~~M. How~~

~~Quad~~

Quadratic Equations

→ Standard Form

$ax^2 + bx + c = 0$

where, $a \neq 0$

and a, b, c are constants

→ Example:

$x^2 - 2 + 5x = 0$

→ Degree

2

→ Finding Roots

for $ax^2 + bx + c$ where D = Discriminant $= b^2 - 4ac$

$$x = \frac{-b \pm \sqrt{D}}{2x}$$

Nature of roots of Q.E.

$D = b^2 - 4ac$	Nature of roots
$D > 0$	2 distinct real roots
$D = 0$	2 equal roots
$D < 0$	No real roots

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Exercise 4.1

Check whether the following are quadratic equations:

(i) $(x+1)^2 = 2(x-3)$

$x^2+1+2x = 2x-6$

$x^2 - 7 = 0$

yes, it is quadratic Equation

(ii) $x^2 - 2x = (-2)(-3-x)$

$x^2 - 2x = -6 + 2x$

$x^2 - 4x - 6 = 0$

yes, it is quadratic equation

(iii) $(x-2)(x+1) = (x-1)(x+3)$

$x(x+1) - 2(x+1) = x(x+3) - 1(x+3)$

$x^2 + x - 2x - 2 = x^2 + 3x - x - 3$

$x^2 - x - 2 = x^2 + 2x - 3$

$-x - 2 - 2x + 3 = 0$

$-3x + 1 = 0$

No, it is not Q.E.

(iv) $(x-3)(2x+1) = x(x+5)$

$= x(2x+1) - 3(2x+1) = x^2 + 5x$

$2x^2 + x - 6x - 3 = x^2 + 5x$

$2x^2 - x^2 - 5x - 5x - 3 = 0$

$x^2 - 10x = 0$

yes, it is Q.E

(v) $(2x-1)(x-3) = (x+5)(x-1)$

$2x(x-3) - 1(x-3) = x(x-1) + 5(x-1)$

$= 2x^2 - x^2 - 6x - x + x - 5x + 5 + 3 = 0$

$= x^2 - 11x + 8 = 0$

yes, it is Q.E

(vi) $x^2 + 3x + 1 = (x-2)^2$

$x^2 + 3x + 1 = x^2 + 4 - 4x$

$x^2 + 3x - 1 - x^2 - 4 + 4x = 0$

$7x - 3 = 0$

No, it is not a Q.E

(vii) $(x+2)^3 = 2x(x^2-1)$

$x^3 + 2^3 + 3x^2 \times 2 + 3x \times 2^2 = 2x^3 - 2x$

$x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x$

$x^3 - 2x^3 + 8 + 6x^2 + 14x = 0$

$-x^3 + 8 + 6x^2 + 14x = 0$

It is not a Q.E.

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

$\cancel{x^3 - 4x^2 - x + 1 = x^3 - 2^3 - 3 \times x^2 \times 2 + 3 \times x(-2)^2}$

$\cancel{x^3 - 4x^2 - x + 1 = x^3 - 8 - [3 \times x^2 \times (-2)] + 3 \times x \times (-2)^2 \}}$

$\cancel{x^3 - 4x^2 - x + 1 = x^3 - 8 + 6x^2 + 12x}$

$\cancel{-10x^2 - 13x + 9}$

$\cancel{\text{Yes, it is Q.E}}$

$\cancel{x^3} - 4x^2 - x + 1 = \cancel{x^3} - 8 - 6x^2 + 12x$

$-2x^2 - 13x + 9 = 0$

Yes, it is Quadratic Equation.

Q. Represent the following situations in the form of quadratic equations:

(i) The area of a rectangular plot is 528 m^2. The length of the plot - - - - - - - - of the plot.

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(ii) Let the Breath of the plot be x m.
Hence, the length of the plot is $(2x+1)$ m.
Area of a rectangle = Length × Breath
$\therefore 528 = x(2x+1)$
$\Rightarrow 2x^2 + x - 528 = 0$

(iii) The product of 2 consecutive positive integers is 306. We need to find the integers.

(ii) Let the consecutive integers be x and $x+1$
It is given that their product is 306
$\therefore x(x+1) = 306$
$\Rightarrow x^2 + x - 306 = 0$

(iii) Rohan's mother is 26 years older than him. The product of ther ages - - - - - - - - - - - - present age.

Let Rohan's age be x
Hence, his mother's age $= x+26$
3 years hence,
Rohan's age $= x+3$
Mother's age $= x+26+3 = x+29$
It is given that the product of their ages after 3 years is 360.
$\therefore (x+3)(x+29) = 360$
$\Rightarrow x^2 + 32x - 273 = 0$

(iv) A train travels a distance of 480 km at a uniform speed. If the - - - - - - - - - - - of the train.

Let the speed of train be x km/hr

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Time taken to travel 480 km = $480/x$ hrs.

In second condition, let the speed of train = $(x-8)$ km/hr

It is also given that the train will take 3 hrs to cover the same distance.

Therefore, time taken to travel 480 km = $\left[\frac{480}{x} + 3\right]$ hrs

Speed × Time = Distance

$$(x-8)\left[\frac{480}{x} + 3\right] = 480$$

$\Rightarrow 3 \times 2 - 24x - 3840 = 0$

$\Rightarrow x^2 - 8x - 1280 = 0$

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Exercise : 4.2

(i) Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$

$= x^2 - 5x + 2x - 10$

$= x(x-5) + 2(x-5)$

$= (x+2)(x-5)$

$x - 5 = 0 \Rightarrow x = 5$

$x + 2 = 0 \Rightarrow x = -2$

(ii) $2x^2 + x - 6 = 0$

$= 2x^2 + 4x - 3x - 6$

$= 2x(x+2) - 3(x+2)$

$= (x+2)(2x-3)$

$x + 2 = 0 \Rightarrow x = -2$

$2x - 3 = 0 \Rightarrow x = 3/2$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$= \sqrt{2}x + 5x + 2x + 5\sqrt{2}$

$= x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5)$

$= (\sqrt{2}x + 5)(x + \sqrt{2})$

$\sqrt{2}x + 5 = 0, \Rightarrow x = -5/\sqrt{2}$

$x + \sqrt{2} = 0 \Rightarrow x = -\sqrt{2}$

(iv) $2x^2 - x + \frac{1}{8} = 0$

$= \frac{1}{8}(16x^2 - 8x + 1)$

$= \frac{1}{8}(16x^2 - 4x - 4x + 1)$

$= \frac{1}{8}[4x[4x-1] - 1[4x-1]]$

$= \frac{1}{8}[4x-1]^2$

$= [4x-1] = 0 \Rightarrow x = 1/4$

$[4x-1] = 0 \Rightarrow x = 1/4$

(v) $100x^2 - 20x + 1 = 0$

$= 10x[10x-1] - 1[10x-1]$

$= (10x-1)^2$

$10x - 1 = 0 \Rightarrow x = 1/10$

$10x - 1 = 0 \Rightarrow x = 1/10$

2. Solve the problem in Ex: 1

~~Let one of the numbers be x~~

~~∴ Others be x~~

~~$x(27-x) = 182$~~

(i) Let the no. of John's Marbles be x

∴ Jivanti's Marble $= 45 - x$

After losing 5 marbles

No. of John's marbles $= x - 5$

No. of Jivanti's Marbles $= 45 - x - 5 = 40 - x$

It is given that the product of their marbles is 124.

$\therefore (x-5)(40-x) = 124$

$\Rightarrow x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 45x + 324 = 0$

$\Rightarrow x(x-36) - 9(x-36) = 0$

$\Rightarrow (x-36)(x-9) = 0$

Either $x - 36 = 0$ or $x - 9 = 0$

$x = 36$ or $x = 9$

If the number of John's marbles $= 36$

∴ Jivanti's marbles $= 45 - 36 = 9$

If number of John's marbles $= 9$

Then, Jivanti's marbles $= 45 - 9 = 36$

(ii) Let the number of toys produced be x

∴ Cost of production of each toy $=$ Rs $(55 - x)$

It is given, total production of the toys = Rs 750.

$\therefore x(55-x) = 750$

$x^2 - 55x + 750 = 0$

$x^2 - 25x - 30x + 750 = 0$

$(x-25)(x-30) = 0$

Either $x - 25 = 0$ or $x - 30 = 0$

ie; $x = 25$ or $x = 30$

Q3 Find two numbers whose sum is 27 and product is 182.

Let one of the number be x.

Others be $27 - x$

$x(27-x) = 182$

$27x - x = 182$

$x^2 - 27x + 182 = 0$

$x(x-13) - 14(x-13) = 0$

$(x-13)(x-14) = 0$

Either $x - 13 = 0$ or $x - 14 = 0$

If first no = 13

2nd no = 14

If first no = 14

2nd no = 13

Q4. Find 2 consecutive positive integers, sum of whose square is 365.

Let the 2 consecutive positive 2 integers can be x & $x+1$

$x^2 + (x+1)^2 = 365$

$x^2 + x^2 + 1 + 2x = 365$

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$2x^2 + 2x - 364 = 0$

$x^2 + x - 182 = 0$

$(x+14)(x-13) = 0$

$x + 14 = 0 \Rightarrow x = -14$

$x - 13 = 0 \Rightarrow x = 13$

Neglecting $x = -14$

1 number $= x + 13$

2nd no. $= x + 1 = 13 + 1 = 14$

Q5. The altitude of a right triangle is 7cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Let the base of the right triangle be x cm

Its altitude $= (x-7)$ cm

$H^2 = P^2 + B^2$ (Pythagoras)

$13^2 = x^2 + (x-7)^2$

$169 = 2x^2 - 14x - 60$

$0 = x(x-12) + 5(x-12)$

$0 = (x-12)(x+5)$

$\therefore$ Base $= 12$ cm, $P = 5$ cm

Q6 A cottage industry produces a certain number of ... cost of each article.

Let the articles produced be n

$\therefore$ Cost of production $= ₹90$

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$\therefore \; x[2x+3] = 90$

$\therefore \; (2x+15)(x-6) = 0$

$2x + 15 = 0 \quad , \quad x = -15/2$

$x - 6 = 0 \quad , \quad x = 6$

No. of articles produced $x = 6$

cost article $= 3 + 2x$

$= 3 + 2 \times 6$

$= ₹15$

Key Points

i) 2 consecutive odd numbers are $2x+1, 2x+3$

ii) 2 consecutive even no. are $2x$ & $2x+2$

iii) 2 consecutive positive integers are $x, x+1$.

Ex: 4.3

1. Find the nature of the roots of the following quadratic equations.

(i) $2x^2 - 3x + 5 = 0$

$ax^2 + bx + c = 0$

$\therefore a = 2, b = -3, c = 5$

$D = b^2 - 4ac$

$= (-3)^2 - 4 \times 2 \times 5$

$\Rightarrow 9 - 40 = -31 < 0$

$\therefore$ No real roots exist

$13^2 = 21^2 + (u-7)^2$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

$ax^2 + bx + c = 0$

$\therefore a = 3, b = -4\sqrt{3}, c = 4$

$D = b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4 \times 3 \times 4$

$48 - 48 = 0$

$\therefore$ Roots are real & equal

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{[-4\sqrt{3}] + \sqrt{0}}{2 \times 3}$

$= \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{6}$

(iii) $2x^2 - 6x + 3 = 0$ $\quad a = 2, b = -6, c = 3$

$ax^2 + bx + c = 0$

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$D = b^2 - 4ac$
$= (-6)^2 - 4 \times 2 \times 3$
$= 36 - 24$
$= 12 > 0$

$\therefore$ Roots are distinct and Real

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$x = \frac{-[-6] \pm \sqrt{12}}{2 \times 2}$$

$$= \frac{6 \pm 2\sqrt{3}}{4}$$

$$= \frac{3 \pm \sqrt{3}}{2} \Rightarrow \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$$

2. Find the value of k for each of the following quadratic Equation.

(ii) $kx(x-2)+6=0$
$kx^2 - 2kx + 6 = 0$
$ax^2 + bx + c = 0$

$a = k, b = -2k, c = 6$
For equal root
$D = 0$

$b^2 - 4ac = 0$
$(-2k)^2 - 24k = 0$
$4k[k-6] = 0$
$4k = 0$
$k = 0, k = 6$

Neglecting $k = 0$
$\therefore k = 6$

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3. Is it possible to design a rectangular mango grove whose length is twice its breath, and area is $800\ m^2$? If so find its length & breath.

breath $= l$ Length $= 2l$

Area $= 2l^2 = 800$

$= l^2 - 400 = 0$

$a = 1,\ b = 0,\ c = -400$

$D = b^2 - 4ac > 0$

$= 0 + 1600$

$= 1600$

$l = \pm 20$

$\because$ Length, cannot be Negative

$\therefore$ Breath $= 20\ m$, Length $= 40m$

Q4. Is the following situation possible? - - - - - - - - in years was 48.

Let age $= x$ yr

Other friend's age $= (20 - x)$ yr

4 yr ago $= (x - 4)$ years

2nd friend $= (16 - 4)$ yrs

Given: $(x-4)(16-x) = 48$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

$a = 1,\ b = -20,\ c = 112$

$D = b^2 - 4ac$

$= 400 - 448 = -48$

$\because b^2 - 4ac < 0$

$\therefore$ This situation is not possible.

Q5. Is it possible to design a rectangular - - - - length & breath.

Perimeter $= 2(l + b) = 80$

$\Rightarrow l + b = 40$

$b = 40 - l$

Area $= l \times b = l(40 - l)$

$400 = 40l - l^2$

$l^2 - 40l + 400 = 0$

$a = 1,\ b = -40,\ c = 400$

$D = b^2 - 4ac$

$= 1600 - 1600 = 0$

$\because b^2 - 4ac = 0$ real

$\therefore$ Eq. has equal roots.

$l = \frac{b}{2a} = \frac{-40}{2} = 20m,\ b = 20m$

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MCQs

1. Let P be a prime number. The quadratic equation having its root as factors of P is

a) $x^2 - px + p = 0$
b) $x^2 - [p+1]x + p = 0$
c) $x^2 + [p+1]x + p = 0$
d) $x^2 - px + p + 1 = 0$

2. Which of the following equation has 2 as its roots?

a) $x^2 - 4x + 5 = 0$
b) $x^2 + 3x - 12 = 0$
c) $2x^2 - 7x + 6 = 0$
d) $3x^2 - 6x - 2 = 0$

3. If 1/2 is a root of the equation $x^2 + Kx - \frac{5}{4} = 0$, then the value of K is:-

a) 2 b) -2 c) 1/2 d) -1/2

4. The roots of the equation $x^2 + 7x + 10 = 0$ are

a) -5, -2 b) 5, 2 c) 5, -2 d) -5, 2

5. which of the following equation has two distinct real roots.

a) $2x - 3$
b) $2x + 9 = 0$
c) $x^2 + 3x + 2, -2 = 0$
d) $5x^2 - 3x^4 + 1 = 0$

7. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

a) Two ~~dist~~ distinct real roots
b) Two equal real roots
c) No real roots
d) More than 2 real roots.

8. $(x^2 + 1^2) - x^2 = 0$ has

a) ~~No~~ real roots
b) four real roots
d) One real roots
b) Two real roots

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9. The value [s] of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots. is

a) 4 b) ±4 c) −4 d) 0

6. Value of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

a) 0 only b) 4 c) 3 only d) 0, 8

10. Assertion [A]: If the coefficient of x^2 and the constant term have the same sign and if the coefficient of x term is zero then the quadratic equation has no real roots.

Reason [R]: The equation $13\sqrt{3}x^2 + 10x + \sqrt{3} = 0$ has no real roots.

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Q1. The speed of a motorboat is 20 km/hr, for covering the distance of 15 Km. The motorboat took 1 hour more for upstream than downstream. Answer the following :-

i) Let the speed of the stream be x km/hr then the speed of the motorboat in upstream will be ____

ii) What is the relation between distance, speed and time?

iii) Find the speed of the stream

iv) How much time motorboat took in downstream?

Q2. A parabolic arch is in the shape of a parabola. In structures of parabolic shaped objects, curves represent parabola in the form of bridges and architectures. Answer the following questions:

i) The graph of the parabolic curve $x^2+1=0$

(a) intersects x-axis at two distinct points.

(b) touches x-axis at a point

(c) Neither touches nor intersect x-axis

(d) Either touches or intersects x-axis

ii) If α and $\frac{1}{\alpha}$ are the zeroes of the quadratic polynomial $2x^2 - x + 8k = 0$. Find the value of k.

iii) The graph of a parabola is:

a) Linear ~~polynial~~ polynomial

b) Quadratic polynomial

c) Cubic polynomial

d) Biquadratic polynomial

iv) If α and β are the ~~ratio~~ roots of the equation $x^2 - 5x + 3(k+1) = 0$, and $\alpha - \beta = 1$. Find k.

Ans: i) Speed of motorboat in upstream = Speed of boat − speed of stream

$(20 - x)$ km/hr

ii) Speed × time = Distance

iii) Upstream = Time taken = $\frac{15}{20-x}$

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Down stream = Time taken = $\frac{15}{20+x}$

$\therefore \frac{15}{20-x} - \frac{15}{20+x} = 1$

$= \frac{20-x-(20-x)}{(20-x)(20+x)} = \frac{1}{15}$

$= \frac{2x}{400-x^2} = \frac{1}{15}$

$\Rightarrow 30x = 400 - x$

$\Rightarrow x^2 + 30x - 400 = 0.$

$\Rightarrow (x+40)(x-10) = 0$

$x = -40, \; x = 10$

Speed of stream = 10 km/hr

(iv) Time $= \frac{15}{20+x}$

$= \frac{15}{20+10} = \frac{15}{30} = 30$ min.

Ans 2

(ii) $2x^2 - x + 8k$

$= \alpha + \frac{1}{\alpha} = \frac{8k}{2}$

$\Rightarrow 1 = 4k$

$\Rightarrow k = \frac{1}{4}$

(iv) $x^2 - 5x + 3(k-1) = 0$

$\alpha + \beta = \frac{-[-5]}{1}$

$\Rightarrow 2\alpha = 6$

$\Rightarrow \alpha = 3$

$\alpha + \beta = 5$

$\beta = 2$

$\alpha\beta = \frac{c}{a}$

$3 \times 2 = 3[k-1]$

$k = 3$

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Solutions :-

Ans 2) $p(x) = 2x^2 - 7x + 6 = 0$

$p(2) = 2(2) - 7 \times 2 + 6 = 0$

$= 8 - 14 + 6 = 0$

$= 14 - 14 = 0$

Ans 3) $p(x) = x^2 + kx - \frac{5}{4} = 0$

$\Rightarrow p(1/2) = \left(\frac{1}{2}\right)^2 + k\left[\frac{1}{2}\right] - \frac{5}{4} = 0$

$\Rightarrow \frac{1}{2}k = \frac{5}{4} - \frac{1}{4}$

$\Rightarrow \frac{1}{2}k = 1$

$\Rightarrow k = 2$

Ans 4) $P(x) = x^2 + 7x + 10 = 0$

$p(x) = x^2 + 5x + 2x + 10 = 0$

$\Rightarrow x(x+5) + 2(x+5) = 0$

$\Rightarrow (x+2)(x+5)$

$x = -2, \; x = -5$

$\alpha = -2, \; \beta = -5$

Ans 5) $D = b^2 - 4ac$

$2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0 \quad \left[a = 2, b = -3\sqrt{2}, c = \frac{9}{4}\right]$

$D = b^2 - 4ac \Rightarrow D = 18 - 4 \times 2 \times \frac{9}{4}$

$\Rightarrow D = 18 - 8 \Rightarrow D = 0$

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b) $x^2 + x - 5 = 0$ $(a = 1, b = 1, c = -5)$

$D = b^2 - 4ac \Rightarrow D = (1)^2 - 4 \times 1 \times (-5)$

$D > 0 \Rightarrow D = 1 + 20 \Rightarrow D = 21$

$\therefore$ two distinct real roots.

Ans 7) $2x^2 - \sqrt{5}x + 1 = 0$ $[a = 2, b = -\sqrt{5}, c = 1)$

$D = b^2 - 4ac = (-\sqrt{5})^2 - 4 \times 2 \times 1$

$\Rightarrow 5 - 8 \Rightarrow -3$

$D < 0$, no real roots.