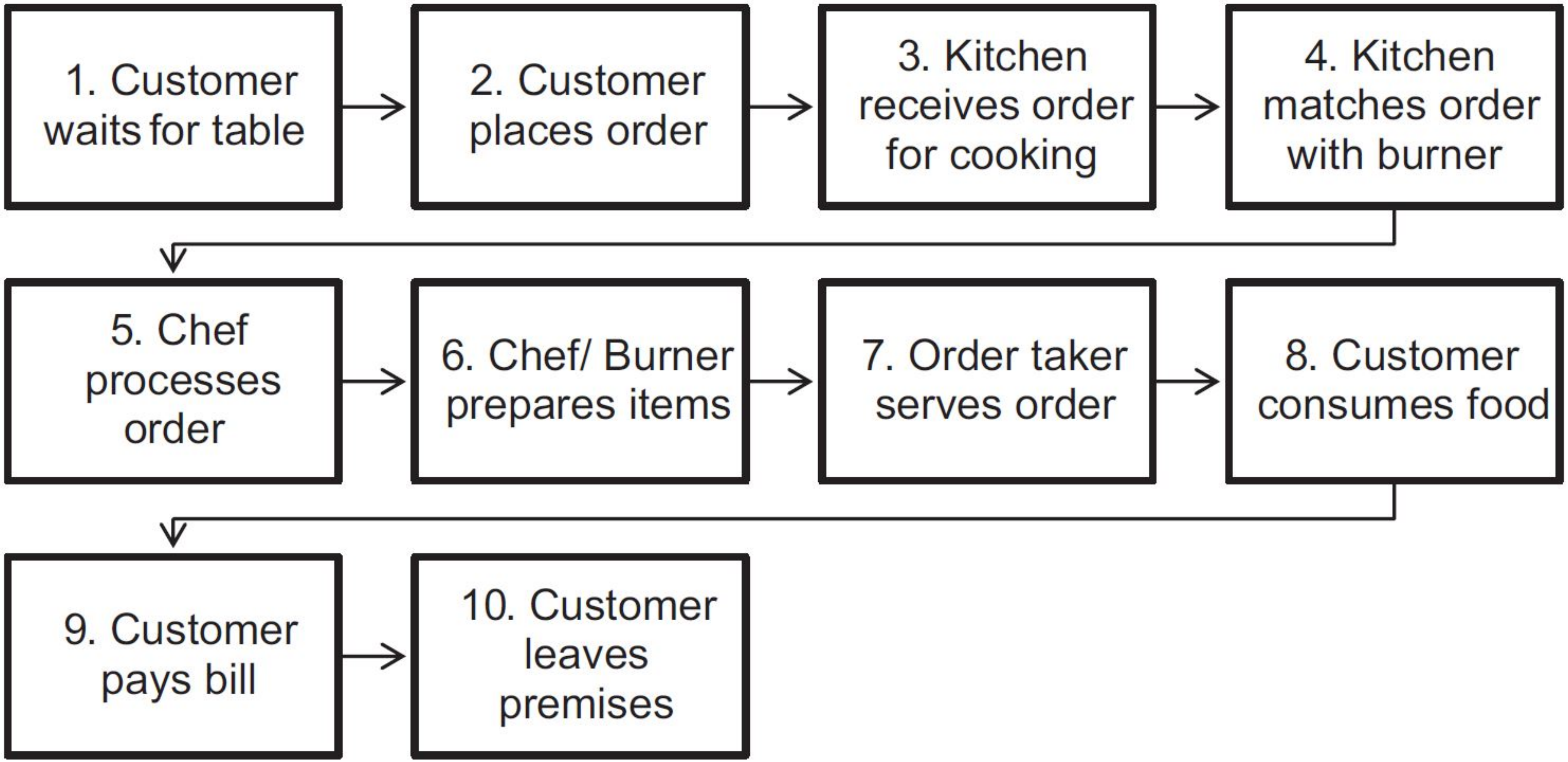
A NESTED SEMI-OPEN QUEUING NETWORK MODEL FOR ANALYZING DINE-IN RESTAURANT PERFORMANCE

Arman Jabbari | IEOR-267

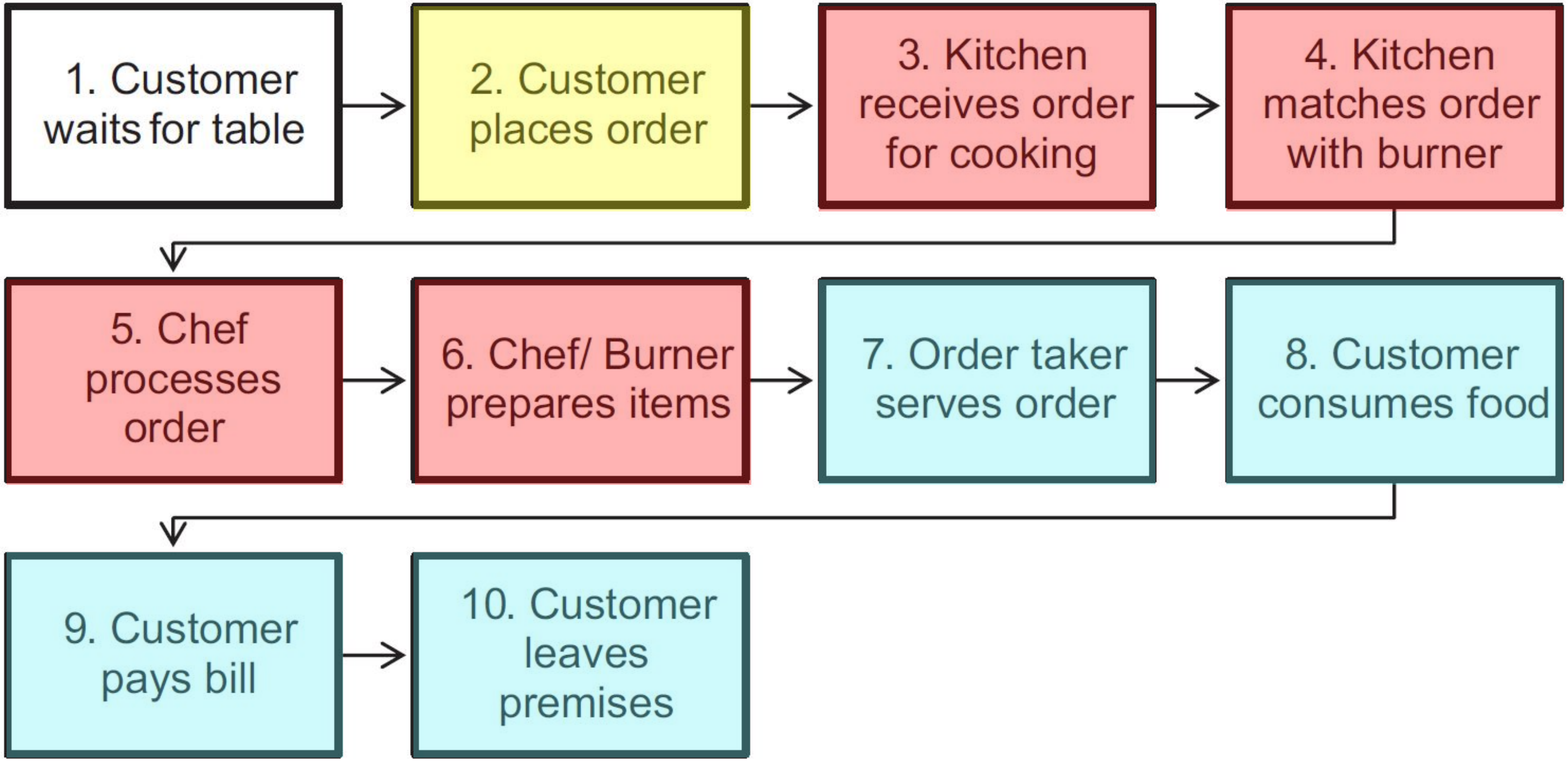
The average waiting time of 92.7% of restaurants is at least **23 min**.

The average waiting time of 8% of restaurants is more than **40 min**.


1. Customer waits for table
2. Customer places order
3. Kitchen receives order for cooking
4. Kitchen matches order with burner
5. Chef processes order
6. Chef/ Burner prepares items
7. Order taker serves order
8. Customer consumes food
9. Customer pays bill
10. Customer leaves premises

Assumptions

- Waiting room has infinite capacity
- Customer arrivals have Poisson distribution
- Tables are homogeneous
- Customer do not have table preference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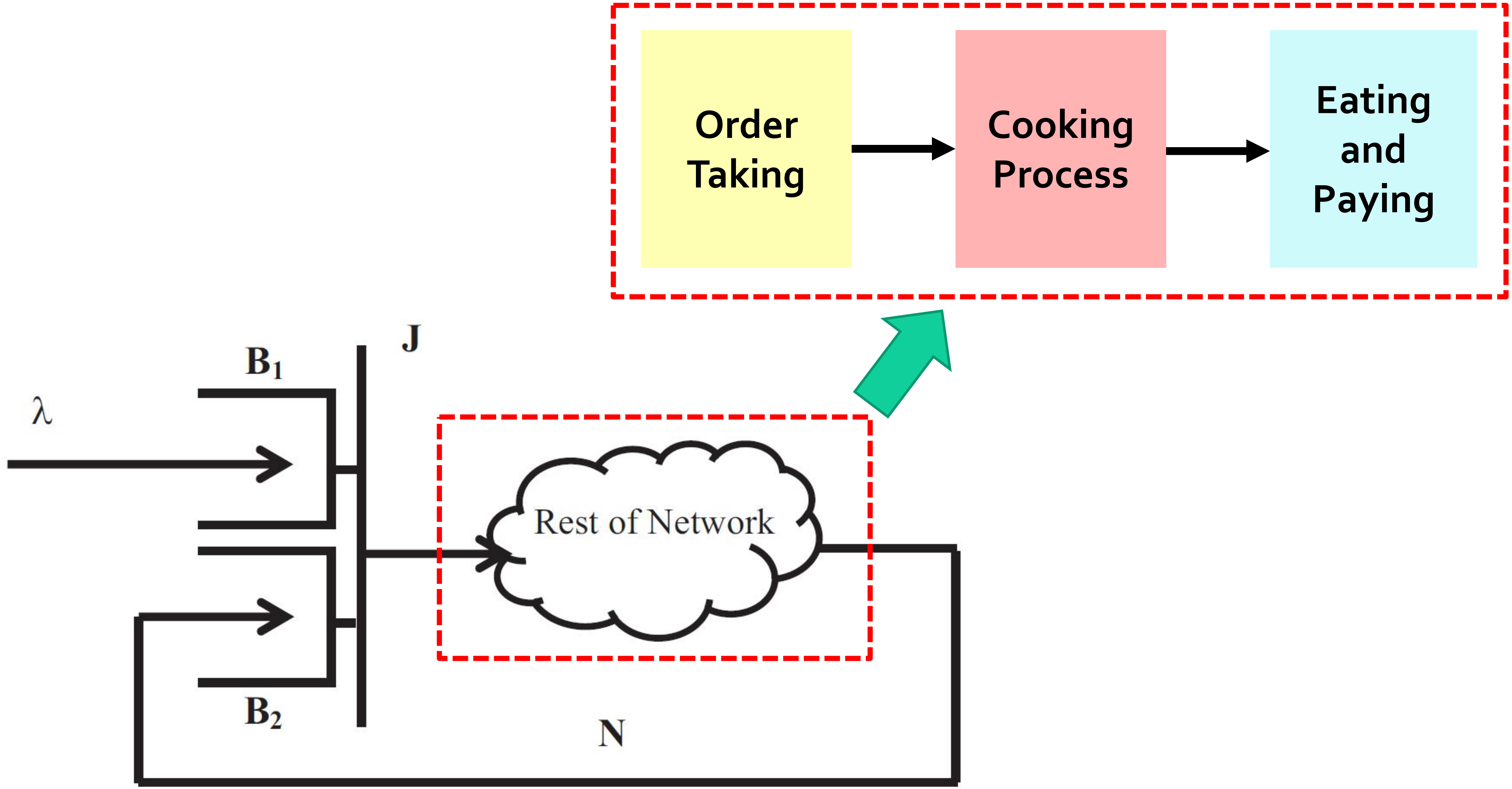
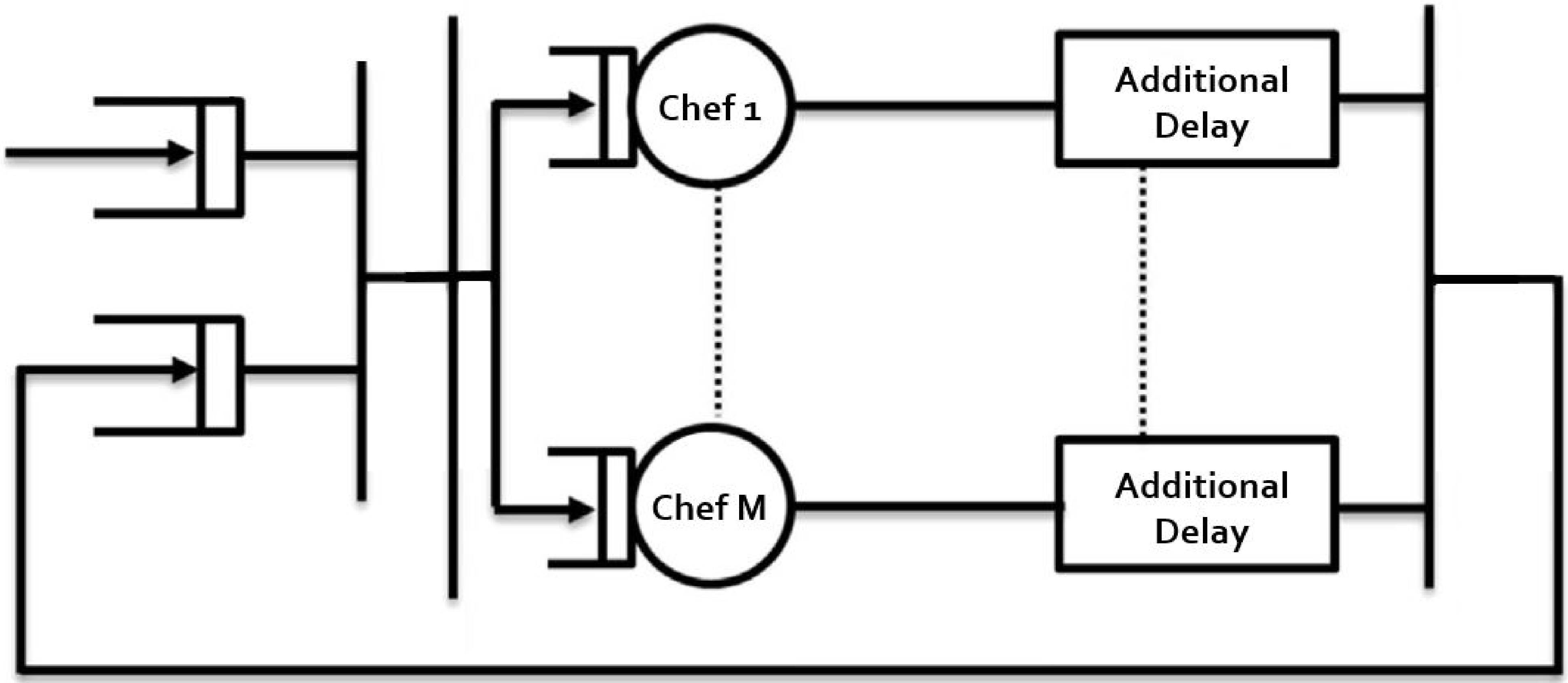
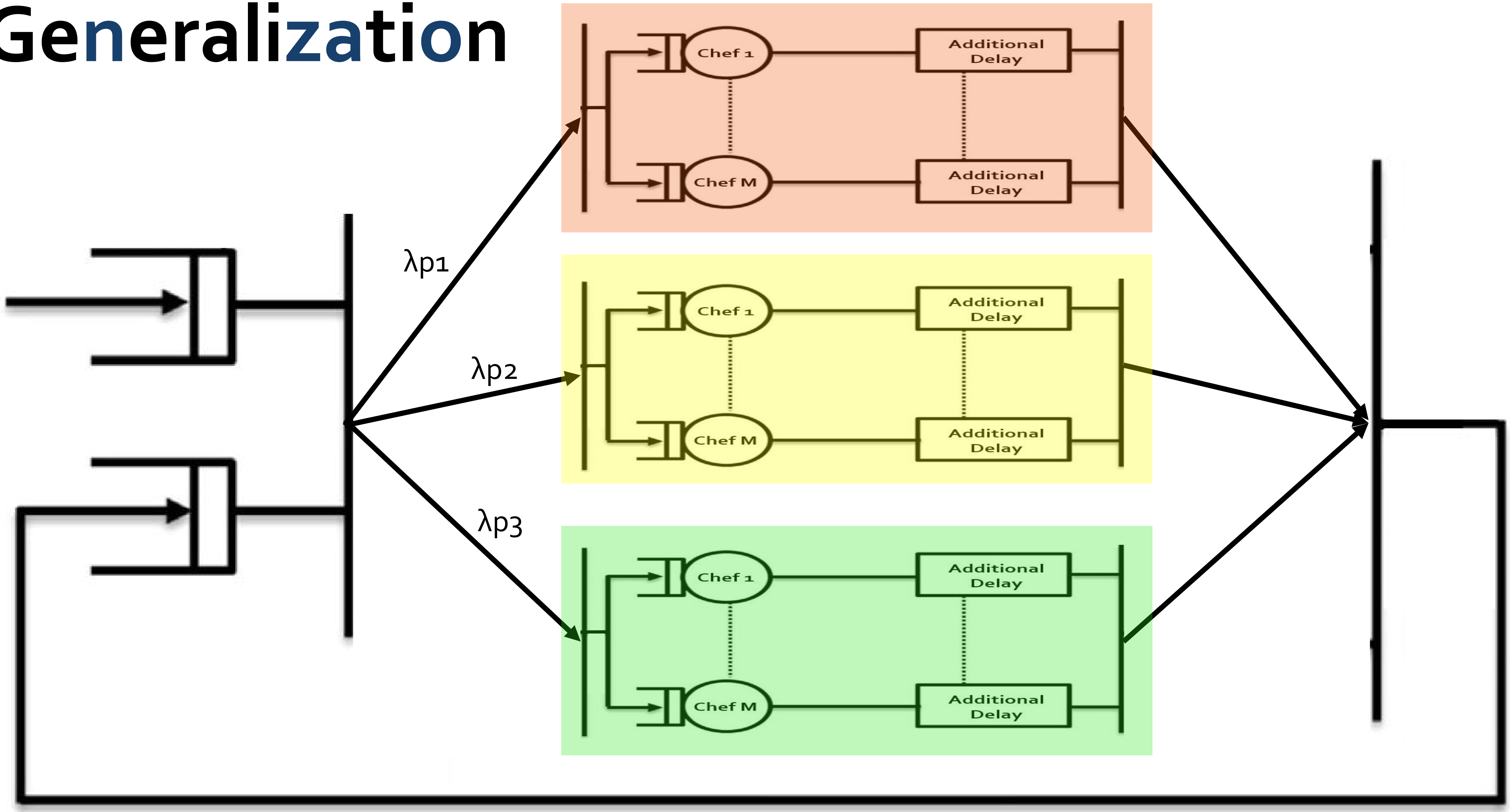

Order Taking
Cooking Process
Eating and Paying
J
B_1
λ
Rest of Network
B_2
N

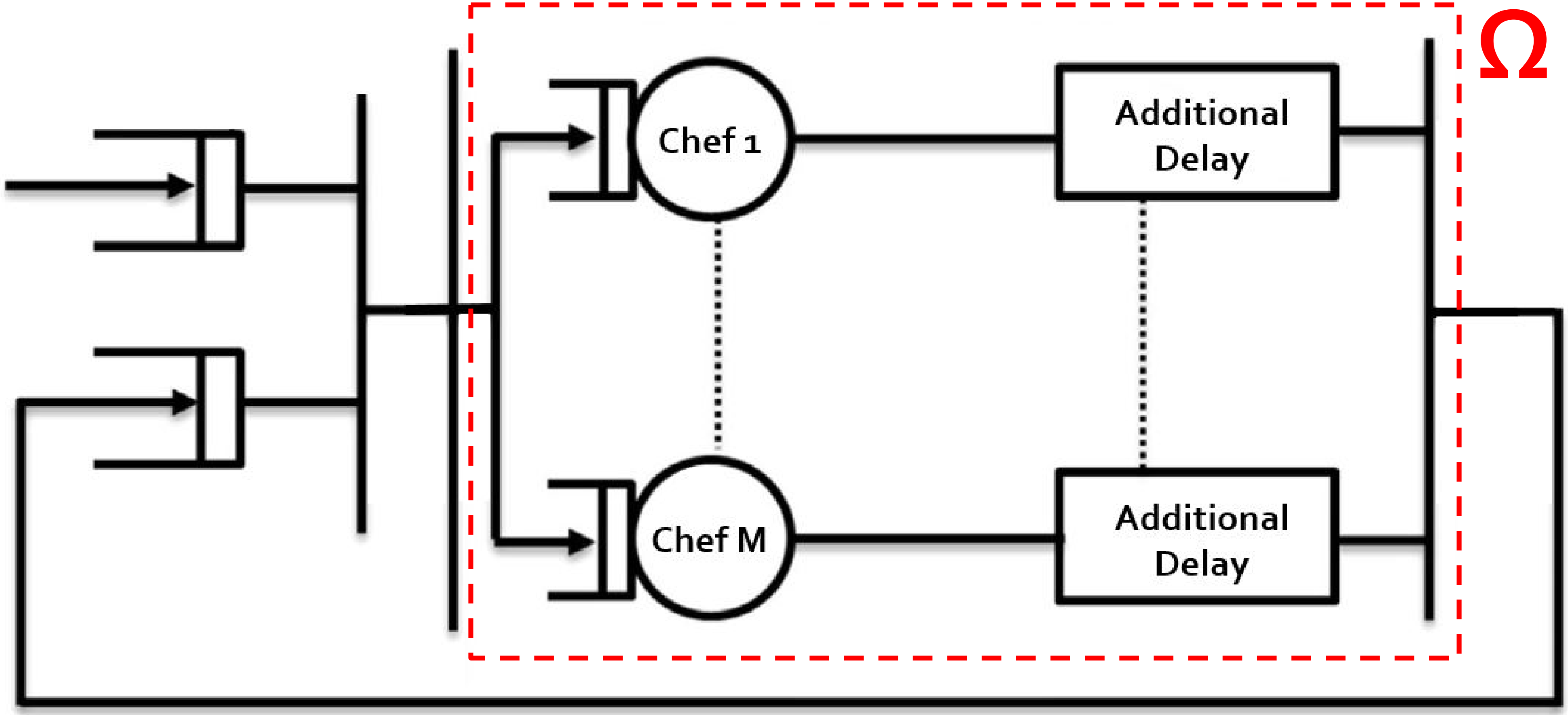
Kitchen



Generalization



Solution to the inner-level kitchen queuing network



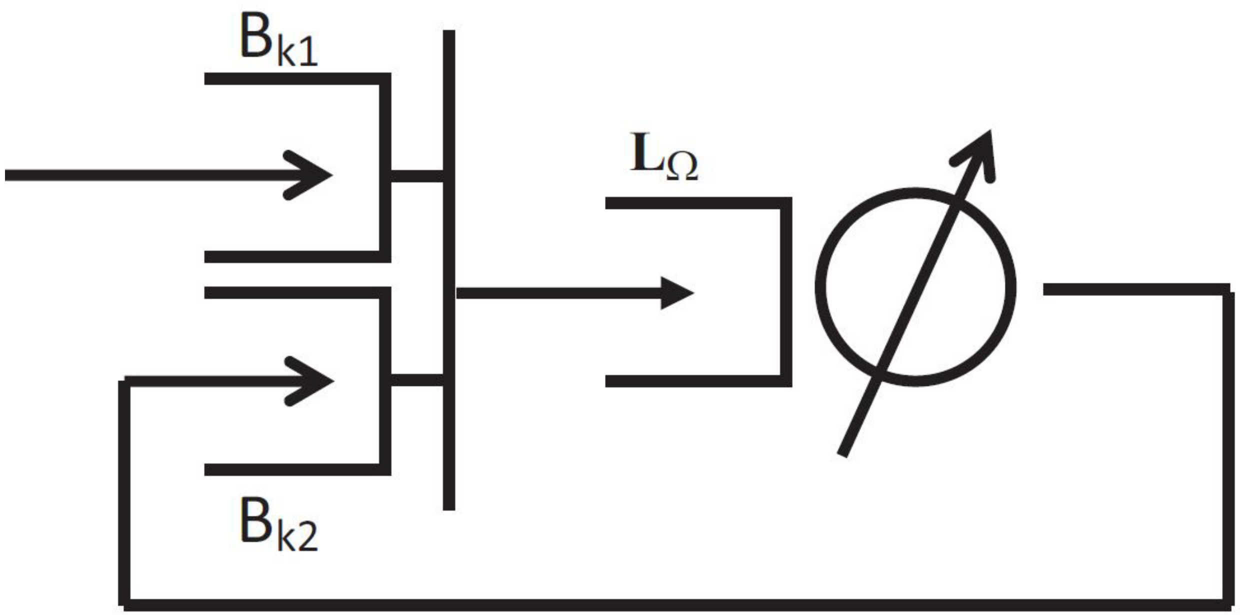
- Step 1: Solve Ω for $n = 1..N$
- Step 2: Develop a CTMC for kitchen
- Step 3: Solve it ☺

Step 1: Solve Ω for n = 1..N

1. Using an approximate mean value analysis algorithm based on:

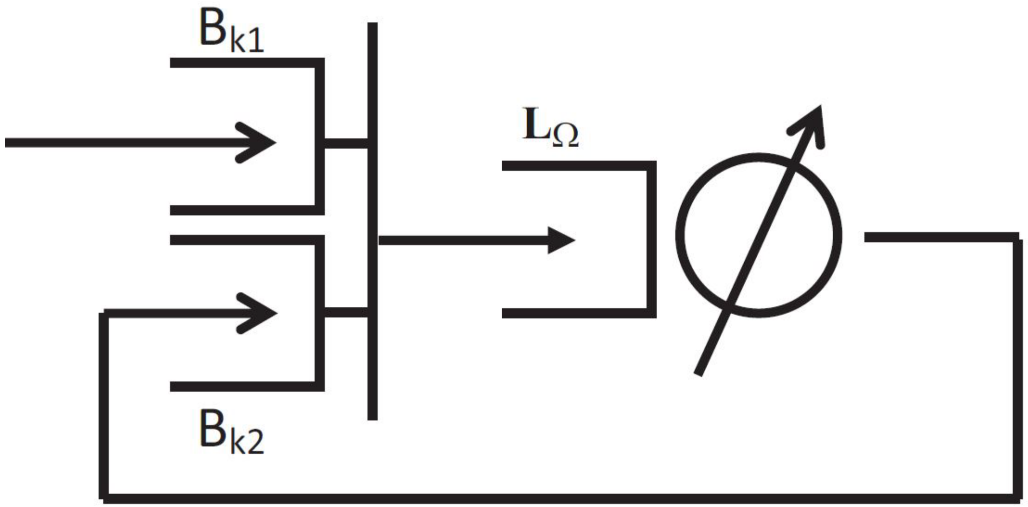
Buitenhek R, van Houtum Geert-Jan, Zijm H. AMVA-based solution procedures for open queueing networks with population constraints. Ann Oper Res 2000;93(1–4):15–40.

2. Replace the sub-network with a load-dependent server



Step 2: Develop a CTMC for kitchen

States: the number of customer orders waiting to be processed in Buffer Bk1 minus the number of ovens idle in Buffer Bk2



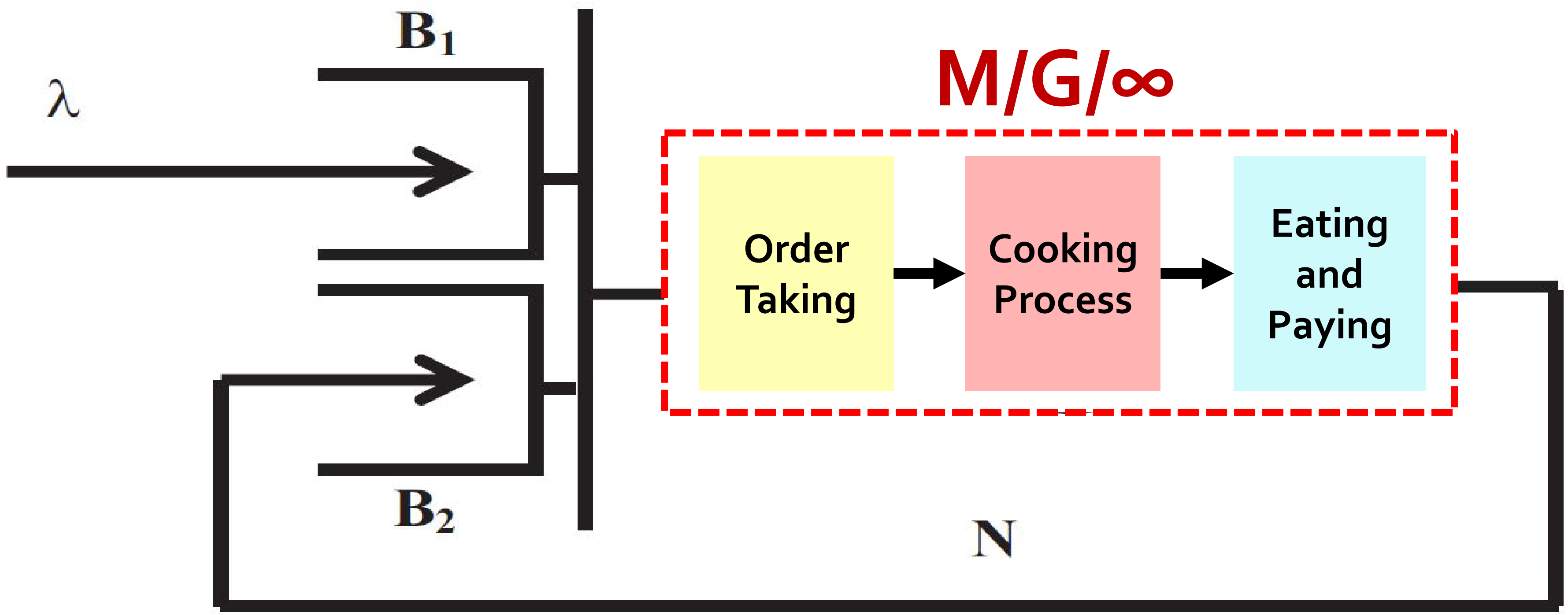
Step 3: Solve it ☺

It is easily solvable because it is reversible.

$$\pi_i = \frac{\lambda^{\mathrm{i+N}}}{X(1)X(2)\ldots X(N\mathrm{+i})} \qquad \text{for } i = \text{- N to } 0$$

$$\pi_i = \frac{\lambda^N}{X(1)X(2)\ldots X(N)}\left(\frac{\lambda}{X(N)}\right)^i \pi_{-N} \qquad \text{for } \quad i = 0 \text{ to } \infty$$

Convolution distribution



The outer IS-SOQN is equivalence to M/G/c

Based on: Roy D, De Koster R. Modeling and design of container terminal operations, ERIM report series reference no. ERS-2014-008-LIS; 2014

For finding L and Sojourn time, M/G/c is approximated using M/M/c as follow:

$$L^{\mathrm{app}} = \left[(1 - U_{\mathrm{T}})\gamma_1 C\mu + \frac{1}{2} U_{\mathrm{T}} (1 + c_{\mathrm{S}}^2) \right] L_q \text{ (exp)}$$

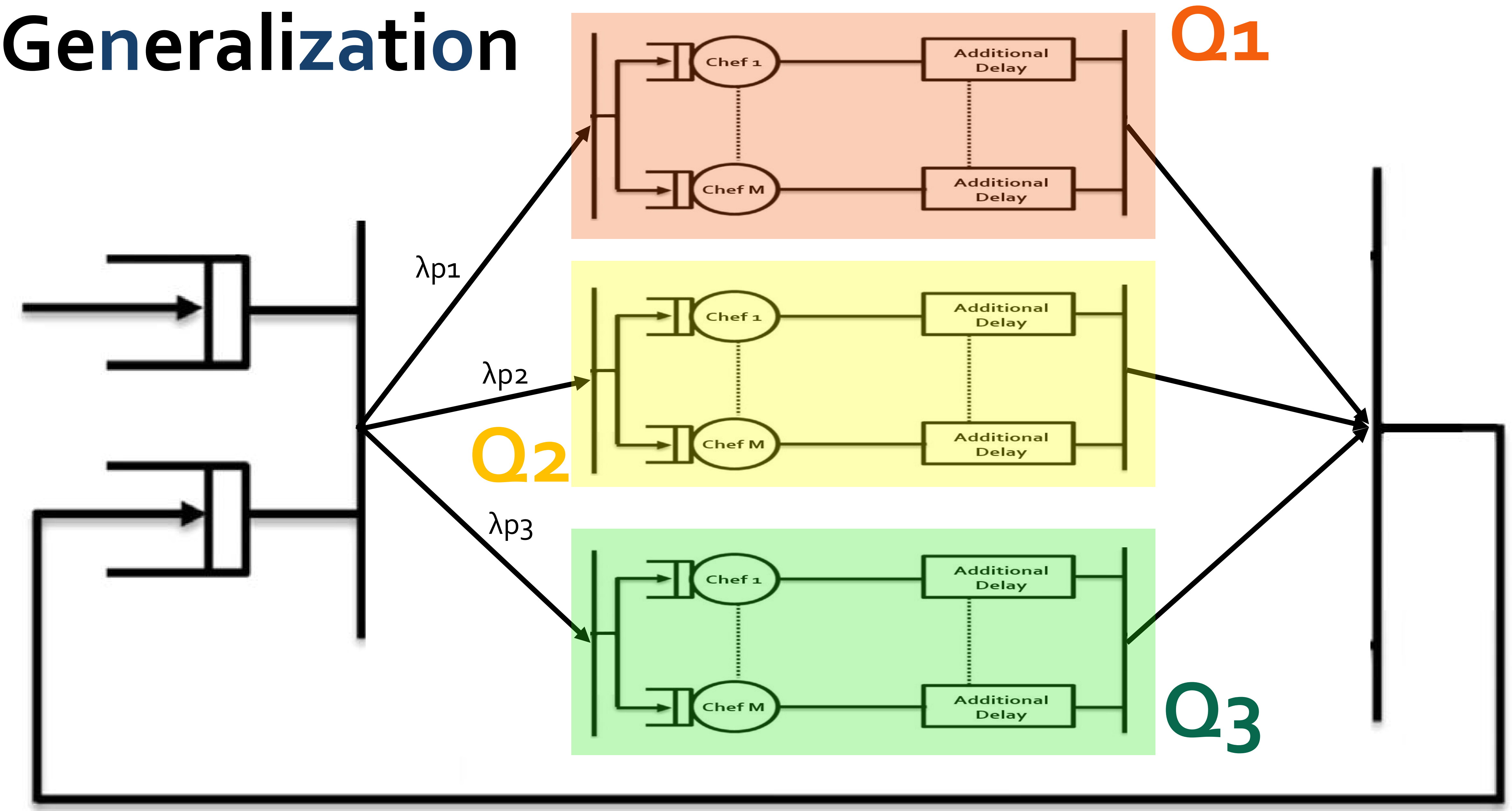
$$E[T_{\mathrm{C}}] = \frac{L^{\mathrm{app}}}{\lambda} + \mu^{-1}$$

Where :

c_{S}^2 : The square of coefficient variation of service

$U_{\mathrm{T}} = \lambda / C\mu$ $\qquad$ $\gamma_1 = (c_{\mathrm{S}}^2 \mu^{-1}/C - 1) + (1 - c_{\mathrm{S}}^2 / C\mu)$

Generalization



Why should we do this?

1. This model is appropriate for group arrival with different type of order
2. This approximation is more realistic
3. The previous model did not consider the variation of service times
4. In the restaurants, resources of different dishes are not the same

Convolution

$$P(T^{Q1} < t) = \int_0^t \left(1 - \frac{\lambda^{N_{Q1}+1}}{\left(X(1)X(2)\ldots X(N_{Q1}) + \lambda X(2)\ldots X(N_{Q1}) + \cdots + \frac{\lambda^{N_{Q1}} X(N_{Q1})}{X(N_{Q1}) - \lambda} \right) \left(X(N_{Q1}) - \lambda p_1 \right)} \right) e^{-(X(N_{Q1})-\lambda)(t-s)} dG^1(s)$$

Order	Ratio of that order to all order
(1,0,0)	O(1) = 0.2
(1,0,1)	O(2) = 0.5
(1,1,2)	O(3) = 0.2
(0,3,1)	O(4) = 0.1

Here:

$$p_1 = O(1) + O(2) + O(3) = 0.9$$
$$p_2 = O(3) + O(4) = 0.3$$
$$p_3 = O(2) + O(3) + O(4) = 0.8$$

Convolution

Order	Ratio of that order to all order	Distribution of sojourn time for that order
(1,0,0)	0.2	$P(T^{Q1} < t)$
(1,0,1)	0.5	$P(T^{Q1} < t) \times P(T^{Q3} < t)$
(1,1,2)	0.2	$P(T^{Q1} < t) \times P(T^{Q2} < t) \times P(T^{Q3} < t)$
(0,3,1)	0.1	$P(T^{Q2} < t) \times P(T^{Q3} < t)$

So for this example:

$$Y(t) = P(T < t)$$
$$= 0.2 \times P(T^{Q1} < t) + 0.5 \times P(T^{Q1} < t) \times P(T^{Q3} < t) + 0.2 \times P(T^{Q1} < t)$$
$$\times P(T^{Q2} < t) \times P(T^{Q3} < t) + 0.1 \times P(T^{Q2} < t) \times P(T^{Q3} < t)$$

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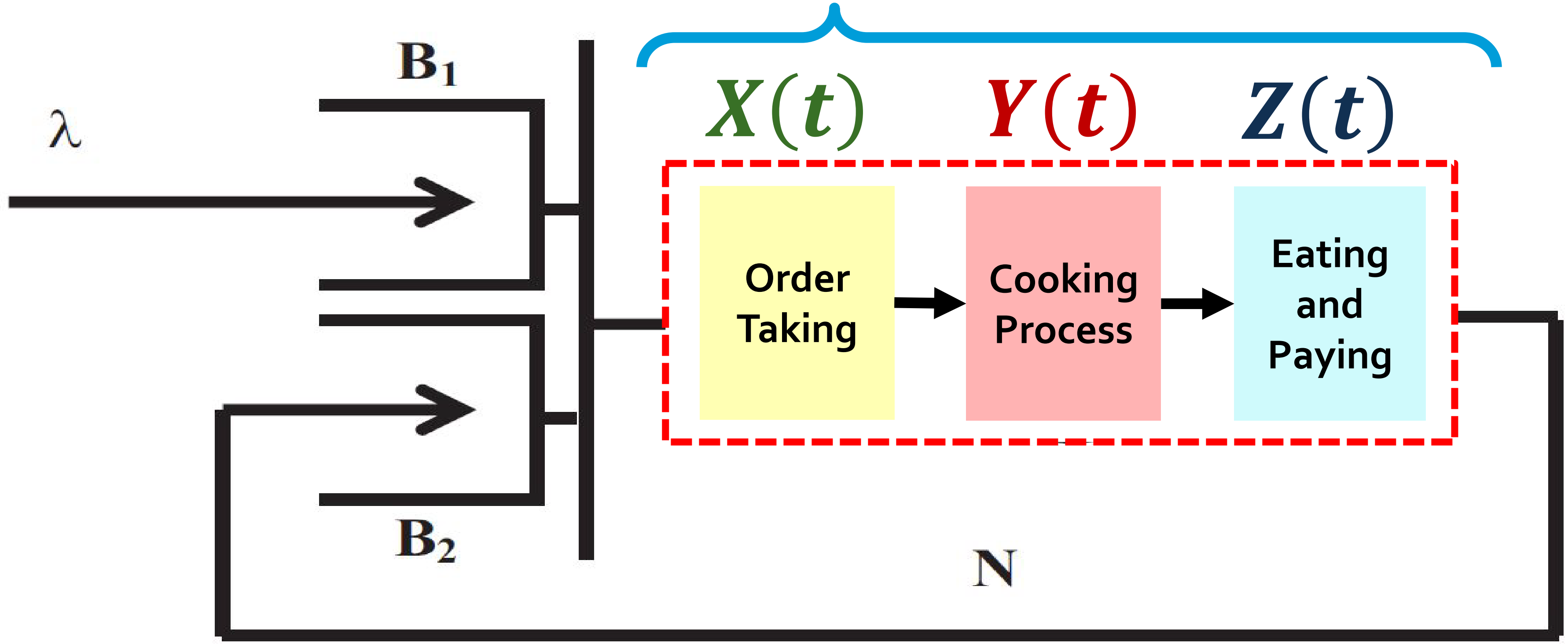
So for this example:

$$
\begin{aligned}
&Y(t) = P(T < t) \\
&= 0.2 \times P(T^{Q1} < t) + 0.5 \times P(T^{Q1} < t) \times P(T^{Q3} < t) + 0.2 \times P(T^{Q1} < t) \\
&\times P(T^{Q2} < t) \times P(T^{Q3} < t) + 0.1 \times P(T^{Q2} < t) \times P(T^{Q3} < t)
\end{aligned}
$$

Convolution distribution of M/G/ ∞

$$\text{M/G/}\infty \longrightarrow G(t) = X(t) * Y(t) * Z(t)$$



References

- Roy, D., Bandyopadhyay, A., & Banerjee, P. (2016). A nested semi-open queuing network model for analyzing dine-in restaurant performance.*Computers & Operations Research*, *65*, 29-41. **(Main Reference)**
- Buitenhek, R., van Houtum, G. J., & Zijm, H. (2000). AMVA-based solution procedures for open queueing networks with population constraints. *Annals of Operations Research*, *93*(1-4), 15-40.
- Roy, D., & De Koster, M. B. M. (2014). Modeling and design of container terminal operations. ERIM Report Series Reference No. ERS-2014-008-LIS.