General Physics I
Homework Chapter 4

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02/22/2016

Homework: Chapter 4

Problem (1)

An ion's position vector is initially

$$\vec{r} = (-3.1\ m)\hat{i} + (5.7\ m)\hat{j} + (-3.9\ m)\hat{k}$$

And 4.0 s late it is

$$\vec{r} = (-5.2\ m)\hat{i} + (7.3\ m)\hat{j} + (-6.8\ m)\hat{k}$$

In unit-vector notation, what is its average velocity during the 4.0 s?

R:

$$\begin{aligned}
\vec{\bar{v}} &= \left(\frac{\Delta x}{\Delta t}\right)\hat{i} + \left(\frac{\Delta y}{\Delta t}\right)\hat{j} + \left(\frac{\Delta z}{\Delta t}\right)\hat{k} \\
\vec{\bar{v}} &= \left(\frac{(-5.2\ m)-(-3.1\ m)}{4.0\ s}\right)\hat{i} + \left(\frac{(-7.3\ m)-(5.7\ m)}{4.0\ s}\right)\hat{j} + \left(\frac{(-6.8\ m)-(-3.9\ m)}{4.0\ s}\right)\hat{k} \\
\vec{\bar{v}} &= (-0.525\ m/s)\hat{i} + (-3.25\ m/s)\hat{j} + (-0.725\ m/s)\hat{k} \qquad (1)
\end{aligned}$$

Problem (2)

A particle moves so that its position (in feet) as a function of time (in seconds) is

$$\vec{r} = (7)\hat{i} + (5t^2)\hat{j} + (4t)\hat{k}$$

Question (a)

Write expression (in unit vector notation) for its velocity as function of time:
R:

$$\begin{aligned}
\vec{v} &= \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j} + \left(\frac{dz}{dt}\right)\hat{k} \\
\vec{v} &= \left(\frac{d}{dt}[7]\right)\hat{i} + \left(\frac{d}{dt}\left[5t^2\right]\right)\hat{j} + \left(\frac{d}{dt}[4t]\right)\hat{k} \\
\vec{v} &= \left[\left(10\ ft/s^2\right)t\right]\hat{j} + (4\ ft/s)\hat{k} \qquad (2)
\end{aligned}$$

Question (b)

Write expression (in unit vector notation) for its acceleration as function of time:
R:

$$\begin{aligned}
\vec{a} &= \left(\frac{dv_x}{dt}\right)\hat{i} + \left(\frac{dv_y}{dt}\right)\hat{j} + \left(\frac{dv_z}{dt}\right)\hat{k} \\
\vec{a} &= \left(\frac{d}{dt}[10t]\right)\hat{j} + \left(\frac{d}{dt}[4]\right)\hat{k} \\
\vec{a} &= \left(10\ ft/s^2\right)\hat{j} \qquad (3)
\end{aligned}$$

Problem (3)

A small ball rolls horizontally off the edge of a tabletop that is 2.7 ft high. It strikes the floor at a point 4.2 ft horizontally away from the edge of the table.

Question (a)

How long is the ball in the air?

R:

x-values	**y-values**
$x_0 = 0\ ft$	$y_0 = 2.7\ ft$
$x = 4.2\ ft$	$y = 0\ ft$
$v_{0_x} = ?\ ft/s$	$v_{0_y} = 0\ ft/s$
$v_x = v_{0_x}$	$v_y = ?\ ft/s$
$a_x = 0$	$a_y = -g = -32.2\ ft/s^2$

$$\begin{aligned} y &= y_0 + v_{0_y} t + \frac{1}{2} a_y t^2 \\ 0 &= (2.7\ ft) + \left(-16.1\ ft/s^2\right) t^2 \\ t &= \sqrt{\frac{-2.7\ ft}{-16.1\ ft/s^2}} \\ &= \sqrt{0.1677\ s^2} \\ &= 0.4095\ s \end{aligned} \tag{4}$$

Question (b)

What is its speed at the instant it leaves the table?

R:

$$\begin{aligned} x &= x_0 + v_{0_x} t + \frac{1}{2} a_x t^2 \\ (4.2\ ft) &= v_{0_x}(0.4095\ s) \\ v_{0_x} &= \frac{4.2\ ft}{0.4095\ s} \\ &= 10.2564\ ft/s \\ v_{0_y} &= 0\ ft/s \\ |\vec{v_0}| &= v_{0_x} = 10.2564\ ft/s \end{aligned} \tag{5}$$

Problem (4)

In the fig. 1, a stone is projected at a cliff of height h with an initial speed of 39 m/s directed at an angle $\theta_0 = 66^o$ above the horizontal. The stone strikes at A, 4.1 s after launching.

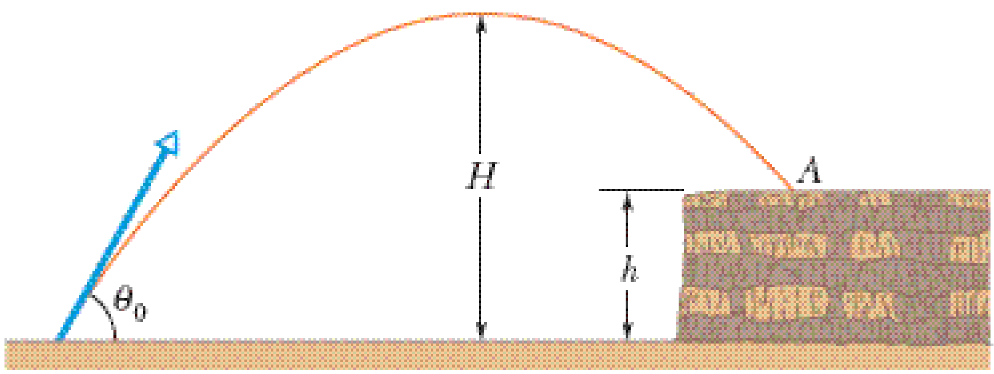

Figure 1: Illustration of Problem 4

Question (a)

Find the height h of the cliff:
R:

$$
\begin{aligned}
y &= y_0 + v_{0_y} t + \frac{1}{2} a_y t^2 \\
y &= h \\
v_{0_y} &= v_0 \sin 66^o \\
&= (39\ m/s)(0.9135) = 35.6265\ m/s \\
h &= (35.6265\ m/s)(4.1\ s) + \left(-4.9\ m/s^2\right)(4.1\ s)^2 \\
&= (150.1687\ m) - (82.3690\ m) \\
&= 67.7997\ m
\end{aligned} \tag{6}
$$

Question (b)

Find the speed of the stone just before impact at A:

R:

$$
\begin{aligned}
v_x &= v_{0_x} + a_x t = v_{0_x} \\
v_x &= v_0 \cos 66^o \\
&= (39\ m/s)(0.4067) = 15.8613\ m/s \\
v_y &= v_{0_y} + a_y t \\
&= (35.6265\ m/s) + \left(-9.8\ m/s^2\right)(4.1\ s) \\
&= (35.6265\ m/s) - (40.1800\ m/s) = -4.5535\ m/s \\
|\vec{v}| &= \sqrt{(15.8613\ m/s)^2 + (-4.5535\ m/s)^2} \\
&= 16.5020\ m/s
\end{aligned} \tag{7}
$$

Question (c)

Find the maximum height H reached above the ground.

R:

$$
\begin{aligned}
v_y &= v_{0_y} + a_x t \\
0 &= (35.6265\ m/s) + \left(-9.8\ m/s^2\right) t \\
t &= \frac{-35.6265\ m/s}{-9.8\ m/s^2} = 3.6354\ s \\
y &= y_0 + v_{0_y} t + \frac{1}{2} a_y t^2 \\
y &= H \\
H &= (35.6265\ m/s)(3.6354\ s) + \left(-4.9\ m/s^2\right)(3.6354\ s)^2 \\
&= (133.1520\ m) - (64.7591\ m) \\
&= 68.3929\ m
\end{aligned} \tag{8}
$$

Problem (5)

You throw a ball toward a wall at speed 25 m/s and at angle $\theta_0 = 38^o$ above the horizontal. The wall is distance $d = 17\ m$ from the release point of the ball.

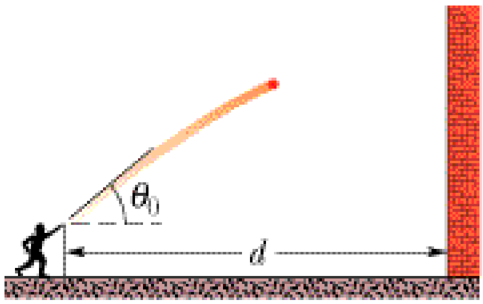

Figure 2: Illustration of Problem 5

Question (a)

How far above the release point does the ball hit the wall?

R:

$$
\begin{aligned}
v_{0_x} &= v\cos 38^o \\
&= (25\ m/s)(0.7880) = 19.7000\ m/s \\
x &= x_0 + v_{0_x}t + \frac{1}{2}a_x t^2 \\
17\ m &= (19.7\ m/s)t \\
t &= \frac{17\ m}{19.7\ m/s} = 0.8629\ s \\
v_{0_y} &= v\sin 38^o \\
&= (25\ m/s)(0.6157) = 15.3925\ m/s \\
y &= y_0 + v_{0_y}t + \frac{1}{2}a_y t^2 \\
&= (15.3925\ m/s)(0.8629\ s) + \left(-4.9\ m/s^2\right)(0.8629\ s)^2 \\
&= (13.2821\ m) - (3.6485\ m) = 9.6336\ m \qquad (9)
\end{aligned}
$$

Question (b)

What is the horizontal component of its velocity as it hits the wall?

R:

$$\begin{aligned} v_x &= v_{0_x} + a_x t = v_{0_x} \\ &= 19.7000\ m/s \end{aligned} \tag{10}$$

Question (c)

What is the vertical component of its velocity as it hits the wall?
R:

$$\begin{aligned} v_y &= v_{0_y} + a_y t \\ &= (15.3925\ m/s) + \left(-9.8\ m/s^2\right)(0.8629\ s) \\ &= (15.3925\ m/s) - (8.4564\ m/s) = 6.9361\ m/s \end{aligned} \tag{11}$$

Problem (6)

What is the magnitude of the acceleration of a sprinter running at 27 ft/s when rounding a turn with a radius of 55 ft?
R:

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ a_c &= \frac{(27\ ft/s)^2}{55\ ft} = 13.2545\ ft/s^2 \end{aligned} \tag{12}$$

Problem (7)

A carnival merry-go-round rotates about a vertical axis at a constant rate. A man standing on the edge has a constant speed of 3.2 m/s and a centripetal acceleration of magnitude 2.1 m/s^2. How far is the man from the center of the merry-go-round?
R:

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ 2.1\ m/s^2 &= \frac{(3.2\ m/s)^2}{r} \\ r &= \frac{10.24\ m^2/s^2}{2.1\ m/s^2} = 4.8762\ m \end{aligned} \tag{13}$$