IMPEDÂNCIA DE ENTRADA DE ANTENAS LINEARES

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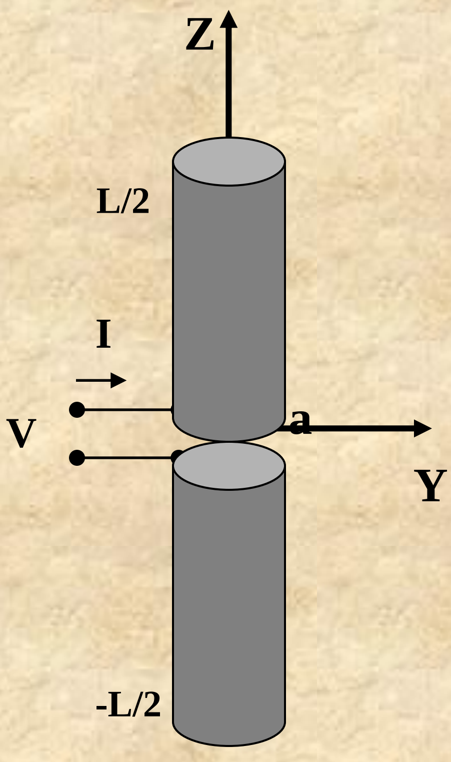
Conteúdo

1. Geometria da Antena Linear



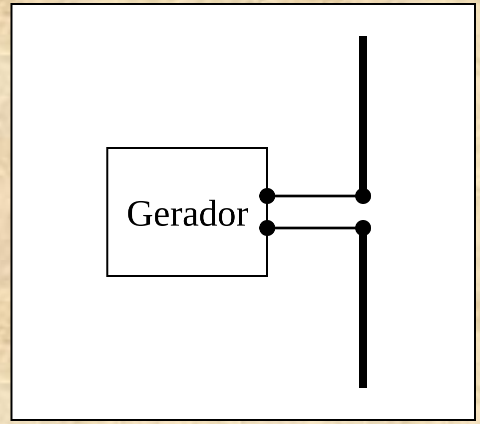
a: raio da antena

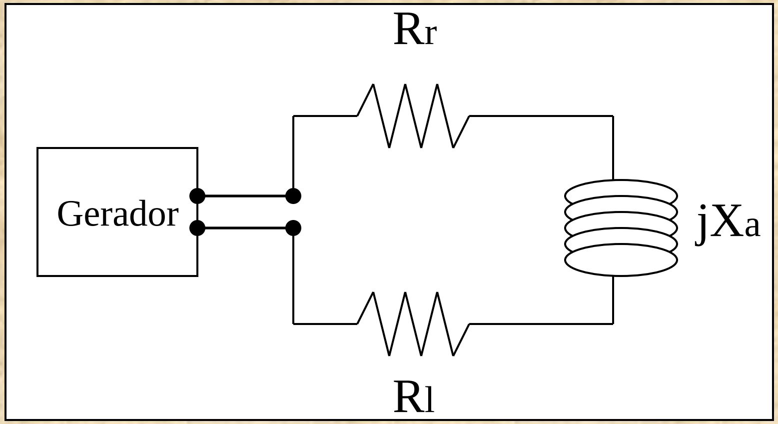
L: comprimento da antena

σ: condutividade do metal

$$\sigma = \infty$$

2. Impedância de Entrada





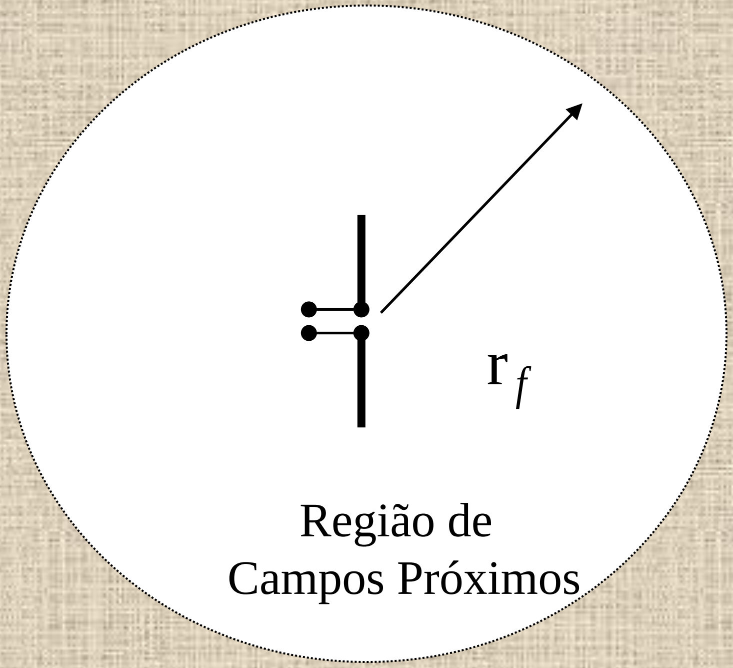
Impedância de Entrada: $Z_i = R_i + jX_i$

- **R_r: Resistência de Radiação**
- **R_l: Resistência de Perdas (=0)**
- **X_i: Reatância da Antena**

2.1 Determinação de Ri através dos Campos Distantes

Região de Fraunhofer

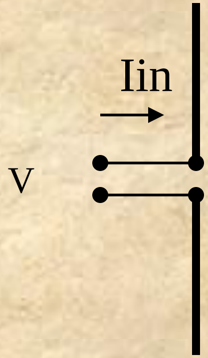
$$\begin{cases} r \gg 2\,\dfrac{L^2}{\lambda} \\ r \gg L \\ r \gg \lambda \end{cases}$$



Ondas TEM: $\overline{H} = \dfrac{1}{\eta}\,\hat{a}_r \times \overline{E}$

$$R_i = \frac{2P_r}{|I_{in}|^2} = \frac{1}{|I_{in}|^2} \oiint_{S_\infty} \left(\overline{E} \times \overline{H}^*\right) \bullet \hat{a}_r \, ds$$

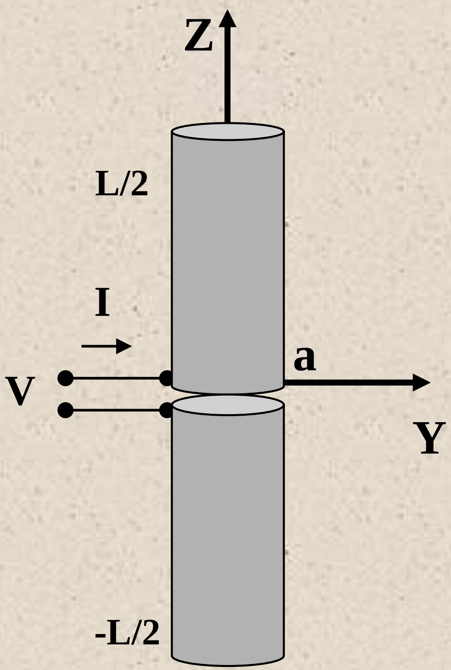
2.2 <u>Determinação de Zi através da Corrente de Entrada</u>



$$Z_i = R_i + jX_i = \frac{V}{I_{in}}$$

Etapas: **1. Assumir uma fonte de tensão V**
2. Equação Integral
3. Determinar I(z')
4. Achar Zi

3. Equação Integral



Aproximações:

i) Fio muito fino ($a<<\lambda$)

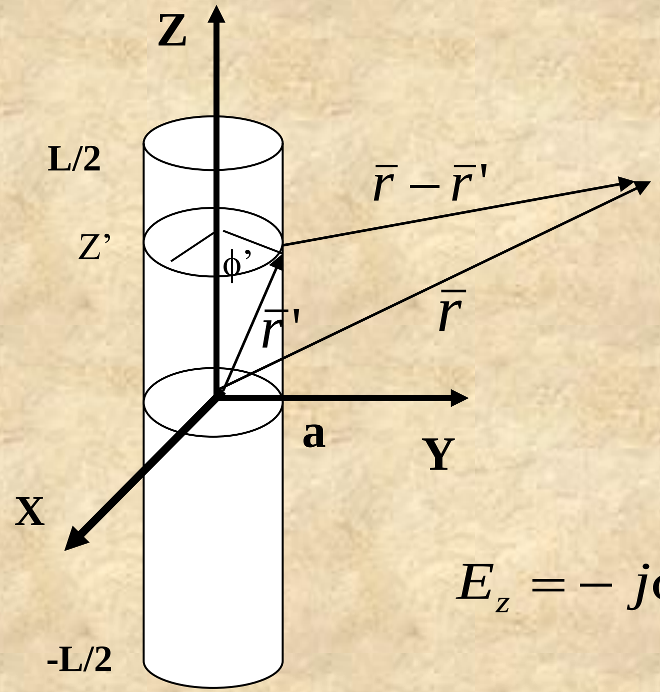
ii) A corrente superficial tem direção z e não depende de ϕ

$$\bar{J}(\phi', z') = J_z(z')\,\hat{a}_z$$
$$I_z(z') = 2\pi a J_z(z')$$

iii) As correntes no topo e base são desprezadas

iv) Fonte de Tensão ideal em z=0

- **Campo Elétrico:**

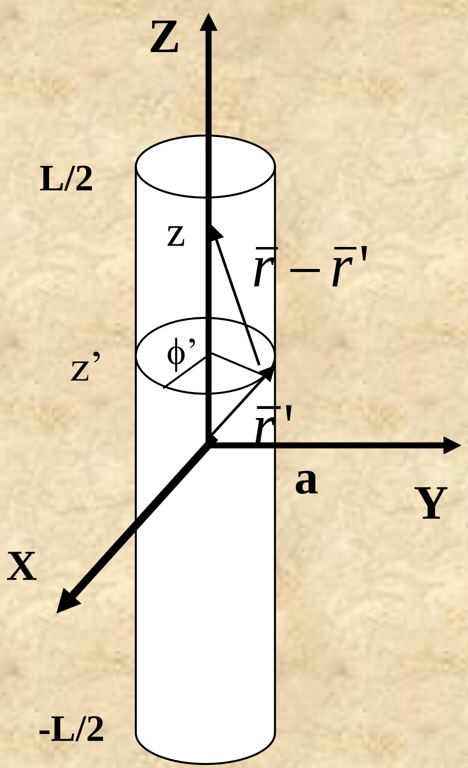


$$\bar{E} = -j\omega\bar{A} + \frac{1}{j\omega\varepsilon\mu}\nabla\left(\nabla \bullet \bar{A}\right)$$

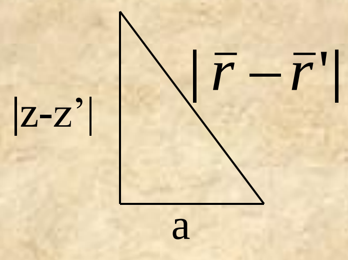
$$A_z(\bar{r}) = \frac{\mu}{4\pi}\oiint_{S'} J_z\left(z'\right)\frac{e^{jk|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|}\,ds'$$

$$E_z = -j\omega A_z + \frac{1}{j\omega\varepsilon\mu}\frac{\partial^2 A_z}{\partial z^2} = \frac{1}{j\omega\varepsilon\mu}\left(k^2 + \frac{\partial^2}{\partial z^2}\right)A_z$$

- **<u>Potencial Vetor no eixo da antena:</u>**



$$\begin{cases} \bar r = z\,\hat a_z \\ \bar r' = a\cos(\phi')\,\hat a_x + a\,sin(\phi')\,\hat a_y + z'\,\hat a_z \end{cases}$$



$$|\bar r - \bar r'| = \sqrt{(z-z')^2 + a^2}$$

$$A_z(z\,\hat a_z) = \frac{\mu}{4\pi}\int_0^{2\pi}\int_{-l/2}^{l/2} J_z(z')\,\frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{\sqrt{(z-z')^2+a^2}}\,a\,d\phi'\,dz'$$

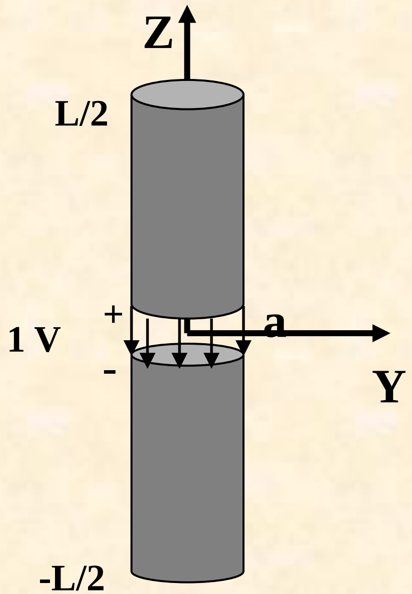
Da simetria cilíndrica da distribuição de corrente:

$$A_z(z\,\hat{a}_z) = \frac{\mu}{4\pi}\int_{-l/2}^{l/2} I_z(z')\,\frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{\sqrt{(z-z')^2+a^2}}\,dz'$$

- **<u>Campo Elétrico no eixo da antena:</u>**

$$E_z(z\,\hat{a}_z) = \frac{1}{j\omega\varepsilon}\left(k^2+\frac{\partial^2}{\partial z^2}\right)\int_{-l/2}^{l/2} I_z(z')\,\frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{4\pi\sqrt{(z-z')^2+a^2}}\,dz'$$

- **<u>Excitação</u>**



- **Votage-gap generator**

$$E_z(z\,\hat{a}_z) = -\delta(z)$$

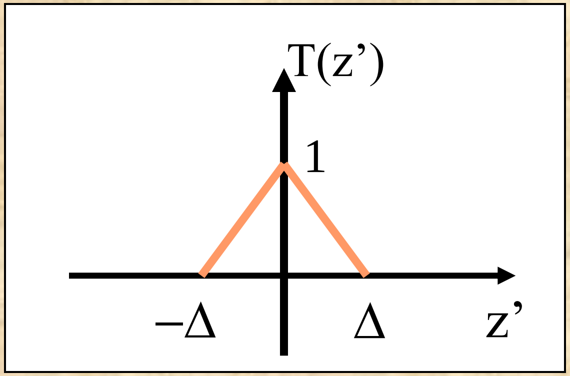
$$V = -\int_{0^-}^{0^+} \bar{E} \bullet d\bar{l} = 1$$

$$\left(k^2 + \frac{\partial^2}{\partial z^2}\right) \int_{-l/2}^{l/2} I_z(z') \frac{e^{-jk\sqrt{(z-z')^2 + a^2}}}{4\pi\sqrt{(z-z')^2 + a^2}}\, dz' = -j\omega\varepsilon\,\delta(z)$$

<u>Equação Integral</u>

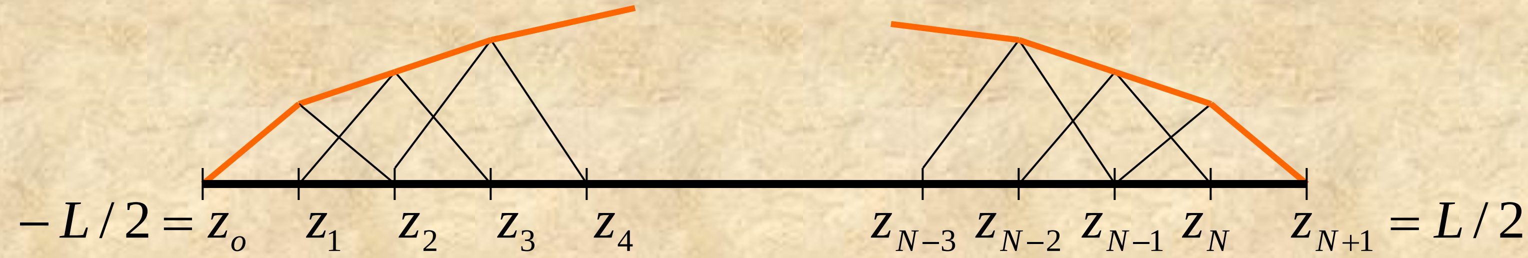
4. Expansão em Funções de Base

$$I_z(z') \approx \sum_{n=1}^{N} I_n U_n(z') = \sum_{n=1}^{N} I_n T(z'-z_n)$$



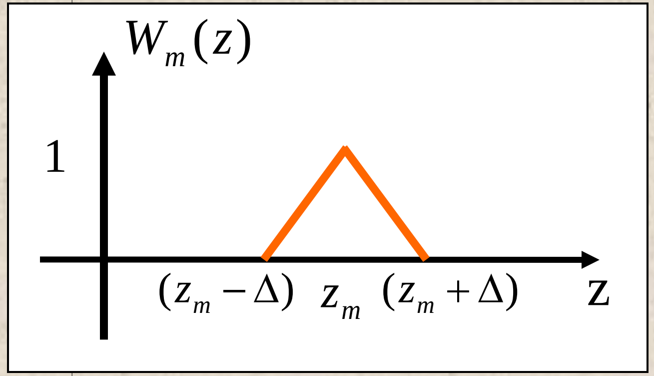
$$z_n = -L/2 + n\Delta \qquad \Delta = \frac{L}{N+1}$$

$$T(z') = 1 - |z'|/\Delta \qquad |z'| \leq \Delta$$



$$\sum_{n=1}^{N} I_n \left(k^2 + \frac{\partial^2}{\partial z^2} \right) \int_{-l/2}^{l/2} U_n(z') \frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{4\pi\sqrt{(z-z')^2+a^2}} \, dz' = -j\omega\varepsilon\,\delta(z)$$

5. Testando com Funções de Peso



$$W_m(z) = T(z - z_m)$$

$$\sum_{n=1}^{N} I_n \int_{-l/2}^{l/2} W_m(z) \left(k^2 + \frac{\partial^2}{\partial z^2} \right) \int_{-l/2}^{l/2} U_n(z') \frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{4\pi\sqrt{(z-z')^2+a^2}} \, dz' \, dz =$$

$$= -j\omega\varepsilon \int_{-l/2}^{l/2} W_m(z)\,\delta(z)\,dz \qquad m = 1,\ldots, N$$

- **Organizando em forma matricial:**

$$\boxed{[Z][I]=[V]}$$

$$\boxed{Z_{mn} = \int_{-l/2}^{l/2} W_m(z)\left(k^2 + \frac{\partial^2}{\partial z^2}\right) \int_{-l/2}^{l/2} U_n(z')\,\frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{4\pi\sqrt{(z-z')^2+a^2}}\,dz'\,dz}$$

$$\boxed{[I]_n = I_n}$$

$$\boxed{V_m = -j\omega\varepsilon \int_{-l/2}^{l/2} W_m(z)\,\delta(z)\,dz = -j\omega\varepsilon W_m(0)}$$

$\Rightarrow$ Equação Integral foi transformada num sistema linear de equações

6. Avaliação de Zmn

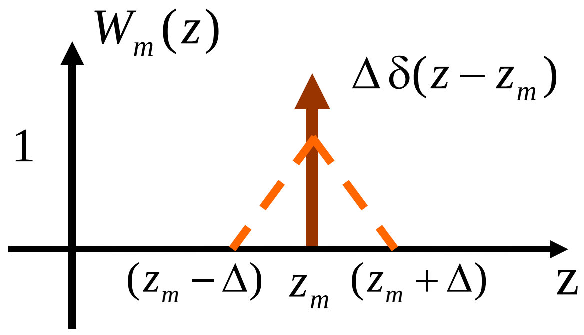
$$Z_{mn} = k^2 \int_{-l/2}^{l/2} W_m(z) \int_{-l/2}^{l/2} U_n\left(z'\right)\; G(z-z')\, dz'\, dz +$$

$$+ \int_{-l/2}^{l/2} W_m(z) \frac{\partial}{\partial z} \int_{-l/2}^{l/2} U_n\left(z'\right)\left(-\frac{\partial}{\partial z'}\right) G(z-z')\, dz'\, dz$$

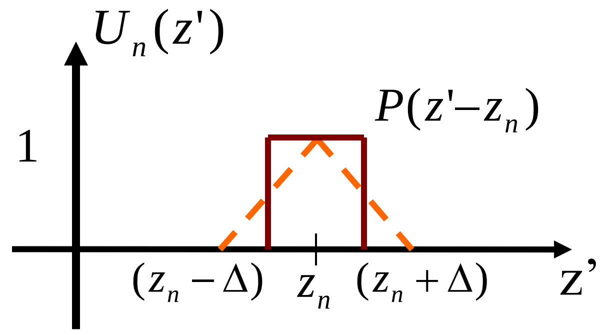
Onde: $$G(z-z') = \frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{4\pi\sqrt{(z-z')^2+a^2}}$$

$$\boxed{\begin{aligned} Z_{mn} &= k^2 \int_{-l/2}^{l/2} W_m(z) \int_{-l/2}^{l/2} U_n\left(z'\right)\; G(z-z')\, dz'\, dz + \\ &- \int_{-l/2}^{l/2} \frac{\partial W_m(z)}{\partial z} \int_{-l/2}^{l/2} \frac{\partial U_n\left(z'\right)}{\partial z'}\; G(z-z')\, dz'\, dz = \\ &= k^2 A_{mn} - \Phi_{mn} \end{aligned}}$$

- **Aproximações:**



$$W_m(z) \approx \Delta\,\delta(z - z_m)$$



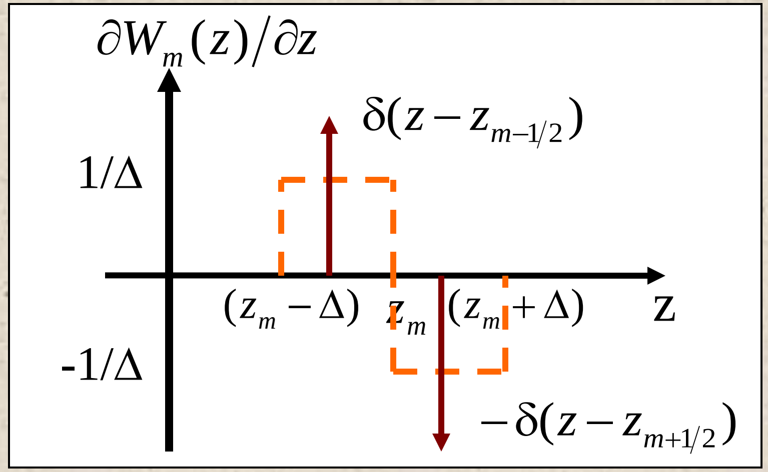
$$U_n(z') \approx P(z' - z_n)$$

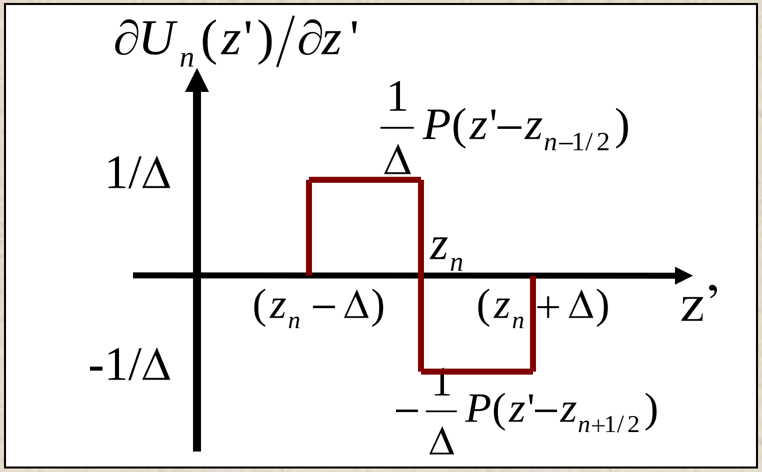
$$A_{mn} = \int_{-l/2}^{l/2} W_m(z) \int_{-l/2}^{l/2} U_n(z')\; G(z - z')\, dz'\, dz =$$

$$\approx \Delta \int_{z_{n-1/2}}^{z_{n+1/2}} G(z_m - z')\, dz' = \Delta^2\; \Psi(m,n)$$

onde:

$$\Psi(m,n) = \frac{1}{\Delta} \int_{z_{n-1/2}}^{z_{n+1/2}} G(z_m - z')\, dz' = \frac{1}{\Delta} \int_{z_{n-1/2}}^{z_{n+1/2}} \frac{e^{-jk\sqrt{(z_m - z')^2 + a^2}}}{4\pi\sqrt{(z_m - z')^2 + a^2}}\, dz'$$





$$\frac{\partial W_m(z)}{\partial z} \approx \delta(z-z_{m-1/2}) - \delta(z-z_{m+1/2})$$

$$\frac{\partial U_n(z')}{\partial z'} = \frac{1}{\Delta}\left[P(z'-z_{n-1/2}) - P(z'-z_{n+1/2})\right]$$

$$\Phi_{mn} = \int_{-l/2}^{l/2} \frac{\partial W_m(z)}{\partial z} \int_{-l/2}^{l/2} \frac{\partial U_n(z')}{\partial z'} \, G(z-z')\,dz'\,dz =$$

$$\approx \int_{-l/2}^{l/2} \left[\delta(z-z_{m-1/2}) - \delta(z-z_{m+1/2})\right] \int_{-l/2}^{l/2} \frac{1}{\Delta}\left[P(z'-z_{n-1/2}) - P(z'-z_{n+1/2})\right] G(z-z')\,dz'\,dz$$

$$\boxed{\Phi_{mn} \approx \Psi(m-1/2, n-1/2) - \Psi(m-1/2, n+1/2) - \Psi(m+1/2, n-1/2) + \Psi(m+1/2, n+1/2)}$$

- **<u>Aproximação para Ψ(m,n)</u>**

i) Se m=n

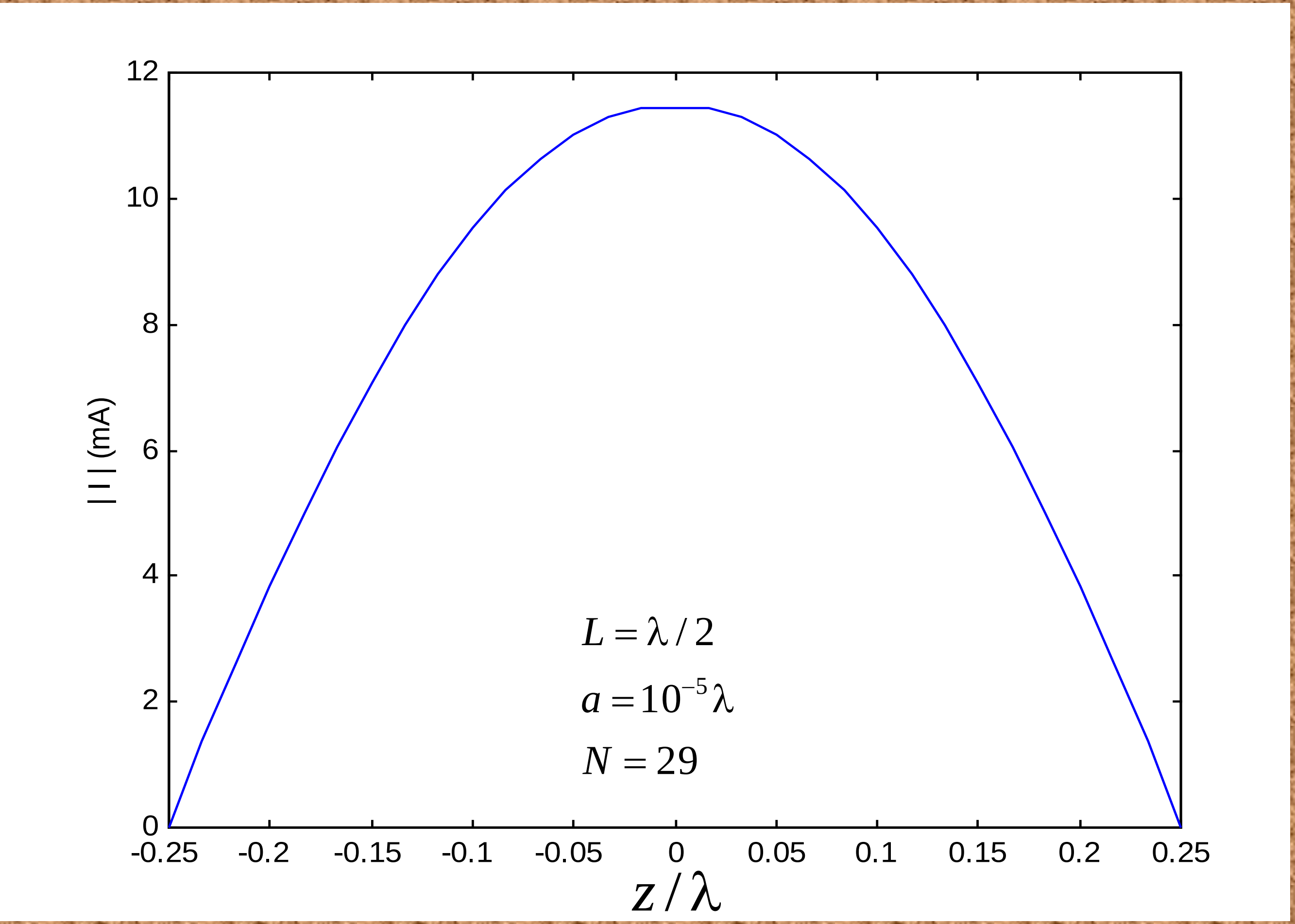
$$e^{-jk\sqrt{(z_m - z')^2 + a^2}} \approx 1 - jk\sqrt{(z_m - z')^2 + a^2}$$

$$\Psi(m,n) \approx \frac{1}{\Delta}\int_{z_{n-1/2}}^{z_{n+1/2}} \frac{1 - jk\sqrt{(z_m - z')^2 + a^2}}{4\pi\sqrt{(z_m - z')^2 + a^2}}\,dz' \approx \frac{1}{2\pi\Delta}\ln(\Delta/a) - \frac{jk}{4\pi}$$

ii) Se m ≠ n

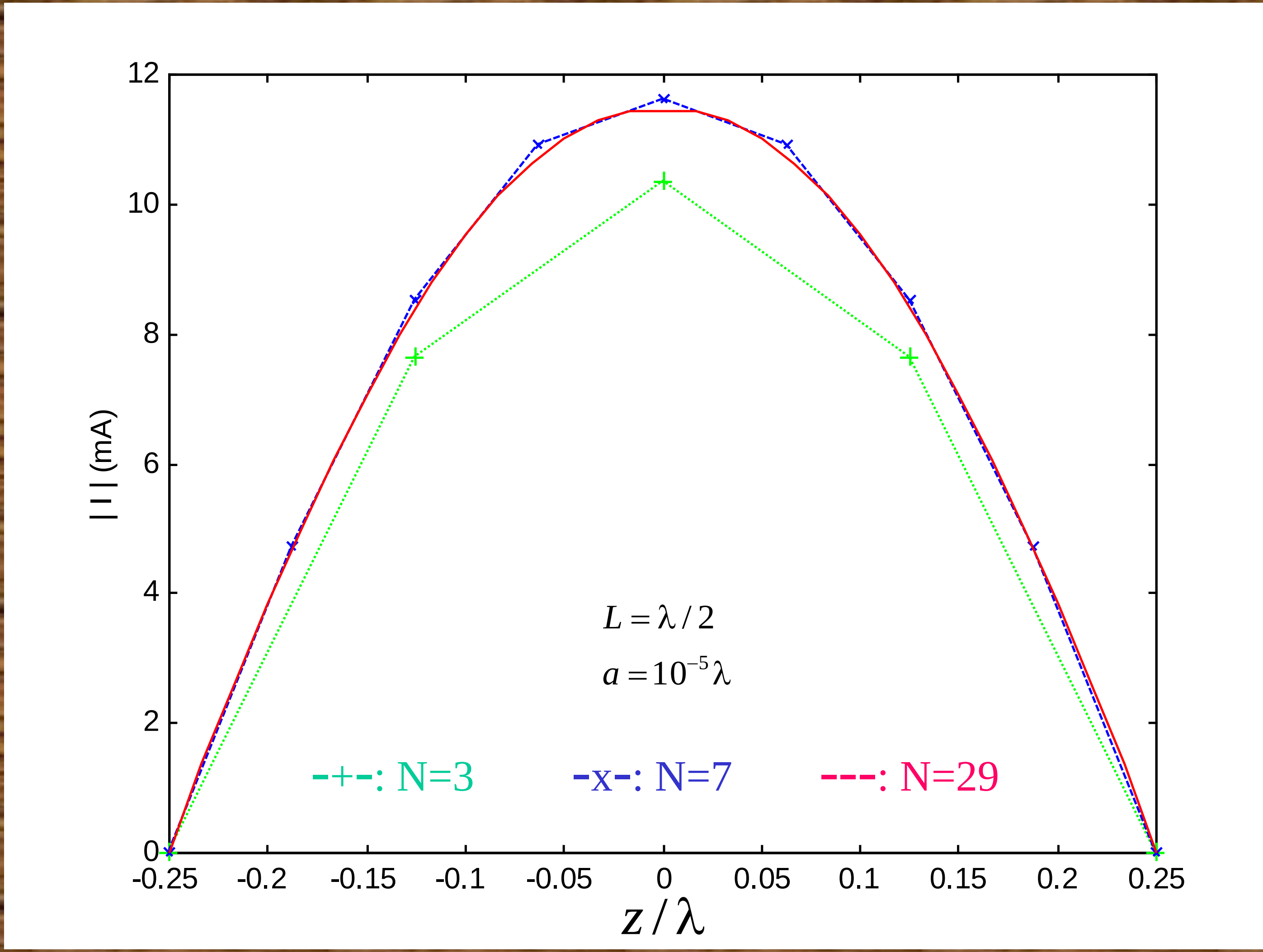
$$\Psi(m,n) \approx \frac{e^{-jk\sqrt{(z_m - z_n)^2 + a^2}}}{4\pi\sqrt{(z_m - z_n)^2 + a^2}}$$

7. Impedância de Entrada

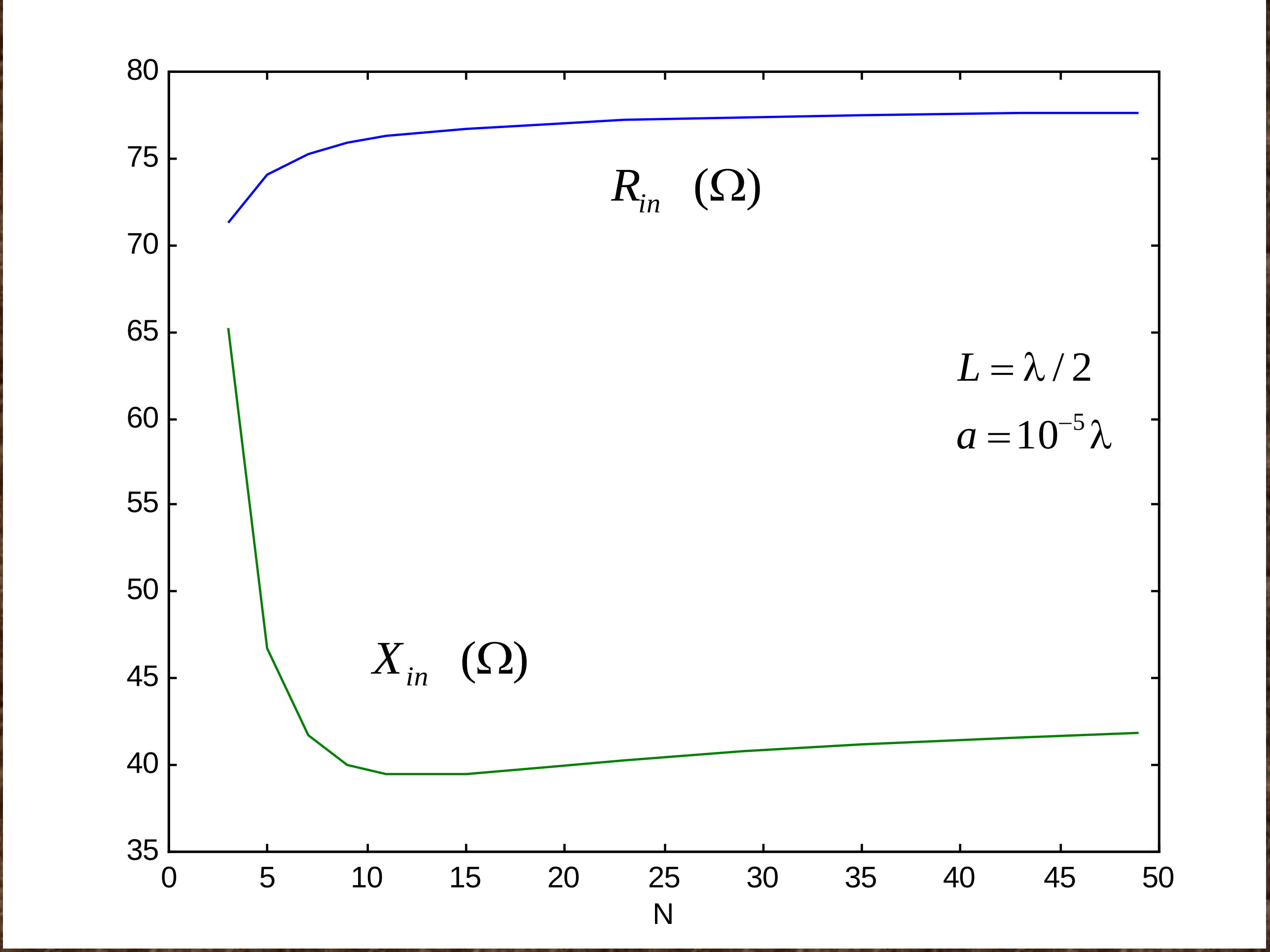


$$Z_{in}=\frac{1}{I_{in}}=77.36+j40.76 \quad (\Omega)$$

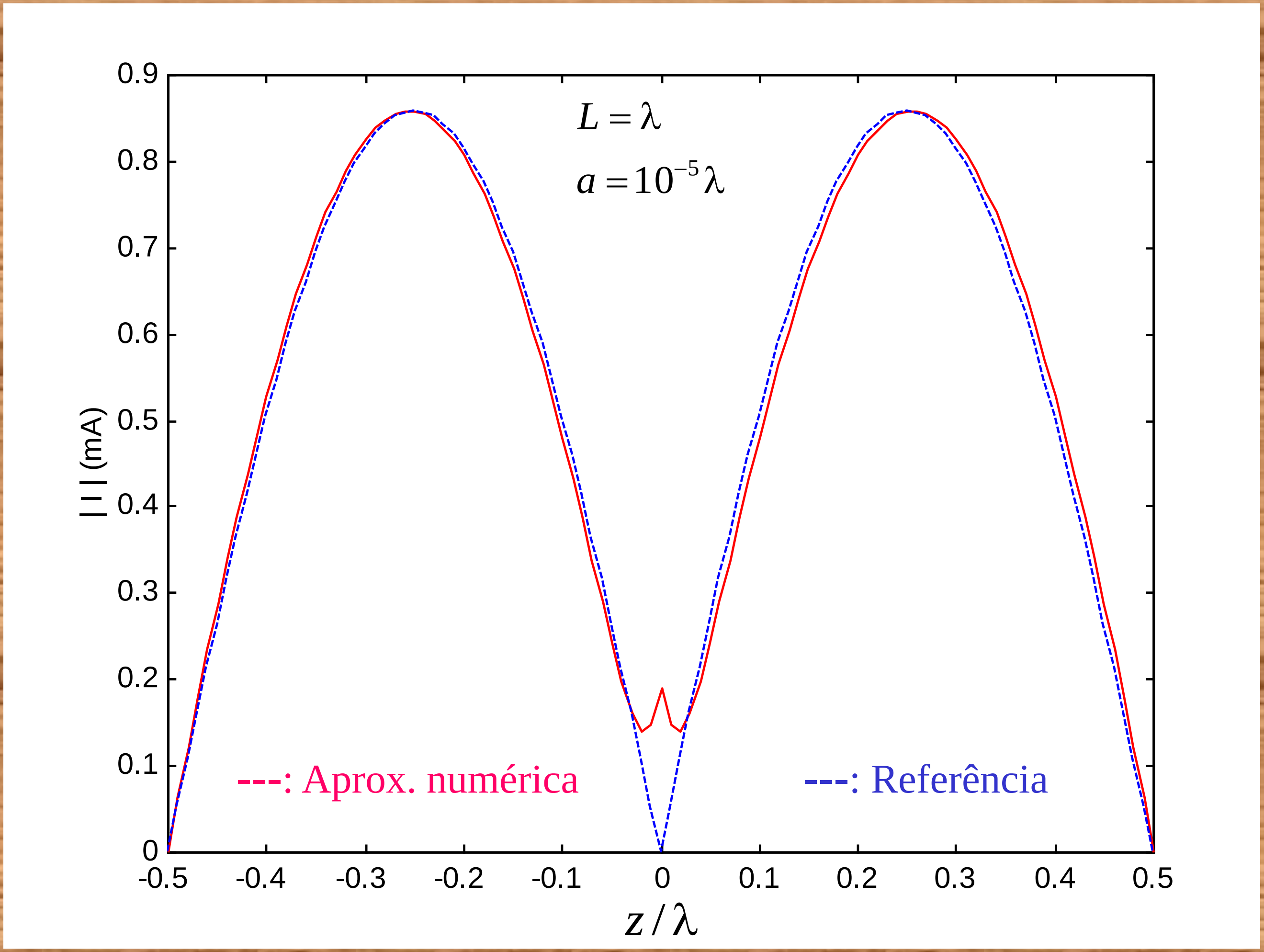
• **Convergência da Distribuição de Corrente**



• **Convergência da Impedância de Entrada**



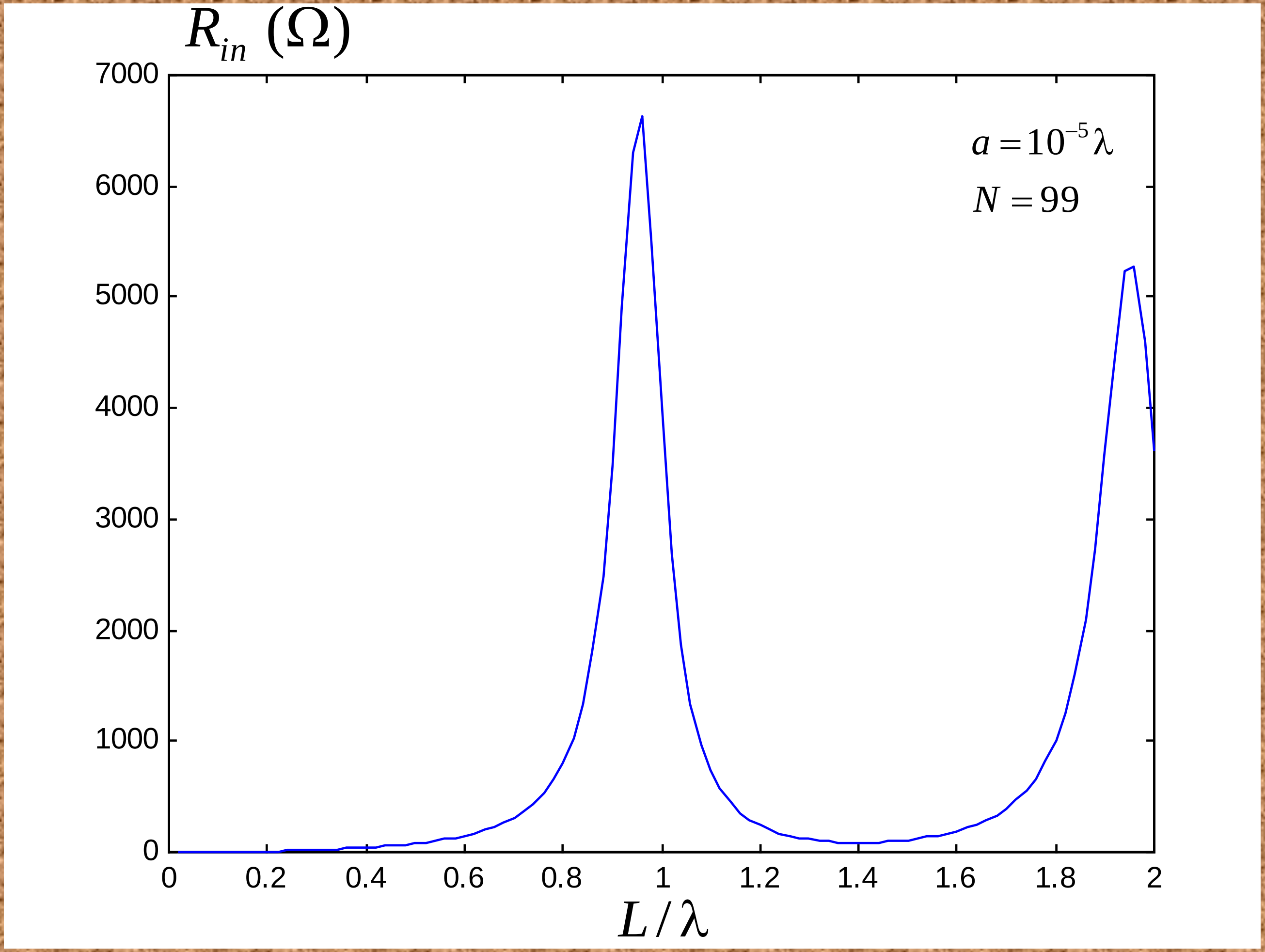
• **Distribuição de Corrente para antena de 1 comprimento de onda**



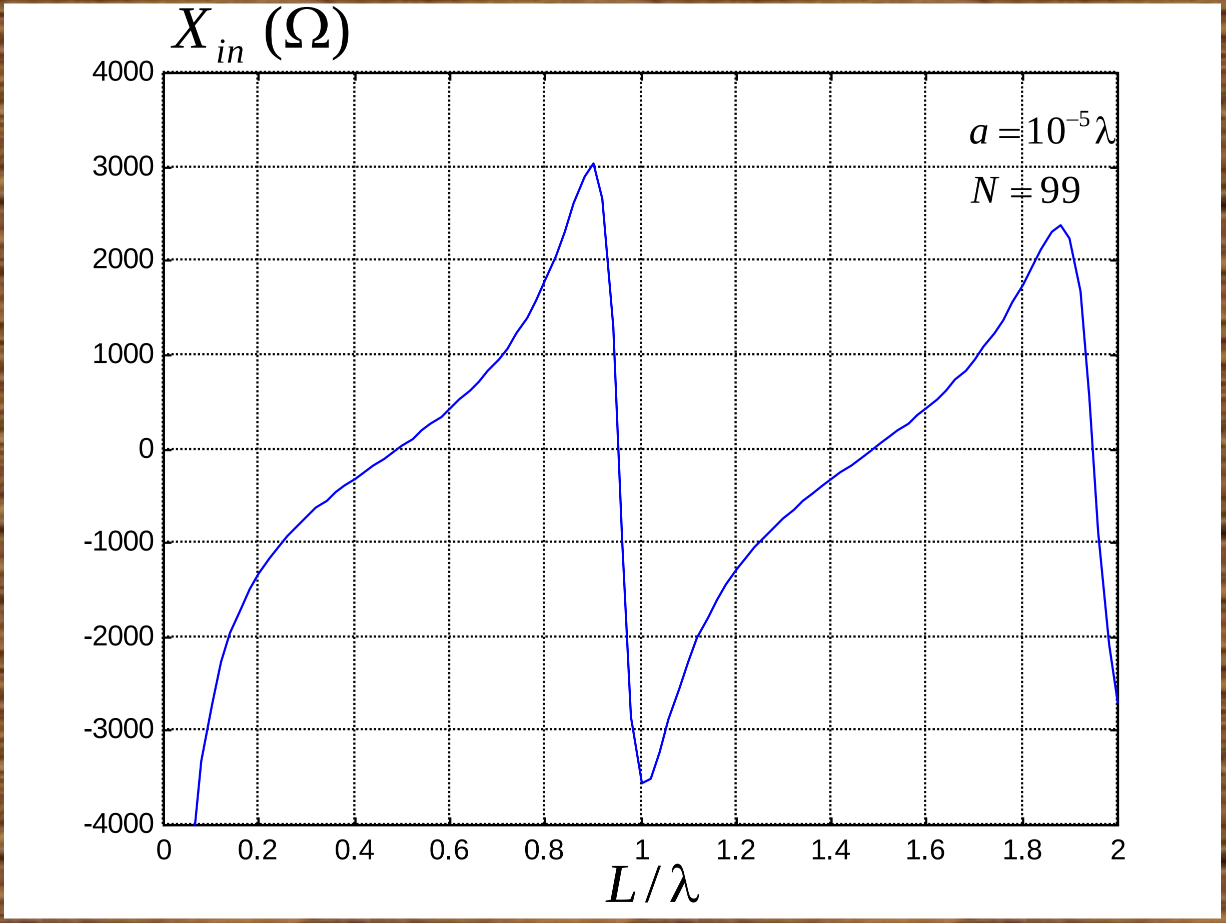
Referência:

$$I(z) = I_o sink(\tfrac{L}{2} - |z|)$$

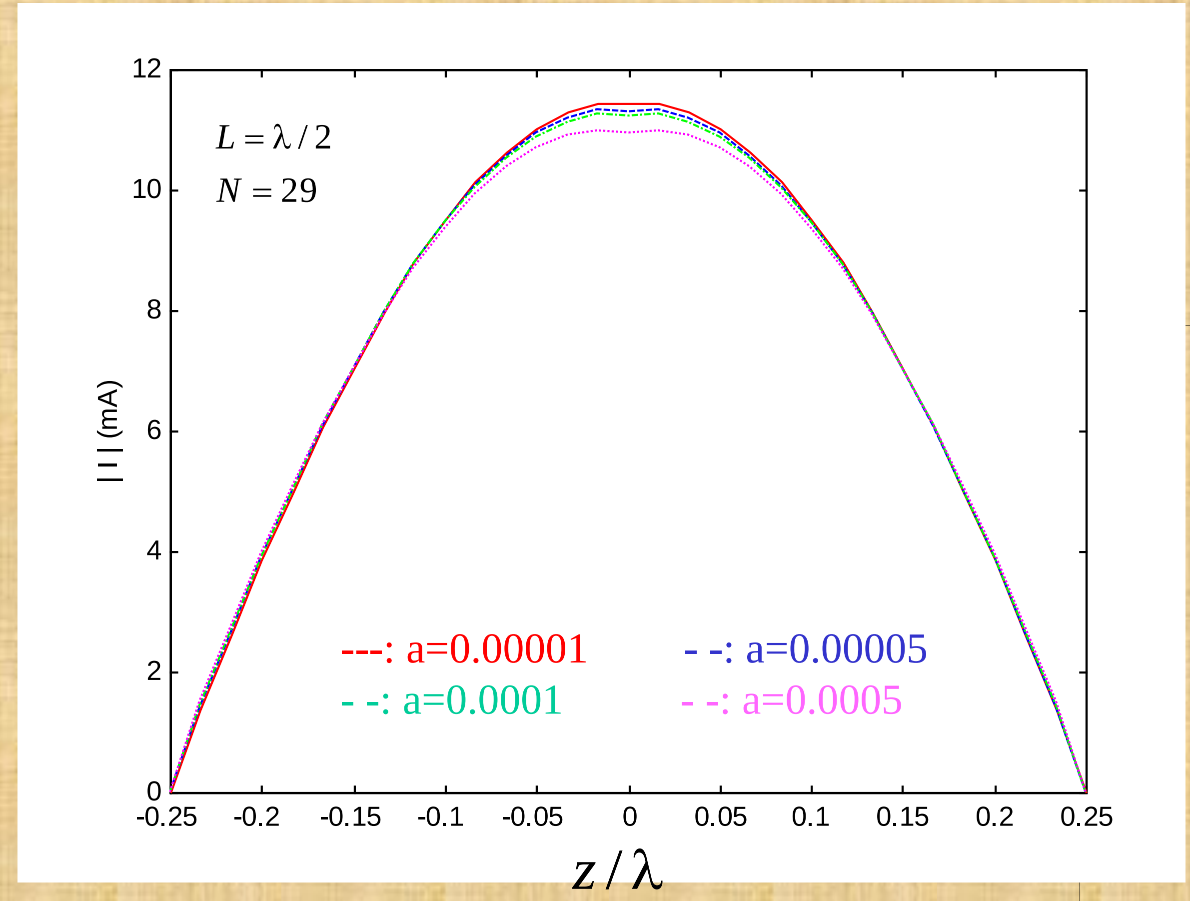
• Resistência de Entrada em função do comprimento L



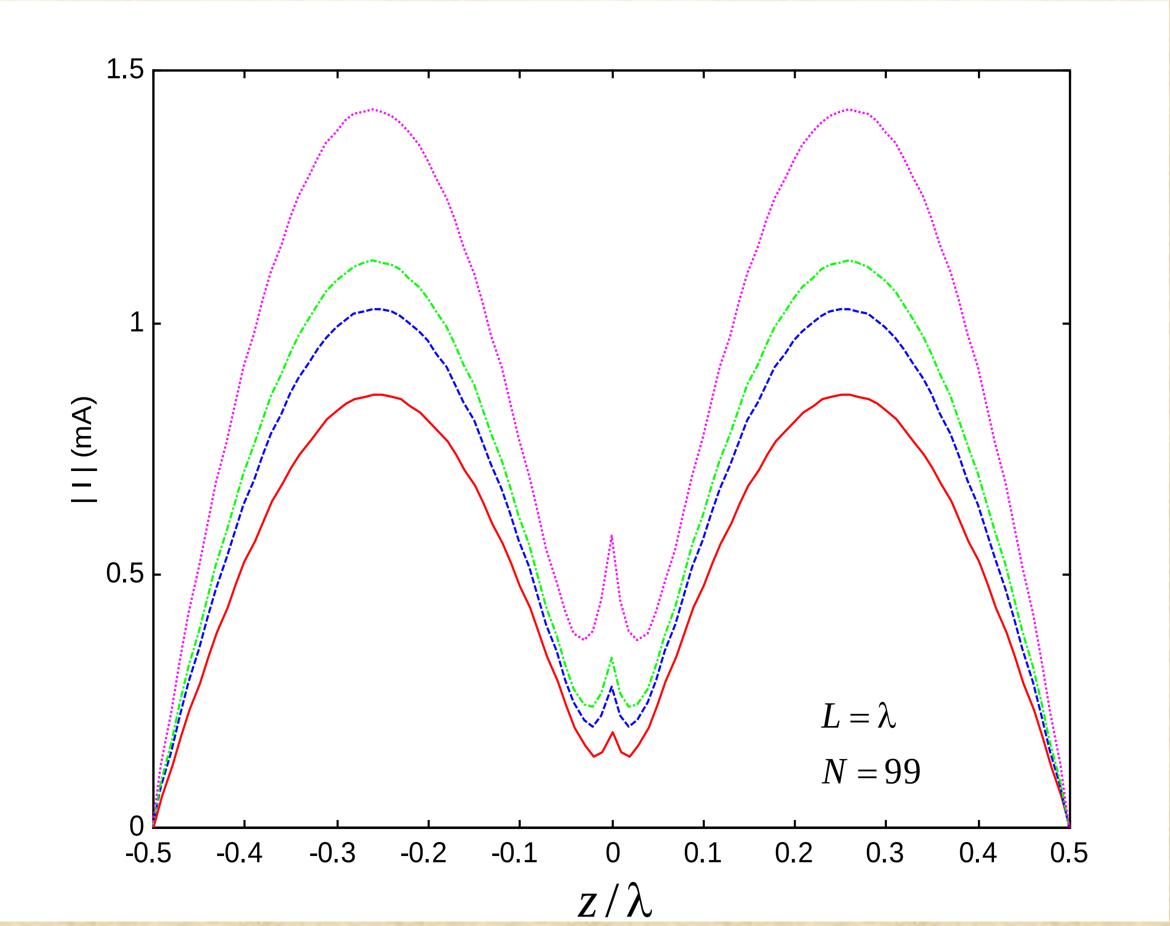
• **Reatância de Entrada em função do comprimento L**



- **Variação com o raio da antena**



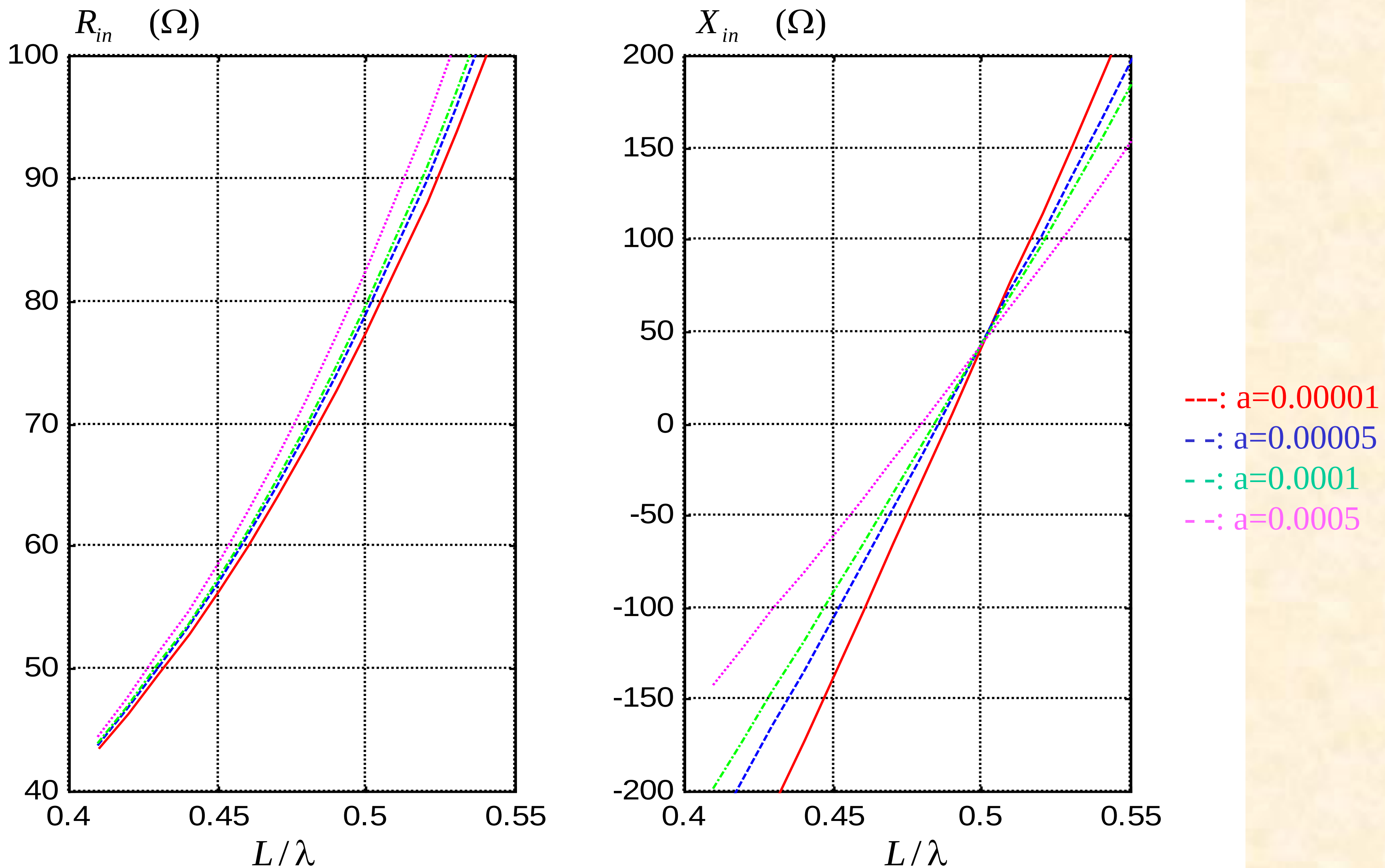
a/λ	R_{in} (Ω)	X_{in} (Ω)
10^{-5}	77.36	40.76
$5.\ 10^{-5}$	78.50	40.73
10^{-4}	79.20	40.73
$5.\ 10^{-4}$	81.82	40.75



a/λ	R_{in} (Ω)	X_{in} (Ω)
10^{-5}	3923	-3553
$5.\ 10^{-5}$	2576	-2523
10^{-4}	2079	-2130
$5.\ 10^{-4}$	1115	-1330

---: a=0.00001 - -: a=0.00005
- -: a=0.0001 - -: a=0.0005

• **<u>Variação do Comprimento de Ressonância com o raio</u>**



8. Conclusões

i) Modelamento de Correntes em Antenas Lineares utilizando o Método dos Momentos.

ii) Cálculo da Impedância de Entrada.

iii) Dependência com o comprimento e raio.

iv) Comparações com referências analíticas.