat{j} \end{aligned}$$

Question (a)

In unit-vector notation, what is the sum of vectors $\vec{a}$ and $\vec{b}$?

R:

$$\begin{aligned} \vec{r} &= \vec{a} + \vec{b} \\ \vec{r} &= [(4.0\ m) + (-13.0\ m)]\,\hat{i} + [(3.0\ m) + (7.0\ m)]\,\hat{j} \\ \vec{r} &= (-9.0\ m)\hat{i} + (10.0\ m)\hat{j} \end{aligned} \tag{7}$$

Question (b)

What is the magnitude of $\vec{a} + \vec{b}$?

R:

$$\begin{aligned} r = |\vec{r}| &= \sqrt{(-9.0\ m)^2 + (10.0\ m)^2} \\ r &= \sqrt{(81.0\ m^2) + (100.0\ m^2)} \\ &= \sqrt{181.0\ m^2} \\ &= 13.5\ m \end{aligned} \tag{8}$$

Question (c)

What is the direction of $\vec{a} + \vec{b}$ (relative to $\hat{i}$)?

R:

$$\begin{aligned} \theta_r &= \tan^{-1}\left(\frac{r_y}{r_x}\right) \\ \theta_r &= \tan^{-1}\left(\frac{10.0\ m}{-9.0\ m}\right) \\ &= \tan^{-1}(-1.1) \\ &= -47.7^o\ (+\ 180.0^o) \\ &= 132.3^o \end{aligned} \tag{9}$$

Problem (5)

The two vectors $\vec{a}$ and $\vec{b}$ in fig. 1 have equal magnitudes of 42 ft and the angles are $\theta_1 = 37^o$ and $\theta_2 = 102^o$.

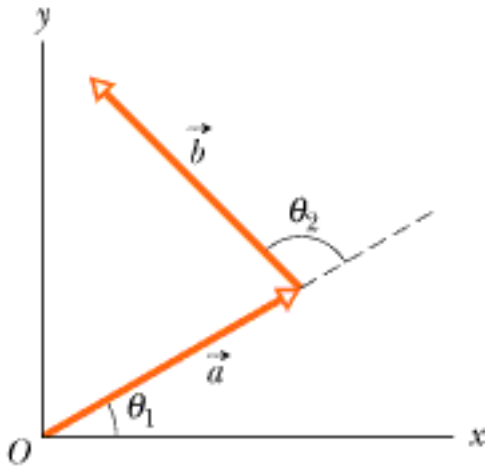

Figure 1: Plot of y versus x

Question (a)

Find the x component of their vector sum $\vec{r}$.

R:

$$
\begin{aligned}
r_x &= (a\ \cos\theta_a) + (b\ \cos\theta_b) \\
a &= b = 42\ ft \\
\theta_a &= \theta_1 = 37^o \\
\theta_b &= \theta_a + \theta_2 = 37^o + 102^o = 139^o \\
r_x &= [(42\ ft)\cos 37^o] + [(42\ ft)\cos 139^o] \\
&= [(42\ ft) \times (0.799)] + [(42\ ft) \times (-0.755)] \\
&= (33.558\ ft) + (-31.710\ ft) \\
&= 1.848\ ft
\end{aligned} \tag{10}
$$

Question (b)

Find the y component of their vector sum $\vec{r}$.

R:

$$\begin{aligned} r_y &= (a\ \sin\theta_a) + (b\ \sin\theta_b) \\ r_y &= [(42\ ft)\sin 37^o] + [(42\ ft)\sin 139^o] \\ &= [(42\ ft) \times (0.602)] + [(42\ ft) \times (0.656)] \\ &= (25.284\ ft) + (27.552\ ft) \\ &= 52.836\ ft \end{aligned} \tag{11}$$

Question (c)

Find the magnitude of $\vec{r}$.

R:

$$\begin{aligned} r &= |\vec{r}| = \sqrt{(1.848\ ft)^2 + (52.836\ ft)^2} \\ r &= \sqrt{(3.415\ ft^2) + (2791.643\ ft^2)} \\ &= \sqrt{2795.058\ ft^2} \\ &= 52.868\ ft \end{aligned} \tag{12}$$

Question (d)

Find the angle $\vec{r}$ makes with the positive x axis.

R:

$$\begin{aligned} \theta_r &= \tan^{-1}\left(\frac{52.836\ ft}{1.848\ ft}\right) \\ &= \tan^{-1}(28.591) \\ &= 88.000^o \end{aligned} \tag{13}$$