

# THE STATA JOURNAL

## Editors

H. JOSEPH NEWTON  
Department of Statistics  
Texas A&M University  
College Station, Texas  
editors@stata-journal.com

NICHOLAS J. COX  
Department of Geography  
Durham University  
Durham, UK  
editors@stata-journal.com

## Associate Editors

CHRISTOPHER F. BAUM, Boston College  
NATHANIEL BECK, New York University  
RINO BELLOCCO, Karolinska Institutet, Sweden, and  
University of Milano-Bicocca, Italy  
MAARTEN L. BUIS, University of Konstanz, Germany  
A. COLIN CAMERON, University of California–Davis  
MARIO A. CLEVES, University of Arkansas for  
Medical Sciences  
WILLIAM D. DUPONT, Vanderbilt University  
PHILIP ENDER, University of California–Los Angeles  
DAVID EPSTEIN, Columbia University  
ALLAN GREGORY, Queen's University  
JAMES HARDIN, University of South Carolina  
BEN JANN, University of Bern, Switzerland  
STEPHEN JENKINS, London School of Economics and  
Political Science  
ULRICH KOHLER, University of Potsdam, Germany

FRAUKE KREUTER, Univ. of Maryland–College Park  
PETER A. LACHENBRUCH, Oregon State University  
JENS LAURITSEN, Odense University Hospital  
STANLEY LEMESHOW, Ohio State University  
J. SCOTT LONG, Indiana University  
ROGER NEWSON, Imperial College, London  
AUSTIN NICHOLS, Urban Institute, Washington DC  
MARCELLO PAGANO, Harvard School of Public Health  
SOPHIA RABE-HESKETH, Univ. of California–Berkeley  
J. PATRICK ROYSTON, MRC Clinical Trials Unit,  
London  
PHILIP RYAN, University of Adelaide  
MARK E. SCHAFFER, Heriot-Watt Univ., Edinburgh  
JEROEN WEESIE, Utrecht University  
IAN WHITE, MRC Biostatistics Unit, Cambridge  
NICHOLAS J. G. WINTER, University of Virginia  
JEFFREY WOOLDRIDGE, Michigan State University

## Stata Press Editorial Manager

LISA GILMORE

## Stata Press Copy Editors

DAVID CULWELL, SHELBI SEINER, and DEIRDRE SKAGGS

The *Stata Journal* publishes reviewed papers together with shorter notes or comments, regular columns, book reviews, and other material of interest to Stata users. Examples of the types of papers include 1) expository papers that link the use of Stata commands or programs to associated principles, such as those that will serve as tutorials for users first encountering a new field of statistics or a major new technique; 2) papers that go “beyond the Stata manual” in explaining key features or uses of Stata that are of interest to intermediate or advanced users of Stata; 3) papers that discuss new commands or Stata programs of interest either to a wide spectrum of users (e.g., in data management or graphics) or to some large segment of Stata users (e.g., in survey statistics, survival analysis, panel analysis, or limited dependent variable modeling); 4) papers analyzing the statistical properties of new or existing estimators and tests in Stata; 5) papers that could be of interest or usefulness to researchers, especially in fields that are of practical importance but are not often included in texts or other journals, such as the use of Stata in managing datasets, especially large datasets, with advice from hard-won experience; and 6) papers of interest to those who teach, including Stata with topics such as extended examples of techniques and interpretation of results, simulations of statistical concepts, and overviews of subject areas.

The *Stata Journal* is indexed and abstracted by *CompuMath Citation Index*, *Current Contents/Social and Behavioral Sciences*, *RePEc: Research Papers in Economics*, *Science Citation Index Expanded* (also known as *SciSearch*), *Scopus*, and *Social Sciences Citation Index*.

For more information on the *Stata Journal*, including information for authors, see the webpage

<http://www.stata-journal.com>

**Subscriptions** are available from StataCorp, 4905 Lakeway Drive, College Station, Texas 77845, telephone 979-696-4600 or 800-STATA-PC, fax 979-696-4601, or online at

<http://www.stata.com/bookstore/sj.html>

**Subscription rates** listed below include both a printed and an electronic copy unless otherwise mentioned.

| U.S. and Canada                   |       | Elsewhere                         |       |
|-----------------------------------|-------|-----------------------------------|-------|
| <b>Printed &amp; electronic</b>   |       | <b>Printed &amp; electronic</b>   |       |
| 1-year subscription               | \$115 | 1-year subscription               | \$145 |
| 2-year subscription               | \$210 | 2-year subscription               | \$270 |
| 3-year subscription               | \$285 | 3-year subscription               | \$375 |
| 1-year student subscription       | \$ 85 | 1-year student subscription       | \$115 |
| 1-year institutional subscription | \$345 | 1-year institutional subscription | \$375 |
| 2-year institutional subscription | \$625 | 2-year institutional subscription | \$685 |
| 3-year institutional subscription | \$875 | 3-year institutional subscription | \$965 |
| <b>Electronic only</b>            |       | <b>Electronic only</b>            |       |
| 1-year subscription               | \$ 85 | 1-year subscription               | \$ 85 |
| 2-year subscription               | \$155 | 2-year subscription               | \$155 |
| 3-year subscription               | \$215 | 3-year subscription               | \$215 |
| 1-year student subscription       | \$ 55 | 1-year student subscription       | \$ 55 |

Back issues of the *Stata Journal* may be ordered online at

<http://www.stata.com/bookstore/sjj.html>

Individual articles three or more years old may be accessed online without charge. More recent articles may be ordered online.

<http://www.stata-journal.com/archives.html>

The *Stata Journal* is published quarterly by the Stata Press, College Station, Texas, USA.

Address changes should be sent to the *Stata Journal*, StataCorp, 4905 Lakeway Drive, College Station, TX 77845, USA, or emailed to [sj@stata.com](mailto:sj@stata.com).



Copyright © 2015 by StataCorp LP

**Copyright Statement:** The *Stata Journal* and the contents of the supporting files (programs, datasets, and help files) are copyright © by StataCorp LP. The contents of the supporting files (programs, datasets, and help files) may be copied or reproduced by any means whatsoever, in whole or in part, as long as any copy or reproduction includes attribution to both (1) the author and (2) the *Stata Journal*.

The articles appearing in the *Stata Journal* may be copied or reproduced as printed copies, in whole or in part, as long as any copy or reproduction includes attribution to both (1) the author and (2) the *Stata Journal*.

Written permission must be obtained from StataCorp if you wish to make electronic copies of the insertions. This precludes placing electronic copies of the *Stata Journal*, in whole or in part, on publicly accessible websites, file servers, or other locations where the copy may be accessed by anyone other than the subscriber.

Users of any of the software, ideas, data, or other materials published in the *Stata Journal* or the supporting files understand that such use is made without warranty of any kind, by either the *Stata Journal*, the author, or StataCorp. In particular, there is no warranty of fitness of purpose or merchantability, nor for special, incidental, or consequential damages such as loss of profits. The purpose of the *Stata Journal* is to promote free communication among Stata users.

The *Stata Journal* (ISSN 1536-867X) is a publication of Stata Press. Stata, **stata**, Stata Press, Mata, **mata**, and NetCourse are registered trademarks of StataCorp LP.

# Fixed-effect panel threshold model using Stata

Qunyong Wang  
Institute of Statistics and Econometrics  
Nankai University  
Tianjin, China  
QunyongWang@outlook.com

**Abstract.** Threshold models are widely used in macroeconomics and financial analysis for their simple and obvious economic implications. With these models, however, estimation and inference is complicated by the existence of nuisance parameters. To combat this issue, Hansen (1999, *Journal of Econometrics* 93: 345–368) proposed the fixed-effect panel threshold model. In this article, I introduce a new command (`xthreg`) for implementing this model. I also use Monte Carlo simulations to show that, although the size distortion of the threshold-effect test is small, the coverage rate of the confidence interval estimator is unsatisfactory. I include an example on financial constraints (originally from Hansen [1999, *Journal of Econometrics* 93: 345–368]) to further demonstrate the use of `xthreg`.

**Keywords:** st0373, xthreg, panel threshold, fixed effect

## 1 Introduction

Heterogeneity is a common problem of panel data. That is to say, each individual in a study is different, and structural relationships may vary across individuals. The classical fixed effect or random effect reflects only the heterogeneity in intercepts. Hsiao (2003) considers many varying slope models for this problem. Among these models, Hansen's (1999) panel threshold model has a simple specification but obvious implications for economic policy. Though threshold models are familiar in time-series analysis, their use with panel data has been limited.

The threshold model describes the jumping character or structural break in the relationship between variables. This model type is popular in nonlinear time series, one example being the threshold autoregressive (TAR) model (Tong 1983). This model can capture many economic phenomena. For example, using five-year interval averages of standard measures of financial development, inflation, and growth for 84 countries from 1960 to 1995, Rousseau and Wachtel (2009) showed that there is an inflation threshold for the finance and growth relationship that lies between 13–25%. When inflation exceeds the threshold, finance ceases to increase economic growth. Inflation's effect on economic growth depends on the inflation level. High levels of inflation are harmful to economic growth, while low levels of inflation are beneficial to economic growth. As another example, the technical spillover of foreign direct investment (FDI) has been widely studied. Girma (2005) found that the productivity benefit from FDI increases with absorptive capacity until some threshold level, at which point it becomes less pronounced. There is also a minimum absorptive capacity threshold level below which productivity spillovers from FDI are negligible or even negative.

This article is arranged as follows. In section 2, I review some basic theories about fixed-effect panel threshold models. I then describe the new `xthreg` command in section 3. In section 4, I perform Monte Carlo simulations to study test-power distortion and the coverage rate of confidence interval estimators in finite samples. I illustrate use of the command with an example from Hansen (1999) in section 5. In section 6, I conclude the article.

## 2 Fixed-effect panel threshold models

### 2.1 Single-threshold model

Consider the following single-threshold model:

$$y_{it} = \mu + \mathbf{X}_{it}(q_{it} < \gamma)\beta_1 + \mathbf{X}_{it}(q_{it} \geq \gamma)\beta_2 + u_i + e_{it} \quad (1)$$

The variable  $q_{it}$  is the threshold variable, and  $\gamma$  is the threshold parameter that divides the equation into two regimes with coefficients  $\beta_1$  and  $\beta_2$ . The parameter  $u_i$  is the individual effect, while  $e_{it}$  is the disturbance. We can also write (1) as

$$y_{it} = \mu + \mathbf{X}_{it}(q_{it}, \gamma)\beta + u_i + e_{it}$$

where

$$\mathbf{X}_{it}(q_{it}, \gamma) = \begin{cases} \mathbf{X}_{it}I(q_{it} < \gamma) \\ \mathbf{X}_{it}I(q_{it} \geq \gamma) \end{cases}$$

Given  $\gamma$ , the ordinary least-squares estimator of  $\beta$  is

$$\hat{\beta} = \{\mathbf{X}^*(\gamma)' \mathbf{X}^*(\gamma)\}^{-1} \{\mathbf{X}^*(\gamma)' \mathbf{y}^*\}$$

where  $\mathbf{y}^*$  and  $\mathbf{X}^*$  are within-group deviations. The residual sum of squares (RSS) is equal to  $\hat{e}^{*'}\hat{e}^*$ . To estimate  $\gamma$ , one can search over a subset of the threshold variable  $q_{it}$ . Instead of searching over the whole sample, we restrict the range within the interval  $(\underline{\gamma}, \bar{\gamma})$ , which are quantiles of  $q_{it}$ .  $\gamma$ 's estimator is the value that minimizes the RSS, that is,

$$\hat{\gamma} = \arg \min_{\gamma} S_1(\gamma)$$

If  $\gamma$  is known, the model is no different from the ordinary linear model. But if  $\gamma$  is unknown, there is a nuisance parameter problem, which makes the  $\gamma$  estimator's distribution nonstandard. Hansen (1999) proved that  $\hat{\gamma}$  is a consistent estimator for  $\gamma$ , and he argued that the best way to test  $\gamma = \gamma_0$  is to form the confidence interval using the “no-rejection region” method with a likelihood-ratio (LR) statistic, as follows:

$$\begin{aligned} \text{LR}_1(\gamma) &= \frac{\{\text{LR}_1(\gamma) - \text{LR}_1(\hat{\gamma})\}}{\hat{\sigma}^2} \xrightarrow{\text{Pr}} \xi \\ \text{Pr}(x < \xi) &= (1 - e^{-\frac{x}{2}})^2 \end{aligned} \quad (2)$$

Given significance level  $\alpha$ , the lower limit corresponds to the maximum value in the LR series, which is less than the  $\alpha$  quantile, and the upper limit corresponds to the minimum value in the LR series, which is less than the  $\alpha$  quantile. The  $\alpha$  quantile can be computed from the following inverse function of (2):

$$c(\alpha) = -2 \log(1 - \sqrt{1 - \alpha})$$

For example, for  $\alpha = 0.1, 0.05$ , and  $0.01$ , the quantiles are 6.53, 7.35, and 10.59, respectively. If  $LR_1(\gamma_0)$  exceeds  $c(\alpha)$ , then we reject  $H_0$ .

Testing for a threshold effect is the same as testing for whether the coefficients are the same in each regime. The null hypothesis and the alternative hypothesis (the linear versus the single-threshold model) are

$$H_0: \beta_1 = \beta_2 \quad H_a: \beta_1 \neq \beta_2$$

The  $F$  statistic is constructed as

$$F_1 = \frac{(S_0 - S_1)}{\hat{\sigma}^2} \quad (3)$$

Under  $H_0$ , the threshold  $\gamma$  is not identified, and  $F_1$  has nonstandard asymptotic distribution. We use bootstrap on the critical values of the  $F$  statistic to test the significance of the threshold effect.  $S_0$  is the RSS of the linear model. Hansen (1996) suggested the following bootstrap design:

Step 1: Fit the model under  $H_a$  and obtain the residual  $\hat{e}_{it}^*$ .

Step 2: Make a cluster resampling  $\hat{e}_{it}^*$  with replacement, and obtain the new residual  $v_{it}^*$ .

Step 3: Generate a new series under the  $H_a$  data-generating process (DGP),  $y_{it}^* = \mathbf{X}_{it}^* \beta + v_{it}^*$ , where  $\beta$  can take arbitrary values.

Step 4: Fit the model under  $H_0$  and  $H_a$ , and compute the  $F$  statistic using (3).

Step 5: Repeat steps 1–4  $B$  times, and the probability of  $F$  is  $\Pr = I(F > F_1)$ , namely, the proportion of  $F > F_1$  in bootstrap number  $B$ .

## 2.2 Multiple-thresholds model

If there are multiple thresholds (that is, multiple regimes), we fit the model sequentially. We use a double-threshold model as an example.

$$y_{it} = \mu + \mathbf{X}_{it}(q_{it} < \gamma_1)\beta_1 + \mathbf{X}_{it}(\gamma_1 \leq q_{it} < \gamma_2)\beta_2 + \mathbf{X}_{it}(q_{it} \geq \gamma_2)\beta_3 + u_i + e_{it}$$

Here,  $\gamma_1$  and  $\gamma_2$  are the thresholds that divide the equation into three regimes with coefficients  $\beta_1, \beta_2$ , and  $\beta_3$ . We need to compute this  $(N \times T)^2$  times using the grid search method, which is infeasible in practice. According to Bai (1997) and Bai and Perron (1998), the sequential estimator is consistent, so we estimate the thresholds as follows:

Step 1: Fit the single-threshold model to obtain the threshold estimator  $\gamma_1$  and the RSS  $S_1(\hat{\gamma}_1)$ .

Step 2: Given  $\hat{\gamma}_1$ , the second threshold and its confidence interval are

$$\begin{aligned}\hat{\gamma}_2^r &= \arg \min_{\gamma_2} \{S_2^r(\gamma_2)\} \\ S_2^r &= S \{ \min(\hat{\gamma}_1, \gamma_2) \max(\hat{\gamma}_1, \gamma_2) \} \\ \text{LR}_2^r(\gamma_2) &= \frac{\{S_2^r(\gamma_2) - S_2^r(\hat{\gamma}_2^r)\}}{\hat{\sigma}_{22}^2}\end{aligned}$$

Step 3:  $\hat{\gamma}_2^r$  is efficient but  $\hat{\gamma}_1^r$  is not. We reestimate the first threshold as

$$\begin{aligned}\hat{\gamma}_1^r &= \arg \min_{\gamma_1} \{S_1^r(\gamma_1)\} \\ S_1^r &= S \{ \min(\gamma_1, \hat{\gamma}_2) \max(\gamma_1, \hat{\gamma}_2) \} \\ \text{LR}_1^r(\gamma_1) &= \frac{\{S_1^r(\gamma_1) - S_1^r(\hat{\gamma}_1^r)\}}{\hat{\sigma}_{21}^2}\end{aligned}$$

The threshold-effect test is also sequential; that is, if we reject the null hypothesis in a single-threshold model, then we must test the double-threshold model. The null hypothesis is a single-threshold model, and the alternative hypothesis is a double-threshold model. The  $F$  statistic is constructed as

$$F_2 = \frac{\{S_1(\hat{\gamma}_1) - S_2^r(\hat{\gamma}_2^r)\}}{\hat{\sigma}_{22}^2} \quad (4)$$

The bootstrapping design for this is similar to that in the single-threshold model. In step 3, we generate a new series under the  $H_0$  DGP,  $y_{it}^* = \mathbf{X}_{it}^* \beta_S + v_{it}^*$ . The estimator  $\beta_S$  is the estimator in a single-threshold model, that is, under the  $H_a$  DGP. We use the predicted values.

For models with more than two threshold parameters, the process is similar. Chan (1993) and Hansen (1999) show that the dependence of the estimation and inference of  $\beta$  on the threshold estimate is not of first-order asymptotic importance, so the inference of  $\beta$  can proceed because  $\gamma$  is given.

## 3 The xthreg command

### 3.1 Syntax

```
xthreg depvar [indepvars] [if] [in], rx(varlist) qx(varname) [thnum(#)
    grid(#) trim(numlist) bs(numlist) thlevel(#) gen(newvarname) noreg
    nobslog thgiven options]
```

where *depvar* is the dependent variable and *indepvars* are the regime-independent variables.

### 3.2 Options

*rx*(*varlist*) is the regime-dependent variable. Time-series operators are allowed. *rx*() is required.

*qx*(*varname*) is the threshold variable. Time-series operators are allowed. *qx*() is required.

*thnum*(#) is the number of thresholds. In the current version (Stata 13), # must be equal to or less than 3. The default is *thnum*(1).

*grid*(#) is the number of grid points. *grid*() is used to avoid consuming too much time when computing large samples. The default is *grid*(300).

*trim*(*numlist*) is the trimming proportion to estimate each threshold. The number of trimming proportions must be equal to the number of thresholds specified in *thnum*(). The default is *trim*(0.01) for all thresholds. For example, to fit a triple-threshold model, you may set *trim*(0.01 0.01 0.05).

*bs*(*numlist*) is the number of bootstrap replications. If *bs*() is not set, *xthreg* does not use bootstrap for the threshold-effect test.

*thlevel*(#) specifies the confidence level, as a percentage, for confidence intervals of the threshold. The default is *thlevel*(95).

*gen*(*newvarname*) generates a new categorical variable with 0, 1, 2, ... for each regime. The default is *gen*(\_cat).

*noreg* suppresses the display of the regression result.

*nobslog* suppresses the iteration process of the bootstrap.

*thgiven* fits the model based on previous results.

*options* are any options available for *xtreg* (see [XT] *xtreg*).

Time-series operators are allowed in *depvar*, *indepvars*, *rx*(), and *qx*().

### 3.3 Stored results

`xthreg` uses `xtreg` (see [XT] `xtreg`) to fit the fixed-effect panel threshold model given the threshold estimator. Along with the standard stored results from `xtreg`, `xthreg` also stores the following results in `e()`:

#### Scalars

|                       |                              |
|-----------------------|------------------------------|
| <code>e(thnum)</code> | number of thresholds         |
| <code>e(grid)</code>  | number of grid search points |

#### Macros

|                        |                              |
|------------------------|------------------------------|
| <code>e(depvar)</code> | name of dependent variable   |
| <code>e(ix)</code>     | regime-independent variables |
| <code>e(rx)</code>     | regime-dependent variables   |
| <code>e(qx)</code>     | threshold variable           |

#### Matrices

|                        |  |
|------------------------|--|
| <code>e(Thrss)</code>  | threshold estimator and confidence interval                  |
| <code>e(Fstat)</code>  | threshold-effect test result                                 |
| <code>e(bs)</code>     | bootstrap number   |
| <code>e(trim)</code>   | trimming proportion  |
| <code>e(LR)</code>     | LR statistics for single-threshold model                     |
| <code>e(LR_2.1)</code> | LR statistics for first threshold in double-threshold model  |
| <code>e(LR_2.2)</code> | LR statistics for second threshold in double-threshold model |
| <code>e(LR3)</code>    | LR statistics for third threshold in triple-threshold model  |

The fixed-effect panel threshold model requires balanced panel data, which is checked automatically by `xthreg`. The estimation and test of the threshold effect are computed in Mata.

You can fit the model with more thresholds using the previous result. For example, assume you use `xthreg` to fit a single-threshold model by using `thnum(1)`, but the single threshold is not sufficient to capture the nonlinear effect. To now fit a double-threshold model, you can run the `xthreg` command using `thgiven`. Stata will search the second threshold using the previous result and will not fit the single-threshold model. I illustrate this in the example below.

## 4 Monte Carlo simulation

`xthreg` implements the method by Hansen (1999), in which the bootstrap method is used to test the null hypothesis of no threshold. Under the null, the distribution is continuous, so the bootstrap method can be applied. However, there is no formal justification for using the bootstrap test with a multiple-threshold model. Moreover, if bootstrap is used to create confidence intervals for the threshold model, then there may be a problem.

Enders, Falk, and Siklos (2007) compared the finite-sample performance of the following three methods in TAR: inverting the LR statistic using the asymptotic critical values, using the bootstrapped distribution of the LR to determine the critical values, and using the bootstrap percentile method. They found that none of the three methods performs satisfactorily for the discontinuous-TAR model. All three methods are too conservative, creating confidence intervals that are too wide. Of the methods, the bootstrapped LR method performs the worst.



However, new bootstrap methods for the TAR model have recently been introduced. Gonzalo and Wolf (2005) proposed to use a subsampling method—like that introduced by Politis, Romano, and Wolf (1999)—to improve the finite-sample performance of the threshold estimator in the self-exciting TAR model. Andrews and Guggenberger (2009) then introduced a hybrid subsampling method and size-corrected methods of constructing tests and confidence intervals that have correct asymptotic size. However, literature concerning the panel threshold model is still very limited, and there is still no theoretical development for overcoming wide confidence intervals. Following Hansen (1999), I invert the LR statistic to construct the confidence interval of the threshold estimator. Hansen (1999) did not perform Monte Carlo simulations to study the coverage rate for this method.

In this section, I perform simulations to study the size of the threshold-effect test and the coverage rate of the threshold estimator. For the real significance level of the threshold-effect test, the DGP for the null hypothesis (without the threshold effect) is simply a linear regression model, that is,

$$y_{it} = 1 + z_{it} + x_{it} + u_i + e_{it}$$

The variables are generated according to  $z_{it}, x_{it} \sim \chi^2(1)$ ,  $u_i \sim \chi^2(1) - 1$ , and  $e_{it} \sim N(0, 1)$ , where  $z$  is regime independent and  $x$  is regime dependent. The alternative hypothesis is that the coefficient of  $x$  is regime dependent on some threshold variable  $q$ . We set  $n = 50, 100, 200, 500$  and  $T = 5, 20, 50$ . The bootstrap iteration number is set to 300 for this single-threshold model. The iteration number of Monte Carlo simulation is set to 500.

The simulation is time consuming because bootstrap is used to test the threshold effect for each simulation. On a computer with Inter Core i7, 2.22 GHz processor, and 8 GB RAM, the simulation takes about 26 hours. Table 1 lists the result.

The test size distortion is small, even for small  $n$  and  $T$ . However, there is no convergence of the bias with increasing sample sizes. As Hansen (1997) noted, “the bootstrap procedure attains the first-order asymptotic distribution, so  $p$ -values constructed from the bootstrap are asymptotically valid. Because the asymptotic distribution is nonpivotal, bootstrap size will not have an accelerated rate of convergence relative to conventional asymptotic approximations.”

Table 1. Simulation result of the size of the threshold-effect test

| $n$ | $T$ | 1%    | 5%    | 10%   |
|-----|-----|-------|-------|-------|
| 50  | 5   | 0.010 | 0.064 | 0.118 |
| 100 | 5   | 0.024 | 0.062 | 0.114 |
| 200 | 5   | 0.022 | 0.058 | 0.094 |
| 500 | 5   | 0.008 | 0.048 | 0.106 |
| 50  | 20  | 0.012 | 0.050 | 0.108 |
| 100 | 20  | 0.016 | 0.046 | 0.090 |
| 200 | 20  | 0.008 | 0.040 | 0.098 |
| 500 | 20  | 0.014 | 0.058 | 0.122 |
| 50  | 50  | 0.014 | 0.058 | 0.108 |
| 100 | 50  | 0.012 | 0.044 | 0.114 |
| 200 | 50  | 0.010 | 0.048 | 0.102 |
| 500 | 50  | 0.014 | 0.056 | 0.122 |

The following DGP is used to simulate the coverage rate where  $q_{it} \sim \chi^2(1)$ :

$$y_{it} = 1 + z_{it} + x_{it}(q_{it} < 1) + 2x_{it}(q_{it} \geq 1) + u_i + e_{it}$$

The iteration number of Monte Carlo simulation is set to 500. The result is given in table 2.

Table 2. Simulation result of the coverage rate

| $n$ | $T$ | RMSE   | Rate  | $n$ | $T$ | RMSE    | Rate  | $n$ | $T$ | RMSE    | Rate  |
|-----|-----|--------|-------|-----|-----|---------|-------|-----|-----|---------|-------|
| 50  | 5   | 0.0103 | 0.460 | 50  | 20  | 0.00040 | 0.808 | 50  | 50  | 0.00010 | 0.952 |
| 100 | 5   | 0.0026 | 0.636 | 100 | 20  | 0.00005 | 0.920 | 100 | 50  | 0.00005 | 0.994 |
| 200 | 5   | 0.0005 | 0.768 | 200 | 20  | 0.00003 | 0.988 | 200 | 50  | 0.00003 | 1.000 |
| 500 | 5   | 0.0001 | 0.948 | 500 | 20  | 0.00001 | 0.998 | 500 | 50  | 0.00002 | 1.000 |

Obviously, the root of the mean squared error decreases with larger  $n$  or larger  $T$ . Figure 1 is the kernel density plot of the estimators for  $n = 50, 100, 200, 500$  and  $T = 5$ , which demonstrates the consistency of the estimator. Because of the consistency of the threshold estimator, the estimators of the regression coefficient given the threshold are consistent with the ordinary linear fixed-effect model. However, as  $n$  or  $T$  increases, the confidence interval gets wider and the coverage rate of the threshold estimator gets bigger than the nominal level. This means that the confidence interval gets too wide for large  $n$  and  $T$ , because the inverse LR method is not a very efficient method for constructing confidence intervals. As previously discussed, this problem is not specific to the panel threshold model; it is common to almost all threshold models. In sum, the inverse LR method does not perform that well for small sample sizes, and more research is needed to improve the coverage rate for the panel threshold model.

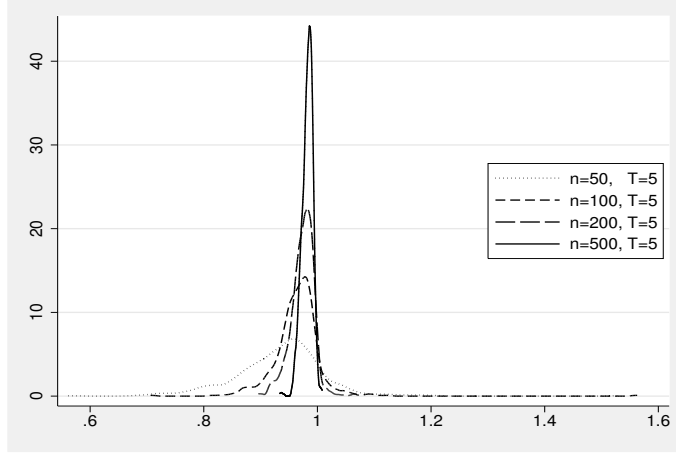


Figure 1. Kernel density of threshold estimator for different samples with real  $\gamma = 1$

## 5 Example

Financing constraints are widely researched. The basic idea is that a firm's cash flow will be positively related to its investment rate only when the firm faces constraints on external financing. If a firm is free to borrow on external financial markets, cash flow will be irrelevant for investment. The dividend-to-income ratio is often used as an indicator; that is, a financially constrained firm will choose to retain earnings rather than pay dividends. Hence, the firms that have low levels of dividend payments are the financially constrained firms.

Hansen (1999) applied the fixed-effect panel threshold model to a 15-year sample of 565 U.S. firms to test whether financial constraints affect investment decisions. Hansen fit the following model:

$$I_{it} = \beta_0 + \beta_1 q_{it-1} + \beta_2 q_{it-1}^2 + \beta_3 q_{it-1}^3 + \beta_4 d_{it-1} + \beta_5 q_{it-1} d_{it-1} + \beta_6 c_{it-1} I(d_{it-1} < \gamma_1) \\ + \beta_7 c_{it-1} I(\gamma_1 \leq d_{it-1} < \gamma_2) + \beta_8 c_{it-1} I(d_{it-1} \geq \gamma_2) + u_i + e_{it}$$

The variable  $I_{it}$  is the investment-to-capital ratio,  $q_{it}$  is the ratio of total market value to assets,  $c_{it}$  is the ratio of cash flow to assets, and  $d_{it}$  is the long-term debt-to-asset ratio.

First, we fit a single-threshold model. The threshold variable  $d1$  is trimmed off 5% at both sides to be searched for the threshold estimator. We use `grid(400)` to reduce the computation cost. The bootstrap number is set to `bs(300)`.

```
. use hansen1999, clear
(The Value and Performance of U.S. Corporations (B.H.Hall & R.E.Hall, 1993))
. xthreg i q1 q2 q3 d1 qd1, rx(c1) qx(d1) thnum(1) grid(400) trim(0.01) bs(300)
Estimating the threshold parameters: 1st ..... Done
Bootstrap for single threshold
..... + 50
..... + 100
..... + 150
..... + 200
..... + 250
..... + 300
```

Threshold estimator (level = 95):

| model | Threshold | Lower  | Upper  |
|-------|-----------|--------|--------|
| Th-1  | 0.0154    | 0.0141 | 0.0167 |

Threshold effect test (bootstrap = 300):

| Threshold | RSS     | MSE    | Fstat | Prob   | Crit10  | Crit5   | Crit1   |
|-----------|---------|--------|-------|--------|---------|---------|---------|
| Single    | 17.7818 | 0.0023 | 35.20 | 0.0033 | 11.9749 | 14.0259 | 22.8402 |

Fixed-effects (within) regression

Group variable: id

R-sq: within = 0.0951

between = 0.0692

overall = 0.0660

Number of obs = 7910

Number of groups = 565

Obs per group: min = 14

avg = 14.0

max = 14

F(7,7338) = 110.21

Prob > F = 0.0000

corr(u\_i, Xb) = -0.3972

| i         | Coef.     | Std. Err.                         | t     | P> t  | [95% Conf. Interval] |           |
|-----------|-----------|-----------------------------------|-------|-------|----------------------|-----------|
| q1        | .0105555  | .0008917                          | 11.84 | 0.000 | .0088075             | .0123035  |
| q2        | -.0202872 | .0025602                          | -7.92 | 0.000 | -.025306             | -.0152683 |
| q3        | .0010785  | .0001952                          | 5.53  | 0.000 | .0006959             | .0014612  |
| d1        | -.0229482 | .0042381                          | -5.41 | 0.000 | -.031256             | -.0146403 |
| qd1       | .0007392  | .0014278                          | 0.52  | 0.605 | -.0020597            | .0035381  |
| _cat#c.c1 |           |                                   |       |       |                      |           |
| 0         | .0552454  | .0053343                          | 10.36 | 0.000 | .0447885             | .0657022  |
| 1         | .0862498  | .0052022                          | 16.58 | 0.000 | .076052              | .0964476  |
| _cons     | .0628165  | .0016957                          | 37.05 | 0.000 | .0594925             | .0661405  |
| sigma_u   | .03980548 |                                   |       |       |                      |           |
| sigma_e   | .04922656 |                                   |       |       |                      |           |
| rho       | .39535508 | (fraction of variance due to u_i) |       |       |                      |           |

F test that all u\_i=0: F(564, 7338) = 6.90 Prob > F = 0.0000

The output consists of four parts. The first part outputs the estimation and bootstrap results. The second part outputs the threshold estimators and their confidence intervals. **Th-1** denotes the estimator in single-threshold models. In the **threshold estimator** table, **Th-21** and **Th-22** denote the two estimators in a double-threshold model. Sometimes, **Th-21** is the same as **Th-1**. The third part lists the threshold-effect test, including the RSS, the mean squared error (MSE), the  $F$  statistic (**Fstat**), the probability value of the  $F$  statistic (**Prob**), and critical values at 10%, 5%, and 1% significance levels (**Crit10**, **Crit5**, and **Crit1**, respectively). The fourth part outputs the fixed-effect regression.

In this example, the single-threshold model's estimator is 0.0154 with 95% confidence interval [0.0141, 0.0167]. The  $F$  statistic is highly significant. Therefore, we reject the linear model and fit a double- or triple-threshold model.

Next, we directly fit a triple-threshold model based on the result above. The trimming values are set to be 0.01 and 0.05 for the estimation of the second and third thresholds. Note that the trimming proportion (0.01) for the single-threshold model still needs to be set because **xthreg** searches the second threshold using the trimmed series in the single-threshold model. We set the bootstrap number to 300 for the double- and triple-threshold models; however, we set this to 0 for the single-threshold model because there is no need to use bootstrap for it again (the full option is **bs(0 300 300)**). We suppress the output of bootstrap replications and the fixed-effect regression.

```
. xthreg i q1 q2 q3 d1 qd1, rx(c1) qx(d1) thnum(3) grid(400) trim(0.01 0.01 0.05
> ) bs(0 300 300) thgiven nob slog noreg
Estimating the threshold parameters: 2nd ..... 3rd ..... Done
Bootstrapping for threshold effect test: 2nd ..... 3rd ..... Done
Threshold estimator (level = 95):
```

| model | Threshold | Lower  | Upper  |
|-------|-----------|--------|--------|
| Th-1  | 0.0154    | 0.0141 | 0.0167 |
| Th-21 | 0.0154    | 0.0141 | 0.0167 |
| Th-22 | 0.5418    | 0.5268 | 0.5473 |
| Th-3  | 0.4778    | 0.4755 | 0.4823 |

```
Threshold effect test (bootstrap = 0 300 300):
```

| Threshold | RSS     | MSE    | Fstat | Prob   | Crit10  | Crit5   | Crit1   |
|-----------|---------|--------|-------|--------|---------|---------|---------|
| Single    | 17.7818 | 0.0023 | 35.20 | 0.0033 | 11.9749 | 14.0259 | 22.8402 |
| Double    | 17.7258 | 0.0022 | 24.97 | 0.0133 | 10.8901 | 12.8409 | 27.6450 |
| Triple    | 17.7119 | 0.0022 | 6.20  | 0.5933 | 16.8006 | 21.8740 | 31.6187 |

Note that **xthreg** gets a similar but not identical  $F$  statistic because of the randomness of bootstrap sampling. Of course, this does not affect the conclusion. In the threshold-effect test table, **Single** corresponds to  $H_0$  (linear model) and  $H_a$  (single-threshold model), **Double** corresponds to  $H_0$  (single-threshold model) and  $H_a$  (double-threshold model), and so forth. Obviously, the double-threshold model is accepted with probability value 0.59.

We can view the threshold confidence interval by plotting the LR statistic.

```
. _matplot e(LR21), columns(1 2) yline(7.35, lpattern(dash))
>      connect(direct) msize(small) mlabp(0) mlabs(zero)
>      ylabel("LR Statistics") xtitle("First Threshold") recast(line) name(L
> LR21) nodraw
. _matplot e(LR22), columns(1 2) yline(7.35, lpattern(dash))
>      connect(direct) msize(small) mlabp(0) mlabs(zero)
>      ylabel("LR Statistics") xtitle("Second Threshold") recast(line) name(
> LR22) nodraw
. graph combine LR21 LR22, cols(1)
```

In figure 2, the dashed line denotes the critical value (7.35) at the 95% confidence level.

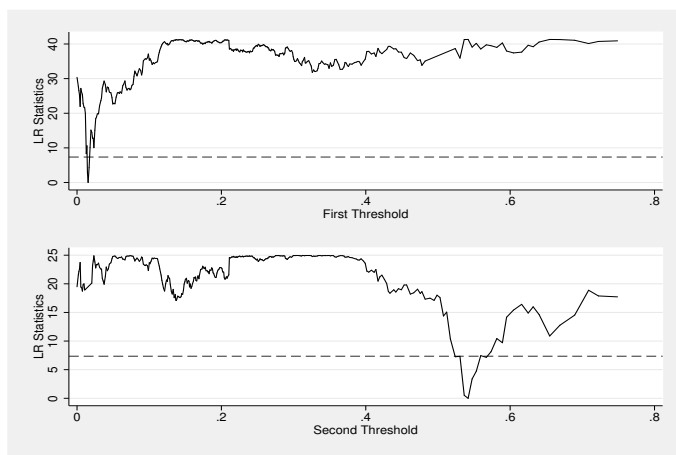


Figure 2. LR statistic of two thresholds

We can also directly fit a triple-threshold model by using the following command:

```
. xthreg i q1 q2 q3 d1 qd1, rx(c1) qx(d1) thnum(3) grid(400)
> trim(0.01 0.01 0.05) bs(300 300 300) nob slog noreg
(output omitted)
```

## 6 Conclusion

The threshold model is a valuable tool for studying many economic phenomena, and the panel threshold model has been widely used in financial and macroeconomic fields. In this article, I introduced the new **xthreg** command, which fits the fixed-effect panel threshold model—a threshold model that could also be a useful tool in financial and economic research. I performed Monte Carlo simulations to exemplify the effectiveness of using the bootstrap method in the threshold model, as originally suggested by Hansen (1999). When using these methods, the significance test level of the threshold effect is very near to the nominal significance test level. I also showed that the threshold

estimator is consistent but the confidence interval gets wider and the coverage rate of the threshold estimator gets bigger than the nominal level as  $n$  or  $T$  increases.

## 7 Acknowledgments

I thank H. Joseph Newton (the editor) for providing advice and encouragement. Professional and valuable comments from Zongwu Cai, Bruce E. Hansen, Ted Juhl, and an anonymous referee substantially helped the revision of this article. The article is financed by the Natural Science Foundation of China (Grant Number 71101075).

## 8 References

- Andrews, D. W. K., and P. Guggenberger. 2009. Hybrid and size-corrected subsampling methods. *Econometrica* 77: 721–762.
- Bai, J. 1997. Estimating multiple breaks one at a time. *Econometric Theory* 13: 315–352.
- Bai, J., and P. Perron. 1998. Estimating and testing linear models with multiple structural changes. *Econometrica* 66: 47–78.
- Chan, K. S. 1993. Consistency and limiting distribution of the least squares estimator of a threshold autoregressive model. *Annals of Statistics* 21: 520–533.
- Enders, W., B. L. Falk, and P. Siklos. 2007. A threshold model of real U.S. GDP and the problem of constructing confidence intervals in TAR models. *Studies in Nonlinear Dynamics and Econometrics* 11: 1–28.
- Girma, S. 2005. Absorptive capacity and productivity spillovers from FDI: A threshold regression analysis export. *Oxford Bulletin of Economics and Statistics* 67: 281–306.
- Gonzalo, J., and M. Wolf. 2005. Subsampling inference in threshold autoregressive models. *Journal of Econometrics* 127: 201–224.
- Hansen, B. E. 1996. Inference when a nuisance parameter is not identified under the null hypothesis. *Econometrica* 64: 413–430.
- . 1997. Inference in TAR models. *Studies in Nonlinear Dynamics and Econometrics* 2: 1–16.
- . 1999. Threshold effects in non-dynamic panels: Estimation, testing, and inference. *Journal of Econometrics* 93: 345–368.
- Hsiao, C. 2003. *Analysis of Panel Data*. 2nd ed. Cambridge: Cambridge University Press.
- Politis, D. N., J. P. Romano, and M. Wolf. 1999. *Subsampling*. New York: Springer.

Rousseau, P. L., and P. Wachtel. 2009. What is happening to the impact of financial deepening on economic growth? Vanderbilt University Department of Economics Working Papers No. 0915, Vanderbilt University Department of Economics. <http://ideas.repec.org/p/van/wpaper/0915.html>.

Tong, H. 1983. Threshold models in non-linear time series analysis. In *Lecture Notes in Statistics*, 21, ed. D. Brillinger, S. Fienberg, J. Gani, J. Hartigan, and K. Krickeberg. Berlin: Springer.

#### **About the author**

Qunying Wang received his PhD from Nankai University. He works at the Institute of Statistics and Econometrics at Nankai University. He is currently a visiting professor at the University of Kansas under the financial support of the China Scholarship Council.