1 Multiproduct UPP

Let $\Delta q\,(p)$ be the demand jacobian for all products of all firms, D_{11} be the submatrix of $\Delta q\,(p)$ of elements where i and j are products of firm 1 and D_{12} be the submatrix of terms where i is a product of firm 1 and j is a product of firm 2. (As all derivatives will be at pre-merger prices, the dependence of these submatrices on p is surpressed.) The FOC for the set of products of firm 1 can be expressed as

$$D_{11}(p_1 - c_1) + q_1 = 0 \tag{1}$$

As above, subscripts indicate sub-vectors of the products that belong to firm 1 prior to the merger. If firm 1 buys firm 2, these equations are modified as follows

$$D_{11}(p_1 - (1 - e_1)c_1) + D_{12}(p_2 - c_2) + q_1 \tag{2}$$

At pre-merger prices this expression will not be zero. Subtracting, we can define the vector:

$$UPP_{12} = -e_1 c_1 - D_{11}^{-1} D_{12}(p_2 - c_2)$$

The invertability of D_{11} is guaranteed by the existence of a unique pre-merger equilibrium. The $j \times k$ matrix

$$Div_{12} = -D_{11}^{-1} D_{12}$$

are the diversion ratios for each product j of firm 1 and k of firm 2.

We can, along the lines of Schmalensee also define

$$UPP_{12}^{*} = -e_1 c_1 - D_{11}^{-1} D_{12}(p_2 - (1 - e_2)\,c_2)$$

1.1 Interpretation

Let be the optimal price conditional on p_2 and other firms

$$D_{11}(p_1 + \Delta p_1 - (1 - e_1)c_1) + D_{12}(p_2 - c_2) + q_1 = 0$$

Substituting q_1 from (1), we get

$$D_{11}(\Delta p_1 + e_1 c_1) + D_{12}(p_2 - c_2) = 0$$

Thus

$$UPP_{12} = \Delta p_1$$

UPP_{12} is thus the optimal price change for the products of firm 1, given that prices of other products including the products formerly owned by firm 2 are unchanged. The measure is the absolute price change rather than the relative change measured in merger simulations, as the focus is on the direction of change rather than the magnitude.

1.2 UPP and first iteration of fixed point

The fixed point algorithm solves the following equation for prices post-merger:

$$\begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} \begin{pmatrix} p_1 - c_1 \\ p_2 - c_2 \end{pmatrix} + \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = 0$$

The FOC for other products are solved seperately as ownership creates a block diagonal matrix D.

Block inverting the matrix D, the prices of the pre-merger products in the first iteration of the fixed point algorithm is:

$$p_1 = (1-e_1)c_1 - \left(D_{11} - D_{12}D_{22}^{-1}D_{21}\right)^{-1} q_1 - D_{11}^{-1}D_{12}\left(-D_{22} + D_{21}D_{11}^{-1}D_{12}\right)^{-1} q_2$$

The difference between UPP and the first iteration for firm 1 is that for the new products, UPP takes the prices as given by the pre-merger level, whereas the first stage fixed point calculates these as being optimal given the prices of other firms.

The difference between these expressions are given by the second terms within the two parenthesises with inverses ($D_{12}D_{22}^{-1}D_{21}$ and $D_{21}D_{11}^{-1}D_{12}$). This can be seen as follows. Setting these to zero and using the fact that the pre-merger FOC for products 2 can be written as

$$D_{22}\left(p_2 - c_2\right) = -q_2$$

we get the UPP equation above.

1.3 Average UPP

Assuming that firm 1 has the same marginal costs for all products $c_1 = \bar{c}_1 i_1$, and firm 2 has the same markup $\bar{m}_2 i_2 = p_2 - c_2$ on all products, wherei_k is a column vector of ones corresponding to the number of products of firm k, we can define the vector:

$$-\frac{(i_1' D_{11} i_1)}{J_1}\bar{c}_1 e_1 - \frac{(i_1' D_{12} i_2)}{J_1}\bar{m}_2$$

where J_1 is the number of products of firm 1. Under these assumptions we can get an average UPP that depends on a scalar diversion ratio.

$$AUPP_{12} = -\bar{c}_1 e_1 - \frac{(i_1' D_{12} i_2)}{(i_1' D_{11} i_1)}\bar{m}_2 = -\bar{c}_1 e_1 - \frac{(i_1' D_{12} i_2)}{(i_1' D_{11} i_1)}\bar{m}_2$$

We can then define the diversion ratio as

$$Div_{12} = -\frac{i_1' D_{12} i_2}{i_1' D_{11} i_1}$$

Both UPP_{12} and $AUPP_{12}$ correspond to the single market UPP_{12} when both firms 1 and 2 are single product firms. Note that

$$Div_{12} = -\frac{i_1' \Delta q\left(p\right) i_2}{i_1' \Delta q\left(p\right) i_1}$$

if vectors i_k are redefined as containing 1 if product j belongs to k, otherwise 0.

1.4 Painkiller UPP

Testing these formulas on the two level nested logit specification and 25% efficiencies, we get $UPP_{12} = (-0.0588, -0.0328)'$ for GSK with average value -0.0458. The corresponding $AUPP_{12} = -0.0427$. For AZ, we have $UPP_{21} = (-0.0964, -0.0678, -0.0911, -0.0608)'$ with an average of -0.0790 and $AUPP_{21} = -0.0651$.