General Physics I
Classnotes

Jonathan Henrique Maia de Moraes (ID: 1620855)

01/27/2016

January 22

1 Metric Prefixes

10^{12}	$= 1,000,000,000,000$	tera	T
10^{9}	$= 1,000,000,000$	giga	G
10^{6}	$= 1,000,000$	mega	M
10^{3}	$= 1,000$	kilo	k
10^{0}	$= 1$	$-$	$-$
10^{-2}	$= 0.01$	centi	c
10^{-3}	$= 0.001$	milli	m
10^{-6}	$= 0.000,001$	micro	μ
10^{-9}	$= 0.000,000,001$	nano	n
10^{-12}	$= 0.000,000,000,001$	pico	p

2 Basic Quantities

	Metric	English
Length	$m =$ meter	$ft =$ foot
Mass	$kg =$ kilogram	$sl =$ slug
Time	$s =$ second	$s =$ second

$$1\text{day} = 24 \times 60 \times 60 = 86,400\ s$$
$$1\text{day} = 10 \times 100 \times 100 = 100,000\ s$$

3 Derived Quantities

velocity/speed	mi/s	km/h	m/min	$\ldots$	$[L]/[T]$
area	cm^2	m^2	$\ldots$	$\ldots$	$[L]^2$
density	g/cm^3	kg/m^3	$\ldots$	$\ldots$	$[M]/[L]^3$

4 Conversions

$$\begin{aligned}
1\ min &\equiv 60\ s \\
1\ h &\equiv 60\ min \\
1\ ft &\equiv 12\ in \\
1\ mi &\equiv 5280\ ft \\
1\ L &\equiv 1,000\ cm^3 \\
1\ mi^2 &\equiv 640\ acres \\
1\ in &\equiv 2.54\ cm
\end{aligned}$$

Example:

$$\begin{aligned}
70\ mi/h &= ?\ m/s \\
&= 70\ mi/h \times \left(\frac{5280\ ft}{1\ mi}\right) \times \left(\frac{12\ in}{1\ ft}\right) \times \left(\frac{2.54\ cm}{1\ in}\right) \times \left(\frac{1\ m}{100\ cm}\right) \\
&\quad \times \left(\frac{1\ h}{60\ min}\right) \times \left(\frac{1\ min}{60\ s}\right) \\
&= 31.2928\ m/s
\end{aligned}$$

Example:

$$\begin{aligned}
350 in^3 &= ?\ L \\
&= 350\ in^3 \times \left(\frac{2.54\ cm}{1\ in}\right)^3 \times \left(\frac{1\ L}{1000\ cm^3}\right) \\
&= 5.7355\ L
\end{aligned}$$

Homework:

$$
\begin{aligned}
1acre &= ?\ in^2 \\
&= 1\ acre \times \left(\frac{1\ mi^2}{640\ acres}\right) \times \left(\frac{5280\ ft}{1\ mi}\right)^2 \times \left(\frac{12\ in}{1\ ft}\right)^2 \\
&= 6,272,640\ in^2
\end{aligned}
$$

January 25

5 Position

Let

$$\begin{aligned} x &= \text{position} \\ x_i &= \text{initial position} \\ x_f &= \text{final position} \\ \Delta x &= \text{Displacement} \\ &= x_f - x_i \end{aligned}$$

Example:

$$\begin{aligned} x_i &= \ +3\ ft \\ x_f &= \ +5\ ft \\ \Delta x &= x_f - x_i \\ &= 5\ ft - 3\ ft \\ &= \ +2\ ft \end{aligned}$$

Example:

$$\begin{aligned} x_i &= \ +5\ ft \\ x_f &= \ -1\ ft \\ \Delta x &= x_f - x_i \\ &= \ -1\ ft - 5\ ft \\ &= \ -6\ ft \end{aligned}$$

Example:

$$\begin{aligned} x_i &= \ +3\ ft \\ x_2 &= \ +5\ ft \\ x_f &= \ -1\ ft \\ \Delta x &= x_f - x_i \\ &= \ -1\ ft - 3\ ft \\ &= \ -4\ ft \\ \text{Distance Traveled} &= 2\ ft + 6\ ft \\ &= 8\ ft \end{aligned}$$

6 Velocity

$$\begin{aligned} \bar{v} &= \text{average velocity} \\ \bar{v} &\equiv \frac{\Delta x}{\Delta t} = \frac{\text{displacement}}{\text{time elapsed}} \\ \text{average speed} &= \frac{\text{distance travelled}}{\text{time elapsed}} \end{aligned}$$

Example:

$$\begin{aligned} \text{Start at } x = &\ +3\ ft \\ \text{Move to } x = &\ +5\ ft \\ \text{End at } x = &\ -1\ ft \\ \text{Trip takes } &4\ s \\ \text{Find } &a) average velocity \\ &b) average speed \end{aligned}$$

$$\begin{aligned} \bar{v} &\equiv \frac{\Delta x}{\Delta t} \\ &= \frac{-1\ ft - 3\ ft}{4\ s} = \frac{-4\ ft}{4\ s} = -1\ ft/s \\ \text{average speed} &= \frac{\text{distance}}{\text{time}} = \frac{8\ ft}{4\ s} = 2\ ft/s \end{aligned}$$

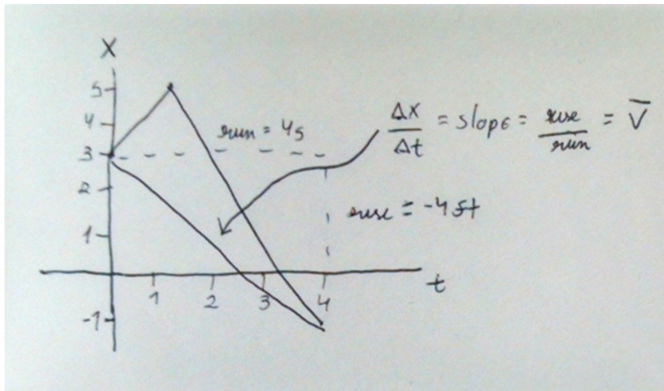

Figure 1: Graphic of the Average Speed

$$
\begin{aligned}
v &= \text{instantaneous velocity} \\
&= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \\
v &\equiv \frac{dx}{dt}
\end{aligned}
$$

Example:

$$
\begin{aligned}
&x = 3\ m + (17\ m/s)t + (7\ m/s^3)t^3 \\
&\text{Find} \\
&\qquad a)\text{position at } t = 2\ s \\
&\qquad b)\text{position at } t = 4\ s \\
&\qquad c)\text{average velocity from} 2\ s \rightarrow 4\ s
\end{aligned}
$$

a)

$$
\begin{aligned}
x &= 3\ m + (17\ m/s)(2\ s) + (7\ m/s^3)(2\ s)^3 \\
&= 3\ m + 34\ m + 56\ m \\
&= 93\ m
\end{aligned}
\tag{1}
$$

b)

$$\begin{aligned} x &= 3\ m + (17\ m/s)(4\ s) + (7\ m/s^3)(4\ s)^3 \\ &= 3\ m + 68\ m + 448\ m \\ &= 519\ m \end{aligned}$$

c)

$$\begin{aligned} \bar{v} &= \frac{\Delta x}{\Delta t} = \frac{519\ m - 93\ m}{4\ s - 2\ s} \\ &= \frac{426\ m}{2\ s} \\ &= 213\ m/s \end{aligned}$$

d)

$$\begin{aligned} v &= \frac{dx}{dt} \\ &= \frac{d}{dt}\left[3\ m + (17\ m/s)t + (7\ m/s^3)t^3\right] \\ &= 0 + 17\ m/s + (21\ m/s^3)t^2 \\ v(3\ s) &= 17\ m/s + (21\ m/s^3)(3\ s)^2 \\ &= 17\ m/s + 189\ m/s = 208\ m/s \end{aligned}$$

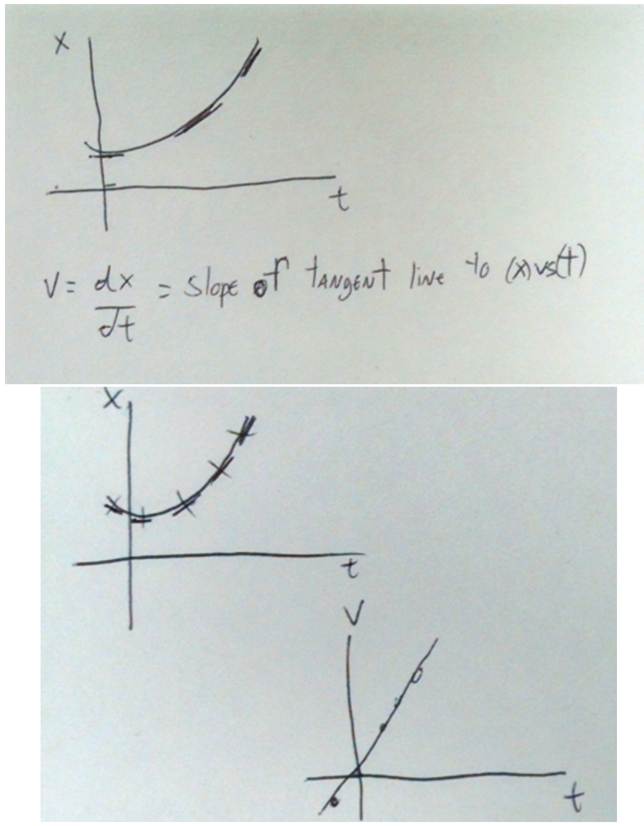

Figure 2: Graphics of Instantaneous Velocity

January 27

7 Summary

$$
\begin{aligned}
x &= \text{position} \\
\Delta x &= \text{displacement} \\
&= x_f - x_i \\
\bar{v} &= \text{average velocity} \\
&= \frac{\Delta x}{\Delta t} \\
v &= \text{instantaneous velocity} \\
&= \frac{dx}{dt} = \text{slope of x vs. t} \\
\text{Avg Speed} &= \frac{\text{distance traveled}}{\text{time elapsed}}
\end{aligned}
$$

8 Acceleration

$$
\begin{aligned}
\text{Let} \qquad \bar{a} =& \qquad \text{average acceleration} \\
\bar{a} \equiv& \qquad \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{\text{change in velocity}}{\text{time elapsed}}
\end{aligned}
$$

Example: A car goes from 20 mph to 60 mph in 8 s. What is its average acceleration?

$$
\begin{aligned}
v_i &= 20\ mi/h \\
v_f &= 60\ mi/h \\
\Delta t &= 8\ s \\
\bar{a} &= \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} \\
&= \frac{60\ mi/h - 20\ mi/h}{8\ s} \\
&= \frac{40\ mi/h}{8\ s} \\
&= 5\ \frac{mi}{h \times s}
\end{aligned}
$$

Example: Justin Bieber's Limo goes from 30 m/s to a stop in 0.10 s. What is its average acceleration?

$$
\begin{aligned}
v_i &= 30\ m/s \\
v_f &= 0\ m/s \\
\Delta t &= 0.10\ s \\
\bar{a} &= \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} \\
&= \frac{0\ m/s - 30\ m/s}{0.10\ s} \\
&= \frac{-30\ m/s}{0.10\ s} \\
&= -300\ \frac{m/s}{s} = -300 m/s^2
\end{aligned}
$$

($-$ means slowing)

Let

$$
\begin{aligned}
a &= \text{instantaneous acceleration} \\
&= lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \\
a &\equiv \frac{dv}{dt} = \text{rate of change of velocity} \\
&= \text{slope of tangent line to v vs. t}
\end{aligned}
$$

Example:

$$
\begin{aligned}
x &= 3\ m + (17\ m/s)\, t + \left(7\ m/s^3\right) t^3 \\
Find &: \text{a) velocity at } 3\ s \\
&\quad \text{b) velocity at } 5\ s \\
&\quad \text{c) average acceleration from } 3\ s \to 5\ s \\
&\quad \text{c) instantaneous acceleration at } 4\ s
\end{aligned}
$$

a)

$$
\begin{aligned}
v &= \frac{dx}{dt} = 17\ m/s + \left(21\ m/s^3\right) t^2 \\
v(3\ s) &= 17\ m/s + \left(21\ m/s^3\right)(3\ s)^2 \\
&= 17\ m/s + 189\ m/s \\
&= 206\ m/s
\end{aligned}
$$

b)

$$
\begin{aligned}
v(5\ s) &= 17\ m/s + \left(21\ m/s^3\right)(5\ s)^2 \\
&= 17\ m/s + 525\ m/s \\
&= 542\ m/s
\end{aligned}
$$

c)

$$
\begin{aligned}
\bar{a} &= \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{542\ m/s - 206\ m/s}{5\ s - 3\ s} \\
&= \frac{336\ m/s}{2\ s} = 168\ m/s^2
\end{aligned}
$$

d)

$$
\begin{aligned}
a &= \frac{dv}{dt} \\
&= \frac{d}{dt}\left[17\ m/s + (21\ m/s^3)t^2\right] \\
&= 0 + (42\ m/s^3)t \\
a(4\ s) &= (42\ m/s^3)(4\ s) \\
&= 168\ m/s^2
\end{aligned}
$$

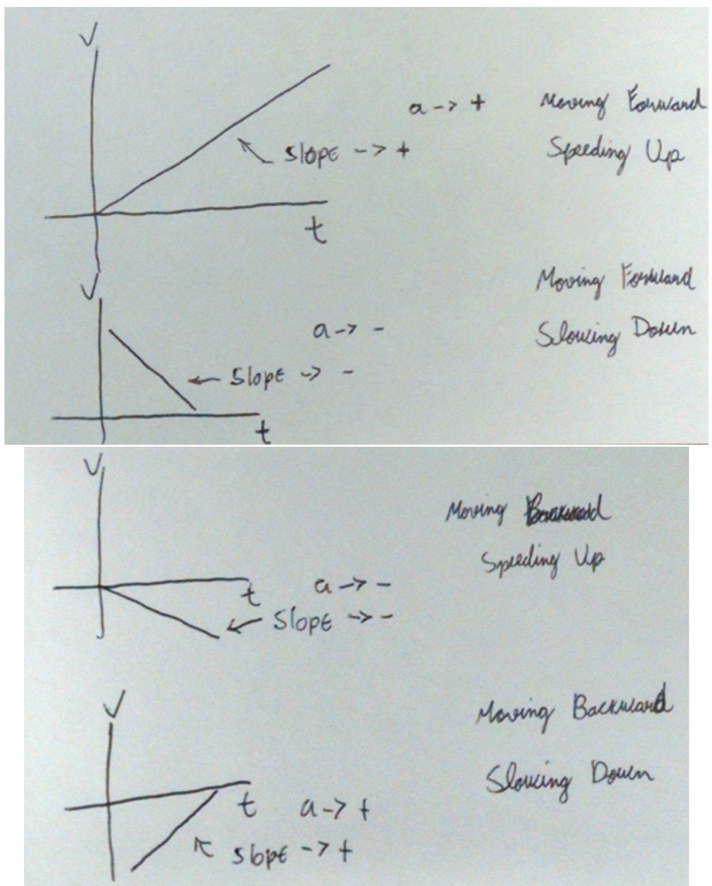

Figure 3: Graphics of Acceleration

$$
\begin{array}{lcl}
t_i & \to & 0 \\
t_f & \to & t \\
x_i & \to & x_0 \\
x_f & \to & x \\
v_i & \to & v_0 \\
v_f & \to & v
\end{array}
$$

Suppose $a =$ constant

$$
\begin{aligned}
\bar{a} &= a \\
\frac{v - v_0}{t} &= a \\
v - v_0 &= at \\
v &= v_0 + at \,:\, v(t) && (2) \\
x &= x_0 + v_0 t + \frac{1}{2} a t^2 \,:\, x(t) && (3) \\
x &= x_0 + \frac{1}{2}(v_0 + v)t \,:\, \text{no } a && (4) \\
2a(x - x_0) &= v^2 - v_0^2 \,:\, \text{no } t && (5)
\end{aligned}
$$

February 01

9 Summary

Iff $a =$ constant?

$$
\begin{array}{lll}
(1) & v = v_0 + at & v(t) \\
(2) & x = x_0 + v_0 t + \frac{1}{2} a t^2 & x(t) \\
(3) & x = x_0 + \frac{1}{2}(v_0 + v)t & \text{No a} \\
(4) & 2a(x - x_0) = v^2 - v_0^2 & \text{No t}
\end{array}
$$

10 Gravity

In the absence of air resistante ($\alpha = 0$) gravity produces a constant acceleration of $g = 9.80\ m/s^2 = 32.2\ ft/s^2 = 22.0\ \frac{mi/h}{s}$.

Example: Drop an object from a height. What is the velocity and the distance fallen after 0 s, 1 s, 2 s, and 3 s?

$$\text{speed} = |v| \tag{6}$$

$$
\begin{aligned}
x_0 &= 0 \\
v_0 &= 0 \\
a &= +g \\
v &= v_0 + at \\
&= 0 + gt \\
&= \left(9.80\ m/s^2\right) t \\
&= \left(32.2\ ft/s^2\right) t \\
&= \left(22\ \frac{mi/h}{s}\right) t \\
x &= x_0 + v_0 t + \frac{1}{2} a t^2 \\
&= 0 + (0)t + \frac{1}{2} g t^2 \\
&= \frac{1}{2}\left(9.80\ m/s^2\right) t^2 = \left(4.90\ m/s^2\right) t^2 \\
&= \frac{1}{2}\left(32.2\ ft/s^2\right) t^2 = \left(16.1\ ft/s^2\right) t^2 \\
&= \frac{1}{2}\left(22\ \frac{mi/h}{s}\right) t^2 = \left(11\ \frac{mi/h}{s}\right) t^2
\end{aligned}
$$

t	v	v	v	x	x
(s)	(m/s)	(ft/s)	(mi/h)	m	ft
0	0	0	0	0	0
1	9.80	32.2	22	4.9	16.1
2	19.6	64.4	44	19.6	64.4
3	29.4	96.6	66	44.1	144.9

→ Example 3 - PH1300 Examples (Dr. Rex Joyner) - 2015-2016

February 03

11 Summary

Iff $a =$ constant?

$$
\begin{aligned}
&(1) \qquad & v &= v_0 + at & \qquad & v(t) \\
&(2) \qquad & x &= x_0 + v_0 t + \frac{1}{2} a t^2 & \qquad & x(t) \\
&(3) \qquad & x &= x_0 + \frac{1}{2}(v_0 + v)t & \qquad & \text{No a} \\
&(4) \qquad & 2a(x - x_0) &= v^2 - v_0^2 & \qquad & \text{No t}
\end{aligned}
$$

→ Example 3 - PH1300 Examples (Dr. Rex Joyner) - 2015-2016 - Continuation

→ Example 4 - PH1300 Examples (Dr. Rex Joyner) - 2015-2016

→ Chapter Two Homework due Monday. Chapter Three due next Friday. Test 1 next friday.

February 05

12 Vector

Definition: A vector is a quantity with both magnitude (size) and direction.

Definition: A vector is a quantity with magnitude only.

Vectors	Scalars	
10 ft left	10 ft	
Displacement	Distance	
70 mi/h south	70 mi/h	
Velocity	Speed	
18 m/s^2 down	18 m/s^2	
		(7)

Let:

$$\vec{A} = \text{vector } A$$

Book uses boldface:

$$\mathbf{A} = \text{vector } A$$

$$A = |\vec{A}| = \text{magnitude of } \vec{A} \tag{8}$$

$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \tag{9}$$

Vector addition is commutative.

Two vectors are equal if they have the same magnitude and direction. Location does not matter.

$$\vec{A} - \vec{B} = ?$$
$$\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$$

$$A_x = x\text{-component of } \vec{A}$$
$$A_y = y\text{-component of } \vec{A}$$

$$\cos\theta_A = \frac{\text{adj}}{\text{hip}} = \frac{A_x}{A} \to A_x = A\cos\theta_A$$
$$\sin\theta_A = \frac{\text{opp}}{\text{hip}} = \frac{A_y}{A} \to A_y = A\sin\theta_A$$

If we know A_x and A_y. What are A and θ_A?

$$\tan\theta_A = \frac{\text{opp}}{\text{adj}} = \frac{A_y}{A_x}$$

$$A^2 = A_x^2 + A_y^2$$
$$A = \sqrt{A_x^2 + A_y^2}$$

February 08

13 Vector

$$\begin{aligned} &\hat{i} && unit vector in + x direction \\ &\hat{j} && unit vector in + y direction \\ &\hat{k} && unit vector in + z direction \end{aligned}$$

A unit vector has length 1 unit.

$$|\hat{i}| = 1$$
$$|\hat{j}| = 1$$
$$|\hat{k}| = 1$$

$$A_x = 4$$
$$A_y = 3$$
$$\vec{A} = 4\hat{i} + 3\hat{j}$$

Example:

$$\vec{A} = 8\ m@30^o$$

Find $A_x,\ A_y$

$$\begin{aligned}
A_x &= A\cos\theta_A \\
&= (8\ m)\cos(30^o) \\
&= 6.928\ m \\
A_y &= A\sin\theta_A \\
&= (8\ m)\sin(30^o) \\
&= 4.00\ m \\
\vec{A} &= (6.928\ m)\hat{i} + (4.00\ m)\hat{j}
\end{aligned}$$

Example:

$$\begin{aligned}
\vec{B} &= 12\ m@140^o \\
B_x &= B\cos\theta_B \\
&= (12\ m)\cos(140^o) \\
&= -9.19\ m \\
B_y &= B\sin\theta_B \\
&= (12\ m)\sin(140^o) \\
&= 7.71\ m \\
\vec{B} &= (-9.19\ m)\hat{i} + (7.71\ m)\hat{j}
\end{aligned}$$

Example:

$$\vec{C} = 4\hat{i} + 3\hat{j}$$

Find $c,\ \theta_C$

$$
\begin{aligned}
C &= \sqrt{C_x^2 + C_y^2} \\
&= \sqrt{4^2 + 3^2} \\
&= \sqrt{16 + 9} \\
&= \sqrt{25} \\
&= 5.0 \\
\tan\theta_C &= \frac{C_y}{C_x} = \frac{3}{4} = 0.75 \\
\theta_C &= \tan^{-1}(0.75) \\
&= 36.87^o \\
\vec{C} &= 5.0@36.87^o
\end{aligned}
$$

Example:

$$
\begin{aligned}
\vec{D} &= -9\hat{\imath} + 12\hat{\jmath} \\
D &= \sqrt{D_x^2 + D_y^2} \\
&= \sqrt{(-9)^2 + 12^2} \\
&= \sqrt{81 + 144} \\
&= \sqrt{225} \\
&= 15 \\
\tan\theta_D &= \frac{D_y}{D_x} = \frac{12}{-9} = -\frac{4}{3} \\
\theta_D &= \tan^{-1}\left(-\frac{3}{4}\right) \\
&= -53.13^o + 180^o \\
&= 126.87^o \\
\vec{D} &= 15.0@126.87^o
\end{aligned}
$$

Rule: when x-component is negative:

$$\theta = \theta + 180^o \tag{10}$$

$$
\begin{aligned}
\vec{R} &= \vec{A} + \vec{B} \\
\vec{R}_x &= \vec{A}_x + \vec{B}_x \\
\vec{R}_y &= \vec{A}_y + \vec{B}_y
\end{aligned}
\tag{11}
$$

Example:

$$
\begin{aligned}
\vec{A} &= 10 \ @ \ 37^o \\
\vec{B} &= 12 \ @ \ -60^o
\end{aligned}
$$

Find magnitude and direction of $\vec{R} = \vec{A} + \vec{B}$

$$
\begin{aligned}
A_x &= A \ \cos\theta_A \\
&= 10 \ \cos(37^o) \\
&= 7.98 \\
A_y &= A \ \sin\theta_A \\
&= 10 \ \sin(37^o) \\
&= 6.02
\end{aligned}
$$

$$
\begin{aligned}
B_x &= B \ \cos\theta_B \\
&= 12 \ \cos(-60^o) \\
&= 6.00 \\
B_x &= B \ \sin\theta_B \\
&= 12 \ \sin(-60^o) \\
&= -10.39
\end{aligned}
$$

$$
\begin{aligned}
\vec{R}_x &= \vec{A}_x + \vec{B}_x \\
&= 7.98 + 6.00 \\
&= 13.98 \\
\vec{R}_y &= \vec{A}_y + \vec{B}_y \\
&= 6.02 + (-10.39) \\
&= \ -4.37
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
\vec{R} &= 13.98\hat{\imath} - 4.37\hat{\jmath} \\
R &= \sqrt{(13.98)^2 + (-4.37)^2} \\
&= 14.6 \\
\tan\theta_R &= \frac{R_y}{R_x} = \frac{-4.37}{13.98} = -0.313 \\
\theta_R &= \ \tan^{-1}(-0.313) = -17.4^o \\
\vec{R} &= 14.6 \ @ \ -17.4^o
\end{aligned}
\tag{13}
$$

February 10

14 Kinematics in 2-D

Let:

$$\vec{r} = \text{position}$$
$$= x\hat{i} + y\hat{j} \qquad \vec{r_i} = x_i\hat{i} + y_i\hat{j} = \text{initial}$$
$$\vec{r_f} = x_f\hat{i} + y_f\hat{j} = \text{final}$$

$$\begin{aligned}
\Delta\vec{r} &= \vec{r_f} - \vec{r_i} \\
&= (x_f\hat{i} + y_f\hat{j}) - (x_i\hat{i} + y_i\hat{j}) \\
&= (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j} \\
&= \Delta x\hat{i} + \Delta y\hat{j}
\end{aligned}$$

$$\begin{aligned}
\vec{\bar{v}} &= \text{average velocity} \\
&= \frac{\text{displacement}}{\text{time}} \\
&= \frac{\Delta\vec{r}}{\Delta t} \\
&= \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t} \\
&= \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} \\
&= \vec{\bar{v}}_x\hat{i} + \vec{\bar{v}}_y\hat{j}
\end{aligned}$$

$$\begin{aligned}\vec{v} &= \text{instantaneous velocity}\\ &= \frac{d\vec{r}}{dt}\\ &= \frac{d}{dt}\left(x\hat{i} + y\hat{j}\right)\\ &= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}\\ &= \vec{v_x}\hat{i} + \vec{v_y}\hat{j}\end{aligned}$$

$$\begin{aligned}\vec{\bar{a}} &= \text{average acceleration}\\ &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{\Delta\left(v_x\hat{i} + v_y\hat{j}\right)}{\Delta t}\\ &= \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}\\ &= \vec{\bar{a}}_x\hat{i} + \vec{\bar{a}}_y\hat{j}\end{aligned}$$

$$\begin{aligned}\vec{a} &= \text{instantaneous acceleration}\\ &= \frac{d\vec{v}}{dt}\\ &= \frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j}\right)\\ &= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}\\ &= \vec{a_x}\hat{i} + \vec{a_y}\hat{j}\end{aligned}$$

15 Projectiles

$$\begin{aligned}v_x &= \text{constant}\\ a_x &= 0 \qquad\qquad\qquad a_y = -g\end{aligned}$$

(1-D) iff $a = \text{constant}$

$$
\begin{aligned}
&(1) \qquad && v = v_0 + at \\
&(2) \qquad && x = x_0 + v_0 t + \frac{1}{2} a t^2 \\
&(3) \qquad && x = x_0 + \frac{1}{2}\left(v_0 + v\right) t \\
&(4) \qquad && 2a\left(x - x_0\right) = v^2 - v_0^2
\end{aligned}
$$

iff $\vec{a} = \text{constant}$

$$
\begin{aligned}
&(1x) \qquad && v_x = v_{0_x} + a_x t \\
&(1y) \qquad && v_y = v_{0_y} + a_y t \\
&(2x) \qquad && x = x_0 + v_{0_x} t + \frac{1}{2} a_x t^2 \\
&(2y) \qquad && y = y_0 + v_{0_y} t + \frac{1}{2} a_y t^2
\end{aligned}
$$