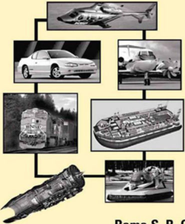
Turbomachinery
Design and Theory



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Turbomachinery
Design and Theory

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To my parents, Tirupelamma and Subba Reddy Gorla,
who encouraged me to strive for excellence in education

—R. S. R. G.

To my wife, Tahseen Ara,
and to my daughters, Shumaila, Sheema, and Afifa

—A. A. K.

Preface

Turbomachinery: Design and Theory offers an introduction to the subject of turbomachinery and is intended to be a text for a single-semester course for senior undergraduate and beginning graduate students in mechanical engineering, aerospace engineering, chemical engineering, design engineering, and manufacturing engineering. This book is also a valuable reference to practicing engineers in the fields of propulsion and turbomachinery.

A basic knowledge of thermodynamics, fluid dynamics, and heat transfer is assumed. We have introduced the relevant concepts from these topics and reviewed them as applied to turbomachines in more detail. An introduction to dimensional analysis is included. We applied the basic principles to the study of hydraulic pumps, hydraulic turbines, centrifugal compressors and fans, axial flow compressors and fans, steam turbines, and axial flow and radial flow gas turbines. A brief discussion of cavitation in hydraulic machinery is presented.

Each chapter includes a large number of solved illustrative and design example problems. An intuitive and systematic approach is used in the solution of these example problems, with particular attention to the proper use of units, which will help students understand the subject matter easily. In addition, we have provided several exercise problems at the end of each chapter, which will allow students to gain more experience. We urge students to take these exercise problems seriously: they are designed to help students fully grasp each topic and to lead them toward a more concrete understanding and mastery of the techniques presented.

This book has been written in a straightforward and systematic manner, without including irrelevant details. Our goal is to offer an engineering textbook on turbomachinery that will be read by students with enthusiasm and interest—we have made special efforts to touch students' minds and assist them in exploring the exciting subject matter.

R.S.R.G. would like to express thanks to his wife, Vijaya Lakshmi, for her support and understanding during the preparation of this book. A.A.K. would like to extend special recognition to his daughter, Shumaila, a practicing computer engineer, for her patience and perfect skills in the preparation of figures; to Sheema Aijaz, a civil engineer who provided numerous suggestions for enhancement of the material on hydraulic turbomachines; and to M. Sadiq, who typed some portions of the manuscript. A.A.K. is also indebted to Aftab Ahmed, Associate Professor of Mechanical Engineering at N.E.D. University of Engineering and Technology, for his many helpful discussions during the writing of this book.

We would like to thank Shirley Love for her assistance in typing portions of the manuscript. We also thank the reviewers for their helpful comments, and we are grateful to John Corrigan, editor at Marcel Dekker, Inc., for encouragement and assistance.

Rama S. R. Gorla
Aijaz A. Khan

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Introduction: Dimensional Analysis—Basic Thermodynamics and Fluid Mechanics

1.1 INTRODUCTION TO TURBOMACHINERY

A turbomachine is a device in which energy transfer occurs between a flowing fluid and a rotating element due to dynamic action, and results in a change in pressure and momentum of the fluid. Mechanical energy transfer occurs inside or outside of the turbomachine, usually in a steady-flow process. Turbomachines include all those machines that produce power, such as turbines, as well as those types that produce a head or pressure, such as centrifugal pumps and compressors. The turbomachine extracts energy from or imparts energy to a continuously moving stream of fluid. However in a positive displacement machine, it is intermittent.

The turbomachine as described above covers a wide range of machines, such as gas turbines, steam turbines, centrifugal pumps, centrifugal and axial flow compressors, windmills, water wheels, and hydraulic turbines. In this text, we shall deal with incompressible and compressible fluid flow machines.

1.2 TYPES OF TURBOMACHINES

There are different types of turbomachines. They can be classified as:

1. Turbomachines in which (i) work is done by the fluid and (ii) work is done on the fluid.

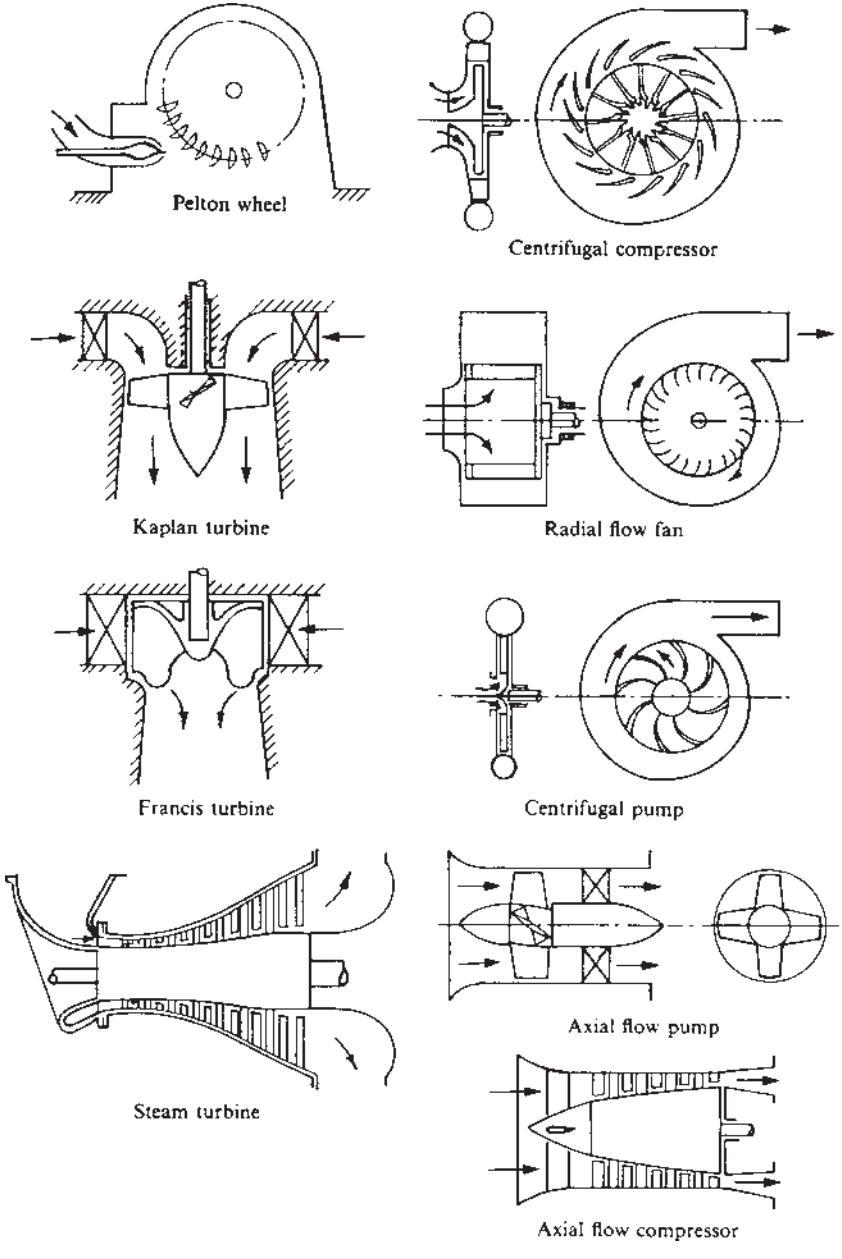


Figure 1.1 Types and shapes of turbomachines.

2. Turbomachines in which fluid moves through the rotating member in axial direction with no radial movement of the streamlines. Such machines are called axial flow machines whereas if the flow is essentially radial, it is called a radial flow or centrifugal flow machine. Some of these machines are shown in Fig. 1.1, and photographs of actual machines are shown in Figs. 1.2–1.6. Two primary points will be observed: first, that the main element is a rotor or runner carrying blades or vanes; and secondly, that the path of the fluid in the rotor may be substantially axial, substantially radial, or in some cases a combination of both. Turbomachines can further be classified as follows:

   Turbines: Machines that produce power by expansion of a continuously flowing fluid to a lower pressure or head.

   Pumps: Machines that increase the pressure or head of flowing fluid.

   Fans: Machines that impart only a small pressure-rise to a continuously flowing gas; usually the gas may be considered to be incompressible.

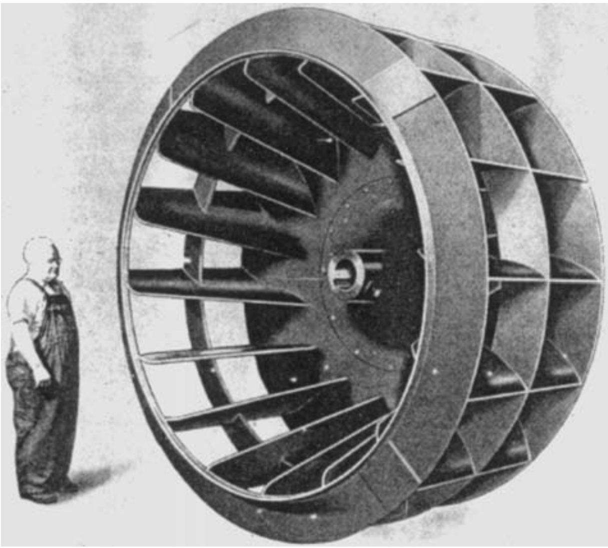


Figure 1.2 Radial flow fan rotor. (Courtesy of the Buffalo Forge Corp.)

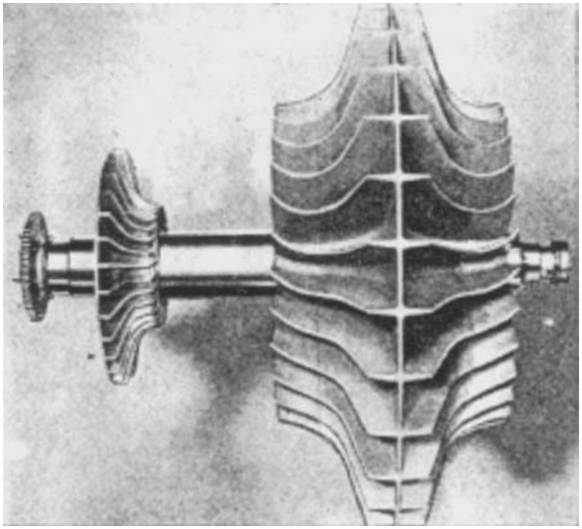


Figure 1.3 Centrifugal compressor rotor (the large double-sided impellar on the right is the main compressor and the small single-sided impellar is an auxiliary for cooling purposes). (Courtesy of Rolls-Royce, Ltd.)

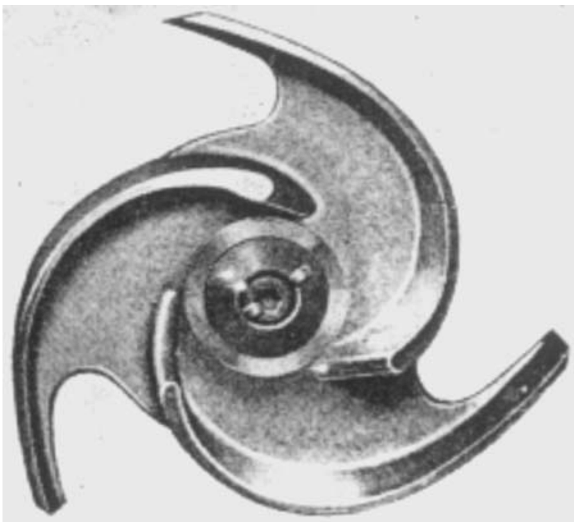


Figure 1.4 Centrifugal pump rotor (open type impeller). (Courtesy of the Ingersoll-Rand Co.)

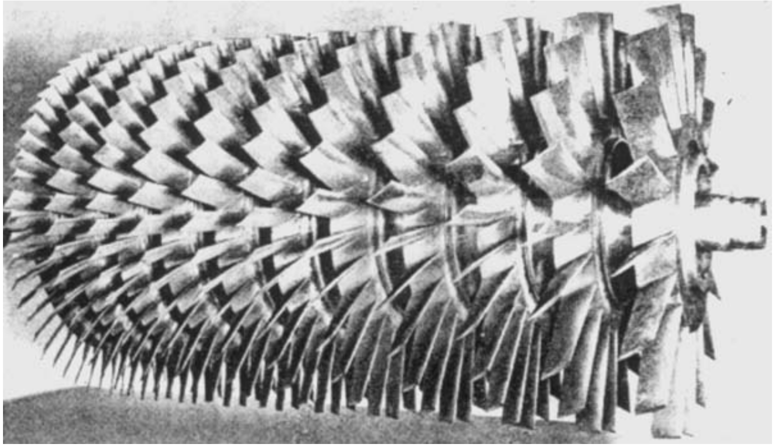


Figure 1.5 Multi-stage axial flow compressor rotor. (Courtesy of the Westinghouse Electric Corp.)

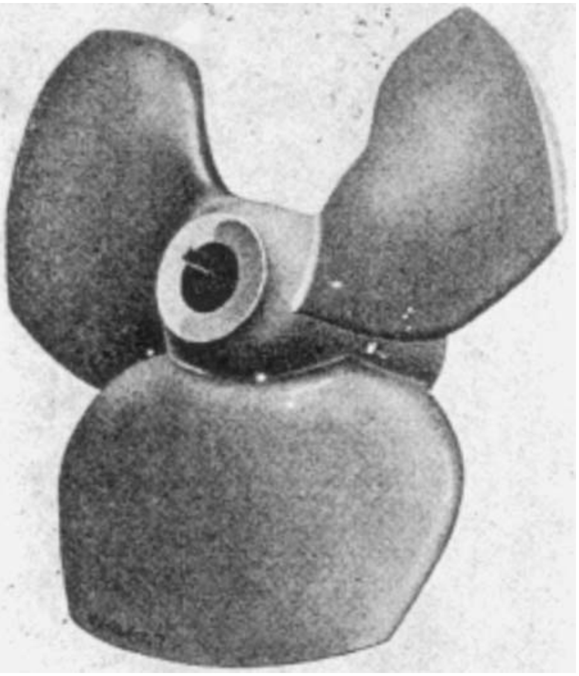


Figure 1.6 Axial flow pump rotor. (Courtesy of the Worthington Corp.)

Compressors: Machines that impart kinetic energy to a gas by compressing it and then allowing it to rapidly expand. Compressors can be axial flow, centrifugal, or a combination of both types, in order to produce the highly compressed air. In a dynamic compressor, this is achieved by imparting kinetic energy to the air in the impeller and then this kinetic energy is converted into pressure energy in the diffuser.

1.3 DIMENSIONAL ANALYSIS

To study the performance characteristics of turbomachines, a large number of variables are involved. The use of dimensional analysis reduces the variables to a number of manageable dimensional groups. Usually, the properties of interest in regard to turbomachine are the power output, the efficiency, and the head. The performance of turbomachines depends on one or more of several variables. A summary of the physical properties and dimensions is given in Table 1.1 for reference.

Dimensional analysis applied to turbomachines has two more important uses: (1) prediction of a prototype's performance from tests conducted on a scale

Table 1.1 Physical Properties and Dimensions

Property	Dimension
Surface	L^2
Volume	L^3
Density	M/L^3
Velocity	L/T
Acceleration	L/T^2
Momentum	ML/T
Force	ML/T^2
Energy and work	ML^2/T^2
Power	ML^2/T^3
Moment of inertia	ML^2
Angular velocity	I/T
Angular acceleration	I/T^2
Angular momentum	ML^2/T
Torque	ML^2/T^2
Modules of elasticity	M/LT^2
Surface tension	M/T^2
Viscosity (absolute)	M/LT
Viscosity (kinematic)	L^2/T

model (similitude), and (2) determination of the most suitable type of machine, on the basis of maximum efficiency, for a specified range of head, speed, and flow rate. It is assumed here that the student has acquired the basic techniques of forming nondimensional groups.

1.4 DIMENSIONS AND EQUATIONS

The variables involved in engineering are expressed in terms of a limited number of basic dimensions. For most engineering problems, the basic dimensions are:

1. *SI system*: mass, length, temperature and time.
2. *English system*: mass, length, temperature, time and force.

The dimensions of pressure can be designated as follows

$$P = \frac{F}{L^2} \tag{1.1}$$

Equation (1.1) reads as follows: "The dimension of P equals force per length squared." In this case, L^2 represents the dimensional characteristics of area. The left hand side of Eq. (1.1) must have the same dimensions as the right hand side.

1.5 THE BUCKINGHAM Π THEOREM

In 1915, Buckingham showed that the number of independent dimensionless group of variables (dimensionless parameters) needed to correlate the unknown variables in a given process is equal to $n - m$, where n is the number of variables involved and m is the number of dimensionless parameters included in the variables. Suppose, for example, the drag force F of a flowing fluid past a sphere is known to be a function of the velocity (v) mass density (ρ) viscosity (μ) and diameter (D). Then we have five variables (F, v, ρ, μ, and D) and three basic dimensions (L, F, and T) involved. Then, there are $5 - 3 = 2$ basic grouping of variables that can be used to correlate experimental results.

1.6 HYDRAULIC MACHINES

Consider a control volume around the pump through which an incompressible fluid of density ρ flows at a volume flow rate of Q.

Since the flow enters at one point and leaves at another point the volume flow rate Q can be independently adjusted by means of a throttle valve. The discharge Q of a pump is given by

$$Q = f(N, D, g, H, \mu, \rho) \tag{1.2}$$

where H is the head, D is the diameter of impeller, g is the acceleration due to gravity, ρ is the density of fluid, N is the revolution, and μ is the viscosity of fluid.

In Eq. (1.2), primary dimensions are only four. Taking N, D, and ρ as repeating variables, we get

$$\Pi_1 = (N)^{a}(D)^{b}(\rho)^{c}(Q)$$

$$\mathrm{M^0L^0T^0} = (\mathrm{T}^{-1})^{a}(\mathrm{L})^{b}(\mathrm{ML}^{-3})^{c}(\mathrm{L^3T^{-1}})$$

For dimensional homogeneity, equating the powers of M, L, and T on both sides of the equation: for M, $0 = c$ or $c = 0$; for T, $0 = -a - 1$ or $a = -1$; for L, $0 = b - 3c + 3$ or $b = -3$.
Therefore,

$$\Pi_1 = N^{-1}D^{-3}\rho^{0}Q = \frac{Q}{ND^3} \tag{1.3}$$

Similarly,

$$\Pi_2 = (N)^{d}(D)^{e}(\rho)^{f}(g)$$

$$\mathrm{M^0L^0T^0} = (\mathrm{T}^{-1})^{d}(\mathrm{L})^{e}(\mathrm{ML}^{-3})^{f}(\mathrm{LT^{-2}})$$

Now, equating the exponents: for M, $0 = f$ or $f = 0$; for T, $0 = -d - 2$ or $d = -2$; for L, $0 = e - 3f + 1$ or $e = -1$.
Thus,

$$\Pi_2 = N^{-2}D^{-1}\rho^{0}g = \frac{g}{N^2D} \tag{1.4}$$

Similarly,

$$\Pi_3 = (N)^{g}(D)^{h}(\rho)^{i}(H)$$

$$\mathrm{M^0L^0T^0} = (\mathrm{T}^{-1})^{g}(\mathrm{L})^{h}(\mathrm{ML}^{-3})^{i}(\mathrm{L})$$

Equating the exponents: for M, $0 = i$ or $i = 0$; for T, $0 = -g$ or $g = 0$; for L, $0 = h - 3i + 1$ or $h = -1$.
Thus,

$$\Pi_3 = N^{0}D^{-1}\rho^{0}H = \frac{H}{D} \tag{1.5}$$

and,

$$\Pi_4 = (N)^{j}(D)^{k}(\rho)^{l}(\mu)$$

$$\mathrm{M^0L^0T^0} = (\mathrm{T}^{-1})^{j}(\mathrm{L})^{k}(\mathrm{ML}^{-3})^{l}(\mathrm{ML^{-1}T^{-1}})$$

Equating the exponents: for M, $0 = l + 1$ or $l = -1$; for T, $0 = -j - 1$ or $j = -1$; for L, $0 = k\text{-}3l - 1$ or $k = -2$.

Thus,

$$\Pi_4 = N^{-1}D^{-2}\rho^{-1}\mu = \frac{\mu}{ND^2\rho} \tag{1.6}$$

The functional relationship may be written as

$$f\left(\frac{Q}{ND^3}, \frac{g}{N^2D}, \frac{H}{D}, \frac{\mu}{ND^2\rho}\right) = 0$$

Since the product of two Π terms is dimensionless, therefore replace the terms Π_2 and Π_3 by gh/N^2D^2

$$f\left(\frac{Q}{ND^3}, \frac{gH}{N^2D^2}, \frac{\mu}{ND^2\rho}\right) = 0$$

or

$$Q = ND^3 f\left(\frac{gH}{N^2D^2}, \frac{\mu}{ND^2\rho}\right) = 0 \tag{1.7}$$

A dimensionless term of extremely great importance that may be obtained by manipulating the discharge and head coefficients is the specific speed, defined by the equation

$$N_s = \sqrt{\frac{\text{Flow coefficient}}{\text{Head coefficient}}} = N\sqrt{Q}/(gH)^{3/4} \tag{1.8}$$

The following few dimensionless terms are useful in the analysis of incompressible fluid flow machines:

1. *The flow coefficient and speed ratio*: The term $Q/(ND^3)$ is called the flow coefficient or specific capacity and indicates the volume flow rate of fluid through a turbomachine of unit diameter runner, operating at unit speed. It is constant for similar rotors.
2. *The head coefficient*: The term gH/N^2D^2 is called the specific head. It is the kinetic energy of the fluid spouting under the head H divided by the kinetic energy of the fluid running at the rotor tangential speed. It is constant for similar impellers.

$$\psi = H/(U^2/g) = gH/(\pi^2N^2D^2) \tag{1.9}$$

3. *Power coefficient or specific power*: The dimensionless quantity $P/(\rho N^2D^2)$ is called the power coefficient or the specific power. It shows the relation between power, fluid density, speed and wheel diameter.
4. *Specific speed*: The most important parameter of incompressible fluid flow machinery is specific speed. It is the non-dimensional term. All turbomachineries operating under the same conditions of flow and head

having the same specific speed, irrespective of the actual physical size of the machines. Specific speed can be expressed in this form

$$N_s = N\sqrt{Q}/(gH)^{3/4} = N\sqrt{P}/\lfloor \rho^{1/2}(gH)^{5/4}\rfloor \tag{1.10}$$

The specific speed parameter expressing the variation of all the variables N, Q and H or N,P and H, which cause similar flows in turbomachines that are geometrically similar. The specific speed represented by Eq. (1.10) is a nondimensional quantity. It can also be expressed in alternate forms.
These are

$$N_s = N\sqrt{Q}/H^{3/4} \tag{1.11}$$

and

$$N_s = N\sqrt{P}/H^{5/4} \tag{1.12}$$

Equation (1.11) is used for specifying the specific speeds of pumps and Eq. (1.12) is used for the specific speeds of turbines. The turbine specific speed may be defined as the speed of a geometrically similar turbine, which develops 1 hp under a head of 1 meter of water. It is clear that N_s is a dimensional quantity. In metric units, it varies between 4 (for very high head Pelton wheel) and 1000 (for the low-head propeller on Kaplan turbines).

1.7 THE REYNOLDS NUMBER

Reynolds number is represented by

$$Re = D^2N/\nu$$

where ν is the kinematic viscosity of the fluid. Since the quantity D^2N is proportional to DV for similar machines that have the same speed ratio. In flow through turbomachines, however, the dimensionless parameter D^2N/ν is not as important since the viscous resistance alone does not determine the machine losses. Various other losses such as those due to shock at entry, impact, turbulence, and leakage affect the machine characteristics along with various friction losses.

Consider a control volume around a hydraulic turbine through which an incompressible fluid of density ρ flows at a volume flow rate of Q, which is controlled by a valve. The head difference across the control volume is H, and if the control volume represents a turbine of diameter D, the turbine develops a shaft power P at a speed of rotation N. The functional equation may be written as

$$P = f(\rho, N, \mu, D, Q, gH) \tag{1.13}$$

Equation (1.13) may be written as the product of all the variables raised to a power and a constant, such that

$$P = \text{const.}\left(\rho^a N^b \mu^c D^d Q^e (gH)^f\right) \tag{1.14}$$

Substituting the respective dimensions in the above Eq. (1.14),

$$\left(\mathrm{ML^2/T^3}\right) = \text{const.}(\mathrm{M/L^3})^a(1/\mathrm{T})^b(\mathrm{M/LT})^c(\mathrm{L})^d(\mathrm{L^3/T})^e(\mathrm{L^2/T^2})^f \tag{1.15}$$

Equating the powers of M, L, and T on both sides of the equation: for M, $1 = a + c$; for L, $2 = -3a - c + d + 3e + 2f$; for T, $-3 = -b - c - e - 2f$.

There are six variables and only three equations. It is therefore necessary to solve for three of the indices in terms of the remaining three. Solving for a, b, and d in terms of c, e, and f we have:

$$a = 1 - c$$

$$b = 3 - c - \mathrm{e} - 2f$$

$$d = 5 - 2c - 3e - 2f$$

Substituting the values of a, b, and d in Eq. (1.13), and collecting like indices into separate brackets,

$$P = \text{const.}\left[\left(\rho N^3 D^5\right), \left(\frac{\mu}{\rho N D^2}\right)^c, \left(\frac{Q}{ND^3}\right)^e, \left(\frac{gH}{N^2 D^2}\right)^f\right] \tag{1.16}$$

In Eq. (1.16), the second term in the brackets is the inverse of the Reynolds number. Since the value of c is unknown, this term can be inverted and Eq. (1.16) may be written as

$$P/\rho N^3 D^5 = \text{const.}\left[\left(\frac{\rho N D^2}{\mu}\right)^c, \left(\frac{Q}{ND^3}\right)^{\mathrm{e}}, \left(\frac{gH}{N^2 D^2}\right)^f\right] \tag{1.17}$$

In Eq. (1.17) each group of variables is dimensionless and all are used in hydraulic turbomachinery practice, and are known by the following names: the power coefficient $\left(P/\rho N^3 D^5 = \overline{P}\right)$; the flow coefficient $\left(Q/ND^3 = \phi\right)$; and the head coefficient $\left(gH/N^2 D^2 = \psi\right)$.

Eqution (1.17) can be expressed in the following form:

$$\overline{P} = f\left(Re, \phi, \psi\right) \tag{1.18}$$

Equation (1.18) indicates that the power coefficient of a hydraulic machine is a function of Reynolds number, flow coefficient and head coefficient. In flow through hydraulic turbomachinery, Reynolds number is usually very high. Therefore the viscous action of the fluid has very little effect on the power output of the machine and the power coefficient remains only a function of ϕ and ψ.

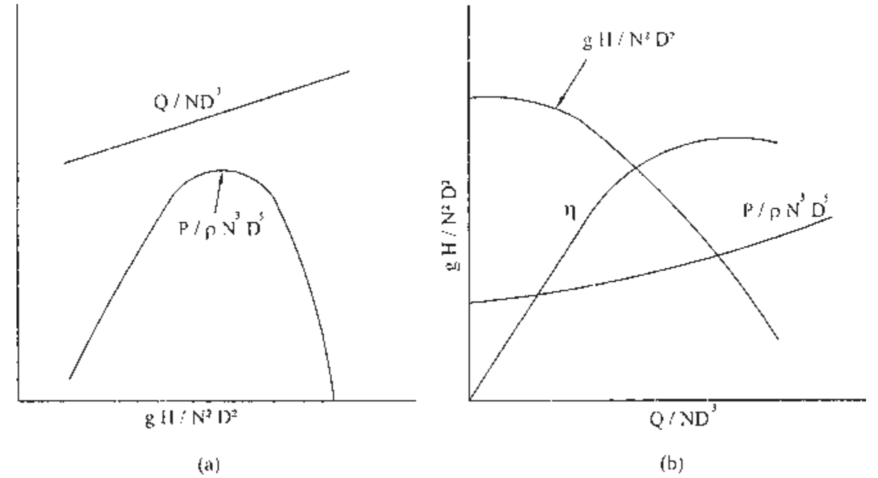


Figure 1.7 Performance characteristics of hydraulic machines: (a) hydraulic turbine, (b) hydraulic pump.

Typical dimensionless characteristic curves for a hydraulic turbine and pump are shown in Fig. 1.7 (a) and (b), respectively. These characteristic curves are also the curves of any other combination of P, N, Q, and H for a given machine or for any other geometrically similar machine.

1.8 MODEL TESTING

Some very large hydraulic machines are tested in a model form before making the full-sized machine. After the result is obtained from the model, one may transpose the results from the model to the full-sized machine. Therefore if the curves shown in Fig 1.7 have been obtained for a completely similar model, these same curves would apply to the full-sized prototype machine.

1.9 GEOMETRIC SIMILARITY

For geometric similarity to exist between the model and prototype, both of them should be identical in shape but differ only in size. Or, in other words, for geometric similarity between the model and the prototype, the ratios of all the corresponding linear dimensions should be equal.

Let L_p be the length of the prototype, B_p, the breadth of the prototype, D_p, the depth of the prototype, and L_m, B_m, and D_m the corresponding dimensions of the model. For geometric similarity, linear ratio (or scale ratio) is given by

$$L_r = \frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{D_p}{D_m} \tag{1.19}$$

Similarly, the area ratio between prototype and model is given by

$$A_r = \left(\frac{L_p}{L_m}\right)^2 = \left(\frac{B_p}{B_m}\right)^2 = \left(\frac{D_p}{D_m}\right)^2 \tag{1.20}$$

and the volume ratio

$$V_r = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{B_p}{B_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3 \tag{1.21}$$

1.10 KINEMATIC SIMILARITY

For kinematic similarity, both model and prototype have identical motions or velocities. If the ratio of the corresponding points is equal, then the velocity ratio of the prototype to the model is

$$V_r = \frac{V_1}{v_1} = \frac{V_2}{v_2} \tag{1.22}$$

where V_1 is the velocity of liquid in the prototype at point 1, V_2, the velocity of liquid in the prototype at point 2, v_1, the velocity of liquid in the model at point 1, and v_2 is the velocity of liquid in the model at point 2.

1.11 DYNAMIC SIMILARITY

If model and prototype have identical forces acting on them, then dynamic similarity will exist. Let F_1 be the forces acting on the prototype at point 1, and F_2 be the forces acting on the prototype at point 2. Then the force ratio to establish dynamic similarity between the prototype and the model is given by

$$F_r = \frac{F_{p1}}{F_{m1}} = \frac{F_{p2}}{F_{m2}} \tag{1.23}$$

1.12 PROTOTYPE AND MODEL EFFICIENCY

Let us suppose that the similarity laws are satisfied, η_p and η_m are the prototype and model efficiencies, respectively. Now from similarity laws, representing the model and prototype by subscripts m and p respectively,

$$\frac{H_p}{(N_p D_p)^2} = \frac{H_m}{(N_m D_m)^2} \quad \text{or} \quad \frac{H_p}{H_m} = \left(\frac{N_p}{N_m}\right)^2 \left(\frac{D_p}{D_m}\right)^2$$

$$\frac{Q_p}{N_p D_p^3} = \frac{Q_m}{N_m D_m^3} \quad \text{or} \quad \frac{Q_p}{Q_m} = \left(\frac{N_p}{N_m}\right)\left(\frac{D_p}{D_m}\right)^3$$

$$\frac{P_p}{N_p^3 D_p^5} = \frac{P_m}{N_m^3 D_m^5} \quad \text{or} \quad \frac{P_p}{P_m} = \left(\frac{N_p}{N_m}\right)^3\left(\frac{D_p}{D_m}\right)^5$$

Turbine efficiency is given by

$$\eta_t = \frac{\text{Power transferred from fluid}}{\text{Fluid power available.}} = \frac{P}{\rho g Q H}$$

Hence, $\dfrac{\eta_m}{\eta_p} = \left(\dfrac{P_m}{P_p}\right)\left(\dfrac{Q_p}{Q_m}\right)\left(\dfrac{H_p}{H_m}\right) = 1.$

Thus, the efficiencies of the model and prototype are the same providing the similarity laws are satisfied.

1.13 PROPERTIES INVOLVING THE MASS OR WEIGHT OF THE FLUID

1.13.1 Specific Weight (γ)

The weight per unit volume is defined as specific weight and it is given the symbol γ (gamma). For the purpose of all calculations relating to hydraulics, fluid machines, the specific weight of water is taken as 1000 l/m^3. In S.I. units, the specific weight of water is taken as 9.80 kN/m^3.

1.13.2 Mass Density (ρ)

The mass per unit volume is mass density. In S.I. systems, the units are kilograms per cubic meter or NS^2/m^4. Mass density, often simply called density, is given the greek symbol ρ (rho). The mass density of water at 15.5° is 1000 kg/m^3.

1.13.3 Specific Gravity (sp.gr.)

The ratio of the specific weight of a given liquid to the specific weight of water at a standard reference temperature is defined as specific gravity. The standard reference temperature for water is often taken as 4°C Because specific gravity is a ratio of specific weights, it is dimensionless and, of course, independent of system of units used.

1.13.4 Viscosity (μ)

We define viscosity as the property of a fluid, which offers resistance to the relative motion of fluid molecules. The energy loss due to friction in a flowing fluid is due to the viscosity. When a fluid moves, a shearing stress develops in it. The magnitude of the shearing stress depends on the viscosity of the fluid. Shearing stress, denoted by the symbol τ (tau) can be defined as the force required to slide on unit area layers of a substance over another. Thus τ is a force divided by an area and can be measured in units N/m^2 or Pa. In a fluid such as water, oil, alcohol, or other common liquids, we find that the magnitude of the shearing stress is directly proportional to the change of velocity between different positions in the fluid. This fact can be stated mathematically as

$$\tau = \mu\left(\frac{\Delta v}{\Delta y}\right) \tag{1.24}$$

where $\frac{\Delta v}{\Delta y}$ is the velocity gradient and the constant of proportionality μ is called the dynamic viscosity of fluid.

Units for Dynamic Viscosity

Solving for μ gives

$$\mu = \frac{\tau}{\Delta v/\Delta y} = \tau\left(\frac{\Delta \mathrm{y}}{\Delta v}\right)$$

Substituting the units only into this equation gives

$$\mu = \frac{\mathrm{N}}{\mathrm{m}^2}\times\frac{\mathrm{m}}{\mathrm{m/s}} = \frac{\mathrm{N}\times\mathrm{s}}{\mathrm{m}^2}$$

Since Pa is a shorter symbol representing N/m^2, we can also express μ as

$$\mu = \mathrm{Pa}\cdot\mathrm{s}$$

1.13.5 Kinematic Viscosity (ν)

The ratio of the dynamic viscosity to the density of the fluid is called the kinematic viscosity υ (nu). It is defined as

$$\nu = \frac{\mu}{\rho} = \mu(1/\rho) = \frac{\mathrm{kg}}{\mathrm{ms}}\times\frac{\mathrm{m}^3}{\mathrm{kg}} = \frac{\mathrm{m}^2}{\mathrm{s}} \tag{1.25}$$

Any fluid that behaves in accordance with Eq. (1.25) is called a Newtonian fluid.

1.14 COMPRESSIBLE FLOW MACHINES

Compressible fluids are working substances in gas turbines, centrifugal and axial flow compressors. To include the compressibility of these types of fluids (gases), some new variables must be added to those already discussed in the case of hydraulic machines and changes must be made in some of the definitions used. The important parameters in compressible flow machines are pressure and temperature.

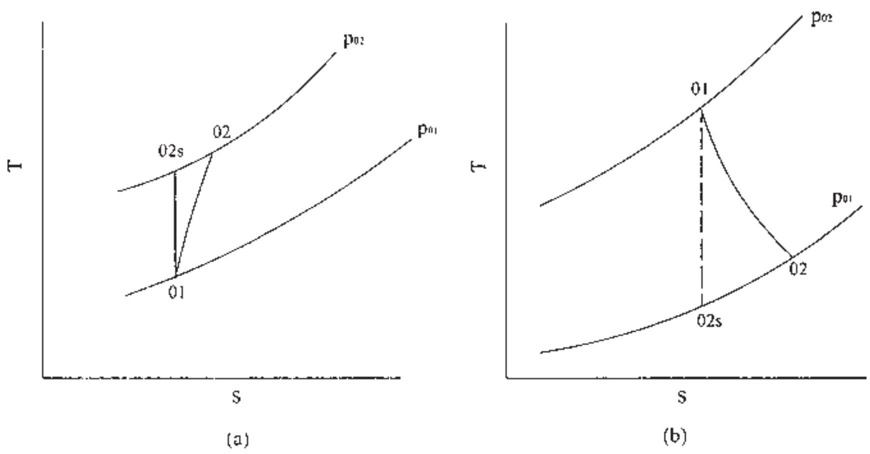


Figure 1.8 Compression and expansion in compressible flow machines: (a) compression, (b) expansion.

In Fig. 1.8 T-s charts for compression and expansion processes are shown.

Isentropic compression and expansion processes are represented by s and the subscript 0 refers to stagnation or total conditions. 1 and 2 refer to the inlet and outlet states of the gas, respectively. The pressure at the outlet, P_{02}, can be expressed as follows

$$P_{02} = f\left(D, N, m, P_{01}, T_{01}, T_{02}, \rho_{01}, \rho_{02}, \mu\right) \tag{1.26}$$

The pressure ratio P_{02}/P_{01} replaces the head H, while the mass flow rate m (kg/s) replaces Q. Using the perfect gas equation, density may be written as $\rho = P/RT$. Now, deleting density and combining R with T, the functional relationship can be written as

$$P_{02} = f(P_{01}, RT_{01}, RT_{02}, m, N, D, \mu) \tag{1.27}$$

Substituting the basic dimensions and equating the indices, the following fundamental relationship may be obtained

$$\frac{P_{02}}{P_{01}} = f\left(\left(\frac{RT_{02}}{RT_{01}}\right), \left(\frac{\left(\frac{m}{RT_{01}}\right)^{1/2}}{P_{01}D^2}\right), \left(\frac{ND}{(RT_{01})^{1/2}}\right), \mathrm{Re}\right) \tag{1.28}$$

In Eq. (1.28), R is constant and may be eliminated. The Reynolds number in most cases is very high and the flow is turbulent and therefore changes in this parameter over the usual operating range may be neglected. However, due to

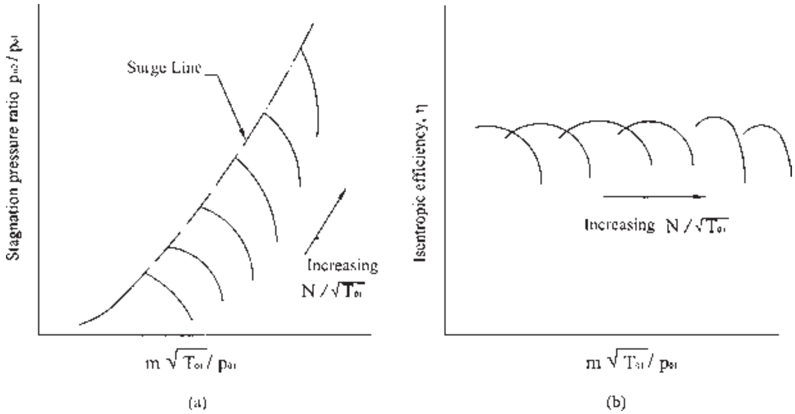


Figure 1.9 Axial flow compressor characteristics: (a) pressure ratio, (b) efficiency.

large changes of density, a significant reduction in *Re* can occur which must be taken into consideration. For a constant diameter machine, the diameter D may be ignored, and hence Eq. (1.28) becomes

$$\frac{P_{02}}{P_{01}} = f\left(\left(\frac{T_{02}}{T_{01}}\right), \left(\frac{mT_{01}^{1/2}}{P_{01}}\right), \left(\frac{N}{T_{01}^{1/2}}\right)\right) \tag{1.29}$$

In Eq. (1.29) some of the terms are new and no longer dimensionless. For a particular machine, it is typical to plot P_{02}/P_{01} and T_{02}/T_{01} against the mass flow

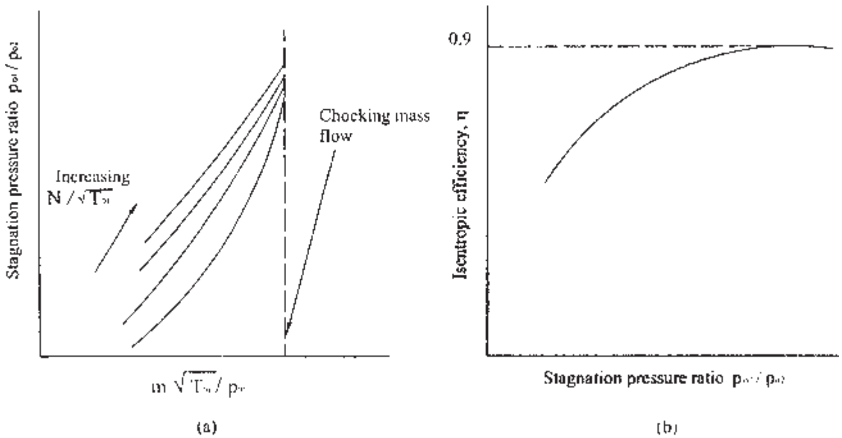


Figure 1.10 Axial flow gas turbine characteristics: (a) pressure ratio, (b) efficiency.

rate parameter $mT_{01}^{1/2}/P_{01}$ for different values of the speed parameter $N/T_{01}^{1/2}$. Equation (1.28) must be used if it is required to change the size of the machine. The term $ND/(RT_{01})^{1/2}$ indicates the Mach number effect. This occurs because the impeller velocity $v \propto ND$ and the acoustic velocity $a_{01} \propto RT_{01}$, while the Mach number

$$M = V/a_{01} \tag{1.30}$$

The performance curves for an axial flow compressor and turbine are shown in Figs. 1.9 and 1.10.

1.15 BASIC THERMODYNAMICS, FLUID MECHANICS, AND DEFINITIONS OF EFFICIENCY

In this section, the basic physical laws of fluid mechanics and thermodynamics will be discussed. These laws are:

1. The continuity equation.
2. The First Law of Thermodynamics.
3. Newton's Second Law of Motion.
4. The Second Law of Thermodynamics.

The above items are comprehensively dealt with in books on thermodynamics with engineering applications, so that much of the elementary discussion and analysis of these laws need not be repeated here.

1.16 CONTINUITY EQUATION

For steady flow through a turbomachine, m remains constant. If A_1 and A_2 are the flow areas at Secs. 1 and 2 along a passage respectively, then

$$\dot{m} = \rho_1 A_1 C_1 = \rho_2 A_2 C_2 = \text{constant} \tag{1.31}$$

where ρ_1, is the density at section 1, ρ_2, the density at section 2, C_1, the velocity at section 1, and C_2, is the velocity at section 2.

1.17 THE FIRST LAW OF THERMODYNAMICS

According to the First Law of Thermodynamics, if a system is taken through a complete cycle during which heat is supplied and work is done, then

$$\oint (\delta Q - \delta W) = 0 \tag{1.32}$$

where $\oint \delta Q$ represents the heat supplied to the system during this cycle and $\oint \delta W$

the work done by the system during the cycle. The units of heat and work are taken to be the same. During a change of state from 1 to 2, there is a change in the internal energy of the system

$$U_2 - U_1 = \int_1^2 (\delta Q - \delta W) \tag{1.33}$$

For an infinitesimal change of state

$$dU = \delta Q - \delta W \tag{1.34}$$

1.17.1 The Steady Flow Energy Equation

The First Law of Thermodynamics can be applied to a system to find the change in the energy of the system when it undergoes a change of state. The total energy of a system, E may be written as:

$$E = \text{Internal Energy} + \text{Kinetic Energy} + \text{Potential Energy}$$

$$E = U + K.E. + P.E. \tag{1.35}$$

where U is the internal energy. Since the terms comprising E are point functions, we can write Eq. (1.35) in the following form

$$dE = dU + d(\text{K.E.}) + d(\text{P.E.}) \tag{1.36}$$

The First Law of Thermodynamics for a change of state of a system may therefore be written as follows

$$\delta Q = dU + d(\text{KE}) + d(\text{PE}) + \delta W \tag{1.37}$$

Let subscript 1 represents the system in its initial state and 2 represents the system in its final state, the energy equation at the inlet and outlet of any device may be written

$$Q_{1-2} = U_2 - U_1 + \frac{m(C_2^2 - C_1^2)}{2} + mg(Z_2 - Z_1) + W_{1-2} \tag{1.38}$$

Equation (1.38) indicates that there are differences between, or changes in, similar forms of energy entering or leaving the unit. In many applications, these differences are insignificant and can be ignored. Most closed systems encountered in practice are stationary; i.e. they do not involve any changes in their velocity or the elevation of their centers of gravity during a process. Thus, for stationary closed systems, the changes in kinetic and potential energies are negligible (i.e. $\triangle(\text{K.E.}) = \triangle(\text{P.E.}) = 0$), and the first law relation reduces to

$$Q - W = \Delta E \tag{1.39}$$

If the initial and final states are specified the internal energies 1 and 2 can easily be determined from property tables or some thermodynamic relations.

1.17.2 Other Forms of the First Law Relation

The first law can be written in various forms. For example, the first law relation on a unit-mass basis is

$$q - w = \Delta e(\text{kJ/kg}) \tag{1.40}$$

Dividing Eq. (1.39) by the time interval Δt and taking the limit as $\Delta t \to 0$ yields the rate form of the first law

$$\dot{Q} - \dot{W} = \frac{\mathrm{d}E}{\mathrm{d}t} \tag{1.41}$$

where $\dot{Q}$ is the rate of net heat transfer, $\dot{W}$ the power, and $\frac{\mathrm{d}E}{\mathrm{d}t}$ is the rate of change of total energy. Equations. (1.40) and (1.41) can be expressed in differential form

$$\delta Q - \delta W = \mathrm{d}E(\text{kJ}) \tag{1.42}$$

$$\delta q - \delta w = \mathrm{d}e(\text{kJ/kg}) \tag{1.43}$$

For a cyclic process, the initial and final states are identical; therefore, $\Delta E = E_2 - E_1$.

Then the first law relation for a cycle simplifies to

$$Q - W = 0(\text{kJ}) \tag{1.44}$$

That is, the net heat transfer and the net work done during a cycle must be equal. Defining the stagnation enthalpy by: $h_0 = h + \frac{1}{2}c^2$ and assuming $g\ (Z_2 - Z_1)$ is negligible, the steady flow energy equation becomes

$$\dot{Q} - \dot{W} = \dot{m}(h_{02} - h_{01}) \tag{1.45}$$

Most turbomachinery flow processes are adiabatic, and so $\dot{Q} = 0$. For work producing machines, $\dot{W} > 0$; so that

$$\dot{W} = \dot{m}(h_{01} - h_{02}) \tag{1.46}$$

For work absorbing machines (compressors) $W < 0$; so that

$$\dot{W} \to -\dot{W} = \dot{m}(h_{02} - h_{01}) \tag{1.47}$$

1.18 NEWTON'S SECOND LAW OF MOTION

Newton's Second Law states that the sum of all the forces acting on a control volume in a particular direction is equal to the rate of change of momentum of the fluid across the control volume. For a control volume with fluid entering with uniform velocity C_1 and leaving with uniform velocity C_2, then

$$\sum F = \dot{m}(C_2 - C_1) \tag{1.48}$$

Equation (1.48) is the one-dimensional form of the steady flow momentum equation, and applies for linear momentum. However, turbomachines have impellers that rotate, and the power output is expressed as the product of torque and angular velocity. Therefore, angular momentum is the most descriptive parameter for this system.

1.19 THE SECOND LAW OF THERMODYNAMICS: ENTROPY

This law states that for a fluid passing through a cycle involving heat exchanges

$$\oint \frac{\delta Q}{T} \leq 0 \tag{1.49}$$

where δQ is an element of heat transferred to the system at an absolute temperature T. If all the processes in the cycle are reversible, so that $\delta Q = \delta Q_{\mathrm{R}}$, then

$$\oint \frac{\delta Q_{\mathrm{R}}}{T} = 0 \tag{1.50}$$

The property called entropy, for a finite change of state, is then given by

$$S_2 - S_1 = \int_1^2 \frac{\delta Q_{\mathrm{R}}}{T} \tag{1.51}$$

For an incremental change of state

$$\mathrm{d}S = m\mathrm{d}s = \frac{\delta Q_{\mathrm{R}}}{T} \tag{1.52}$$

where m is the mass of the fluid. For steady flow through a control volume in which the fluid experiences a change of state from inlet 1 to outlet 2,

$$\int_1^2 \frac{\delta \dot{Q}}{T} \leq \dot{m}(s_2 - s_1) \tag{1.53}$$

For adiabatic process, $\delta Q = 0$ so that

$$s_2 \geq s_1 \tag{1.54}$$

For reversible process

$$s_2 = s_1 \tag{1.55}$$

In the absence of motion, gravity and other effects, the first law of thermodynamics, Eq. (1.34) becomes

$$T\mathrm{d}s = \mathrm{d}u + p\mathrm{d}v \tag{1.56}$$

Putting $h = u + pv$ and $\mathrm{d}h = \mathrm{d}u + p\mathrm{d}v + v\mathrm{d}p$ in Eq. (1.56) gives

$$T\mathrm{d}s = \mathrm{d}h - v\mathrm{d}p \tag{1.57}$$

1.20 EFFICIENCY AND LOSSES

Let H be the head parameter (m), Q discharge (m^3/s)

The waterpower supplied to the machine is given by

$$P = \rho QgH(\text{in watts}) \tag{1.58}$$

and letting $\rho = 1000\,\mathrm{kg/m^3}$,

$$= QgH(\text{in kW})$$

Now, let ΔQ be the amount of water leaking from the tail race. This is the amount of water, which is not providing useful work.

Then:

$$\text{Power wasted} = \Delta Q(gH)(\mathrm{kW})$$

For volumetric efficiency, we have

$$\eta_\nu = \frac{Q - \Delta Q}{Q} \tag{1.59}$$

Net power supplied to turbine

$$= (Q - \Delta Q)gH(\mathrm{kW}) \tag{1.60}$$

If H_r is the runner head, then the hydraulic power generated by the runner is given by

$$P_\mathrm{h} = (Q - \Delta Q)gH_\mathrm{r}(\mathrm{kW}) \tag{1.61}$$

The hydraulic efficiency, η_h is given by

$$\eta_\mathrm{h} = \frac{\text{Hydraulic output power}}{\text{Hydraulic input power}} = \frac{(Q - \Delta Q)gH_\mathrm{r}}{(Q - \Delta Q)gH} = \frac{H_\mathrm{r}}{H} \tag{1.62}$$

If P_m represents the power loss due to mechanical friction at the bearing, then the available shaft power is given by

$$P_s = P_h - P_m \tag{1.63}$$

Mechanical efficiency is given by

$$\eta_m = \frac{P_s}{P_h} = \frac{P_s}{P_m - P_s} \tag{1.64}$$

The combined effect of all these losses may be expressed in the form of overall efficiency. Thus

$$\eta_0 = \frac{P_s}{WP} = \eta_m \frac{P_h}{WP}$$

$$= \eta_m \frac{WP(Q - \Delta Q)}{WPQ\Delta H} = \eta_m \eta_v \eta_h \tag{1.65}$$

1.21 STEAM AND GAS TURBINES

Figure 1.11 shows an enthalpy–entropy or Mollier diagram. The process is represented by line 1–2 and shows the expansion from pressure P_1 to a lower pressure P_2. The line 1–2s represents isentropic expansion. The actual

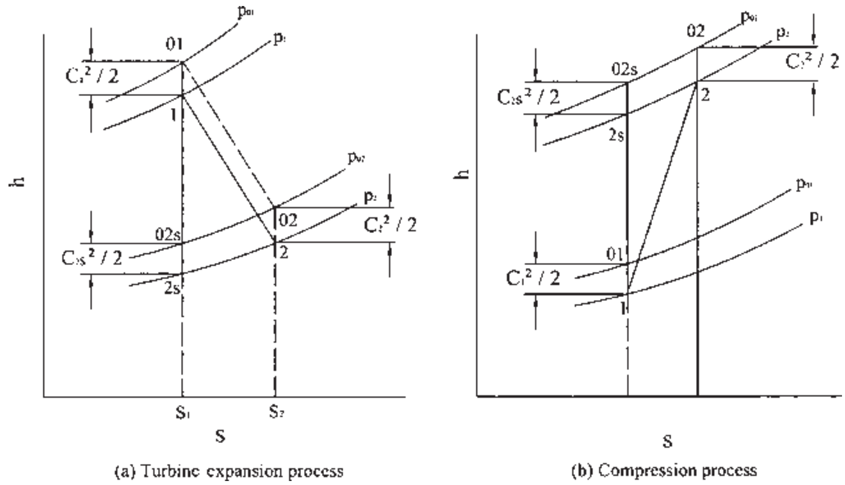


Figure 1.11 Enthalpy–entropy diagrams for turbines and compressors: (a) turbine expansion process, (b) compression process.

turbine-specific work is given by

$$W_t = h_{01} - h_{02} = (h_1 - h_2) + \frac{1}{2}(C_1^2 - C_2^2) \tag{1.66}$$

Similarly, the isentropic turbine rotor specific work between the same two pressures is

$$W_t' = h_{01} - h_{02s} = (h_1 - h_{2s}) + \frac{1}{2}\left(C_1^2 - C_{2s}^2\right) \tag{1.67}$$

Efficiency can be expressed in several ways. The choice of definitions depends largely upon whether the kinetic energy at the exit is usefully utilized or wasted. In multistage gas turbines, the kinetic energy leaving one stage is utilized in the next stage. Similarly, in turbojet engines, the energy in the gas exhausting through the nozzle is used for propulsion. For the above two cases, the turbine isentropic efficiency η_{tt} is defined as

$$\eta_{tt} = \frac{W_t}{W_t'} = \frac{h_{01} - h_{02}}{h_{01} - h_{02s}} \tag{1.68}$$

When the exhaust kinetic energy is not totally used but not totally wasted either, the total-to-static efficiency, η_{ts}, is used. In this case, the ideal or isentropic turbine work is that obtained between static points 01 and 2s. Thus

$$\eta_{ts} = \frac{h_{01} - h_{02}}{h_{01} - h_{02s} + \frac{1}{2}C_{2s}^2} = \frac{h_{01} - h_{02}}{h_{01} - h_{2s}} \tag{1.69}$$

If the difference between inlet and outlet kinetic energies is small, Eq. (1.69) becomes

$$\eta_{ts} = \frac{h_1 - h_2}{h_1 - h_{2s} + \frac{1}{2}C_{1s}^2}$$

An example where the outlet kinetic energy is wasted is a turbine exhausting directly to the atmosphere rather than exiting through a diffuser.

1.22 EFFICIENCY OF COMPRESSORS

The isentropic efficiency of the compressor is defined as

$$\eta_c = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} \tag{1.70}$$

If the difference between inlet and outlet kinetic energies is small, $\frac{1}{2}C_1^2 = \frac{1}{2}C_2^2$ and

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \tag{1.71}$$

1.23 POLYTROPIC OR SMALL-STAGE EFFICIENCY

Isentropic efficiency as described above can be misleading if used for compression and expansion processes in several stages. Turbomachines may be used in large numbers of very small stages irrespective of the actual number of stages in the machine. If each small stage has the same efficiency, then the isentropic efficiency of the whole machine will be different from the small stage efficiency, and this difference is dependent upon the pressure ratio of the machine.

Isentropic efficiency of compressors tends to decrease and isentropic efficiency of turbines tends to increase as the pressure ratios for which the machines are designed are increased. This is made more apparent in the following argument.

Consider an axial flow compressor, which is made up of several stages, each stage having equal values of η_c, as shown in Fig. 1.12.

Then the overall temperature rise can be expressed by

$$\Delta T = \sum \frac{\Delta T_s'}{\eta_s} = \frac{1}{\eta_s} \sum \Delta T_s'$$

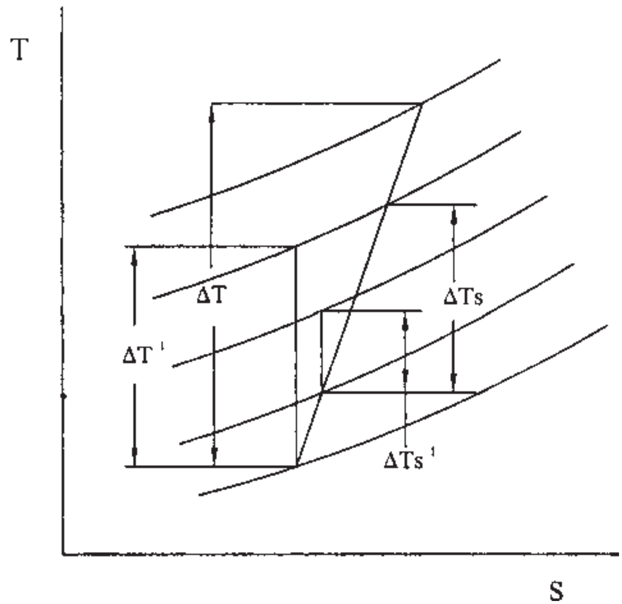


Figure 1.12 Compression process in stages.

(Prime symbol is used for isentropic temperature rise, and subscript s is for stage temperature).

Also, $\Delta T = \Delta^{T'}/\eta_c$ by definition of η_c, and thus: $\eta_s/\eta_c = \sum \Delta T_s{}'/\Delta T'$. It is clear from Fig. 1.12 that $\sum \Delta T'_s > \Delta T'$. Hence, $\eta_c < \eta_s$ and the difference will increase with increasing pressure ratio. The opposite effect is obtained in a turbine where η_s (i.e., small stage efficiency) is less than the overall efficiency of the turbine.

The above discussions have led to the concept of polytropic efficiency, η_∞, which is defined as the isentropic efficiency of an elemental stage in the process such that it is constant throughout the entire process.

The relationship between a polytropic efficiency, which is constant through the compressor, and the overall efficiency η_c may be obtained for a gas of constant specific heat.

For compression,

$$\eta_{\infty c} = \frac{\mathrm{d}T'}{\mathrm{d}T} = \text{constant}$$

But, $\frac{T}{P^{(\gamma-1)/\gamma}} =$ constant for an isentropic process, which in differential form is

$$\frac{\mathrm{d}T'}{\mathrm{d}T} = \frac{\gamma - 1}{\gamma}\frac{\mathrm{d}P}{P}$$

Now, substituting $\mathrm{d}T'$ from the previous equation, we have

$$\eta_{\infty c}\frac{\mathrm{d}T'}{\mathrm{d}T} = \frac{\gamma - 1}{\gamma}\frac{\mathrm{d}P}{P}$$

Integrating the above equation between the inlet 1 and outlet 2, we get

$$\eta_{\infty c} = \frac{\ln(P_2/P_1)^{\frac{\gamma-1}{\gamma}}}{\ln(T_2/T_1)} \tag{1.72}$$

Equation (1.72) can also be written in the form

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma\eta_{\infty c}}} \tag{1.73}$$

The relation between $\eta_{\infty c}$ and η_c is given by

$$\eta_c = \frac{(T'_2/T_1) - 1}{(T_2/T_1) - 1} = \frac{(P_2/P_1)^{\frac{\gamma-1}{\gamma}} - 1}{(P_2/P_1)^{\frac{\gamma-1}{\gamma\eta_{\infty c}}} - 1} \tag{1.74}$$

From Eq. (1.74), if we write $\frac{\gamma-1}{\gamma\eta_{\infty c}}$ as $\frac{n-1}{n}$, Eq. (1.73) is the functional relation between P and T for a polytropic process, and thus it is clear that the non isentropic process is polytropic.

Similarly, for an isentropic expansion and polytropic expansion, the following relations can be developed between the inlet 1 and outlet 2:

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\eta_{\infty t}(\gamma-1)}{\gamma}}$$

and

$$\eta_t = \frac{1 - \left(\frac{1}{P_1/P_2}\right)^{\frac{\eta_{\infty t}(\gamma-1)}{\gamma}}}{1 - \left(\frac{1}{P_1/P_2}\right)^{\frac{(\gamma-1)}{\gamma}}} \tag{1.75}$$

where $\eta_{\infty t}$ is the small-stage or polytropic efficiency for the turbine.

Figure 1.13 shows the overall efficiency related to the polytropic efficiency for a constant value of $\gamma = 1.4$, for varying polytropic efficiencies and for varying pressure ratios.

As mentioned earlier, the isentropic efficiency for an expansion process exceeds the small-stage efficiency. Overall isentropic efficiencies have been

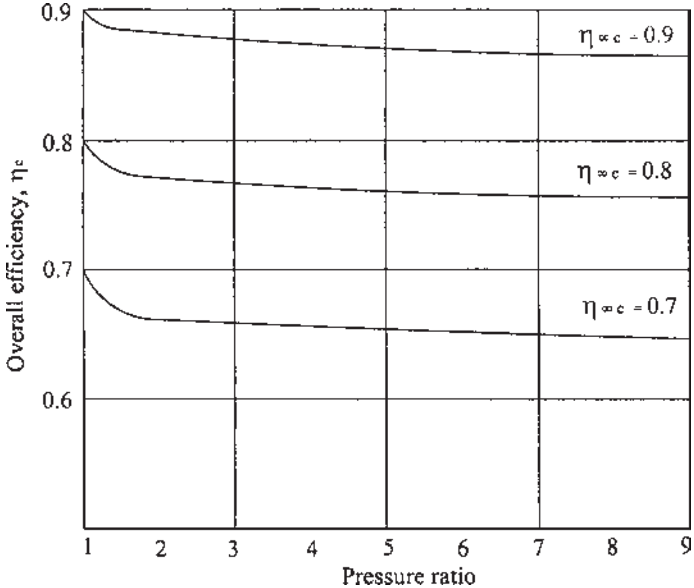


Figure 1.13 Relationships among overall efficiency, polytropic efficiency, and pressure ratio.

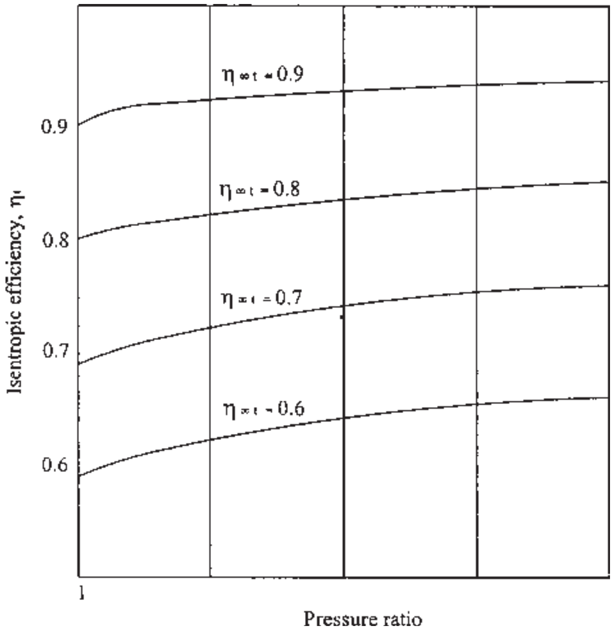


Figure 1.14 Turbine isentropic efficiency against pressure ratio for various polytropic efficiencies ($\gamma = 1.4$).

calculated for a range of pressure ratios and different polytropic efficiencies. These relationships are shown in Fig. 1.14.

1.24 NOZZLE EFFICIENCY

The function of the nozzle is to transform the high-pressure temperature energy (enthalpy) of the gasses at the inlet position into kinetic energy. This is achieved by decreasing the pressure and temperature of the gasses in the nozzle.

From Fig. 1.15, it is clear that the maximum amount of transformation will result when we have an isentropic process between the pressures at the entrance and exit of the nozzle. Such a process is illustrated as the path 1–2s. Now, when nozzle flow is accompanied by friction, the entropy will increase. As a result, the path is curved as illustrated by line 1–2. The difference in the enthalpy change between the actual process and the ideal process is due to friction. This ratio is known as the nozzle adiabatic efficiency and is called nozzle efficiency (η_n) or jet

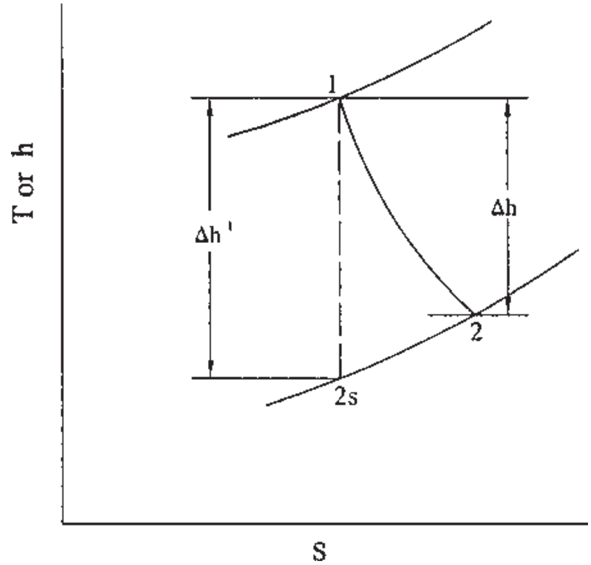


Figure 1.15 Comparison of ideal and actual nozzle expansion on a T-s or h–s plane.

pipe efficiency (η_j). This efficiency is given by:

$$\eta_j = \frac{\Delta h}{\Delta^{h\prime}} = \frac{h_{01} - h_{02}}{h_{01} - h_{02}{}'} = \frac{c_p(T_{01} - T_{02})}{c_p(T_{01} - T_{02}{}')} \tag{1.76}$$

1.25 DIFFUSER EFFICIENCY

The diffuser efficiency η_d is defined in a similar manner to compressor efficiency (see Fig. 1.16):

$$\eta_d = \frac{\text{Isentropic enthalpy rise}}{\text{Actual enthalpy rise}}$$
$$= \frac{h_{2s} - h_1}{h_2 - h_1} \tag{1.77}$$

The purpose of diffusion or deceleration is to convert the maximum possible kinetic energy into pressure energy. The diffusion is difficult to achieve and is rightly regarded as one of the main problems of turbomachinery design. This problem is due to the growth of boundary layers and the separation of the fluid molecules from the diverging part of the diffuser. If the rate of diffusion is too rapid, large losses in stagnation pressure are inevitable. On the other hand, if

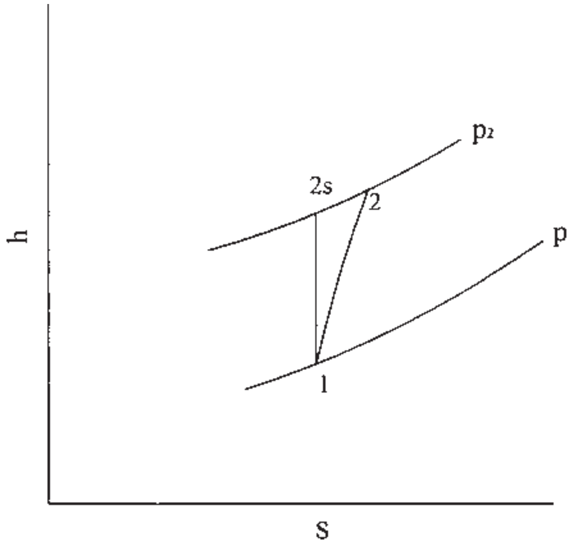


Figure 1.16 Mollier diagram for the diffusion process.

the rate of diffusion is very low, the fluid is exposed to an excessive length of wall and friction losses become predominant. To minimize these two effects, there must be an optimum rate of diffusion.

1.26 ENERGY TRANSFER IN TURBOMACHINERY

This section deals with the kinematics and dynamics of turbomachines by means of definitions, diagrams, and dimensionless parameters. The kinematics and dynamic factors depend on the velocities of fluid flow in the machine as well as the rotor velocity itself and the forces of interaction due to velocity changes.

1.27 THE EULER TURBINE EQUATION

The fluid flows through the turbomachine rotor are assumed to be steady over a long period of time. Turbulence and other losses may then be neglected, and the mass flow rate m is constant. As shown in Fig. 1.17, let ω (omega) be the angular velocity about the axis A–A.

Fluid enters the rotor at point 1 and leaves at point 2.

In turbomachine flow analysis, the most important variable is the fluid velocity and its variation in the different coordinate directions. In the designing of blade shapes, velocity vector diagrams are very useful. The flow in and across

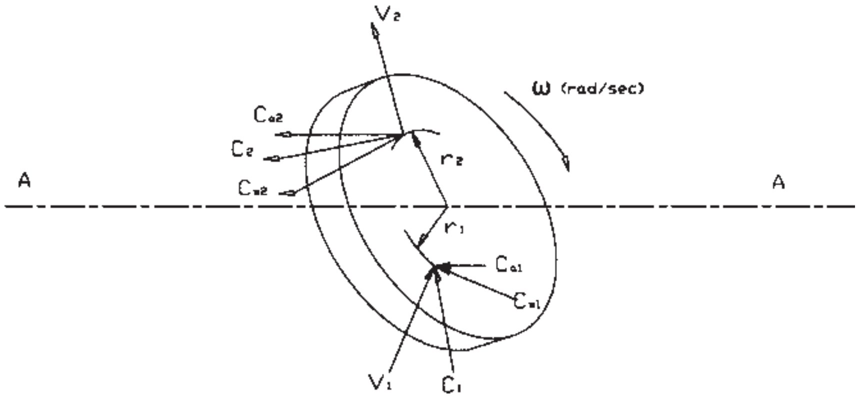


Figure 1.17 Velocity components for a generalized rotor.

the stators, the absolute velocities are of interest (i.e., C). The flow velocities across the rotor relative to the rotating blade must be considered. The fluid enters with velocity C_1, which is at a radial distance r_1 from the axis A–A. At point 2 the fluid leaves with absolute velocity (that velocity relative to an outside observer). The point 2 is at a radial distance r_2 from the axis A–A. The rotating disc may be either a turbine or a compressor. It is necessary to restrict the flow to a steady flow, i.e., the mass flow rate is constant (no accumulation of fluid in the rotor). The velocity C_1 at the inlet to the rotor can be resolved into three components; viz.;

C_{a1} — Axial velocity in a direction parallel to the axis of the rotating shaft.

C_{r1} — Radial velocity in the direction normal to the axis of the rotating shaft.

C_{w1} — whirl or tangential velocity in the direction normal to a radius.

Similarly, exit velocity C_2 can be resolved into three components; that is, C_{a2}, C_{r2}, and C_{w2}. The change in magnitude of the axial velocity components through the rotor gives rise to an axial force, which must be taken by a thrust bearing to the stationary rotor casing. The change in magnitude of the radial velocity components produces radial force. Neither has any effect on the angular motion of the rotor. The whirl or tangential components C_w produce the rotational effect. This may be expressed in general as follows:

The unit mass of fluid entering at section 1 and leaving in any unit of time produces:

The angular momentum at the inlet: $C_{w1}r_1$

The angular momentum at the outlet: $C_{w2}r_2$

And therefore the rate of change of angular momentum $= C_{w1}r_1 - C_{w2}r_2$

By Newton's laws of motion, this is equal to the summation of all the applied forces on the rotor; i.e., the net torque of the rotor τ (tau). Under steady flow conditions, using mass flow rate m, the torque exerted by or acting on the rotor will be:

$$\tau = m(C_{w1}r_1 - C_{w2}r_2)$$

Therefore the rate of energy transfer, W, is the product of the torque and the angular velocity of the rotor ω (omega), so:

$$W = \tau\omega = m\omega(C_{w1}r_1 - C_{w2}r_2)$$

For unit mass flow, energy will be given by:

$$W = \omega(C_{w1}r_1 - C_{w2}r_2) = (C_{w1}r_1\omega - C_{w2}r_2\omega)$$

But, $\omega\, r_1 = U_1$ and $\omega\, r_2 = U_2$.

$$\text{Hence, } W = (C_{w1}U_1 - C_{w2}U_2), \tag{1.78}$$

where, W is the energy transferred per unit mass, and U_1 and U_2 are the rotor speeds at the inlet and the exit respectively. Equation (1.78) is referred to as Euler's turbine equation. The standard thermodynamic sign convention is that work done by a fluid is positive, and work done on a fluid is negative. This means the work produced by the turbine is positive and the work absorbed by the compressors and pumps is negative. Therefore, the energy transfer equations can be written separately as

$$W = (C_{w1}U_1 - C_{w2}U_2) \text{ for turbine}$$

and

$$W = (C_{w2}U_2 - C_{w1}U_1) \text{ for compressor and pump.}$$

The Euler turbine equation is very useful for evaluating the flow of fluids that have very small viscosities, like water, steam, air, and combustion products.

To calculate torque from the Euler turbine equation, it is necessary to know the velocity components C_{w1}, C_{w2}, and the rotor speeds U_1 and U_2 or the velocities V_1, V_2, C_{r1}, C_{r2} as well as U_1 and U_2. These quantities can be determined easily by drawing the velocity triangles at the rotor inlet and outlet, as shown in Fig. 1.18. The velocity triangles are key to the analysis of turbomachinery problems, and are usually combined into one diagram. These triangles are usually drawn as a vector triangle:

Since these are vector triangles, the two velocities U and V are relative to one another, so that the tail of V is at the head of U. Thus the vector sum of U and V is equal to the vector C. The flow through a turbomachine rotor, the absolute velocities C_1 and C_2 as well as the relative velocities V_1 and V_2 can have three

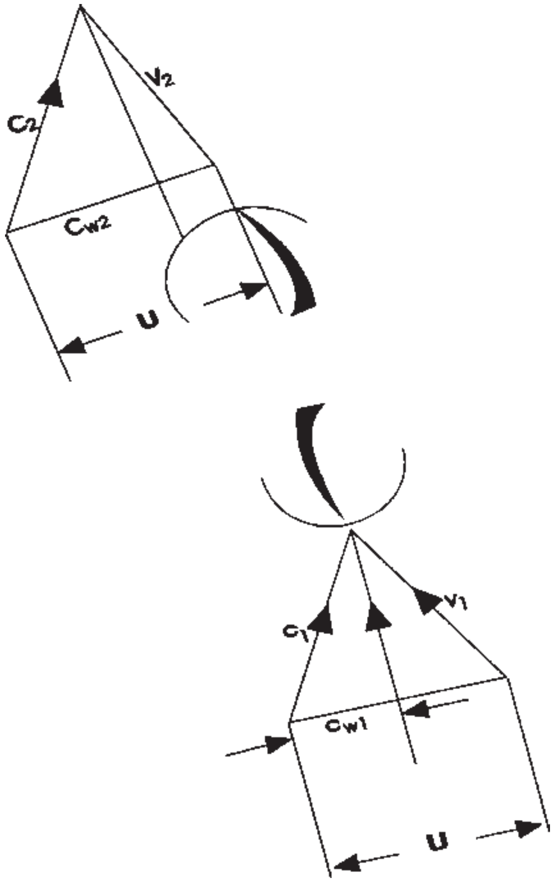


Figure 1.18 Velocity triangles for a rotor.

components as mentioned earlier. However, the two velocity components, one tangential to the rotor (C_w) and another perpendicular to it are sufficient. The component C_r is called the meridional component, which passes through the point under consideration and the turbomachine axis. The velocity components C_{r1} and C_{r2} are the flow velocity components, which may be axial or radial depending on the type of machine.

1.28 COMPONENTS OF ENERGY TRANSFER

The Euler equation is useful because it can be transformed into other forms, which are not only convenient to certain aspects of design, but also useful in understanding the basic physical principles of energy transfer. Consider the fluid velocities at the inlet and outlet of the turbomachine, again designated by the subscripts 1 and 2, respectively. By simple geometry,

$$C_{r2}^2 = C_2^2 - C_{w2}^2$$

and

$$C_{r2}^2 = V_2^2 - (U_2 - C_{w2})^2$$

Equating the values of C_{r2}^2 and expanding,

$$C_2^2 - C_{w2}^2 = V_2^2 - U_2^2 + 2U_2C_{w2} - C_{w2}^2$$

and

$$U_2C_{w2} = \frac{1}{2}\left(C_2^2 + U_2^2 - V_2^2\right)$$

Similarly,

$$U_1C_{w1} = \frac{1}{2}(C_1^2 + U_1^2 - V_1^2)$$

Inserting these values in the Euler equation,

$$E = \frac{1}{2}\left[(C_1^2 - C_2^2) + (U_1^2 - U_2^2) + (V_1^2 - V_2^2)\right] \tag{1.79}$$

The first term, $\frac{1}{2}(C_1^2 - C_2^2)$, represents the energy transfer due to change of absolute kinetic energy of the fluid during its passage between the entrance and exit sections. In a pump or compressor, the discharge kinetic energy from the rotor, $\frac{1}{2}C_2^2$, may be considerable. Normally, it is static head or pressure that is required as useful energy. Usually the kinetic energy at the rotor outlet is converted into a static pressure head by passing the fluid through a diffuser. In a turbine, the change in absolute kinetic energy represents the power transmitted from the fluid to the rotor due to an impulse effect. As this absolute kinetic energy change can be used to accomplish rise in pressure, it can be called a "virtual pressure rise" or "a pressure rise" which is possible to attain. The amount of pressure rise in the diffuser depends, of course, on the efficiency of the diffuser. Since this pressure rise comes from the diffuser, which is external to the rotor, this term, i.e., $\frac{1}{2}(C_1^2 - C_2^2)$, is sometimes called an "external effect."

The other two terms of Eq. (1.79) are factors that produce pressure rise within the rotor itself, and hence they are called "internal diffusion." The centrifugal effect, $\frac{1}{2}(U_1^2 - U_2^2)$, is due to the centrifugal forces that are developed as the fluid particles move outwards towards the rim of the machine. This effect is produced if the fluid changes radius as it flows from the entrance to the exit section. The third term, $\frac{1}{2}(V_1^2 - V_2^2)$, represents the energy transfer due to the change of the relative kinetic energy of the fluid. If $V_2 > V_1$, the passage acts like a nozzle and if $V_2 < V_1$, it acts like a diffuser. From the above discussions, it is apparent that in a turbocompresser, pressure rise occurs due to both external effects and internal diffusion effect. However, in axial flow compressors, the centrifugal effects are not utilized at all. This is why the pressure rise per stage is less than in a machine that utilizes all the kinetic energy effects available. It should be noted that the turbine derives power from the same effects.

Illustrative Example 1.1: A radial flow hydraulic turbine produces 32 kW under a head of 16 m and running at 100 rpm. A geometrically similar model producing 42 kW and a head of 6 m is to be tested under geometrically similar conditions. If model efficiency is assumed to be 92%, find the diameter ratio between the model and prototype, the volume flow rate through the model, and speed of the model.

Solution:

Assuming constant fluid density, equating head, flow, and power coefficients, using subscripts 1 for the prototype and 2 for the model, we have from Eq. (1.19),

$$\frac{P_1}{(\rho_1 N_1^3 D_1^5)} = \frac{P_2}{(\rho_2 N_2^3 D_2^5)}, \text{ where } \rho_1 = \rho_2.$$

Then, $\frac{D_2}{D_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{5}} \left(\frac{N_1}{N_2}\right)^{\frac{3}{5}}$ or $\frac{D_2}{D_1} = \left(\frac{0.032}{42}\right)^{\frac{1}{5}} \left(\frac{N_1}{N_2}\right)^{\frac{3}{5}} = 0.238 \left(\frac{N_1}{N_2}\right)^{\frac{3}{5}}$

Also, we know from Eq. (1.19) that

$$\frac{gH_1}{(N_1 D_1)^2} = \frac{gH_2}{(N_2 D_2)^2} \text{ (gravity remains constant)}$$

Then

$$\frac{D_2}{D_1} = \left(\frac{H_2}{H_1}\right)^{\frac{1}{2}} \left(\frac{N_1}{N_2}\right) = \left(\frac{6}{16}\right)^{\frac{1}{2}} \left(\frac{N_1}{N_2}\right)$$

Equating the diameter ratios, we get

$$0.238 \left(\frac{N_1}{N_2}\right)^{\frac{3}{5}} = \left(\frac{6}{16}\right)^{\frac{1}{2}} \left(\frac{N_1}{N_2}\right)$$

or

$$\left(\frac{N_2}{N_1}\right)^{\frac{2}{5}} = \frac{0.612}{0.238} = 2.57$$

Therefore the model speed is

$$N_2 = 100 \times (2.57)^{\frac{5}{2}} = 1059 \text{ rpm}$$

Model scale ratio is given by

$$\frac{D_2}{D_1} = (0.238)\left(\frac{100}{1059}\right)^{\frac{3}{5}} = 0.238(0.094)^{0.6} = 0.058.$$

$$\text{Model efficiency is } \eta_\mathrm{m} = \frac{\text{Power output}}{\text{Water power input}}$$

or,

$$0.92 = \frac{42 \times 10^3}{\rho g Q H},$$

or,

$$Q = \frac{42 \times 10^3}{0.92 \times 10^3 \times 9.81 \times 6} = 0.776 \ \mathrm{m^3/s}$$

Illustrative Example 1.2: A centrifugal pump delivers 2.5 $\mathrm{m^3/s}$ under a head of 14 m and running at a speed of 2010 rpm. The impeller diameter of the pump is 125 mm. If a 104 mm diameter impeller is fitted and the pump runs at a speed of 2210 rpm, what is the volume rate? Determine also the new pump head.

Solution:

First of all, let us assume that dynamic similarity exists between the two pumps. Equating the flow coefficients, we get [Eq. (1.3)]

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \quad \text{or} \quad \frac{2.5}{2010 \times (0.125)^3} = \frac{Q_2}{2210 \times (0.104)^3}$$

Solving the above equation, the volume flow rate of the second pump is

$$Q_2 = \frac{2.5 \times 2210 \times (0.104)^3}{2010 \times (0.125)^3} = 1.58 \ \mathrm{m^3/s}$$

Now, equating head coefficients for both cases gives [Eq. (1.9)]

$$gH_1/N_1^2 D_1^2 = gH_2/N_2^2 D_2^2$$

Substituting the given values,

$$\frac{9.81 \times 14}{(2010 \times 125)^2} = \frac{9.81 \times H_2}{(2210 \times 104)^2}$$

Therefore, $H_2 = 11.72$ m of water.

Illustrative Example 1.3: An axial flow compressor handling air and designed to run at 5000 rpm at ambient temperature and pressure of 18°C and 1.013 bar, respectively. The performance characteristic of the compressor is obtained at the atmosphere temperature of 25°C. What is the correct speed at which the compressor must run? If an entry pressure of 65 kPa is obtained at the point where the mass flow rate would be 64 kg/s, calculate the expected mass flow rate obtained in the test.

Solution:
Since the machine is the same in both cases, the gas constant R and diameter can be cancelled from the operating equations. Using first the speed parameter,

$$\frac{N_1}{\sqrt{T_{01}}} = \frac{N_2}{\sqrt{T_{02}}}$$

Therefore,

$$N_2 = 5000\left(\frac{273+25}{273+18}\right)^{\frac{1}{2}} = 5000\left(\frac{298}{291}\right)^{0.5} = 5060 \text{ rpm}$$

Hence, the correct speed is 5060 rpm. Now, considering the mass flow parameter,

$$\frac{m_1\sqrt{T_{01}}}{p_{01}} = \frac{m_2\sqrt{T_{02}}}{p_{02}}$$

Therefore,

$$m_2 = 64 \times \left(\frac{65}{101.3}\right)\left(\frac{291}{298}\right)^{0.5} = 40.58 \text{ kg/s}$$

Illustrative Example 1.4: A pump discharges liquid at the rate of Q against a head of H. If specific weight of the liquid is w, find the expression for the pumping power.

Solution:
Let Power P be given by:

$$P = f(w, Q, H) = kw^aQ^bH^c$$

where k, a, b, and c are constants. Substituting the respective dimensions in the above equation,

$$\mathrm{ML^2T^{-3}} = k(\mathrm{ML^{-2}T^{-2}})^a(\mathrm{L^3T^{-1}})^b(\mathrm{L})^c$$

Equating corresponding indices, for M, $1 = a$ or $a = 1$; for L, $2 = -2a + 3b + c$; and for T, $-3 = -2a - b$ or $b = 1$, so $c = 1$.

Therefore,

$$P = kwQH$$

Illustrative Example 1.5: Prove that the drag force F on a partially submerged body is given by:

$$F = V^2 l^2 \rho f\left(\frac{k}{l}, \frac{lg}{V^2}\right)$$

where V is the velocity of the body, l is the linear dimension, ρ, the fluid density, k is the rms height of surface roughness, and g is the gravitational acceleration.

Solution:

Let the functional relation be:

$$F = f(V, l, k, \rho, g)$$

Or in the general form:

$$F = f(F, V, l, k, \rho, g) = 0$$

In the above equation, there are only two primary dimensions. Thus, $m = 2$. Taking V, l, and ρ as repeating variables, we get:

$$\Pi_1 = (V)^{\mathrm{a}}(l)^{\mathrm{b}}(\rho)^{\mathrm{c}} F$$

$$\mathrm{M^oL^oT^o} = (\mathrm{LT}^{-1})^a(\mathrm{L})^b(\mathrm{ML}^{-3})^c(\mathrm{MLT}^{-2})$$

Equating the powers of M, L, and T on both sides of the equation, for M, $0 = c + 1$ or $c = -1$; for T, $0 = -a - 2$ or $a = -2$; and for L, $0 = a + b - 3c + 1$ or $b = -2$.

Therefore,

$$\Pi_1 = (V)^{-2}(l)^{-2}(\rho)^{-1}F = \frac{F}{V^2 l^2 \rho}$$

Similarly,

$$\Pi_2 = (V)^d(l)^e(\rho)^f(k)$$

Therefore,

$$\mathrm{M^0L^0T^0} = (\mathrm{LT}^{-1})^d(\mathrm{L})^e(\mathrm{ML}^{-3})^f(\mathrm{L})$$

for M, $0 = f$ or $f = 0$; for T, $0 = -d$ or $d = 0$; and for L, $0 = d + e - 3f + 1$ or $e = -1$.

Thus,

$$\Pi_2 = (V)^0(l)^{-1}(\rho)^0 k = \frac{k}{l}$$

and

$$\Pi_3 = (V)^g (l)^h (\rho)^i (g)$$

$$M^0L^0T^0 = (LT^{-1})^g (L)^h (ML^{-3})^i (LT^{-2})$$

Equating the exponents gives, for M, $0 = i$ or $i = 0$; for T, $0 = -g-2$ or $g = -2$; for L, $0 = g + h - 3i + 1$ or $h = 1$.

$$\text{Therefore,}\ \ \Pi_3 = V^{-2} l^1 \rho^0 g = \frac{lg}{V^2}$$

Now the functional relationship may be written as:

$$f\left(\frac{F}{V^2 l^2 \rho}, \frac{k}{l}, \frac{lg}{V^2}\right) = 0$$

Therefore,

$$F = V^2 l^2 \rho f\left(\frac{k}{l}, \frac{lg}{V^2}\right)$$

Illustrative Example 1.6: Consider an axial flow pump, which has rotor diameter of 32 cm that discharges liquid water at the rate of 2.5 m^3/min while running at 1450 rpm. The corresponding energy input is 120 J/kg, and the total efficiency is 78%. If a second geometrically similar pump with diameter of 22 cm operates at 2900 rpm, what are its (1) flow rate, (2) change in total pressure, and (3) input power?

Solution:
Using the geometric and dynamic similarity equations,

$$\frac{Q_1}{N_1 D_1^2} = \frac{Q_2}{N_2 D_2^2}$$

Therefore,

$$Q_2 = \frac{Q_1 N_2 D_2^2}{N_1 D_1^2} = \frac{(2.5)(2900)(0.22)^2}{(1450)(0.32)^2} = 2.363\ \text{m}^3/\text{min}$$

As the head coefficient is constant,

$$W_2 = \frac{W_1 N_2^2 D_2^2}{N_1^2 D_1^2} = \frac{(120)(2900)^2(0.22)^2}{(1450)^2(0.32)^2} = 226.88\ \text{J/kg}$$

The change in total pressure is:

$$\Delta P = W_2 \eta_{tt} \rho = (226.88)(0.78)(1000)\ \text{N/m}^2$$

$$= (226.88)(0.78)(1000)10^{-5} = 1.77\ \text{bar}$$

Input power is given by

$$P = \dot{m}W_2 = \frac{(1000)(2.363)(0.22688)}{60} = 8.94 \text{ kW}$$

Illustrative Example 1.7: Consider an axial flow gas turbine in which air enters at the stagnation temperature of 1050 K. The turbine operates with a total pressure ratio of 4:1. The rotor turns at 15500 rpm and the overall diameter of the rotor is 30 cm. If the total-to-total efficiency is 0.85, find the power output per kg per second of airflow if the rotor diameter is reduced to 20 cm and the rotational speed is 12,500 rpm. Take $\gamma = 1.4$.

Solution:
Using the isentropic P–T relation:

$$T_{02}^{'} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{(\gamma-1)}{2}} = (1050)\left(\frac{1}{4}\right)^{0.286} = 706.32\text{K}$$

Using total-to-total efficiency,

$$T_{01} - T_{02} = \left(T_{01} - T_{02}'\right)\eta_{\text{tt}} = (343.68)(0.85) = 292.13 \text{ K}$$

and

$$W_1 = c_{\text{p}}\Delta T_0 = (1.005)(292.13) = 293.59 \text{ kJ/kg}$$

$$W_2 = \frac{W_1 N_2^2 D_2^2}{N_1^2 D_1^2} = \frac{(293.59 \times 10^3)(12,500)^2(0.20)^2}{(15,500)^2(0.30)^2}$$

$$= 84,862 \text{ J/kg}$$

$\therefore$ Power output $= 84.86$ kJ/kg

Illustrative Example 1.8: At what velocity should tests be run in a wind tunnel on a model of an airplane wing of 160 mm chord in order that the Reynolds number should be the same as that of the prototype of 1000 mm chord moving at 40.5 m/s. Air is under atmospheric pressure in the wind tunnel.

Solution:
Let

Velocity of the model: V_{m}

Length of the model: $L_{\text{m}} = 160$ mm

Length of the prototype: $L_{\text{p}} = 1000$ mm

Velocity of the prototype: $V_{\text{p}} = 40.5$ m/s

According to the given conditions:

$$(\mathrm{Re})_\mathrm{m} = (\mathrm{Re})_\mathrm{p}$$

$$\frac{V_\mathrm{m} L_\mathrm{m}}{\nu_\mathrm{m}} = \frac{V_\mathrm{p} L_\mathrm{p}}{\nu_\mathrm{p}}, \text{ Therefore, } \nu_\mathrm{m} = \nu_\mathrm{p} = \nu_\mathrm{air}$$

Hence

$$V_\mathrm{m} L_\mathrm{m} = V_\mathrm{p} L_\mathrm{p},$$

or

$$V_\mathrm{m} = L_\mathrm{p} V_\mathrm{p}/L_\mathrm{m} = 40.5 \times 1000/160 = 253.13 \text{ m/s}$$

Illustrative Example 1.9: Show that the kinetic energy of a body equals kmV^2 using the method of dimensional analysis.

Solution:
Since the kinetic energy of a body depends on its mass and velocity,

$$\text{K.E.} = f(V, m), \text{ or K.E.} = \mathrm{k}V^a m^b.$$

Dimensionally,

$$\mathrm{FLT}^0 = (\mathrm{LT}^{-1})^a (\mathrm{FT}^2\mathrm{L}^{-1})^b$$

Equating the exponents of F, L, and T, we get:

$$\text{F: } 1 = b; \quad \text{L: } 1 = a - b; \quad \text{T: } 0 = -a + 2b$$

This gives $b = 1$ and $a = 2$. So, K.E. $= kV^2 m$, where k is a constant.

Illustrative Example 1.10: Consider a radial inward flow machine, the radial and tangential velocity components are 340 m/s and 50 m/s, respectively, and the inlet and the outlet radii are 14 cm and 7 cm, respectively. Find the torque per unit mass flow rate.

Solution:
Here,

$$r_1 = 0.14 \text{ m}$$

$$C_{\mathrm{w}1} = 340 \text{ m/s},$$

$$r_2 = 0.07 \text{ m}$$

$$C_{\mathrm{w}2} = 50 \text{ m/s}$$

Torque is given by:

$$T = r_1 C_{w1} - r_2 C_{w2}$$
$$= (0.14 \times 340 - 0.07 \times 50)$$
$$= (47.6 - 3.5) = 44.1 \text{ N-m per kg/s}$$

PROBLEMS

1.1 Show that the power developed by a pump is given by

$$P = kwQH$$

where $k =$ constant, $w =$ specific weight of liquid, $Q =$ rate of discharge, and $H =$ head dimension.

1.2 Develop an expression for the drag force on a smooth sphere of diameter D immersed in a liquid (of density ρ and dynamic viscosity μ) moving with velocity V.

1.3 The resisting force F of a supersonic plane in flight is given by:

$$F = f(L, V, \rho, \mu, k)$$

where $L =$ the length of the aircraft, $V =$ velocity, $\rho =$ air density, $\mu =$ air viscosity, and $k =$ the bulk modulus of air.

1.4 Show that the resisting force is a function of Reynolds number and Mach number.

1.5 The torque of a turbine is a function of the rate of flow Q, head H, angular velocity ω, specific weight w of water, and efficiency. Determine the torque equation.

1.6 The efficiency of a fan depends on density ρ, dynamic viscosity μ of the fluid, angular velocity ω, diameter D of the rotor and discharge Q. Express efficiency in terms of dimensionless parameters.

1.7 The specific speed of a Kaplan turbine is 450 when working under a head of 12 m at 150 rpm. If under this head, 30,000 kW of energy is generated, estimate how many turbines should be used.

(7 turbines).

1.8 By using Buckingham's Π theorem, show that dimensionless expression ΔP is given by:

$$\Delta P = \frac{4 f V^2 \rho l}{2D}$$

where $\triangle P$ = pressure drop in a pipe, V = mean velocity of the flow, l = length of the pipe, D = diameter of the pipe, μ = viscosity of the fluid, k = average roughness of the pipe, and ρ = density of the fluid.

1.9 If H_f is the head loss due to friction ($\triangle P/w$) and w is the specific weight of the fluid, show that

$$H_f = \frac{4fV^2 l}{2gD}$$

(other symbols have their usual meaning).

1.10 Determine the dimensions of the following in M.L.T and F.L.T systems: (1) mass, (2) dynamic viscosity, and (3) shear stress.

$$\left(\mathrm{M}, \mathrm{FT}^2\mathrm{L}^{-1}, \mathrm{ML}^{-1}\mathrm{T}^{-1}, \mathrm{FTL}^{-2}, \mathrm{ML}^{-1}\mathrm{T}^{-2}, \mathrm{FL}^{-3}\right)$$

NOTATION

A_r	area ratio
a	sonic velocity
B_r	breadth of prototype
C	velocity of gas, absolute velocity of turbo machinery
D	diameter of pipe, turbine runner, or pump
D_p	depth of the prototype
E	energy transfer by a rotor or absorbed by the rotor
F	force
F_r	force ratio
g	local acceleration due to gravity
H	head
h	specific enthalpy
h_0	stagnation enthalpy
K.E.	kinetic energy
L	length
L_p	length of prototype
L_r	scale ratio
M	Mach number
m	mass rate of flow
N	speed
N_s	specific speed
P	power
P_h	hydraulic power
P_m	power loss due to mechanical friction at the bearing
P_s	shaft power
P.E.	potential energy

p	fluid pressure
p_0	stagnation pressure
Q	volume rate of flow, heat transfer
R	gas constant
Re	Reynolds number
r	radius of rotor
s	specific entropy
sp . gr	specific gravity of fluid
T	temperature, time
T_0	stagnation temperature
t	time
U	rotor speed
V	relative velocity, mean velocity
W	work
V_r	volume ratio, velocity ratio
W_t	actual turbine work output
W_t'	isentropic turbine work output
α	absolute air angle
β	relative air angle
γ	specific weight, specific heat ratio
η	efficiency
$\eta_{\infty c}$	polytropic efficiency of compressor
$\eta_{\infty t}$	polytropic efficiency of turbine
η_c	compressor efficiency
η_d	diffuser efficiency
η_h	hydraulic efficiency
η_j	jet pipe or nozzle efficiency
η_m	mechanical efficiency
η_o	overall efficiency
η_p	prototype efficiency
η_s	isentropic efficiency
η_t	turbine efficiency
η_{ts}	total-to-static efficiency
η_{tt}	total-to-total efficiency
η_v	volumetric efficiency
μ	absolute or dynamic viscosity
ν	kinematic viscosity
Π	dimensionless parameter
ρ	mass density
τ	shear stress, torque exerted by or acting on the rotor
ω	angular velocity

SUFFIXES

0	stagnation conditions
1	inlet to rotor
2	outlet from the rotor
3	outlet from the diffuser
a	axial
h	hub
r	radial
t	tip
w	whirl or tangential

2

Hydraulic Pumps

2.1 INTRODUCTION

Hydraulics is defined as the science of the conveyance of liquids through pipes. The pump is often used to raise water from a low level to a high level where it can be stored in a tank. Most of the theory applicable to hydraulic pumps has been derived using water as the working fluid, but other liquids can also be used. In this chapter, we will assume that liquids are totally incompressible unless otherwise specified. This means that the density of liquids will be considered constant no matter how much pressure is applied. Unless the change in pressure in a particular situation is very great, this assumption will not cause a significant error in calculations. Centrifugal and axial flow pumps are very common hydraulic pumps. Both work on the principle that the energy of the liquid is increased by imparting kinetic energy to it as it flows through the pump. This energy is supplied by the impeller, which is driven by an electric motor or some other drive. The centrifugal and axial flow pumps will be discussed separately in the following sections.

2.2 CENTRIFUGAL PUMPS

The three important parts of centrifugal pumps are (1) the impeller, (2) the volute casing, and (3) the diffuser.

2.2.1 Impeller

The centrifugal pump is used to raise liquids from a lower to a higher level by creating the required pressure with the help of centrifugal action. Whirling motion is imparted to the liquid by means of backward curved blades mounted on a wheel known as the impeller. As the impeller rotates, the fluid that is drawn into the blade passages at the impeller inlet or eye is accelerated as it is forced radially outwards. In this way, the static pressure at the outer radius is much higher than at the eye inlet radius. The water coming out of the impeller is then lead through the pump casing under high pressure. The fluid has a very high velocity at the outer radius of the impeller, and, to recover this kinetic energy by changing it into pressure energy, diffuser blades mounted on a diffuser ring may be used. The stationary blade passages have an increasing cross-sectional area. As the fluid moves through them, diffusion action takes place and hence the kinetic energy is converted into pressure energy. Vaneless diffuser passages may also be used. The fluid moves from the diffuser blades into the volute casing. The functions of a volute casing can be summarized as follows: It collects water and conveys it to the pump outlet. The shape of the casing is such that its area of cross-section gradually increases towards the outlet of the pump. As the flowing water progresses towards the delivery pipe, more and more water is added from the outlet periphery of the impeller. Figure 2.1 shows a centrifugal pump impeller with the velocity triangles at inlet and outlet.

For the best efficiency of the pump, it is assumed that water enters the impeller radially, i.e., $\alpha_1 = 90°$ and $C_{w1} = 0$. Using Euler's pump equation, the work done per second on the water per unit mass of fluid flowing

$$E = \frac{W}{m} = (U_2 C_{w2} - U_1 C_{w1}) \tag{2.1}$$

Where C_w is the component of absolute velocity in the tangential direction. E is referred to as the Euler head and represents the ideal or theoretical head developed by the impeller only. The flow rate is

$$Q = 2\pi r_1 C_{r1} b_1 = 2\pi r_2 C_{r2} b_2 \tag{2.2}$$

Where C_r is the radial component of absolute velocity and is perpendicular to the tangent at the inlet and outlet and b is the width of the blade. For shockless entry and exit to the vanes, water enters and leaves the vane tips in a direction parallel to their relative velocities at the two tips.

As discussed in Chapter 1, the work done on the water by the pump consists of the following three parts:

1. The part $(C_2^2 - C_1^2)/2$ represents the change in kinetic energy of the liquid.
2. The part $(U_2^2 - U_1^2)/2$ represents the effect of the centrifugal head or energy produced by the impeller.

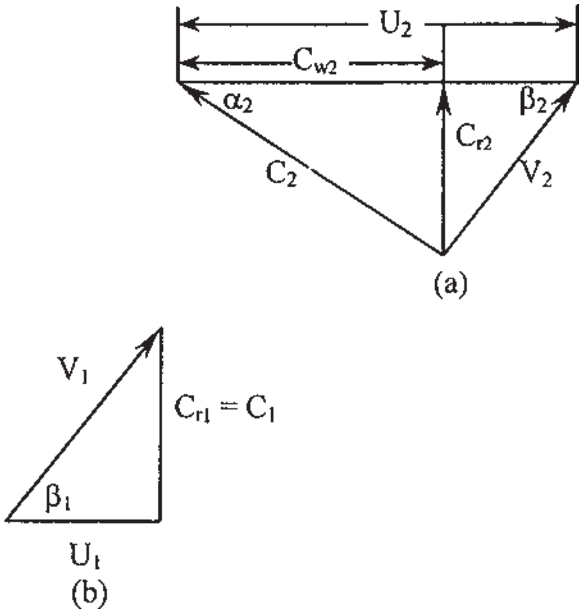


Figure 2.1 Velocity triangles for centrifugal pump impeller.

3. The part $(V_2^2 - V_1^2)/2$ represents the change in static pressure of the liquid, if the losses in the impeller are neglected.

2.3 SLIP FACTOR

From the preceding section, it may be seen that there is no assurance that the actual fluid will follow the blade shape and leave the impeller in a radial direction. There is usually a slight slippage of the fluid with respect to the blade rotation. Figure 2.2 shows the velocity triangles at impeller tip.

In Fig. 2.2, β_2' is the angle at which the fluid leaves the impeller, and β_2 is the actual blade angle, and C_{w2} and C_{w2}' are the tangential components of absolute velocity corresponding to the angles β_2 and β_2', respectively. Thus, C_{w2} is reduced to C_{w2}' and the difference ΔC_w is defined as the slip. The slip factor is defined as

$$\text{Slip factor, } \sigma = \frac{C_{w2}'}{C_{w2}}$$

According to Stodola's theory, slip in centrifugal pumps and compressors is due to relative rotation of fluid in a direction opposite to that of impeller with the same

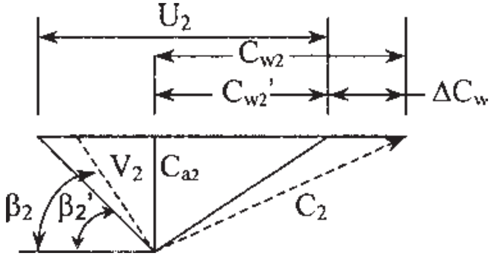


Figure 2.2 Velocity triangle at impeller outlet with slip.

angular velocity as that of an impeller. Figure 2.3 shows the leading side of a blade, where there is a high-pressure region while on the trailing side of the blade there is a low-pressure region.

Due to the lower pressure on the trailing face, there will be a higher velocity and a velocity gradient across the passage. This pressure distribution is associated with the existence of circulation around the blade, so that low velocity on the high-pressure side and high velocity on the low-pressure side and velocity distribution is not uniform at any radius. Due to this fact, the flow may separate from the suction surface of the blade. Thus, C_{w2} is less than C_{w2}' and the difference is defined as the slip. Another way of looking at this effect, as given by Stodola, is shown in Fig. 2.4, the impeller itself has an angular velocity ω so that, relative to the impeller, the fluid must have an angular velocity of $-\omega$; the result of this being a circulatory motion relative to the channel or relative eddy. The net result of the previous discussion is that the fluid is discharged from the impeller at an angle relative to the impeller, which is less than the vane angle as mentioned earlier.

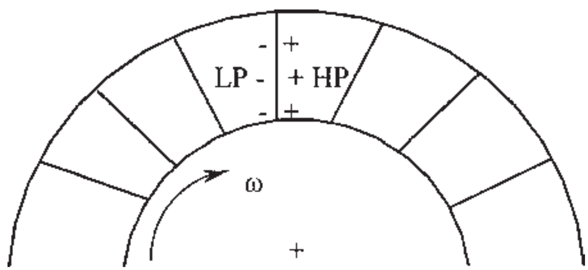


Figure 2.3 Pressure distribution on impeller vane. LP = low pressure, HP = high pressure.

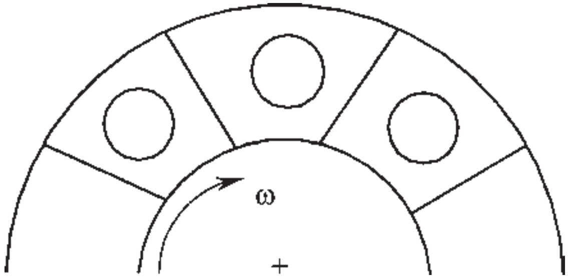


Figure 2.4 Relative eddy in impeller channel.

Hence, the slip factor σ is defined as

$$\sigma = \frac{C'_{w2}}{C_{w2}} \tag{2.3}$$

For purely radial blades, which are often used in centrifugal compressors, β_2 will be 90° and the Stodola slip factor becomes

$$\sigma = 1 - \frac{\pi}{n} \tag{2.4}$$

where n is the number of vanes. The Stanitz slip factor is given by

$$\sigma = 1 - \frac{0.63\pi}{n} \tag{2.5}$$

When applying a slip factor, the Euler pump equation becomes

$$\frac{W}{m} = \sigma U_2 C_{w2} - U_1 C_{w1} \tag{2.6}$$

Typically, the slip factor lies in the region of 0.9, while the slip occurs even if the fluid is ideal.

2.4 PUMP LOSSES

The following are the various losses occurring during the operation of a centrifugal pump.

1. Eddy losses at entrance and exit of impeller, friction losses in the impeller, frictional and eddy losses in the diffuser, if provided.
2. Losses in the suction and delivery pipe. The above losses are known as hydraulic losses.
3. Mechanical losses are losses due to friction of the main bearings, and stuffing boxes. Thus, the energy supplied by the prime mover to

impeller is equal to the energy produced by impeller plus mechanical losses. A number of efficiencies are associated with these losses.

Let ρ be the density of liquid; Q, flow rate; H, total head developed by the pump; P_s, shaft power input; H_i, total head across the impeller; and h_i, head loss in the impeller. Then, the overall efficiency η_o is given by:

$$\eta_o = \frac{\text{Fluid power developed by pump}}{\text{Shaft power input}} = \frac{\rho g Q H}{P_s} \tag{2.7}$$

Casing efficiency η_c is given by:

$$\begin{aligned}\eta_c &= \text{Fluid power at casing outlet/fluid power at casing inlet}\\ &= \text{Fluid power at casing outlet/(fluid power developed by}\\ &\quad\ \text{impeller} - \text{leakage loss)}\\ &= \rho g Q H / \rho g Q H_i = H/H_i\end{aligned} \tag{2.8}$$

Impeller efficiency η_i is given by:

$$\begin{aligned}\eta_i &= \text{Fluid power at impeller exit/fluid}\\ &\qquad \text{power supplied to impeller}\\ &= \text{Fluid power at impeller exit/(fluid power}\\ &\qquad \text{developed by impeller}\\ &\qquad + \text{impeller loss)}\\ &= \rho g Q_i H_i / \left[\rho g Q_i (H_i + h_i)\right] = H_i/(H_i + h_i)\end{aligned} \tag{2.9}$$

Volumetric efficiency η_v is given by:

$$\begin{aligned}\eta_v &= \text{Flow rate through pump/flow rate through impeller}\\ &= Q/(Q+q)\end{aligned} \tag{2.10}$$

Mechanical efficiency η_m is given by:

$$\begin{aligned}\eta_m &= \text{Fluid power supplied to the impeller/power}\\ &\qquad \text{input to the shaft}\\ &= \rho g Q_i (h_i + H_i)/P_s\end{aligned} \tag{2.11}$$

Therefore,

$$\eta_o = \eta_c \eta_i \eta_v \eta_m \tag{2.12}$$

A hydraulic efficiency may be defined as

$$\eta_h = \frac{\text{Actual head developed by pump}}{\text{Theoretical head developed by impeller}}$$
$$= \frac{H}{(H_i + h_i)} \tag{2.13}$$

The head H is also known as manometric head.

2.5 THE EFFECT OF IMPELLER BLADE SHAPE ON PERFORMANCE

The various blade shapes utilized in impellers of centrifugal pumps/compressors are shown in Fig. 2.5. The blade shapes can be classified as:

1. Backward-curved blades ($\beta_2 < 90°$)
2. Radial blades ($\beta_2 = 90°$)
3. Forward-curved blades ($\beta_2 > 90°$)

As shown in Fig. 2.5, for backward-curved vanes, the value of C_{w2} (whirl component at outlet) is much reduced, and thus, such rotors have a low energy transfer for a given impeller tip speed, while forward-curved vanes have a high value of energy transfer. Therefore, it is desirable to design for high values of β_2 (over 90°), but the velocity diagrams show that this also leads to a very high value of C_2. High kinetic energy is seldom required, and its reduction to static pressure by diffusion in a fixed casing is difficult to perform in a reasonable sized casing. However, radial vanes ($\beta_2 = 90°$) have some particular advantages for very high-speed compressors where the highest possible pressure is required. Radial vanes are relatively easy to manufacture and introduce no complex bending stresses (Fig. 2.6).

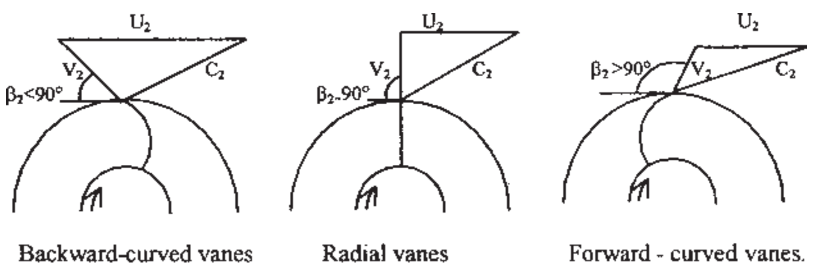


Figure 2.5 Centrifugal pump outlet velocity triangles for varying blade outlet angle.

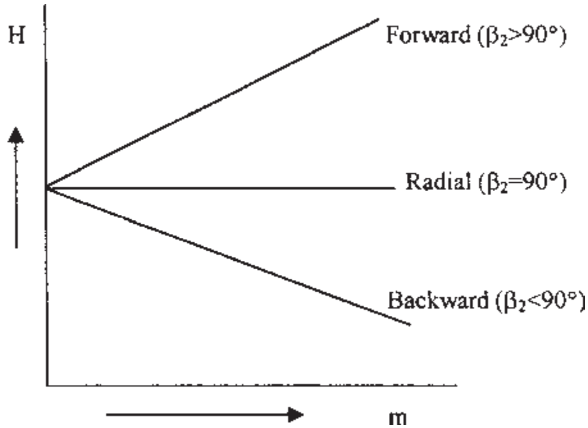


Figure 2.6 Characteristics for varying outlet blade angle.

2.6 VOLUTE OR SCROLL COLLECTOR

A volute or scroll collector consists of a circular passage of increasing cross-sectional area (Fig. 2.7). The advantage of volute is its simplicity and low cost. The cross-sectional area increases as the increment of discharge increases around the periphery of the impeller, and, if the velocity is constant in the volute,

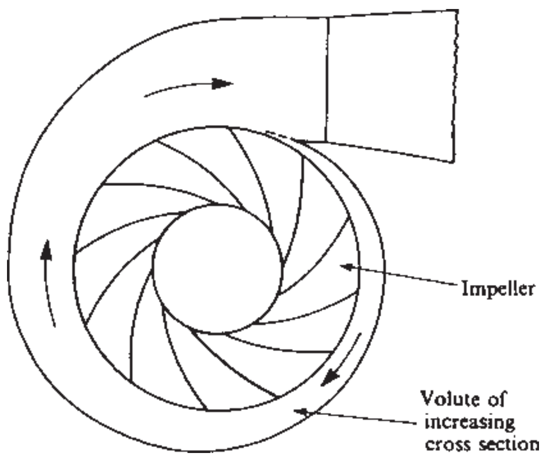


Figure 2.7 Volute or scroll collector.

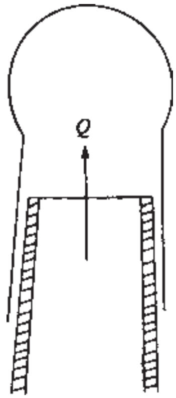


Figure 2.8 Cross-section of volute casing.

then the static pressure is likewise constant and the radial thrust will be zero. Any deviation in capacity (i.e., flow rate) from the design condition will result in a radial thrust which if allowed to persist could result in shaft bending.

The cross-sectional shape of the volute is generally similar to that shown in Fig. 2.8, with the sidewalls diverging from the impeller tip and joined by a semicircular outer wall. The circular section is used to reduce the losses due to friction and impact when the fluid hits the casing walls on exiting from the impeller.

2.7 VANELESS DIFFUSER

For the diffusion process, the vaneless diffuser is reasonably efficient and is best suited for a wide range of operations. It consists simply of an annular passage without vanes surrounding the impeller. A vaneless diffuser passage is shown in Fig. 2.9. The size of the diffuser can be determined by using the continuity equation. The mass flow rate in any radius is given by

$$m = \rho A C_r = 2\pi r b \rho C_r \tag{2.14}$$

where b is the width of the diffuser passage,

$$C_r = \frac{r_2 b_2 \rho_2 C_{r2}}{r b \rho} \tag{2.15}$$

where subscripted variables represent conditions at the impeller outlet and the unsubscripted variables represent conditions at any radius r in the vaneless diffuser. Assuming the flow is frictionless in the diffuser, angular momentum

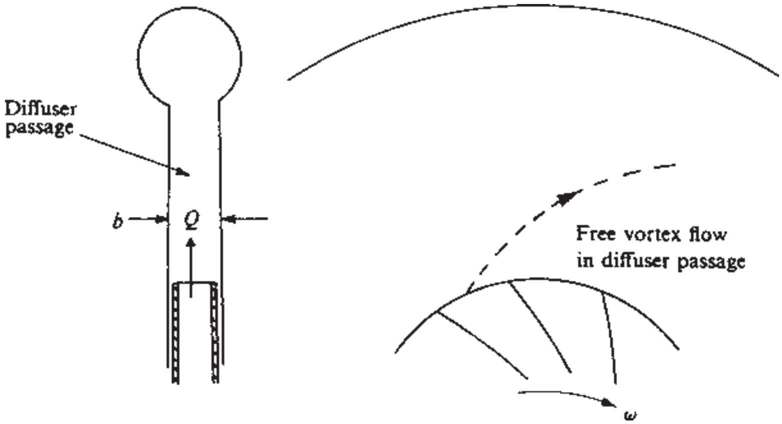


Figure 2.9 Vaneless diffuser.

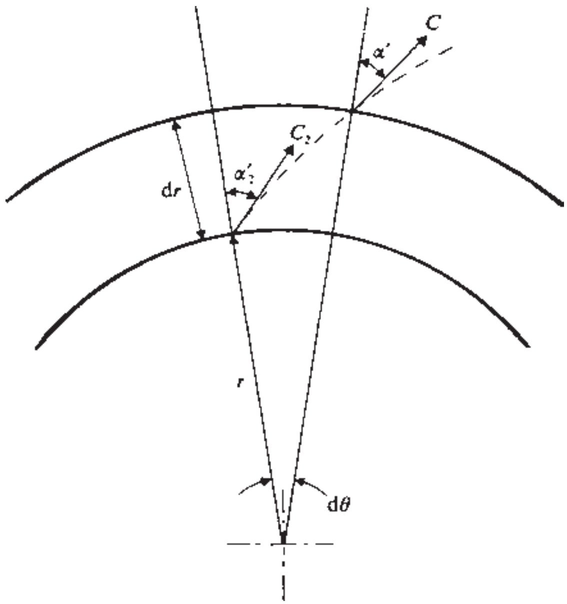


Figure 2.10 Logarithmic spiral flow in vaneless space.

is constant and

$$C_w = (C_{w2} r_2)/r \tag{2.16}$$

But the tangential velocity component (C_w) is usually very much larger than the radial velocity component C_r, and, therefore, the ratio of the inlet to outlet diffuser velocities $\frac{C_2}{C_3} = \frac{r_3}{r_2}$.

It means that for a large reduction in the outlet kinetic energy, a diffuser with a large radius is required. For an incompressible flow, rC_r is constant, and, therefore, $\tan \alpha = C_w/C_r =$ constant. Thus, the flow maintains a constant inclination to radial lines, the flow path traces a logarithmic spiral.

As shown in Fig. 2.10, for an incremental radius dr, the fluid moves through angle dθ, then $r\mathrm{d}\theta = \mathrm{d}r \tan \alpha$.

Integrating we have

$$\theta - \theta_2 = \tan \alpha \log(r/r_2) \tag{2.17}$$

Substituting $\alpha = 78°$ and $(r/r_2) = 2$, the change in angle of the diffuser is equal to 180°. Because of the long flow path with this type of diffuser, friction effects are high and the efficiency is low.

2.8 VANED DIFFUSER

The vaned diffuser is advantageous where small size is important. In this type of diffuser, vanes are used to diffuse the outlet kinetic energy of the fluid at a much higher rate than is possible by a simple increase in radius, and hence it is possible to reduce the length of flow path and diameter. The vane number, the angle of divergence is smaller, and the diffuser becomes more efficient, but greater is the friction. The cross section of the diffuser passage should be square to give a maximum hydraulic radius. However, the number of diffuser vanes should have no common factor with the number of impeller vanes. The collector and diffuser operate at their maximum efficiency at the design point only. Any deviation from the design discharge will change the outlet velocity triangle and the subsequent flow in the casing.

2.9 CAVITATION IN PUMPS

Cavitation is caused by local vaporization of the fluid, when the local static pressure of a liquid falls below the vapor pressure of the liquid. Small bubbles or cavities filled with vapor are formed, which suddenly collapse on moving forward with the flow into regions of high pressure. These bubbles collapse with tremendous force, giving rise to pressure as high as 3500 atm. In a centrifugal pump, these low-pressure zones are generally at the impeller inlet, where the fluid is locally accelerated over the vane surfaces. In turbines, cavitation is most likely to occur at the downstream outlet end of a blade on the low-pressure leading face. When cavitation occurs, it causes the following undesirable effects:

1. Local pitting of the impeller and erosion of the metal surface.
2. Serious damage can occur from this prolonged cavitation erosion.
3. Vibration of machine and noise is also generated in the form of sharp cracking sounds when cavitation takes place.
4. A drop in efficiency due to vapor formation, which reduces the effective flow areas.

The avoidance of cavitation in conventionally designed machines can be regarded as one of the essential tasks of both pump and turbine designers. This cavitation imposes limitations on the rate of discharge and speed of rotation of the pump.

A cavitation parameter is defined as $\sigma_c =$ pump total inlet head above vapor pressure/head developed by the pump or at the inlet flange

$$\sigma_c = \left(\frac{p_1}{\rho g} + \frac{V_1^2}{2g} - \frac{p_v}{\rho g}\right) \Big/ H \tag{2.18}$$

The numerator of Eq. (2.18) is a suction head and is called the net positive suction

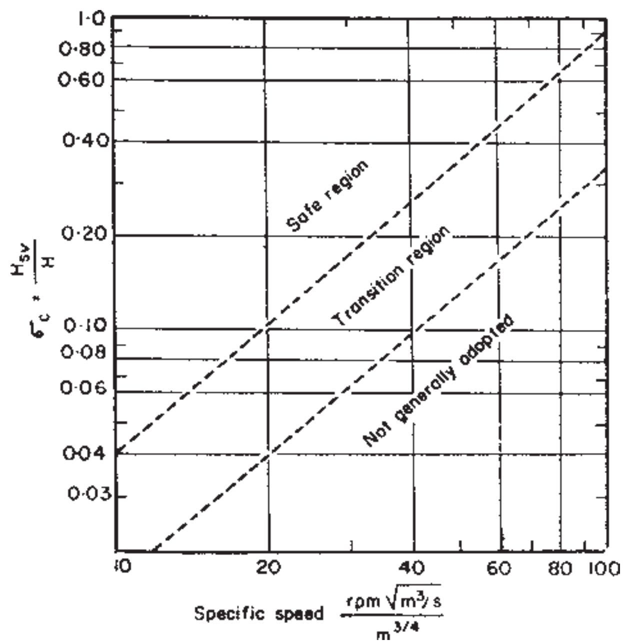


Figure 2.11 Cavitation limits for radial flow pumps.

head (NPSH) of the pump. It is a measure of the energy available on the suction side of the pump, and H is the manometric head. The cavitation parameter is a function of specific speed, efficiency of the pump, and number of vanes. Figure 2.11 shows the relationship between σ_c and N_s. It may be necessary in the selection of pumps that the value of σ_c does not fall below the given value by the plots in Fig. 2.11 for any condition of operation.

2.10 SUCTION SPECIFIC SPEED

The efficiency of the pump is a function of flow coefficient and suction specific speed, which is defined as

$$N_{suc} = NQ^{1/2}\left[g(\mathrm{NPSH})\right]^{-3/4}$$

Thus,

$$\eta = f\left(Q, N_{suc}\right)$$

The cavitation parameter may also be determined by the following equation

$$\begin{aligned} N_s/N_{suc} &= (\mathrm{NPSH})^{3/4}/H^{3/4} \\ &= \sigma_c^{3/4} \end{aligned} \tag{2.19}$$

2.11 AXIAL FLOW PUMP

In an axial flow pump, pressure is developed by flow of liquid over blades of airfoil section. It consists of a propeller-type of impeller running in a casing. The advantage of an axial flow pump is its compact construction as well as its ability to run at extremely high speeds. The flow area is the same at inlet and outlet and the minimum head for this type of pump is the order of 20 m.

2.12 PUMPING SYSTEM DESIGN

Proper pumping system design is the most important single element in minimizing the life cycle cost. All pumping systems are comprised of a pump, a driver, pipe installation, and operating controls. Proper design considers the interaction between the pump and the rest of the system and the calculation of the operating duty point(s) (Fig. 2.12). The characteristics of the piping system must be calculated in order to determine required pump performance. This applies to both simple systems as well as to more complex (branched) systems.

Both procurement costs and the operational costs make up the total cost of an installation during its lifetime. A number of installation and operational costs are directly dependent on the piping diameter and the components in the piping system.

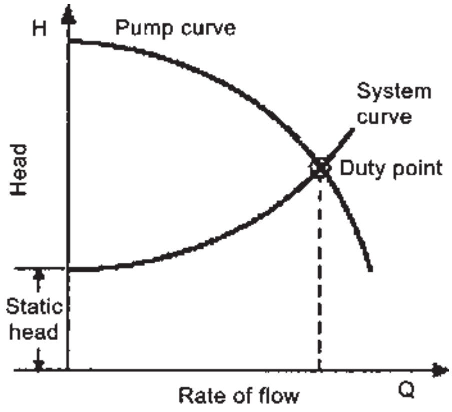


Figure 2.12 The duty point of the pump is determined by the intersection of the system curve and the pump curve as shown above.

A considerable amount of the pressure losses in the system are caused by valves, in particular, control valves in throttle-regulated installations. In systems with several pumps, the pump workload is divided between the pumps, which together, and in conjunction with the piping system, deliver the required flow.

The piping diameter is selected based on the following factors:

- Economy of the whole installation (pumps and system)
- Required lowest flow velocity for the application (e.g., avoid sedimentation)
- Required minimum internal diameter for the application (e.g., solid handling)
- Maximum flow velocity to minimize erosion in piping and fittings
- Plant standard pipe diameters.

Decreasing the pipeline diameter has the following effects:

- Piping and component procurement and installation costs will decrease.
- Pump installation procurement costs will increase as a result of increased flow losses with consequent requirements for higher head pumps and larger motors. Costs for electrical supply systems will therefore increase.
- Operating costs will increase as a result of higher energy usage due to increased friction losses.

Some costs increase with increasing pipeline size and some decrease. Because of this, an optimum pipeline size may be found, based on minimizing costs over the life of the system. A pump application might need to cover several duty points, of which the largest flow and/or head will determine the rated duty for the pump. The pump user must carefully consider the duration of operation at the individual duty points to properly select the number of pumps in the installation and to select output control.

2.12.1 Methods for Analyzing Existing Pumping Systems

The following steps provide an overall guideline to improve an existing pumping system.

- Assemble a complete document inventory of the items in the pumping system.
- Determine the flow rates required for each load in the system.
- Balance the system to meet the required flow rates of each load.
- Minimize system losses needed to balance the flow rates.
- Affect changes to the pump to minimize excessive pump head in the balanced system.
- Identify pumps with high maintenance cost.

One of two methods can be used to analyze existing pumping systems. One consists of observing the operation of the actual piping system, and the second consists of performing detailed calculations using fluid analysis techniques. The first method relies on observation of the operating piping system (pressures, differential pressures, and flow rates), the second deals with creating an accurate mathematical model of the piping system and then calculating the pressure and flow rates with the model.

The following is a checklist of some useful means to reduce the life cycle cost of a pumping system.

- Consider all relevant costs to determine the life cycle cost.
- Procure pumps and systems using life cycle cost considerations.
- Optimize total cost by considering operational costs and procurement costs.
- Consider the duration of the individual pump duty points.
- Match the equipment to the system needs for maximum benefit.
- Match the pump type to the intended duty.
- Do not oversize the pump.
- Match the driver type to the intended duty.
- Specify motors to have high efficiency.
- Monitor and sustain the pump and the system to maximize benefit.
- Consider the energy wasted using control valves.

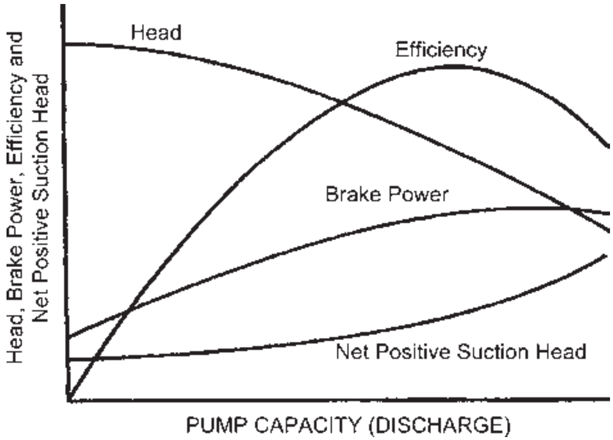


Figure 2.13 Typical pump characteristics.

2.12.2 Pump System Interaction

The actual operating point on the pump system characteristic curve is defined by its interaction with the operating characteristics of the installed system (Fig. 2.13).

The system characteristics will consist of:

- The total static head, being the difference in elevation between the upstream and downstream controls (generally represented by reservoir levels),
- The energy losses in the system (generally pipe friction), which are normally expressed as a function of velocity head.
- The interaction point of these curves represents the actual operating point (as shown later), defining the *Head* supplied by the pump and the *Discharge* of the system. The efficiency of the pump under these conditions will also be defined.

Note that the efficiency of the pump at this operating point is the critical parameter in pump selection and suitability for a particular system (Figs. 2.14 and 2.15).

2.13 LIFE CYCLE ANALYSIS

Over the economic life of a pumped supply system, a number of design parameter will change. System behavior for all possible operating environments is needed (Fig. 2.16). Parameters that will change include:

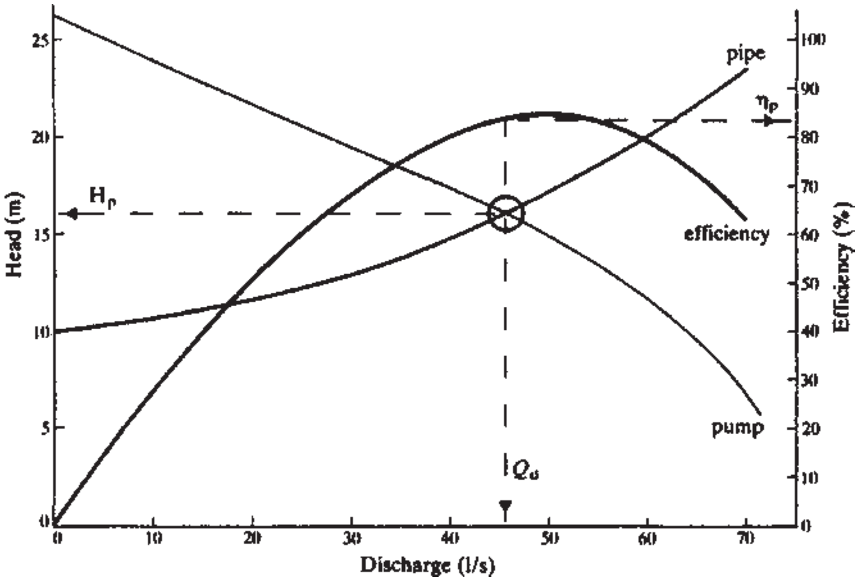


Figure 2.14 Pump–system interaction point and pump efficiency.

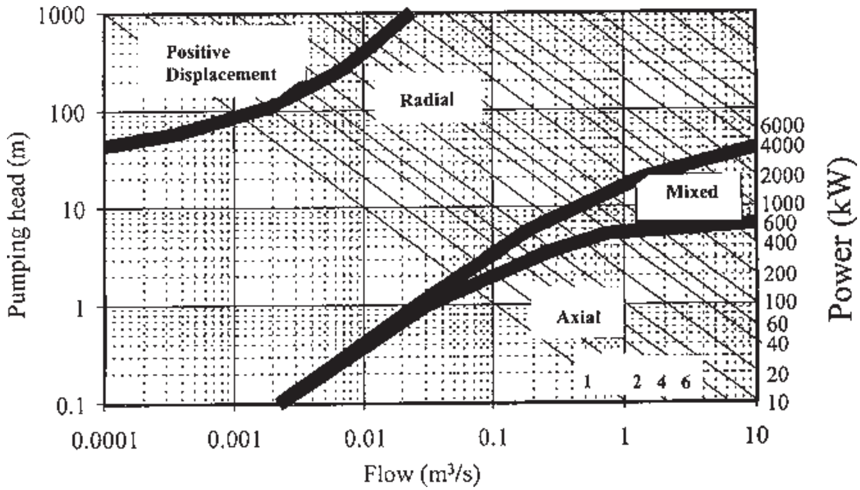


Figure 2.15 Selection of pump type.

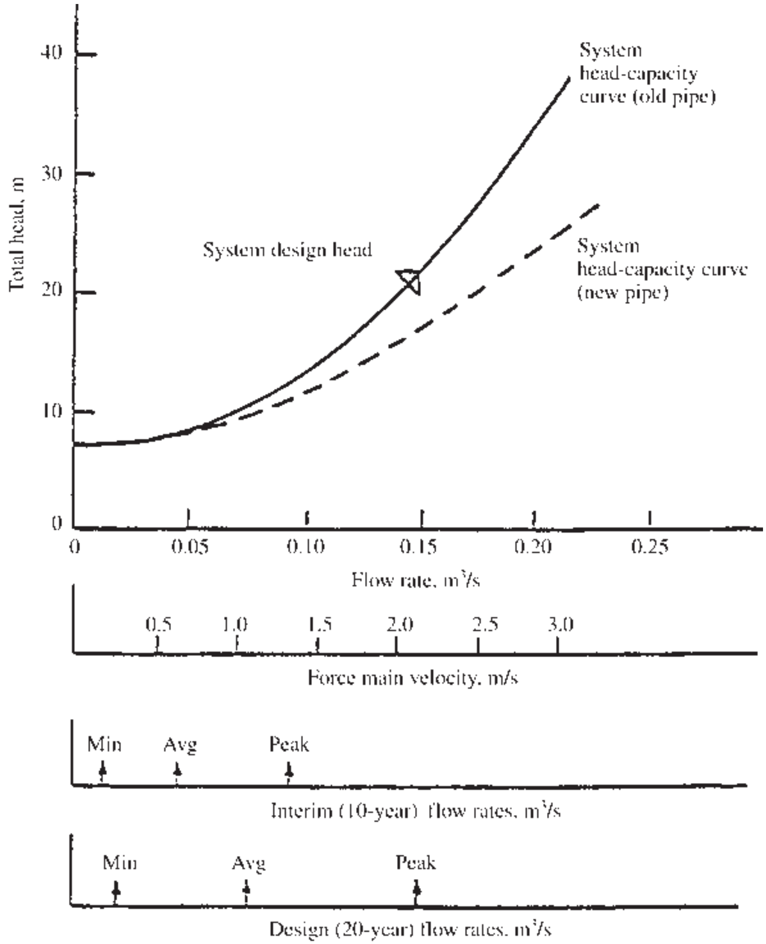


Figure 2.16 Variations in demand and operating characteristics.

- Seasonal variations in demand.
- Water demand increases as the system population expands.
- Increasing pipe friction as the system ages.

For all operating conditions, it is necessary to maintain pump operation close to peak efficiency. This can be achieved using multiple pumps and timed pumping periods.

2.14 CHANGING PUMP SPEED

The most common pump–motor combination employed in water supply operations is a close coupled system powered by an electric motor. These units can only operate at a speed related to the frequency of the A.C. supply (50 cycles/s or 3000 cycles/min), with the number of pairs of poles in the motor (N) reducing the pump speed to $3000/N$ revolutions per minute.

Pumps driven through belt drives or powered by petrol or diesel motors are more flexible allowing the pump speed to be adjusted to suit the operational requirements. Analysis of system operation will require the head–discharge-efficiency characteristic for the particular operating speed.

Given the head–discharge-efficiency characteristics for speed N (in tabular form), the corresponding characteristics for any speed N' can be established as follows:

$$Q' = \left(\frac{N'}{N}\right)Q \quad \text{flow points} \tag{2.20}$$

$$H' = \left(\frac{N'}{N}\right)^2 H \quad \text{head points} \tag{2.21}$$

$$\eta' = \eta \quad \text{efficiency points} \tag{2.22}$$

The data set for the new pump speed can then be matched to the system characteristics.

2.15 MULTIPLE PUMP OPERATION

The most common type of pumping station has two or more pumps operating in parallel. This provides flexibility of operation in coping with a range of flow conditions and allows maintenance of individual units while other units are in operation.

Occasionally situations will be encountered where pumps are operated in series to boost outputs. This is generally a temporary measure as any failure of one unit will severely affect system operation.

Composite characteristics (head–discharge curves) are obtained by combining the individual curves. The composite curve is employed in the same manner (i.e., intersection with system curve) *s* an individual curve (Fig. 2.17).

Where pumps operate in parallel, the composite curve is obtained by adding the *flow rates* for a given *head*.

Where pumps operate in series, the composite is obtained by adding *heads* for a given *flow rate*.

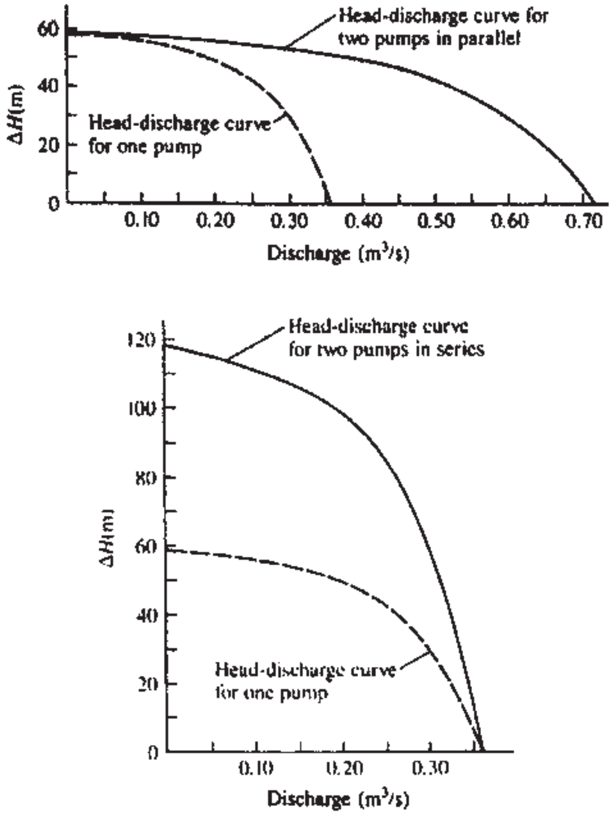


Figure 2.17 Composite pump characteristics.

2.15.1 Net Positive Suction Head

Simply, NPSH is the minimum suction condition (pressure) required to prevent pump cavitation. Conceptually, NPSH can be imagined as the pressure drop between the pump inlet flange and the point inside the pump where the fluid dynamic action, as it leaves the impeller, causes a pressure rise. Sufficient NPSH allows for pumping without liquid vaporizing in the pump first-stage impeller eye as the fluid pressure drops due to pump suction losses (Fig. 2.18).

The NPSH required is reported in head of fluid (being pumped) required at the pump inlet. As such, NPSH required has units of length (height). Usually, the datum line for pump NPSH is the centerline of the inlet. This is satisfactory for small pumps. For larger pumps, the NPSH requirements reported by the manufacturer should be verified for the datum line being discussed. The NPSH

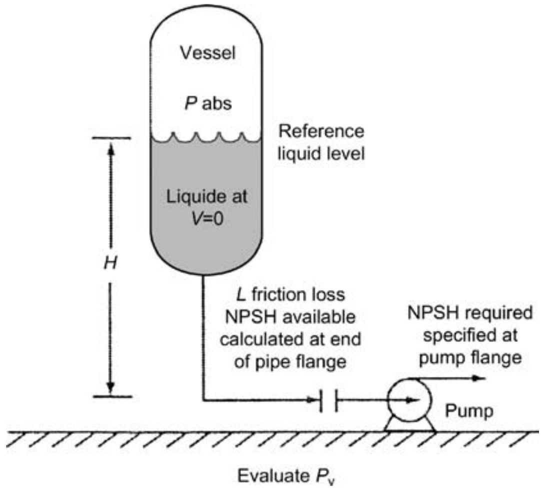


Figure 2.18 The elements of Eq. (2.24) are illustrated with a pump taking suction from a tower.

available differs from NPSH required. The NPSH required determined during the manufacturers test and shown on the vendor's pump curve is based upon a 3% head pump differential loss. The NPSH available must be large enough to eliminate head loss. The NPSH available is the excess of pressure over the liquid's vapor pressure at the pump suction flange. Except in rare circumstances, centrifugal pumps require the NPSH available to be greater than NPSH required to avoid cavitation during operation. Determining the NPSH available is the process engineer's job. Determining the NPSH required is the job of the pump vendor.

Our concern here is the process system side of determining what NPSH is available. Pressure balance and NPSH available derive from Bernoulli's equation for a stationary conduit where the total energy (head) in a system is the same for any point along a streamline (assuming no friction losses). Incorporating friction losses and restating the formula in a form familiar to process engineers, the NPSH available in a system can be expressed as:

$$\mathrm{NPSH_a} = \frac{2.31(P + P_a - P_v)}{\gamma} + \left(S - B - L + \frac{V^2}{2g}\right) \tag{2.23}$$

where $\mathrm{NPSH_a}$ is the net positive suction head available (ft); P, pressure above liquid (psi gage); P_a, atmospheric pressure (psi); P_v, vapor pressure of liquid at pumping conditions (psia); γ, specific gravity of liquid at pumping conditions;

S, static height of liquid from grade (ft); B, distance of pump centerline (suction nozzle centerline for vertical pumps); L, suction system friction losses (ft of pumping liquid); and V is the average liquid velocity at pump suction nozzle (ft/s).

Converting to absolute pressures, fluid density and resetting the datum line to the pump centerline results in:

$$\mathrm{NPSH_a} = \frac{144(P_{\mathrm{abs}} - P_{\mathrm{v}})}{\rho} + H - L + \frac{V^2}{2g} \tag{2.24}$$

where P_{abs} is the pressure above liquids (psia); ρ, fluid density (lb/ft^3); and H is the static height of liquid between liquid level and pump suction centerline (datum line), ft.

Illustrative Example 2.1: A centrifugal pump runs at a tip speed of 12 m/s and a flow velocity of 1.5 m/s. The impeller diameter is 1.2 m and delivers 3.8 m^3/min of water. The outlet blade angle is 28° to the tangent at the impeller periphery. Assuming that the fluid enters in the axial direction and zero slip, calculate the torque delivered by the impeller.

Solution:

From Fig. 2.2, for zero slip $\beta_2 = \beta_2^1$. Using Eq. (2.1), the Euler head $H = E = (U_2C_{w2} - U_1C_{w1})/g$. Since $C_{w1} = 0$, as there is no inlet whirl component, head H is given by

$$H = \frac{U_2C_{w2}}{g} = \frac{U_2}{g}\left(U_2 - \frac{1.5}{\tan 28^\circ}\right) = \frac{12}{9.81}\left(12 - \frac{1.5}{\tan 28^\circ}\right)$$

$$= 11.23\ \mathrm{m}$$

$$\text{Power delivered} = \rho g Q H \frac{J}{s} = \frac{1000(9.81)(3.8)(11.23)}{60(1000)}$$

$$= 6.98\ \mathrm{kW}$$

Torque delivered = Power/angular velocity = 6980(0.6)/12 = 349 Nm.

Illustrative Example 2.2: A fluid passes through an impeller of 0.22 m outlet diameter and 0.1 m inlet diameter. The impeller is rotating at 1250 rpm, and the outlet vane angle is set back at an angle of 22° to the tangent. Assuming that the fluid enters radially with velocity of flow as 3.5 m/s, calculate the head imparted to a fluid.

Solution:

Since fluid enters in the radial direction, $C_{w1} = 0$, $\alpha_1 = 90^\circ$, $\beta_2 = 22^\circ$, $C_{a1} = 3.5\ \mathrm{m/s} = C_{a2}$

Head developed $H = C_{w2}U_2/g$

Impeller tip speed, $U_2 = \frac{\pi DN}{60} = \frac{\pi(0.22)(1250)}{60} = 14.40\,\text{m/s}$

Whirl velocity at impeller outlet, from velocity diagram,

$$C_{w2} = U_2 - (C_{a2}/\tan\beta_2)$$
$$= 14.40 - (3.5/\tan 22^\circ) = 5.74\,\text{m/s}$$

Therefore, the head imparted is given by

$$H = 5.74(14.40)/9.81 = 8.43\,\text{m}$$

Design Example 2.3: A centrifugal pump impeller runs at 1400 rpm, and vanes angle at exit is 25°. The impeller has an external diameter of 0.4 m and an internal diameter of 0.2 m. Assuming a constant radial flow through the impeller at 2.6 m/s, calculate (1) the angle made by the absolute velocity of water at exit with the tangent, (2) the inlet vane angle, and (3) the work done per kg of water (Fig. 2.19).

Solution:

1. Impeller tip speed is given by

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.4)(1400)}{60} = 29.33\,\text{m/s}$$

Whirl velocity at impeller tip

$$C_{w2} = U_2 - \frac{C_{r2}}{\tan\beta_2} = 29.33 - \frac{2.6}{\tan 25^\circ} = 23.75\,\text{m/s}$$

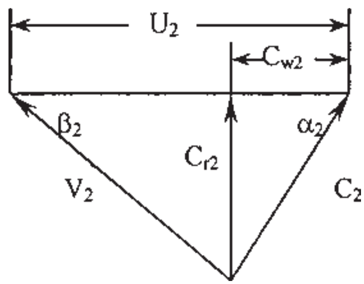


Figure 2.19 Velocity triangle at outlet.

Now, from the velocity triangle at impeller tip,

$$\tan \alpha_2 = \frac{C_{r2}}{C_{w2}} = \frac{2.6}{23.75} = 0.1095$$

Therefore, $\alpha_2 = 6.25°$.

2. Impeller velocity at inlet

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi(0.2)1400}{60} = 14.67\,\text{m/s}$$

$$\tan \beta_1 = \frac{C_{r1}}{U_1} = \frac{2.6}{14.67} = 0.177$$

Therefore, $\beta_1 = 10.05°$.

3. Work done per kg of water is given by

$$C_{w2}U_2 = 23.75(29.33) = 696.59\,\text{Nm} = 696.59\,\text{J}.$$

Design Example 2.4: A centrifugal pump impeller has a diameter of 1.2 m; rpm 210; area at the outer periphery 0.65 m^2; angle of vane at outlet 25°, and ratio of external to internal diameter 2:1. Calculate (1) the hydraulic efficiency, (2) power, and (3) minimum speed to lift water against a head of 6.2 m. Assume that the pump discharges 1550 l/s (Fig. 2.20).

Solution:

1. Here, $Q = 1550\,\text{l/s}$, $\beta_2 = 25°$, $H = 6.2\,\text{m}$, $D_2/D_1 = 2$, $D_2 = 1.2\,\text{m}$, $N = 210$ rpm, $A = 0.65\,\text{m}^2$.
   Velocity of flow at impeller tip

$$C_{r2} = \frac{Q}{A} = \frac{1550}{1000(0.65)} = 2.385\,\text{m/s}$$

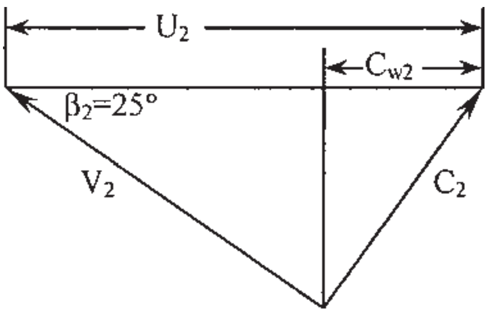


Figure 2.20 Velocity triangle at impeller outlet.

Impeller tip speed,

$$U_2 = \frac{\pi(1.2)(210)}{60} = 13.2\,\text{m/s}$$

$$C_{w2} = U_2 - \frac{C_{r2}}{\tan\beta_2} = 13.2 - \frac{2.385}{\tan 25°} = 8.09\,\text{m/s}$$

Assuming radial entry, theoretical head is given by

$$H = \frac{C_{w2}U_2}{9.81} = \frac{8.09(13.2)}{9.81} = 10.89\,\text{m}$$

Assuming slip factor, $\sigma = 1$, hydraulic efficiency is given by

$$\eta_h = \frac{6.2(100)}{10.89} = 56.9\%.$$

2. Power $P = (1550)(10.89)(9.81)/1000 = 165.59\,\text{kW}$.
3. Centrifugal head is the minimum head. Therefore,

$$\frac{U_2^2 - U_1^2}{2g} = 6.2$$

It is given that $U_1 = U_2/2$. Therefore,

$$U_2^2 - 0.25U_2^2 = 2(9.81)(6.2)$$

i.e., $U_2 = 12.74\,\text{m/s}$
Hence, minimum speed is $= 12.74(60)/\pi(1.2) = 203\,\text{rpm}$.

Illustrative Example 2.5: A centrifugal pump is required to pump water against a total head of 35 m at the rate of 45 l/s. Find the horsepower of the pump, if the overall efficiency is 60%.

Solution:

Total head, $H = 35\text{m}$

Discharge, $Q = 45\text{l/s} = 0.045\,\text{m}^3/\text{s}$

Overall efficiency, $\eta_o = 60\% = 0.60$

$$\text{Power, } P = \frac{\rho g Q H}{\eta_o}\frac{J}{s} = \frac{\rho g Q H}{1000\eta_o}\text{kW}$$

$$= \frac{9.81(0.045)(35)}{0.746(0.60)}$$

$$= 34.5\,\text{hp}$$

Illustrative Example 2.6: A centrifugal pump impeller has 0.3 m inlet diameter and 0.6 m external diameters. The pump runs at 950 rpm, and the entry

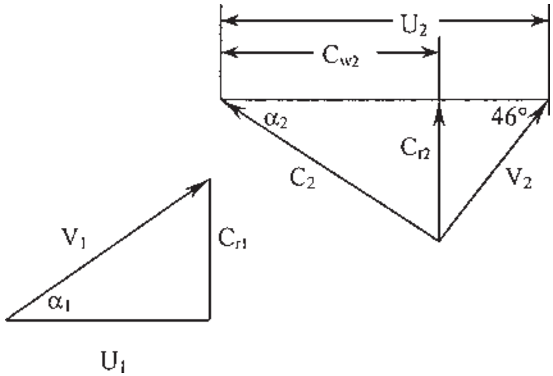


Figure 2.21 Velocity triangle at impeller outlet and inlet.

of the pump is radial. The velocity of flow through the impeller is constant at 3.5 m/s. The impeller vanes are set back at angle of 46° to the outer rim. Calculate (1) the vane angle at inlet of a pump, (2) the velocity direction of water at outlet, and (3) the work done by the water per kg of water (Fig. 2.21).

Solution:

1. Velocity of flow, $C_{r1} = C_{r2} = 3.5$ m/s. Let α_1 be the vane angle at inlet. Tangential velocity of impeller at inlet

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi(0.3)(950)}{60} = 14.93 \text{ m/s}$$

From inlet velocity triangle

$$\tan \alpha_1 = \frac{C_{r1}}{U_1} = \frac{3.5}{14.93} = 0.234$$

Therefore, $\alpha_1 = 13.19°$.

2. Tangential velocity of impeller at outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.6)(950)}{60} = 29.86 \text{ m/s}$$

For velocity of whirl at impeller outlet, using velocity triangle at outlet

$$C_{w2} = U_2 - \frac{C_{r2}}{\tan 46°} = 29.86 - \frac{3.5}{\tan 46°} = 26.48 \text{ m/s}$$

and $C_2^2 = C_{r2}^2 + C_{w2}^2 = 3.5^2 + 26.48^2, \quad C_2 = 26.71$ m/s

where C_2 is the velocity of water at outlet. Let α_2 be the direction of water at outlet, and, therefore, α_2 is given by

$$\tan \alpha_2 = \frac{C_{r2}}{C_{w2}} = \frac{3.5}{26.48} = 0.132$$

i.e., $\alpha_2 = 7.53°$.

3. Work done by the wheel per kg of water

$$W = C_{w2}U_2 = 26.48(29.86) = 790.69\,\text{Nm}$$

Design Example 2.7: A centrifugal pump delivers water at the rate of 8.5 m^3/min against a head of 10 m. It has an impeller of 50 cm outer diameter and 25 cm inner diameter. Vanes are set back at outlet at an angle of 45°, and impeller is running at 500 rpm. The constant velocity of flow is 2 m/s. Determine (1) the manometric efficiency, (2) vane angle at inlet, and (3) minimum starting speed of the pump (Fig. 2.22).

Solution:

1. The manometric efficiency is given by

$$\eta_{\text{man}} = \frac{H}{(C_{w2}U_2/g)}$$

From outlet velocity triangle

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.5)(500)}{60} = 13.0\,\text{m/s}$$

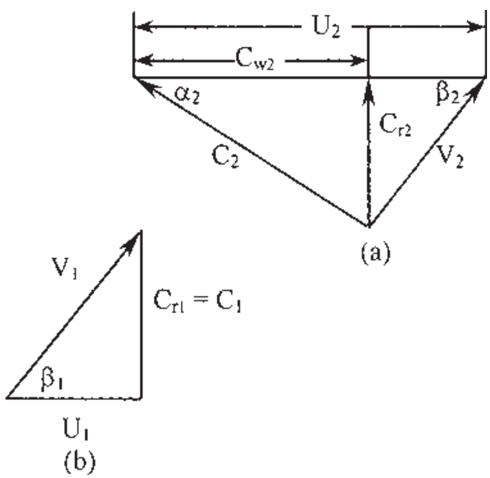


Figure 2.22 Velocity triangle (a) outlet, (b) inlet.

Now, $\tan\beta_2 = C_{r2}/(U_2 - C_{w2})$, or $\tan 45° = 2/(U_2 - C_{w2})$, or $1 = 2/(13 - C_{w2})$, $C_{w2} = 11$ m/s.

Hence, $\eta_{man} = \dfrac{H}{(C_{w2}U_2/g)} = \dfrac{10(9.81)}{11(13)} = 68.6\%$

2. Vane angle at inlet β_1

$$\tan\beta_1 = \frac{C_{r1}}{U_1} \text{ and } U_1 = 0.5 \times U_2 = 6.5 \text{ m/s}$$

$$\therefore \tan\beta_1 = \frac{2}{6.5} = 0.308$$

i.e., $\beta_1 = 17°$.

3. The minimum starting speed is

$$(U_2^2 - U_1^2)/2g = H \quad \text{or} \quad \frac{\left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2}{2(9.81)} = 10$$

Therefore, $N = 618$ rpm.

Illustrative Example 2.8: A centrifugal pump impeller has 0.6 m outside diameter and rotated at 550 rpm. Vanes are radial at exit and 8.2 cm wide. Velocity of flow through the impeller is 3.5 m/s, and velocity in the delivery pipe is 2.5 m/s. Neglecting other losses, calculate head and power of the pump (Fig. 2.23).

Solution:

1. $D_2 = 0.6$ m, $N = 550$ rpm, $C_{r2} = 3.5$ m/s, $C_{w2} = U_2$.
   Impeller speed at outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.6)(550)}{60} = 17.29 \text{ m/s}.$$

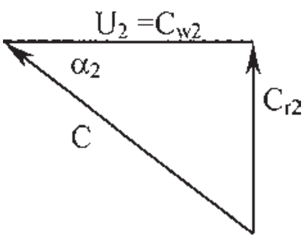


Figure 2.23 Velocity triangle for Example 2.8.

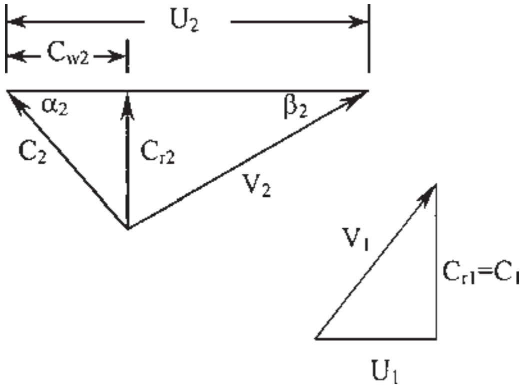


Figure 2.24 Velocity triangles for Example 2.9.

Head, through which the water can be lifted,

$$H = \frac{C_{w2}U_2}{g} - \frac{V^2}{2g} \text{(neglecting all losses)}$$

$$= \frac{(17.29)(17.29)}{9.81} - \frac{2.5^2}{2(9.81)} = 30.47 - 0.319$$

$$= 30.2 \text{ m of water.}$$

2. Power $= \dfrac{\rho g Q H}{1000}$ kW
   where $Q = \pi D_2 b_2 C_{r2}$ (where b_2 is width)

   $$= \pi(0.6)(0.082)(3.5) = 0.54 \text{ m}^3/\text{s}$$

   Therefore, power is given by

   $$P = \frac{\rho g Q H}{1000} \text{kW} = \frac{1000(9.81)(0.54)(30.2)}{1000} = 160\,\text{kW}.$$

Illustrative Example 2.9: A centrifugal pump impeller has a diameter of 1 m and speed of 11 m/s. Water enters radially and discharges with a velocity whose radial component is 2.5 m/s. Backward vanes make an angle of 32° at exit. If the discharge through the pump is 5.5 m^3/min, determine (1) h.p. of the pump and (2) turning moment of the shaft (Fig. 2.24).

Solution:

1. Data

   $D_2 = 1$ m,

   $U_2 = 11$ m/s,

$\alpha_1 = 90°$,

$C_{r2} = 2.5$ m/s,

$\beta_2 = 32°$,

$Q = 5.5\,\text{m}^3/\text{min}$.

First, consider outlet velocity triangle

$$C_{w2} = U_2 - \frac{C_{r2}}{\tan \beta_2}$$
$$= 11 - \frac{2.5}{\tan 32°} = 7\,\text{m/s}$$

Power of the pump is given by

$$P = \frac{\rho Q C_{w2} U_2}{1000}\,\text{kW}$$

$$P = \frac{(1000)(5.5)(7)(11)}{(60)(1000)} = 7\ \text{kW}.$$

2. Now, h.p. $= \dfrac{2\pi NT}{60}$

where T is the torque of the shaft. Therefore,

$$\text{T} = \frac{\text{h.p.} \times 60}{2\pi N}$$

But $U_2 = \frac{\pi D_2 N}{60}$ or $N = \frac{60 \times U_2}{\pi D_2} = \frac{60(11)}{\pi(1)} = 210\,\text{rpm}$

Hence,

$$T = \frac{(7)(1000)(60)}{2\pi(210)} = 318\,\text{Nm/s}.$$

Illustrative Example 2.10: A centrifugal pump running at 590 rpm and discharges 110 l/min against a head of 16 m. What head will be developed and quantity of water delivered when the pump runs at 390 rpm?

Solution:

$N_1 = 590$,

$Q_1 = 110\,\text{l/min} = 1.83\,\text{l/s}$,

$H_1 = 16\,\text{m}$,

$N_2 = 390\,\text{rpm}$,

$H_2 = ?$

As $\frac{\sqrt{H_1}}{N_1} = \frac{\sqrt{H_2}}{N_2}$

Then, $\frac{\sqrt{16}}{590} = \frac{\sqrt{H_2}}{390}$

Therefore, $H_2 = 6.98\,\text{m}$

Therefore, head developed by the pump at 390 rpm = 6.98 m. In order to find discharge through the pump at 390 rpm, using the Eq. (1.11)

$$\frac{N_1\sqrt{Q_1}}{H_1^{3/4}} = \frac{N_2\sqrt{Q_2}}{H_2^{3/4}}$$

$$\frac{590\sqrt{1.83}}{(16)^{3/4}} = \frac{390\sqrt{Q_2}}{(6.98)^{3/4}} \quad \text{or} \quad \frac{798.14}{8} = \frac{390\sqrt{Q_2}}{4.29}$$

$$\sqrt{Q_2} = 1.097$$

i.e., $Q = 1.203\,\text{l/s}$

Illustrative Example 2.11: The impeller of a centrifugal pump has outlet diameter of 0.370 m, runs at 800 rpm, and delivers 30 l/s of water. The radial velocity at the impeller exit is 2.5 m/s. The difference between the water levels at the overhead tank and the pump is 14 m. The power required to drive the pump is 8 hp, its mechanical and volumetric effectiveness being 0.96 and 0.97, respectively. The impeller vanes are backward curved with an exit angle of 45°. Calculate (1) ideal head developed with no slip and no hydraulic losses and (2) the hydraulic efficiency.

Solution:

1. Impeller tip speed

$$U_2 = \frac{\pi D_2 N}{60}$$

or

$$U_2 = \frac{\pi(0.37)(800)}{60} = 15.5\,\text{m/s}.$$

As the radial velocity at the impeller exit = 2.5 m/s.
Therefore, $C_{w2} = U_2 - \frac{C_{r2}}{\tan\beta_2} = 15.5 - \frac{2.5}{\tan 45°} = 13\,\text{m/s}.$
When there is no slip, the head developed will be

$$H = \frac{C_{w2}U_2}{g} = \frac{(13)(15.5)}{9.81} = 20.54\,\text{m}$$

If there are no hydraulic internal losses, the power utilized by the pump will be:

$$P = (0.96)(8) = 7.68\,\text{hp}$$

$$\text{Theoretical flow rate} = \frac{Q}{\eta_v} = \frac{0.03}{0.97} = 0.031\,\text{m}^3/\text{s}$$

Ideal head, H_i, is given by

$$H_i = \frac{(7.68)(0.746)}{(9.81)(0.031)} = 18.84\,\text{m}.$$

2. The hydraulic efficiency is

$$\eta_h = \frac{H}{H_i} = \frac{14}{18.84} = 0.746 \quad \text{or} \quad 74.3\%.$$

Illustrative Example 2.12: The impeller of a centrifugal pump has outer diameter of 1.06 m and speed is 56 m/s. The blades are backward curved and they make an angle of 20° with the wheel tangent at the blade tip. If the radial velocity of the flow at the tip is 7.5 m/s and the slip factor is 0.88. Determine (1) the actual work input per kg of water flow and (2) the absolute velocity of fluid at the impeller.

Solution:

1. Exit blade angle, $\beta_2 = 20°$

$$\therefore C_{w2} = U_2 - \frac{C_{r2}}{\tan\beta_2} = 56 - \frac{7.5}{\tan 20°} = 35.4\,\text{m/s}$$

Using slip factor, $\sigma = 0.88$, the velocity whirl at exit is, $C_{w2} = \sigma \times 35.4 = 0.88 \times 35.4 = 31.2\,\text{m/s}$.
Work input per kg of water flow

$$W = \frac{C_{w2}U_2}{1000} = \frac{(56)(31.2)}{1000} = 1.75\,\text{kJ/kg}.$$

2. Absolute velocity at impeller tip

$$C_2 = \sqrt{\left(C_{r2}^2 + C_{W2}^2\right)} = \sqrt{\left(7.5^2 + 31.2^2\right)}$$

$$C_2 = \sqrt{56.25 + 973.44} = 32.09\,\text{m/s}$$

Design Example 2.13: A centrifugal pump impeller of 260 mm diameter runs at 1400 rpm, and delivers 0.03 m³/s of water. The impeller has a backward curved facing blades inclined at 30° to the tangent at outlet. The blades are 20 mm in depth at the outlet, and a slip factor of 0.78 may be assumed. Calculate the theoretical head developed by the impeller, and the number of impeller blades.

Solution:

Assuming the blades are of infinitesimal thickness, the flow area is given by

$$A = \text{impeller periphery} \times \text{blade depth}$$
$$= \pi \times 0.26 \times 0.02 = 0.0163\ \text{m}^2$$

Flow velocity is given by

$$C_{r2} = \frac{Q}{A} = \frac{0.03}{0.0163} = 1.84\ \text{m/s}$$

Impeller tip speed, U_2, is

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.26)(1400)}{60} = 19.07\ \text{m/s}$$

Absolute whirl component, C_{w2} is given by

$$C_{w2} = U_2 - \frac{C_{r2}}{\tan 30^\circ} = 19.07 - \frac{1.84}{\tan 30^\circ} = 15.88\ \text{m/s}$$

Using Euler's equation, and assuming $C_{w1} = 0$ (i.e., no whirl at inlet)

$$H = \frac{U_2 C_{w2}}{g} = \frac{(19.07)(15.88)}{9.81} = 30.87\ \text{m}$$

Theoretical head with slip is $H = 0.78 \times 30.87 = 24.08$ m.

To find numbers of impeller blades, using Stanitz formula

Slip factor, $\sigma = 1 - \dfrac{0.63\pi}{n}$ or $0.78 = 1 - \dfrac{0.63\pi}{n}$ or $\dfrac{0.63\pi}{n} = 1 - 0.78 = 0.22$

$$\therefore\ n = \frac{0.63\pi}{0.22} = 9$$

Number of blades required = 9

Design Example 2.14: A centrifugal pump impeller runs at 1500 rpm, and has internal and external diameter of 0.20 m and 0.4 m, respectively, assuming constant radial velocity at 2.8 m/s and the vanes at the exit are set back at an angle of 30°. Calculate (1) the angle absolute velocity of water at exit makes with the tangent, (2) inlet vane angle, and (3) the work done per kg of water.

Solution:

1. $D_1 = 0.2\,\text{m}$, $D_2 = 0.4\,\text{m}$, $N = 1500\,\text{rpm}$, $C_{r2} = 2.8\,\text{m/s}$, $\beta_2 = 30°$.
   Impeller tip speed, U_2, is

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.4)(1500)}{60} = 31.43\,\text{m/s}$$

   Whirl component of absolute velocity at impeller exit is

$$C_{w2} = U_2 - \frac{C_{r2}}{\tan 30°} = 31.43 - \frac{2.8}{\tan 30°} = 26.58\,\text{m/s}$$

$$\tan \alpha_2 = \frac{2.8}{26.58} = 0.1053$$

   i.e., $\alpha_2 = 6°$.

2. Impeller speed at inlet

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi(0.2)(1500)}{60} = 15.7\,\text{m/s}$$

$$\tan \beta_1 = \frac{2.8}{15.7} = 0.178$$

   i.e., $\beta_1 = 10.1°$.

3. Work done per kg of water

$$C_{w2}U_2 = 26.58 \times 31.43 = 835.4\,\text{Nm}.$$

Design Example 2.15: An axial flow pump discharges water at the rate of $1.30\,\text{m}^3/\text{s}$ and runs at 550 rpm. The total head is 10 m. Assume blade velocity = 22 m/s, the flow velocity = 4.5 m/s, hydraulic efficiency = 0.87, and the overall pump efficiency = 0.83, find (1) the power delivered to the water, and power input, (2) the impeller hub diameter and tip diameter, and (3) the inlet and outlet blade angles for the rotor.

Solution:

1. Power delivered to the water

$$P = \rho g H Q / 1000\,\text{kW}$$
$$= (9.81)(1.30)(10) = 127.53\,\text{kW}$$

   Power input to the pump

$$P = \frac{127.53}{0.83} = 153.65\,\text{kW}.$$

2. Rotor tip diameter is given by

$$D_2 = \frac{60U_2}{\pi N} = \frac{(60)(22)}{\pi(550)} = 0.764\,\text{m}$$

Rotor hub diameter

$$D_1^2 = D_2^2 - \frac{Q}{(\pi/4) \times C_a} = 0.764^2 - \frac{1.3}{(\pi/4)(4.5)} = 0.216\,\text{m}$$

i.e., $D_1 = 0.465\,\text{m}$.

3. Rotor velocity at hub is given by

$$U_1 = \frac{D_1}{D_2} U_2 = \frac{(0.465)(22)}{0.764} = 13.39\,\text{m/s}$$

Since, the axial velocity is constant, we have: rotor inlet angle at tip

$$\alpha_{1t} = \tan^{-1}(C_a/U_1) = \tan^{-1}(4.5/13.39) = 18.58°$$

Rotor outlet angle

$$\alpha_{2t} = \tan^{-1}(C_a/U_2) = \tan^{-1}(4.5/22) = 11.56°.$$

Design Example 2.16: A single stage, radial flow, and double suction centrifugal pump having the following data:

Discharge	72 l/s
Inner diameter	90 mm
Outer diameter	280 mm
Revolution/minute	1650
Head	25 m
Width at inlet	20 mm/side
Width at outlet	18 mm/side
Absolute velocity angle at inlet	90°
Leakage losses	2 l/s
Mechanical losses	1.41 kW
Contraction factor due to vane thickness	0.85
Relative velocity angle measured from tangential direction	35°
Overall efficiency of the pump	0.56

Determine (1) inlet vane angle, (2) the angle at which the water leaves the wheel, (3) the absolute velocity of water leaving impeller, (4) manometric efficiency, and (5) the volumetric and mechanical efficiencies.

Solution:

Total quantity of water to be handled by the pump

$$Q_t = Q_{del} + Q_{leak}$$
$$= 72 + 2 = 74$$

Total quantity of water per side = 74/2 = 37 l/s

1. Impeller speed at inlet

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi(0.09)(1650)}{60} = 7.78 \text{ m/s}$$

Flow area at inlet = $\Pi D_1 b_1 \times$ contraction factor

$$= (\Pi)(0.09)(0.02)(0.85) = 0.0048 \text{ m}^2$$

Therefore, the velocity of flow at inlet

$$C_{r1} = \frac{Q}{\text{Area of flow}} = \frac{37 \times 10^{-3}}{0.0048} = 7.708 \text{ m/s}$$

From inlet velocity triangle

$$\tan\beta_1 = \frac{C_{r1}}{U_1} = \frac{7.708}{7.78} = 0.9907$$

$$\beta_1 = 44.73°$$

2. Area of flow at outlet

$$A_2 = \Pi \times D_2 \times b_2 \times \text{contraction factor}$$

Where $b_2 = 18/2 = 9$ mm for one side.
So, $A_2 = (\Pi)(0.28)(0.009)\,(0.85) = 0.0067 \text{ m}^2$.
Therefore, the velocity of flow at outlet

$$C_{r2} = \frac{Q}{\text{Area of flow}} = \frac{37 \times 10^{-3}}{0.0067} = 5.522 \text{ m/s}$$

The impeller speed at outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.28)(1650)}{60} = 24.2 \text{ m/s}$$

Now using velocity triangle at outlet

$$\tan \beta_2 = \frac{C_{r2}}{U_2 - C_{w2}} = \frac{5.522}{24.2 - C_{w2}}$$

$$C_{w2} = 16.99\,\text{m/s}$$

Further,

$$\tan \alpha_2 = \frac{C_{r2}}{C_{w2}} = \frac{5.522}{16.99} = 0.325$$

$$\alpha_2 = 18^\circ.$$

3. The absolute velocity of water leaving the impeller

$$C_2 = \frac{C_{w2}}{\text{Cos}\,\alpha_2} = \frac{16.99}{\text{Cos}\,18^\circ} = 17.8\,\text{m/s}.$$

4. The manometric efficiency

$$\eta_{\text{mano}} = \frac{(g)(H_{\text{mano}})}{(U_2)(C_{W2})} = \frac{9.81 \times 25}{24.2 \times 16.99} = 0.596.$$

5. The volumetric efficiency

$$\eta_v = \frac{Q_2}{Q_{\text{Total}}} = \frac{72}{74} = 0.973$$

$$\begin{aligned}\text{Water power} &= \rho g Q H \\ &= 1000 \times 9.81 \times 72 \times 25/1000 \\ &= 17.66\,\text{kW}\end{aligned}$$

$$\text{Shaft power} = \frac{\text{Water power}}{\eta_o} = \frac{17.66}{0.56} = 31.54\,\text{kW}$$

Mechanical efficiency is

$$\eta_m = \frac{P_s - P_{\text{loss}}}{P_s} = \frac{31.54 - 1.41}{31.54} = 0.955 \quad \text{or} \quad 95.5\%.$$

Illustrative Example 2.17: A single stage centrifugal pump is designed to give a discharge of Q when working against a manometric head of 20 m. On test, it was found that head actually generated was 21.5 m for the designed discharge, Q. If it is required to reduce the original diameter 32 cm without reducing the speed of the impeller, compute the required diameter to be reduced.

Solution:

Head generated by the pump

$$H = \frac{U^2}{2g} = \frac{(\pi DN/60)^2}{2g}$$

or

$$H \propto D^2$$

$$\frac{H}{H'} = \left(\frac{D}{D'}\right)^2$$

$$H = 21.5\,\text{m},\ \ H' = 20\,\text{m},\ \ D = 32\,\text{cm}$$

So,

$$D' = D\left(\frac{H'}{H}\right)^{1/2} = 32\left(\frac{20}{21.5}\right)^2 = 30.86\,\text{cm}$$

Design Example 2.18: A two stage centrifugal pump is designed to discharge 55 l/s at a head of 70 m. If the overall efficiency is 76% and specific speed per stage about 38, find (1) the running speed in rpm and (2) the power required to run pump.

If the actual manometric head developed is 65% of the theoretical head, assuming no slip, the outlet angle of the blades 28°, and radial velocity at exit 0.14 times the impeller tip speed at exit, find the diameter of impeller.

Solution:

1. The specific speed is

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$N = \frac{N_s H^{3/4}}{\sqrt{Q}} = \frac{38(70/2)^{3/4}}{\sqrt{55 \times 10^{-3}}} = \frac{546.81}{0.235} = 2327\ \text{rpm}.$$

2. $Q = 55 \times 10^{-3}\ \text{m}^3/\text{s}$

Power required to drive pump

$$= \frac{\rho g Q H}{0.76} = \frac{1000 \times 9.81 \times 55 \times 10^{-3} \times 70}{0.76 \times 1000}$$

$$= 49.7\ \text{kW}$$

$H_{\text{mano}} = 0.65$ H.
Here $\beta_2 = 28°$ and $C_{r2} = 0.14U_2$.

From velocity triangle at outlet

$$\tan\beta_2 = \frac{C_{r2}}{U_2 - C_{W2}}$$

$$\text{or } \tan 28° = \frac{0.14U_2}{U_2 - C_{W2}}$$

$$\frac{U_2}{U_2 - C_{W2}} = \frac{0.5317}{0.14} = 3.798 \qquad \text{(A)}$$

As the flow at entrance is radial and $\alpha_1 = 90°$, the fundamental equation of pump would be

$$\frac{H_{\text{mano}}}{\eta_{\text{mano}}} = \frac{U_2 C_{W2}}{g}$$

Where η_{mano} manometric efficiency of pump which is 65%. Therefore, $\frac{35}{0.65} = \frac{U_2 C_{W2}}{g}$

$$U_2 C_{W2} = \frac{35 \times 9.81}{0.65}$$

$$C_{W2} = \frac{528.23}{U_2} \qquad \text{(B)}$$

Substituting for C_{W2} in Eq. (A) and solving

$$\frac{U_2}{U_2 - \frac{528.23}{U_2}} = 3.798$$

$$U_2 = 26.78 \text{ m/s.}$$

Also,

$$U_2 = \frac{\pi D_2 N}{60}$$

or $26.78 = \frac{\pi \times D_2 \times 2327}{60}$

$$\text{D}_2 = 0.2197 \text{ m} \quad \text{or} \quad 21.97 \text{ cm.}$$

Design Example 2.19: Two multistage centrifugal pumps are used in series to handle water flow rate of 0.0352 m^3/s, and total head required is 845 m. Each pump is required to produce a head of half the total and run at 1445 rpm. If the impeller in all the stages is identical and specific speed is 14, determine (1) head developed per stage and the required number of stages in each pump, (2) The required impeller diameters assuming the speed ratio based on the outer tip diameter to be 0.96 and the shaft power input, if the overall efficiency of each pump is 0.75.

Solution:

Head developed in each stage is

$$H^{3/4} = \frac{N\sqrt{Q}}{N_s} = \frac{1445\sqrt{0.0352}}{14}$$

$$H = 51.93\,\text{m}$$

Total head required $= 845\,\text{m}$ (of water)

Number of stages needed $= \frac{845}{51.93} = 16$

Number of stages in each pump $= 8$

Impeller speed at tip is

$$\begin{aligned} U_2 &= 0.96(2gH)^{0.5} \\ &= 0.96[2 \times 9.81 \times 51.93]^{0.5} \\ &= 30.6\,\text{m/s} \end{aligned}$$

Impeller diameter at tip, $D_2 = \pi \times 60 \times 30.6 \times 1445$. But

$$U_2 = \frac{\pi D_2 N}{60}$$

or

$$D_2 = \frac{U_2 \times 60}{\pi \times 1445} = \frac{30.6 \times 60}{\pi \times 1445} = 0.4043\,\text{m} \quad \text{or} \quad 40.43\,\text{cm}.$$

Design Example 2.20: A centrifugal pump is required to be made to lift water through 105 m heights from a well. Number of identical pumps having their designed speed 900 rpm and specific speed 700 rpm with a rated discharge of 5500 l/min are available. Determine the number of pumps required and how they should be connected?

Solution:

Specific speed for a single impeller is given by

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

Given, $N_s = 700, \quad H = 105 \quad N = 900, \quad \text{and} \quad Q = \frac{5500}{60} = 91.67\ \text{l/s}$

Substituting,

$$700 = \frac{900\sqrt{91.67}}{H^{3/4}}, \quad H = 28\,\text{m}$$

Hence number of stages required

$$= \frac{\text{Total head to be developed}}{\text{Head per stage}}$$

$$= \frac{105}{28} = 4 \text{ stages in series}$$

Design Example 2.21: The specific speed of an axial flow pump impeller is 1150 and velocity of flow is 2.5 m/s. The outer and inner diameters of the impeller are 0.90 m and 0.45 m, respectively. Calculate the suitable speed of the pump to give a head of 5.5 m. Also, calculate vane angle at the entry of the pump.

Solution:
Given,

$$D_2 = 0.9\,\text{m},\ D_1 = 0.45\,\text{m},\ N_s = 1150,\ C_r = 2.5\,\text{m/s},\ H = 5.5\,\text{m}.$$

As discharge, Q = area of flow × velocity of flow

$$= \tfrac{\pi}{4}(0.9^2 - 0.45^2) \times 2.5 = 1.193\,\text{m}^3/\text{s}$$

$$= 1193\,\text{l/s}$$

Also,

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

or

$$1150 = \frac{N\sqrt{1193}}{(5.5)^{3/4}}$$

$$N = \frac{(5.5)^{3/4} \times 1150}{\sqrt{1193}} = 120\,\text{rpm}$$

In order to find vane angle at entry, using velocity triangle at inlet,

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.45 \times 120}{60} = 2.82\,\text{m/s}$$

$$\tan \alpha_1 \frac{C_{r1}}{U_1} = \frac{2.5}{2.82} = 0.8865$$

i.e.,

$$\alpha = 41.56°.$$

PROBLEMS

2.1 A centrifugal pump of 25 cm impeller diameter running at 1450 rpm, develops a head of 15 m. If the outlet flow area is 480 cm^2, and discharging water 0.12 m^3/s, and loss of head in the pump can be taken as $0.003C_1^2$, find the outlet blade angle.

(14°)

2.2 A centrifugal pump having vane angles at inlet and outlet are 25° and 30°, respectively. If internal and external diameters of impeller are 0.15 and 0.30 m, respectively, calculate the work done per kg of water. Assume velocity of flow constant.

(197.18 Nm)

2.3 A centrifugal pump discharges 50 liters/second of water against a total head of 40 m. Find the horsepower of the pump, if the overall efficiency is 62%.

(42 hp)

2.4 A centrifugal pump delivers 26 l/s against a total head of 16 m at 1450 rpm. The impeller diameter is 0.5 m. A geometrically similar pump of 30 cm diameter is running at 2900 rpm. Calculate head and discharge required assuming equal efficiencies between the two pumps.

(11.52 m, 11.23 l/s)

2.5 A centrifugal pump is built to work against a head of 20 m. A model of this pump built to one-fourth its size is found to generate a head of 7 m when running at its best speed of 450 rpm and requires 13.5 hp to run it. Find the speed of the prototype.

(190 rpm)

2.6 Derive the expression for power required for a pump when it discharges a liquid of specific weight w at the rate of Q against a head of H.

2.7 Show that the pressure rise in an impeller of a centrifugal pump is given by $\frac{C_{r1}^2 + U_2^2 - C_{r2}^2 \operatorname{cosec}^2 \beta_2}{2g}$ (where C_{r_1} = velocity of flow at inlet, U_2 = blade velocity at outlet, C_{r_2} = velocity of flow at outlet, and β_2 = blade angle at outlet). Assuming that friction and other losses are neglected.

2.8 Derive an expression for static head developed by a centrifugal pump having radial flow at inlet.

2.9 A centrifugal pump discharges 0.15 m^3/s of water against a head of 15 m. The impeller has outer and inner diameter of 35 and 15 cm, respectively. The outlet vanes are set back at an angle 40°. The area of flow is constant from inlet to outlet and is 0.06 m^2. Calculate the manometric efficiency and vane angle at inlet if the speed of the pump is 960 rpm. Take slip factor = 1.

(57.3%, 18°)

2.10 A centrifugal pump of 35 cm diameter running at 1000 rpm develops a head of 18 m. The vanes are curved back at an angle of 30° to the tangent at outlet. If velocity flow is constant at 2.4 m/s, find the manometric efficiency of the pump.

(76.4%)

2.11 An axial flow pump is required to deliver 1 m^3/s at 7 m head while running at 960 rpm. Its outer diameter is 50 and hub diameter is 25 cm. Find (1) flow velocity, which is assumed to be constant from hub to tip and (2) power required to drive the pump if overall efficiency is 84%.

(6.791 m/s, 81.75 kW)

2.12 An axial flow pump has the following data:

Rotational speed	750 rpm
Discharge of water	1.75 m^3/s
Head	7.5 m
Hub to runner diameter ratio	0.45

Through flow velocity is 0.35 times the peripheral velocity. Find the diameter and minimum speed ratio.

(0.59 m, 0.83)

2.13 In an axial flow pump, the rotor has an outer diameter of 75 cm and an inner diameter of 40 cm; it rotates at 500 rpm. At the mean blade radius, the inlet blade angle is 12° and the outlet blade angle is 15°. Sketch the corresponding velocity diagrams at inlet and outlet, and estimate from them (1) the head the pump will generate, (2) the discharge or rate of flow in l/s, (3) the shaft h.p. input required to drive the pump, and (4) the specific speed of the pump. Assume a manometric or hydraulic efficiency of 88% and a gross or overall efficiency of 81%.

(19.8 m; 705 l/s; 230 hp; 45)

2.14 If an axial flow pump delivers a discharge Q against a head H when running at a speed N, deduce an expression for the speed of a geometrically similar pump of such a size that when working against unit head, it will transmit unit power to the water flowing through it. Show that this value is proportional to the specific speed of the pump.

NOTATION

b	width of the diffuser passage
C_{w2}	tangential components of absolute velocity corresponding to the angle β_2
E	Euler head
H	total head developed by the pump
H_i	total head across the impeller
N_{suc}	Suction specific speed
m	mass flow rate
n	number of vanes
P_s	shaft power input
Q	flow rate
r	radius
U	impeller speed
V	relative velocity
α	absolute velocity angle
β	relative velocity angle
η_c	casing efficiency
η_R	hydraulic efficiency
η_i	impeller efficiency
η_m	mechanical efficiency
η_o	overall efficiency
η_v	volumetric efficiency
ρ	density of liquid
σ	slip factor
ω	angular velocity

SUFFIXES

1	inlet to impeller
2	outlet from the impeller
3	outlet from the diffuser
a	axial
r	radial
w	whirl

3

Hydraulic Turbines

3.1 INTRODUCTION

In a hydraulic turbine, water is used as the source of energy. Water or hydraulic turbines convert kinetic and potential energies of the water into mechanical power. The main types of turbines are (1) impulse and (2) reaction turbines. The predominant type of impulse machine is the Pelton wheel, which is suitable for a range of heads of about 150–2,000 m. The reaction turbine is further subdivided into the Francis type, which is characterized by a radial flow impeller, and the Kaplan or propeller type, which is an axial-flow machine. In the sections that follow, each type of hydraulic turbine will be studied separately in terms of the velocity triangles, efficiencies, reaction, and method of operation.

3.2 PELTON WHEEL

An American Engineer Lester A. Pelton discovered this (Fig. 3.1) turbine in 1880. It operates under very high heads (up to 1800 m.) and requires comparatively less quantity of water. It is a pure impulse turbine in which a jet of fluid delivered is by the nozzle at a high velocity on the buckets. These buckets are fixed on the periphery of a circular wheel (also known as runner), which is generally mounted on a horizontal shaft. The primary feature of the impulse

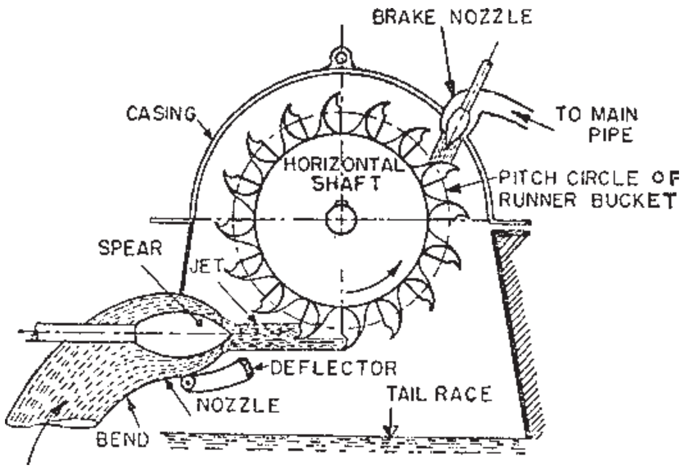


Figure 3.1 Single-jet, horizontal shaft Pelton turbine.

turbine with respect to fluid mechanics is the power production as the jet is deflected by the moving vane(s).

The impact of water on the buckets causes the runner to rotate and thus develops mechanical energy. The buckets deflect the jet through an angle of about 160 and 165° in the same plane as the jet. After doing work on the buckets water is discharged in the tailrace, and the whole energy transfer from nozzle outlet to tailrace takes place at constant pressure.

The buckets are so shaped that water enters tangentially in the middle and discharges backward and flows again tangentially in both the directions to avoid thrust on the wheel. The casing of a Pelton wheel does not perform any hydraulic function. But it is necessary to safeguard the runner against accident and also to prevent the splashing water and lead the water to the tailrace.

3.3 VELOCITY TRIANGLES

The velocity diagrams for the Pelton wheel are shown in Fig. 3.2.

Since the angle of entry of the jet is nearly zero, the inlet velocity triangle is a straight line, as shown in Fig. 3.2. If the bucket is brought to rest, then the relative fluid velocity, V_1, is given by

$$\begin{aligned} V_1 &= \text{jet velocity} - \text{bucket speed} \\ &= C_1 - U_1 \end{aligned}$$

The angle turned through by the jet in the horizontal plane during its passage over the bucket surface is α and the relative velocity at exit is V_2. The absolute

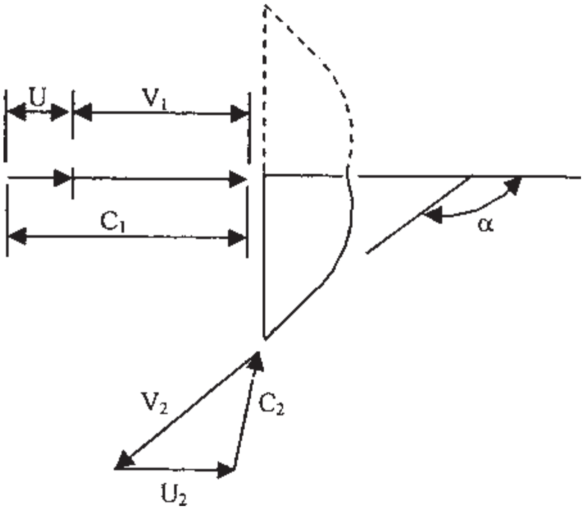


Figure 3.2 Velocity triangles for a Pelton wheel.

velocity, C_2, at exit can be obtained by adding bucket speed vector U_2 and relative velocity, V_2, at exit.

Now using Euler's turbine Eq. (1.78)

$$W = U_1 C_{W1} - U_2 C_{W2}$$

Since in this case C_{W2} is in the negative x direction,

$$W = U\{(U + V_1) + [V_1 \cos(180 - \alpha) - U]\}$$

Neglecting loss due to friction across the bucket surface, that is, $V_1 = V_2$, then

$$W = U(V_1 - V_1 \cos \alpha)$$

Therefore

$$E = U(C_1 - U)(1 - \cos \alpha)/g \tag{3.1}$$

the units of E being Watts per Newton per second weight of flow.

Eq. (3.1) can be optimized by differentiating with respect to U, and equating it to zero.

Therefore

$$\frac{dE}{dU} = (1 - \cos \alpha)(C_1 - 2U)/g = 0$$

Then

$$C_1 = 2U \text{ or } \quad U = C_1/2 \tag{3.2}$$

Substituting Eq. (3.2) into Eq. (3.1) we get

$$E_{\max} = C_1^2(1 - \cos\alpha)/4g$$

In practice, surface friction is always present and $V_1 \neq V_2$, then Eq. (3.1) becomes

$$E = U(C_1 - U)(1 - k\cos\alpha)/g \tag{3.3}$$

where $k = \frac{V_2}{V_1}$

Introducing hydraulic efficiency as

$$\eta_h = \frac{\text{Energy Transferred}}{\text{Energy Available in jet}}$$

$$\text{i.e. } \eta_h = \frac{E}{(C_1^2/2g)} \tag{3.4}$$

if $\alpha = 180°$, the maximum hydraulic efficiency is 100%. In practice, deflection angle is in the order of 160–165°.

3.4 PELTON WHEEL (LOSSES AND EFFICIENCIES)

Head losses occur in the pipelines conveying the water to the nozzle due to friction and bend. Losses also occur in the nozzle and are expressed by the velocity coefficient, C_v.

The jet efficiency (η_j) takes care of losses in the nozzle and the mechanical efficiency (η_m) is meant for the bearing friction and windage losses. The overall efficiency (η_o) for large Pelton turbine is about 85–90%. Following efficiency is usually used for Pelton wheel.

$$\text{Pipeline transmission efficiency} = \frac{\text{Energy at end of the pipe}}{\text{Energy available at reservoir}}$$

Figure 3.3 shows the total headline, where the water supply is from a reservoir at a head H_1 above the nozzle. The frictional head loss, h_f, is the loss as the water flows through the pressure tunnel and penstock up to entry to the nozzle.

Then the transmission efficiency is

$$\eta_{\text{trans}} = (H_1 - h_f)/H_1 = H/H_1 \tag{3.5}$$

The nozzle efficiency or jet efficiency is

$$\eta_j = \frac{\text{Energy at nozzle outlet}}{\text{Energy at nozzle inlet}} = C_1^2/2gH \tag{3.6}$$

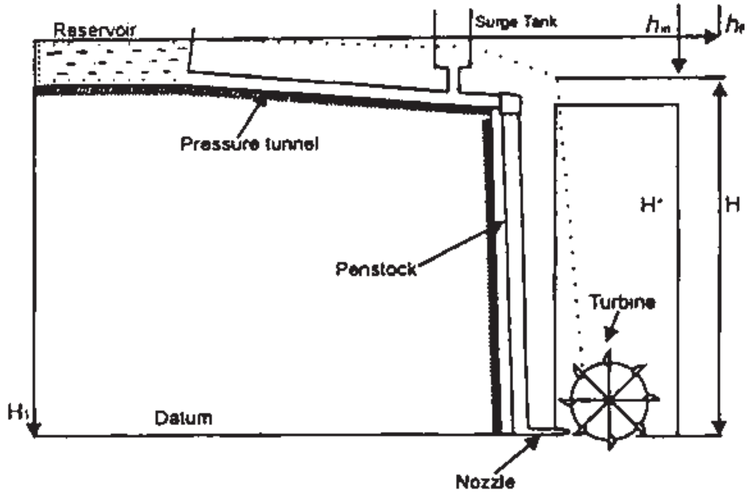


Figure 3.3 Schematic layout of hydro plant.

Nozzle velocity coefficient

$$C_v = \frac{\text{Actual jet velocity}}{\text{Theoretical jet velocity}} = C_1 / \sqrt{2gH}$$

Therefore the nozzle efficiency becomes

$$\eta_j = C_1^2 / 2gH = C_V^2 \tag{3.7}$$

The characteristics of an impulse turbine are shown in Fig. 3.4.

Figure 3.4 shows the curves for constant head and indicates that the peak efficiency occurs at about the same speed ratio for any gate opening and that

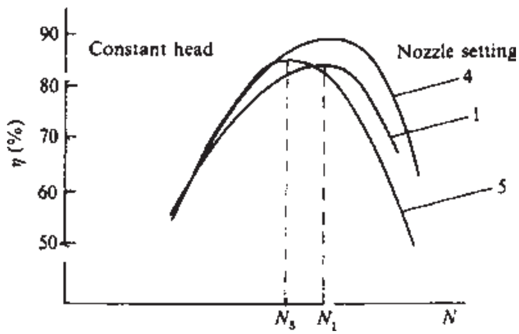


Figure 3.4 Efficiency vs. speed at various nozzle settings.

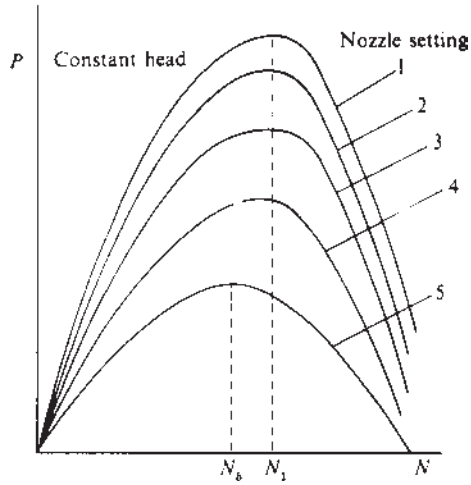


Figure 3.5 Power vs. speed of various nozzle setting.

the peak values of efficiency do not vary much. This happens as the nozzle velocity remaining constant in magnitude and direction as the flow rate changes, gives an optimum value of U/C_1 at a fixed speed. Due to losses, such as windage, mechanical, and friction cause the small variation. Fig. 3.5 shows the curves for power vs. speed. Fixed speed condition is important because generators are usually run at constant speed.

Illustrative Example 3.1: A generator is to be driven by a Pelton wheel with a head of 220 m and discharge rate of 145 L/s. The mean peripheral velocity of wheel is 14 m/s. If the outlet tip angle of the bucket is 160°, find out the power developed.

Solution:

$$\text{Dischargerate}, Q = 145\,\text{L/s}$$

$$\text{Head}, H = 220\,\text{m}$$

$$U_1 = U_2 = 14\,\text{m/s}$$

$$\beta_2 = 180 - 160° = 20°$$

Refer to Fig. 3.6
Using Euler's equation, work done per weight mass of water per sec.

$$= (C_{w1}U_1 - C_{w2}U_2)$$

But for Pelton wheel C_{w2} is negative

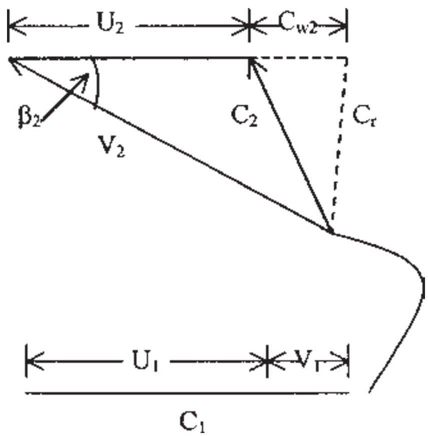


Figure 3.6 Inlet and outlet velocity triangles.

Therefore

$$\text{Work done / s} = (C_{w1}U_1 + C_{w2}U_2)\ \text{Nm / s}$$

From inlet velocity triangle

$$C_{w1} = C_1 \text{ and } \frac{C_1^2}{2g} = H$$

Hence, $C_1 = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 220} = 65.7$ m/s
Relative velocity at inlet is

$$V_1 = C_1 - U_1 = 65.7 - 14 = 51.7 \text{ m/s}$$

From outlet velocity triangle

$$V_1 = V_2 = 51.7 \text{ m/s(neglecting friction)}$$

and $\cos\ \beta_2 = \frac{U_2 + C_{w2}}{V_2}$ or

$$\cos(20) = \frac{14 + C_{w2}}{51.7}$$

Therefore

$$C_{w2} = 34.58 \text{ m/s}$$

Hence, work done per unit mass of water per sec.

$$= (65.7)(14) + (34.58)(14) = 1403.92 \text{ Nm}$$

$$\text{Power developed} = \frac{(1403.92)(145)}{1000} = 203.57 \text{ kW}$$

Design Example 3.2: A Pelton wheel is supplied with 0.035 m^3/s of water under a head of 92 m. The wheel rotates at 725 rpm and the velocity coefficient of the nozzle is 0.95. The efficiency of the wheel is 82% and the ratio of bucket speed to jet speed is 0.45. Determine the following:

1. Speed of the wheel
2. Wheel to jet diameter ratio
3. Dimensionless power specific speed of the wheel

Solution:

Overall efficiency $\eta_{\mathrm{o}} = \dfrac{\text{Power developed}}{\text{Power available}}$

$$\therefore\ P = \rho g Q H \eta_o \ \mathrm{J/s} = \tfrac{\rho g Q H \eta_o}{1000}\ \mathrm{kW}$$
$$= 9.81(0.035)(92)(0.82) = 25.9\,\mathrm{kW}$$

Velocity coefficient

$$C_{\mathrm{v}} = \frac{C_1}{\sqrt{2gH}}$$

or $C_1 = C_{\mathrm{v}}\sqrt{2gH} = 0.95[(2)(9.81)(92)]^{1/2} = 40.36\,\mathrm{m/s}$

1. Speed of the wheel is given by

$$U = 0.45(40.36) = 18.16\,\mathrm{m/s}$$

2. If D is the wheel diameter, then

$$U = \frac{\omega D}{2} \quad \mathit{or} \quad D = \frac{2U}{\omega} = \frac{(2)(18.16)(60)}{725(2\pi)} = 0.478\,\mathrm{m}$$

Jet area $A = \dfrac{Q}{C_1} = \dfrac{0.035}{40.36} = 0.867 \times 10^{-3}\,\mathrm{m}^2$

and Jet diameter, d, is given by

$$d = \left(\frac{4A}{\pi}\right)^{1/2} = \left(\frac{(4)(0.867 \times 10^{-3})}{\pi}\right)^{1/2} = 0.033\,\mathrm{m}$$

Diameter ratio $\dfrac{D}{d} = \dfrac{0.478}{0.033} = 14.48$

3. Dimensionless specific speed is given by Eq. (1.10)

$$
\begin{aligned}
N\mathrm{sp} &= \frac{NP^{1/2}}{\rho^{1/2}(gH)^{5/4}} \\
&= \left(\frac{725}{60}\right) \times \left(\frac{(25.9)(1000)}{10^3}\right)^{1/2} \times \left(\frac{1}{(9.81)\times(92)}\right)^{5/4} \\
&= (12.08)(5.09)(0.0002) \\
&= 0.0123 \text{ rev} \\
&= (0.0123)(2\pi) \text{ rad} \\
&= 0.077 \text{ rad}
\end{aligned}
$$

Illustrative Example 3.3: The speed of Pelton turbine is 14 m/s. The water is supplied at the rate of 820 L/s against a head of 45 m. If the jet is deflected by the buckets at an angle of 160°, find the hP and the efficiency of the turbine.

Solution:

Refer to Fig. 3.7

$U_1 = U_2 = 14$ m/s

$Q = 820$ L/s $= 0.82$ m^3/s

$H = 45$ m

$\beta_2 = 180 - 160° = 20°$

Velocity of jet

$$
\begin{aligned}
C_1 &= C_V\sqrt{2gH}, \text{ assuming } C_V = 0.98 \\
&= 0.98\sqrt{(2)(9.81)(45)} = 29.12 \text{ m/s}
\end{aligned}
$$

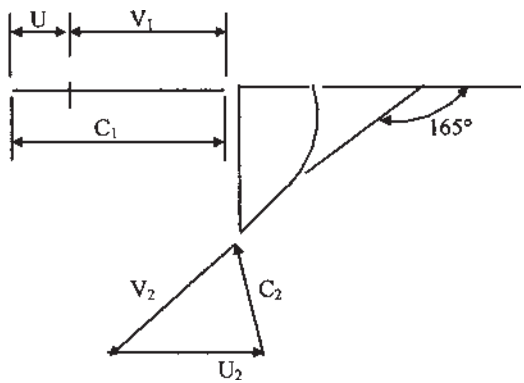


Figure 3.7 Velocity triangle for Example 3.3.

Assuming

$$\beta_1 = 180°$$
$$\beta_2 = 180 - 160° = 20°$$
$$C_{w1} = C_1 = 29.12\,\text{m/s}$$
$$V_1 = C_1 - U_1 = 29.12 - 14 = 15.12\,\text{m/s}$$

From outlet velocity triangle,

$$U_1 = U_2 (\text{neglecting losses on buckets})$$

$$V_2 = 15.12\,\text{m/s} \quad \text{and} \quad U_2 = 14\,\text{m/s}$$

$$C_{w2} = V_2 \cos\alpha_2 - U_2 = 15.12 \ \cos 20° - 14$$
$$= 0.208\,\text{m/s}$$

Work done per weight mass of water per sec

$$= (C_{w1} + C_{w2})U$$
$$= (29.12 + 0.208) \times (14) = 410.6\,\text{Nm/s}$$

$$\therefore \text{ Power developed} = \frac{(410.6)(0.82 \times 10^3)}{1000} = 336.7\,\text{kW}$$
$$= 451\,\text{hP}$$

$$\text{Efficiency}\,\eta_1 = \frac{\text{Power developed}}{\text{Available Power}}$$
$$= \frac{(1000)(336.7)}{(1000)(9.81)(0.82)(45)} = 0.930 \text{ or } 93.0\%$$

Illustrative Example 3.4: A Pelton wheel develops 12,900 kW at 425 rpm under a head of 505 m. The efficiency of the machine is 84%. Find (1) discharge of the turbine, (2) diameter of the wheel, and (3) diameter of the nozzle. Assume $C_v = 0.98$, and ratio of bucket speed to jet speed = 0.46.

Solution:

Head, $H = 505$ m.

Power, $P = 12{,}900$ kW

Speed, $N = 425$ rpm

Efficiency, $\eta_o = 84\%$

1. Let Q be the discharge of the turbine

$$\text{Using the relation } \eta_o = \frac{P}{9.81QH}$$

or

$$0.84 = \frac{12,900}{(9.81)(505)Q} = \frac{2.60}{Q}$$

or

$$Q = 3.1\ \mathrm{m^3/s}$$

2. Velocity of jet

$$C = C_V\sqrt{2gH}\ (\text{assume}\ \ C_V = 0.98)$$

or

$$C = 0.98\sqrt{(2)(9.81)(505)} = 97.55\ \mathrm{m/s}$$

Tangential velocity of the wheel is given by

$$U = 0.46C = (0.46)(97.55) = 44.87\ \mathrm{m/s}$$

and

$$U = \frac{\pi DN}{60},\ \text{hence wheel diameter is}$$

$$D = \frac{60U}{\pi N} = \frac{(60)(44.87)}{(\pi)(425)} = 2.016\ \mathrm{m}$$

3. Let d be the diameter of the nozzle
The discharge through the nozzle must be equal to the discharge of the turbine. Therefore

$$Q = \tfrac{\pi}{4} \times d^2 \times C$$

$$3.1 = (\tfrac{\pi}{4})(d^2)(97.55) = 76.65\,d^2$$

$$\therefore\ d = \sqrt{\frac{3.1}{76.65}} = 0.20\ \mathrm{m}$$

Illustrative Example 3.5: A double Overhung Pelton wheel unit is to operate at 12,000 kW generator. Find the power developed by each runner if the generator is 95%.

Solution:
Output power = 12,000 kW

Efficiency, $\eta = 95\%$

Therefore, power generated by the runner

$$= \frac{12,000}{0.95} = 12,632\ \mathrm{kW}$$

Since there are two runners, power developed by each runner

$$= \frac{12,632}{2} = 6316\,\text{kW}$$

Design Example 3.6: At the power station, a Pelton wheel produces 1260 kW under a head of 610 m. The loss of head due to pipe friction between the reservoir and nozzle is 46 m. The buckets of the Pelton wheel deflect the jet through an angle of 165°, while relative velocity of the water is reduced by 10% due to bucket friction. The bucket/jet speed ratio is 0.46. The bucket circle diameter of the wheel is 890 mm and there are two jets. Find the theoretical hydraulic efficiency, speed of rotation of the wheel, and diameter of the nozzle if the actual hydraulic efficiency is 0.9 times that calculated above. Assume nozzle velocity coefficient, $C_v = 0.98$.

Solution:

Refer to Fig. 3.8.

$$\text{Hydraulic efficiency } \eta_h = \frac{\text{Power output}}{\text{Energy available in the jet}} = \frac{P}{0.5mC_1^2}$$

At entry to nozzle

$$H = 610 - 46 = 564\,\text{m}$$

Using nozzle velocity coefficient

$$C_1 = C_V\sqrt{2gH} = 0.98\sqrt{(2)(9.81)(564)} = 103.1\,\text{m/s}$$

Now

$$\begin{aligned}\frac{W}{m} &= U_1C_{w1} - U_2C_{w2}\\ &= U\{(U+V_1) - [U - V_2\cos(180° - \alpha)]\}\\ &= U[(C_1 - U)(1 - k\cos\alpha)] \text{ where } V_2 = kV_1\end{aligned}$$

Therefore, $W/m = 0.46C_1(C_1 - 0.46C_1)(1 - 0.9\cos 165°)$

Substitute the value of C_1

$$W/m = 5180.95$$

$$\begin{aligned}\text{Theoretical hydraulic efficiency} &= \frac{\text{Power output}}{\text{Energy available in the jet}}\\ &= \frac{5180.95}{0.5 \times 103^2} = 98\%\end{aligned}$$

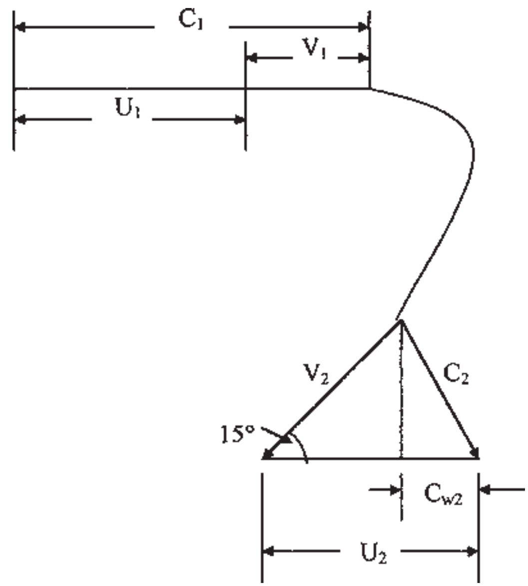


Figure 3.8 Velocity triangle for Example 3.6.

$$\text{Actual hydraulic efficiency} = (0.9)(0.98) = 0.882$$

$$\text{Wheel bucket speed} = (0.46)(103) = 47.38\,\text{m/s}$$

$$\text{Wheel rotational speed} = N = \frac{(47.38)(60)}{(0.445)(2\pi)} = 1016\,\text{rpm}$$

$$\text{Actual hydraulic efficiency} = \frac{\text{Actual power}}{\text{energy in the jet}} = \frac{(1260 \times 10^3)}{0.5\,mC_1^2}$$

Therefore, $m = \dfrac{(1260 \times 10^3)}{(0.882)(0.5)(103^2)} = 269\,\text{kg/s}$

For one nozzle, $m = 134.5\,\text{kg/s}$

For nozzle diameter, using continuity equation, $m = \rho C_1 A = \dfrac{\rho C_1 \pi d^2}{4}$

Hence, $d = \sqrt{\dfrac{(134.5)(4)}{(\pi)(103 \times 10^3)}} = 0.041\,\text{m} = 41\,\text{mm}$

Illustrative Example 3.7: A Pelton wheel has a head of 90 m and head lost due to friction in the penstock is 30 m. The main bucket speed is 12 m/s and the nozzle discharge is 1.0 m^3/s. If the bucket has an angle of 15° at the outlet and $C_v = 0.98$, find the power of Pelton wheel and hydraulic efficiency.

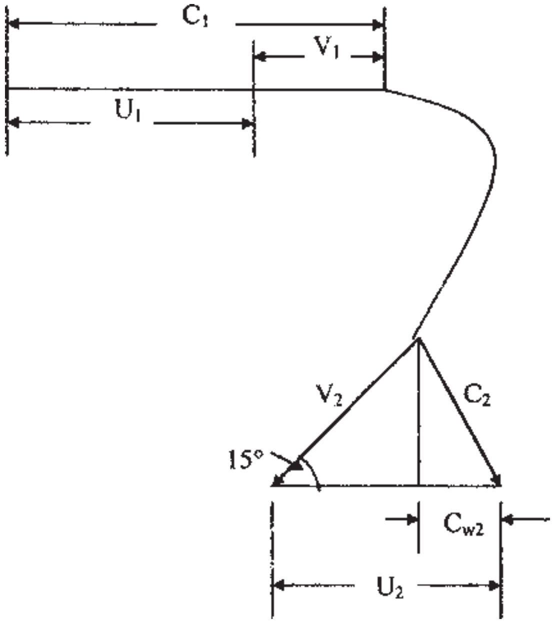


Figure 3.9 Velocity triangle for Example 3.7.

Solution: (Fig. 3.9)
Head $= 90\,\text{m}$
Head lost due to friction $= 30\,\text{m}$
Head available at the nozzle $= 90 - 30 = 60\,\text{m}$
$Q = 1\,\text{m}^3/\text{s}$
From inlet diagram

$$C_1 = C_V\sqrt{2gH} = 0.98 \times \sqrt{(2)(9.81)(60)} = 33.62\,\text{m/s}$$

Therefore, $V_1 = C_1 - U_1 = 33.62 - 12 = 21.62\,\text{m/s}$
From outlet velocity triangle

$$V_2 = V_1 = 21.16\,\text{m/s}\,(\text{neglecting losses})$$

$$U_2 = U_1 = 12\,\text{m/s}$$

$$C_{w2} = V_2 \cos\alpha - U_2 = 21.62\cos 15° - 12 = 8.88\,\text{m/s}$$

and

$$Cr_2 = V_2 \sin\alpha = 21.62 \sin 15° = 5.6\,\text{m/s}$$

Therefore,

$$C_2 = \sqrt{C_{w2}^2 + Cr_2^2} = \sqrt{(8.88)^2 + (5.6)^2} = 10.5\,\text{m/s}$$

$$\therefore \text{ Work done} = \frac{C_1^2 - C_2^2}{2} = \frac{(33.62)^2 - (10.5)^2}{2} = 510\,\text{kJ/kg}$$

Note Work done can also be found by using Euler's equation ($C_{w1}U_1 + C_{w2}U_2$)

Power = 510 kW

Hydraulic efficiency

$$\eta_h = \frac{\text{work done}}{\text{kinetic energy}} = \frac{(510)(2)}{(33.62)^2} = 90.24\%$$

Design Example 3.8: A single jet Pelton wheel turbine runs at 305 rpm against a head of 515 m. The jet diameter is 200 mm, its deflection inside the bucket is 165° and its relative velocity is reduced by 12% due to friction. Find (1) the waterpower, (2) resultant force on the bucket, (3) shaft power if the mechanical losses are 4% of power supplied, and (4) overall efficiency. Assume necessary data.

Solution: (Fig. 3.10)
Velocity of jet, $C_1 = C_V\sqrt{2gH} = 0.98\sqrt{(2)(9.81)(515)} = 98.5\,\text{m/s}$
Discharge, Q is given by

$$Q = \text{Area of jet} \times \text{Velocity} = \frac{\pi}{4} \times (0.2)^2(98.5) = 3.096\,\text{m}^3/\text{s}$$

1. Water power is given by

$$P = \rho g Q H = (9.81)(3.096)(515) = 15641.5\,\text{kW}$$

2. Bucket velocity, U_1, is given by

$$U_1 = C_V\sqrt{2gH}$$
$$= 0.46\sqrt{(2)(9.81)(515)} = 46\,\text{m/s} \text{ (assuming } C_V = 0.46)$$

Relative velocity, V_1, at inlet is given by

$$V_1 = C_1 - U_1 = 98.5 - 46 = 52.5\,\text{m/s}$$

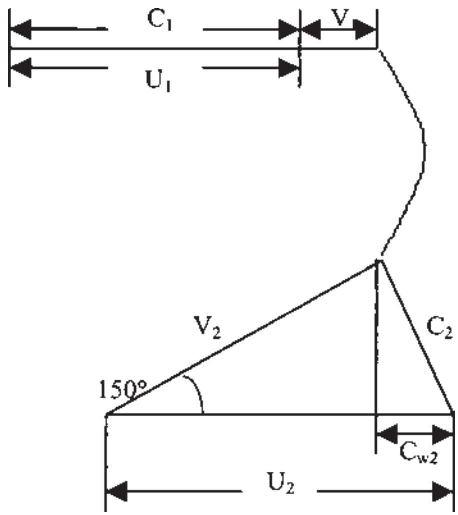


Figure 3.10 Velocity triangles for Example 3.8.

and

$$V_2 = 0.88 \times 52.5 = 46.2\,\text{m/s}$$

From the velocity diagram

$$C_{w2} = U_2 - V_2 \cos 15 = 46 - 46.2 \times 0.966 = 1.37\,\text{m/s}$$

Therefore force on the bucket

$$= \rho Q(C_{w1} - C_{w2}) = 1000 \times 3.096(98.5 - 1.37)$$
$$= 300714\,\text{N}$$

3. Power produced by the Pelton wheel

$$= \frac{(300714)(46)}{1000} = 13832.8\,\text{kW}$$

Taking mechanical loss = 4%

Therefore, shaft power produced = 0.96 × 13832.8 = 13279.5 kW

4. Overall efficiency

$$\eta_o = \frac{13279.5}{15641.5} = 0.849 \text{ or } 84.9\%$$

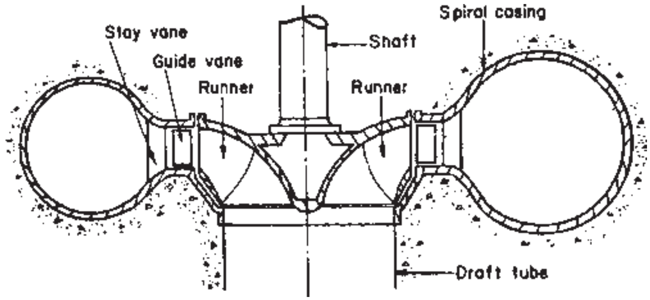


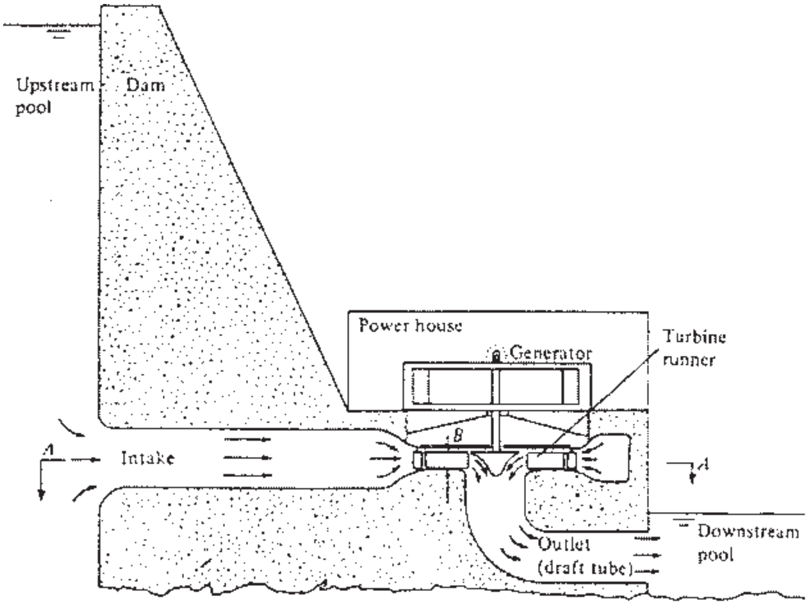
Figure 3.11 Outlines of a Francis turbine.

3.5 REACTION TURBINE

The radial flow or Francis turbine is a reaction machine. In a reaction turbine, the runner is enclosed in a casing and therefore, the water is always at a pressure other than atmosphere. As the water flows over the curved blades, the pressure head is transformed into velocity head. Thus, water leaving the blade has a large relative velocity but small absolute velocity. Therefore, most of the initial energy of water is given to the runner. In reaction turbines, water leaves the runner at atmospheric pressure. The pressure difference between entrance and exit points of the runner is known as reaction pressure.

The essential difference between the reaction rotor and impulse rotor is that in the former, the water, under a high station head, has its pressure energy converted into kinetic energy in a nozzle. Therefore, part of the work done by the fluid on the rotor is due to reaction from the pressure drop, and part is due to a change in kinetic energy, which represents an impulse function. Fig. 3.11 shows a cross-section through a Francis turbine and Fig. 3.12 shows an energy distribution through a hydraulic reaction turbine. In reaction turbine, water from the reservoir enters the turbine casing through penstocks.

Hence, the total head is equal to pressure head plus velocity head. Thus, the water enters the runner or passes through the stationary vanes, which are fixed around the periphery of runners. The water then passes immediately into the rotor where it moves radially through the rotor vanes and exits from the rotor blades at a smaller diameter, after which it turns through 90° into the draft tube. The draft tube is a gradually increasing cross-sectional area passage. It helps in increasing the work done by the turbine by reducing pressure at the exit. The penstock is a waterway, which carries water from the reservoir to the turbine casing. The inlet and outlet velocity triangles for the runner are shown in Fig. 3.13.



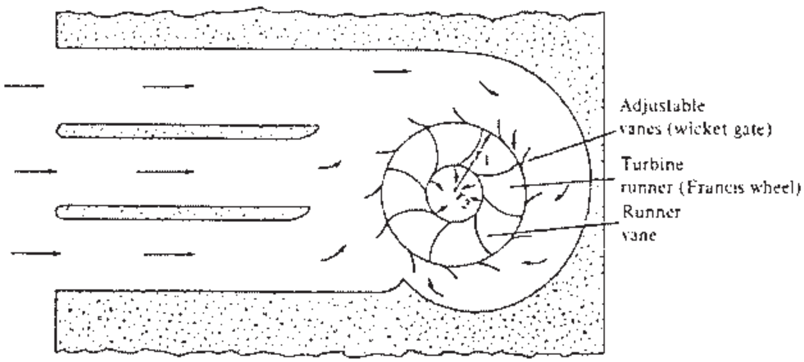
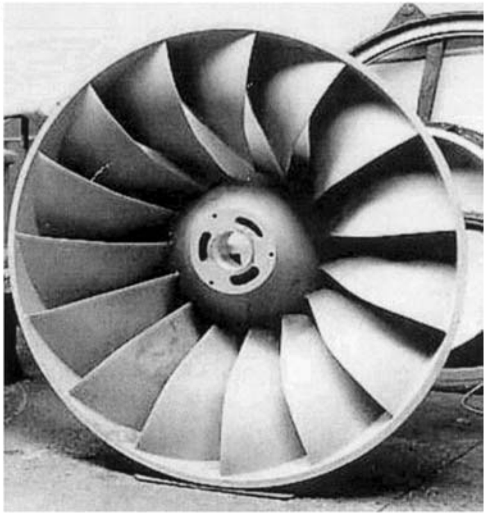


Figure 3.12 Reaction turbine installation.

(a)



(b)

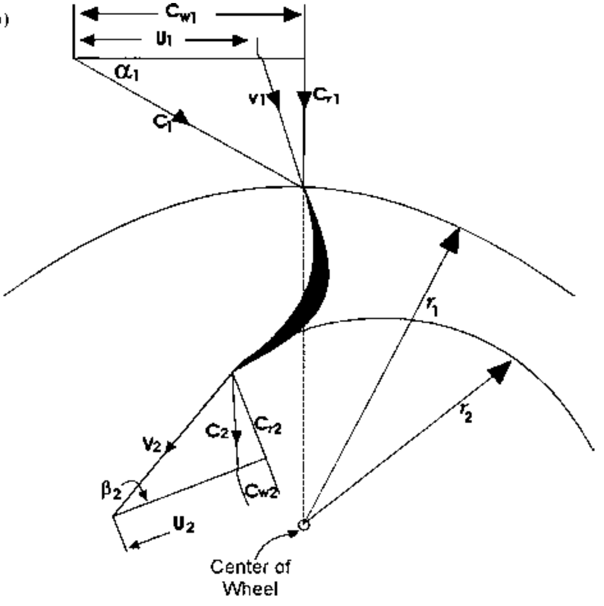


Figure 3.13 (a) Francis turbine runner and (b) velocity triangles for inward flow reaction turbine.

Let

C_1 = Absolute velocity of water at inlet

D_1 = Outer diameter of the runner

N = Revolution of the wheel per minute

U_1 = Tangential velocity of wheel at inlet

V_1 = Relative velocity at inlet

C_{r1} = radial velocity at inlet

α_1 = Angle with absolute velocity to the direction of motion

β_1 = Angle with relative velocity to the direction of motion

H = Total head of water under which turbine is working

$C_2, D_2, U_2, V_2, C_{r2}$ = Corresponding values at outlet

Euler's turbine equation Eq. (1.78) and E is maximum when C_{w2} (whirl velocity at outlet) is zero that is when the absolute and flow velocities are equal at the outlet.

3.6 TURBINE LOSSES

Let

P_s = Shaft power output

P_m = Mechanical power loss

P_r = Runner power loss

P_c = Casing and draft tube loss

P_l = Leakage loss

P = Water power available

$P_h = P_r + P_c + P_l$ = Hydraulic power loss

Runner power loss is due to friction, shock at impeller entry, and flow separation. If h_f is the head loss associated with a flow rate through the runner of Q_r, then

$$P_S = \rho g Q_r h_f \text{ (Nm/s)} \tag{3.8}$$

Leakage power loss is due to leakage in flow rate, q, past the runner and therefore not being handled by the runner. Thus

$$Q = Q_r + q \tag{3.9}$$

If H_r is the head across the runner, the leakage power loss becomes

$$P_l = \rho g H_r q \,(\mathrm{Nm} / \mathrm{s}) \tag{3.10}$$

Casing power loss, P_c, is due to friction, eddy, and flow separation losses in the casing and draft tube. If h_c is the head loss in casing then

$$P_c = \rho g Q h_c \,(\mathrm{Nm} / \mathrm{s}) \tag{3.11}$$

From total energy balance we have

$$\rho g Q H = P_m + \rho g (h_f Q_r + h_c Q + H_r q + P_s)$$

Then overall efficiency, η_o, is given by

$$\eta_o = \frac{\text{Shaft power output}}{\text{Fluid power available at inlet}}$$

or

$$\eta_o = \frac{P_s}{\rho g Q H} \tag{3.12}$$

Hydraulic efficiency, η_h, is given by

$$\eta_h = \frac{\text{Power available at runner}}{\text{Fluid power available at inlet}}$$

or

$$\eta_h = \frac{(P_s + P_m)}{\rho g Q H} \tag{3.13}$$

Eq. (3.13) is the theoretical energy transfer per unit weight of fluid. Therefore the maximum efficiency is

$$\eta_h = U_1 C_{w1} / gH \tag{3.14}$$

3.7 TURBINE CHARACTERISTICS

Part and overload characteristics of Francis turbines for specific speeds of 225 and 360 rpm are shown in Fig. 3.14

Figure 3.14 shows that machines of low specific speeds have a slightly higher efficiency. It has been experienced that the Francis turbine has unstable characteristics for gate openings between 30 to 60%, causing pulsations in output and pressure surge in penstocks. Both these problems were solved by Paul Deriaz by designing a runner similar to Francis runner but with adjustable blades.

The part-load performance of the various types are compared in Fig. 3.15 showing that the Kaplan and Pelton types are best adopted for a wide range of load but are followed fairly closely by Francis turbines of low specific speed.

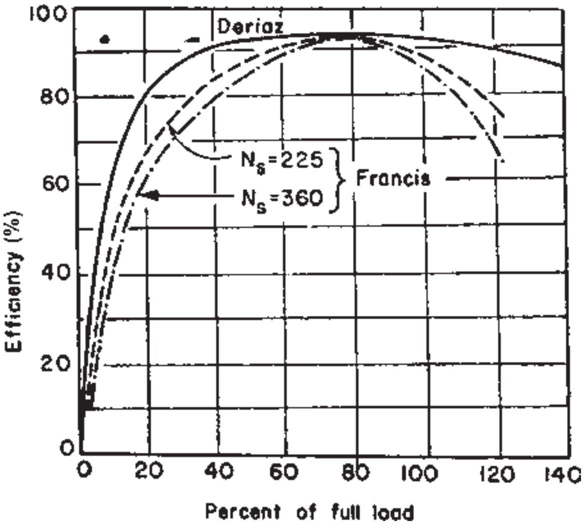


Figure 3.14 Variation of efficiency with load for Francis turbines.

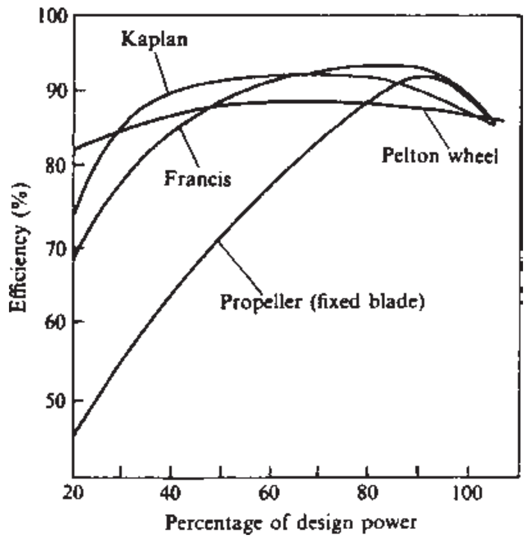


Figure 3.15 Comparison of part-load efficiencies of various types of hydraulic turbine.

3.8 AXIAL FLOW TURBINE

In an axial flow reaction turbine, also known as Kaplan turbine, the flow of water is parallel to the shaft.

A Kaplan turbine is used where a large quantity of water is available at low heads and hence the blades must be long and have large chords so that they are strong enough to transmit the very high torque that arises. Fig. 3.16 and 3.17 shows the outlines of the Kaplan turbine. The water from the scroll flows over the guide blades and then over the vanes. The inlet guide vanes are fixed and are situated at a plane higher than the runner blades such that fluid must turn through 90° to enter the runner in the axial direction. The function of the guide vanes is to impart whirl to the fluid so that the radial distribution of velocity is the same as in a free vortex.

Fig. 3.18 shows the velocity triangles and are usually drawn at the mean radius, since conditions change from hub to tip. The flow velocity is axial at inlet and outlet, hence $C_{r1} = C_{r2} = C_a$

C_1 is the absolute velocity vector at angle α_1 to U_1, and V_1 is the relative velocity at an angle β_1. For maximum efficiency, the whirl component $C_{w2} = 0$, in which case the absolute velocity at exit is axial and then $C_2 = C_{r2}$

Using Euler's equation

$$E = U(C_{w1} - C_{w2})/g$$

and for zero whirl ($C_{w2} = 0$) at exit

$$E = UC_{w1}/g$$

3.9 CAVITATION

In the design of hydraulic turbine, cavitation is an important factor. As the outlet velocity V_2 increases, then p_2 decreases and has its lowest value when the vapor pressure is reached.

At this pressure, cavitation begins. The Thoma parameter $\sigma = \frac{\text{NPSH}}{H}$ and Fig. 3.19 give the permissible value of σ_c in terms of specific speed.

The turbines of high specific speed have a high critical value of σ, and must therefore be set lower than those of smaller specific speed (N_s).

Illustrative Example 3.9: Consider an inward flow reaction turbine in which velocity of flow at inlet is 3.8 m/s. The 1 m diameter wheel rotates at 240 rpm and absolute velocity makes an angle of 16° with wheel tangent. Determine (1) velocity of whirl at inlet, (2) absolute velocity of water at inlet, (3) vane angle at inlet, and (4) relative velocity of water at entrance.

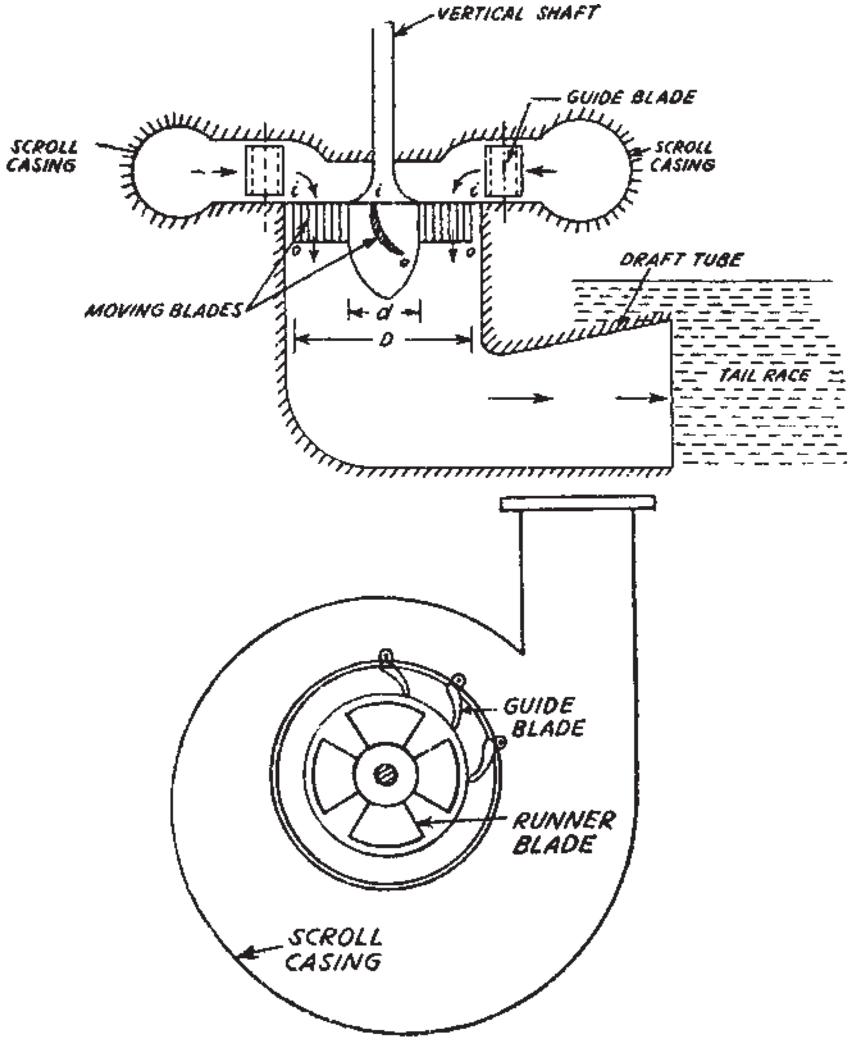


Figure 3.16 Kaplan turbine of water is available at low heads.

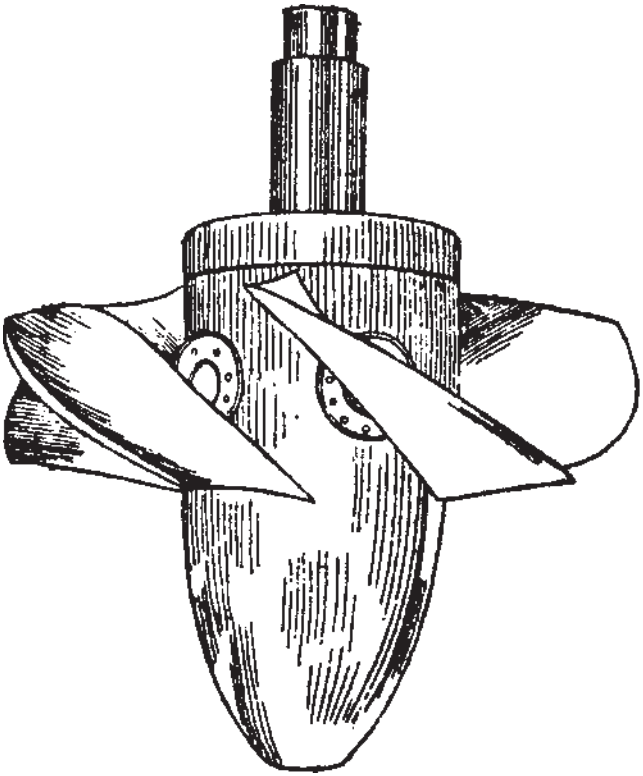


Figure 3.17 Kaplan turbine runner.

Solution: From Fig. 3.13b

1. From inlet velocity triangle (subscript 1)

$$\tan\alpha_1 = \frac{C_{r1}}{C_{w1}} \quad \text{or} \quad C_{w1} = \frac{C_{r1}}{\tan\alpha_1} = \frac{3.8}{\tan 16^\circ} = 13.3\,\text{m/s}$$

2. Absolute velocity of water at inlet, C_1, is

$$\sin\alpha_1 = \frac{C_{r1}}{C_1} \quad \textit{or} \quad C_1 = \frac{C_{r1}}{\sin\alpha_1} = \frac{3.8}{\sin 16^\circ} = 13.79\,\text{m/s}$$

3.

$$U_1 = \frac{(\pi D_1)(N)}{60} = \frac{(\pi)(1)(240)}{60} = 12.57\,\text{m/s}$$

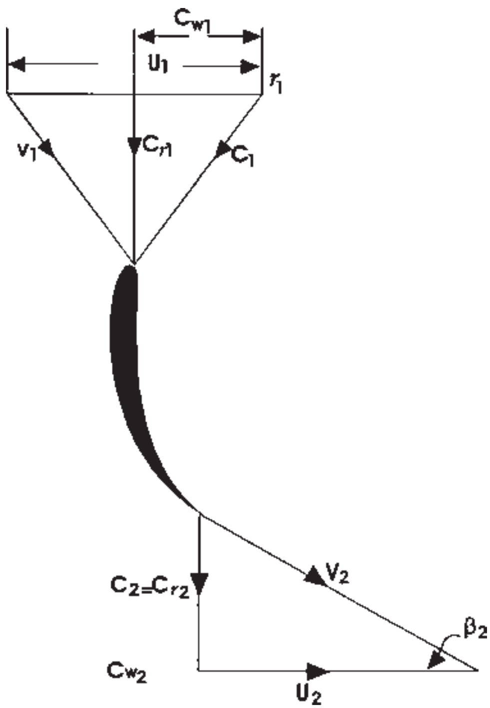


Figure 3.18 Velocity triangles for an axial flow hydraulic turbine.

and

$$\tan \beta_1 = \frac{C_{r1}}{(C_{w1} - U_1)} = \frac{3.8}{(13.3 - 12.57)} = \frac{3.8}{0.73} = 5.21$$

$$\therefore \ \beta_1 = 79^\circ \text{ nearby}$$

4. Relative velocity of water at entrance

$$\sin \beta_1 = \frac{C_{r1}}{V_1} \quad or \quad V_1 = \frac{C_{r1}}{\sin \beta_1} = \frac{3.8}{\sin 79^\circ} = 3.87\text{m/s}$$

Illustrative Example 3.10: The runner of an axial flow turbine has mean diameter of 1.5 m, and works under the head of 35 m. The guide blades make an angle of 30° with direction of motion and outlet blade angle is 22°. Assuming axial discharge, calculate the speed and hydraulic efficiency of the turbine.

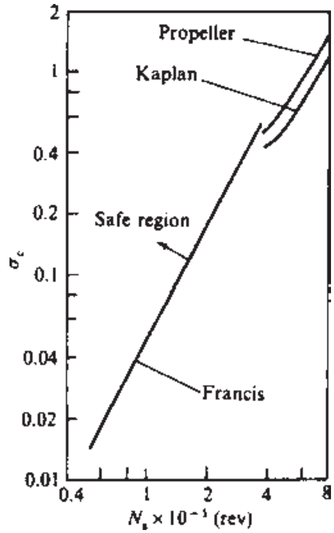


Figure 3.19 Cavitation limits for reaction turbines.

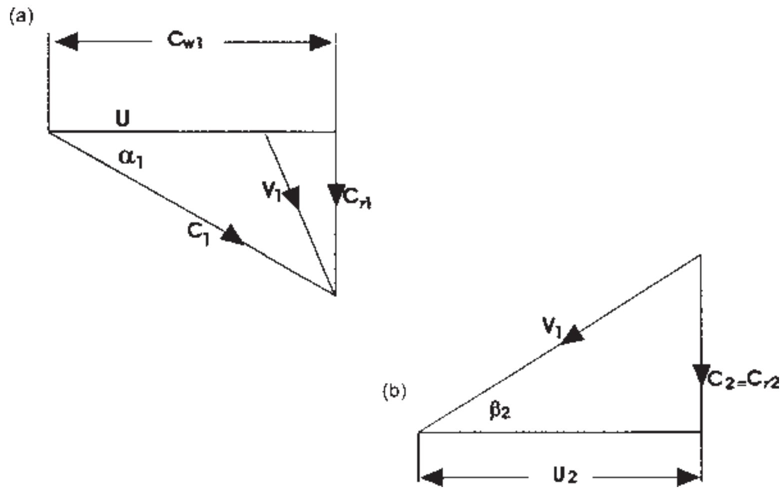


Figure 3.20 Velocity triangles (a) inlet and (b) outlet.

Solution:

Since this is an impulse turbine, assume coefficient of velocity $= 0.98$
Therefore the absolute velocity at inlet is

$$C_1 = 0.98\sqrt{2gH} = 0.98\sqrt{(2)(9.81)(35)} = 25.68 \text{ m/s}$$

The velocity of whirl at inlet

$$C_{w1} = C_1 \cos\alpha_1 = 25.68 \cos 30° = 22.24 \text{ m/s}$$

Since $U_1 = U_2 = U$
Using outlet velocity triangle

$$C_2 = U_2 \tan\beta_2 = U \tan\beta_2 = U \tan 22°$$

Hydraulic efficiency of turbine (neglecting losses)

$$\eta_h = \frac{C_{w1}U_1}{gH} = \frac{H - C_2^2/2g}{H}$$

$$\frac{22.24U}{g} = H - \frac{(U \tan 22°)^2}{2g}$$

or

$$\frac{22.24U}{g} + \frac{(U \tan 22)^2}{2g} = H$$

or

$$22.24U + 0.082U^2 - 9.81H = 0$$

or

$$0.082U^2 + 22.24U - 9.81H = 0$$

or

$$U = \frac{-22.24 \pm \sqrt{(22.24)^2 + (4)(0.082)(9.81)(35)}}{(2)(0.082)}$$

As U is positive,

$$U = \frac{-22.24 + \sqrt{494.62 + 112.62}}{0.164}$$
$$= \frac{-22.24 + 24.64}{0.164} = 14.63 \text{ m/s}$$

Now using relation

$$U = \frac{\pi DN}{60}$$

or

$$N = \frac{60U}{\pi D} = \frac{(60)(14.63)}{(\pi)(1.5)} = 186\,\text{rpm}$$

Hydraulic efficiency

$$\eta_{\text{h}} = \frac{C_{\text{w1}}U}{gH} = \frac{(22.24)(14.63)}{(9.81)(35)} = 0.948 \text{ or } 94.8\%$$

Illustrative Example 3.11: A Kaplan runner develops 9000 kW under a head of 5.5 m. Assume a speed ratio of 2.08, flow ratio 0.68, and mechanical efficiency 85%. The hub diameter is 1/3 the diameter of runner. Find the diameter of the runner, and its speed and specific speed.

Solution:

$$U_1 = 2.08\sqrt{2gH} = 2.08\sqrt{(2)(9.81)(5.5)} = 21.61\,\text{m/s}$$

$$C_{\text{r1}} = 0.68\sqrt{2gH} = 0.68\sqrt{(2)(9.81)(5.5)} = 7.06\,\text{m/s}$$

Now power is given by

$$9000 = (9.81)(5.5)(0.85)Q$$

Therefore,

$$Q = 196.24\,\text{m}^3/\text{s}$$

If D is the runner diameter and, d, the hub diameter

$$Q = \frac{\pi}{4}(D^2 - d^2)C_{\text{r1}}$$

or

$$\frac{\pi}{4}\left(D^2 - \frac{1}{9}D^2\right)7.06 = 196.24$$

Solving

$$D = \sqrt{\frac{(196.24)(4)(9)}{(\pi)(7.06)(8)}} = 6.31\,\text{m}$$

$$N_{\text{S}} = \frac{N\sqrt{P}}{H^{5/4}} = \frac{65\sqrt{9000}}{5.5^{5/4}} = 732\,\text{rpm}$$

Design Example 3.12: A propeller turbine develops 12,000 hp, and rotates at 145 rpm under a head of 20 m. The outer and hub diameters are 4 m and 1.75 m, respectively. Calculate the inlet and outlet blade angles measured at mean radius if overall and hydraulic efficiencies are 85% and 93%, respectively.

Solution:

$$\text{Mean diameter} = \frac{4 + 1.75}{2} = 2.875\,\text{m}$$

$$U_1 = \frac{\pi DN}{60} = \frac{(\pi)(2.875)(145)}{60} = 21.84\,\text{m/s}$$

Using hydraulic efficiency

$$\eta_\text{h} = \frac{C_{\text{w}1}U_1}{gH} = \frac{(C_{\text{w}1})(21.84)}{(9.81)(20)} = 0.93C_{\text{w}1}$$

or

$$C_{\text{w}1} = 8.35\,\text{m/s}$$

$$\text{Power} = (12,000)(0.746) = 8952\,\text{kW}$$

$$\text{Power} = \rho g Q H \eta_\text{o}$$

or

$$8952 = 9.81 \times Q \times 20 \times 0.85$$

Therefore, $Q = \dfrac{8952}{(9.81)(20)(0.85)} = 53.68\,\text{m}^3/\text{s}$

Discharge, $Q = 53.68 = \frac{\pi}{4}(4^2 - 1.75^2)C_{\text{r}1}$

$$\therefore\ C_{\text{r}1} = 5.28\,\text{m/s}$$

$$\tan\beta_1 = \frac{C_{\text{r}1}}{U_1 - C_{\text{w}1}} = \frac{5.28}{21.84 - 8.35} = \frac{5.28}{13.49} = 0.3914$$

$$\beta_1 = 21.38^\circ$$

and

$$\tan\beta_2 = \frac{C_{r2}}{U_2} = \frac{5.28}{21.84} = 0.2418$$

$$\beta_2 = 13.59^\circ$$

Illustrative Example 3.13: An inward flow reaction turbine wheel has outer and inner diameter are 1.4 m and 0.7 m respectively. The wheel has radial vanes and discharge is radial at outlet and the water enters the vanes at an angle of 12°. Assuming velocity of flow to be constant, and equal to 2.8 m/s, find

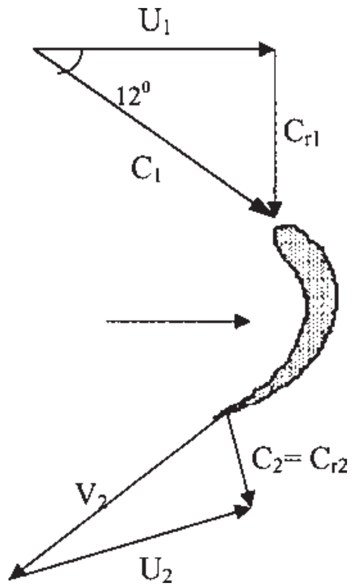


Figure 3.21 Velocity triangles at inlet and outlet for Example 3.13.

1. The speed of the wheel, and
2. The vane angle at outlet.

Solution:
Outer diameter, $D_2 = 1.4\,\text{m}$
Inner diameter, $D_1 = 0.7\,\text{m}$
Angle at which the water enters the vanes, $\alpha_1 = 12°$
Velocity of flow at inlet,

$$C_{r1} = C_{r2} = 2.8\,\text{m/s}$$

As the vanes are radial at inlet and outlet end, the velocity of whirl at inlet and outlet will be zero, as shown in Fig. 3.21.
Tangential velocity of wheel at inlet,

$$U_1 = \frac{C_{r1}}{\tan 12°} = \frac{2.8}{0.213} = 13.15\,\text{m/s}$$

Also, $U_1 = \frac{\pi D_2 N}{60}$ or

$$N = \frac{60U_1}{\pi D_2} = \frac{(60)(13.15)}{(\pi)(1.4)} = 179\,\text{rpm}$$

Let β_2 is the vane angle at outlet

$$U_2 = \frac{\pi D_1 N}{60} = \frac{(\pi)(0.7)(179)}{60} = 6.56\,\text{m/s}$$

From Outlet triangle,

$$\tan\beta_2 = \frac{C_{r2}}{U_2} = \frac{2.8}{6.56} = 0.4268 \text{ i.e. } \beta_2 = 23.11°$$

Illustrative Example 3.14: Consider an inward flow reaction turbine in which water is supplied at the rate of 500 L/s with a velocity of flow of 1.5 m/s. The velocity periphery at inlet is 20 m/s and velocity of whirl at inlet is 15 m/s. Assuming radial discharge, and velocity of flow to be constant, find

1. Vane angle at inlet, and
2. Head of water on the wheel.

Solution:

Discharge, $Q = 500\,\text{L/s} = 0.5\,\text{m}^3/\text{s}$

Velocity of flow at inlet, $C_{r1} = 1.5\,\text{m/s}$

Velocity of periphery at inlet, $U_1 = 20\,\text{m/s}$

Velocity of whirl at inlet, $C_{w1} = 15\,\text{m/s}$

As the velocity of flow is constant, $C_{r1} = C_{r2} = 1.5\,\text{m/s}$

Let β_1 = vane angle at inlet

From inlet velocity triangle

$$\tan(180 - \beta_1) = \frac{C_{r1}}{U_1 - C_{w1}} = \frac{1.5}{20 - 15} = 0.3$$

$$\therefore\ (180 - \beta_1) = 16°41'$$

or

$$\beta_1 = 180° - 16°41' = 163°19'$$

Since the discharge is radial at outlet, ad so the velocity of whirl at outlet is zero
Therefore,

$$\frac{C_{w1}U_1}{g} = H - \frac{C_1^2}{2g} = H - \frac{C_{r1}^2}{2g}$$

or

$$\frac{(15)(20)}{9.81} = H - \frac{1.5^2}{(2)(9.81)}$$

$$\therefore\ H = 30.58 - 0.1147 = 30.47\ \text{m}$$

Design Example 3.15: Inner and outer diameters of an outward flow reaction turbine wheel are 1 m and 2 m respectively. The water enters the vane at angle of 20° and leaves the vane radially. Assuming the velocity of flow remains constant at 12 m/s and wheel rotates at 290 rpm, find the vane angles at inlet and outlet.

Solution:

Inner diameter of wheel, $D_1 = 1\ \text{m}$

Outer diameter of wheel, $D_2 = 2\ \text{m}$

$\alpha_1 = 20°$

Velocity of flow is constant

That is, $C_{r1} = C_{r2} = 12\ \text{m/s}$

Speed of wheel, $N = 290\ \text{rpm}$

Vane angle at inlet $= \beta_1$

U_1 is the velocity of periphery at inlet.

Therefore, $U_1 = \dfrac{\pi D_1 N}{60} = \dfrac{(\pi)(1)(290)}{60} = 15.19\ \text{m/s}$

From inlet triangle, velocity of whirl is given by

$$C_{w1} = \frac{12}{\tan 20} = \frac{12}{0.364} = 32.97\ \text{m/s}$$

Hence, $\tan\beta_1 = \dfrac{C_{r1}}{C_{w1} - U_1} = \dfrac{12}{32.97 - 15.19} = \dfrac{12}{17.78} = 0.675$

i.e. $\beta_1 = 34°$

Let $\beta_2 =$ vane angle at outlet

$U_2 =$ velocity of periphery at outlet

Therefore $U_2 = \dfrac{\pi D_2 N}{60} = \dfrac{(\pi)(2)(290)}{60} = 30.38\ \text{m/s}$

From the outlet triangle

$$\tan\beta_2 = \frac{C_{r2}}{U_2} = \frac{12}{30.38} = 0.395$$

i.e.,

$$\beta_2 = 21°33'$$

Illustrative Example 3.16: An inward flow turbine is supplied with 245 L of water per second and works under a total head of 30 m. The velocity of wheel periphery at inlet is 16 m/s. The outlet pipe of the turbine is 28 cm in diameter. The radial velocity is constant. Neglecting friction, calculate

1. The vane angle at inlet
2. The guide blade angle
3. Power.

Solution:
If D_1 is the diameter of pipe, then discharge is

$$Q = \frac{\pi}{4} D_1^2 C_2$$

or

$$C_2 = \frac{(4)(0.245)}{(\pi)(0.28)^2} = 3.98 \text{ m/s}$$

But $C_2 = C_{r1} = C_{r2}$
Neglecting losses, we have

$$\frac{C_{w1} U_1}{gH} = \frac{H - C_2^2/2g}{H}$$

or

$$C_{W1} U_1 = gH - C_2^2/2$$

$$= [(9.81)(30)] - \frac{(3.98)^2}{2} = 294.3 - 7.92 = 286.38$$

Power developed

$$P = (286.38)(0.245) \text{ kW} = 70.16 \text{ kW}$$

and $C_{w1} = \dfrac{286.38}{16} = 17.9$ m/s

$$\tan \alpha_1 = \frac{3.98}{17.9} = 0.222$$

i.e. $\alpha_1 = 12°31'$

$$\tan \beta_1 = \frac{C_{r1}}{C_{w1} - U_1} = \frac{3.98}{17.9 - 16} = \frac{3.98}{1.9} = 2.095$$

i.e. $\beta_1 = 64.43$ or $\beta_1 = 64°25'$

Design Example 3.17: A reaction turbine is to be selected from the following data:

Discharge $= 7.8\,\mathrm{m^3/s}$

Shaft power $= 12,400\,\mathrm{kW}$

Pressure head in scroll casing

at the entrance to turbine $= 164\,\mathrm{m}$ of water

Elevation of turbine casing above tail water level $= 5.4\,\mathrm{m}$

Diameter of turbine casing $= 1\,\mathrm{m}$

Velocity in tail race $= 1.6\,\mathrm{m/s}$

Calculate the effective head on the turbine and the overall efficiency of the unit.

Solution:

Velocity in casing at inlet to turbine

$$C_c = \frac{\text{Discharge}}{\text{Cross} - \text{ sectional area of casing}}$$

$$= \frac{7.8}{(\pi/4)(1)^2} = 9.93\,\mathrm{m/s}$$

The net head on turbine

$$= \text{Pressure head} + \text{Head due to turbine position} + \frac{C_c^2 - C_1^2}{2g}$$

$$= 164 + 5.4 + \frac{(9.93)^2 - (1.6)^2}{2g}$$

$$= 164 + 5.4 + \frac{98.6 - 2.56}{19.62} = 174.3\,\mathrm{m} \text{ of water}$$

Waterpower supplied to turbine $= QgH$ kW

$$= (7.8)(9.81)(174.3) = 13,337\,\mathrm{kW}$$

Hence overall efficiency,

$$\eta_o = \frac{\text{Shaft Power}}{\text{Water Power}} = \frac{12,400}{13,337} = 0.93 \text{ or } 93\%$$

Design Example 3.18: A Francis turbine wheel rotates at 1250 rpm and net head across the turbine is 125 m. The volume flow rate is $0.45\,\mathrm{m^3/s}$, radius of the runner is 0.5 m. The height of the runner vanes at inlet is 0.035 m. and the angle of the inlet guide vanes is set at 70° from the radial direction. Assume that the absolute flow velocity is radial at exit, find the torque and power exerted by the water. Also calculate the hydraulic efficiency.

Solution:
For torque, using angular momentum equation

$$T = m(C_{W2}r_2 - C_{W1}r_1)$$

As the flow is radial at outlet, $C_{w2} = 0$ and therefore

$$\begin{aligned} T &= -mC_{W1}r_1 \\ &= -\rho QC_{W1}r_1 \\ &= -(10^3)(0.45)(0.5C_{W1}) \\ &= -225C_{W1}\text{Nm} \end{aligned}$$

If h_1 is the inlet runner height, then inlet area, A, is

$$\begin{aligned} A &= 2\pi r_1 h_1 \\ &= (2)(\pi)(0.5)(0.035) = 0.11\text{m}^2 \end{aligned}$$

$$C_{r1} = Q/A = \frac{0.45}{0.11} = 4.1\text{ m/s}$$

From velocity triangle, velocity of whirl

$$C_{w1} = C_{r1}\tan 70° = (4.1)(2.75) = 11.26\text{m/s}$$

Substituting C_{w1}, torque is given by

$$T = -(225)(11.26) = -2534\text{ Nm}$$

Negative sign indicates that torque is exerted on the fluid. The torque exerted by the fluid is $+2534$ Nm
Power exerted

$$\begin{aligned} P &= T\omega \\ &= \frac{(2534)(2)(\pi)(1250)}{(60)(1000)} \\ &= 331.83\text{ kW} \end{aligned}$$

Hydraulic efficiency is given by

$$\eta_h = \frac{\text{Power exerted}}{\text{Power available}}$$
$$= \frac{(331.83)(10^3)}{\rho g Q H}$$
$$= \frac{331.83 \times 10^3}{(10^3)(9.81)(0.45)(125)}$$
$$= 0.6013 = 60.13\%$$

Design Example 3.19: An inward radial flow turbine develops 130 kW under a head of 5 m. The flow velocity is 4 m/s and the runner tangential velocity at inlet is 9.6 m/s. The runner rotates at 230 rpm while hydraulic losses accounting for 20% of the energy available. Calculate the inlet guide vane exit angle, the inlet angle to the runner vane, the runner diameter at the inlet, and the height of the runner at inlet. Assume radial discharge, and overall efficiency equal to 72%.

Solution:
Hydraulic efficiency is

$$\eta_h = \frac{\text{Power deleloped}}{\text{Power available}}$$
$$= \frac{m(C_{w1}U_1 - C_{w2}U)}{\rho g Q H}$$

Since flow is radial at outlet, then $C_{w2} = 0$ and $m = \rho Q$, therefore

$$\eta_h = \frac{C_{w1}U_1}{gH}$$

$$0.80 = \frac{(C_{w1})(9.6)}{(9.81)(5)}$$

$$C_{w1} = \frac{(0.80)(9.81)(5)}{9.6} = 4.09 \text{ m/s}$$

Radial velocity $C_{r1} = 4$ m/s

$$\tan \alpha_1 = C_{r1}/C_{W1} \text{ (from velocity triangle)}$$
$$= \frac{4}{4.09} = 0.978$$

i.e., inlet guide vane angle $\alpha_1 = 44°21'$

$$\tan \beta_1 = \frac{C_{r1}}{(C_{w1} - U_1)}$$

$$= \frac{4}{(4.09 - 9.6)} = \frac{4}{-5.51} = -0.726$$

i.e., $\beta_1 = -35.98°$ or $180° - 35.98 = 144.02°$

Runner speed is

$$U_1 = \frac{\pi D_1 N}{60}$$

or

$$D_1 = \frac{60 U_1}{\pi N} = \frac{(60)(9.6)}{(\pi)(230)}$$

$$D_1 = 0.797\,\text{m}$$

Overall efficiency

$$\eta_o = \frac{\text{Power output}}{\text{Power available}}$$

or

$$\rho g Q H = \frac{(130)(10^3)}{0.72}$$

or

$$Q = \frac{(130)(10^3)}{(0.72)(10^3)(9.81)(5)} = 3.68\,\text{m}^3/\text{s}$$

But

$$Q = \pi D_1 h_1 C_{r1} (\text{where } h_1 \text{ is the height of runner})$$

Therefore,

$$h_1 = \frac{3.68}{(\pi)(0.797)(4)} = 0.367\,\text{m}$$

Illustrative Example 3.20: The blade tip and hub diameters of an axial hydraulic turbine are 4.50 m and 2 m respectively. The turbine has a net head of 22 m across it and develops 22 MW at a speed of 150 rpm. If the hydraulic efficiency is 92% and the overall efficiency 84%, calculate the inlet and outlet blade angles at the mean radius assuming axial flow at outlet.

Solution:
Mean diameter, D_m, is given by

$$D_m = \frac{D_h + D_t}{2} = \frac{2 + 4.50}{2} = 3.25\,\text{m}$$

Overall efficiency, η_o, is given by

$$\eta_o = \frac{\text{Power develpoed}}{\text{Power available}}$$

$$\therefore\ \text{Power available} = \frac{22}{0.84} = 26.2\,\text{MW}$$

Also, available power $= \rho g Q H$

$$(26.2)(10^6) = (10^3)(9.81)(22)Q$$

Hence flow rate, Q, is given by

$$Q = \frac{(26.2)(10^6)}{(10^3)(9.81)(22)} = 121.4\,\text{m}^3/\text{s}$$

Now rotor speed at mean diameter

$$U_m = \frac{\pi D_m N}{60} = \frac{(\pi)(3.25)(150)}{60} = 25.54\,\text{m/s}$$

$$\begin{aligned}\text{Power given to runner} &= \text{Power available} \times \eta_h \\ &= 26.2 \times 10^6 \times 0.92 \\ &= 24.104\,\text{MW}\end{aligned}$$

Theoretical power given to runner can be found by using

$$P = \rho Q U_m C_{w1} (C_{w2} = 0)$$

$$(24.104)(10^6) = (10^3)(121.4)(25.54)(C_{w1})$$

$$\therefore\ C_{w1} = \frac{(24.104)(10^6)}{(10^3)(121.4)(25.54)} = 7.77\,\text{m/s}$$

Axial velocity is given by

$$C_r = \frac{Q \times 4}{\pi(D_t^2 - D_h^2)} = \frac{(121.4)(4)}{\pi(4.50^2 - 2^2)} = 9.51\,\text{m/s}$$

Using velocity triangle

$$\tan(180 - \beta_1) = \frac{C_r}{U_m - C_{w1}} = \frac{9.51}{25.54 - 7.77}$$

Inlet angle,

$$\beta_1 = 151.85°$$

At outlet

$$\tan \beta_2 = \frac{C_r}{V_{cw_2}}$$

But V_{cw2} equals to U_m since C_{w2} is zero. Hence

$$\tan \beta_2 = \frac{9.51}{25.54} = 0.3724$$

that is,

$$\beta_2 = 20.43°$$

Design Example 3.21: The following design data apply to an inward flow radial turbine:

Overall efficiency	75%
Net head across the turbine	6 m
Power output	128 kW
The runner tangential velocity	10.6 m/s
Flow velocity	4 m/s
Runner rotational speed	235 rpm
Hydraulic losses	18% of energy available

Calculate the inlet guide vane angle, the inlet angle of the runner vane, the runner diameter at inlet, and height of the runner at inlet. Assume that the discharge is radial.

Solution:

Hydraulic efficiency, η_h, is given by

$$\eta_h = \frac{\text{Power given to runner}}{\text{Water Power available}} = \frac{m(U_1 C_{w1} - U_2 C_{w2})}{\rho g Q H}$$

Since flow is radial at exit, $C_{w2} = 0$ and $m = \rho Q$. Therefore

$$\eta_h = \frac{U_1 C_{w1}}{gH}$$

$$0.82 = \frac{(10.6)(C_{w1})}{(9.81)(6)} \quad \text{or} \quad C_{w1} = 4.6\,\text{m/s}$$

Now

$$\tan \alpha_1 = C_{r1}/C_{w1} = \frac{4}{4.6} = 0.8695$$

that is, $\alpha_1 = 41°$

From Figs. 3.22 and 3.23

$$\tan(180 - \beta_1) = \frac{C_{r1}}{U_1 - C_{w1}} = \frac{4}{10.6 - 4.6} = 0.667$$

that is, $\beta_1 = 33.69°$

Hence blade angle, β_1, is given by

$$180° - 33.69° = 146.31°$$

Runner speed at inlet

$$U_1 = \frac{\pi D_1 N}{60}$$

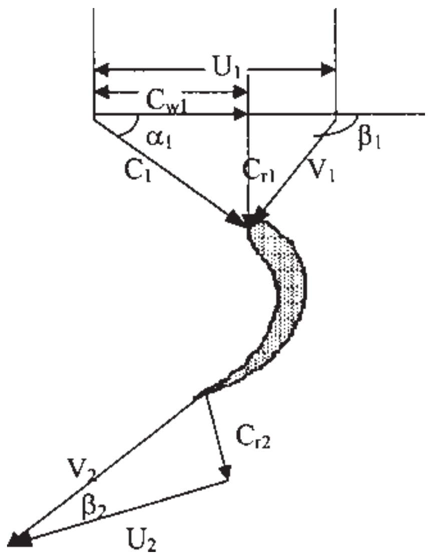


Figure 3.22 Velocity triangles for Example 3.14.

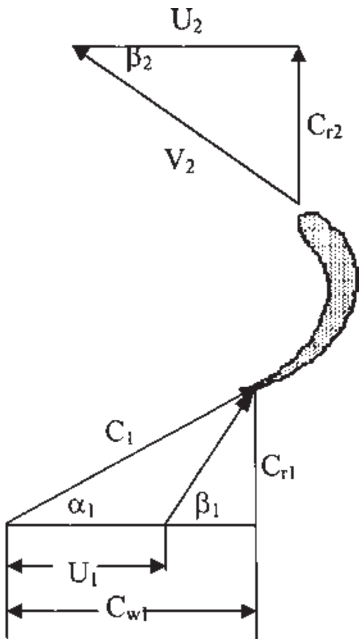


Figure 3.23 Velocity triangles at inlet and outlet for Example 3.15.

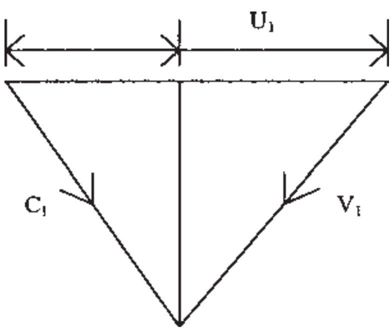


Figure 3.24 Inlet velocity triangle.

or

$$D_1 = \frac{U_1(60)}{\pi N} = \frac{(10.6)(60)}{(\pi)(235)} = 0.86\,\text{m}$$

Overall efficiency

$$\eta_O = \frac{\text{Power output}}{\text{Power available}}$$

$$\rho g Q H = \frac{(128)(10^3)}{0.75}$$

From which flow rate

$$Q = \frac{(128)(10^3)}{(0.75)(10^3)(9.81)(6)} = 2.9\,\text{m}^3/\text{s}$$

Also,

$$Q = \pi D_1 h C_{r1}$$

where h_1 is the height of runner
Therefore,

$$h_1 = \frac{2.9}{(\pi)(0.86)(4)} = 0.268\,\text{m}$$

Design Example 3.22: A Kaplan turbine develops 10,000 kW under an effective head 8 m. The overall efficiency is 0.86, the speed ratio 2.0, and flow ratio 0.60. The hub diameter of the wheel is 0.35 times the outside diameter of the wheel. Find the diameter and speed of the turbine.

Solution:
Head, H = 8 m, Power, P = 10,000 kW
Overall efficiency, $\eta_o = 0.86$
Speed ratio

$$2 = \frac{U_1}{(2gH)^{1/2}}, \quad \text{or } U_l = \sqrt{2 \times 9.81 \times 8} = 25.06\,\text{m/s}$$

Flow ratio

$$\frac{C_{r1}}{(2gH)^{1/2}} = 0.60 \text{ or } C_{r1} = 0.60\sqrt{2 \times 9.81 \times 8} = 7.52\,\text{m/s}$$

Hub diameter, $D_1 = 0.35\ D_2$

Overall efficiency,

$$\eta_O = \frac{P}{\rho g Q H}$$

Or

$$0.86 = \frac{10000}{1000 \times 9.81 \times Q \times 8}$$

$$\therefore \ \mathrm{Q} = 148.16\,\mathrm{m^3/s}$$

Now using the relation

$$\mathrm{Q} = C_{r1} \times \frac{\pi}{4}\left[D_1^2 - D_2^2\right]$$

Or

$$148.16 = 7.52 \times \frac{\pi}{4}\left[D_1^2 - \left(0.35 D_1^2\right)\right]$$

$$\therefore \ \mathrm{D_1} = 5.35\,\mathrm{m}$$

The peripheral velocity of the turbine at inlet

$$25.06 = \frac{\pi D_1 N}{60} = \frac{\pi \times 5.35 \times N}{60}$$

$$\therefore \ \mathrm{N} = \frac{60 \times 25.06}{\pi \times 5.35} = 89\,\mathrm{rpm}$$

Design Example 3.23: An inward flow reaction turbine, having an inner and outer diameter of 0.45 m and 0.90 m, respectively. The vanes are radial at inlet and the discharge is radial at outlet and the water enters the vanes at an angle of 12°. Assuming the velocity of flow as constant and equal to 2.8 m/s, find the speed of the wheel and the vane angle at outlet.

Solution:
Inner Diameter, $\mathrm{D_2} = 0.45\,\mathrm{m}$
Outer Diameter, $\mathrm{D_1} = 0.9\,\mathrm{m}$

$$\alpha_2 = 90^\circ (\text{radial discharge})$$

$$\alpha_1 = 12^\circ, \mathrm{C_{r1}} = \mathrm{C_{r2}} = 2.8\,\mathrm{m/s}$$

From velocity triangle at inlet (see Fig. 3.11), The peripheral velocity of the wheel at inlet

$$U_1 = \frac{C_{r1}}{\tan \alpha_1} = \frac{2.8}{\tan 12^\circ} = 13.173\,\text{m/s}$$

Now,

$$U_1 = \frac{\pi D_1 N}{60}$$

or

$$N = \frac{60 U_1}{\pi D_1} = \frac{60 \times 13.173}{\pi \times 0.9} = 279\,\text{rpm}$$

Considering velocity triangle at outlet peripheral velocity at outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.45 \times 279}{60} = 6.58\,\text{m/s}$$

$$\tan \beta_2 = \frac{C_{r2}}{U_2} = \frac{2.8}{6.58} = 0.426$$

$$\therefore\ \beta_2 = 23.05^\circ$$

Design Example 3.24: An inward flow reaction turbine develops 70 kW at 370 rpm. The inner and outer diameters of the wheel are 40 and 80 cm, respectively. The velocity of the water at exit is 2.8 m/s. Assuming that the discharge is radial and that the width of the wheel is constant, find the actual and theoretical hydraulic efficiencies of the turbine and the inlet angles of the guide and wheel vanes. Turbine discharges 545 L/s under a head of 14 m.

Solution:

$Q = 545\,\text{L/s} = 0.545\,\text{m}^3/\text{s}$

$D_1 = 80\,\text{cm}$, $D_2 = 40\,\text{cm}$

$H = 14\,\text{m}$, $\alpha_2 = 90^\circ$ (radial discharge)

$\beta_1 = \beta_2$

Peripheral velocity of the wheel at inlet

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.80 \times 370}{60} = 15.5\,\text{m/s}$$

Velocity of flow at the exit, $C_{r2} = 2.8\,\text{m/s}$

As $\alpha_2 = 90^\circ$, $C_{r2} = C_2$

Work done/s by the turbine per kg of water $= \frac{C_w \times U_1}{g}$

But this is equal to the head utilized by the turbine, i.e.

$$\frac{C_{w1}U_1}{g} = H - \frac{C_2}{2g}$$

(Assuming there is no loss of pressure at outlet) or

$$\frac{C_{w1} \times 15.5}{9.81} = 14 - \frac{(2.8)^2}{2 \times 9.81} = 13.6\,\text{m}$$

or

$$C_{w1} = \frac{13.6 \times 9.81}{15.5} = 8.6\,\text{m/s}$$

Work done per second by turbine

$$= \frac{\rho Q}{g} C_{w1} U_1$$

$$= \frac{1000 \times 0.545 \times 8.6 \times 15.5}{1000}$$

$$= 72.65\text{kW}$$

$$\text{Available power or water power} = \frac{\rho g Q H}{1000}$$

$$= 74.85$$

Actual available power = 70 kW

$$\text{Overall turbine efficiency is } \eta_t = \frac{70}{74.85} \times 100$$

$$\eta_t = 93.52\%$$

This is the actual hydraulic efficiency as required in the problem.
Hydraulic Efficiency is

$$\eta_h = \frac{72.65}{75.85} \times 100 = 97.06\%$$

This is the theoretical efficiency

$$Q = \pi D_1 b_1 C_{r1} = \pi D_2 b_2 C_{r2}$$

(Neglecting blade thickness)

$$C_{r1} = C_{r2}\frac{D_2}{D_1} = 2.8 \times \frac{40}{20} = 1.4\,\text{m/s}$$

Drawing inlet velocity triangle

$$\tan \beta_1 = \frac{C_{r1}}{U_1 - C_{w1}} = \frac{1.4}{15.5 - 8.6} = \frac{1.4}{6.9} = 0.203$$

i.e., $\beta_1 = 11.47°$

$$C_1 = \sqrt{C_{w1} + C_{r1}} = \left(8.6^2 + 1.4^2\right)^{0.5} = 8.64\,\text{m/s}$$

and

$$\cos\alpha_1 = \frac{C_{w1}}{C_1} = \frac{8.6}{8.64} = 0.995$$

i.e., $\alpha_1 = 5.5°$

Design Example 3.25: An inward flow Francis turbine, having an overall efficiency of 86%, hydraulic efficiency of 90%, and radial velocity of flow at inlet $0.28\sqrt{2gH}$. The turbine is required to develop 5000 kW when operating under a net head of 30 m, specific speed is 270, assume guide vane angle 30°, find

1. rpm of the wheel,
2. the diameter and the width of the runner at inlet, and
3. the theoretical inlet angle of the runner vanes.

Solution:

Power, $P = 5000\,\text{kW}, \alpha_1 = 30°, H = 30\,\text{m}, C_{r1} = 0.28\sqrt{2gH}$, $N_s = 270$, $\eta_h = 0.90$, $\eta_o = 0.86$

1. Specific speed of the turbine is

$$N_S = \frac{N\sqrt{P}}{H^{5/4}}$$

or

$$N = \frac{N_S H^{5/4}}{\sqrt{P}} = \frac{270 \times (30)^{1.25}}{\sqrt{5000}} = \frac{18957}{71} = 267\,\text{rpm}$$

2. Velocity of Flow:

$$C_{r1} = 0.28\sqrt{2 \times 9.81 \times 30} = 6.79\text{m/s}$$

From inlet velocity triangle

$$C_{r1} = C_1 \sin\alpha_1$$

or

$$6.79 = C_1 \sin 30°$$

or

$$C_1 = \frac{6.79}{0.5} = 13.58\,\text{m/s}$$

$$C_{w1} = C_1 \cos 30° = 13.58 \times 0.866 = 11.76\,\text{m/s}$$

Work done per (sec) (kg) of water

$$= \frac{C_{w1} \times U_1}{g} = \eta_h \times \mathrm{H}$$
$$= 0.9 \times 30$$
$$= 27\,\mathrm{mkg/s}$$

Peripheral Velocity,

$$U_1 = \frac{27 \times 9.81}{11.76} = 22.5\,\mathrm{m/s}$$

But $U_1 = \dfrac{\pi D_1 N}{60}$
or

$$D_1 = \frac{60 U_1}{\pi N} = \frac{60 \times 22.5}{\pi \times 267} = 1.61\,\mathrm{m}$$

Power, $P = \rho g Q H \eta_o$
or
$5000 = 1000 \times 9.81 \times Q \times 30 \times 0.86$
or
$Q = 19.8\,\mathrm{m^3/s}$
Also $Q = k\pi D_1 b_1 C_{r1}$ (where k is the blade thickness coefficient and b_1 is the breath of the wheel at inlet) or

$$b_1 = \frac{Q}{k\pi D_1 C_{r1}} = \frac{19.8}{0.95 \times \pi \times 1.61 \times 6.79} = 0.61\,\mathrm{m}$$

3. From inlet velocity triangle

$$\tan\beta_1 = \frac{C_{r1}}{U_1 - C_{w1}} = \frac{6.79}{22.5 - 11.76} = \frac{6.79}{10.74} = 0.632$$

i.e. $\beta_1 = 32.30°$

Design Example 3.26: A 35 MW generator is to operate by a double overhung Pelton wheel. The effective head is 350 m at the base of the nozzle. Find the size of jet, mean diameter of the runner and specific speed of wheel. Assume Pelton wheel efficiency 84%, velocity coefficient of nozzle 0.96, jet ratio 12, and speed ratio 0.45.

Solution:
In this case, the generator is fed by two Pelton turbines.

Power developed by each turbine,

$$P_{\mathrm{T}} = \frac{35,000}{2} = 17,500\,\mathrm{kW}$$

Using Pelton wheel efficiency in order to find available power of each turbine

$$P = \frac{17,500}{0.84} = 20,833\,\mathrm{kW}$$

But, $P = \rho gQH$

$$Q = \frac{P}{\rho gH} = \frac{20833}{1000 \times 9.81 \times 350} = 6.07\,\mathrm{m^3/s}$$

Velocity of jet, $C_{\mathrm{j}} = C_{\mathrm{V}}\sqrt{2gH} = 0.96\sqrt{2 \times 9.81 \times 350}$

$$C_{\mathrm{j}} = 79.6\,\mathrm{m/s}$$

Area of jet, $A = \frac{Q}{C_j} = \frac{6.07}{79.6} = 0.0763\,\mathrm{m^2}$

$$\therefore \text{ Diameter of jet}, d = \left(\frac{4A}{\pi}\right)^{0.5} = \left(\frac{4 \times 0.0763}{\pi}\right)^{0.5} = 0.312\,\mathrm{m}$$

$$d = 31.2\,\mathrm{cm}$$

Diameter of wheel D = d × jet ratio = 0.312 × 12 = 3.744 m
Peripheral velocity of the wheel

$$U = \text{speed ratio}\sqrt{2gH}$$

$$= 0.45 \times \sqrt{2 \times 9.81 \times 350} = 37.29\,\mathrm{m/s}$$

But $U = \frac{\pi DN}{60}$ or

$$N = \frac{60U}{\pi D} = \frac{60 \times 37.29}{\pi \times 3.744} = 190\,\mathrm{rpm}$$

Specific speed,

$$N_{\mathrm{S}} = \frac{N\sqrt{P_T}}{H^{5/4}} = \frac{190\sqrt{17,500}}{(350)^{1.25}} = 16.6$$

PROBLEMS

3.1 A Pelton wheel produces 4600 hP under a head of 95 m, and with an overall efficiency of 84%. Find the diameter of the nozzle if the coefficient of velocity for the nozzle is 0.98.

(0.36 m)

3.2 Pelton wheel develops 13,500 kW under a head of 500 m. The wheel rotates at 430 rpm. Find the size of the jet and the specific speed. Assume 85% efficiency.

(0.21 m, 21)

3.3 A Pelton wheel develops 2800 bhP under a head of 300 m at 84% efficiency. The ratio of peripheral velocity of wheel to jet velocity is 0.45 and specific speed is 17. Assume any necessary data and find the jet diameter.

(140 mm)

3.4 A Pelton wheel of power station develops 30,500 hP under a head of 1750 m while running at 760 rpm. Calculate (1) the mean diameter of the runner, (2) the jet diameter, and (3) the diameter ratio.

(2.14 m, 0.104 m, 20.6)

3.5 Show that in an inward flow reaction turbine, when the velocity of flow is constant and wheel vane angle at entrance is 90°, the best peripheral velocity is

$$\sqrt{2gH}/\sqrt{2 + \tan^2 \alpha}$$

where H is the head and α the angle of guide vane.

3.6 A Pelton wheel develops 740 kW under a head of 310 m. Find the jet diameter if its efficiency is 86% and

$$C_V = 0.98.$$

(0.069 m)

3.7 A reaction turbine runner diameter is 3.5 m at inlet and 2.5 m at outlet. The turbine discharge 102 m^3 per second of water under a head of 145 m. Its inlet vane angle is 120°. Assume radial discharge at 14 m/s, breadth of wheel constant and hydraulic efficiency of 88%, calculate the power developed and speed of machine.

(128 MW, 356 rpm)

3.8 Show that in a Pelton wheel, where the buckets deflect the water through an angle of (180° − α) degrees, the hydraulic efficiency of the wheel is given by

$$\eta_h = \frac{2U(C - U)(1 + \cos \alpha)}{C^2}$$

where C is the velocity of jet and U is mean blade velocity.

3.9 A Kaplan turbine produces 16000 kW under a head of 20 m, while running at 166 rpm. The diameter of the runner is 4.2 m while the hub diameter is 2 m, the discharge being 120 m^3/s. Calculate (1) the turbine efficiency,

(2) specific speed, (3) the speed ratio based on the tip diameter of the blade, and (4) the flow ratio.

(78%, 497, 1.84, 0.48)

3.10 Evolve a formula for the specific speed of a Pelton wheel in the following form

$$N_s = k \cdot \sqrt{\eta \cdot} \frac{d}{D}$$

where N_s = specific speed, η = overall efficiency, d = diameter of jet, D = diameter of bucket circle, and k = a constant.

NOTATION

C	jet velocity, absolute
C_v	nozzle velocity coefficient
C_w	velocity of whirl
D	wheel diameter
d	diameter of nozzle
E	energy transfer by bucket
H_r	head across the runner
h_f	frictional head loss
N_s	specific speed
P	water power available
P_c	casing and draft tube losses
P_h	hydraulic power loss
P_l	leakage loss
P_m	mechanical power loss
P_r	runner power loss
P_s	shaft power output
U	bucket speed
W	work done
α	angle of the blade tip at outlet
β	angle with relative velocity
η_i	nozzle efficiency
η_{trans}	transmission efficiency
κ	relative velocity ratio

4

Centrifugal Compressors and Fans

4.1 INTRODUCTION

This chapter will be concerned with power absorbing turbomachines, used to handle compressible fluids. There are three types of turbomachines: fans, blowers, and compressors. A fan causes only a small rise in stagnation pressure of the flowing fluid. A fan consists of a rotating wheel (called the impeller), which is surrounded by a stationary member known as the housing. Energy is transmitted to the air by the power-driven wheel and a pressure difference is created, providing airflow. The air feed into a fan is called induced draft, while the air exhausted from a fan is called forced draft. In blowers, air is compressed in a series of successive stages and is often led through a diffuser located near the exit. The overall pressure rise may range from 1.5 to 2.5 atm with shaft speeds up to 30,000 rpm or more.

4.2 CENTRIFUGAL COMPRESSOR

The compressor, which can be axial flow, centrifugal flow, or a combination of the two, produces the highly compressed air needed for efficient combustion. In turbocompressors or dynamic compressors, high pressure is achieved by imparting kinetic energy to the air in the impeller, and then this kinetic energy converts into pressure in the diffuser. Velocities of airflow are quite high and the Mach number of the flow may approach unity at many points in the air stream. Compressibility effects may have to be taken into account at every stage of the compressor. Pressure ratios of 4:1 are typical in a single stage, and ratios of 6:1 are possible if materials such as titanium are used. There is renewed interest in the centrifugal stage, used in conjunction with one or more axial stages, for small turbofan and turboprop aircraft engines. The centrifugal compressor is not suitable when the pressure ratio requires the use of more than one stage in series because of aerodynamic problems. Nevertheless, two-stage centrifugal compressors have been used successfully in turbofan engines.

Figure 4.1 shows part of a centrifugal compressor. It consists of a stationary casing containing an impeller, which rotates and imparts kinetic energy to the air and a number of diverging passages in which the air decelerates. The deceleration converts kinetic energy into static pressure. This process is known as diffusion, and the part of the centrifugal compressor containing the diverging passages is known as the diffuser. Centrifugal compressors can be built with a double entry or a single entry impeller. Figure 4.2 shows a double entry centrifugal compressor.

Air enters the impeller eye and is whirled around at high speed by the vanes on the impeller disc. After leaving the impeller, the air passes through a diffuser in which kinetic energy is exchanged with pressure. Energy is imparted to the air by the rotating blades, thereby increasing the static pressure as it moves from eye radius r_1 to tip radius r_2. The remainder of the static pressure rise is achieved in the diffuser. The normal practice is to design the compressor so that about half the pressure rise occurs in the impeller and half in the diffuser. The air leaving the diffuser is collected and delivered to the outlet.

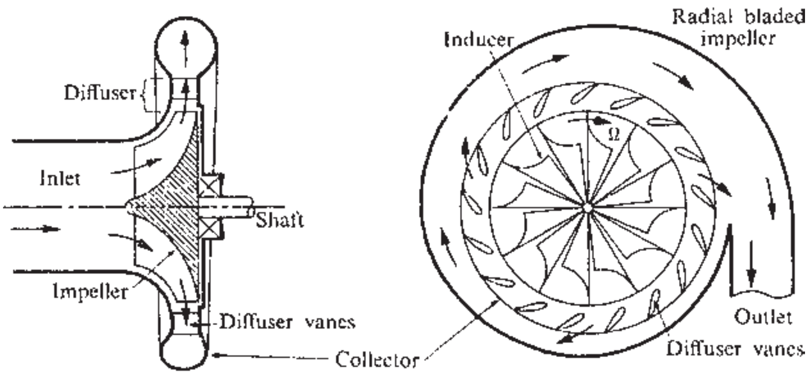


Figure 4.1 Typical centrifugal compressor.

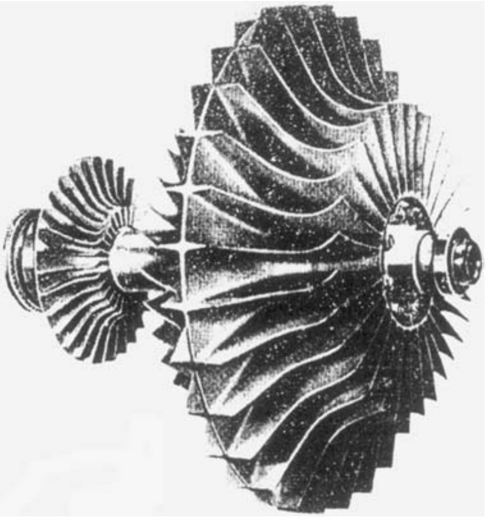


Figure 4.2 Double-entry main stage compressor with side-entry compressor for cooling air. (Courtesy of Rolls-Royce, Ltd.)

4.3 THE EFFECT OF BLADE SHAPE ON PERFORMANCE

As discussed in Chapter 2, there are three types of vanes used in impellers. They are: forward-curved, backward-curved, and radial vanes, as shown in Fig. 4.3.

The impellers tend to undergo high stress forces. Curved blades, such as those used in some fans and hydraulic pumps, tend to straighten out due to centrifugal force and bending stresses are set up in the vanes. The straight radial

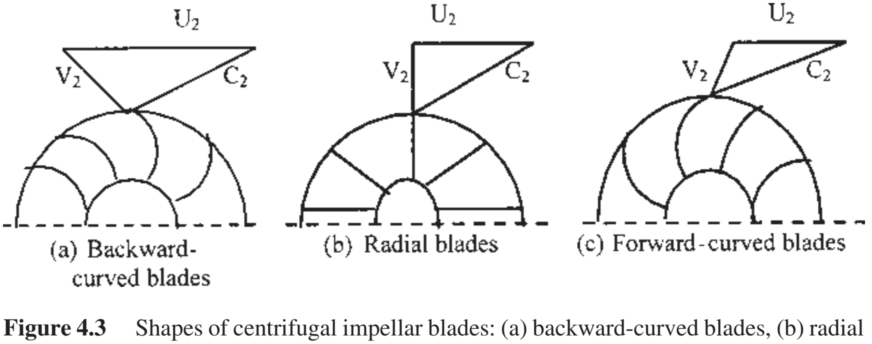


Figure 4.3 Shapes of centrifugal impellar blades: (a) backward-curved blades, (b) radial blades, and (c) forward-curved blades.

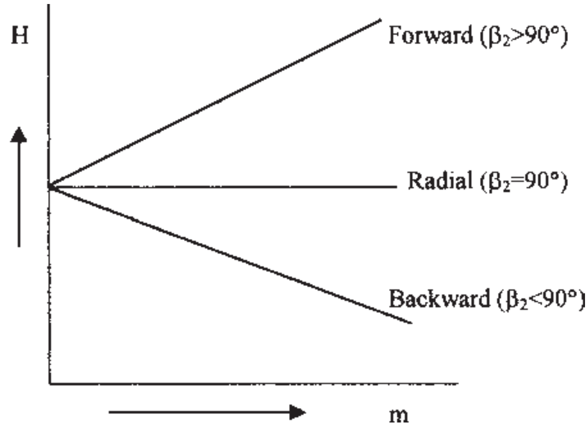


Figure 4.4 Pressure ratio or head versus mass flow or volume flow, for the three blade shapes.

blades are not only free from bending stresses, they may also be somewhat easier to manufacture than curved blades.

Figure 4.3 shows the three types of impeller vanes schematically, along with the velocity triangles in the radial plane for the outlet of each type of vane. Figure 4.4 represents the relative performance of these types of blades. It is clear that increased mass flow decreases the pressure on the backward blade, exerts the same pressure on the radial blade, and increases the pressure on the forward blade. For a given tip speed, the forward-curved blade impeller transfers maximum energy, the radial blade less, and the least energy is transferred by the backward-curved blades. Hence with forward-blade impellers, a given pressure ratio can be achieved from a smaller-sized machine than those with radial or backward-curved blades.

4.4 VELOCITY DIAGRAMS

Figure 4.5 shows the impeller and velocity diagrams at the inlet and outlet. Figure 4.5a represents the velocity triangle when the air enters the impeller in the axial direction. In this case, absolute velocity at the inlet, $C_1 = C_{a1}$. Figure 4.5b represents the velocity triangle at the inlet to the impeller eye and air enters through the inlet guide vanes. Angle θ is made by C_1 and C_{a1} and this angle is known as the angle of prewhirl. The absolute velocity C_1 has a whirl component C_{w1}. In the ideal case, air comes out from the impeller tip after making an angle of 90° (i.e., in the radial direction), so $C_{w2} = U_2$. That is, the whirl component is exactly equal to the impeller tip velocity. Figure 4.5c shows the ideal velocity

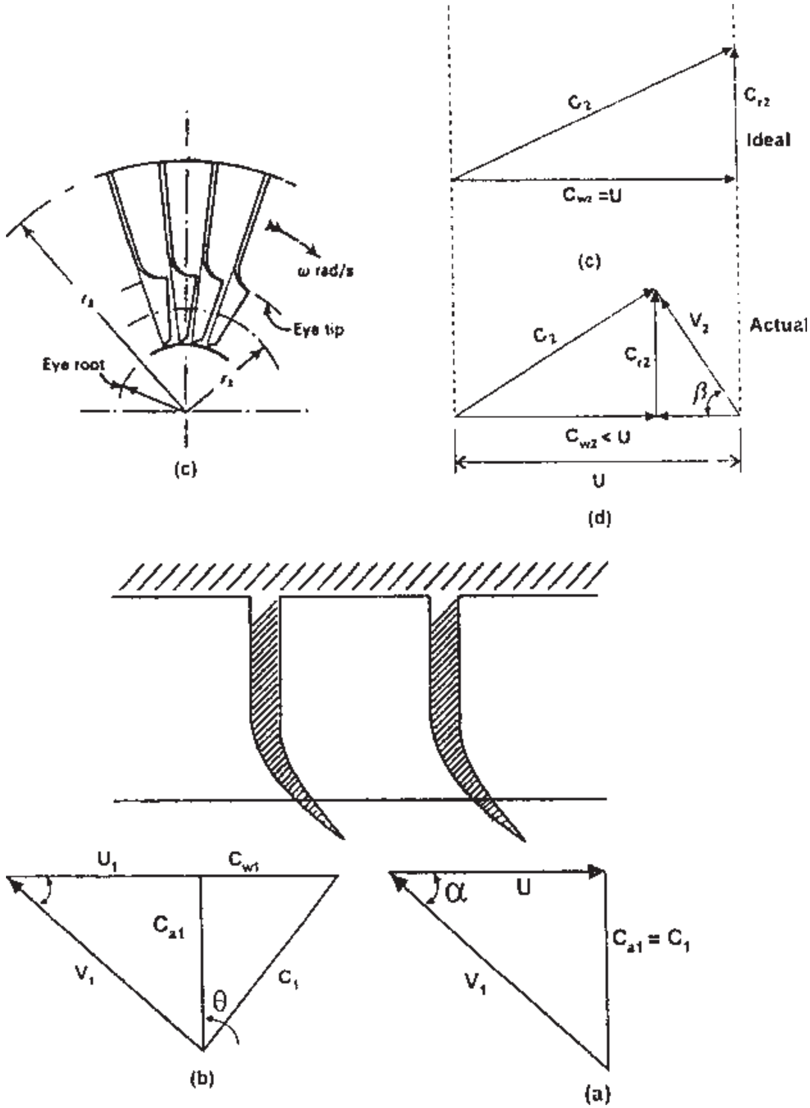


Figure 4.5 Centrifugal impellar and velocity diagrams.

triangle. But there is some slip between the impeller and the fluid, and actual values of C_{w1} are somewhat less than U_2. As we have already noted in the centrifugal pump, this results in a higher static pressure on the leading face of a vane than on the trailing face. Hence, the air is prevented from acquiring a whirl velocity equal to the impeller tip speed. Figure 4.5d represents the actual velocity triangle.

4.5 SLIP FACTOR

From the above discussion, it may be seen that there is no assurance that the actual fluid will follow the blade shape and leave the compressor in a radial direction. Thus, it is convenient to define a slip factor σ as:

$$\sigma = \frac{C_{w2}}{U_2} \tag{4.1}$$

Figure 4.6 shows the phenomenon of fluid slip with respect to a radial blade. In this case, C_{w2} is not equal to U_2; consequently, by the above definition, the slip factor is less than unity. If radial exit velocities are to be achieved by the actual fluid, the exit blade angle must be curved forward about 10–14 degrees. The slip factor is nearly constant for any machine and is related to the number of vanes on the impeller. Various theoretical and empirical studies of the flow in an impeller channel have led to formulas for

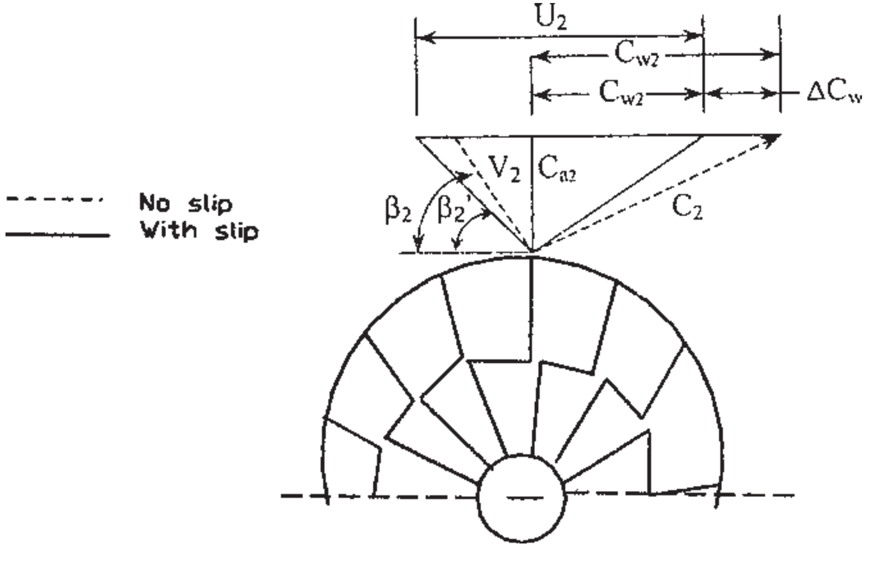


Figure 4.6 Centrifugal compressor impeller with radial vanes.

slip factors: For radial vaned impellers, the formula for σ is given by Stanitz as follows:

$$\sigma = 1 - \frac{0.63\pi}{n} \tag{4.2}$$

where n is the number of vanes. The velocity diagram indicates that C_{w2} approaches U_2 as the slip factor is increased. Increasing the number of vanes may increase the slip factor but this will decrease the flow area at the inlet. A slip factor of about 0.9 is typical for a compressor with 19–21 vanes.

4.6 WORK DONE

The theoretical torque will be equal to the rate of change of angular momentum experienced by the air. Considering a unit mass of air, this torque is given by theoretical torque,

$$\tau = C_{w2} r_2 \tag{4.3}$$

where, C_{w2} is whirl component of C_2 and r_2 is impeller tip radius.

Let ω = angular velocity. Then the theoretical work done on the air may be written as:

Theoretical work done $W_c = C_{w2} r_2 \omega = C_{w2} U_2$.

Using the slip factor, we have theoretical $W_c = \sigma U_2^2$ (treating work done on the air as positive)

In a real fluid, some of the power supplied by the impeller is used in overcoming losses that have a braking effect on the air carried round by the vanes. These include windage, disk friction, and casing friction. To take account of these losses, a power input factor can be introduced. This factor typically takes values between 1.035 and 1.04. Thus the actual work done on the air becomes:

$$W_c = \psi \sigma U_2^2 \tag{4.4}$$

(assuming $C_{w1} = 0$, although this is not always the case.)

Temperature equivalent of work done on the air is given by:

$$T_{02} - T_{01} = \frac{\psi \sigma U_2^2}{C_p}$$

where T_{01} is stagnation temperature at the impeller entrance; T_{02} is stagnation temperature at the impeller exit; and C_p is mean specific heat over this temperature range. As no work is done on the air in the diffuser, $T_{03} = T_{02}$, where T_{03} is the stagnation temperature at the diffuser outlet.

The compressor isentropic efficiency (η_c) may be defined as:

$$\eta_c = \frac{T_{03'} - T_{01}}{T_{03} - T_{01}}$$

(where $T_{03'}$ = isentropic stagnation temperature at the diffuser outlet) or

$$\eta_c = \frac{T_{01}\left(T_{03'}/T_{01} - 1\right)}{T_{03} - T_{01}}$$

Let P_{01} be stagnation pressure at the compressor inlet and; P_{03} is stagnation pressure at the diffuser exit. Then, using the isentropic P–T relationship, we get:

$$\frac{P_{03}}{P_{01}} = \left(\frac{T_{03'}}{T_{01}}\right)^{\gamma/(\gamma-1)} = \left[1 + \frac{\eta_c(T_{03} - T_{01})}{T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{\eta_c \psi \sigma U_2{}^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)} \tag{4.5}$$

Equation (4.5) indicates that the pressure ratio also depends on the inlet temperature T_{01} and impeller tip speed U_2. Any lowering of the inlet temperature T_{01} will clearly increase the pressure ratio of the compressor for a given work input, but it is not under the control of the designer. The centrifugal stresses in a rotating disc are proportional to the square of the rim. For single sided impellers of light alloy, U_2 is limited to about 460 m/s by the maximum allowable centrifugal stresses in the impeller. Such speeds produce pressure ratios of about 4:1. To avoid disc loading, lower speeds must be used for double-sided impellers.

4.7 DIFFUSER

The designing of an efficient combustion system is easier if the velocity of the air entering the combustion chamber is as low as possible. Typical diffuser outlet velocities are in the region of 90 m/s. The natural tendency of the air in a diffusion process is to break away from the walls of the diverging passage, reverse its direction and flow back in the direction of the pressure gradient, as shown in Fig. 4.7. Eddy formation during air deceleration causes loss by reducing the maximum pressure rise. Therefore, the maximum permissible included angle of the vane diffuser passage is about 11°. Any increase in this angle leads to a loss of efficiency due to

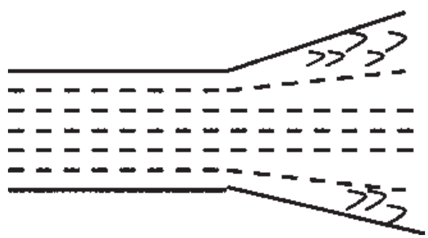


Figure 4.7 Diffusing flow.

boundary layer separation on the passage walls. It should also be noted that any change from the design mass flow and pressure ratio would also result in a loss of efficiency. The use of variable-angle diffuser vanes can control the efficiency loss. The flow theory of diffusion, covered in Chapter 2, is applicable here.

4.8 COMPRESSIBILITY EFFECTS

If the relative velocity of a compressible fluid reaches the speed of sound in the fluid, separation of flow causes excessive pressure losses. As mentioned earlier, diffusion is a very difficult process and there is always a tendency for the flow to break away from the surface, leading to eddy formation and reduced pressure rise. It is necessary to control the Mach number at certain points in the flow to mitigate this problem. The value of the Mach number cannot exceed the value at which shock waves occur. The relative Mach number at the impeller inlet must be less than unity.

As shown in Fig. 4.8a, the air breakaway from the convex face of the curved part of the impeller, and hence the Mach number at this point, will be very important and a shock wave might occur. Now, consider the inlet velocity triangle again (Fig. 4.5b). The relative Mach number at the inlet will be given by:

$$\mathrm{M}_1 = \frac{V_1}{\sqrt{\gamma \mathrm{R} T_1}} \tag{4.6}$$

where T_1 is the static temperature at the inlet.

It is possible to reduce the Mach number by introducing the prewhirl. The prewhirl is given by a set of fixed intake guide vanes preceding the impeller.

As shown in Fig. 4.8b, relative velocity is reduced as indicated by the dotted triangle. One obvious disadvantage of prewhirl is that the work capacity of

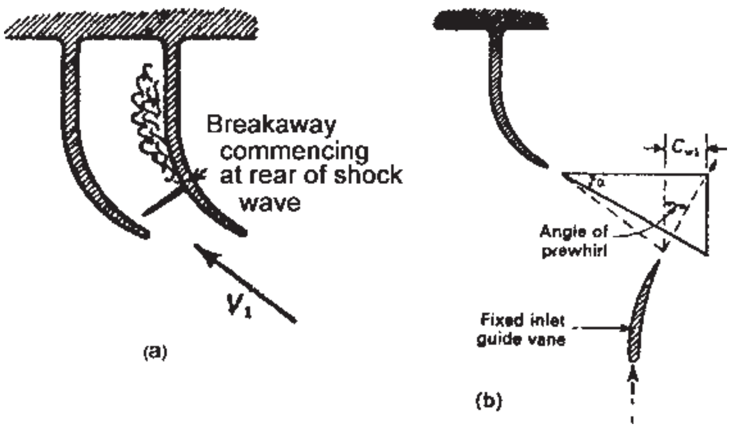


Figure 4.8 a) Breakaway commencing at the aft edge of the shock wave, and b) Compressibility effects.

the compressor is reduced by an amount U_1C_{w1}. It is not necessary to introduce prewhirl down to the hub because the fluid velocity is low in this region due to lower blade speed. The prewhirl is therefore gradually reduced to zero by twisting the inlet guide vanes.

4.9 MACH NUMBER IN THE DIFFUSER

The absolute velocity of the fluid becomes a maximum at the tip of the impeller and so the Mach number may well be in excess of unity. Assuming a perfect gas, the Mach number at the impeller exit M_2 can be written as:

$$\mathrm{M}_2 = \frac{C_2}{\sqrt{\gamma \mathrm{R} T_2}} \tag{4.7}$$

However, it has been found that as long as the radial velocity component (C_{r2}) is subsonic, Mach number greater than unity can be used at the impeller tip without loss of efficiency. In addition, supersonic diffusion can occur without the formation of shock waves provided constant angular momentum is maintained with vortex motion in the vaneless space. High Mach numbers at the inlet to the diffuser vanes will also cause high pressure at the stagnation points on the diffuser vane tips, which leads to a variation of static pressure around the circumference of the diffuser. This pressure variation is transmitted upstream in a radial direction through the vaneless space and causes cyclic loading of the impeller. This may lead to early fatigue failure when the exciting frequency is of the same order as one of the natural frequencies of the impeller vanes. To overcome this concern, it is a common a practice to use prime numbers for the impeller vanes and an even number for the diffuser vanes.

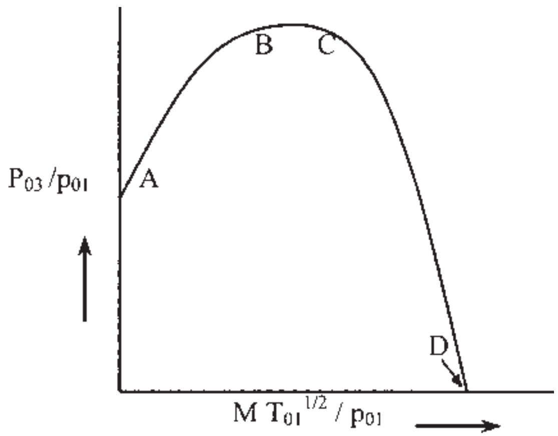


Figure 4.9 The theoretical centrifugal compressor characteristic.

4.10 CENTRIFUGAL COMPRESSOR CHARACTERISTICS

The performance of compressible flow machines is usually described in terms of the groups of variables derived in dimensional analysis (Chapter 1). These characteristics are dependent on other variables such as the conditions of pressure and temperature at the compressor inlet and physical properties of the working fluid. To study the performance of a compressor completely, it is necessary to plot P_{03}/P_{01} against the mass flow parameter $m\frac{\sqrt{T_{01}}}{P_{01}}$ for fixed speed intervals of $\frac{\mathrm{N}}{\sqrt{T_{01}}}$. Figure 4.9 shows an idealized fixed speed characteristic. Consider a valve placed in the delivery line of a compressor running at constant speed. First, suppose that the valve is fully closed. Then the pressure ratio will have some value as indicated by Point A. This pressure ratio is available from vanes moving the air about in the impeller. Now, suppose that the valve is opened and airflow begins. The diffuser contributes to the pressure rise, the pressure ratio increases, and at Point B, the maximum pressure occurs. But the compressor efficiency at this pressure ratio will be below the maximum efficiency. Point C indicates the further increase in mass flow, but the pressure has dropped slightly from the maximum possible value. This is the design mass flow rate pressure ratio. Further increases in mass flow will increase the slope of the curve until point D. Point D indicates that the pressure rise is zero. However, the above-described curve is not possible to obtain.

4.11 STALL

Stalling of a stage will be defined as the aerodynamic stall, or the breakaway of the flow from the suction side of the blade airfoil. A multistage compressor may operate stably in the unsurged region with one or more of the stages stalled, and the rest of the stages unstalled. Stall, in general, is characterized by reverse flow near the blade tip, which disrupts the velocity distribution and hence adversely affects the performance of the succeeding stages.

Referring to the cascade of Fig. 4.10, it is supposed that some nonuniformity in the approaching flow or in a blade profile causes blade B to stall. The air now flows onto blade A at an increased angle of incidence due to blockage of channel AB. The blade A then stalls, but the flow on blade C is now at a lower incidence, and blade C may unstall. Therefore the stall may pass along the cascade in the direction of lift on the blades. Rotating stall may lead to vibrations resulting in fatigue failure in other parts of the gas turbine.

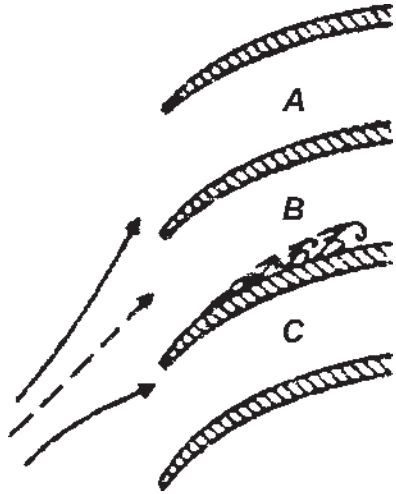


Figure 4.10 Mechanism of stall propagation.

4.12 SURGING

Surging is marked by a complete breakdown of the continuous steady flow throughout the whole compressor, resulting in large fluctuations of flow with time and also in subsequent mechanical damage to the compressor. The phenomenon of surging should not be confused with the stalling of a compressor stage.

Figure 4.11 shows typical overall pressure ratios and efficiencies η_c of a centrifugal compressor stage. The pressure ratio for a given speed, unlike the temperature ratio, is strongly dependent on mass flow rate, since the machine is usually at its peak value for a narrow range of mass flows. When the compressor is running at a particular speed and the discharge is gradually reduced, the pressure ratio will first increase, peaks at a maximum value, and then decreased. The pressure ratio is maximized when the isentropic efficiency has the maximum value. When the discharge is further reduced, the pressure ratio drops due to fall in the isentropic efficiency. If the downstream pressure does not drop quickly there will be backflow accompanied by further decrease in mass flow. In the mean time, if the downstream pressure drops below the compressor outlet pressure, there will be increase in mass flow. This phenomenon of sudden drop in delivery pressure accompanied by pulsating flow is called surging. The point on the curve where surging starts is called the surge point. When the discharge pipe of the compressor is completely choked (mass flow is zero) the pressure ratio will have some value due to the centrifugal head produced by the impeller.

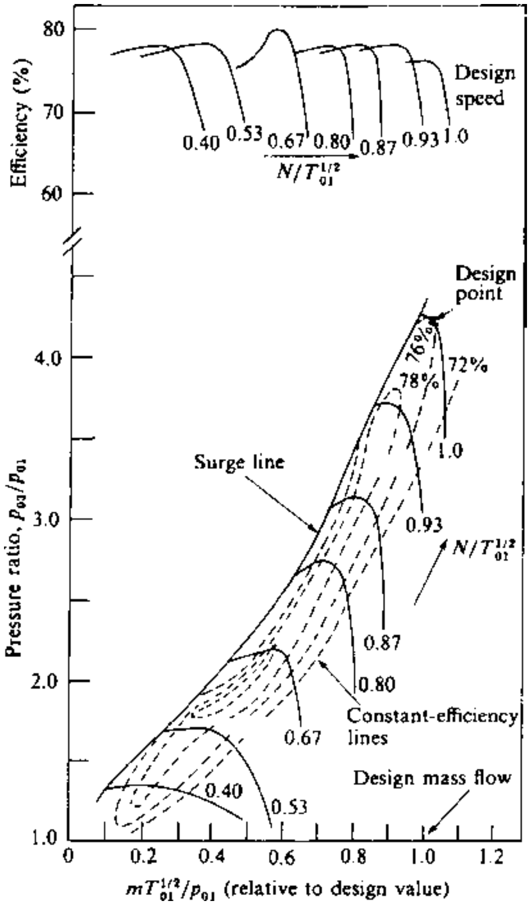


Figure 4.11 Centrifugal compressor characteristics.

Between the zero mass flow and the surge point mass flow, the operation of the compressor will be unstable. The line joining the surge points at different speeds gives the surge line.

4.13 CHOKING

When the velocity of fluid in a passage reaches the speed of sound at any cross-section, the flow becomes choked (air ceases to flow). In the case of inlet flow passages, mass flow is constant. The choking behavior of rotating passages differs from that of the stationary passages, and therefore it is necessary to make separate analysis for impeller and diffuser, assuming one dimensional, adiabatic flow, and that the fluid is a perfect gas.

4.13.1 Inlet

When the flow is choked, $C^2 = a^2 = \gamma RT$. Since $h_0 = h + \frac{1}{2}C^2$, then $C_p T_0 = C_p T + \frac{1}{2}\gamma\, RT$, and

$$\frac{T}{T_0} = \left(1 + \frac{\gamma R}{2C_p}\right)^{-1} = \frac{2}{\gamma + 1} \tag{4.8}$$

Assuming isentropic flow, we have:

$$\left(\frac{\rho}{\rho_0}\right) = \left(\frac{P}{P_0}\right)\left(\frac{T_0}{T}\right) = \left[1 + \frac{1}{2}(\gamma - 1)M^2\right]^{(1-\gamma)/(\gamma-1)} \tag{4.9}$$

and when $C = \mathrm{a}$, $M = 1$, so that:

$$\left(\frac{\rho}{\rho_0}\right) = \left[\frac{2}{(\gamma + 1)}\right]^{1/(\gamma-1)} \tag{4.10}$$

Using the continuity equation, $\left(\frac{\dot{m}}{\mathrm{A}}\right) = \rho C = \rho\left[\gamma RT\right]^{1/2}$, we have

$$\left(\frac{\dot{m}}{\mathrm{A}}\right) = \rho_0 a_0 \left[\frac{2}{\gamma + 1}\right]^{(\gamma+1)/2(\gamma-1)} \tag{4.11}$$

where (ρ_0 and a_0 refer to inlet stagnation conditions, which remain unchanged. The mass flow rate at choking is constant.

4.13.2 Impeller

When choking occurs in the impeller passages, the relative velocity equals the speed of sound at any section. The relative velocity is given by:

$$V^2 = a^2 = \left[\gamma RT\right] \text{ and } T_{01} = T + \left(\frac{\gamma RT}{2C_p}\right) - \frac{U^2}{2C_p}$$

Therefore,

$$\left(\frac{T}{T_{01}}\right) = \left(\frac{2}{\gamma + 1}\right)\left(1 + \frac{U^2}{2C_p T_{01}}\right) \tag{4.12}$$

Using isentropic conditions,

$$\left(\frac{\rho}{\rho_{01}}\right) = \left[\frac{T}{T_{01}}\right]^{1/(\gamma-1)} \quad \text{and, from the continuity equation:}$$

$$\begin{aligned}
\left(\frac{\dot{m}}{\mathrm{A}}\right) &= \rho_0 a_{01}\left[\frac{T}{T_{01}}\right]^{(\gamma+1)/2(\gamma-1)} \\
&= \rho_{01} a_{01}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{U^2}{2C_{\mathrm{p}}T_{01}}\right)\right]^{(\gamma+1)/2(\gamma-1)} \\
&= \rho_{01} a_{01}\left(\frac{2+(\gamma-1)U^2/a_{01}^2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}
\end{aligned} \tag{4.13}$$

Equation (4.13) indicates that for rotating passages, mass flow is dependent on the blade speed.

4.13.3 Diffuser

For choking in the diffuser, we use the stagnation conditions for the diffuser and not the inlet. Thus:

$$\left(\frac{\dot{m}}{\mathrm{A}}\right) = \rho_{02} a_{02}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)} \tag{4.14}$$

It is clear that stagnation conditions at the diffuser inlet are dependent on the impeller process.

Illustrative Example 4.1: Air leaving the impeller with radial velocity 110 m/s makes an angle of 25°30′ with the axial direction. The impeller tip speed is 475 m/s. The compressor efficiency is 0.80 and the mechanical efficiency is 0.96. Find the slip factor, overall pressure ratio, and power required to drive the compressor. Neglect power input factor and assume $\gamma = 1.4$, $T_{01} = 298$ K, and the mass flow rate is 3 kg/s.

Solution:

From the velocity triangle (Fig. 4.12),

$$\tan(\beta_2) = \frac{U_2 - C_{\mathrm{w}2}}{C_{\mathrm{r}2}}$$

$$\tan(25.5°) = \frac{475 - C_{\mathrm{w}2}}{110}$$

Therefore, $C_{\mathrm{w}2} = 422.54$ m/s.

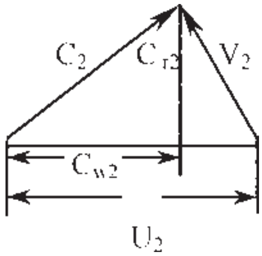


Figure 4.12 Velocity triangle at the impeller tip.

Now, $\sigma = \dfrac{C_{w2}}{U_2} = \dfrac{422.54}{475} = 0.89$

The overall pressure ratio of the compressor:

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma \psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)} = \left[1 + \frac{(0.80)(0.89)(475^2)}{(1005)(298)}\right]^{3.5} = 4.5$$

The theoretical power required to drive the compressor:

$$P = \left[\frac{m\sigma\psi U_2^2}{1000}\right] \mathrm{kW} = \left[\frac{(3)(0.89)(475^2)}{1000}\right] = 602.42\,\mathrm{kW}$$

Using mechanical efficiency, the actual power required to drive the compressor is: $P = 602.42/0.96 = 627.52$ kW.

Illustrative Example 4.2: The impeller tip speed of a centrifugal compressor is 370 m/s, slip factor is 0.90, and the radial velocity component at the exit is 35 m/s. If the flow area at the exit is 0.18 m^2 and compressor efficiency is 0.88, determine the mass flow rate of air and the absolute Mach number at the impeller tip. Assume air density = 1.57 kg/m^3 and inlet stagnation temperature is 290 K. Neglect the work input factor. Also, find the overall pressure ratio of the compressor.

Solution:

Slip factor: $\sigma = \dfrac{C_{w2}}{U_2}$

Therefore: $C_{w2} = U_2\sigma = (0.90)(370) = 333$ m/s

The absolute velocity at the impeller exit:

$$C_2 = \sqrt{C_{r2}^2 + C_{\omega 2}^2} = \sqrt{333^2 + 35^2} = 334.8\ \mathrm{m/s}$$

The mass flow rate of air: $\dot{m} = \rho_2 A_2 C_{r2} = 1.57 * 0.18 * 35 = 9.89$ kg/s

The temperature equivalent of work done (neglecting ψ):

$$T_{02} - T_{01} = \frac{\sigma U_2^2}{C_p}$$

Therefore, $T_{02} = T_{01} + \dfrac{\sigma U_2^2}{C_p} = 290 + \dfrac{(0.90)(370^2)}{1005} = 412.6\,\text{K}$

The static temperature at the impeller exit,

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 412.6 - \frac{334.8^2}{(2)(1005)} = 356.83\,\text{K}$$

The Mach number at the impeller tip:

$$M_2 = \frac{C_2}{\sqrt{\gamma R T_2}} = \frac{334.8}{\sqrt{(1.4)(287)(356.83)}} = 0.884$$

The overall pressure ratio of the compressor (neglecting ψ):

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma \psi U_2^2}{C_p T_{01}}\right]^{3.5} = \left[1 + \frac{(0.88)(0.9)(370^2)}{(1005)(290)}\right]^{3.5} = 3.0$$

Illustrative Example 4.3: A centrifugal compressor is running at 16,000 rpm. The stagnation pressure ratio between the impeller inlet and outlet is 4.2. Air enters the compressor at stagnation temperature of 20°C and 1 bar. If the impeller has radial blades at the exit such that the radial velocity at the exit is 136 m/s and the isentropic efficiency of the compressor is 0.82. Draw the velocity triangle at the exit (Fig. 4.13) of the impeller and calculate slip. Assume axial entrance and rotor diameter at the outlet is 58 cm.

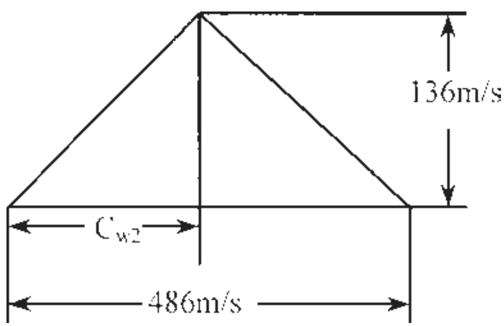


Figure 4.13 Velocity triangle at exit.

Solution:
Impeller tip speed is given by:

$$U_2 = \frac{\pi DN}{60} = \frac{(\pi)(0.58)(16000)}{60} = 486\,\text{m/s}$$

Assuming isentropic flow between impeller inlet and outlet, then

$$T_{02'} = T_{01}(4.2)^{0.286} = 441.69\,\text{K}$$

Using compressor efficiency, the actual temperature rise

$$T_{02} - T_{01} = \frac{\left(T_{02'} - T_{01}\right)}{\eta_c} = \frac{(441.69 - 293)}{0.82} = 181.33\text{K}$$

Since the flow at the inlet is axial, $C_{w1} = 0$

$$W = U_2 C_{w2} = C_p(T_{02} - T_{01}) = 1005(181.33)$$

Therefore: $C_{w2} = \dfrac{1005(181.33)}{486} = 375\,\text{m/s}$

Slip $= 486 - 375 = 111\,\text{m/s}$

Slip factor: $\sigma = \dfrac{C_{w2}}{U_2} = \dfrac{375}{486} = 0.772$

Illustrative Example 4.4: Determine the adiabatic efficiency, temperature of the air at the exit, and the power input of a centrifugal compressor from the following given data:

Impeller tip diameter $= 1\,\text{m}$

Speed $= 5945\,\text{rpm}$

Mass flow rate of air $= 28\,\text{kg/s}$

Static pressure ratio $p_3/p_1 = 2.2$

Atmospheric pressure $= 1\,\text{bar}$

Atmospheric temperature $= 25°\text{C}$

Slip factor $= 0.90$

Neglect the power input factor.

Solution:
The impeller tip speed is given by:

$$U_2 = \frac{\pi DN}{60} = \frac{(\pi)(1)(5945)}{60} = 311\,\text{m/s}$$

The work input: $W = \sigma U_2^2 = \dfrac{(0.9)(311^2)}{1000} = 87\,\text{kJ/kg}$

Using the isentropic P–T relation and denoting isentropic temperature by $T_{3'}$, we get:

$$T_{3'} = T_1 \left(\frac{P_3}{P_1}\right)^{0.286} = (298)(2.2)^{0.286} = 373.38\,\mathrm{K}$$

Hence the isentropic temperature rise:

$$T_{3'} - T_1 = 373.38 - 298 = 75.38\,\mathrm{K}$$

The temperature equivalent of work done:

$$T_3 - T_1 = \left(\frac{W}{C_\mathrm{p}}\right) = 87/1.005 = 86.57\,\mathrm{K}$$

The compressor adiabatic efficiency is given by:

$$\eta_\mathrm{c} = \frac{(T_{3'} - T_1)}{(T_3 - T_1)} = \frac{75.38}{86.57} = 0.871 \text{ or } 87.1\%$$

The air temperature at the impeller exit is:

$$T_3 = T_1 + 86.57 = 384.57\,\mathrm{K}$$

Power input:

$$P = \dot{m}W = (28)(87) = 2436\,\mathrm{kW}$$

Illustrative Example 4.5: A centrifugal compressor impeller rotates at 9000 rpm. If the impeller tip diameter is 0.914 m and $\alpha_2 = 20°$, calculate the following for operation in standard sea level atmospheric conditions: (1) U_2, (2) C_{w2}, (3) C_{r2}, (4) β_2, and (5) C_2.

1. Impeller tip speed is given by $U_2 = \frac{\pi DN}{60} = \frac{(\pi)(0.914)(9000)}{60} = 431$ m/s
2. Since the exit is radial and no slip, $C_{w2} = U_2 = 431$ m/s
3. From the velocity triangle,
   $C_{r2} = U_2 \tan(\alpha_2) = (431)\,(0.364) = 156.87$ m/s
4. For radial exit, relative velocity is exactly perpendicular to rotational velocity U_2. Thus the angle β_2 is 90° for radial exit.
5. Using the velocity triangle (Fig. 4.14),

$$C_2 = \sqrt{U_2^2 + C_{r2}^2} = \sqrt{431^2 + 156.87^2} = 458.67\,\mathrm{m/s}$$

Illustrative Example 4.6: A centrifugal compressor operates with no prewhirl is run with a rotor tip speed of 457 m/s. If C_{w2} is 95% of U_2 and $\eta_\mathrm{c} = 0.88$,

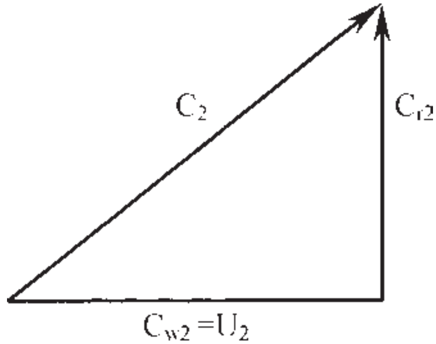


Figure 4.14 Velocity triangle at impeller exit.

calculate the following for operation in standard sea level air: (1) pressure ratio, (2) work input per kg of air, and (3) the power required for a flow of 29 k/s.

Solution:

1. The pressure ratio is given by (assuming $\sigma = \psi = 1$):

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma \psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{(0.88)(0.95)(457^2)}{(1005)(288)}\right]^{3.5} = 5.22$$

2. The work per kg of air

$$W = U_2 C_{w2} = (457)(0.95)(457) = 198.4\,\text{kJ/kg}$$

3. The power for 29 kg/s of air

$$P = \dot{m}W = (29)(198.4) = 5753.6\,\text{kW}$$

Illustrative Example 4.7: A centrifugal compressor is running at 10,000 rpm and air enters in the axial direction. The inlet stagnation temperature of air is 290 K and at the exit from the impeller tip the stagnation temperature is 440 K. The isentropic efficiency of the compressor is 0.85, work input factor $\psi = 1.04$, and the slip factor $\sigma = 0.88$. Calculate the impeller tip diameter, overall pressure ratio, and power required to drive the compressor per unit mass flow rate of air.

Solution:

Temperature equivalent of work done:

$$T_{02} - T_{01} = \frac{\sigma\psi U_2^2}{\mathrm{C_p}} \text{ or } \frac{(0.88)(1.04)(U_2^2)}{1005}.$$

Therefore, $U_2 = 405.85\,\text{m/s}$

$$\text{and} \quad D = \frac{60U_2}{\pi N} = \frac{(60)(405.85)}{(\pi)(10{,}000)} = 0.775\,\text{m}$$

The overall pressure ratio is given by:

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma\psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{(0.85)(0.88)(1.04)(405.85^2)}{(1005)(290)}\right]^{3.5} = 3.58$$

Power required to drive the compressor per unit mass flow:

$$P = m\psi\sigma U_2^2 = \frac{(1)(0.88)(1.04)(405.85^2)}{1000} = 150.75\,\text{kW}$$

Design Example 4.8: Air enters axially in a centrifugal compressor at a stagnation temperature of 20°C and is compressed from 1 to 4.5 bars. The impeller has 19 radial vanes and rotates at 17,000 rpm. Isentropic efficiency of the compressor is 0.84 and the work input factor is 1.04. Determine the overall diameter of the impeller and the power required to drive the compressor when the mass flow is 2.5 kg/s.

Solution:

Since the vanes are radial, using the Stanitz formula to find the slip factor:

$$\sigma = 1 - \frac{0.63\pi}{\mathrm{n}} = 1 - \frac{0.63\pi}{19} = 0.8958$$

The overall pressure ratio

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma\psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}, \text{ or } 4.5$$

$$= \left[1 + \frac{(0.84)(0.8958)(1.04)(U_2^2)}{(1005)(293)}\right]^{3.5}, \text{ so } U_2 = 449.9\,\text{m/s}$$

The impeller diameter, $D = \dfrac{60U_2}{\pi N} = \dfrac{(60)(449.9)}{\pi(17{,}000)} = 0.5053\,\text{m} = 50.53\,\text{cm}.$

The work done on the air $W = \frac{\psi\sigma U_2^2}{1000} = \frac{(0.8958)(1.04)(449.9^2)}{1000} = 188.57\,\text{kJ/kg}$

Power required to drive the compressor: $P = \dot{m}W = (2.5)(188.57) = 471.43\,\text{kW}$

Design Example 4.9: Repeat problem 4.8, assuming the air density at the impeller tip is $1.8\,\text{kg/m}^3$ and the axial width at the entrance to the diffuser is 12 mm. Determine the radial velocity at the impeller exit and the absolute Mach number at the impeller tip.

Solution:
Slip factor: $\sigma = \frac{C_{w2}}{U_2}$, or $C_{w2} = (0.8958)(449.9) = 403\,\text{m/s}$
Using the continuity equation,

$$\dot{m} = \rho_2 A_2 C_{r2} = \rho_2 2\pi r_2 b_2 C_{r2}$$

where:

b_2 = axial width
r_2 = radius

Therefore:

$$C_{r2} = \frac{2.5}{(1.8)(2\pi)(0.25)(0.012)} = 73.65\,\text{m/s}$$

Absolute velocity at the impeller exit

$$C_2 = \sqrt{C_{r2}^2 + C_{w2}^2} = \sqrt{73.65^2 + 403^2} = 409.67\,\text{m/s}$$

The temperature equivalent of work done:

$$T_{02} - T_{01} = 188.57/C_p = 188.57/1.005 = 187.63\,\text{K}$$

Therefore, $T_{02} = 293 + 187.63 = 480.63\,\text{K}$
Hence the static temperature at the impeller exit is:

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 480.63 - \frac{409.67^2}{(2)(1005)} = 397\,\text{K}$$

Now, the Mach number at the impeller exit is:

$$\text{M}_2 = \frac{C_2}{\sqrt{\gamma R T_2}} = \frac{409.67}{\sqrt{(1.4)(287)(397)}} = 1.03$$

Design Example 4.10: A centrifugal compressor is required to deliver 8 kg/s of air with a stagnation pressure ratio of 4 rotating at 15,000 rpm. The air enters the compressor at 25°C and 1 bar. Assume that the air enters axially with velocity of 145 m/s and the slip factor is 0.89. If the compressor isentropic efficiency is 0.89, find the rise in stagnation temperature, impeller tip speed, diameter, work input, and area at the impeller eye.

Solution:

Inlet stagnation temperature:

$$T_{01} = T_{a} + \frac{C_1^2}{2C_p} = 298 + \frac{145^2}{(2)(1005)} = 308.46\,\mathrm{K}$$

Using the isentropic P–T relation for the compression process,

$$T_{03'} = T_{01}\left(\frac{P_{03}}{P_{01}}\right)^{(\gamma-1)/\gamma} = (308.46)(4)^{0.286} = 458.55K$$

Using the compressor efficiency,

$$T_{02} - T_{01} = \frac{\left(T_{02'} - T_{01}\right)}{\eta_c} = \frac{(458.55 - 308.46)}{0.89} = 168.64\,\mathrm{K}$$

Hence, work done on the air is given by:

$$W = C_p(T_{02} - T_{01}) = (1.005)(168.64) = 169.48\,\mathrm{kJ/kg}$$

But,

$$W = \sigma U_2^2 = \frac{(0.89)(U_2)}{1000}, \text{ or :}169.48 = 0.89U_2^2/1000$$

or:

$$U_2 = \sqrt{\frac{(1000)(169.48)}{0.89}} = 436.38\,\mathrm{m/s}$$

Hence, the impeller tip diameter

$$D = \frac{60U_2}{\pi N} = \frac{(60)(436.38)}{\pi(15,000)} = 0.555\,\mathrm{m}$$

The air density at the impeller eye is given by:

$$\rho_1 = \frac{\mathrm{P}_1}{\mathrm{R}T_1} = \frac{(1)(100)}{(0.287)(298)} = 1.17\,\mathrm{kg/m^3}$$

Using the continuity equation in order to find the area at the impeller eye,

$$A_1 = \frac{\dot{m}}{\rho_1 C_1} = \frac{8}{(1.17)(145)} = 0.047\,\mathrm{m^2}$$

The power input is:

$$P = \dot{m}\ W = (8)(169.48) = 1355.24\,\mathrm{kW}$$

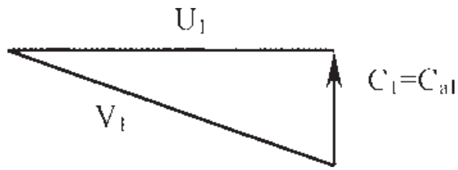


Figure 4.15 The velocity triangle at the impeller eye.

Design Example 4.11: The following data apply to a double-sided centrifugal compressor (Fig. 4.15):

Impeller eye tip diameter:	0.28 m
Impeller eye root diameter:	0.14 m
Impeller tip diameter:	0.48 m
Mass flow of air:	10 kg/s
Inlet stagnation temperature:	290 K
Inlet stagnation pressure:	1 bar
Air enters axially with velocity:	145 m/s
Slip factor:	0.89
Power input factor:	1.03
Rotational speed:	15,000 rpm

Calculate (1) the impeller vane angles at the eye tip and eye root, (2) power input, and (3) the maximum Mach number at the eye.

Solution:

(1) Let U_{er} be the impeller speed at the eye root. Then the vane angle at the eye root is:

$$\alpha_{er} = \tan^{-1}\left(\frac{C_a}{U_{er}}\right)$$

and

$$U_{er} = \frac{\pi D_{er} N}{60} = \frac{\pi(0.14)(15,000)}{60} = 110\,\text{m/s}$$

Hence, the vane angle at the impeller eye root:

$$\alpha_{er} = \tan^{-1}\left(\frac{C_a}{U_{er}}\right) = \tan^{-1}\left(\frac{145}{110}\right) = 52^\circ 48'$$

Impeller velocity at the eye tip:

$$U_{\text{et}} = \frac{\pi D_{\text{et}} N}{60} = \frac{\pi(0.28)(15,000)}{60} = 220\,\text{m/s}$$

Therefore vane angle at the eye tip:

$$\alpha_{\text{et}} = \tan^{-1}\left(\frac{C_{\text{a}}}{U_{\text{et}}}\right) = \tan^{-1}\left(\frac{145}{220}\right) = 33^{\circ}23'$$

(2) Work input:

$$W = \dot{m}\psi\sigma U_2^2 = (10)(0.819)(1.03U_2^2)$$

but:

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.48)(15,000)}{60} = 377.14\,\text{m/s}$$

Hence,

$$W = \frac{(10)(0.89)(1.03)(377.14^2)}{1000} = 1303.86\,\text{kW}$$

(3) The relative velocity at the eye tip:

$$V_1 = \sqrt{U_{\text{et}}^2 + C_{\text{a}}^2} = \sqrt{220^2 + 145^2} = 263.5\,\text{m/s}$$

Hence, the maximum relative Mach number at the eye tip:

$$\text{M}_1 = \frac{V_1}{\sqrt{\gamma \text{R} T_1}},$$

where T_1 is the static temperature at the inlet

$$T_1 = T_{01} - \frac{C_1^2}{2C_{\text{p}}} = 290 - \frac{145^2}{(2)(1005)} = 279.54\,\text{K}$$

The Mach number at the inlet then is:

$$\text{M}_1 = \frac{V_1}{\sqrt{\gamma \text{R} T_1}} = \frac{263/5}{\sqrt{(1.4)(287)(279.54)}} = 0.786$$

Design Example 4.12: Recalculate the maximum Mach number at the impeller eye for the same data as in the previous question, assuming prewhirl angle of 20°.

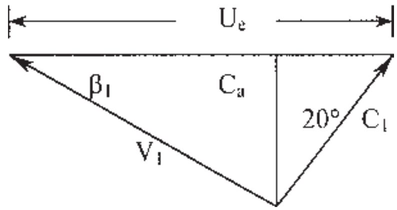


Figure 4.16 The velocity triangle at the impeller eye.

Solution:
Figure 4.16 shows the velocity triangle with the prewhirl angle.
From the velocity triangle:

$$C_1 = \frac{145}{\cos(20^{\circ})} = 154.305\,\text{m/s}$$

Equivalent dynamic temperature:

$$\frac{C_1^2}{2C_\text{p}} = \frac{154.305^2}{(2)(1005)} = 11.846\,\text{K}$$

$$C_{\text{w}1} = \tan(20^{\circ})\,C_{\text{a}1} = (0.36)(145) = 52.78\,\text{m/s}$$

Relative velocity at the inlet:

$$V_1^2 = C_\text{a}^2 + (U_\text{e} - C_{\text{w}1})^2 = 145^2 + (220 - 52.78)^2, \text{ or } V_1$$
$$= 221.3\,\text{m/s}$$

Therefore the static temperature at the inlet:

$$T_1 = T_{01} - \frac{C_1^2}{2C_\text{p}} = 290 - 11.846 = 278.2\,\text{K}$$

Hence,

$$\text{M}_1 = \frac{V_1}{\sqrt{\gamma \text{R} T_1}} = \frac{221.3}{\sqrt{(1.4)(287)(278.2)}} = 0.662$$

Note the reduction in Mach number due to prewhirl.

Design Example 4.13: The following data refers to a single-sided centrifugal compressor:

Ambient Temperature:	288 K
Ambient Pressure:	1 bar
Hub diameter:	0.125 m
Eye tip diameter:	0.25 m
Mass flow:	5.5 kg/s
Speed:	16,500 rpm

Assume zero whirl at the inlet and no losses in the intake duct. Calculate the blade inlet angle at the root and tip and the Mach number at the eye tip.

Solution:

Let: r_h = hub radius
r_t = tip radius

The flow area of the impeller inlet annulus is:

$$A_1 = \pi\left(r_t^2 - r_h^2\right) = \pi\left(0.125^2 - 0.0625^2\right) = 0.038\ \mathrm{m}^2$$

Axial velocity can be determined from the continuity equation but since the inlet density (ρ_1) is unknown a trial and error method must be followed. Assuming a density based on the inlet stagnation condition,

$$\rho_1 = \frac{P_{01}}{\mathrm{R}T_{01}} = \frac{(1)(10^5)}{(287)(288)} = 1.21\ \mathrm{kg/m^3}$$

Using the continuity equation,

$$C_a = \frac{\dot{m}}{\rho_1 A_1} = \frac{5.5}{(1.21)(0.038)} = 119.6\ \mathrm{m/s}$$

Since the whirl component at the inlet is zero, the absolute velocity at the inlet is $C_1 = C_a$.
The temperature equivalent of the velocity is:

$$\frac{C_1^2}{2C_p} = \frac{119.6^2}{(2)(1005)} = 7.12\ \mathrm{K}$$

Therefore:

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 288 - 7.12 = 280.9\ \mathrm{K}$$

Using isentropic P–T relationship,

$$\frac{P_1}{P_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\gamma/(\gamma-1)}, \text{ or } P_1 = 10^5\left(\frac{280.9}{288}\right)^{3.5} = 92\,\text{kPa}$$

and:

$$\rho_1 = \frac{P_1}{\mathrm{R}T_1} = \frac{(92)(10^3)}{(287)(280.9)} = 1.14\,\text{kg/m}^3, \text{ and}$$

$$C_a = \frac{5.5}{(1.14)(0.038)} = 126.96\,\text{m/s}$$

Therefore:

$$\frac{C_1^2}{2C_p} = \frac{(126.96)^2}{2(1005)} = 8.02\,\text{K}$$

$$T_1 = 288 - 8.02 = 279.98°\,\text{K}$$

$$P_1 = 10^5\left(\frac{279.98}{288}\right)^{3.5} = 90.58\,\text{kPa}$$

$$\rho_1 = \frac{(90.58)(10^3)}{(287)(279.98)} = 1.13\,\text{kg/m}^3$$

Further iterations are not required and the value of $\rho_1 = 1.13\,\text{kg/m}^3$ may be taken as the inlet density and $C_a = C_1$ as the inlet velocity. At the eye tip:

$$U_{et} = \frac{2\pi r_{et} N}{60} = \frac{2\pi(0.125)(16,500)}{60} = 216\,\text{m/s}$$

The blade angle at the eye tip:

$$\beta_{et} = \tan^{-1}\left(\frac{U_{et}}{C_a}\right) = \tan^{-1}\left(\frac{216}{126.96}\right) = 59.56°$$

At the hub,

$$U_{eh} = \frac{2\pi(0.0625)(16,500)}{60} = 108\,\text{m/s}$$

The blade angle at the hub:

$$\beta_{eh} = \tan^{-1}\left(\frac{108}{126.96}\right) = 40.39°$$

The Mach number based on the relative velocity at the eye tip using the inlet velocity triangle is:

$$V_1 = \sqrt{C_a^2 + U_1^2} = \sqrt{126.96^2 + 216^2}, \text{ or } V_1 = 250.6\ \text{m/s}$$

The relative Mach number

$$\text{M} = \frac{V_1}{\sqrt{\gamma \text{R} T_1}} = \frac{250.6}{\sqrt{(1.4)(287)(279.98)}} = 0.747$$

Design Example 4.14: A centrifugal compressor compresses air at ambient temperature and pressure of 288 K and 1 bar respectively. The impeller tip speed is 364 m/s, the radial velocity at the exit from the impeller is 28 m/s, and the slip factor is 0.89. Calculate the Mach number of the flow at the impeller tip. If the impeller total-to-total efficiency is 0.88 and the flow area from the impeller is 0.085 m^2, calculate the mass flow rate of air. Assume an axial entrance at the impeller eye and radial blades.

Solution:
The absolute Mach number of the air at the impeller tip is:

$$\text{M}_2 = \frac{C_2}{\sqrt{\gamma \text{R} T_2}}$$

where T_2 is the static temperature at the impeller tip. Let us first calculate C_2 and T_2.
Slip factor:

$$\sigma = \frac{C_{w2}}{U_2}$$

Or:

$$C_{w2} = \sigma U_2 = (0.89)(364) = 323.96\ \text{m/s}$$

From the velocity triangle,

$$C_2^2 = C_{r2}^2 + C_{w2}^2 = 28^2 + 323.96^2 = (1.06)(10^5)\ \text{m}^2/\text{s}^2$$

With zero whirl at the inlet

$$\frac{W}{m} = \sigma U_2^2 = C_p(T_{02} - T_{01})$$

Hence,

$$T_{02} = T_{01} + \frac{\sigma U_2^2}{C_p} = 288 + \frac{(0.89)(364^2)}{1005} = 405.33\ \text{K}$$

Static Temperature

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 405.33 - \frac{106000}{(2)(1005)} = 352.6\,\text{K}$$

Therefore,

$$\text{M}_2 = \left(\frac{(1.06)(10^5)}{(1.4)(287)(352.6)}\right)^{\frac{1}{2}} = 0.865$$

Using the isentropic P–T relation:

$$\left(\frac{P_{02}}{P_{01}}\right) = \left[1 + \eta_c\left(\frac{T_{02}}{T_{01}} - 1\right)\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + 0.88\left(\frac{405.33}{288} - 1\right)\right]^{3.5} = 2.922$$

$$\left(\frac{P_2}{P_{02}}\right) = \left(\frac{T_2}{T_{02}}\right)^{3.5} = \left(\frac{352.6}{405.33}\right)^{3.5} = 0.614$$

Therefore,

$$P_2 = \left(\frac{P_2}{P_{02}}\right)\left(\frac{P_{02}}{P_{01}}\right)P_{01}$$

$$= (0.614)(2.922)(1)(100)$$

$$= 179.4\,\text{kPa}$$

$$\rho_2 = \frac{179.4(1000)}{287(352.6)} = 1.773\,\text{kg/m}^3$$

Mass flow:

$$\dot{m} = (1.773)(0.085)(28) = 4.22\,\text{kg/s}$$

Design Example 4.15: The impeller of a centrifugal compressor rotates at 15,500 rpm, inlet stagnation temperature of air is 290 K, and stagnation pressure at inlet is 101 kPa. The isentropic efficiency of impeller is 0.88, diameter of the impellar is 0.56 m, axial depth of the vaneless space is 38 mm, and width of the vaneless space is 43 mm. Assume ship factor as 0.9, power input factor 1.04, mass flow rate as 16 kg/s. Calculate

1. Stagnation conditions at the impeller outlet, assume no fore whirl at the inlet,
2. Assume axial velocity approximately equal to 105 m/s at the impeller outlet, calculate the Mach number and air angle at the impeller outlet,

3. The angle of the diffuser vane leading edges and the Mach number at this radius if the diffusion in the vaneless space is isentropic.

Solution:

1. Impeller tip speed

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.56 \times 15500}{60}$$

$$U_2 = 454.67 \text{ m/s}$$

Overall stagnation temperature rise

$$T_{03} - T_{01} = \frac{\psi \sigma U_2^2}{1005} = \frac{1.04 \times 0.9 \times 454.67^2}{1005}$$
$$= 192.53\text{K}$$

Since $T_{03} = T_{02}$
Therefore, $T_{02} - T_{01} = 192.53\text{K}$ and $T_{02} = 192.53 + 290 = 482.53\text{K}$
Now pressure ratio for impeller

$$\frac{p_{02}}{p_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{3.5} = \left(\frac{482.53}{290}\right)^{3.5} = 5.94$$

then, $p_{02} = 5.94 \times 101 = 600$ KPa

2.

$$\sigma = \frac{C_{w2}}{U_2}$$

$$C_{w2} = \sigma U_2$$

or

$$C_{w2} = 0.9 \times 454.67 = 409 \text{ m/s}$$

Let $C_{r2} = 105$ m/s
Outlet area normal to periphery

$$A_2 = \pi D_2 \times \text{impeller depth}$$
$$= \pi \times 0.56 \times 0.038$$
$$A_2 = 0.0669 \text{ m}^2$$

From outlet velocity triangle

$$C_2{}^2 = C_{r2}^2 + C_{w2}^2$$
$$= 105^2 + 409^2$$
$$C_2{}^2 = 178306$$

i.e. $C_2 = 422.26\,\text{m/s}$

$$T_2 = T_{02} - \frac{C_2^2}{2C_\text{p}} = 482.53 - \frac{422.26^2}{2 \times 1005}$$

$$T_2 = 393.82\,\text{K}$$

Using isentropic P–T relations

$$P_2 = P_{02}\left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}} = 600\left(\frac{393.82}{482.53}\right)^{3.5} = 294.69\,\text{kPa}$$

From equation of state

$$\rho_2 = \frac{P_2}{\text{R}T_2} = \frac{293.69 \times 10^3}{287 \times 393.82} = 2.61\,\text{kg/m}^3$$

The equation of continuity gives

$$C_{\text{r2}} = \frac{\dot{m}}{A_2 P_2} = \frac{16}{0.0669 \times 2.61} = 91.63\,\text{m/s}$$

Thus, impeller outlet radial velocity $= 91.63\,\text{m/s}$
Impeller outlet Mach number

$$\text{M}_2 = \frac{C_2}{\sqrt{\gamma \text{R} T_2}} = \frac{422.26}{(1.4 \times 287 \times 393.82)^{0.5}}$$

$$\text{M}_2 = 1.06$$

From outlet velocity triangle

$$\text{Cos}\alpha_2 = \frac{C_{\text{r2}}}{C_2} = \frac{91.63}{422.26} = 0.217$$

i.e., $\alpha_2 = 77.47°$

3. Assuming free vortex flow in the vaneless space and for convenience denoting conditions at the diffuser vane without a subscript ($r = 0.28 + 0.043 = 0.323$)

$$C_\text{w} = \frac{C_{\text{w2}} r_2}{r} = \frac{409 \times 0.28}{0.323}$$

$$C_\text{w} = 354.55\,\text{m/s}$$

The radial component of velocity can be found by trial and error. Choose as a first try, $C_r = 105$ m/s

$$\frac{C^2}{2C_p} = \frac{105^2 + 354.55^2}{2 \times 1005} = 68\,\mathrm{K}$$

$T = 482.53 - 68$ (since $T = T_{02}$ in vaneless space)

$$T = 414.53\mathrm{K}$$

$$p = p_{02}\left(\frac{T_2}{T_{02}}\right)^{3.5} = 600\left(\frac{419.53}{482.53}\right)^{3.5} = 352.58\,\mathrm{kPa}$$

$$\rho = \frac{p_2}{RT_2} = \frac{294.69}{287 \times 393.82}$$

$$\rho = 2.61\,\mathrm{kg/m^3}$$

The equation of continuity gives

$$A = 2\pi r \times \text{depth of vanes}$$
$$= 2\pi \times 0.323 \times 0.038$$
$$= 0.0772\,\mathrm{m^2}$$

$$C_r = \frac{16}{2.61 \times 0.0772} = 79.41\,\mathrm{m/s}$$

Next try $C_r = 79.41$ m/s

$$\frac{C^2}{2C_p} = \frac{79.41^2 + 354.55^2}{2 \times 1005} = 65.68$$

$$T = 482.53 - 65.68 = 416.85\mathrm{K}$$

$$p = p_{02}\left(\frac{T}{T_{02}}\right)^{3.5} = 600\left(\frac{416.85}{482.53}\right)^{3.5}$$

$$p = 359.54\,\mathrm{Pa}$$

$$\rho = \frac{359.54}{416.85 \times 287} = 3\,\mathrm{kg/m^3}$$

$$C_r = \frac{16}{3.0 \times 0.772} = 69.08\,\text{m/s}$$

Try $C_r = 69.08\,\text{m/s}$

$$\frac{C^2}{2C_p} = \frac{69.08^2 + 354.55^2}{2 \times 1005} = 64.9$$

$$T = 482.53 - 64.9 = 417.63\,\text{K}$$

$$p = p_{02}\left(\frac{T}{T_{02}}\right)^{3.5} = 600\left(\frac{417.63}{482.53}\right)^{3.5}$$

$$p = 361.9\,\text{Pa}$$

$$\rho = \frac{361.9}{417.63 \times 287} = 3.02\,\text{kg/m}^3$$

$$C_r = \frac{16}{3.02 \times 0.772} = 68.63\,\text{m/s}$$

Taking C_r as 62.63 m/s, the vane angle

$$\tan\,\alpha = \frac{C_w}{C_r}$$

$$= \frac{354.5}{68.63} = 5.17$$

i.e. $\alpha = 79°$

Mach number at vane

$$\text{M} = \left(\frac{65.68 \times 2 \times 1005}{1.4 \times 287 \times 417.63}\right)^{1/2} = 0.787$$

Design Example 4.16: The following design data apply to a double-sided centrifugal compressor:

Impeller eye root diameter:	18 cm
Impeller eye tip diameter:	31.75 cm
Mass flow:	18.5 kg/s
Impeller speed:	15500 rpm
Inlet stagnation pressure:	1.0 bar
Inlet stagnation temperature:	288 K
Axial velocity at inlet (constant):	150 m/s

Find suitable values for the impeller vane angles at root and tip of eye if the air is given 20° of prewhirl at all radii, and also find the maximum Mach number at the eye.

Solution:

At eye root, $C_a = 150\,\text{m/s}$

$$\therefore C_1 = \frac{C_a}{\cos 20^\circ} = \frac{150}{\cos 20^\circ} = 159.63\,\text{m/s}$$

and $C_{w1} = 150 \tan 20^\circ = 54.6\,\text{m/s}$

Impeller speed at eye root

$$U_{er} = \frac{\pi D_{er} N}{60} = \frac{\pi \times 0.18 \times 15500}{60}$$

$$U_{er} = 146\,\text{m/s}$$

From velocity triangle

$$\tan \beta_{er} = \frac{C_a}{U_{er} - C_{w1}} = \frac{150}{146 - 54.6} = \frac{150}{91.4} = 1.641$$

i.e., $\beta_{er} = 58.64^\circ$

At eye tip from Fig. 4.17(b)

$$U_{et}\frac{\pi D_{et} N}{60} = \frac{\pi \times 0.3175 \times 15500}{60}$$

$$U_{et} = 258\,\text{m/s}$$

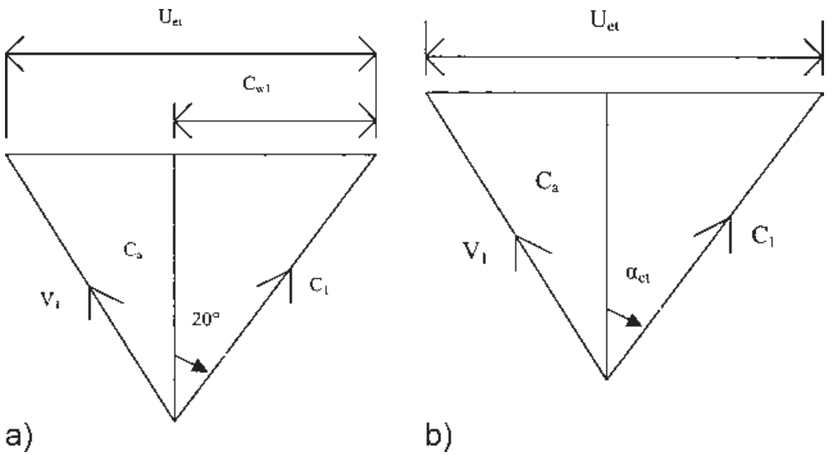


Figure 4.17 Velocity triangles at (a) eye root and (b) eye tip.

$$\tan\ \alpha_{et} = \frac{150}{258 - 54.6} = \frac{150}{203.4} = 0.7375$$

i.e. $\alpha_{et} = 36.41°$

Mach number will be maximum at the point where relative velocity is maximum.
Relative velocity at eye root is:

$$V_{er} = \frac{C_a}{\sin \beta_{er}} = \frac{150}{\sin 58.64°} = \frac{150}{0.8539}$$

$$V_{er} = 175.66\,\text{m/s}$$

Relative velocity at eye tip is:

$$V_{et} = \frac{C_a}{\sin \alpha_{et}} = \frac{150}{\sin 36.41°} = \frac{150}{0.5936}$$

$$V_{et} = 252.7\,\text{m/s}$$

Relative velocity at the tip is maximum.
Static temperature at inlet:

$$T_1 = T_{01} = \frac{V_{et}^2}{2C_p} = 288 - \frac{252.7^2}{2 \times 1005} = 288 - 31.77$$

$$T_1 = 256.23\,\text{K}$$

$$\mathrm{M}_{max} = \frac{V_{et}}{\left(\gamma R T_1\right)^{1/2}} = \frac{252.7}{(1.4 \times 287 \times 256.23)^{1/2}} = \frac{252.7}{320.86}$$

$$\mathrm{M}_{max} = 0.788$$

Design Example 4.17: In a centrifugal compressor air enters at a stagnation temperature of 288 K and stagnation pressure of 1.01 bar. The impeller has 17 radial vanes and no inlet guide vanes. The following data apply:

Mass flow rate:	2.5 kg/s
Impeller tip speed:	475 m/s
Mechanical efficiency:	96%
Absolute air velocity at diffuser exit:	90 m/s
Compressor isentropic efficiency:	84%
Absolute velocity at impeller inlet:	150 m/s

Diffuser efficiency:	82%
Axial depth of impeller:	6.5 mm
Power input factor:	1.04
γ for air:	1.4

Determine:

1. shaft power
2. stagnation and static pressure at diffuser outlet
3. radial velocity, absolute Mach number and stagnation and static pressures at the impeller exit, assume reaction ratio as 0.5, and
4. impeller efficiency and rotational speed

Solution:

1. Mechanical efficiency is

$$\eta_m = \frac{\text{Work transferred to air}}{\text{Work supplied to shaft}}$$

$$\text{or shaft power} = \frac{W}{\eta_m}$$

for vaned impeller, slip factor, by Stanitz formula is

$$\sigma = 1 - \frac{0.63\pi}{n} = 1 - \frac{0.63 \times \pi}{17}$$

$$\sigma = 0.884$$

Work input per unit mass flow

$$W = \psi\sigma U_2 C_{w2}$$

Since $C_{w1} = 0$

$$= \psi\sigma U_2^2$$

$$= 1.04 \times 0.884 \times 475^2$$

Work input for 2.5 kg/s

$$W = 1.04 \times 0.884 \times 2.5 \times 475^2$$

$$W = 518.58\text{K}$$

$$\text{Hence, Shaft Power} = \frac{518.58}{0.96} = 540.19\text{kW}$$

2. The overall pressure ratio is

$$\frac{p_{03}}{p_{01}} = \left[1 + \frac{\eta_c \psi \sigma U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

$$= \left[1 + \frac{0.84 \times 1.04 \times 0.884 \times 475^2}{1005 \times 288}\right]^{3.5} = 5.2$$

Stagnation pressure at diffuser exit

$$P_{03} = p_{01} \times 5.20 = 1.01 \times 5.20$$

$$P_{03} = 5.25\,\text{bar}$$

$$\frac{p_3}{p_{03}} = \left(\frac{T_3}{T_{03}}\right)^{\gamma/\gamma-1}$$

$$W = m \times C_p(T_{03} - T_{01})$$

$$\therefore T_{03} = \frac{W}{mC_p} + T_{01} = \frac{518.58 \times 10^3}{2.5 \times 1005} + 288 = 494.4\,\text{K}$$

Static temperature at diffuser exit

$$T_3 = T_{03} - \frac{C_3^2}{2C_p} = 494.4 - \frac{90^2}{2 \times 1005}$$

$$T_3 = 490.37\,\text{K}$$

Static pressure at diffuser exit

$$p_3 = p_{03}\left(\frac{T_3}{T_{03}}\right)^{\gamma/\gamma-1} = 5.25\left(\frac{490.37}{494.4}\right)^{3.5}$$

$$p_3 = 5.10\,\text{bar}$$

3. The reaction is

$$0.5 = \frac{T_2 - T_1}{T_3 - T_1}$$

and

$$T_3 - T_1 = (T_{03} - T_{01}) + \left(\frac{C_1{}^2 - C_3{}^2}{2C_p}\right) = \frac{W}{mC_p} + \frac{150^2 - 90^2}{2 \times 1005}$$

$$= \frac{518.58 \times 10^3}{2.5 \times 1005} + 7.164 = 213.56\,K$$

Substituting

$$T_2 - T_1 = 0.5 \times 213.56$$
$$= 106.78\,\mathrm{K}$$

Now

$$T_2 = T_{01} - \frac{C_1^2}{2C_p} + (T_2 - T_1)$$
$$= 288 - 11.19 + 106.78$$
$$T_2 = 383.59\,\mathrm{K}$$

At the impeller exit

$$T_{02} = T_2 + \frac{C_2{}^2}{2C_p}$$

or

$$T_{03} = T_2 + \frac{C_2{}^2}{2C_p} (\text{Since } T_{02} = T_{03})$$

Therefore,

$$C_2{}^2 = 2C_p[(T_{03} - T_{01}) + (T_{01} - T_2)]$$
$$= 2 \times 1005(206.4 + 288 + 383.59)$$
$$C_2 = 471.94\,\mathrm{m/s}$$

Mach number at impeller outlet

$$\mathrm{M}_2 = \frac{C_2}{(1.4 \times 287 \times 383.59)^{1/2}}$$

$$\mathrm{M}_2 = 1.20$$

Radial velocity at impeller outlet

$$C_{r2}{}^2 = C_2{}^2 - C_{w2}^2$$
$$= (471.94)^2 - (0.884 \times 475)^2$$

$$C_{r2}{}^2 = 215.43\,\text{m/s}$$

Diffuser efficiency is given by

$$\eta_D = \frac{h_{3'} - h_2}{h_3 - h_2} = \frac{\text{isentropic enthalpy increase}}{\text{actual enthalpy increase}} = \frac{T_{3'} - T_2}{T_3 - T_2}$$

$$= \frac{T_2\left(\frac{T_{3'}}{T_2} - 1\right)}{T_3 - T_2} = \frac{\left[T_2\left(\frac{p_3}{p_2}\right)^{\gamma-1/\gamma} - 1\right]}{(T_3 - T_2)}$$

Therefore

$$\frac{p_3}{p_2} = \left[1 + \eta_D\left(\frac{T_3 - T_2}{T_2}\right)\right]^{3.5}$$

$$= \left(1 + \frac{0.821 \times 106.72}{383.59}\right)^{3.5}$$

$$= 2.05$$

$$\text{or } p_2 = \frac{5.10}{2.05} = 2.49\,\text{bar}$$

From isentropic $P{-}T$ relations

$$p_{02} = p_2\left(\frac{T_{02}}{T_2}\right)^{3.5} = 2.49\left(\frac{494.4}{383.59}\right)^{3.5}$$

$$p_{02} = 6.05\,\text{bar}$$

4. Impeller efficiency is

$$\eta_i = \frac{T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{03} - T_{01}}$$

$$= \frac{288\left[\left(\frac{6.05}{1.01}\right)^{0.286} - 1\right]}{494.4 - 288}$$

$$= 0.938$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{2.49 \times 10^5}{287 \times 383.59}$$

$$\rho_2 = 2.27\,\text{kg/m}^3$$

$$\dot{m} = \rho_2 A_2 C_{r2}$$
$$= 2\pi r_2 \rho_2 b_2$$

But

$$U_2 = \frac{\pi N D_2}{60} = \frac{\pi N \dot{m}}{\rho_2 \pi C_{r2} b_2 \times 60}$$

$$N = \frac{475 \times 2.27 \times 246.58 \times 0.0065 \times 60}{2.5}$$

$$N = 41476\,\text{rpm}$$

PROBLEMS

4.1 The impeller tip speed of a centrifugal compressor is 450 m/s with no prewhirl. If the slip factor is 0.90 and the isentropic efficiency of the compressor is 0.86, calculate the pressure ratio, the work input per kg of air, and the power required for 25 kg/s of airflow. Assume that the compressor is operating at standard sea level and a power input factor of 1.

(4.5, 182.25 kJ/kg, 4556.3 kW)

4.2 Air with negligible velocity enters the impeller eye of a centrifugal compressor at 15°C and 1 bar. The impeller tip diameter is 0.45 m and rotates at 18,000 rpm. Find the pressure and temperature of the air at the compressor outlet. Neglect losses and assume $\gamma = 1.4$.

(5.434 bar, 467 K)

4.3 A centrifugal compressor running at 15,000 rpm, overall diameter of the impeller is 60 cm, isentropic efficiency is 0.84 and the inlet stagnation temperature at the impeller eye is 15°C. Calculate the overall pressure ratio, and neglect losses.

(6)

4.4 A centrifugal compressor that runs at 20,000 rpm has 20 radial vanes, power input factor of 1.04, and inlet temperature of air is 10°C. If the pressure ratio is 2 and the impeller tip diameter is 28 cm, calculate the isentropic efficiency of the compressor. Take $\gamma = 1.4$ (77.4%)

4.5 Derive the expression for the pressure ratio of a centrifugal compressor:

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c \sigma \psi U_2^2}{C_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

4.6 Explain the terms "slip factor" and "power input factor."

4.7 What are the three main types of centrifugal compressor impellers? Draw the exit velocity diagrams for these three types.

4.8 Explain the phenomenon of stalling, surging and choking in centrifugal compressors.

4.9 A centrifugal compressor operates with no prewhirl and is run with a tip speed of 475 the slip factor is 0.89, the work input factor is 1.03, compressor efficiency is 0.88, calculate the pressure ratio, work input per kg of air and power for 29 airflow. Assume $T_{01} = 290\,\text{K}$ and $C_p = 1.005$ kJ/kg K.

(5.5, 232.4 kJ/kg, 6739 kW)

4.10 A centrifugal compressor impeller rotates at 17,000 rpm and compresses 32 kg of air per second. Assume an axial entrance, impeller trip radius is 0.3 m, relative velocity of air at the impeller tip is 105 m/s at an exit angle of 80°. Find the torque and power required to drive this machine.

(4954 Nm, 8821 kW)

4.11 A single-sided centrifugal compressor designed with no prewhirl has the following dimensions and data:

Total head/pressure ratio:	3.8:1
Speed:	12, 000 rpm
Inlet stagnation temperature:	293 K
Inlet stagnation pressure:	1.03 bar
Slip factor:	0.9
Power input factor:	1.03
Isentropic efficiency:	0.76
Mass flow rate:	20 kg/s

Assume an axial entrance. Calculate the overall diameter of the impeller and the power required to drive the compressor.

(0.693 m, 3610 kW)

4.12 A double-entry centrifugal compressor designed with no prewhirl has the following dimensions and data:

Impeller root diameter:	0.15 m
Impeller tip diameter:	0.30 m
Rotational speed:	15, 000 rpm
Mass flow rate:	18 kg/s

Ambient temperature:	25°C
Ambient pressure:	1.03 bar
Density of air at eye inlet:	1.19 kg/m^3

Assume the axial entrance and unit is stationary. Find the inlet angles of the vane at the root and tip radii of the impeller eye and the maximum Mach number at the eye.

(α_1 at root $= 50.7°$, $\alpha_1 = 31.4°$ at tip, 0.79)

4.13 In Example 4.12, air does not enter the impeller eye in an axial direction but it is given a prewhirl of 20° (from the axial direction). The remaining values are the same. Calculate the inlet angles of the impeller vane at the root and tip of the eye.

(α_1 at root $=$ 65.5°, α_1 at tip $= 38.1°$, 0.697)

NOTATION

C	absolute velocity
r	radius
U	impeller speed
V	relative velocity
α	vane angle
σ	slip factor
ω	angular velocity
ψ	power input factor

SUFFIXES

1	inlet to rotor
2	outlet from the rotor
3	outlet from the diffuser
a	axial, ambient
r	radial
w	whirl

5

Axial Flow Compressors and Fans

5.1 INTRODUCTION

As mentioned in Chapter 4, the maximum pressure ratio achieved in centrifugal compressors is about 4:1 for simple machines (unless multi-staging is used) at an efficiency of about 70–80%. The axial flow compressor, however, can achieve higher pressures at a higher level of efficiency. There are two important characteristics of the axial flow compressor—high-pressure ratios at good efficiency and thrust per unit frontal area. Although in overall appearance, axial turbines are very similar, examination of the blade cross-section will indicate a big difference. In the turbine, inlet passage area is greater than the outlet. The opposite occurs in the compressor, as shown in Fig. 5.1.

Thus the process in turbine blades can be described as an accelerating flow, the increase in velocity being achieved by the nozzle. However, in the axial flow compressor, the flow is decelerating or diffusing and the pressure rise occurs when the fluid passes through the blades. As mentioned in the chapter on diffuser design (Chapter 4, Sec. 4.7), it is much more difficult to carry out efficient diffusion due to the breakaway of air molecules from the walls of the diverging passage. The air molecules that break away tend to reverse direction and flow back in the direction of the pressure gradient. If the divergence is too rapid, this may result in the formation of eddies and reduction in useful pressure rise. During acceleration in a nozzle, there is a natural tendency for the air to fill the passage

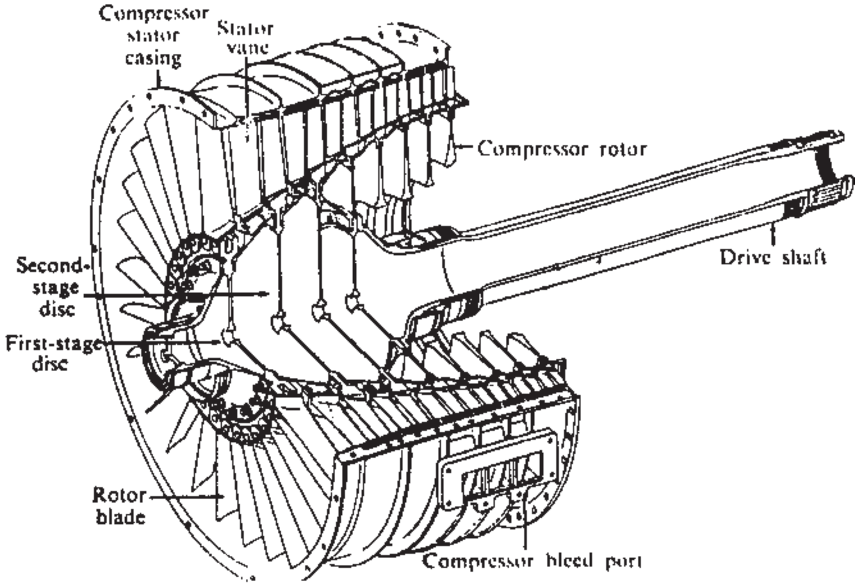


Figure 5.1 Cutaway sketch of a typical axial compressor assembly: the General Electric J85 compressor. (Courtesy of General Electric Co.)

walls closely (only the normal friction loss will be considered in this case). Typical blade sections are shown in Fig. 5.2. Modern axial flow compressors may give efficiencies of 86–90%—compressor design technology is a well-developed field. Axial flow compressors consist of a number of stages, each stage being formed by a stationary row and a rotating row of blades.

Figure 5.3 shows how a few compressor stages are built into the axial compressor. The rotating blades impart kinetic energy to the air while increasing air pressure and the stationary row of blades redirect the air in the proper direction and convert a part of the kinetic energy into pressure. The flow of air through the compressor is in the direction of the axis of the compressor and, therefore, it is called an axial flow compressor. The height of the blades is seen to decrease as the fluid moves through the compressor. As the pressure increases in the direction of flow, the volume of air decreases. To keep the air velocity the same for each stage, the blade height is decreased along the axis of the compressor. An extra row of fixed blades, called the inlet guide vanes, is fitted to the compressor inlet. These are provided to guide the air at the correct angle onto the first row of moving blades. In the analysis of the highly efficient axial flow compressor, the 2-D flow through the stage is very important due to cylindrical symmetry.

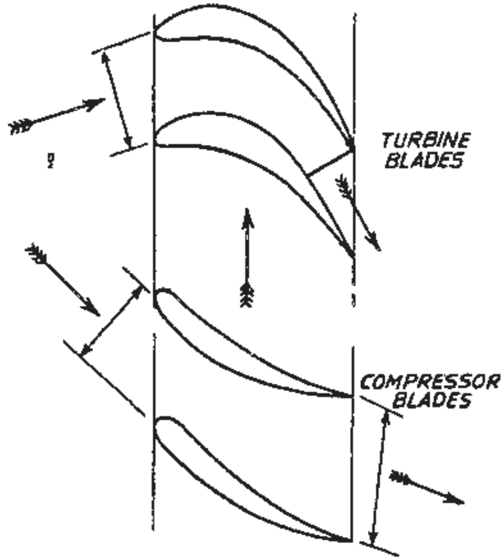


Figure 5.2 Compressor and turbine blade passages: turbine and compressor housing.

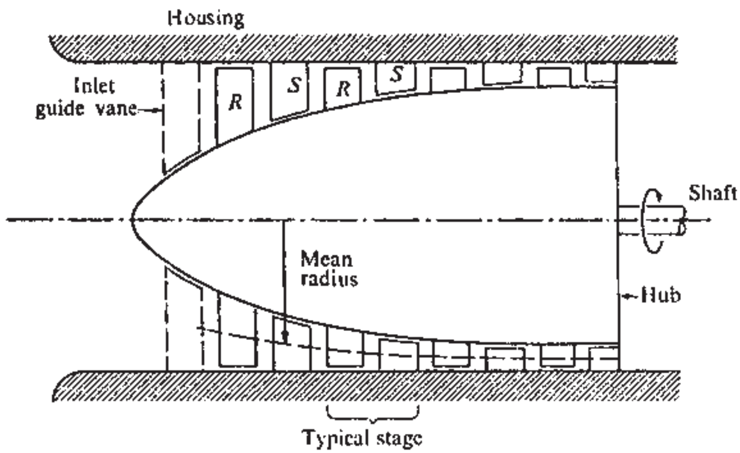


Figure 5.3 Schematic of an axial compressor section.

The flow is assumed to take place at a mean blade height, where the blade peripheral velocities at the inlet and outlet are the same. No flow is assumed in the radial direction.

5.2 VELOCITY DIAGRAM

The basic principle of axial compressor operation is that kinetic energy is imparted to the air in the rotating blade row, and then diffused through passages of both rotating and stationary blades. The process is carried out over multiple numbers of stages. As mentioned earlier, diffusion is a deceleration process. It is efficient only when the pressure rise per stage is very small. The blading diagram and the velocity triangle for an axial flow compressor stage are shown in Fig. 5.4.

Air enters the rotor blade with absolute velocity C_1 at an angle α_1 measured from the axial direction. Air leaves the rotor blade with absolute velocity C_2 at an angle α_2. Air passes through the diverging passages formed between the rotor blades. As work is done on the air in the rotor blades, C_2 is larger than C_1. The rotor row has tangential velocity U. Combining the two velocity vectors gives the relative velocity at inlet V_1 at an angle β_1. V_2 is the relative velocity at the rotor outlet. It is less than V_1, showing diffusion of the relative velocity has taken place with some static pressure rise across the rotor blades. Turning of the air towards the axial direction is brought about by the camber of the blades. Euler's equation

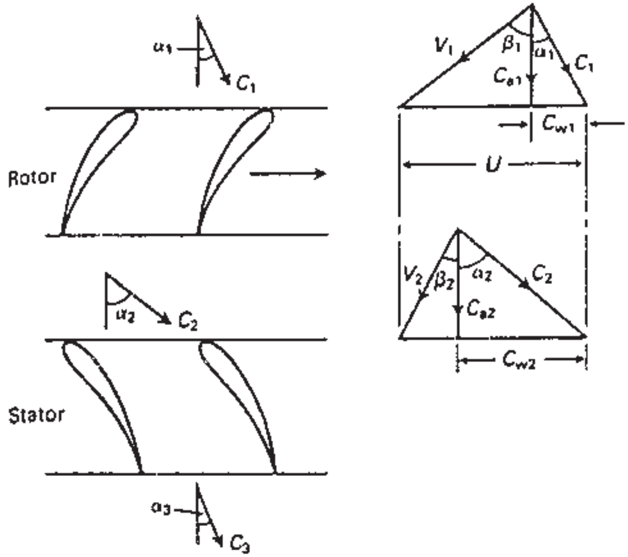


Figure 5.4 Velocity diagrams for a compressor stage.

provides the work done on the air:

$$W_c = U(C_{w2} - C_{w1}) \tag{5.1}$$

Using the velocity triangles, the following basic equations can be written:

$$\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1 \tag{5.2}$$

$$\frac{U}{C_a} = \tan\alpha_2 + \tan\beta_2 \tag{5.3}$$

in which $C_a = C_{a1} = C_2$ is the axial velocity, assumed constant through the stage. The work done equation [Eq. (5.1)] may be written in terms of air angles:

$$W_c = UC_a(\tan\alpha_2 - \tan\alpha_1) \tag{5.4}$$

also,

$$W_c = UC_a(\tan\beta_1 - \tan\beta_2) \tag{5.5}$$

The whole of this input energy will be absorbed usefully in raising the pressure and velocity of the air and for overcoming various frictional losses. Regardless of the losses, all the energy is used to increase the stagnation temperature of the air, $\triangle T_{0s}$. If the velocity of air leaving the first stage C_3 is made equal to C_1, then the stagnation temperature rise will be equal to the static temperature rise, $\triangle T_s$. Hence:

$$T_{0s} = \Delta T_s = \frac{UC_a}{C_p}(\tan\beta_1 - \tan\beta_2) \tag{5.6}$$

Equation (5.6) is the theoretical temperature rise of the air in one stage. In reality, the stage temperature rise will be less than this value due to 3-D effects in the compressor annulus. To find the actual temperature rise of the air, a factor λ, which is between 0 and 100%, will be used. Thus the actual temperature rise of the air is given by:

$$T_{0s} = \frac{\lambda UC_a}{C_p}(\tan\beta_1 - \tan\beta_2) \tag{5.7}$$

If R_s is the stage pressure ratio and η_s is the stage isentropic efficiency, then:

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\gamma/(\gamma-1)} \tag{5.8}$$

where T_{01} is the inlet stagnation temperature.

5.3 DEGREE OF REACTION

The degree of reaction, Λ, is defined as:

$$\Lambda = \frac{\text{Static enthalpy rise in the rotor}}{\text{Static enthalpy rise in the whole stage}} \tag{5.9}$$

The degree of reaction indicates the distribution of the total pressure rise into the two types of blades. The choice of a particular degree of reaction is important in that it affects the velocity triangles, the fluid friction and other losses.

Let:

ΔT_A = the static temperature rise in the rotor

ΔT_B = the static temperature rise in the stator

Using the work input equation [Eq. (5.4)], we get:

$$\left.\begin{aligned} W_c &= C_p(\Delta T_A + \Delta T_B) = \Delta T_S \\ &= UC_a(\tan\beta_1 - \tan\beta_2) \\ &= UC_a(\tan\alpha_2 - \tan\alpha_1) \end{aligned}\right\} \tag{5.10}$$

But since all the energy is transferred to the air in the rotor, using the steady flow energy equation, we have:

$$W_c = C_p\Delta T_A + \frac{1}{2}(C_2^2 - C_1^2) \tag{5.11}$$

Combining Eqs. (5.10) and (5.11), we get:

$$C_p\Delta T_A = UC_a(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}(C_2^2 - C_1^2)$$

from the velocity triangles,

$$C_2 = C_a\cos\alpha_2 \quad \text{and} \quad C_1 = C_a\cos\alpha_1$$

Therefore,

$$\begin{aligned} C_p\Delta T_A &= UC_a(\tan\alpha_2 - \tan\alpha_1) - \tfrac{1}{2}C_a^2(\sec^2\alpha_2 - \sec^2\alpha_1) \\ &= UC_a(\tan\alpha_2 - \tan\alpha_1) - \tfrac{1}{2}C_a^2(\tan^2\alpha_2 - \tan^2\alpha_1) \end{aligned}$$

Using the definition of degree of reaction,

$$\begin{aligned} \Lambda &= \frac{\Delta T_A}{\Delta T_A + \Delta T_B} \\ &= \frac{UC_a(\tan\alpha_2 - \tan\alpha_1) - \tfrac{1}{2}C_a^2(\tan^2\alpha_2 - \tan^2\alpha_1)}{UC_a(\tan\alpha_2 - \tan\alpha_1)} \\ &= 1 - \tfrac{C_a}{U}(\tan\alpha_2 + \tan\alpha_1) \end{aligned}$$

But from the velocity triangles, adding Eqs. (5.2) and (5.3),

$$\frac{2U}{C_a} = (\tan \alpha_1 + \tan \beta_1 + \tan \alpha_2 + \tan \beta_2)$$

Therefore,

$$\Lambda = \frac{C_a}{2U}\left(\frac{2U}{C_a} - \frac{2U}{C_a} + \tan \beta_1 + \tan \beta_2\right)$$

$$= \frac{C_a}{2U}(\tan \beta_1 + \tan \beta_2) \tag{5.12}$$

Usually the degree of reaction is set equal to 50%, which leads to this interesting result:

$$(\tan \beta_1 + \tan \beta_2) = \frac{U}{C_a}.$$

Again using Eqs. (5.1) and (5.2),

$$\tan \alpha_1 = \tan \beta_2, \quad \text{i.e.,} \quad \alpha_1 = \beta_2$$

$$\tan \beta_1 = \tan \alpha_2, \quad \text{i.e.,} \quad \alpha_2 = \beta_1$$

As we have assumed that C_a is constant through the stage,

$$C_a = C_1 \cos \alpha_1 = C_3 \cos \alpha_3.$$

Since we know $C_1 = C_3$, it follows that $\alpha_1 = \alpha_3$. Because the angles are equal, $\alpha_1 = \beta_2 = \alpha_3$, and $\beta_1 = \alpha_2$. Under these conditions, the velocity triangles become symmetric. In Eq. (5.12), the ratio of axial velocity to blade velocity is called the flow coefficient and denoted by Φ. For a reaction ratio of 50%, $(h_2 - h_1) = (h_3 - h_1)$, which implies the static enthalpy and the temperature increase in the rotor and stator are equal. If for a given value of C_a/U, β_2 is chosen to be greater than α_2 (Fig. 5.5), then the static pressure rise in the rotor is greater than the static pressure rise in the stator and the reaction is greater than 50%.

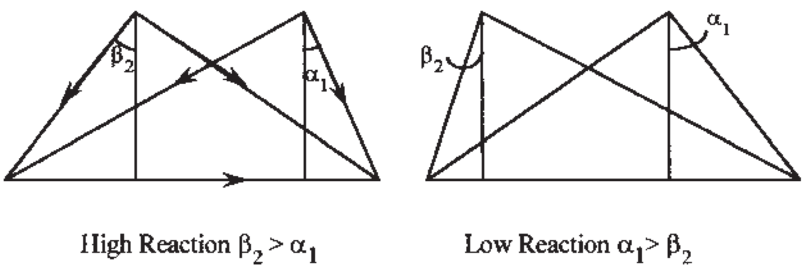


Figure 5.5 Stage reaction.

Conversely, if the designer chooses β_2 less than β_1, the stator pressure rise will be greater and the reaction is less than 50%.

5.4 STAGE LOADING

The stage-loading factor Ψ is defined as:

$$\Psi = \frac{W_c}{mU^2} = \frac{h_{03} - h_{01}}{U^2}$$
$$= \frac{\lambda(C_{w2} - C_{w1})}{U}$$
$$= \frac{\lambda C_a}{U}(\tan\alpha_2 - \tan\alpha_1) \tag{5.13}$$
$$\Psi = \lambda\Phi\,(\tan\alpha_2 - \tan\alpha_1)$$

5.5 LIFT-AND-DRAG COEFFICIENTS

The stage-loading factor Ψ may be expressed in terms of the lift-and-drag coefficients. Consider a rotor blade as shown in Fig. 5.6, with relative velocity vectors V_1 and V_2 at angles β_1 and β_2. Let $\tan(\beta_m) = (\tan(\beta_1) + \tan(\beta_2))/2$. The flow on the rotor blade is similar to flow over an airfoil, so lift-and-drag forces will be set up on the blade while the forces on the air will act on the opposite direction.

The tangential force on each moving blade is:

$$F_x = L\cos\beta_m + D\sin\beta_m$$

$$F_x = L\cos\beta_m\left[1 + \left(\frac{C_D}{C_L}\right)\tan\beta_m\right] \tag{5.14}$$

where: L = lift and D = drag.

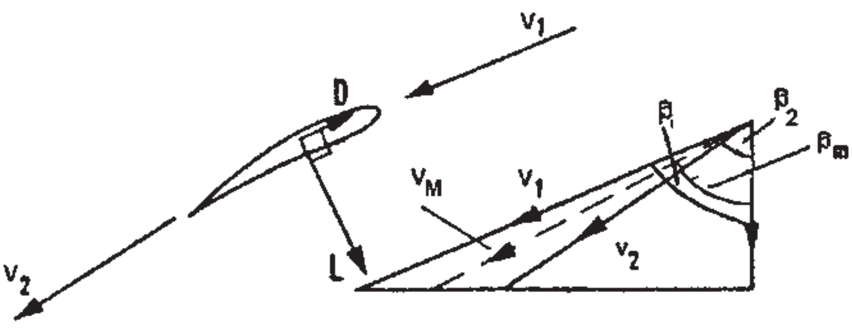


Figure 5.6 Lift-and-drag forces on a compressor rotor blade.

The lift coefficient is defined as:

$$C_L = \frac{L}{0.5\rho V_m^2 A} \tag{5.15}$$

where the blade area is the product of the chord c and the span l.

Substituting $V_m = \frac{C_a}{\cos\beta_m}$ into the above equation,

$$F_x = \frac{\rho C_a^2 c l C_L}{2} \sec\beta_m \left[1 + \left(\frac{C_D}{C_L}\right)\tan\beta_m\right] \tag{5.16}$$

The power delivered to the air is given by:

$$\begin{aligned} UF_x &= m(h_{03} - h_{01}) \\ &= \rho C_a l s (h_{03} - h_{01}) \end{aligned} \tag{5.17}$$

considering the flow through one blade passage of width s.

Therefore,

$$\begin{aligned} &= \frac{h_{03} - h_{01}}{U^2} \\ &= \frac{F_x}{\rho C_a l s U} \\ &= \frac{1}{2}\left(\frac{C_a}{U}\right)\left(\frac{c}{s}\right)\sec\beta_m(C_L + C_D\tan\beta_m) \\ &= \frac{1}{2}\left(\frac{c}{s}\right)\sec\beta_m(C_L + C_D\tan\beta_m) \end{aligned} \tag{5.18}$$

For a stage in which $\beta_m = 45°$, efficiency will be maximum. Substituting this back into Eq. (5.18), the optimal blade-loading factor is given by:

$$\Psi_{opt} = \frac{\varphi}{\sqrt{2}}\left(\frac{c}{s}\right)(C_L + C_D) \tag{5.19}$$

For a well-designed blade, C_D is much smaller than C_L, and therefore the optimal blade-loading factor is approximated by:

$$\Psi_{opt} = \frac{\varphi}{\sqrt{2}}\left(\frac{c}{s}\right)C_L \tag{5.20}$$

5.6 CASCADE NOMENCLATURE AND TERMINOLOGY

Studying the 2-D flow through cascades of airfoils facilitates designing highly efficient axial flow compressors. A cascade is a row of geometrically similar blades arranged at equal distance from each other and aligned to the flow direction. Figure 5.7, which is reproduced from Howell's early paper on cascade theory and performance, shows the standard nomenclature relating to airfoils in cascade.

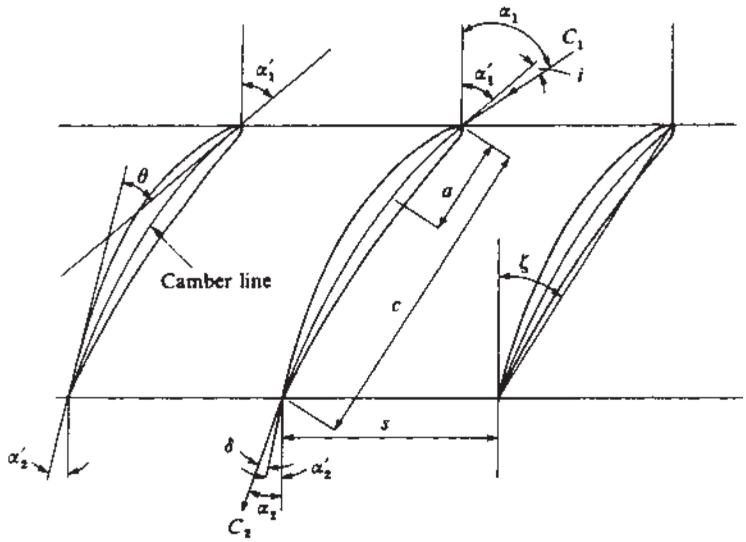


Figure 5.7 Cascade nomenclature.

$\alpha_1{}'$ and $\alpha_2{}'$ are the camber angles of the entry and exit tangents the camber line makes with the axial direction. The blade camber angle $\theta = \alpha_1' - \alpha_2'$. The chord c is the length of the perpendicular of the blade profile onto the chord line. It is approximately equal to the linear distance between the leading edge and the trailing edge. The stagger angle ξ is the angle between the chord line and the axial direction and represents the angle at which the blade is set in the cascade. The pitch s is the distance in the direction of rotation between corresponding points on adjacent blades. The incidence angle i is the difference between the air inlet angle (α_1) and the blade inlet angle $\left(\alpha_1'\right)$. That is, $i = \alpha_1 - \alpha_1'$. The deviation angle (δ) is the difference between the air outlet angle (α_2) and the blade outlet angle $\left(\alpha_2'\right)$. The air deflection angle, $\varepsilon = \alpha_1 - \alpha_2$, is the difference between the entry and exit air angles.

A cross-section of three blades forming part of a typical cascade is shown in Fig. 5.7. For any particular test, the blade camber angle θ, its chord c, and the pitch (or space) s will be fixed and the blade inlet and outlet angles α_1' and α_2' are determined by the chosen setting or stagger angle ξ. The angle of incidence, i, is then fixed by the choice of a suitable air inlet angle α_1, since $i = \alpha_1 - \alpha_1'$. An appropriate setting of the turntable on which the cascade is mounted can accomplish this. With the cascade in this position the pressure and direction measuring instruments are then traversed along the blade row in the upstream and downstream position. The results of the traverses are usually presented as shown

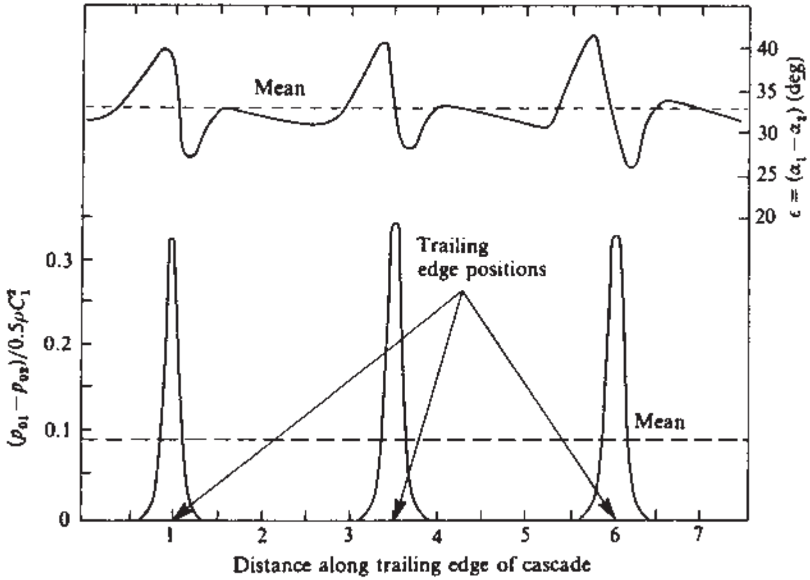


Figure 5.8 Variation of stagnation pressure loss and deflection for cascade at fixed incidence.

in Fig. 5.8. The stagnation pressure loss is plotted as a dimensionless number given by:

$$\text{Stagnation pressure loss coefficient} = \frac{P_{01} - P_{02}}{0.5\rho C_1^2} \tag{5.21}$$

This shows the variation of loss of stagnation pressure and the air deflection, $\varepsilon = \alpha_1 - \alpha_2$, covering two blades at the center of the cascade. The curves of Fig. 5.8 can now be repeated for different values of incidence angle, and the whole set of results condensed to the form shown in Fig. 5.9, in which the mean loss and mean deflection are plotted against incidence for a cascade of fixed geometrical form.

The total pressure loss owing to the increase in deflection angle of air is marked when i is increased beyond a particular value. The stalling incidence of the cascade is the angle at which the total pressure loss is twice the minimum cascade pressure loss. Reducing the incidence i generates a negative angle of incidence at which stalling will occur.

Knowing the limits for air deflection without very high (more than twice the minimum) total pressure loss is very useful for designers in the design of efficient compressors. Howell has defined nominal conditions of deflection for

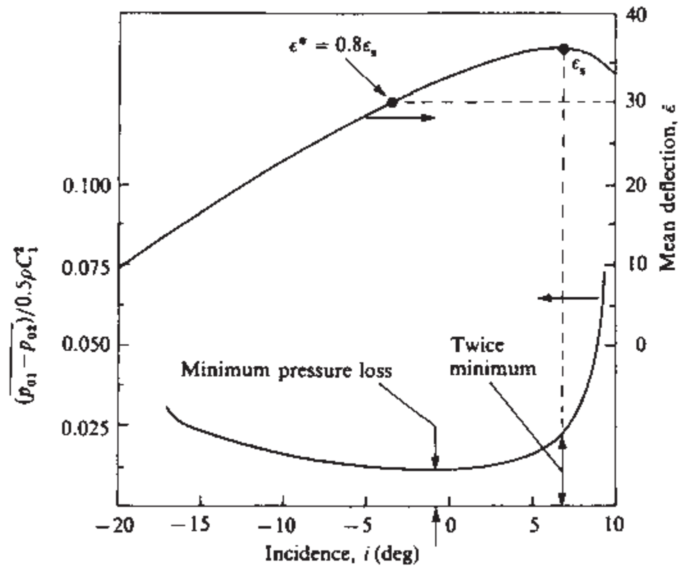


Figure 5.9 Cascade mean deflection and pressure loss curves.

a cascade as 80% of its stalling deflection, that is:

$$\varepsilon^* = 0.8\varepsilon_s \tag{5.22}$$

where ε_s is the stalling deflection and ε^* is the nominal deflection for the cascade.

Howell and Constant also introduced a relation correlating nominal deviation δ^* with pitch chord ratio and the camber of the blade. The relation is given by:

$$\delta^* = m\theta\left(\frac{s}{l}\right)^n \tag{5.23}$$

For compressor cascade, $n = \frac{1}{2}$, and for the inlet guide vane in front of the compressor, $n = 1$. Hence, for a compressor cascade, nominal deviation is given by:

$$\delta^* = m\theta\left(\frac{s}{l}\right)^{\frac{1}{2}} \tag{5.24}$$

The approximate value suggested by Constant is 0.26, and Howell suggested a modified value for m:

$$m = 0.23\left(\frac{2a}{l}\right)^2 + 0.1\left(\frac{\alpha_2^*}{50}\right) \tag{5.25}$$

where the maximum camber of the cascade airfoil is at a distance a from the leading edge and α_2^* is the nominal air outlet angle.

Then,

$$\alpha_2^* = \beta_2 + \delta^*$$

$$= \beta_2 + m\theta\left(\frac{s}{l}\right)^{\frac{1}{2}}$$

and,

$$\alpha_1^* - \alpha_2^* = \varepsilon^*$$

or:

$$\alpha_1^* = \alpha_2^* + \varepsilon^*$$

Also,

$$i^* = \alpha_1^* - \beta_1 = \alpha_2^* + \varepsilon^* - \beta_1$$

5.7 3-D CONSIDERATION

So far, all the above discussions were based on the velocity triangle at one particular radius of the blading. Actually, there is a considerable difference in the velocity diagram between the blade hub and tip sections, as shown in Fig. 5.10.

The shape of the velocity triangle will influence the blade geometry, and, therefore, it is important to consider this in the design. In the case of a compressor with high hub/tip ratio, there is little variation in blade speed from root to tip. The shape of the velocity diagram does not change much and, therefore, little variation in pressure occurs along the length of the blade. The blading is of the same section at all radii and the performance of the compressor stage is calculated from the performance of the blading at the mean radial section. The flow along the compressor is considered to be 2-D. That is, in 2-D flow only whirl and axial flow velocities exist with no radial velocity component. In an axial flow compressor in which high hub/tip radius ratio exists on the order of 0.8, 2-D flow in the compressor annulus is a fairly reasonable assumption. For hub/tip ratios lower than 0.8, the assumption of two-dimensional flow is no longer valid. Such compressors, having long blades relative to the mean diameter, have been used in aircraft applications in which a high mass flow requires a large annulus area but a small blade tip must be used to keep down the frontal area. Whenever the fluid has an angular velocity as well as velocity in the direction parallel to the axis of rotation, it is said to have "vorticity." The flow through an axial compressor is vortex flow in nature. The rotating fluid is subjected to a centrifugal force and to balance this force, a radial pressure gradient is necessary. Let us consider the pressure forces on a fluid element as shown in Fig. 5.10. Now, resolve

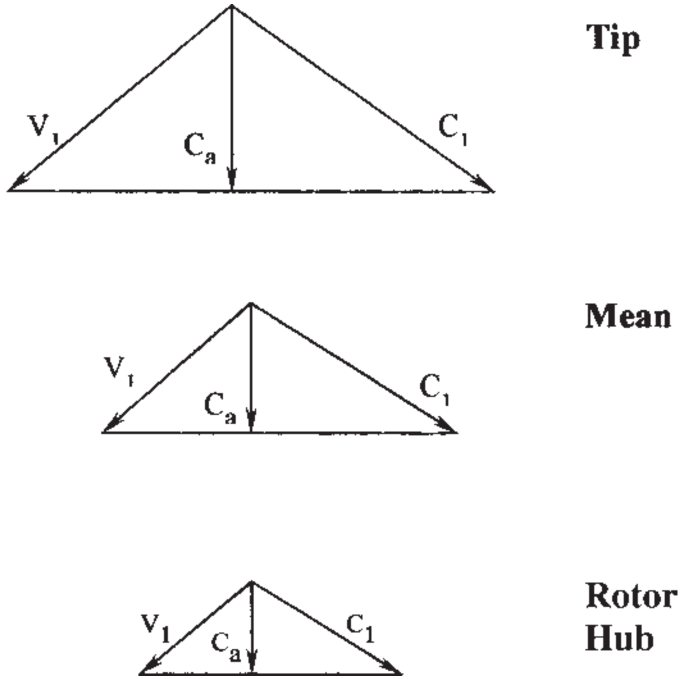


Figure 5.10 Variation of velocity diagram along blade.

the forces in the radial direction Fig. 5.11:

$$\mathrm{d}\theta(P+\mathrm{d}P)(r+\mathrm{d}r) - Pr\,\mathrm{d}\theta - 2\left(P+\frac{\mathrm{d}P}{2}\right)\mathrm{d}r\frac{\mathrm{d}\theta}{2}$$

$$= \rho\,\mathrm{d}r\,r\,\mathrm{d}\theta\frac{C_\mathrm{w}^2}{r} \tag{5.26}$$

or

$$(P+\mathrm{d}P)(r+\mathrm{d}r) - Pr - \left(P+\frac{\mathrm{d}P}{2}\right)\mathrm{d}r = \rho\,\mathrm{d}r\,C_\mathrm{w}^2$$

where: P is the pressure, ρ, the density, C_w, the whirl velocity, r, the radius.

After simplification, we get the following expression:

$$Pr + P\,\mathrm{d}r + r\,\mathrm{d}P + \mathrm{d}P\,\mathrm{d}r - Pr + \rho\,\mathrm{d}r - \frac{1}{2}\mathrm{d}P\,\mathrm{d}r = \rho\,\mathrm{d}r\,C_\mathrm{w}^2$$

or:

$$r\,\mathrm{d}P = \rho\,\mathrm{d}r\,C_\mathrm{w}^2$$

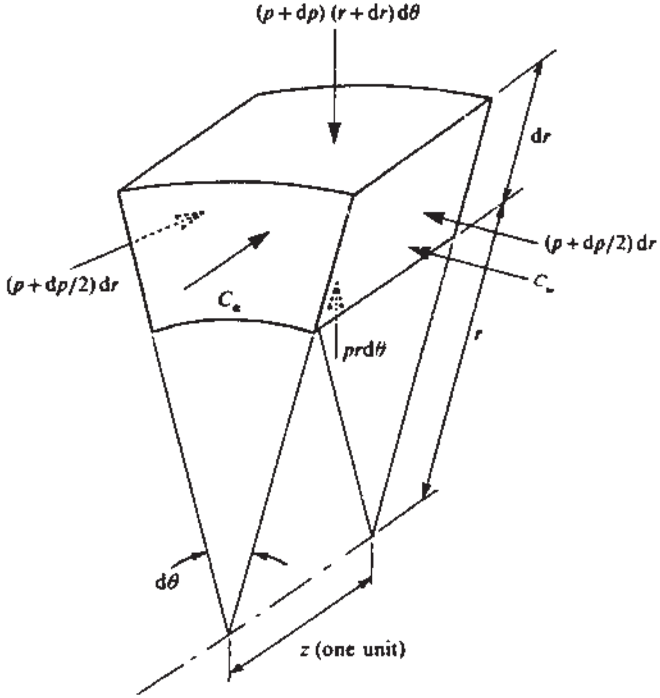


Figure 5.11 Pressure forces on a fluid element.

That is,

$$\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{C_{\mathrm{w}}^2}{r} \tag{5.27}$$

The approximation represented by Eq. (5.27) has become known as radial equilibrium.

The stagnation enthalpy h_0 at any radius r where the absolute velocity is C may be rewritten as:

$$h_0 = h + \frac{1}{2}C_{\mathrm{a}}^2 + \frac{1}{2}C_{\mathrm{w}}^2; \left(h = c_{\mathrm{p}}T, \quad \text{and} \quad C^2 = C_{\mathrm{a}}^2 + C_{\mathrm{w}}^2\right)$$

Differentiating the above equation w.r.t. r and equating it to zero yields:

$$\frac{\mathrm{d}h_0}{\mathrm{d}r} = \frac{\gamma}{\gamma - 1} \times \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} + \frac{1}{2}\left(0 + 2C_{\mathrm{w}}\frac{\mathrm{d}C_{\mathrm{w}}}{\mathrm{d}r}\right)$$

or:

$$\frac{\gamma}{\gamma - 1} \times \frac{1}{\rho} \frac{\mathrm{d}P}{\mathrm{d}r} + C_{\mathrm{w}} \frac{\mathrm{d}C_{\mathrm{w}}}{\mathrm{d}r} = 0$$

Combining this with Eq. (5.27):

$$\frac{\gamma}{\gamma - 1} \frac{C_{\mathrm{w}}^2}{r} + C_{\mathrm{w}} \frac{\mathrm{d}C_{\mathrm{w}}}{\mathrm{d}r} = 0$$

or:

$$\frac{\mathrm{d}C_{\mathrm{w}}}{\mathrm{d}r} = -\frac{\gamma}{\gamma - 1} \frac{C_{\mathrm{w}}}{r}$$

Separating the variables,

$$\frac{\mathrm{d}C_{\mathrm{w}}}{C_{\mathrm{w}}} = -\frac{\gamma}{\gamma - 1} \frac{\mathrm{d}r}{r}$$

Integrating the above equation

$$\int \frac{\mathrm{d}C_{\mathrm{w}}}{C_{\mathrm{w}}} = -\frac{\gamma}{\gamma - 1} \int \frac{\mathrm{d}r}{r}$$

$$-\frac{\gamma}{\gamma - 1} \ln C_{\mathrm{w}} r = c \quad \text{where } c \text{ is a constant.}$$

Taking antilog on both sides,

$$\frac{\gamma}{\gamma - 1} \times C_{\mathrm{w}} \times r = e^{\mathrm{c}}$$

Therefore, we have

$$C_{\mathrm{w}} r = \text{constant} \tag{5.28}$$

Equation (5.28) indicates that the whirl velocity component of the flow varies inversely with the radius. This is commonly known as free vortex. The outlet blade angles would therefore be calculated using the free vortex distribution.

5.8 MULTI-STAGE PERFORMANCE

An axial flow compressor consists of a number of stages. If R is the overall pressure ratio, R_{s} is the stage pressure ratio, and N is the number of stages, then the total pressure ratio is given by:

$$R = (R_{\mathrm{s}})^N \tag{5.29}$$

Equation (5.29) gives only a rough value of R because as the air passes through the compressor the temperature rises continuously. The equation used to find stage pressure is given by:

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}} \tag{5.30}$$

The above equation indicates that the stage pressure ratio depends only on inlet stagnation temperature T_{01}, which goes on increasing in the successive stages. To find the value of R, the concept of polytropic or small stage efficiency is very useful. The polytropic or small stage efficiency of a compressor is given by:

$$\eta_{\infty,c} = \left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{n}{n-1}\right)$$

or:

$$\left(\frac{n}{n-1}\right) = \eta_s\left(\frac{\gamma}{\gamma - 1}\right)$$

where $\eta_s = \eta_{\infty,c}$ = small stage efficiency.

The overall pressure ratio is given by:

$$R = \left[1 + \frac{N\Delta T_{0s}}{T_{01}}\right]^{\frac{n}{n-1}} \tag{5.31}$$

Although Eq. (5.31) is used to find the overall pressure ratio of a compressor, in actual practice the step-by-step method is used.

5.9 AXIAL FLOW COMPRESSOR CHARACTERISTICS

The forms of characteristic curves of axial flow compressors are shown in Fig. 5.12. These curves are quite similar to the centrifugal compressor. However, axial flow compressors cover a narrower range of mass flow than the centrifugal compressors, and the surge line is also steeper than that of a centrifugal compressor. Surging and choking limit the curves at the two ends. However, the surge points in the axial flow compressors are reached before the curves reach a maximum value. In practice, the design points is very close to the surge line. Therefore, the operating range of axial flow compressors is quite narrow.

Illustrative Example 5.1: In an axial flow compressor air enters the compressor at stagnation pressure and temperature of 1 bar and 292K, respectively. The pressure ratio of the compressor is 9.5. If isentropic efficiency of the compressor is 0.85, find the work of compression and the final temperature at the outlet. Assume $\gamma = 1.4$, and $C_p = 1.005$ kJ/kg K.

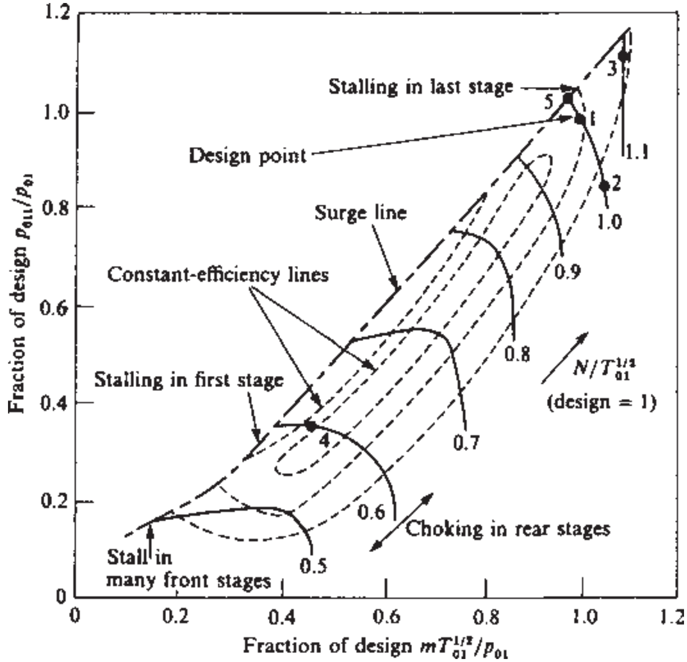


Figure 5.12 Axial flow compressor characteristics.

Solution:

$$T_{01} = 292\text{K}, \quad P_{01} = 1\,\text{bar}, \quad \eta_c = 0.85.$$

Using the isentropic P–T relation for compression processes,

$$\frac{P_{02}}{P_{01}} = \left[\frac{T'_{02}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

where $T_{02'}$ is the isentropic temperature at the outlet.

Therefore,

$$T'_{02} = T_{01}\left[\frac{P_{02}}{P_{01}}\right]^{\frac{\gamma-1}{\gamma}} = 292(9.5)^{0.286} = 555.92\,\text{K}$$

Now, using isentropic efficiency of the compressor in order to find the actual temperature at the outlet,

$$T_{02} = T_{01} + \frac{\left(T'_{02} - T_{01}\right)}{\eta_c} = 292 + \frac{(555.92 - 292)}{0.85} = 602.49\,\text{K}$$

Work of compression:

$$W_c = C_p(T_{02} - T_{01}) = 1.005(602.49 - 292) = 312\,\text{kJ/kg}$$

Illustrative Example 5.2: In one stage of an axial flow compressor, the pressure ratio is to be 1.22 and the air inlet stagnation temperature is 288K. If the stagnation temperature rise of the stages is 21K, the rotor tip speed is 200 m/s, and the rotor rotates at 4500 rpm, calculate the stage efficiency and diameter of the rotor.

Solution:
The stage pressure ratio is given by:

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

or

$$1.22 = \left[1 + \frac{\eta_s(21)}{288}\right]^{3.5}$$

that is,

$$\eta_s = 0.8026 \quad \text{or} \quad 80.26\%$$

The rotor speed is given by:

$$U = \frac{\pi DN}{60}, \quad \text{or} \quad D = \frac{(60)(200)}{\pi(4500)} = 0.85\ \text{m}$$

Illustrative Example 5.3: An axial flow compressor has a tip diameter of 0.95 m and a hub diameter of 0.85 m. The absolute velocity of air makes an angle of 28° measured from the axial direction and relative velocity angle is 56°. The absolute velocity outlet angle is 56° and the relative velocity outlet angle is 28°. The rotor rotates at 5000 rpm and the density of air is 1.2 kg/m^3. Determine:

1. The axial velocity.
2. The mass flow rate.
3. The power required.
4. The flow angles at the hub.
5. The degree of reaction at the hub.

Solution:
1. Rotor speed is given by:

$$U = \frac{\pi DN}{60} = \frac{\pi(0.95)(5000)}{60} = 249\,\text{m/s}$$

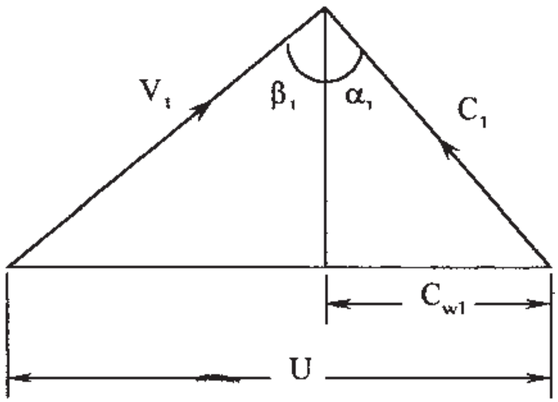


Figure 5.13 Inlet velocity triangle.

Blade speed at the hub:

$$U_{\mathrm{h}} = \frac{\pi D_{\mathrm{h}} N}{60} = \frac{\pi(0.85)(5000)}{60} = 223\ \mathrm{m/s}$$

From the inlet velocity triangle (Fig. 5.13),

$$\tan \alpha_1 = \frac{C_{\mathrm{w1}}}{C_{\mathrm{a}}} \qquad \text{and} \qquad \tan \beta_1 = \frac{(U - C_{\mathrm{w1}})}{C_{\mathrm{a}}}$$

Adding the above two equations:

$$\frac{U}{C_{\mathrm{a}}} = \tan \alpha_1 + \tan \beta_1$$

or:

$$U = C_{\mathrm{a}}(\tan 28^\circ + \tan 56^\circ) = C_{\mathrm{a}}(2.0146)$$

Therefore, $C_{\mathrm{a}} = 123.6\ \mathrm{m/s}$ (constant at all radii)

2. The mass flow rate:

$$\begin{aligned} \dot{m} &= \pi(r_{\mathrm{t}}^2 - r_{\mathrm{h}}^2)\rho C_{\mathrm{a}} \\ &= \pi(0.475^2 - 0.425^2)(1.2)(123.6) = 20.98\ \mathrm{kg/s} \end{aligned}$$

3. The power required per unit kg for compression is:

$$\begin{aligned} W_{\mathrm{c}} &= \lambda U C_{\mathrm{a}}(\tan \beta_1 - \tan \beta_2) \\ &= (1)(249)(123.6)(\tan 56^\circ - \tan 28^\circ)10^{-3} \\ &= (249)(123.6)(1.483 - 0.53) \\ &= 29.268\ \mathrm{kJ/kg} \end{aligned}$$

The total power required to drive the compressor is:

$$W_c = \dot{m}(29.268) = (20.98)(29.268) = 614\,\text{kW}$$

4. At the inlet to the rotor tip:

$$C_{w1} = C_a \tan\alpha_1 = 123.6 \tan 28^\circ = 65.72\,\text{m/s}$$

Using free vortex condition, i.e., $C_w r =$ constant, and using h as the subscript for the hub,

$$C_{w1h} = C_{w1t}\frac{r_t}{r_h} = (65.72)\frac{0.475}{0.425} = 73.452\,\text{m/s}$$

At the outlet to the rotor tip,

$$C_{w2t} = C_a \tan\alpha_2 = 123.6 \tan 56^\circ = 183.24\,\text{m/s}$$

Therefore,

$$C_{w2h} = C_{w2t}\frac{r_t}{r_h} = (183.24)\frac{0.475}{0.425} = 204.8\,\text{m/s}$$

Hence the flow angles at the hub:

$$\tan\alpha_1 = \frac{C_{w1h}}{C_a} = \frac{73.452}{123.6} = 0.594 \text{ or, } \alpha_1 = 30.72^\circ$$

$$\tan\beta_1 = \frac{(U_h)}{C_a} - \tan\alpha_1 = \frac{223}{123.6} - 0.5942 = 1.21$$

i.e., $\beta_1 = 50.43^\circ$

$$\tan\alpha_2 = \frac{C_{w2h}}{C_a} = \frac{204.8}{123.6} = 1.657$$

i.e., $\alpha_2 = 58.89^\circ$

$$\tan\beta_2 = \frac{(U_h)}{C_a} - \tan\alpha_2 = \frac{223}{123.6} - \tan 58.59^\circ = 0.1472$$

i.e., $\beta_2 = 8.37^\circ$

5. The degree of reaction at the hub is given by:

$$\begin{aligned}\Lambda_h &= \frac{C_a}{2U_h}(\tan\beta_1 + \tan\beta_2) = \frac{123.6}{(2)(223)}(\tan 50.43^\circ + \tan 8.37^\circ)\\ &= \frac{123.6}{(2)(223)}(1.21 + 0.147) = 37.61\%\end{aligned}$$

Illustrative Example 5.4: An axial flow compressor has the following data:

Blade velocity at root:	140 m/s
Blade velocity at mean radius:	185 m/s
Blade velocity at tip:	240 m/s
Stagnation temperature rise in this stage:	15K
Axial velocity (constant from root to tip):	140 m/s
Work done factor:	0.85
Degree of reaction at mean radius:	50%

Calculate the stage air angles at the root, mean, and tip for a free vortex design.

Solution:

Calculation at mean radius:

From Eq. (5.1), $W_c = U(C_{w2} - C_{w1}) = U\Delta C_w$

or:

$$C_p(T_{02} - T_{01}) = C_p\Delta T_{0s} = \lambda U \Delta C_w$$

So:

$$\Delta C_w = \frac{C_p \Delta T_{0s}}{\lambda U} = \frac{(1005)(15)}{(0.85)(185)} = 95.87 \text{ m/s}$$

Since the degree of reaction (Fig. 5.14) at the mean radius is 50%, $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$.

From the velocity triangle at the mean,

$$U = \Delta C_w + 2C_{w1}$$

or:

$$C_{w1} = \frac{U - \Delta C_w}{2} = \frac{185 - 95.87}{2} = 44.57 \text{ m/s}$$

Hence,

$$\tan\alpha_1 = \frac{C_{w1}}{C_a} = \frac{44.57}{140} = 0.3184$$

that is,

$$\alpha_1 = 17.66° = \beta_2$$

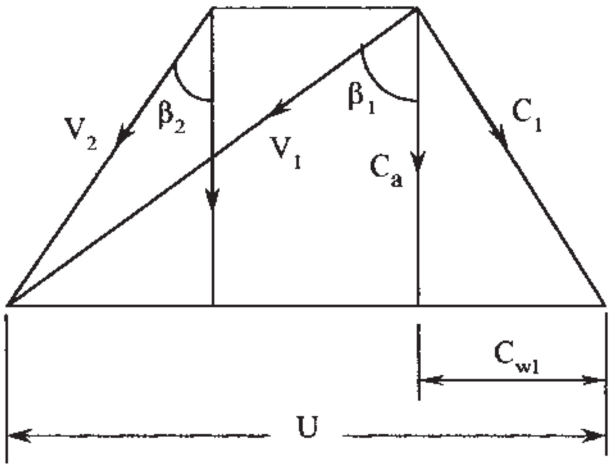


Figure 5.14 Velocity triangle at the mean radius.

and

$$\tan\beta_1 = \frac{(\Delta C_w + C_{w1})}{C_a} = \frac{(95.87 + 44.57)}{140} = 1.003$$

i.e., $\beta_1 = 45.09^\circ = \alpha_2$

Calculation at the blade tip:
Using the free vortex diagram (Fig. 5.15),

$$(\Delta C_w \times U)_t = (\Delta C_w \times U)_m$$

Therefore,

$$\Delta C_w = \frac{(95.87)(185)}{240} = 73.9\,\text{m/s}$$

Whirl velocity component at the tip:

$$C_{w1} \times 240 = (44.57)(185)$$

Therefore:

$$C_{w1} = \frac{(44.57)(185)}{240} = 34.36\,\text{m/s}$$

$$\tan\alpha_1 = \frac{C_{w1}}{C_a} = \frac{34.36}{140} = 0.245$$

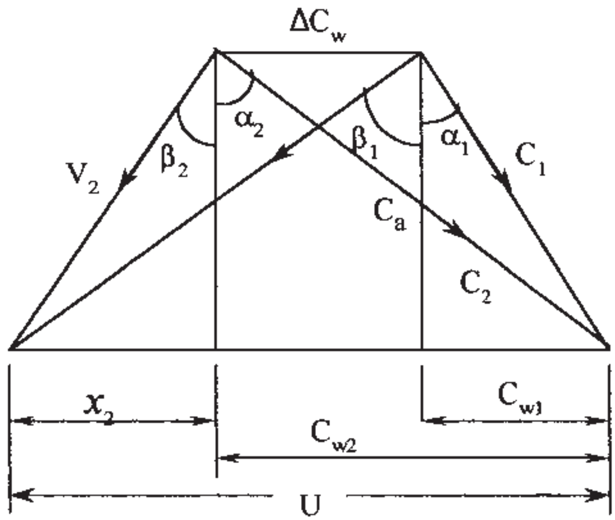


Figure 5.15 Velocity triangles at tip.

Therefore,

$$\alpha_1 = 13.79°$$

From the velocity triangle at the tip,

$$x_2 + \Delta C_w + C_{w1} = U$$

or:

$$x_2 = U - \Delta C_w - C_{w1} = 240 - 73.9 - 34.36 = 131.74$$

$$\tan\beta_1 = \frac{\Delta C_w + x_2}{C_a} = \frac{73.9 + 131.74}{140} = 1.469$$

i.e., $\beta_1 = 55.75°$

$$\tan\alpha_2 = \frac{(C_{w1} + \Delta C_w)}{C_a} = \frac{(34.36 + 73.9)}{140} = 0.7733$$

i.e., $\alpha_2 = 37.71°$

$$\tan\beta_2 = \frac{x_2}{C_a} = \frac{131.74}{140} = 0.941$$

i.e., $\beta_2 = 43.26°$

Calculation at the blade root:

$$(\Delta C_w \times U)_r = (\Delta C_w \times U)_m$$

or:

$$\Delta C_w \times 140 = (95.87)(185) \qquad \text{and} \qquad \Delta C_w = 126.69\,\text{m/s}$$

Also:

$$(C_{w1} \times U)_r = (C_{w1} \times U)_m$$

or:

$$C_{w1} \times 140 = (44.57)(185) \quad \text{and} \quad C_{w1} = 58.9\,\text{m/s}$$

and

$$(C_{w2} \times U)_t = (C_{w2} \times U)_r$$

so:

$$C_{w2,tip} = C_a \tan\alpha_2 = 140 \tan 37.71^{\circ} = 108.24\,\text{m/s}$$

Therefore:

$$C_{w2,root} = \frac{(108.24)(240)}{140} = 185.55\,\text{m/s}$$

$$\tan \alpha_1 = \frac{58.9}{140} = 0.421$$

i.e., $\alpha_1 = 22.82°$

From the velocity triangle at the blade root, (Fig. 5.16)

or:

$$x_2 = C_{w2} - U = 185.55 - 140 = 45.55$$

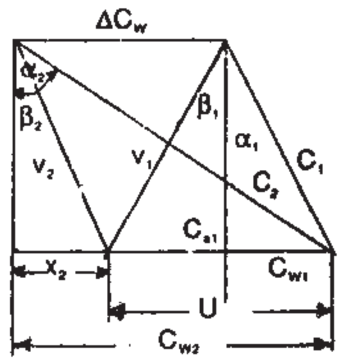


Figure 5.16 Velocity triangles at root.

Therefore:

$$\tan\beta_1 = \frac{U - C_{w1}}{C_a} = \frac{140 - 58.9}{140} = 0.579$$

i.e., $\beta_1 = 30.08°$

$$\tan\alpha_2 = \frac{C_{w2}}{C_a} = \frac{185.55}{140} = 1.325$$

i.e., $\alpha_2 = 52.96°$

$$\tan\beta_2 = -\frac{x_2}{C_a} = -\frac{45.55}{140} = -0.325$$

i.e., $\beta_2 = -18°$

Design Example 5.5: From the data given in the previous problem, calculate the degree of reaction at the blade root and tip.

Solution:

Reaction at the blade root:

$$\Lambda_{root} = \frac{C_a}{2U_r}(\tan\beta_{1r} + \tan\beta_{2r}) = \frac{140}{(2)(140)}(\tan 30.08^{\circ} + \tan(-18^{\circ}))$$

$$= \frac{140}{(2)(140)}(0.579 - 0.325) = 0.127, \quad \text{or} \quad 12.7\%$$

Reaction at the blade tip:

$$\Lambda_{tip} = \frac{C_a}{2U_t}(\tan\beta_{1t} + \tan\beta_{2t}) = \frac{140}{(2)(240)}(\tan 55.75^{\circ} + \tan 43.26^{\circ})$$

$$= \frac{140}{(2)(240)}(1.469 + 0.941) = 0.7029, \quad \text{or} \quad 70.29\%$$

Illustrative Example 5.6: An axial flow compressor stage has the following data:

Air inlet stagnation temperature:	295K
Blade angle at outlet measured from the axial direction:	32°
Flow coefficient:	0.56
Relative inlet Mach number:	0.78
Degree of reaction:	0.5

Find the stagnation temperature rise in the first stage of the compressor.

Solution:

Since the degree of reaction is 50%, the velocity triangle is symmetric as shown in Fig. 5.17. Using the degree of reaction equation [Eq. (5.12)]:

$$\Lambda = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2) \quad \text{and} \quad \varphi = \frac{C_a}{U} = 0.56$$

Therefore:

$$\tan\beta_1 = \frac{2\Lambda}{0.56} - \tan 32^\circ = 1.16$$

i.e., $\beta_1 = 49.24^\circ$

Now, for the relative Mach number at the inlet:

$$M_{r1} = \frac{V_1}{(\gamma R T_1)^{\frac{1}{2}}}$$

or:

$$V_1^2 = \gamma R M_{r1}^2 \left(T_{01} - \frac{C_1^2}{2C_p} \right)$$

From the velocity triangle,

$$V_1 = \frac{C_a}{\cos\beta_1}, \quad \text{and} \quad C_1 = \frac{C_a}{\cos\alpha_1}$$

and:

$$\alpha_1 = \beta_2 (\text{since } \Lambda = 0.5)$$

Therefore:

$$C_1 = \frac{C_a}{\cos 32^\circ} = \frac{C_a}{0.848}$$

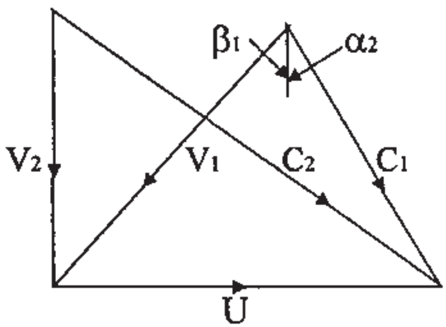


Figure 5.17 Combined velocity triangles for Example 5.6.

and:

$$V_1 = \frac{C_a}{\cos 49.24°} = \frac{C_a}{0.653}$$

Hence:

$$C_1^2 = \frac{C_a^2}{0.719}, \quad \text{and} \quad V_1^2 = \frac{C_a^2}{0.426}$$

Substituting for V_1 and C_1,

$$C_a^2 = 104.41\left(295 - \frac{C_a^2}{1445}\right), \quad \text{so :} \quad C_a = 169.51\,\text{m/s}$$

The stagnation temperature rise may be calculated as:

$$T_{02} - T_{01} = \frac{C_a^2}{C_p \varphi}(\tan\beta_1 - \tan\beta_2)$$

$$= \frac{169.51^2}{(1005)(0.56)}(\tan 49.24° - \tan 32°) = 27.31\text{K}$$

Design Example 5.7: An axial flow compressor has the following design data:

Inlet stagnation temperature:	290K
Inlet stagnation pressure:	1 bar
Stage stagnation temperature rise:	24K
Mass flow of air:	22kg/s
Axialvelocity through the stage:	155.5m/s
Rotational speed:	152rev/s
Work done factor:	0.93
Mean blade speed:	205m/s
Reaction at the mean radius:	50%

Determine: (1) the blade and air angles at the mean radius, (2) the mean radius, and (3) the blade height.

Solution:

(1) The following equation provides the relationship between the temperature rise and the desired angles:

$$T_{02} - T_{01} = \frac{\lambda U C_a}{C_p}(\tan\beta_1 - \tan\beta_2)$$

or:

$$24 = \frac{(0.93)(205)(155.5)}{1005}(\tan\beta_1 - \tan\beta_2)$$

so:

$$\tan\beta_1 - \tan\beta_2 = 0.814$$

Using the degree of reaction equation:

$$\Lambda = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2)$$

Hence:

$$\tan\beta_1 + \tan\beta_2 = \frac{(0.5)(2)(205)}{155.5} = 1.318$$

Solving the above two equations simultaneously for β_1 and β_2,

$$2\tan\beta_1 = 2.132,$$

$$\text{so} : \beta_1 = 46.83° = \alpha_2 \text{ (since the degree of reaction is 50\%)}$$

and:

$$\tan\beta_2 = 1.318 - \tan 46.83° = 1.318 - 1.066,$$

$$\text{so} : \beta_2 = 14.14° = \alpha_1$$

(2) The mean radius, r_m, is given by:

$$r_m = \frac{U}{2\pi N} = \frac{205}{(2\pi)(152)} = 0.215\text{m}$$

(3) The blade height, h, is given by:
$m = \rho A C_{a,}$ where A is the annular area of the flow.

$$C_1 = \frac{C_a}{\cos\alpha_1} = \frac{155.5}{\cos 14.14°} = 160.31 \text{ m/s}$$

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 290 - \frac{160.31^2}{(2)(1005)} = 277.21 \text{ K}$$

Using the isentropic $P-T$ relation:

$$\frac{P_1}{P_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

Static pressure:

$$P_1 = (1)\left(\frac{277.21}{290}\right)^{3.5} = 0.854\,\text{bar}$$

Then:

$$\rho_1 = \frac{P_1}{RT_1} = \frac{(0.854)(100)}{(0.287)(277.21)} = 1.073\,\text{kg/m}^3$$

From the continuity equation:

$$A = \frac{22}{(1.073)(155.5)} = 0.132\text{m}^2$$

and the blade height:

$$h = \frac{A}{2\pi r_\text{m}} = \frac{0.132}{(2\pi)(0.215)} = 0.098\text{m}$$

Illustrative Example 5.8: An axial flow compressor has an overall pressure ratio of 4.5:1, and a mean blade speed of 245 m/s. Each stage is of 50% reaction and the relative air angles are the same (30°) for each stage. The axial velocity is 158 m/s and is constant through the stage. If the polytropic efficiency is 87%, calculate the number of stages required. Assume $T_{01} = 290\text{K}$.

Solution:

Since the degree of reaction at the mean radius is 50%, $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$. From the velocity triangles, the relative outlet velocity component in the x-direction is given by:

$$V_{x2} = C_\text{a}\tan\beta_2 = 158\tan 30° = 91.22\,\text{m/s}$$

$$V_1 = C_2 = \left[(U - V_{x2})^2 + C_\text{a}^2\right]^{\frac{1}{2}}$$

$$= \left[(245 - 91.22)^2 + 158^2\right]^{\frac{1}{2}} = 220.48\,\text{m/s}$$

$$\cos\beta_1 = \frac{C_\text{a}}{V_1} = \frac{158}{220.48} = 0.7166$$

so: $\beta_1 = 44.23°$

Stagnation temperature rise in the stage,

$$\Delta T_{0s} = \frac{UC_a}{C_p}(\tan\beta_1 - \tan\beta_2)$$

$$= \frac{(245)(158)}{1005}(\tan 44.23° - \tan 30°) = 15.21\text{K}$$

Number of stages

$$R = \left[1 + \frac{N\Delta T_{0s}}{T_{01}}\right]^{\frac{n}{n-1}}$$

$$\frac{n}{n-1} = \eta_\infty \frac{\gamma}{\gamma - 1} = 0.87\frac{1.4}{0.4} = 3.05$$

Substituting:

$$4.5 = \left[1 + \frac{N15.21}{290}\right]^{3.05}$$

Therefore,

$$N = 12 \text{ stages.}$$

Design Example 5.9: In an axial flow compressor, air enters at a stagnation temperature of 290K and 1 bar. The axial velocity of air is 180 m/s (constant throughout the stage), the absolute velocity at the inlet is 185 m/s, the work done factor is 0.86, and the degree of reaction is 50%. If the stage efficiency is 0.86, calculate the air angles at the rotor inlet and outlet and the static temperature at the inlet of the first stage and stage pressure ratio. Assume a rotor speed of 200 m/s.

Solution:

For 50% degree of reaction at the mean radius (Fig. 5.18), $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$.

From the inlet velocity triangle,

$$\cos\alpha_1 = \frac{C_a}{C_1} = \frac{180}{185} = 0.973$$

i.e., $\alpha_1 = 13.35° = \beta_2$

From the same velocity triangle,

$$C_{w1} = \left(C_1^2 - C_a^2\right)^{\frac{1}{2}} = \left(185^2 - 180^2\right)^{\frac{1}{2}} = 42.72 \text{ m/s}$$

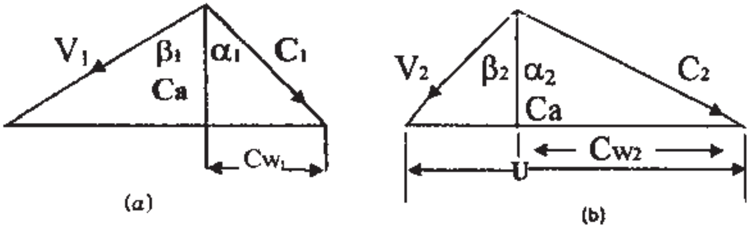


Figure 5.18 Velocity triangles (a) inlet, (b) outlet.

Therefore,

$$\tan \beta_1 = \frac{(U - C_{w1})}{C_a} = \frac{(200 - 42.72)}{180} = 0.874$$

i.e., $\beta_1 = 41.15° = \alpha_2$

Static temperature at stage inlet may be determined by using stagnation and static temperature relationship as given below:

$$T_1 = T_{01} - \frac{C_1}{2C_p} = 290 - \frac{185^2}{2(1005)} = 273\,\text{K}$$

Stagnation temperature rise of the stage is given by

$$\Delta T_{0s} = \frac{\lambda U C_a}{C_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

$$= \frac{0.86(200)(180)}{1005}(0.874 - 0.237) = 19.62\,\text{K}$$

Stage pressure ratio is given by

$$R_s = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\gamma/\gamma-1} = \left[1 + \frac{0.86 \times 19.62}{290}\right]^{3.5} = 1.22$$

Illustrative Example 5.10: Find the isentropic efficiency of an axial flow compressor from the following data:

Pressure ratio:	6
Polytropic efficiency:	0.85
Inlet temperature:	285 K

Solution:

Using the isentropic P–T relation for the compression process,

$$T_{02'} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 285(6)^{0.286} = 475.77\,\text{K}$$

Using the polytropic P–T relation for the compression process:

$$\frac{n-1}{n} = \frac{\gamma-1}{\gamma\eta_{\infty,c}} = \frac{0.4}{1.4(0.85)} = 0.336$$

Actual temperature rise:

$$T_{02} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(n-1)/n} = 285(6)^{0.336} = 520.36\,\text{K}$$

The compressor isentropic efficiency is given by:

$$\eta_c = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}} = \frac{475.77 - 285}{520 - 285} = 0.8105, \quad \text{or} \quad 81.05\%$$

Design Example 5.11: In an axial flow compressor air enters the compressor at 1 bar and 290K. The first stage of the compressor is designed on free vortex principles, with no inlet guide vanes. The rotational speed is 5500 rpm and stagnation temperature rise is 22K. The hub tip ratio is 0.5, the work done factor is 0.92, and the isentropic efficiency of the stage is 0.90. Assuming an inlet velocity of 145 m/s, calculate

1. The tip radius and corresponding rotor air angles, if the Mach number relative to the tip is limited to 0.96.
2. The mass flow at compressor inlet.
3. The stagnation pressure ratio and power required to drive the compressor.
4. The rotor air angles at the root section.

Solution:

(1) As no inlet guide vanes

$$\alpha_1 = 0, C_{w1} = 0$$

Stagnation temperature, T_{01}, is given by

$$T_{01} = T_1 + \frac{C_1}{2C_{p2}}$$

or

$$T_1 = T_{01} - \frac{C_1}{2C_p} = 290 - \frac{145^2}{2 \times 1005} = 288.9\text{K}$$

The Mach number relative to tip is

$$M = \frac{V_1}{\sqrt{\gamma R T_1}}$$

or

$$V_1 = 0.96(1.4 \times 287 \times 288.9)^{0.5} = 340.7 \text{ m/s}$$

i.e., relative velocity at tip = 340.7 m/s

From velocity triangle at inlet (Fig. 5.3)

$$V_1^2 = U_t^2 + C_1^2 \text{ or } U_t = \left(340.7^2 - 145^2\right)^{0.5} = 308.3 \text{ m/s}$$

or tip speed,

$$U_t = \frac{2\pi r_t N}{60}$$

or

$$r_t = \frac{308.3 \times 60}{2\pi \times 5500} = 0.535\text{m}.$$

$$\tan\beta_1 = \frac{U_t}{C_a} = \frac{308.3}{145} = 2.126$$

i.e., $\beta_1 = 64.81°$

Stagnation temperature rise

$$\Delta T_{0s} = \frac{\tau U C_a}{C_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

Substituting the values, we get

$$22 = \frac{0.92 \times 308.3 \times 145}{1005}\left(\tan\beta_1 - \tan\beta_2\right)$$

or

$$\left(\tan\beta_1 - \tan\beta_2\right) = 0.538$$

(2) Therefore, $\tan\beta_2 = 1.588$ and $\beta_2 = 57.8°$

$$\text{root radius/tip radius} = \frac{r_m - h/2}{r_m + h/2} = 0.5$$

(where subscript m for mean and h for height)

or $r_m - h/2 = 0.5\, r_m + 0.25\, h$

$\therefore r_m = 1.5\, h$

but $r_t = r_m + h/2 = 1.5\, h + h/2$

or $0.535 = 2\,h$ or $h = 0.268\,\text{m}$
and $r_\text{m} = 1.5\,h = 0.402\,\text{m}$

Area, $A = 2\pi r_\text{m} h = 2\pi \times\ 0.402 \times 0.268 = 0.677\,\text{m}^2$

Now, using isentropic relationship for $p{-}T$

$$\frac{p_1}{p_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\gamma/(\gamma-1)} \quad \text{or} \quad p_1 = 1 \times \left(\frac{288.9}{290}\right)^{3.5} = 0.987\,\text{bar}$$

and

$$\rho_1 = \frac{p_1}{RT_1} = \frac{0.987 \times 10^5}{287 \times 288.9} = 1.19\,\text{kg/m}^3$$

Therefore, the mass flow entering the stage

$$\dot{m} = \rho A C_a = 1.19 \times 0.677 \times 145 = 116.8\,\text{kg/s}$$

(3) Stage pressure ratio is

$$\begin{aligned} R_\text{s} &= \left[1 + \frac{\eta_\text{s} \Delta T_{0\text{s}}}{T_{01}}\right]^{\gamma/(\gamma-1)} \\ &= \left[1 + \frac{0.90 \times 22}{290}\right]^{3.5} = 1.26 \end{aligned}$$

Now,

$$W = C_\text{p} \Delta T_{0\text{s}} = 1005 \times 22 = 22110\text{J/kg}$$

Power required by the compressor

$$P = \dot{m} W = 116.8 \times 22110 = 2582.4\,\text{kW}$$

(4) In order to find out rotor air angles at the root section, radius at the root can be found as given below.

$$\begin{aligned} r_\text{r} &= r_\text{m} - h/2 \\ &= 0.402 - 0.268/2 = 0.267\text{m}. \end{aligned}$$

Impeller speed at root is

$$\begin{aligned} U_\text{r} &= \frac{2\pi r_\text{r} N}{60} \\ &= \frac{2 \times \pi \times 0.267 \times 5500}{60} = 153.843\ \text{m/s} \end{aligned}$$

Therefore, from velocity triangle at root section

$$\tan\beta_1 = \frac{U_r}{C_a} = \frac{153.843}{145} = 1.061$$

i.e., $\beta_1 = 46.9^\circ$

For β_2 at the root section

$$\Delta T_{0s} = \frac{\tau U_r C_a}{C_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

or

$$22 = \frac{0.92 \times 153.843 \times 145}{1005}\left(\tan\beta_1 - \tan\beta_2\right)$$

or

$$\left(\tan\beta_1 - \tan\beta_2\right) = 1.078$$

$$\therefore\ \beta_2 = -0.974^\circ$$

Design Example 5.12: The following design data apply to an axial flow compressor:

Overall pressure ratio:	4.5
Mass flow:	3.5kg/s
Polytropic efficiency:	0.87
Stagnation temperature rise per stage:	22k
Absolute velocity approaching the last rotor:	160m/s
Absolute velocity angle, measured from the axial direction:	20°
Work done factor:	0.85
Mean diameter of the last stage rotor is:	18.5cm
Ambient pressure:	1.0bar
Ambient temperature:	290K

Calculate the number of stages required, pressure ratio of the first and last stages, rotational speed, and the length of the last stage rotor blade at inlet to the stage. Assume equal temperature rise in all stages, and symmetrical velocity diagram.

Solution:

If N is the number of stages, then overall pressure rise is:

$$R = \left[1 + \frac{N\Delta T_{0s}}{T_{01}}\right]^{\frac{n-1}{n}}$$

where

$$\frac{n-1}{n} = \eta_{\alpha c}\frac{\gamma}{\gamma - 1}$$

(where $\eta_{\alpha c}$ is the polytropic efficiency)
substituting values

$$\frac{n-1}{n} = 0.87 \times \frac{1.4}{0.4} = 3.05$$

overall pressure ratio, R is

$$R = \left[1 + \frac{N \times 22}{290}\right]^{3.05}$$

or

$$(4.5)^{\frac{1}{3.05}} = \left[1 + \frac{N \times 22}{290}\right]$$

$$\therefore \quad N = 8.4$$

Hence number of stages = 8

Stagnation temperature rise, ΔT_{0s}, per stage = 22K, as we took 8 stages, therefore

$$\Delta T_{0s} = \frac{22 \times 8.4}{8} = 23.1$$

From velocity triangle

$$\cos a_8 = \frac{C_{a8}}{C_8}$$

or

$$C_{a8} = 160 \times \cos 20 = 150.35\,\text{m/s}$$

Using degree of reaction, $\Lambda = 0.5$

Then,

$$0.5 = \frac{C_{a8}}{2U}\left(\tan\beta_8 + \tan\beta_9\right)$$

or

$$0.5 = \frac{150.35}{2U}\left(\tan\beta_8 + \tan\beta_9\right) \tag{A}$$

Also,

$$\Delta T_{0s} = \frac{\tau U C_{a8}}{C_p}\left(\tan\beta_8 - \tan\beta_9\right)$$

Now, $\Delta T_{0s} = 22K$ for one stage.

As we took 8 stages, therefore;

$$\Delta T_{0s} = \frac{22 \times 8.4}{8} = 23.1\,\mathrm{K}$$

$$\therefore\ 23.1 = \frac{0.85 \times \mathrm{U} \times 150.35}{1005}\left(\tan\beta_8 - \tan 20\right) \tag{B}$$

Because of symmetry, $\alpha_8 = \beta_9 = 20°$

From Eq. (A)

$$U = 150.35\left(\tan\beta_8 + 0.364\right) \tag{C}$$

From Eq. (B)

$$U = \frac{181.66}{\tan\beta_8 - 0.364} \tag{D}$$

Comparing Eqs. (C) and (D), we have

$$150.35\left(\tan\beta_8 + 0.364\right) = \frac{181.66}{\left(\tan\beta_8 - 0.364\right)}$$

or

$$\left(\tan^2\beta_8 - 0.364^2\right) = \frac{181.66}{150.35} = 1.21$$

or

$$\tan^2\beta_8 = 1.21 + 0.1325 = 1.342$$

$$\therefore\ \tan\beta_8 = \sqrt{1.342} = 1.159$$

i.e., $\beta_8 = 49.20^{\circ}$

Substituting in Eq. (C)

$$\begin{aligned} U &= 150.35(\tan 49.20° + 0.364) \\ &= 228.9\,\mathrm{m/s} \end{aligned}$$

The rotational speed is given by

$$N = \frac{228.9}{2\pi \times 0.0925} = 393.69\text{rps}$$

In order to find the length of the last stage rotor blade at inlet to the stage, it is necessary to calculate stagnation temperature and pressure ratio of the last stage.

Stagnation temperature of last stage: Fig. 5.19

$$\begin{aligned} T_{o8} &= T_{01} + 7 \times T_{0s} \\ &= 290 + 7 \times 23.1 = 451.7\,\text{K} \end{aligned}$$

Pressure ratio of the first stage is:

$$R = \left[\frac{1 + 1 \times 23.1}{451.7}\right]^{3.05}$$

Now,

$$p_{08}/p_{09} = 1.1643$$

$$\frac{p_{09}}{p_{01}} = 4, \quad \text{and} \quad p_{09} = 4\text{bar}$$

$$p_{08} = \frac{4}{1.1643} = 3.44\text{bar}$$

and

$$T_{08} = T_8 + \frac{C_8^2}{2C_p}$$

or

$$\begin{aligned} T_8 &= T_{08} - \frac{C_8^2}{2C_p} \\ &= 451.7 - \frac{160^2}{2 \times 1005} \\ &= 438.96\,\text{K} \end{aligned}$$

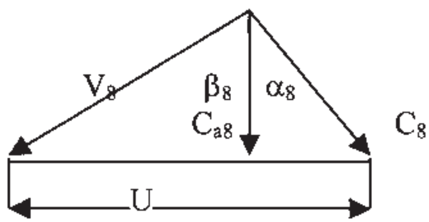


Figure 5.19 Velocity diagram of last stage.

Using stagnation and static isentropic temperature relationship for the last stage, we have

$$\frac{p_8}{p_{08}} = \left(\frac{T_8}{T_{08}}\right)^{1.4/0.4}$$

Therefore,

$$p_8 = 3.44\left(\frac{438.96}{451.7}\right)^{3.5} = 3.112\text{bar}$$

and

$$\rho_8 = \frac{p_8}{RT_8}$$

$$= \frac{3.112 \times 10^5}{287 \times 438.9} = 2.471\ \text{kg/m}^3$$

Using mass flow rate

$$\dot{m} = \rho_8 A_8 C_{a8}$$

or

$$3.5 = 2.471 \times A_8 \times 150.35$$

$$\therefore A_8 = 0.0094m^2$$

$$= 2\pi rh$$

or

$$h = \frac{0.0094}{2\pi \times 0.0925} = 0.0162m$$

i.e., length of the last stage rotor blade at inlet to the stage, $h = 16.17$ mm.

Design Example 5.13: A 10-stage axial flow compressor is designed for stagnation pressure ratio of 4.5:1. The overall isentropic efficiency of the compressor is 88% and stagnation temperature at inlet is 290K. Assume equal temperature rise in all stages, and work done factor is 0.87. Determine the air angles of a stage at the design radius where the blade speed is 218 m/s. Assume a constant axial velocity of 165 m/s, and the degree of reaction is 76%.

Solution:

No. of stages = 10
The overall stagnation temperature rise is:

$$T_0 = \frac{T_{01}\left(R^{\frac{\gamma-1}{\gamma}} - 1\right)}{\eta_c} = \frac{290\left(4.5^{0.286} - 1\right)}{0.88}$$

$$= 155.879\ \text{K}$$

The stagnation temperature rise of a stage

$$T_{0s} = \frac{155.879}{10} = 15.588\,\text{K}$$

The stagnation temperature rise in terms of air angles is:

$$T_{0s} = \frac{\tau U C_a}{C_p}(\tan\alpha_2 - \tan\alpha_1)$$

or

$$(\tan\alpha_2 - \tan\alpha_1) = \frac{T_{0s} \times C_p}{\tau U C_a} = \frac{15.588 \times 1005}{0.87 \times 218 \times 165}$$
$$= 0.501 \qquad \text{(A)}$$

From degree of reaction

$$\wedge = \left[1 - \frac{C_a}{2U}(\tan\alpha_2 + \tan\alpha_1)\right]$$

or

$$0.76 = \left[1 - \frac{165}{2 \times 218}(\tan\alpha_2 + \tan\alpha_1)\right]$$

$$\therefore (\tan\alpha_2 + \tan\alpha_1) = \frac{0.24 \times 2 \times 218}{165} = 0.634 \qquad \text{(B)}$$

Adding (A) and (B), we get

$2\tan\alpha_2 = 1.135$

or $\tan\alpha_2 = 0.5675$

i.e., $\alpha_2 = 29.57°$

and $\tan\alpha_1 = 0.634 - 0.5675 = 0.0665$

i.e., $\alpha_1 = 3.80°$

Similarly, for β_1 and β_2, degree of reaction

$\tan\beta_1 + \tan\beta_2 = 2.01$

and $\tan\beta_1 - \tan\beta_2 = 0.501$

$\therefore 2\tan\beta_1 = 2.511$

i.e., $\beta_1 = 51.46°$

and $\tan\beta_2 = 1.1256 - 0.501 = 0.755$

i.e., $\beta_2 = 37.03°$

Design Example 5.14: An axial flow compressor has a tip diameter of 0.9 m, hub diameter of 0.42 m, work done factor is 0.93, and runs at 5400 rpm. Angles of absolute velocities at inlet and exit are 28 and 58°, respectively and velocity diagram is symmetrical. Assume air density of 1.5 kg/m^3, calculate mass flow rate, work absorbed by the compressor, flow angles and degree of reaction at the hub for a free vortex design.

Solution:

Impeller speed is

$$U = \frac{2\pi rN}{60} = \frac{2\pi \times 0.45 \times 5400}{60} = 254.57 \text{ m/s}$$

From velocity triangle

$$U = C_a(\tan\alpha_1 + \tan\beta_1)$$

$$C_a = \frac{U}{\tan\alpha_1 + \tan\beta_1} = \frac{254.57}{(\tan 28° + \tan 58°)} = 119.47 \text{ m/s}$$

Flow area is

$$A = \pi\left[r_{\text{tip}} - r_{\text{root}}\right]$$
$$= \pi\left[0.45^2 - 0.42^2\right] = 0.0833 \text{ m}^2$$

Mass flow rate is

$$\dot{m} = \rho A C_a = 1.5 \times 0.0833 \times 119.47 = 14.928 \text{ kg/s}$$

Power absorbed by the compressor

$$= \tau U(C_{w2} - C_{w1})$$
$$= \tau U C_a(\tan\alpha_2 - \tan\alpha_1)$$
$$= 0.93 \times 254.57 \times 119.47(\tan 58° - \tan 28°)$$
$$= 30213.7 \text{ Nm}$$

$$\text{Total Power, } P = \frac{\dot{m} \times 30213.7}{1000} \text{ kW}$$
$$= 451 \text{ kW}$$

and whirl velocity at impeller tip $C_{wt} = C_a \tan\alpha_1 = 119.47 \times \tan 28° = 63.52$ m/s

Now using free vortex condition
$r\,C_w =$ constant
$\therefore r_h C_{w1h} = r_t C_{w1t}$ (where subscripts h for hub and t for tip)

or

$$C_{w1h} = \frac{r_t C_{w1t}}{r_h} = \frac{0.45 \times 63.52}{0.4} = 71.46\ \text{m/s}$$

Similarly

$$C_{w2t} = C_a \tan\alpha_2 = 119.47 \tan 58° = 191.2\ \text{m/s}$$

and

$$r_h C_{w2h} = r_t C_{w2t}$$

or

$$C_{w2h} = \frac{r_t C_{w2t}}{r_h} = \frac{0.45 \times 191.2}{0.4} = 215.09\ \text{m/s}$$

Therefore, the flow angles at the hub are

$$\tan\alpha_1 = \frac{C_{w1h}}{C_a} \quad (\text{where } C_a \text{ is constant})$$
$$= \frac{71.46}{119.47} = 0.598$$

i.e., $\alpha_1 = 30.88°$

$$\tan\beta_1 = \frac{U_h - C_a \tan\alpha_1}{C_a}$$

where U_h at the hub is given by

$$U_h = 2\pi r_h N = \frac{2 \times \pi \times 0.4 \times 5400}{60} = 226.29\ \text{m/s}$$

$$\therefore \tan\beta_1 = \frac{226.29 - 119.47 \tan 30.88°}{119.47}$$

i.e., $\beta_1 = 52.34°$

$$\tan\alpha_2 = \frac{C_{w2h}}{C_a} = \frac{215.09}{119.47} = 1.80$$

i.e., $\alpha_2 = 60.95°$
Similarly,

$$\tan\beta_2 = \frac{U_h - C_a \tan\alpha_2}{C_a} = \frac{226.29 - 119.47 \tan 60.95°}{119.47}$$

i.e., $\beta_2 = 5.36°$

Finally, the degree of reaction at the hub is

$$\wedge = \frac{C_a}{2U_h}(\tan\beta_1 + \tan\beta_2) = \frac{119.47}{2 \times 226.29}(\tan 52.34° + \tan 5.36°)$$

$$= 0.367 \text{ or } 36.7\%.$$

Design Example 5.15: An axial flow compressor is to deliver 22 kg of air per second at a speed of 8000 rpm. The stagnation temperature rise of the first stage is 20 K. The axial velocity is constant at 155 m/s, and work done factor is 0.94. The mean blade speed is 200 m/s, and reaction at the mean radius is 50%. The rotor blade aspect ratio is 3, inlet stagnation temperature and pressure are 290 K and 1.0 bar, respectively. Assume C_p for air as 1005 J/kg K and $\gamma = 1.4$. Determine:

1. The blade and air angles at the mean radius.
2. The mean radius.
3. The blade height.
4. The pitch and chord.

Solution:

1. Using Eq. (5.10) at the mean radius

$$T_{02} - T_{01} = \frac{\tau U C_a}{C_p}(\tan\beta_1 - \tan\beta_2)$$

$$20 = \frac{0.94 \times 200 \times 155}{1005}(\tan\beta_1 - \tan\beta_2)$$

$$(\tan\beta_1 - \tan\beta_2) = 0.6898$$

Using Eq. (5.12), the degree of reaction is

$$\wedge = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2)$$

or

$$(\tan\beta_1 + \tan\beta_2) = \frac{0.5 \times 2 \times 200}{155} = 1.29$$

Solving above two equations simultaneously

$$2\tan\beta_1 = 1.98$$

$$\therefore \beta_1 = 44.71° = \alpha_2 \text{ (as the diagram is symmetrical)}$$

$$\tan\beta_2 = 1.29 - \tan 44.71°$$

i.e.,

$$\beta_2 = 16.70° = \alpha_1$$

2. Let r_m be the mean radius

$$r_m = \frac{U}{2\pi N} = \frac{200 \times 60}{2\pi \times 8000} = 0.239\mathrm{m}$$

3. Using continuity equation in order to find the annulus area of flow

$$C_1 = \frac{C_a}{\cos\alpha_1} = \frac{155}{\cos 16.70°} = 162\,\mathrm{m/s}$$

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 290 - \frac{162^2}{2 \times 1005} = 276.94\,\mathrm{K}$$

Using isentropic relationship at inlet

$$\frac{p_1}{p_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

Static pressure is

$$p_1 = 1.0\left(\frac{276.94}{290}\right)^{3.5} = 0.851\mathrm{bars}$$

Density is

$$\rho_1 = \frac{p_1}{RT_1} = \frac{0.851 \times 100}{0.287 \times 276.94} = 1.07\,\mathrm{kg/m^3}$$

From the continuity equation,

$$A = \frac{22}{1.07 \times 155} = 0.133\mathrm{m}^2$$

Blade height is

$$h = \frac{A}{2\pi r_m} = \frac{0.133}{2 \times \pi \times 0.239} = 0.089\mathrm{m}.$$

4. At mean radius, and noting that blades β, an equivalent to cascade, α, nominal air deflection is

$$\varepsilon = \beta_1 - \beta_2$$
$$= 44.71° - 16.70° = 28.01°$$

Using Fig. 5.20 for cascade nominal deflection vs. air outlet angle, the solidity,

$$\frac{s}{c} = 0.5$$

$$\text{Blade aspect ratio} = \frac{\text{span}}{\text{chord}}$$

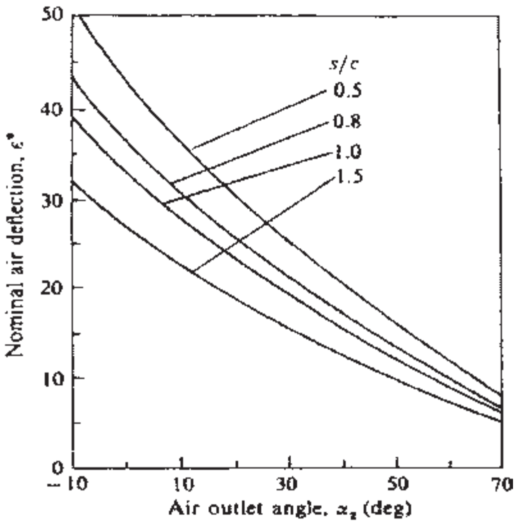


Figure 5.20 Cascade nominal deflection versus air outlet angle.

Blade chord,

$$C = \frac{0.089}{3} = 0.03\text{m}$$

Blade pitch,

$$s = 0.5 \times 0.03 = 0.015\,\text{m}.$$

PROBLEMS

5.1 An axial flow compressor has constant axial velocity throughout the compressor of 152 m/s, a mean blade speed of 162 m/s, and delivers 10.5 kg of air per second at a speed of 10,500 rpm. Each stage is of 50% reaction and the work done factor is 0.92. If the static temperature and pressure at the inlet to the first stage are 288K and 1 bar, respectively, and the stagnation stage temperature rise is 15K, calculate: 1 the mean diameter of the blade row, (2) the blade height, (3) the air exit angle from the rotating blades, and (4) the stagnation pressure ratio of the stage with stage efficiency 0.84.

(0.295 m, 0.062 m, 11.37°, 1.15)

5.2 The following design data apply to an axial flow compressor:

Stagnation temperature rise of the stage:	20 K
Work done factor:	0.90
Blade velocity at root:	155 m/s

Blade velocity at mean radius:	208 m/s
Blade velocity at tip:	255 m/s
Axial velocity (constant through the stage):	155 m/s
Degree of reaction at mean radius:	0.5

Calculate the inlet and outlet air and blade angles at the root, mean radius and tip for a free vortex design.
(18°, 45.5°, 14.84°, 54.07°, 39.71°, 39.18°, 23.56°, 29.42°, 53.75°, −20°)

5.3 Calculate the degree of reaction at the tip and root for the same data as Prob. 5.2.
(66.7%, 10%)

5.4 Calculate the air and blade angles at the root, mean and tip for 50% degree of reaction at all radii for the same data as in Prob. [5.2].
(47.86°, 28.37°, 43.98°, 1.72°)

5.5 Show that for vortex flow,

$$C_w \times r = \text{constant}$$

that is, the whirl velocity component of the flow varies inversely with the radius.

5.6 The inlet and outlet angles of an axial flow compressor rotor are 50 and 15°, respectively. The blades are symmetrical; mean blade speed and axial velocity remain constant throughout the compressor. If the mean blade speed is 200 m/s, work done factor is 0.86, pressure ratio is 4, inlet stagnation temperature is equal to 290 K, and polytropic efficiency of the compressor is 0.85, find the number of stages required.
(8 stages)

5.7 In an axial flow compressor air enters at 1 bar and 15°C. It is compressed through a pressure ratio of four. Find the actual work of compression and temperature at the outlet from the compressor. Take the isentropic efficiency of the compressor to be equal to 0.84
. (167.66 kJ/kg, 454.83 K)

5.8 Determine the number of stages required to drive the compressor for an axial flow compressor having the following data: difference between the tangents of the angles at outlet and inlet, i.e., $\tan\beta_1 - \tan\beta_2 = 0.55$. The isentropic efficiency of the stage is 0.84, the stagnation temperature at the compressor inlet is 288K, stagnation pressure at compressor inlet is 1 bar, the overall stagnation pressure rise is 3.5 bar, and the mass flow rate is 15 kg/s. Assume $C_p = 1.005$ kJ/kg K, $\gamma = 1.4$, $\lambda = 0.86$, $\eta_m = 0.99$
(10 stages, 287.5 kW)

5.9 From the data given below, calculate the power required to drive the compressor and stage air angles for an axial flow compressor.

Stagnation temperature at the inlet:	288 K
Overall pressure ratio:	4
Isentropic efficiency of the compressor:	0.88
Mean blade speed:	170 m/s
Axial velocity:	120 m/s
Degree of reaction:	0.5

(639.4 kW, $\beta_1 = 77.8°$, $\beta_2 = -72.69°$)

5.10 Calculate the number of stages from the data given below for an axial flow compressor:

Air stagnation temperature at the inlet:	288 K
Stage isentropic efficiency:	0.85
Degree of reaction:	0.5
Air angles at rotor inlet:	40°
Air angle at the rotor outlet:	10°
Meanblade speed:	180 m/s
Work done factor:	0.85
Overall pressure ratio:	6

(14 stages)

5.11 Derive the expression for polytropic efficiency of an axial flow compressor in terms of:

(a) n and γ
(b) inlet and exit stagnation temperatures and pressures.

5.12 Sketch the velocity diagrams for an axial flow compressor and derive the expression:

$$\frac{P_{02}}{P_{01}} = \left[1 + \frac{\eta_s \Delta T_{0s}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

5.13 Explain the term "degree of reaction". Why is the degree of reaction generally kept at 50%?

5.14 Derive an expression for the degree of reaction and show that for 50% reaction, the blades are symmetrical; i.e., $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$.

5.16 What is vortex theory? Derive an expression for vortex flow.

5.17 What is an airfoil? Define, with a sketch, the various terms used in airfoil geometry.

NOTATION

C	absolute velocity
C_{L}	lift coefficient
C_{p}	specific heat at constant pressure
D	drag
F_{x}	tangential force on moving blade
h	blade height, specific enthalpy
L	lift
N	number of stage, rpm
n	number of blades
R	overall pressure ratio, gas constant
R_{s}	stage pressure ratio
U	tangential velocity
V	relative velocity
α	angle with absolute velocity, measured from the axial direction
α_2^*	nominal air outlet angle
β	angle with relative velocity, measure from the axial direction
ΔT_{A}	static temperature rise in the rotor
ΔT_{B}	static temperature rise in the stator
$\Delta T_{0\mathrm{s}}$	stagnation temperature rise
ΔT_{s}	static temperature rise
Δ^*	nominal deviation
ϵ^*	nominal deflection
ϵ_{s}	stalling deflection
φ	flow coefficient
Λ	degree of reaction
λ	work done factor
ψ	stage loading factor

SUFFIXES

1	inlet to rotor
2	outlet from the rotor
3	inlet to second stage
a	axial, ambient
m	mean
r	radial, root
t	tip
w	whirl

6

Steam Turbines

6.1 INTRODUCTION

In a steam turbine, high-pressure steam from the boiler expands in a set of stationary blades or vanes (or nozzles). The high-velocity steam from the nozzles strikes the set of moving blades (or buckets). Here the kinetic energy of the steam is utilized to produce work on the turbine rotor. Low-pressure steam then exhausts to the condenser. There are two classical types of turbine stage designs: the impulse stage and the reaction stage.

Steam turbines can be noncondensing or condensing. In noncondensing turbines (or backpressure turbines), steam exhausts at a pressure greater than atmospheric. Steam then leaves the turbine and is utilized in other parts of the plant that use the heat of the steam for other processes. The backpressure turbines have very high efficiencies (range from 67% to 75%). A multi-stage condensing turbine is a turbine in which steam exhausts to a condenser and is condensed by air-cooled condensers. The exhaust pressure from the turbine is less than the atmospheric. In this turbine, cycle efficiency is low because a large part of the steam energy is lost in the condenser.

6.2 STEAM NOZZLES

The pressure and volume are related by the simple expression, $PV^{\gamma} =$ constant, for a perfect gas. Steam deviates from the laws of perfect gases. The *P-V* relationship is given by:

$$PV^{n} = \text{constant}$$

where:

$$n = 1.135 \text{ for saturated steam}$$

$$n = 1.3 \text{ for superheated steam}$$

For wet steam, the Zeuner relation,

$$n = \left(1.035 + \frac{x}{10}\right)$$

(where x is the initial dryness fraction of the steam) may be used.

All nozzles consist of an inlet section, a throat, and an exit. The velocity through a nozzle is a function of the pressure-differential across the nozzle. Consider a nozzle as shown in Fig. 6.1.

Assume that the flow occurs adiabatically under steady conditions. Since no work is transferred, the velocity of the fluid at the nozzle entry is usually very small and its kinetic energy is negligible compared with that at the outlet. Hence, the equation reduces to:

$$C_2 = \sqrt{\{2(h_1 - h_2)\}} \tag{6.1}$$

where h_1 and h_2 are the enthalpies at the inlet and outlet of the nozzle, respectively. As the outlet pressure decreases, the velocity increases. Eventually, a point is reached called the critical pressure ratio, where the velocity is equal to the velocity of sound in steam. Any further reduction in pressure will not produce any further increases in the velocity. The temperature, pressure, and density are called critical temperature, critical pressure, and critical

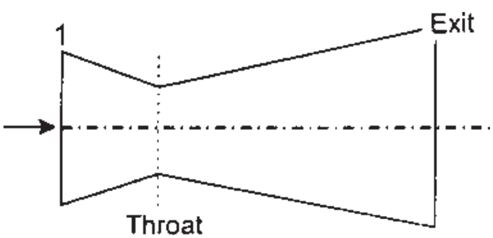


Figure 6.1 Nozzle.

density, respectively. The ratio between nozzle inlet temperature and critical temperature is given by:

$$\frac{T_1}{T_c} = \frac{2}{n+1} \tag{6.2}$$

where T_c is the critical temperature at which section $M = 1$. Assuming isentropic flow in the nozzle, the critical pressure ratio is:

$$\frac{P_1}{P_c} = \left(\frac{T_1}{T_c'}\right)^{\frac{n}{n-1}} \tag{6.3}$$

where T_c' is the temperature, which would have been reached after an isentropic expansion in the nozzle. The critical pressure ratio is approximately 0.55 for superheated steam. When the outlet pressure is designed to be higher than the critical pressure, a simple convergent nozzle may be used. In a convergent nozzle, shown in Fig. 6.2, the outlet cross-sectional area and the throat cross-sectional areas are equal. The operation of a convergent nozzle is not practical in high-pressure applications. In this case, steam tends to expand in all directions and is very turbulent. This will cause increased friction losses as the steam flows through the moving blades. To allow the steam to expand without turbulence, the convergent–divergent nozzle is used. In this type of nozzle, the area of the section from the throat to the exit gradually increases, as shown in Fig. 6.1.

The increase in area causes the steam to emerge in a uniform steady flow. The size of the throat and the length of the divergent section of every nozzle must be specifically designed for the pressure ratio for which the nozzle will be used. If a nozzle is designed to operate so that it is just choked, any other operating condition is an off-design condition. In this respect, the behavior of convergent and convergent–divergent nozzles is different. The temperature at the throat, i.e., the critical temperature, can be found from steam tables at the value of P_c and $s_c = s_1$. The critical velocity is given by the equation:

$$C_c = \sqrt{\{2(h_1 - h_c)\}} \tag{6.4}$$

where h_c is read from tables or the h–s chart at P_c and s_c.

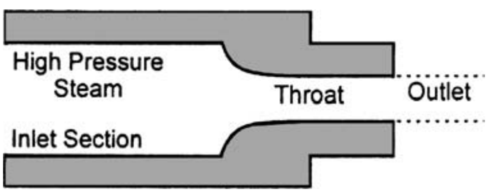


Figure 6.2 Convergent nozzle.

6.3 NOZZLE EFFICIENCY

The expansion process is irreversible due to friction between the fluid and walls of the nozzle, and friction within the fluid itself. However, it is still approximately adiabatic as shown in Fig. 6.3.

$1-2'$ is the isentropic enthalpy drop and $1-2$ is the actual enthalpy drop in the nozzle. Then the nozzle efficiency is defined as

$$\eta_{\mathrm{n}} = \frac{h_1 - h_2}{h_1 - h_2{}'}$$

6.4 THE REHEAT FACTOR

Consider a multi-stage turbine as shown by the Mollier diagram, Fig. 6.4.

The reheat factor is defined by:

$$\begin{aligned}
\text{R.F.} &= \frac{\text{Cumulative stage isentropic enthalpy drop}}{\text{Turbine isentropic enthalpy drop}} \\
&= \frac{\sum\left[\Delta h'\right]_{\text{stage}}}{[\Delta h']_{\text{turbine}}} \\
&= \frac{\left(h_1 - h_2'\right) + \left(h_2 - h_3'\right) + \left(h_3 - h_4'\right)}{\left(h_1 - h_4''\right)} \qquad (6.5)
\end{aligned}$$

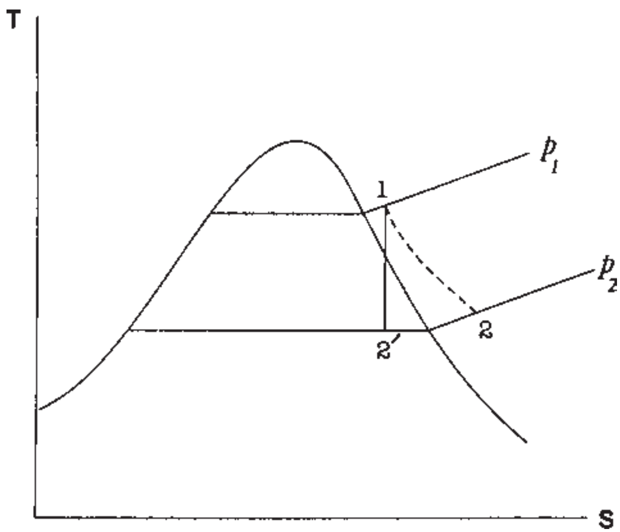


Figure 6.3 Nozzle expansion process for a vapor.

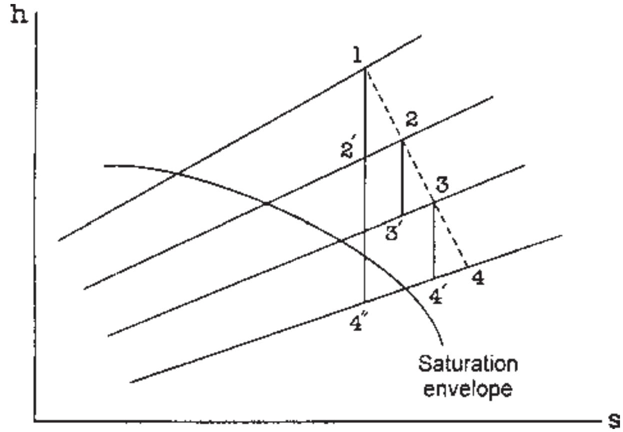


Figure 6.4 Mollier chart for a multi-stage turbine.

Since the isobars diverge, R.F. > 1.

The reheat factor may be used to relate the stage efficiency and the turbine efficiency.

Turbine isentropic efficiency is given by:

$$\eta_t = \frac{\Delta h}{\Delta h'} \tag{6.6}$$

where Δh is the actual enthalpy drop and $\Delta h'$ is the isentropic enthalpy drop. From diagram 6.4 it is clear that:

$$\Delta h = \sum [\Delta h]_{\text{stage}}$$

$$\Delta h_{1-4} = (h_1 - h_2) + (h_2 - h_3) + (h_3 - h_4)$$

if η_s (stage efficiency) is constant, then:

$$\eta_t = \frac{\sum \eta_s [\Delta h']_{\text{stage}}}{[\Delta h']_{\text{turbine}}} = \frac{\eta_s \sum [\Delta h']_{\text{stage}}}{[\Delta h']_{\text{turbine}}}$$

$$\text{or} \quad \eta_t = \eta_s \times (\text{R.F}). \tag{6.7}$$

Equation 6.7 indicates that the turbine efficiency is greater than the stage efficiency. The reheat factor is usually of the order of 1.03–1.04.

6.5 METASTABLE EQUILIBRIUM

As shown in Fig. 6.5, slightly superheated steam at point 1 is expanded in a convergent–divergent nozzle. Assume reversible and adiabatic processes. 1–2 is

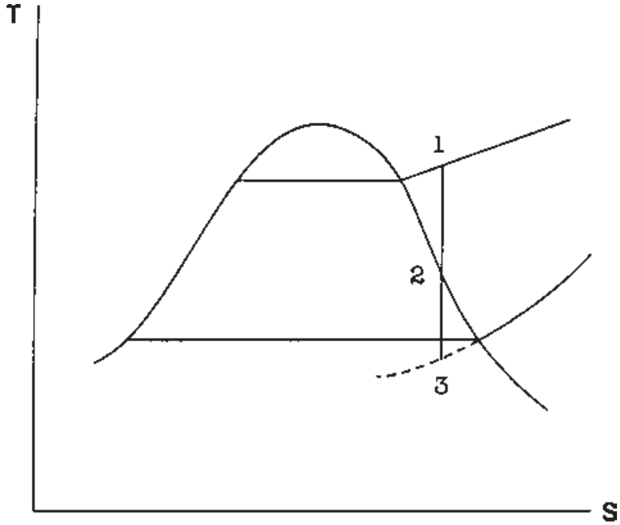


Figure 6.5 Phenomenon of supersaturation on T–S diagram.

the path followed by the steam and point 2 is on the saturated vapor line. Here, we can expect condensation to occur. But, if point 2 is reached in the divergent section of the nozzle, then condensation could not occur until point 3 is reached. At this point, condensation occurs very rapidly. Although the steam between points 2–3 is in the vapor state, the temperature is below the saturation temperature for the given pressure. This is known as the metastable state. In fact, the change of temperature and pressure in the nozzle is faster than the condensation process under such conditions. The condensation lags behind the expansion process. Steam does not condense at the saturation temperature corresponding to the pressure. Degree of undercooling is the difference between the saturation temperature corresponding to pressure at point 3 and the actual temperature of the superheated vapor at point 3. Degree of supersaturation is the actual pressure at point 3 divided by the saturation pressure corresponding to the actual temperature of the superheated vapor at point 3.

Illustrative Example 6.1: Dry saturated steam at 2 MPa enters a steam nozzle and leaves at 0.2 MPa. Find the exit velocity of the steam and dryness fraction. Assume isentropic expansion and neglect inlet velocity.

Solution:

From saturated steam tables, enthalpy of saturated vapor at 2 MPa:

$$h_1 = h_g = 2799.5\,\text{kJ/kg and entropy } s_1 = s_g = 6.3409\,\text{kJ/kg K}$$

Since the expansion is isentropic, $s_1 = s_2$: i.e., $s_1 = s_2 = 6.3409 = s_{f2} + x_2 s_{fg2}$, where x_2 is the dryness fraction after isentropic expansion, s_{f2} is the entropy of saturated liquid at 0.2 MPa, s_{fg2} is the entropy of vaporization at 0.2 MPa. Using tables:

$$x_2 = \frac{6.3409 - 1.5301}{5.5970} = 0.8595$$

Therefore,

$h_2 = h_{f2} + x_2 h_{fg2} = 504.7 + 0.8595 \times 2201.9 = 2397.233\ \text{kJ/kg}$

Using the energy equation:

$$C_2 = \sqrt{\{2(h_1 - h_2)\}}$$

$$= \sqrt{\{(2) \times (1000) \times (2799.5 - 2397.233)\}}$$

or: $C_2 = 897\ \text{m/s}$

Illustrative Example 6.2: Dry saturated steam is expanded in a nozzle from 1.3 MPa to 0.1 MPa. Assume friction loss in the nozzle is equal to 10% of the total enthalpy drop; calculate the mass of steam discharged when the nozzle exit diameter is 10 mm.

Solution:

Enthalpy of dry saturated steam at 1.3 MPa, using steam tables,

$$h_1 = 2787.6\ \text{kJ/kg}, \text{ and entropy } s_1 = 6.4953\ \text{kJ/kg K}.$$

Since the expansion process is isentropic, $s_1 = s_2 = s_{f2} + x_2 s_{fg2}$, hence dryness fraction after expansion:

$$x_2 = \frac{6.4953 - 1.3026}{6.0568} = 0.857$$

Now, the enthalpy at the exit:

$$h_2 = h_{f2} + x_2 h_{fg2} = 417.46 + (0.857) \times (2258)$$

$$= 2352.566\ \text{kJ/kg}$$

Therefore enthalpy drop from 1.3 MPa to 0.1 MPa

$$= h_1 - h_2 = 2787.6 - 2352.566 = 435.034\ \text{kJ/kg}$$

Actual enthalpy drop due to friction loss in the nozzle

$$= 0.90 \times 435.034 = 391.531\ \text{kJ/kg}$$

Hence, the velocity of steam at the nozzle exit:

$$C_2 = \sqrt{\{(2) \times (1000) \times (391.531)\}} = 884.908\ \text{m/s}$$

Specific volume of steam at 0.1 MPa:

$$= x_2 v_{g_2} = (0.857) \times (1.694) = 1.4517\,\mathrm{m^3/kg}$$

(since the volume of the liquid is usually negligible compared to the volume of dry saturated vapor, hence for most practical problems, $v = xv_g$)
Mass flow rate of steam at the nozzle exit:

$$= \frac{AC_2}{x_2 v_{g_2}} = \frac{(\pi) \times (0.01)^2 \times (884.908) \times (3600)}{(4) \times (1.4517)} = 172.42\,\mathrm{kg/h}.$$

Illustrative Example 6.3: Steam at 7.5 MPa and 500°C expands through an ideal nozzle to a pressure of 5 MPa. What exit area is required to accommodate a flow of 2.8 kg/s? Neglect initial velocity of steam and assume isentropic expansion in the nozzle.

Solution:

Initial conditions:

$$P_1 = 7.5\,\mathrm{MPa},\ 500^\circ\mathrm{C}$$
$$h_1 = 3404.3\,\mathrm{kJ/kg}$$
$$s_1 = 6.7598\,\mathrm{kJ/kg\ K}$$

(h_1 and s_1 from superheated steam tables)

At the exit state, $P_2 > P_c = (0.545) \times (7.5) = 4.0875\,\mathrm{MPa}$; and therefore the nozzle is convergent. State 2 is fixed by $P_2 = 5\,\mathrm{MPa}$, $s_1 = s_2 = 6.7598\,\mathrm{kJ/kg\ K}$
$T_2 = 435^\circ\mathrm{K}$, $v_2 = 0.06152\,\mathrm{m^3/kg}$, $h_2 = 3277.9\,\mathrm{kJ/kg}$ (from the superheated steam tables or the Mollier Chart).
The exit velocity:

$$C_2 = \sqrt{\{(2) \times (1000) \times (h_1 - h_2)\}}$$
$$= \sqrt{\{(2) \times (1000) \times (3404.3 - 3277.9)\}} = 502.8\,\mathrm{m/s}$$

Using the continuity equation, the exit area is

$$A_2 = \frac{mv_2}{C_2} = \frac{(2.8) \times (0.06152)}{502.8} = (3.42) \times (10^{-4})\,\mathrm{m^2}$$

Illustrative Example 6.4: Consider a convergent–divergent nozzle in which steam enters at 0.8 MPa and leaves the nozzle at 0.15 MPa. Assuming isentropic expansion and index $n = 1.135$, find the ratio of cross-sectional area, the area at the exit, and the area at the throat for choked conditions (i. e. , for maximum mass flow).

Solution:

Critical pressure for maximum mass flow is given by Fig. 6.6:

$$P_c = P_2 = P_1\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} = 0.8\left(\frac{2}{2.135}\right)^{8.41} = 0.462\,\text{MPa}$$

From the Mollier chart:

$h_1 = 2769\,\text{kJ/kg}$
$h_2 = 2659\,\text{kJ/kg}$
$h_3 = 2452\,\text{kJ/kg}$

Enthalpy drop from 0.8 MPa to 0.15 MPa:

$$\Delta h_{1-3} = h_1 - h_3 = 2769 - 2452 = 317\,\text{kJ/kg}$$

Enthalpy drop from 0.8 MPa to 0.462 MPa:

$$\Delta h_{1-2} = h_1 - h_2 = 2769 - 2659 = 110\,\text{kJ/kg}$$

Dryness fraction: $x_2 = 0.954$
Dryness fraction: $x_3 = 0.902$

The velocity at the exit,

$$C_3 = \sqrt{\{(2)\times(1000)\times(\Delta h_{1-3})\}}$$
$$= \sqrt{\{(2)\times(1000)\times(317)\}} = 796\text{m/s}$$

The velocity at the throat

$$C_2 = \sqrt{\{(2)\times(1000)\times(\Delta h_{1-2})\}} = \sqrt{\{(2)\times(1000)\times(110)\}}$$
$$= 469\text{m/s}$$

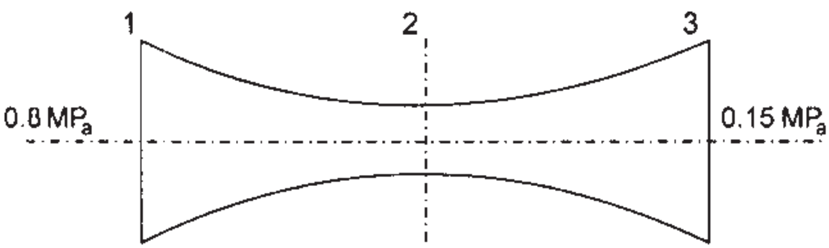


Figure 6.6 Convergent–divergent nozzle.

Mass discharged at the throat:

$$\dot{m}_2 = \frac{A_2 C_2}{x_2 v_{g_2}}$$

Mass discharged at the exit

$$\dot{m}_3 = \frac{A_3 C_3}{x_3 v_{g_3}}$$

Therefore

$$\frac{A_3 C_3}{x_3 v_{g_3}} = \frac{A_2 C_2}{x_2 v_{g_2}}$$

Hence,

$$\frac{A_3}{A_2} = \left[\frac{C_2}{C_3}\right]\left[\frac{x_3 v_{g3}}{x_2 v_{g2}}\right] = \left[\frac{469}{796}\right]\left[\frac{(0.902)(1.1593)}{(0.954)(0.4038)}\right] = 1.599$$

Illustrative Example 6.5: Dry saturated steam enters the convergent–divergent nozzle and leaves the nozzle at 0.1 MPa; the dryness fraction at the exit is 0.85. Find the supply pressure of steam. Assume isentropic expansion (see Fig. 6.7).

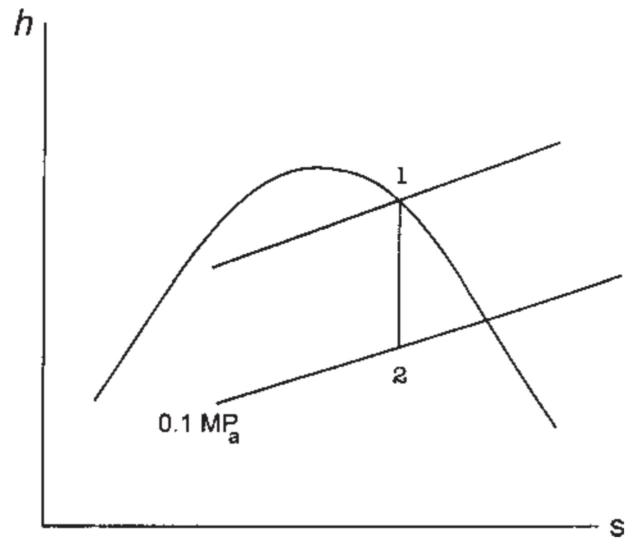


Figure 6.7 *h–s* diagram for Example 6.5.

Solution:

At the state point 2, the dryness fraction is 0.85 and the pressure is 0.1 MPa. This problem can be solved easily by the Mollier chart or by calculations. Enthalpy and entropy may be determined using the following equations:

$$h_2 = h_{f2} + x_2 h_{fg2} \text{ and } s_2 = s_{f2} + x_2 s_{fg2},$$

i.e.: $h_2 = 417.46 + (0.85) \times (2258) = 2336.76\,\text{kJ/kg}$

and $s_2 = 1.3026 + (0.85) \times (6.0568) = 6.451\,\text{kJ/kg K}$

Since $s_1 = s_2$, the state 1 is fixed by $s_1 = 6.451\,\text{kJ/kg K}$, and point 1 is at the dry saturated line. Therefore pressure P_1 may be determined by the Mollier chart or by calculations: i.e.: $P_1 = 1.474\,\text{MPa}$.

6.6 STAGE DESIGN

A turbine stage is defined as a set of stationary blades (or nozzles) followed by a set of moving blades (or buckets or rotor). Together, the two sets of blades allow the steam to perform work on the turbine rotor. This work is then transmitted to the driven load by the shaft on which the rotor assembly is carried. Two turbine stage designs in use are: the impulse stage and reaction stage. The first turbine, designated by DeLaval in 1889, was a single-stage impulse turbine, which ran at 30,000 rpm. Because of its high speed, this type of turbine has very limited applications in practice. High speeds are extremely undesirable due to high blade tip stresses and large losses due to disc friction, which cannot be avoided. In large power plants, the single-stage impulse turbine is ruled out, since alternators usually run speeds around 3000 rpm. Photographs of actual steam turbines are reproduced in Figs. 6.8–6.10.

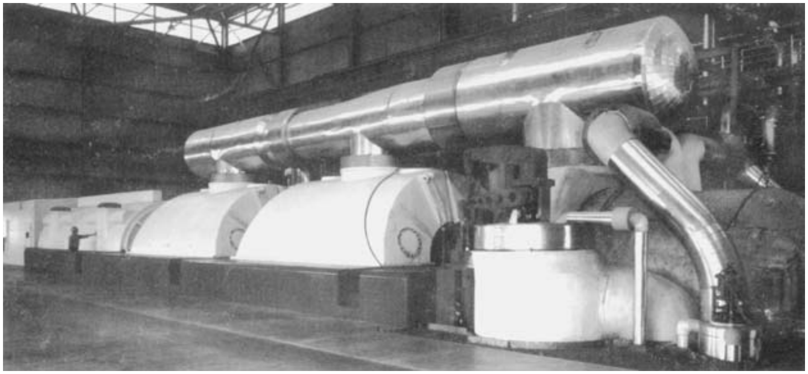


Figure 6.8 Steam turbine.

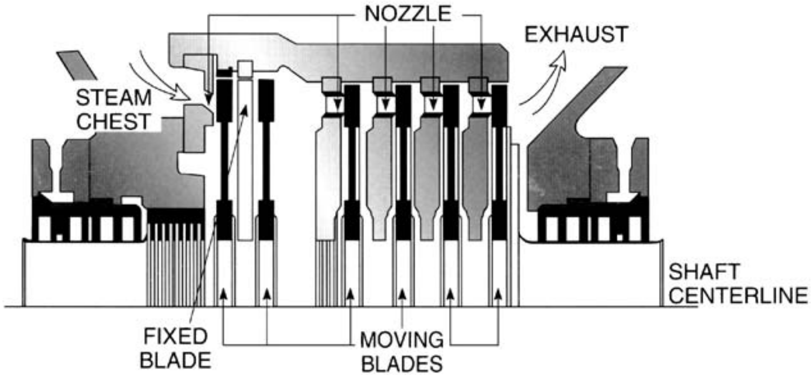


Figure 6.9 Pressure velocity-compounded impulse turbine.

6.7 IMPULSE STAGE

In the impulse stage, the total pressure drop occurs across the stationary blades (or nozzles). This pressure drop increases the velocity of the steam. However, in the reaction stage, the total pressure drop is divided equally across the stationary blades and the moving blades. The pressure drop again results in a corresponding increase in the velocity of the steam flow.

As shown in Figs. 6.10 and 6.11, the shape of the stationary blades or nozzles in both stage designs is very similar. However, a big difference exists in the shapes of the moving blades. In an impulse stage, the shape of the moving blades or buckets is like a cup. The shape of the moving blades in a reaction stage is more like that of an airfoil. These blades look very similar to the stationary blades or nozzles.

6.8 THE IMPULSE STEAM TURBINE

Most of the steam turbine plants use impulse steam turbines, whereas gas turbine plants seldom do. The general principles are the same whether steam or gas is the working substance.

As shown in Fig. 6.12, the steam supplied to a single-wheel impulse turbine expands completely in the nozzles and leaves with absolute velocity C_1 at an angle α_1, and by subtracting the blade velocity vector U, the relative velocity vector at entry to the rotor V_1 can be determined. The relative velocity V_1 makes an angle of β_1 with respect to U. The increase in value of α_1 decreases the value of the useful component, $C_1 \cos \alpha_1$ and increases the value of the axial or flow component $C_a \sin \alpha_1$. The two points of particular interest are the inlet and exit of the blades. As shown in Fig. 6.12, these velocities are V_1 and V_2, respectively.

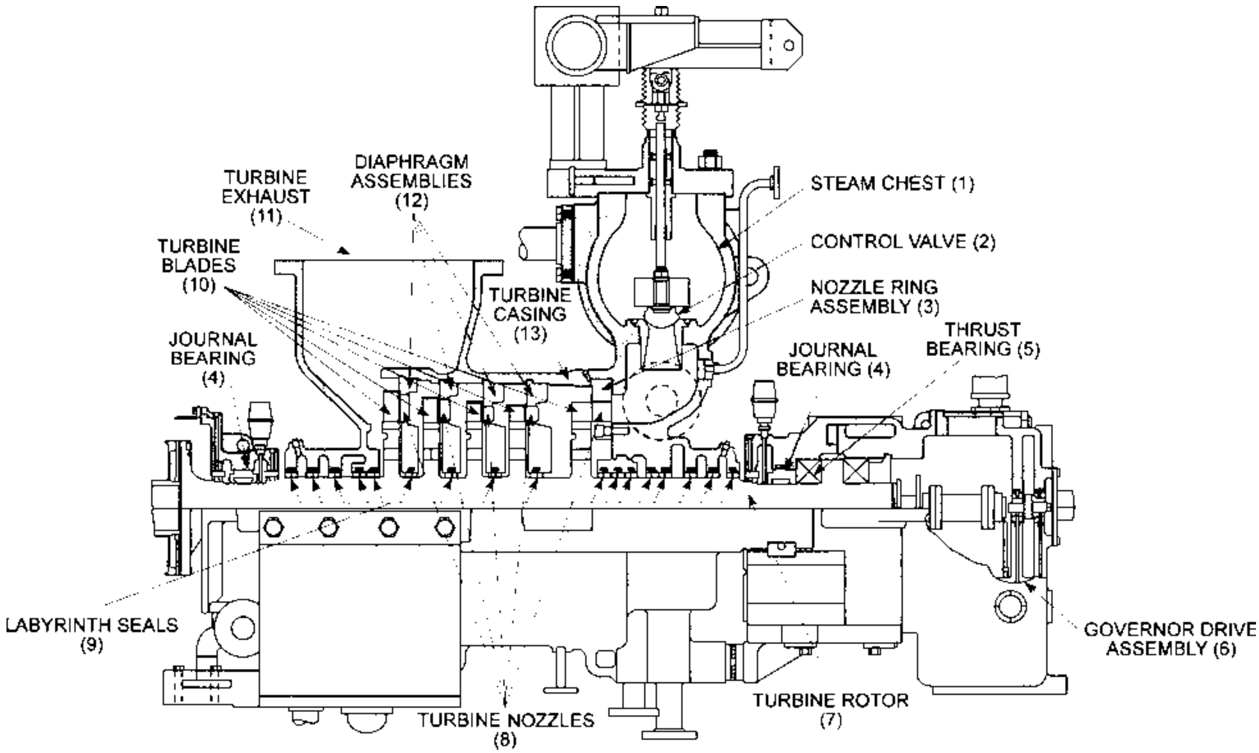


Figure 6.10 Steam turbine cross-sectional view.

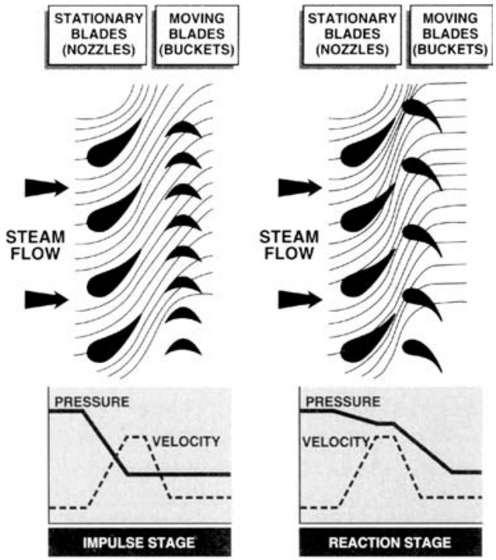


Figure 6.11 Impulse and reaction stage design.

Vectorially subtracting the blade speed results in absolute velocity C_2. The steam leaves tangentially at an angle β_2 with relative velocity V_2. Since the two velocity triangles have the same common side U, these triangles can be combined to give a single diagram as shown in Fig. 6.13.

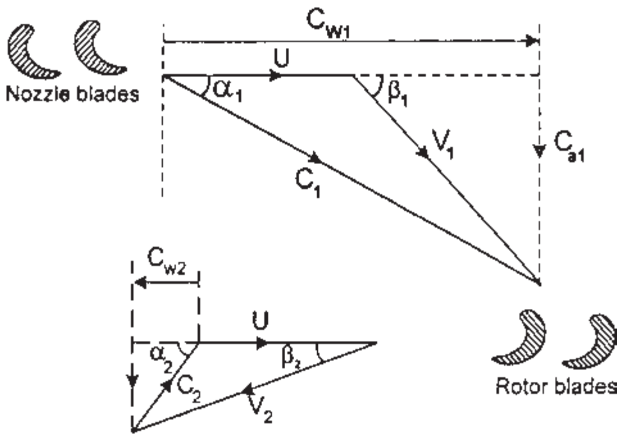


Figure 6.12 Velocity triangles for turbine stage.

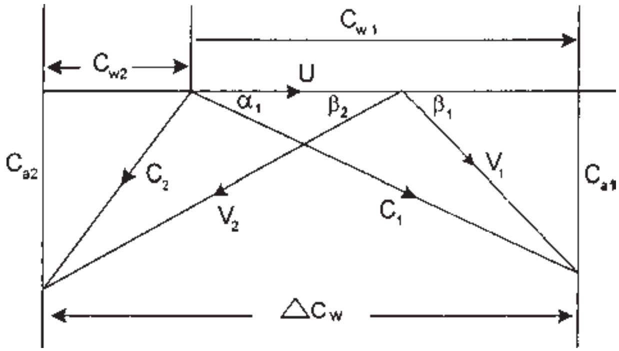


Figure 6.13 Combined velocity diagram.

If the blade is symmetrical then $\beta_1 = \beta_2$ and neglecting the friction effects of blades on the steam, $V_1 = V_2$. In the actual case, the relative velocity is reduced by friction and expressed by a blade velocity coefficient k. That is:

$$k = \frac{V_2}{V_1}$$

From Euler's equation, work done by the steam is given by:

$$W_t = U(C_{w1} + C_{w2})$$

Since C_{w2} is in the negative r direction, the work done per unit mass flow is given by:

$$W_t = U(C_{w1} + C_{w2}) \tag{6.9}$$

If $C_{a1} \neq C_{a2}$, there will be an axial thrust in the flow direction. Assume that C_a is constant. Then:

$$W_t = UC_a(\tan\alpha_1 + \tan\alpha_2) \tag{6.10}$$

$$W_t = UC_a(\tan\beta_1 + \tan\beta_2) \tag{6.11}$$

Equation (6.11) is often referred to as the diagram work per unit mass flow and hence the diagram efficiency is defined as:

$$\eta_d = \frac{\text{Diagram work done per unit mass flow}}{\text{Work available per unit mass flow}} \tag{6.12}$$

Referring to the combined diagram of Fig. 6.13: ΔC_w is the change in the velocity of whirl. Therefore:

$$\text{The driving force on the wheel} = \dot{m}C_w \tag{6.13}$$

The product of the driving force and the blade velocity gives the rate at which work is done on the wheel. From Eq. (6.13):

$$\text{Power output} = \dot{m}U\Delta C_w \tag{6.14}$$

If $C_{a1} - C_{a2} = \Delta C_a$, the axial thrust is given by:

$$\text{Axial thrust} : F_a = \dot{m}\Delta C_a \tag{6.15}$$

The maximum velocity of the steam striking the blades

$$C_1 = \sqrt{\{2(h_0 - h_1)\}} \tag{6.16}$$

where h_0 is the enthalpy at the entry to the nozzle and h_1 is the enthalpy at the nozzle exit, neglecting the velocity at the inlet to the nozzle. The energy supplied to the blades is the kinetic energy of the jet, $C_1^2/2$ and the blading efficiency or diagram efficiency:

$$\eta_d = \frac{\text{Rate of work performed per unit mass flow}}{\text{Energy supplied per unit mass of steam}}$$

$$\eta_d = (U\Delta C_w) \times \frac{2}{C_1^2} = \frac{2U\Delta C_w}{C_1^2} \tag{6.17}$$

Using the blade velocity coefficient $\left(k = \frac{V_2}{V_1}\right)$ and symmetrical blades (i.e., $\beta_1 = \beta_2$), then:

$$\Delta C_w = 2V_1 \cos \alpha_1 - U$$

Hence

$$\Delta C_w = 2(C_1 \cos \alpha_1 - U) \tag{6.18}$$

And the rate of work performed per unit mass $= 2(C_1 \cos \alpha_1 - U)U$

Therefore:

$$\eta_d = 2(C_1 \cos \alpha_1 - U)U \times \frac{2}{C_1^2}$$

$$\eta_d = \frac{4(C_1 \cos \alpha_1 - U)U}{C_1^2}$$

$$\eta_d = \frac{4U}{C_1}\left(\cos \alpha_1 - \frac{U}{C_1}\right) \tag{6.19}$$

where $\frac{U}{C_1}$ is called the blade speed ratio.

Differentiating Eq. (6.19) and equating it to zero provides the maximum diagram efficiency:

$$\frac{d(\eta_d)}{d\left(\frac{U}{C_1}\right)} = 4\cos \alpha_1 - \frac{8U}{C_1} = 0$$

or

$$\frac{U}{C_1} = \frac{\cos\alpha_1}{2} \tag{6.20}$$

i.e., maximum diagram efficiency

$$= \frac{4\cos\alpha_1}{2}\left(\cos\alpha_1 - \frac{\cos\alpha_1}{2}\right)$$

or:

$$\eta_d = \cos^2\alpha_1 \tag{6.21}$$

Substituting this value into Eq. (6.14), the power output per unit mass flow rate at the maximum diagram efficiency:

$$P = 2U^2 \tag{6.22}$$

6.9 PRESSURE COMPOUNDING (THE RATEAU TURBINE)

A Rateau-stage impulse turbine uses one row of nozzles and one row of moving blades mounted on a wheel or rotor, as shown in Fig. 6.14. The total pressure drop is divided in a series of small increments over the stages. In each stage, which consists of a nozzle and a moving blade, the steam is expanded and the kinetic energy is used in moving the rotor and useful work is obtained.

The separating walls, which carry the nozzles, are known as diaphragms. Each diaphragm and the disc onto which the diaphragm discharges its steam is known as a stage of the turbine, and the combination of stages forms a pressure compounded turbine. Rateau-stage turbines are unable to extract a large amount of energy from the steam and, therefore, have a low efficiency. Although the Rateau turbine is inefficient, its simplicity of design and construction makes it well suited for small auxiliary turbines.

6.10 VELOCITY COMPOUNDING (THE CURTIS TURBINE)

In this type of turbine, the whole of the pressure drop occurs in a single nozzle, and the steam passes through a series of blades attached to a single wheel or rotor. The Curtis stage impulse turbine is shown in Fig. 6.15.

Fixed blades between the rows of moving blades redirect the steam flow into the next row of moving blades. Because the reduction of velocity occurs over two stages for the same pressure decreases, a Curtis-stage turbine can extract

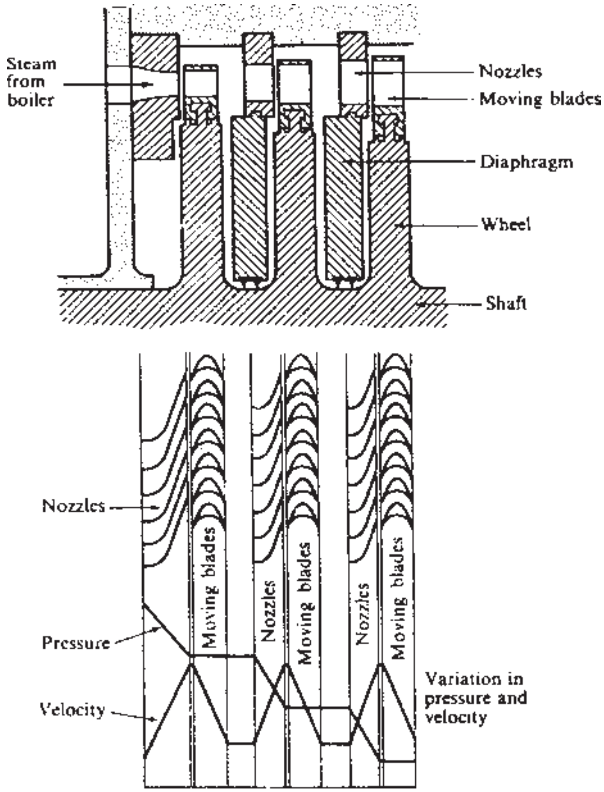


Figure 6.14 Rateau-stage impulse turbine.

more energy from the steam than a Rateau-stage turbine. As a result, a Curtis-stage turbine has a higher efficiency than a Rateau-stage turbine.

6.11 AXIAL FLOW STEAM TURBINES

Sir Charles Parsons invented the reaction steam turbine. The reaction turbine stage consists of a fixed row of blades and an equal number of moving blades fixed on a wheel. In this turbine pressure drop or expansion takes place both in the fixed blades (or nozzles) as well as in the moving blades. Because the pressure drop from inlet to exhaust is divided into many steps through use of alternate rows of fixed and moving blades, reaction turbines that have more than one stage are classified as pressure-compounded turbines. In a reaction turbine, a reactive force is produced on the moving blades when the steam increases in velocity and when the steam changes direction. Reaction turbines are normally used as

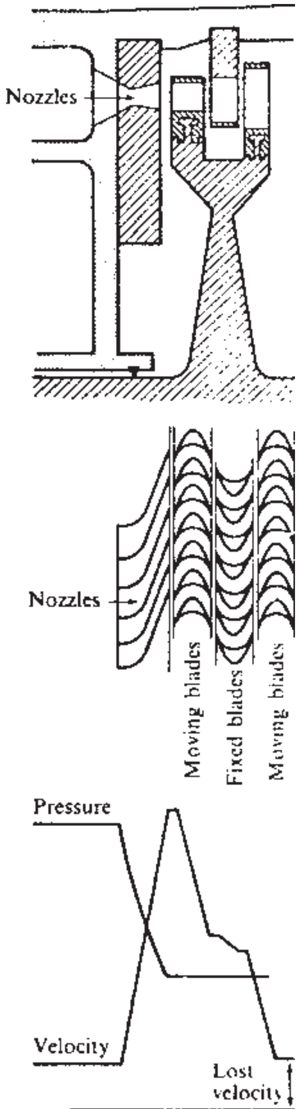


Figure 6.15 The Curtis-stage impulse turbine.

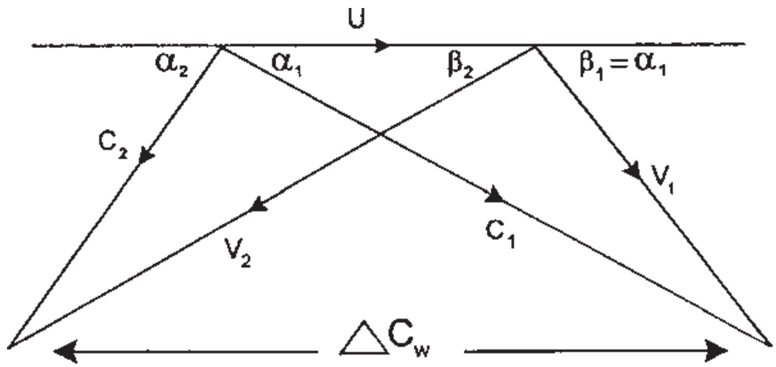


Figure 6.16 Velocity triangles for 50% reaction design.

low-pressure turbines. High-pressure reaction turbines are very costly because they must be constructed from heavy and expensive materials. For a 50% reaction, the fixed and moving blades have the same shape and, therefore, the velocity diagram is symmetrical as shown in Fig. 6.16.

6.12 DEGREE OF REACTION

The degree of reaction or reaction ratio (Λ) is a parameter that describes the relation between the energy transfer due to static pressure change and the energy transfer due to dynamic pressure change. The degree of reaction is defined as the ratio of the static pressure drop in the rotor to the static pressure drop in the stage. It is also defined as the ratio of the static enthalpy drop in the rotor to the static enthalpy drop in the stage. If h_0, h_1, and h_2 are the enthalpies at the inlet due to the fixed blades, at the entry to the moving blades and at the exit from the moving blades, respectively, then:

$$\Lambda = \frac{h_1 - h_2}{h_0 - h_2} \tag{6.23}$$

The static enthalpy at the inlet to the fixed blades in terms of stagnation enthalpy and velocity at the inlet to the fixed blades is given by

$$h_0 = h_{00} - \frac{C_0^2}{2C_\mathrm{p}}$$

Similarly,

$$h_2 = h_{02} - \frac{C_2^2}{2C_\mathrm{p}}$$

Substituting,

$$\Lambda = \frac{(h_1 - h_2)}{\left(h_{00} - \frac{C_0^2}{2C_p}\right) - \left(h_{02} - \frac{C_2^2}{2C_p}\right)}$$

But for a normal stage, $C_0 = C_2$ and since $h_{00} = h_{01}$ in the nozzle, then:

$$\Lambda = \frac{h_1 - h_2}{h_{01} - h_{02}} \tag{6.24}$$

We know that $h_{01\mathrm{Re}1} = h_{02\mathrm{Re}2}$. Then:

$$h_{01\mathrm{Re}1} - h_{02\mathrm{Re}2} = (h_1 - h_2) + \frac{\left(V_1^2 - V_2^2\right)}{2} = 0$$

Substituting for $(h_1 - h_2)$ in Eq. (6.24):

$$\Lambda = \frac{\left(V_2^2 - V_1^2\right)}{[2(h_{01} - h_{02})]}$$

$$\Lambda = \frac{\left(V_2^2 - V_1^2\right)}{[2U(C_{w1} + C_{w2})]} \tag{6.25}$$

Assuming the axial velocity is constant through the stage, then:

$$\Lambda = \frac{\left(V_{w2}^2 - V_{w1}^2\right)}{[2U(U + V_{w1} + V_{w2} - U)]}$$

$$\Lambda = \frac{(V_{w2} - V_{w1})(V_{w2} + V_{w1})}{[2U(V_{w1} + V_{w2})]}$$

$$\Lambda = \frac{C_a\left(\tan\beta_2 - \tan\beta_1\right)}{2U} \tag{6.26}$$

From the velocity triangles, it is seen that

$$C_{w1} = U + V_{w1}, \text{ and } C_{w2} = V_{w2} - U$$

Therefore, Eq. (6.26) can be arranged into a second form:

$$\Lambda = \frac{1}{2} + \frac{C_a}{2U}\left(\tan\beta_2 - \tan\alpha_2\right) \tag{6.27}$$

Putting $\Lambda = 0$ in Eq. (6.26), we get

$$\beta_2 = \beta_1 \text{ and } V_1 = V_2, \text{ and for } \Lambda = 0.5, \beta_2 = \alpha_1.$$

Zero Reaction Stage:

Let us first discuss the special case of zero reaction. According to the definition of reaction, when $\Lambda = 0$, Eq. (6.23) reveals that $h_1 = h_2$ and Eq. (6.26) that $\beta_1 = \beta_2$. The Mollier diagram and velocity triangles for $\Lambda = 0$ are shown in Figs. 6.17 and 6.18:

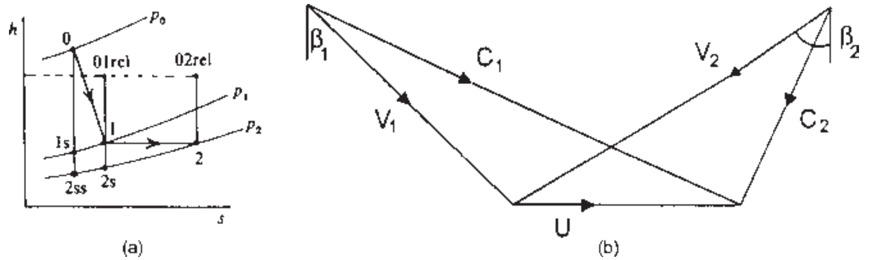


Figure 6.17 Zero reaction (a) Mollier diagram and (b) velocity diagram.

Now, $h_{01r01} = h_{02r01}$ and $h_1 = h_2$ for $\Lambda = 0$. Then, $V_1 = V_2$. In the ideal case, there is no pressure drop in the rotor, and points 1, 2 and 2s on the Mollier chart should coincide. But due to irreversibility, there is a pressure drop through the rotor. The zero reaction in the impulse stage, by definition, means there is no pressure drop through the rotor. The Mollier diagram for an impulse stage is shown in Fig. 6.18, where it can be observed that the enthalpy increases through the rotor.

From Eq. (6.23), it is clear that the reaction is negative for the impulse turbine stage when irreversibility is taken into account.

Fifty-Percent Reaction Stage

From Eq. (6.23), Fig. (6.19) for $\Lambda = 0.5$, $\alpha_1 = \beta_2$, and the velocity diagram is symmetrical. Because of symmetry, it is also clear that $\alpha_2 = \beta_1$. For $\Lambda = 1/2$, the enthalpy drop in the nozzle row equals the enthalpy drop in the rotor. That is:

$$h_0 - h_1 = h_1 - h_2$$

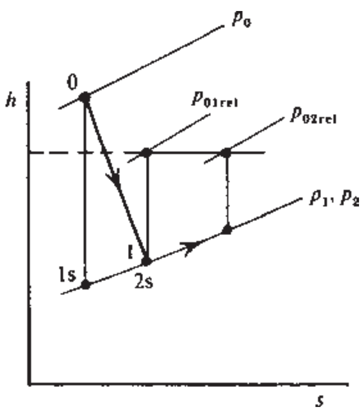


Figure 6.18 Mollier diagram for an impulse stage.

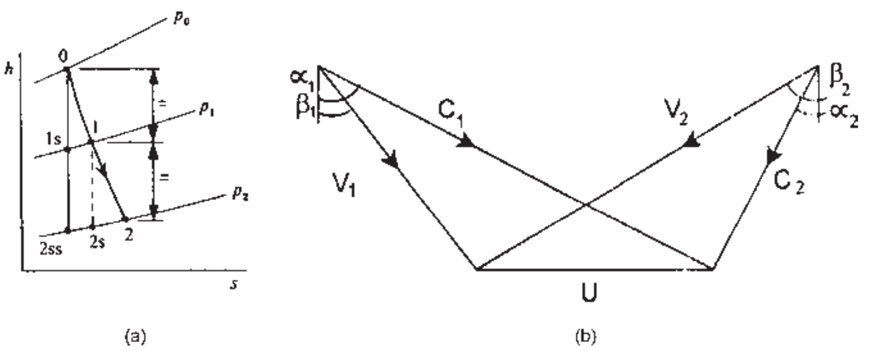


Figure 6.19 A 50% reaction stage (a) Mollier diagram and (b) velocity diagram.

Substituting $\beta_2 = \tan\alpha_2 + \dfrac{U}{C_a}$ into Eq. (6.27) gives

$$\Lambda = 1 + \frac{C_a}{2U}(\tan\alpha_2 - \tan\alpha_1) \tag{6.28}$$

Thus, when $\alpha_2 = \alpha_1$, the reaction is unity (also $C_1 = C_2$). The velocity diagram for $\Lambda = 1$ is shown in Fig. 6.20 with the same value of C_a, U, and W used for $\Lambda = 0$ and $\Lambda = \frac{1}{2}$. It is obvious that if Λ exceeds unity, then $C_1 < C_0$ (i.e., nozzle flow diffusion).

Choice of Reaction and Effect on Efficiency:

Eq. (6.24) can be rewritten as:

$$\Lambda = 1 + \frac{C_{w2} - C_{w1}}{2U}.$$

C_{w2} can be eliminated by using this equation:

$$C_{w2} = \frac{W}{U} - C_{w1},$$

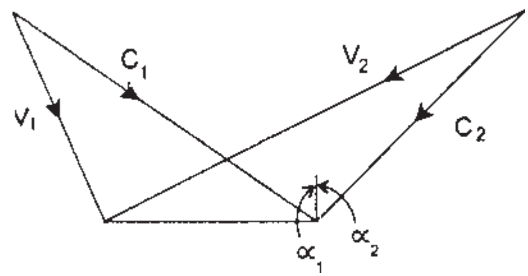


Figure 6.20 Velocity diagram for 100% reaction turbine stage.

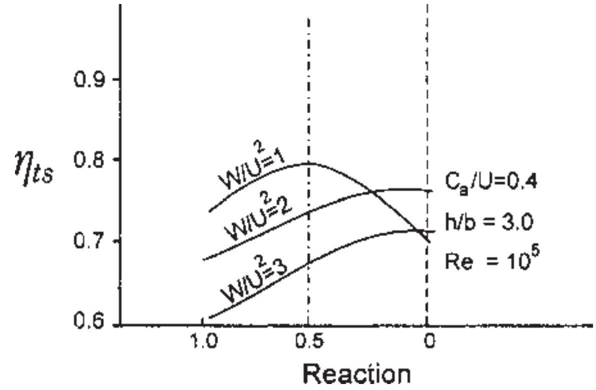


Figure 6.21 Influence of reaction on total-to-static efficiency with fixed values of stage-loading factor.

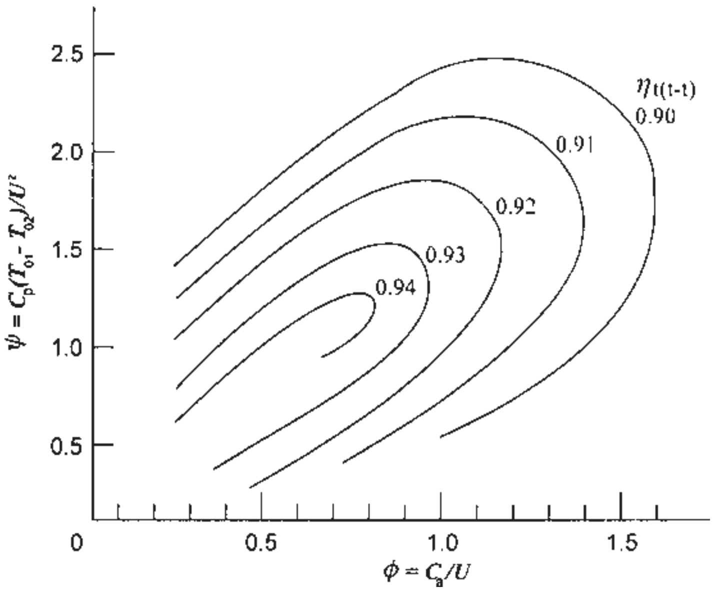


Figure 6.22 Blade loading coefficient vs. flow coefficient.

yielding:

$$\Lambda = 1 + \frac{W}{2U^2} - \frac{C_{w1}}{U} \tag{6.29}$$

In Fig. 6.21 the total-to-static efficiencies are shown plotted against the degree of reaction.

When $\frac{W}{U^2} = 2$, η_{ts} is maximum at $\Lambda = 0$. With higher loading, the optimum η_{ts} is obtained with higher reaction ratios. As shown in Fig. 6.22 for a high total-to-total efficiency, the blade-loading factor should be as small as possible, which implies the highest possible value of blade speed is consistent with blade stress limitations. It means that the total-to-static efficiency is heavily dependent upon the reaction ratio and η_{ts} can be optimized by choosing a suitable value of reaction.

6.13 BLADE HEIGHT IN AXIAL FLOW MACHINES

The continuity equation, $\dot{m} = \rho AC$, may be used to find the blade height h. The annular area of flow $= \pi Dh$. Thus, the mass flow rate through an axial flow compressor or turbine is:

$$\dot{m} = \rho \pi D h C_a \tag{6.30}$$

Blade height will increase in the direction of flow in a turbine and decrease in the direction of flow in a compressor.

Illustrative Example 6.6: The velocity of steam leaving a nozzle is 925 m/s and the nozzle angle is 20°. The blade speed is 250 m/s. The mass flow through the turbine nozzles and blading is 0.182 kg/s and the blade velocity coefficient is 0.7. Calculate the following:

1. Velocity of whirl.
2. Tangential force on blades.
3. Axial force on blades.
4. Work done on blades.
5. Efficiency of blading.
6. Inlet angle of blades for shockless inflow of steam.

Assume that the inlet and outlet blade angles are equal.

Solution:

From the data given, the velocity diagram can be constructed as shown in Fig. 6.23. The problem can be solved either graphically or by calculation.

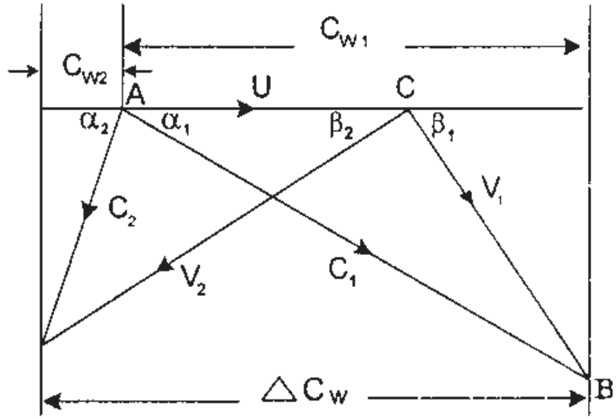


Figure 6.23 Velocity triangles for Example 6.6.

Applying the cosine rule to the $\triangle$ABC,

$$V_1^2 = U^2 + C_1^2 - 2UC_1\cos\alpha_1$$
$$= 250^2 + 925^2 - (2) \times (250) \times (925) \times \cos 20°$$

so: $V_1 = 695.35$ m/s
But,

$$k = \frac{V_2}{V_1}, \text{or} V_2 = (0.70) \times (695.35) = 487 \text{m/s}.$$

Velocity of whirl at inlet:

$$C_{w1} = C_1 cos\alpha_1 = 925\cos 20° = 869.22 \text{m/s}$$

Axial component at inlet:

$$C_{a1} = \text{BD} = C_1 \sin\alpha_1 = 925\sin 20° = 316.37 \text{m/s}$$

Blade angle at inlet:

$$\tan\beta_1 = \frac{C_{a1}}{C_{w1} - U} = \frac{316.37}{619.22} = 0.511$$

Therefore, $\beta_1 = 27.06° = \beta_2 =$ outlet blade angle.

$$\cos\beta_2 = \frac{C_{w2} + U}{V_2},$$

or: $$C_{w2} = V_2 \cos\beta_2 - U = 487 \times \cos 27.06° - 250$$
$$= 433.69 - 250 = 183.69 \text{ m/s}$$

and: $C_{a2} = \text{FE} = (U + C_{w2}) \tan\beta_2 = 433.69 \tan 27.06° = 221.548$ m/s

1. Velocity of whirl at inlet, $C_{w1} = 869.22$ m/s;
   Velocity of whirl at outlet, $C_{w2} = 183.69$ m/s
2. Tangential force on blades
   $$= \dot{m}\,(C_{w1} + C_{w2}) = (0.182)\,(1052.9) = 191.63\,\text{N}.$$
3. Axial force on blades
   $$= \dot{m}(C_{a1} - C_{a2}) = (0.182)(316.37 - 221.548) = 17.26\,\text{N}$$
4. Work done on blades
   $$= \text{tangential force on blades} \times \text{blade velocity}$$
   $$= (191.63) \times (250)/1000 = 47.91\,\text{kW}.$$
5. Efficiency of blading
   $$= \frac{\text{Work done on blades}}{\text{Kinetic energy supplied}}$$
   $$= \frac{47.91}{\frac{1}{2} m C_1^2} = \frac{(47.91)(2)(10^3)}{(0.182)(925^2)}$$
   $$= 0.6153 \text{ or } 61.53\%$$
6. Inlet angle of blades $\beta_1 = 27.06° = \beta_2$.

Design Example 6.7: The steam velocity leaving the nozzle is 590 m/s and the nozzle angle is 20°. The blade is running at 2800 rpm and blade diameter is 1050 mm. The axial velocity at rotor outlet = 155 m/s, and the blades are symmetrical. Calculate the work done, the diagram efficiency and the blade velocity coefficient.

Solution:

Blade speed U is given by:

$$U = \frac{\pi DN}{60} = \frac{(\pi \times 1050) \times (2800)}{(1000) \times (60)} = 154\,\text{m/s}$$

The velocity diagram is shown in Fig. 6.24.
Applying the cosine rule to the triangle ABC,

$$V_1^2 = U^2 + C_1^2 - 2UC_1 \cos\alpha_1$$
$$= 154^2 + 590^2 - (2) \times (154) \times (590) \cos 20°$$

i.e. $V_1 = 448.4$ m/s.

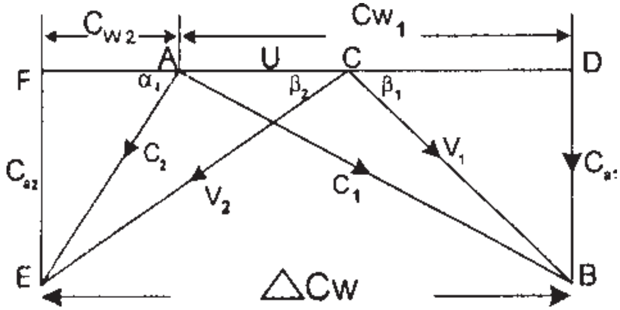


Figure 6.24 Velocity diagram for Example 6.7.

Applying the sine rule to the triangle ABC,

$$\frac{C_1}{\sin\,(\mathrm{ACB})} = \frac{V_1}{\sin(\alpha_1)}$$

But sin (ACB) $= \sin\,(180° - \beta_1) = \sin\,(\beta_1)$

Therefore,

$$\sin(\beta_1) = \frac{C_1 \sin(\alpha_1)}{V_1} = \frac{590\,\sin\,(20°)}{448.4} = 0.450$$

and: $\beta_1 = 26.75°$

From triangle ABD,

$$C_{w1} = C_1\,\cos\,(\alpha_1) = 590\,\cos\,(20°) = 554.42\,\mathrm{m/s}$$

From triangle CEF,

$$\frac{C_{a2}}{U + C_{w2}} = \tan(\beta_2) = \tan(\beta_1) = \tan(26.75°) = 0.504$$

or: $U + C_{w2} = \dfrac{C_{a2}}{0.504} = \dfrac{155}{0.504} = 307.54$

so : $C_{w2} = 307.54 - 154 = 153.54\,\mathrm{m/s}$

Therefore,

$$\Delta C_w = C_{w1} + C_{w2} = 554.42 + 153.54 = 707.96\,\mathrm{m/s}$$

Relative velocity at the rotor outlet is:

$$V_2 = \frac{C_{a2}}{\sin(\beta_2)} = \frac{155}{\sin(26.75°)} = 344.4\,\mathrm{m/s}$$

Blade velocity coefficient is:

$$k = \frac{V_2}{V_1} = \frac{344.4}{448.4} = 0.768$$

Work done on the blades per kg/s:

$$\Delta C_{w2} U = (707.96) \times (154) \times (10^{-3}) = 109\,\text{kW}$$

The diagram efficiency is:

$$\eta_d = \frac{2U\Delta C_w}{C_1^2} = \frac{(2) \times (707.96) \times (154)}{590^2} = 0.6264$$

or, $\eta_d = 62.64\%$

Illustrative Example 6.8: In one stage of an impulse turbine the velocity of steam at the exit from the nozzle is 460 m/s, the nozzle angle is 22° and the blade angle is 33°. Find the blade speed so that the steam shall pass on without shock. Also find the stage efficiency and end thrust on the shaft, assuming velocity coefficient = 0.75, and blades are symmetrical.

Solution:

From triangle ABC (Fig. 6.25):

$$C_{w1} = C_1 \cos 22° = 460 \cos 22° = 426.5\,\text{m/s}$$

and:

$$C_{a1} = C_1 \sin 22° = 460 \sin 22° = 172.32\,\text{m/s}$$

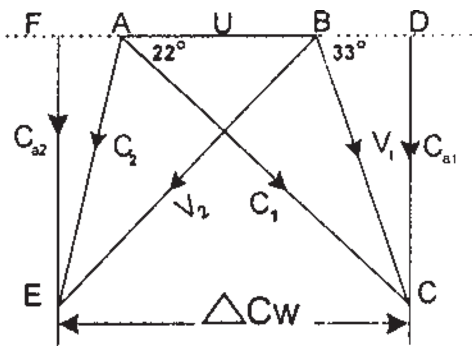


Figure 6.25 Velocity triangles for Example 6.8.

Now, from triangle BCD:

$$\mathrm{BD} = \frac{C_{a1}}{\tan(33^\circ)} = \frac{172.32}{0.649} = 265.5$$

Hence, blade speed is given by:

$$U = C_{w1} - \mathrm{BD} = 426.5 - 265.5 = 161\ \mathrm{m/s}$$

From Triangle BCD, relative velocity at blade inlet is given by:

$$V_1 = \frac{C_{a1}}{\sin(33^\circ)} = \frac{172.32}{0.545} = 316.2\ \mathrm{m/s}$$

Velocity coefficient:

$$k = \frac{V_2}{V_1}, \quad \text{or} \quad V_2 = kV_1 = (0.75) \times (316.2) = 237.2\ \mathrm{m/s}$$

From Triangle BEF,

$$\mathrm{BF} = V_2 \cos(33^\circ) = 237.2 \times \cos(33^\circ) = 198.9$$

and

$$C_{w2} = \mathrm{AF} = \mathrm{BF} - U = 198.9 - 161 = 37.9\ \mathrm{m/s}$$

$$C_{a2} = V_2 \sin(33^\circ) = 237.2 \sin(33^\circ) = 129.2\ \mathrm{m/s}$$

The change in velocity of whirl:

$$\Delta C_w = C_{w1} + C_{w2} = 426.5 + 37.9 = 464.4\ \mathrm{m/s}$$

Diagram efficiency:

$$\eta_d = \frac{2U\Delta C_w}{C_1^2} = \frac{(2) \times (464.4) \times (161)}{460^2} = 0.7067, \text{ or } 70.67\%.$$

End thrust on the shaft per unit mass flow:

$$C_{a1} - C_{a2} = 172.32 - 129.2 = 43.12\ \mathrm{N}$$

Design Example 6.9: In a Parson's turbine, the axial velocity of flow of steam is 0.5 times the mean blade speed. The outlet angle of the blade is 20°, diameter of the ring is 1.30 m and the rotational speed is 3000 rpm. Determine the inlet angles of the blades and power developed if dry saturated steam at 0.5 MPa passes through the blades where blade height is 6 cm. Neglect the effect of the blade thickness.

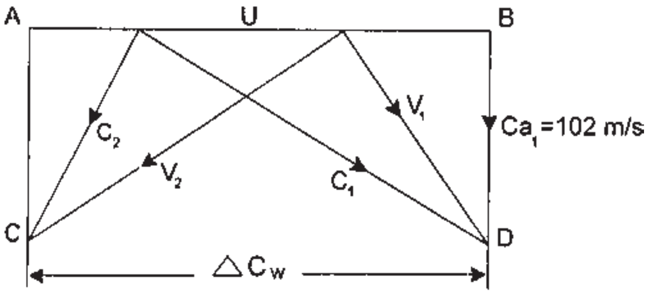


Figure 6.26 Velocity triangles for Example 6.9.

Solution:

The blade speed, $U = \dfrac{\pi DN}{60} = \dfrac{\pi \times (1.30) \times (3000)}{60} = 204\,\text{m/s}$

Velocity of flow, $C_a = (0.5) \times (204) = 102\,\text{m/s}$

Draw lines AB and CD parallel to each other Fig. 6.26 at the distance of 102 m/s, i.e., velocity of flow, $C_{a1} = 102\,\text{m/s}$.

At any point B, construct an angle $\alpha_2 = 20°$ to intersect line CD at point C. Thus, the velocity triangle at the outlet is completed. For Parson's turbine,

$$\alpha_1 = \beta_2, \quad \beta_1 = \alpha_2, \quad C_1 = V_2, \quad \text{and} \quad V_1 = C_2.$$

By measurement,

$$\Delta C_w = C_{w1} + C_{w2} = 280.26 + 76.23 = 356.5\,\text{m/s}$$

The inlet angles are 53.22°.Specific volume of vapor at 0.5 MPa, from the steam tables, is

$$v_g = 0.3749\,\text{m}^3/\text{kg}$$

Therefore the mass flow is given by:

$$\dot{m} = \frac{AC_2}{x_2 v_{g_2}} = \frac{\pi \times (1.30) \times (6) \times (102)}{(100) \times (0.3749)} = 66.7\,\text{kg/s}$$

Power developed:

$$P = \frac{\dot{m} U \Delta C_w}{1000} = \frac{(66.7) \times (356.5) \times (102)}{1000} = 2425.4\,\text{kW}$$

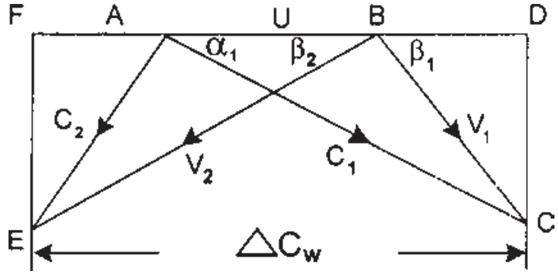


Figure 6.27 Velocity triangles for Example 6.10.

Design Example 6.10: In an impulse turbine, steam is leaving the nozzle with velocity of 950 m/s and the nozzle angle is 20°. The nozzle delivers steam at the rate of 12 kg/min. The mean blade speed is 380 m/s and the blades are symmetrical. Neglect friction losses. Calculate (1) the blade angle, (2) the tangential force on the blades, and (3) the horsepower developed.

Solution:

With the help of α_1, U and C_1, the velocity triangle at the blade inlet can be constructed easily as shown in Fig. 6.27.

Applying the cosine rule to the triangle ABC,

$$V_1^2 = U^2 + C_1^2 - 2UC_1\cos\alpha_1$$
$$= 950^2 + 380^2 - (2) \times (950) \times (380) \times \cos 20° = 607\text{m/s}$$

Now, applying the sine rule to the triangle ABC,

$$\frac{V_1}{\sin(\alpha_1)} = \frac{C_1}{\sin(180° - \beta_1)} = \frac{C_1}{\sin(\beta_1)}$$

or:

$$\sin(\beta_1) = \frac{C_1\sin(\alpha_1)}{V_1} = \frac{(950) \times (0.342)}{607} = 0.535$$

so:

$$\beta_1 = 32.36°$$

From Triangle ACD,

$$C_{w1} = C_1\cos(\alpha_1) = 950 \times \cos(20°) = (950) \times (0.9397)$$
$$= 892.71\text{m/s}$$

As $\beta_1 = \beta_2$, using triangle BEF and neglecting friction loss, i.e,: $V_1 = V_2$

$$\mathrm{BF} = V_2 \cos \beta_2 = 607 \times \cos 32.36^\circ = 512.73$$

Therefore,

$$C_{w2} = \mathrm{BF} - U = 512.73 - 380 = 132.73\ \mathrm{m/s}$$

Change in velocity of whirl:

$$\Delta C_w = C_{w1} + C_{w2} = 892.71 + 132.73 = 1025.44\ \mathrm{m/s}$$

Tangential force on blades:

$$F = \dot{m}\Delta C_w = \frac{(12) \times (1025.44)}{60} = 205\ \mathrm{N}$$

Horsepower, $$P = \dot{m}U\Delta C_w = \frac{(12) \times (1025.44) \times (380)}{(60) \times (1000) \times (0.746)} = 104.47\ \mathrm{hp}$$

Design Example 6.11: In an impulse turbine, the velocity of steam at the exit from the nozzle is 700 m/s and the nozzles are inclined at 22° to the blades, whose tips are both 34°. If the relative velocity of steam to the blade is reduced by 10% while passing through the blade ring, calculate the blade speed, end thrust on the shaft, and efficiency when the turbine develops 1600 kW.

Solution:

Velocity triangles for this problem are shown in Fig. 6.28.
From the triangle ACD,

$$C_{a1} = C_1 \sin \alpha_1 = 700 \times \sin 22^\circ = 262.224\ \mathrm{m/s}$$

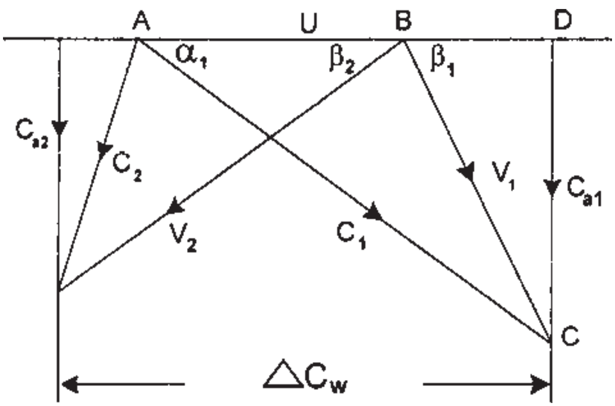


Figure 6.28 Velocity triangles for Example 6.11.

and

$$V_1 = \frac{C_{a1}}{\sin(\beta_1)} = \frac{262.224}{\sin 34^\circ} = 469.32\,\text{m/s}$$

Whirl component of C_1 is given by

$$C_{w1} = C_1 \cos(\alpha_1) = 700 \cos(22^\circ) = 700 \times 0.927 = 649\,\text{m/s}$$

Now, BD $= C_{w1} - U = V_1 \cos\beta_1 = (469.32) \times (0.829) = 389$

Hence, blade speed

$$U = 649 - 389 = 260\,\text{m/s}$$

Using the velocity coefficient to find V_2:

$$\text{i.e.,}\ V_2 = (0.90) \times (469.32) = 422.39\,\text{m/s}$$

From velocity triangle BEF,

$$C_{a2} = V_2 \sin(\beta_2) = 422.39 \sin 34^\circ = 236.2\,\text{m/s}$$

And

$$U + C_{w2} = V_2 \cos 34^\circ = (422.39) \times (0.829) = 350.2\,\text{m/s}$$

Therefore,

$$C_{w2} = 350.2 - 260 = 90.2\,\text{m/s}$$

Then,

$$\Delta C_w = C_{w1} + C_{w2} = 649 + 90.2 = 739.2\,\text{m/s}$$

Mass flow rate is given by:

$$P = \dot{m} U \Delta C_w$$

or

$$\dot{m} = \frac{(1600) \times (1000)}{(739.2) \times (260)} = 8.325\,\text{kg/s}$$

Thrust on the shaft,

$$F = \dot{m}(C_{a1} - C_{a2}) = 8.325(262.224 - 236.2) = 216.65\,\text{N}$$

Diagram efficiency:

$$\eta_d = \frac{2U\Delta C_w}{C_1^2} = \frac{(2) \times (739.2) \times (260)}{700^2} = 0.7844, \quad \text{or} \quad 78.44\%.$$

Illustrative Example 6.12: The moving and fixed blades are identical in shape in a reaction turbine. The absolute velocity of steam leaving the fixed blade is 105 m/s, and the blade velocity is 40 m/s. The nozzle angle is 20°. Assume axial

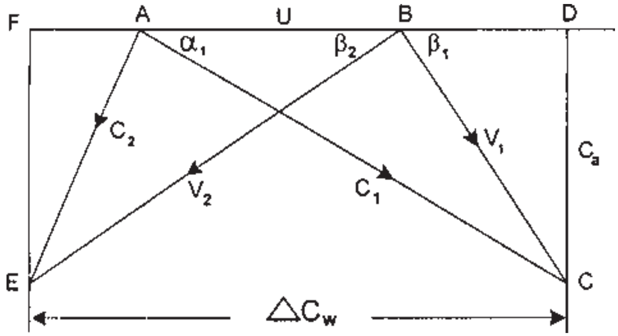


Figure 6.29 Velocity triangles for Example 6.12.

velocity is constant through the stage. Determine the horsepower developed if the steam flow rate is 2 kg/s.

Solution:

For 50% reaction turbine Fig. 6.29, $\alpha_1 = \beta_2$, and $\alpha_2 = \beta_1$.
From the velocity triangle ACD,

$$C_{w1} = C_1 \cos \alpha_1 = 105 \cos 20° = 98.67 \text{ m/s}$$

Applying cosine rule to the Triangle ABC:

$$V_1^2 = C_1^2 + U^2 - 2C_1 U \cos \alpha_1$$

so:

$$V_1 = \sqrt{105^2 + 40^2 - (2) \times (105) \times (40) \times \cos 20°} = 68.79 \text{ m/s}$$

Now,

$$\text{BD} = C_{w1} - U = V_1 \cos \beta_1 = 98.67 - 40 = 58.67$$

Hence,

$$\cos\beta_1 = \frac{58.67}{68.79} = 0.853, \quad \text{and} \quad \beta_1 = 31.47°$$

Change in the velocity of whirl is:

$$\Delta C_w = C_{w1} + C_{w2} = 98.67 + 58.67 = 157.34 \text{ m/s}$$

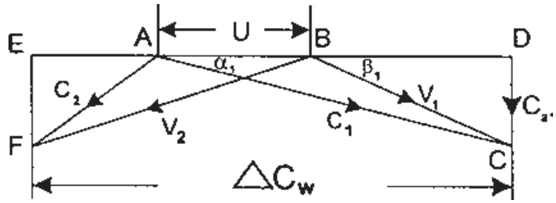


Figure 6.30 Velocity triangles for Example 6.13.

Horsepower developed is:

$$P = \dot{m}U\Delta C_w = \frac{(2)\times(157.34)\times(40)}{(0.746)\times(1000)} = 16.87\,\text{hp}$$

Illustrative Example 6.13: The inlet and outlet angles of blades of a reaction turbine are 25 and 18°, respectively. The pressure and temperature of steam at the inlet to the turbine are 5 bar and 250°C. If the steam flow rate is 10 kg/s and the rotor diameter is 0.72 m, find the blade height and power developed. The velocity of steam at the exit from the fixed blades is 90 m/s.

Solution:

Figure 6.30 shows the velocity triangles.

$$\alpha_1 = \beta_2 = 18°, \text{ and } \alpha_2 = \beta_1 = 25°$$

$$C_1 = 90\,\text{m/s}$$

From the velocity triangle,

$$C_{w1} = C_1 \cos(\alpha_1) = 90 \cos 18° = 85.6\,\text{m/s}$$

$$C_{a1} = \text{CD} = C_1 \sin\alpha_1 = 90 \sin 18° = 27.8\,\text{m/s}$$

From triangle BDC

$$\text{BD} = \frac{C_{a1}}{\sin(\beta_1)} = \frac{27.8}{\sin(25°)} = \frac{27.8}{0.423} = 65.72\,\text{m/s}$$

Hence blade velocity is given by:

$$U = C_{w1} - \text{BD} = 85.6 - 65.62 = 19.98\,\text{m/s}.$$

Applying the cosine rule,

$$V_1^2 = C_1^2 + U^2 - 2C_1U\cos\alpha_1$$

$$= 90^2 + 19.98^2 - (2)\times(90)\times(19.98)\cos 18°$$

$$V_1 = 71.27\,\text{m/s}$$

From triangle AEF,

$$C_{w2} = C_2 \cos(\alpha_2) = 71.27 \cos 25^\circ = 64.59\,\text{m/s}$$

Change in the velocity of whirl:

$$\Delta C_w = C_{w1} + C_{w2} = 85.6 + 64.59 = 150.19\,\text{m/s}$$

Power developed by the rotor:

$$P = \dot{m} U \Delta C_w = \frac{(10)\times(19.98)\times(150.19)}{1000} = 30\,\text{kW}$$

From superheated steam tables at 5 bar, 250°C, the specific volume of steam is:

$$v = 0.4744\,\text{m}^3/\text{kg}$$

Blade height is given by the volume of flow equation:

$$v = \pi D h C_a$$

where C_a is the velocity of flow and h is the blade height.Therefore,

$$0.4744 = \pi \times (0.72) \times (h) \times (27.8), \quad \text{and}$$

$$h = 0.0075\,\text{m} \;\; \text{or} \;\; 0.75\,\text{cm}$$

Design Example 6.14: From the following data, for 50% reaction steam turbine, determine the blade height:

RPM:	440
Power developed:	5.5 MW
Steam mass flow rate:	6.8 kg/kW – h
Stage absolute pressure:	0.90 bar
Steam dryness fraction:	0.95
Exit angles of the blades:	70°

(angle measured from the axial flow direction).

The outlet relative velocity of steam is 1.2 times the mean blade speed. The ratio of the rotor hub diameter to blade height is 14.5.

Solution:

Figure 6.31 shows the velocity triangles.

From the velocity diagram,

$$V_2 = 1.2U$$

$$C_{a2} = V_2 \cos(\beta_2)$$
$$= 1.2U \cos 70°$$
$$= 0.41U \text{ m/s}$$

At mean diameter,

$$U = \frac{\pi DN}{60} = \frac{2\pi N(D_h + h)}{(60) \times (2)}$$

where D_h is the rotor diameter at the hub and h is the blade height. Substituting the value of U in the above equation,

$$C_{a2} = \frac{(0.41) \times (2\pi) \times (440)(14.5h + h)}{(2) \times (60)} = 146.45\, h \text{ m/s}$$

Annular area of flow is given by:

$$A = \pi h(D_h + h) = \pi h(14.5h + h)$$

or

$$A = 15.5\pi h^2$$

Specific volume of saturated steam at 0.90 bar, $v_g = 1.869 \text{ m}^3/\text{kg}$.

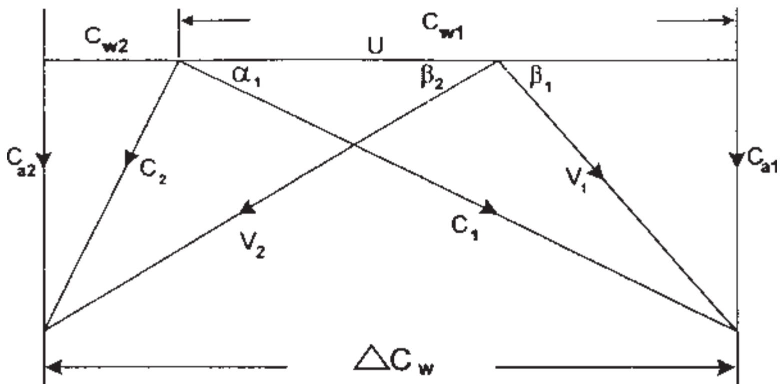


Figure 6.31 Velocity triangles for Example 6.14.

Then the specific volume of steam = (1.869) × (0.95) = 1.776 m^3/kg. The mass flow rate is given by:

$$\dot{m} = \frac{(5.5) \times (10^3) \times (6.8)}{3600} = 10.39 \text{ kg/s}$$

But,

$$\dot{m} = \frac{C_{a2}A}{v} = \frac{C_{a2}15.5\pi h^2}{v}$$

Therefore:

$$10.39 = \frac{(146.45) \times (h) \times (15.5) \times (\pi h^2)}{1.776}$$

or:

$$h^3 = 0.00259, \text{ and } h = 0.137 \text{ m}$$

Design Example 6.15: From the following data for a two-row velocity compounded impulse turbine, determine the power developed and the diagram efficiency:

Blade speed:	115 m/s
Velocity of steam exiting the nozzle:	590 m/s
Nozzle efflux angle:	18°
Outlet angle from first moving blades:	37°
Blade velocity coefficient (all blades):	0.9

Solution:

Figure 6.32 shows the velocity triangles.

Graphical solution:

$U = 115$ m/s

$C_1 = 590$ m/s

$\alpha_1 = 18°$

$\beta_2 = 20°$

The velocity diagrams are drawn to scale, as shown in Fig. 6.33, and the relative velocity:

$V_1 = 482$ m/s using the velocity coefficient

$V_2 = (0.9) \times (482) = 434$ m/s

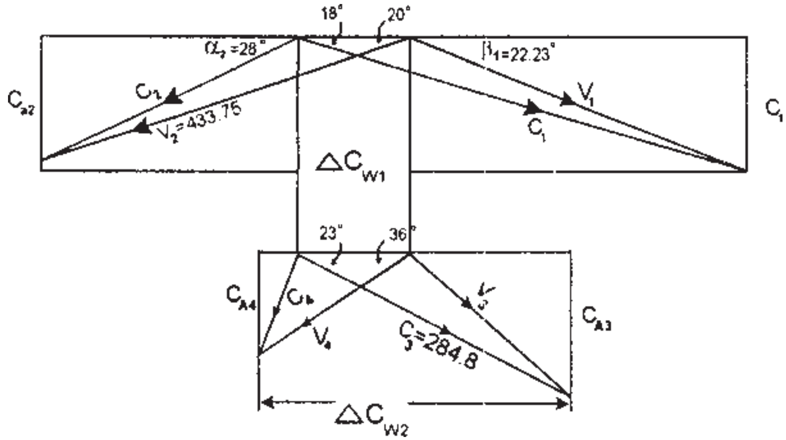


Figure 6.32 Velocity triangle for Example 6.15.

The absolute velocity at the inlet to the second row of moving blades, C_3, is equal to the velocity of steam leaving the fixed row of blades.

$$\text{i.e.,}: C_3 = kC_2 = (0.9) \times (316.4) = 284.8$$

Driving force $= \dot{m}\ \Delta C_w$
For the first row of moving blades, $\dot{m}\Delta C_{w1} = (1) \times (854) = 854\,\text{N}$.
For the second row of moving blades, $\dot{m}\Delta C_{w2} = (1) \times (281.46)$ $\text{N} = 281.46\,\text{N}$
where ΔC_{w1} and ΔC_{w2} are scaled from the velocity diagram.

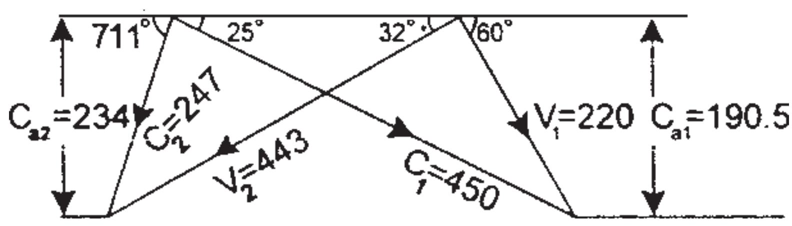


Figure 6.33 Velocity diagram for Example 6.16.

Total driving force $= 854 + 281.46 = 1135.46\,\mathrm{N}$ per kg/s

$$\text{Power} = \text{driving force} \times \text{blade velocity}$$

$$= \frac{(1135.46) \times (115)}{1000} = 130.58\,\mathrm{kW} \text{ per kg/s}$$

Energy supplied to the wheel

$$= \frac{mC_1^2}{2} = \frac{(1) \times (590^2)}{(2) \times (10^3)} = 174.05\mathrm{kW} \text{ per kg/s}$$

Therefore, the diagram efficiency is:

$$\eta_d = \frac{(130.58) \times (10^3) \times (2)}{590^2} = 0.7502, \text{ or } 75.02\%$$

Maximum diagram efficiency:

$$= \cos^2 \alpha_1 = \cos^2 8° = 0.9045, \text{ or } 90.45\%$$

Axial thrust on the first row of moving blades (per kg/s):

$$= \dot{m}(C_{a1} - C_{a2}) = (1) \times (182.32 - 148.4) = 33.9\,\mathrm{N}$$

Axial thrust on the second row of moving blades (per kg/s):

$$= \dot{m}(C_{a3} - C_{a4}) = (1) \times (111.3 - 97.57) = 13.73\,\mathrm{N}$$

Total axial thrust:

$$= 33.9 + 13.73 = 47.63\,\mathrm{N} \text{ per kg/s}$$

Design Example 6.16: In a reaction stage of a steam turbine, the blade angles for the stators and rotors of each stage are: $\alpha_1 = 25°$, $\beta_1 = 60°$, $\alpha_2 = 71.1°$, $\beta_2 = 32°$. If the blade velocity is 300 m/s, and the steam flow rate is 5 kg/s, find the power developed, degree of reaction, and the axial thrust.

Solution:

Figure 6.34 shows the velocity triangles.

The velocity triangles can easily be constructed as the blade velocity and blade angles are given.From velocity triangles, work output per kg is given by:

$$W_t = U(C_{w1} + C_{w2})$$

$$= (300) \times (450 \cos 25° + 247 \cos 71.1°)$$

$$= 14,6,354 \text{ J}$$

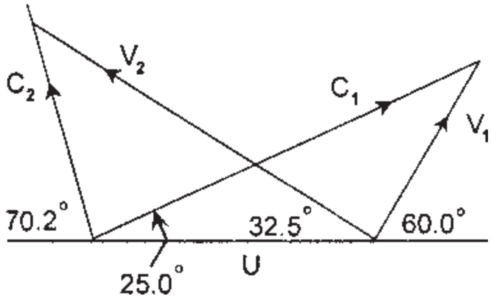


Figure 6.34 Velocity diagram for Example 6.17.

Power output:

$$\dot{m}W_t = \frac{(5) \times (1, 46, 354)}{1000} = 732 \text{ kW}$$

Degree of reaction is given by:

$$\Lambda = \frac{V_2^2 - V_1^2}{2 \times W_t} = \frac{443^2 - 220^2}{(2) \times (14, 6, 354)} = 0.5051, \quad \text{or} \quad 50.51\%$$

Axial thrust:

$$F = \dot{m}(C_{a1} - C_{a2}) = (5) \times (190.5 - 234) = -217.5 \,\text{N}$$

The thrust is negative because its direction is the opposite to the fluid flow.

Design Example 6.17: Steam enters the first row of a series of stages at a static pressure of 10 bars and a static temperature of 300°C. The blade angles for the rotor and stator of each stage are: $\alpha_1 = 25°$, $\beta_1 = 60°$, $\alpha_2 = 70.2°$, $\beta_2 = 32°$. If the blade speed is 250 m/s, and the rotor efficiency is 0.94, find the degree of reaction and power developed for a 5.2 kg/s of steam flow. Also find the static pressures at the rotor inlet and exit if the stator efficiency is 0.93 and the carry-over efficiency is 0.89.

Solution:

Using the given data, the velocity triangles for the inlet and outlet are shown in Fig. 6.34. By measurement, $C_2 = 225$ m/s, $V_2 = 375$ m/s, $C_1 = 400$ m/s, $V_1 = 200$ m/s.

Work done per unit mass flow:

$$W_t = (250) \times (400 \cos 25° + 225 \cos 70.2°) = 1,09,685 \text{ J/kg}$$

Degree of reaction [Eq. (6.25)]

$$\Lambda = \frac{V_2^2 - V_1^2}{2 \times W_t} = \frac{375^2 - 200^2}{(2) \times (1,09,685)} = 0.4587, \quad \text{or} \quad 45.87\%$$

Power output:

$$P = \dot{m}W = \frac{(5.2) \times (1,09,685)}{1000} = 570.37 \text{ kW}$$

Isentropic static enthalpy drop in the stator:

$$\Delta h_s{}' = \frac{\left(C_1^2 - C_2^2\right)}{\eta_s} = \frac{\left(400^2 - (0.89) \times (225^2)\right)}{0.93}$$

$$= 1,23,595 \text{ J/kg}, \quad \text{or} \quad 123.6 \text{ kJ/kg}$$

Isentropic static enthalpy drops in the rotor:

$$\Delta h_r{}' = \frac{W}{\eta_r \eta_s} = \frac{1,09,685}{(0.94) \times (0.93)}$$

$$= 1,25,469 \text{ J/kg}, \quad \text{or} \quad 125.47 \text{ kJ/kg}$$

Since the state of the steam at the stage entry is given as 10 bar, 300°C, Enthalpy at nozzle exit:

$$h_1 - \left[\Delta h'\right]_{\text{stator}} = 3051.05 - 123.6 = 2927.5 \text{kJ/kg}$$

Enthalpy at rotor exit:

$$h_1 - \left[\Delta h'\right]_{\text{rotor}} = 3051.05 - 125.47 = 2925.58 \text{kJ/kg}$$

The rotor inlet and outlet conditions can be found by using the Mollier Chart.

Rotor inlet conditions: $P_1 = 7$ bar, $T_1 = 235$°C
Rotor outlet conditions: $P_2 = 5$ bar, $T_2 = 220$°C

PROBLEMS

6.1 Dry saturated steam is expanded in a steam nozzle from 1 MPa to 0.01 MPa. Calculate dryness fraction of steam at the exit and the heat drop.
(0.79, 686 kJ/kg)

6.2 Steam initially dry and at 1.5 MPa is expanded adiabatically in a nozzle to 7.5 KPa. Find the dryness fraction and velocity of steam at the exit. If the exit diameter of the nozzles is 12.5 mm, find the mass of steam discharged per hour.

(0.756, 1251.26 m/s, 0.376 kg/h)

6.3 Dry saturated steam expands isentropically in a nozzle from 2.5 MPa to 0.30 MPa. Find the dryness fraction and velocity of steam at the exit from the nozzle. Neglect the initial velocity of the steam.

(0.862, 867.68 m/s)

6.4 The nozzles receive steam at 1.75 MPa, 300°C, and exit pressure of steam is 1.05 MPa. If there are 16 nozzles, find the cross-sectional area of the exit of each nozzle for a total discharge to be 280 kg/min. Assume nozzle efficiency of 90%. If the steam has velocity of 120 m/s at the entry to the nozzles, by how much would the discharge be increased?

(1.36 cm^2, 33.42%)

6.5 The steam jet velocity of a turbine is 615 m/s and nozzle angle is 22°, The blade velocity coefficient = 0.70 and the blade is rotating at 3000 rpm. Assume mean blade radius = 600 mm and the axial velocity at the outlet = 160 m/s. Determine the work output per unit mass flow of steam and diagram efficiency.

(93.43 kW, 49.4%)

6.6 Steam is supplied from the nozzle with velocity 400 m/s at an angle of 20° with the direction of motion of moving blades. If the speed of the blade is 200 m/s and there is no thrust on the blades, determine the inlet and outlet blade angles, and the power developed by the turbine. Assume velocity coefficient = 0.86, and mass flow rate of steam is 14 kg/s.

(37° 50′, 45°, 31′, 1234.8 kW)

6.7 Steam expands isentropically in the reaction turbine from 4 MPa, 400°C to 0.225 MPa. The turbine efficiency is 0.84 and the nozzle angles and blade angles are 20 and 36° respectively. Assume constant axial velocity throughout the stage and the blade speed is 160 m/s. How many stages are there in the turbine?

(8 stages)

6.8 Consider one stage of an impulse turbine consisting of a converging nozzle and one ring of moving blades. The nozzles are inclined at 20° to the blades, whose tip angles are both 33°. If the velocity of the steam at the exit from the nozzle is 650 m/s, find the blade speed so that steam passes through without shock and find the diagram efficiency, neglecting losses.

(273 m/s, 88.2%)

6.9 One stage of an impulse turbine consists of a converging nozzle and one ring of moving blades. The nozzle angles are 22° and the blade angles are 35°. The velocity of steam at the exit from the nozzle is 650 m/s. If the relative velocity of steam to the blades is reduced by 14% in passing through the blade ring, find the diagram efficiency and the end thrust on the shaft when the blade ring develops 1650 kW.

(79.2%, 449 N)

6.10 The following refer to a stage of a Parson's reaction turbine:

Mean diameter of the blade ring:	92 cm
Blade speed:	3000 rpm
Inlet absolute velocity of steam:	310 m/s
Blade outlet angle:	20°
Steam flow rate:	6.9 kg/s

Determine the following: (1) blade inlet angle, (2) tangential force, and (3) power developed.

(38°, 2.66 kW, 384.7 kW)

NOTATION

C	absolute velocity, velocity of steam at nozzle exit
D	diameter
h	enthalpy, blade height
h_0	stagnation enthalpy, static enthalpy at the inlet to the fixed blades
h_1	enthalpy at the entry to the moving blades
h_2	enthalpy at the exit from the moving blades
h_{00}	stagnation enthalpy at the entry to the fixed blades
h_{01}	stagnation enthalpy at the entry to the fixed blades
h_{02}	stagnation enthalpy at the exit from the moving blade
k	blade velocity coefficient
N	rotational speed
R. F.	reheat factor
U	blade speed
V	relative velocity
α	angle with absolute velocity
β	angle with relative velocity
ΔC_w	change in the velocity of whirl
Δh	actual enthalpy drop
$\Delta h'$	isentropic enthalpy drop

η_d	diffuser efficiency
η_n	nozzle efficiency
η_s	stage efficiency
η_t	turbine efficiency
η_{ts}	total - to - static efficiency
η_{tt}	total - to - total efficiency
Λ	degree of reaction

SUFFIXES

0	inlet to fixed blades
1	inlet to moving blades
2	outlet from the moving blades
a	axial, ambient
r	radial
w	whirl

7

Axial Flow and Radial Flow Gas Turbines

7.1 INTRODUCTION TO AXIAL FLOW TURBINES

The axial flow gas turbine is used in almost all applications of gas turbine power plant. Development of the axial flow gas turbine was hindered by the need to obtain both a high-enough flow rate and compression ratio from a compressor to maintain the air requirement for the combustion process and subsequent expansion of the exhaust gases. There are two basic types of turbines: the axial flow type and the radial or centrifugal flow type. The axial flow type has been used exclusively in aircraft gas turbine engines to date and will be discussed in detail in this chapter. Axial flow turbines are also normally employed in industrial and shipboard applications. Figure 7.1 shows a rotating assembly of the Rolls-Royce Nene engine, showing a typical single-stage turbine installation. On this particular engine, the single-stage turbine is directly connected to the main and cooling compressors. The axial flow turbine consists of one or more stages located immediately to the rear of the engine combustion chamber. The turbine extracts kinetic energy from the expanding gases as the gases come from the burner, converting this kinetic energy into shaft power to drive the compressor and the engine accessories. The turbines can be classified as (1) impulse and (2) reaction. In the impulse turbine, the gases will be expanded in the nozzle and passed over to the moving blades. The moving blades convert this kinetic energy into mechanical energy and also direct the gas flow to the next stage

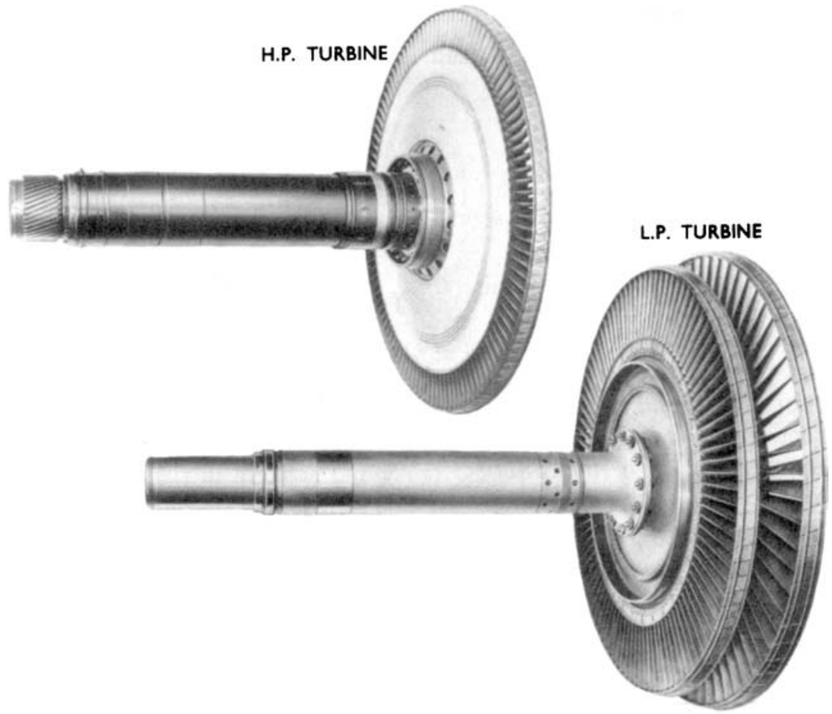


Figure 7.1 Axial flow turbine rotors. (Courtesy Rolls-Royce.)

(multi-stage turbine) or to exit (single-stage turbine). Fig. 7.1 shows the axial flow turbine rotors.

In the case of reaction turbine, pressure drop of expansion takes place in the stator as well as in the rotor-blades. The blade passage area varies continuously to allow for the continued expansion of the gas stream over the rotor-blades. The efficiency of a well-designed turbine is higher than the efficiency of a compressor, and the design process is often much simpler. The main reason for this fact, as discussed in compressor design, is that the fluid undergoes a pressure rise in the compressor. It is much more difficult to arrange for an efficient deceleration of flow than it is to obtain an efficient acceleration. The pressure drop in the turbine is sufficient to keep the boundary layer fluid well behaved, and separation problems, or breakaway of the molecules from the surface, which often can be serious in compressors, can be easily avoided. However, the turbine designer will face much more critical stress problem because the turbine rotors must operate in very high-temperature gases. Since the design principle and concepts of gas turbines are essentially the same as steam turbines, additional information on turbines in general already discussed in Chapter 6 on steam turbines.

7.2 VELOCITY TRIANGLES AND WORK OUTPUT

The velocity diagram at inlet and outlet from the rotor is shown in Fig. 7.2. Gas with an absolute velocity C_1 making an angle α_1, (angle measured from the axial direction) enters the nozzle (in impulse turbine) or stator blades (in reaction turbine). Gas leaves the nozzles or stator blades with an absolute velocity C_2, which makes and an α_2 with axial direction. The rotor-blade inlet angle will be chosen to suit the direction β_2 of the gas velocity V_2 relative to the blade at inlet. β_2 and V_2 are found by subtracting the blade velocity vector U from the absolute velocity C_2.

It is seen that the nozzles accelerate the flow, imparting an increased tangential velocity component. After expansion in the rotor-blade passages, the gas leaves with relative velocity V_3 at angle β_3. The magnitude and direction of the absolute velocity at exit from the rotor C_3 at an angle α_3 are found by vectorial addition of U to the relative velocity V_3. α_3 is known as the swirl angle.

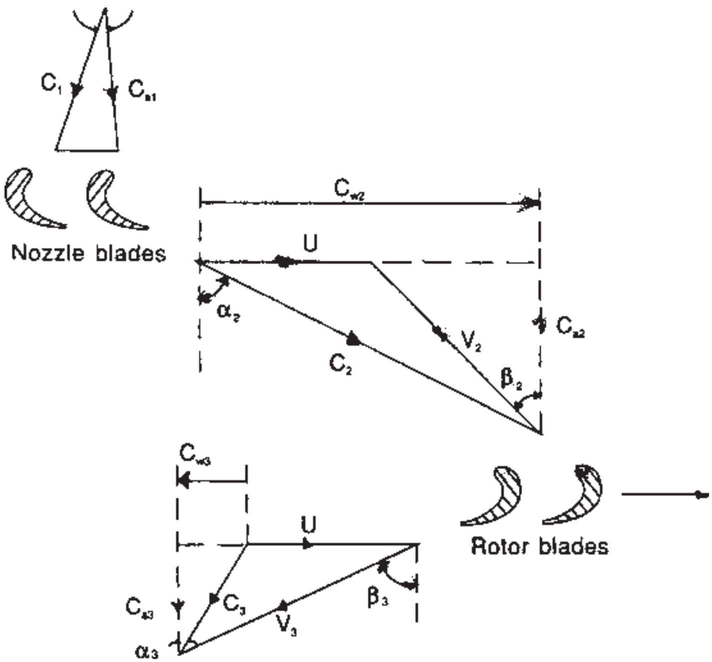


Figure 7.2 Velocity triangles for an axial flow gas turbine.

The gas enters the nozzle with a static pressure p_1 and temperature T_1. After expansion, the gas pressure is p_2 and temperature T_2. The gas leaves the rotor-blade passages at pressure p_3 and temperature T_3. Note that the velocity diagram of the turbine differs from that of the compressor, in that the change in tangential velocity in the rotor, ΔC_w, is in the direction opposite to the blade speed U. The reaction to this change in the tangential momentum of the fluid is a torque on the rotor in the direction of motion. V_3 is either slightly less than V_2 (due to friction) or equal to V_2. But in reaction stage, V_3 will always be greater than V_2 because part of pressure drop will be converted into kinetic energy in the moving blade. The blade speed U increases from root to tip and hence velocity diagrams will be different for root, tip, and other radii points. For short blades, 2-D approach in design is valid but for long blades, 3-D approach in the designing must be considered. We shall assume in this section that we are talking about conditions at the mean diameter of the annulus. Just as with the compressor blading diagram, it is more convenient to construct the velocity diagrams in combined form, as shown in Fig. 7.3. Assuming unit mass flow, work done by the gas is given by

$$W = U(C_{w2} + C_{w3}) \tag{7.1}$$

From velocity triangle

$$\frac{U}{Ca} = \tan\alpha_2 - \tan\beta_2 = \tan\beta_3 - \tan\alpha_3 \tag{7.2}$$

In single-stage turbine, $\alpha_1 = 0$ and $C_1 = Ca_1$. In multi-stage turbine, $\alpha_1 = \alpha_3$ and $C_1 = C_3$ so that the same blade shape can be used. In terms of air angles, the stage

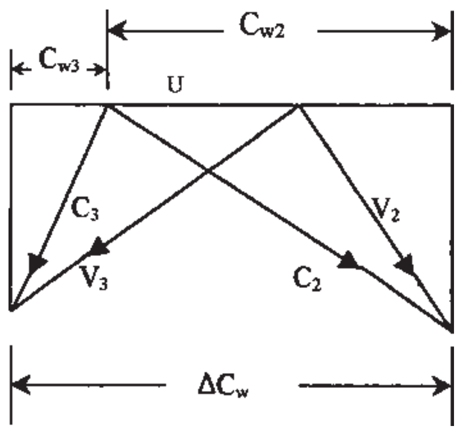


Figure 7.3 Combined velocity diagram.

work output per unit mass flow is given by

$$W = U(C_{w2} + C_{w3}) = UCa(\tan\alpha_2 + \tan\alpha_3) \tag{7.3}$$

or

$$W = UCa\left(\tan\beta_2 + \tan\beta_3\right) \tag{7.4}$$

Work done factor used in the designing of axial flow compressor is not required because in the turbine, flow is accelerating and molecules will not break away from the surface and growth of the boundary layer along the annulus walls is negligible. The stagnation pressure ratio of the stage p_{01}/p_{03} can be found from

$$\Delta T_{0s} = \eta_s T_{01}\left[1 - \left(\frac{1}{p_{01}/p_{03}}\right)^{(\gamma-1)/\gamma}\right] \tag{7.5}$$

where η_s is the isentropic efficiency given by

$$\eta_s = \frac{T_{01} - T_{03}}{T_{01} - T'_{03}} \tag{7.6}$$

The efficiency given by Eq. (7.6) is based on stagnation (or total) temperature, and it is known as total-to-total stage efficiency. Total-to-total stage efficiency term is used when the leaving kinetics energy is utilized either in the next stage of the turbine or in propelling nozzle. If the leaving kinetic energy from the exhaust is wasted, then total-to-static efficiency term is used. Thus total-to-static efficiency,

$$\eta_{ts} = \frac{T_{01} - T_{03}}{T_{01} - T'_3} \tag{7.7}$$

where T'_3 in Eq. (7.7) is the static temperature after an isentropic expansion from p_{01} to p_3.

7.3 DEGREE OF REACTION (Λ)

Degree of reaction is defined as

$$\Lambda = \frac{\text{Enthalpy drop in the moving blades}}{\text{Enthalpy drop in the stage}}$$

$$= \frac{h_2 - h_3}{h_1 - h_3} = \frac{Ca}{2U}\left(\tan\beta_1 - \tan\beta_2\right) \tag{7.8}$$

This shows the fraction of the stage expansion, which occurs in the rotor, and it is usual to define in terms of the static temperature drops, namely

$$\Lambda = \frac{T_2 - T_3}{T_1 - T_3} \tag{7.9}$$

Assuming that the axial velocity is constant throughout the stage, then

$$Ca_2 = Ca_3 = Ca_1, \quad \text{and} \quad C_3 = C_1$$

From Eq. (7.4)

$$C_p(T_1 - T_3) = C_p(T_{01} - T_{03}) = UCa(\tan\beta_2 + \tan\beta_3) \tag{7.10}$$

Temperature drop across the rotor-blades is equal to the change in relative velocity, that is

$$\begin{aligned} C_p(T_2 - T_3) &= \frac{1}{2}\left(V_3^2 - V_2^2\right) \\ &= \frac{1}{2}Ca^2\left(\sec^2\beta_3 - \sec^2\beta_2\right) \\ &= \frac{1}{2}Ca^2\left(\tan^2\beta_3 - \tan^2\beta_2\right) \end{aligned}$$

Thus

$$\Lambda = \frac{Ca}{2U}\left(\tan\beta_3 - \tan\beta_2\right) \tag{7.11}$$

7.4 BLADE-LOADING COEFFICIENT

The blade-loading coefficient is used to express work capacity of the stage. It is defined as the ratio of the specific work of the stage to the square of the blade velocity—that is, the blade-loading coefficient or temperature-drop coefficient ψ is given by

$$\psi = \frac{W}{\frac{1}{2}U^2} = \frac{2C_p\Delta T_{os}}{U^2} = \frac{2Ca}{U}\left(\tan\beta_2 + \tan\beta_3\right) \tag{7.12}$$

Flow Coefficient (ϕ)

The flow coefficient, ϕ, is defined as the ratio of the inlet velocity Ca to the blade velocity U, i.e.,

$$\phi = \frac{Ca}{U} \tag{7.13}$$

This parameter plays the same part as the blade-speed ratio U/C_1 used in the design of steam turbine. The two parameters, ψ and ϕ, are dimensionless and useful to plot the design charts. The gas angles in terms of ψ, Λ, and ϕ can be obtained easily as given below:

Eqs. (7.11) and (7.12) can be written as

$$\psi = 2\phi(\tan\beta_2 + \tan\beta_3) \tag{7.14}$$

$$\Lambda = \frac{\phi}{2}(\tan\beta_3 - \tan\beta_2) \tag{7.15}$$

Now, we may express gas angles β_2 and β_3 in terms of ψ, Λ, and ϕ as follows:

Adding and subtracting Eqs. (7.14) and (7.15), we get

$$\tan\beta_3 = \frac{1}{2\phi}\left(\frac{1}{2}\psi + 2\Lambda\right) \tag{7.16}$$

$$\tan\beta_2 = \frac{1}{2\phi}\left(\frac{1}{2}\psi - 2\Lambda\right) \tag{7.17}$$

Using Eq. (7.2)

$$\tan\alpha_3 = \tan\beta_3 - \frac{1}{\phi} \tag{7.18}$$

$$\tan\alpha_2 = \tan\beta_2 + \frac{1}{\phi} \tag{7.19}$$

It has been discussed in Chapter 6 that steam turbines are usually impulse or a mixture of impulse and reaction stages but the turbine for a gas-turbine power plant is a reaction type. In the case of steam turbine, pressure ratio can be of the order of 1000:1 but for a gas turbine it is in the region of 10:1. Now it is clear that a very long steam turbine with many reaction stages would be required to reduce the pressure by a ratio of 1000:1. Therefore the reaction stages are used where pressure drop per stage is low and also where the overall pressure ratio of the turbine is low, especially in the case of aircraft engine, which may have only three or four reaction stages.

Let us consider 50% reaction at mean radius. Substituting $\Lambda = 0.5$ in Eq. (7.11), we have

$$\frac{1}{\phi} = \tan\beta_3 - \tan\beta_2 \tag{7.20}$$

Comparing this with Eq. (7.2), $\beta_3 = \alpha_2$ and $\beta_2 = \alpha_3$, and hence the velocity diagram becomes symmetrical. Now considering $C_1 = C_3$, we have $\alpha_1 = \alpha_3 = \beta_2$, and the stator and rotor-blades then have the same inlet and outlet angles. Finally, for $\Lambda = 0.5$, we can prove that

$$\psi = 4\phi\tan\beta_3 - 2 = 4\phi\tan\alpha_2 - 2 \tag{7.21}$$

and $$\psi = 4\phi\tan\beta_2 + 2 = 4\phi\tan\alpha_3 + 2 \tag{7.22}$$

and hence all the gas angles can be obtained in terms of ψ and ϕ.

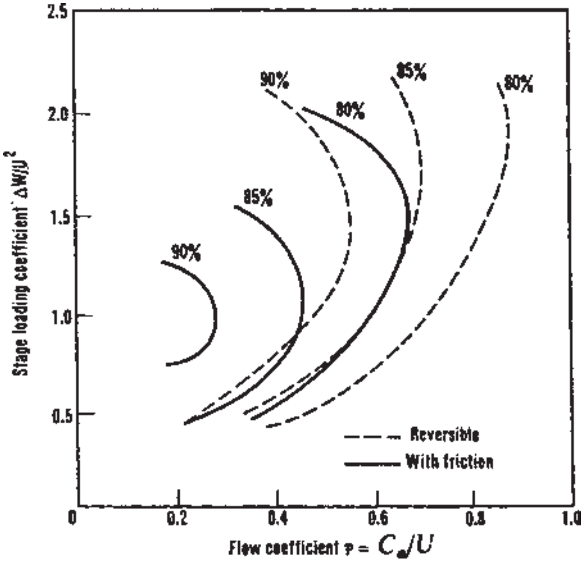


Figure 7.4 Total-to-static efficiency of a 50% reaction axial flow turbine stage.

The low values of ϕ and ψ imply low gas velocities and hence reduced friction losses. But a low value of ψ means more stages for a given overall turbine output, and low ϕ means larger turbine annulus area for a given mass flow. In industrial gas turbine plants, where low sfc is required, a large diameter, relatively long turbine, of low flow coefficient and low blade loading, would be accepted. However, for the gas turbine used in an aircraft engine, the primary consideration is to have minimum weight, and a small frontal area. Therefore it is necessary to use higher values of ψ and ϕ but at the expense of efficiency (see Fig. 7.4).

7.5 STATOR (NOZZLE) AND ROTOR LOSSES

A T–s diagram showing the change of state through a complete turbine stage, including the effects of irreversibility, is given in Fig. 7.5.

In Fig. 7.5, $T_{02} = T_{01}$ because no work is done in the nozzle, $(p_{01} - p_{02})$ represents the pressure drop due to friction in the nozzle. $(T_{01} - T_2')$ represents the ideal expansion in the nozzle, T_2 is the temperature at the nozzle exit due to friction. Temperature, T_2 at the nozzle exit is higher than T_2'. The nozzle loss coefficient, λ_N, in terms of temperature may be defined as

$$\lambda_N = \frac{T_2 - T_2'}{C_2^2/2C_p} \tag{7.23}$$

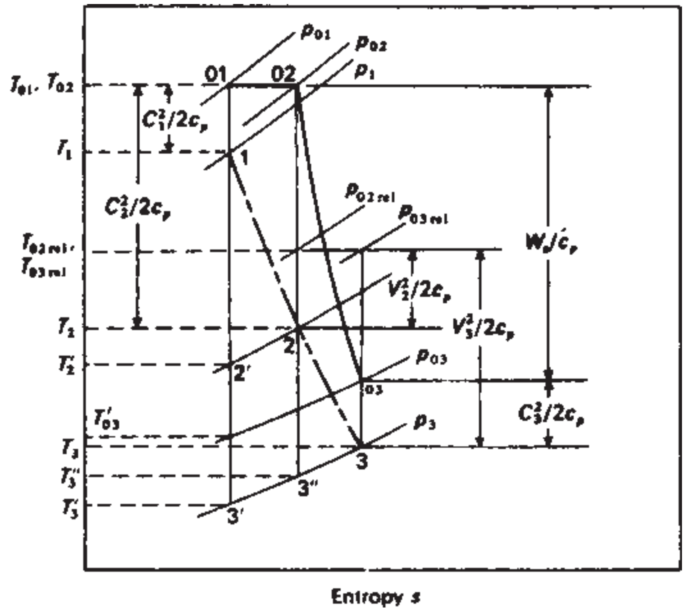


Figure 7.5 $T\text{–}s$ diagram for a reaction stage.

Nozzle loss coefficient in term of pressure

$$y_N = \frac{p_{01} - p_{02}}{p_{01} - p_2} \tag{7.24}$$

λ_N and y_N are not very different numerically. From Fig. 7.5, further expansion in the rotor-blade passages reduces the pressure to p_3. T_3' is the final temperature after isentropic expansion in the whole stage, and T_3'' is the temperature after expansion in the rotor-blade passages alone. Temperature T_3 represents the temperature due to friction in the rotor-blade passages. The rotor-blade loss can be expressed by

$$\lambda_R = \frac{T_3 - T_3''}{V_3^2/2C_p} \tag{7.25}$$

As we know that no work is done by the gas relative to the blades, that is, $T_{03\text{rel}} = T_{02\text{rel}}$. The loss coefficient in terms of pressure drop for the rotor-blades is defined by

$$\lambda_R = \frac{p_{02\,\text{rel}} - p_{03\,\text{rel}}}{p_{03\,\text{rel}} - p_3} \tag{7.26}$$

The loss coefficient in the stator and rotor represents the percentage drop of energy due to friction in the blades, which results in a total pressure and static enthalpy drop across the blades. These losses are of the order of 10–15% but can be lower for very low values of flow coefficient.

Nozzle loss coefficients obtained from a large number of turbine tests are typically 0.09 and 0.05 for the rotor and stator rows, respectively. Figure 7.4 shows the effect of blade losses, determined with Soderberg's correlation, on the total-to-total efficiency of turbine stage for the constant reaction of 50%. It is the evident that exit losses become increasingly dominant as the flow coefficient is increased.

7.6 FREE VORTEX DESIGN

As pointed out earlier, velocity triangles vary from root to tip of the blade because the blade speed U is not constant and varies from root to tip. Twisted blading designed to take account of the changing gas angles is called vortex blading. As discussed in axial flow compressor (Chapter 5) the momentum equation is

$$\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{C_{\mathrm{w}}^2}{r} \tag{7.27}$$

For constant enthalpy and entropy, the equation takes the form

$$\frac{\mathrm{d}h_0}{\mathrm{d}r} = Ca\frac{\mathrm{d}Ca}{\mathrm{d}r} + C_{\mathrm{w}}\frac{\mathrm{d}C_{\mathrm{w}}}{\mathrm{d}r} + \frac{C_{\mathrm{w}}^2}{r} \tag{7.28}$$

For constant stagnation enthalpy across the annulus ($\mathrm{d}h_0/\mathrm{d}r = 0$) and constant axial velocity ($\mathrm{d}Ca/\mathrm{d}r = 0$) then the whirl component of velocity C_{w} is inversely proportional to the radius and radial equilibrium is satisfied. That is,

$$C_{\mathrm{w}} \times r = \text{constant} \tag{7.29}$$

The flow, which follows Eq. (7.29), is called a "free vortex."

Now using subscript m to denote condition at mean diameter, the free vortex variation of nozzle angle α_2 may be found as given below:

$$C_{\mathrm{w}2}r = rCa_2 \tan\alpha_2 = \text{constant}$$

$$Ca_2 = \text{constant}$$

Therefore α_2 at any radius r is related to $\alpha_{2\mathrm{m}}$ at the mean radius r_{m} by

$$\tan\alpha_2 = \left(\frac{r_{\mathrm{m}}}{r}\right)_2 \tan\alpha_{2\mathrm{m}} \tag{7.30}$$

Similarly, α_3 at outlet is given by

$$\tan\alpha_3 = \left(\frac{r_{\mathrm{m}}}{r}\right)_3 \tan\alpha_{3\mathrm{m}} \tag{7.31}$$

The gas angles at inlet to the rotor-blade, from velocity triangle,

$$\tan\beta_3 = \tan\alpha_2 - \frac{U}{Ca}$$

$$= \left(\frac{r_m}{r}\right)_2 \tan\alpha_{2m} - \left(\frac{r}{r_m}\right)_2 \frac{U_m}{Ca_2} \tag{7.32}$$

and β_3 is given by

$$\tan\beta_2 = \left(\frac{r_m}{r}\right)_3 \tan\alpha_{3m} + \left(\frac{r}{r_m}\right)_3 \frac{U_m}{Ca_3} \tag{7.33}$$

7.7 CONSTANT NOZZLE ANGLE DESIGN

As before, we assume that the stagnation enthalpy at outlet is constant, that is, $dh_0/dr = 0$. If α_2 is constant, this leads to the axial velocity distribution given by

$$C_{w2} r^{\sin^2\alpha_2} = \text{constant} \tag{7.34}$$

and since α_2 is constant, then Ca_2 is proportional to C_{w1}. Therefore

$$C_{a2} r^{\sin^2\alpha_2} = \text{constant} \tag{7.35}$$

Normally the change in vortex design has only a small effect on the performance of the blade while secondary losses may actually increase.

Illustrative Example 7.1 Consider an impulse gas turbine in which gas enters at pressure = 5.2 bar and leaves at 1.03 bar. The turbine inlet temperature is 1000 K and isentropic efficiency of the turbine is 0.88. If mass flow rate of air is 28 kg/s, nozzle angle at outlet is 57°, and absolute velocity of gas at inlet is 140 m/s, determine the gas velocity at nozzle outlet, whirl component at rotor inlet and turbine work output. Take, $\gamma = 1.33$, and $C_{pg} = 1.147$ kJ/kgK (see Fig. 7.6).

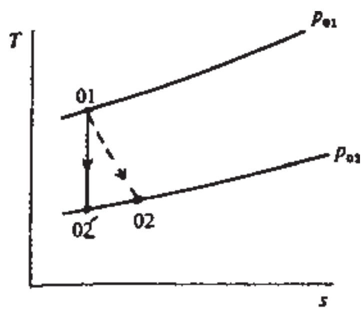


Figure 7.6 *T-s* diagram for Example 7.1.

Solution

From isentropic p–T relation for expansion process

$$\frac{T'_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma}$$

or $$T'_{02} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} = 1000\left(\frac{1.03}{5.2}\right)^{(0.248)} = 669\,\text{K}$$

Using isentropic efficiency of turbine

$$T_{02} = T_{01} - \eta_t\left(T_{01} - T'_{02}\right) = 1000 - 0.88(1000 - 669)$$

$$= 708.72\,\text{K}$$

Using steady-flow energy equation

$$\frac{1}{2}\left(C_2^2 - C_1^2\right) = C_p(T_{01} - T_{02})$$

Therefore, $C_2 = \sqrt{[(2)(1147)(1000 - 708.72) + 19600]} = 829.33\,\text{m/s}$

From velocity triangle, velocity of whirl at rotor inlet

$$C_{w2} = 829.33\ \sin 57° = 695.5\ \ \text{m/s}$$

Turbine work output is given by

$$W_t = mC_{pg}(T_{01} - T_{02}) = (28)(1.147)(1000 - 708.72)$$

$$= 9354.8\ \text{kW}$$

Design Example 7.2 In a single-stage gas turbine, gas enters and leaves in axial direction. The nozzle efflux angle is 68°, the stagnation temperature and stagnation pressure at stage inlet are 800°C and 4 bar, respectively. The exhaust static pressure is 1 bar, total-to-static efficiency is 0.85, and mean blade speed is 480 m/s, determine (1) the work done, (2) the axial velocity which is constant through the stage, (3) the total-to-total efficiency, and (4) the degree of reaction. Assume $\gamma = 1.33$, and $C_{pg} = 1.147$ kJ/kgK.

Solution

(1) The specific work output

$$\begin{aligned} W =\ & C_{pg}(T_{01} - T_{03}) \\ =\ & \eta_{ts} C_{pg} T_{01}\left[1 - (1/4)^{0.33/1.33}\right] \\ =\ & (0.85)(1.147)(1073)\left[1 - (0.25)^{0.248}\right] = 304.42\,\text{kJ/kg} \end{aligned}$$

(2) Since $\alpha_1 = 0$, $\alpha_3 = 0$, $C_{w1} = 0$ and specific work output is given by

$$W = UC_{w2} \quad \text{or} \quad C_{w2} = \frac{W}{U} = \frac{304.42 \times 1000}{480} = 634.21 \text{ m/s}$$

From velocity triangle

$$\sin \alpha_2 = \frac{C_{w2}}{C_2}$$

or

$$C_2 = \frac{C_{w2}}{\sin \alpha_2} = \frac{634.21}{\sin 68^\circ} = 684 \text{ m/s}$$

Axial velocity is given by

$$Ca_2 = 684 \cos 68^\circ = 256.23 \text{ m/s}$$

(3) Total-to-total efficiency, η_{tt}, is

$$\eta_{tt} = \frac{T_{01} - T_{03}}{T_{01} - T'_{03}}$$

$$= \frac{w_s}{T_{01} - \left(T_3 + \frac{C_3^2}{2C_{pg}}\right)} = \frac{w_s}{\frac{w_s}{\eta_{ts}} - \frac{C_3^2}{2C_{pg}}}$$

$$= \frac{304.42}{\frac{304.42}{0.85} - \frac{(256.23)^2}{2 \times 1147}} = 92.4\%$$

(4) The degree of reaction

$$\Lambda = \frac{Ca}{2U}(\tan \beta_3 - \tan \beta_2)$$

$$= \left(\frac{Ca}{2U} \times \frac{U}{Ca}\right) - \left(\frac{Ca}{2U} \tan \alpha_2\right) + \left(\frac{U}{Ca} \times \frac{Ca}{2U}\right)$$

(from velocity triangle)

$$\Lambda = 1 - \frac{Ca}{2U} \tan \alpha_2 = 1 - \frac{256.23}{(2)(480)} \tan 68^\circ = 33.94\%$$

Design Example 7.3 In a single-stage axial flow gas turbine gas enters at stagnation temperature of 1100 K and stagnation pressure of 5 bar. Axial velocity is constant through the stage and equal to 250 m/s. Mean blade speed is 350 m/s. Mass flow rate of gas is 15 kg/s and assume equal inlet and outlet velocities. Nozzle efflux angle is 63°, stage exit swirl angle equal to 9°. Determine the rotor-blade gas angles, degree of reaction, and power output.

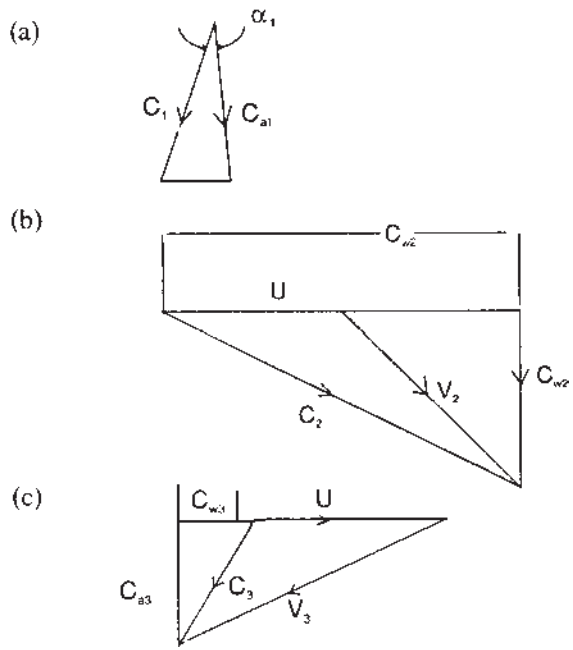


Figure 7.7 Velocity triangles for Example 7.3.

Solution

Refer to Fig. 7.7.

$$Ca_1 = Ca_2 = Ca_3 = Ca = 250 \text{ m/s}$$

From velocity triangle (b)

$$C_2 = \frac{Ca_2}{\cos \alpha_2} = \frac{250}{\cos 63°} = 550.67 \text{ m/s}$$

From figure (c)

$$C_3 = \frac{Ca_3}{\cos \alpha_3} = \frac{250}{\cos 9°} = 253 \text{ m/s}$$

$$C_{w3} = Ca_3 \tan \alpha_3 = 250 \tan 9° = 39.596 \text{ m/s}$$

$$\tan \beta_3 = \frac{U + C_{w3}}{Ca_3} = \frac{350 + 39.596}{250} = 1.5584$$

i.e., $\beta_3 = 57.31°$

From figure (b)

$$C_{w2} = Ca_2 \tan \alpha_2 = 250 \tan 63° = 490.65 \text{ m/s}$$

and

$$\tan \beta_2 = \frac{C_{w2} - U}{Ca_2} = \frac{490.65 - 350}{250} = 0.5626$$

$$\therefore \beta_2 = 29°21'$$

Power output

$$W = mUCa(\tan \beta_2 + \tan \beta_3)$$

$$= (15)(350)(250)(0.5626 + 1.5584)/1000$$

$$= 2784 \text{ kW}$$

The degree of reaction is given by

$$\Lambda = \frac{Ca}{2U}\left(\tan \beta_3 - \tan \beta_2\right)$$

$$= \frac{250}{2 \times 350}(1.5584 - 0.5626)$$

$$= 35.56\%$$

Design Example 7.4 Calculate the nozzle throat area for the same data as in the precious question, assuming nozzle loss coefficient, $T_N = 0.05$. Take $\gamma = 1.333$, and $C_{pg} = 1.147$ kJ/kgK.

Solution

Nozzle throat area, $A = m/\rho_2 Ca_2$

and $\rho_2 = \dfrac{p_2}{RT_2}$

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 1100 - \frac{(550.67)^2}{(2)(1.147)(1000)} \quad (T_{01} = T_{02})$$

i.e., $T_2 = 967.81$ K

From nozzle loss coefficient

$$T_2' = T_2 - \lambda_N \frac{C_2^2}{2C_p} = 967.81 - \frac{0.05 \times (550.67)^2}{(2)(1.147)(1000)} = 961.2 \text{ K}$$

Using isentropic $p-T$ relation for nozzle expansion

$$p_2 = p_{01}/\left(T_{01}/T_2'\right)^{\gamma/(\gamma-1)} = 5/(1100/961.2)^4 = 2.915 \text{ bar}$$

Critical pressure ratio

$$p_{01}/p_c = \left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)} = \left(\frac{2.333}{2}\right)^4 = 1.852$$

or $\quad p_{01}/p_2 = 5/2.915 = 1.715$

Since $\frac{p_{01}}{p_2} < \frac{p_{01}}{p_c}$, and therefore nozzle is unchoked.

Hence nozzle gas velocity at nozzle exit

$$C_2 = \sqrt{[2C_{pg}(T_{01} - T_2)]}$$
$$= \sqrt{[(2)(1.147)(1000)(1100 - 967.81)]} = 550.68 \text{ m/s}$$

Therefore, nozzle throat area

$$A = \frac{m}{\rho_2 C_2}, \text{ and } \rho_2 = \frac{p_2}{RT_2} = \frac{(2.915)(10^2)}{(0.287)(967.81)} = 1.05 \text{ kg/m}^3$$

Thus

$$A = \frac{15}{(1.05)(550.68)} = 0.026 \text{ m}^2$$

Design Example 7.5 In a single-stage turbine, gas enters and leaves the turbine axially. Inlet stagnation temperature is 1000 K, and pressure ratio is 1.8 bar. Gas leaving the stage with velocity 270 m/s and blade speed at root is 290 m/s. Stage isentropic efficiency is 0.85 and degree of reaction is zero. Find the nozzle efflux angle and blade inlet angle at the root radius.

Solution

Since $\Lambda = 0$, therefore

$$\Lambda = \frac{T_2 - T_3}{T_1 - T_3},$$

hence

$$T_2 = T_3$$

From isentropic p–T relation for expansion

$$T'_{03} = \frac{T_{01}}{(p_{01}/p_{03})^{(\gamma-1)/\gamma}} = \frac{1000}{(1.8)^{0.249}} = 863.558 \text{ K}$$

Using turbine efficiency

$$T_{03} = T_{01} - \eta_t\left(T_{01} - T'_{03}\right)$$

$$= 1000 - 0.85(1000 - 863.558) = 884\,\text{K}$$

In order to find static temperature at turbine outlet, using static and stagnation temperature relation

$$T_3 = T_{03} - \frac{C_3^2}{2C_{pg}} = 884 - \frac{270^2}{(2)(1.147)(1000)} = 852\,\text{K} = T_2$$

Dynamic temperature

$$\frac{C_2^2}{2C_{pg}} = 1000 - T_2 = 1000 - 852 = 148\,\text{K}$$

$$C_2 = \sqrt{[(2)(1.147)(148)(1000)]} = 582.677 \text{ m/s}$$

Since, $\quad C_{pg}\Delta T_{os} = U(C_{w3} + C_{w2}) = UC_{w2} \;\; (C_{w3} = 0)$

Therefore, $\quad C_{w2} = \dfrac{(1.147)(1000)(1000 - 884)}{290} = 458.8 \text{ m/s}$

From velocity triangle

$$\sin\alpha_2 = \frac{C_{w2}}{C_2} = \frac{458.8}{582.677} = 0.787$$

That is, $\alpha_2 = 51°54'$

$$\tan\beta_2 = \frac{C_{w2} - U}{Ca_2} = \frac{458.8 - 290}{C_2\cos\alpha_2}$$

$$= \frac{458.8 - 290}{582.677\cos 51.90°} = 0.47$$

i.e., $\beta_2 = 25°9'$

Design Example 7.6 In a single-stage axial flow gas turbine, gas enters the turbine at a stagnation temperature and pressure of 1150 K and 8 bar, respectively. Isentropic efficiency of stage is equal to 0.88, mean blade speed is 300 m/s, and rotational speed is 240 rps. The gas leaves the stage with velocity 390 m/s. Assuming inlet and outlet velocities are same and axial, find the blade height at the outlet conditions when the mass flow of gas is 34 kg/s, and temperature drop in the stage is 145 K.

Solution

Annulus area A is given by

$$A = 2\pi r_m h$$

where $h =$ blade height

$r_m =$ mean radius

As we have to find the blade height from the outlet conditions, in this case annulus area is A_3.

$$\therefore \quad h = \frac{A_3}{2\pi r_m}$$

$$U_m = \pi D_m N$$

or $D_m = \dfrac{(U_m)}{\pi N} = \dfrac{300}{(\pi)(240)} = 0.398$

i.e., $r_m = 0.199\,\text{m}$

Temperature drop in the stage is given by

$$T_{01} - T_{03} = 145\,\text{K}$$

Hence $T_{03} = 1150 - 145 = 1005\,\text{K}$

$$T_3 = T_{03} - \frac{C_3^2}{2C_{pg}} = 1005 - \frac{390^2}{(2)(1.147)(1000)} = 938.697\ \text{K}$$

Using turbine efficiency to find isentropic temperature drop

$$T'_{03} = 1150 - \frac{145}{0.88} = 985.23\ \text{K}$$

Using isentropic p–T relation for expansion process

$$p_{03} = \frac{p_{01}}{\left(T_{01}/T'_{03}\right)^{\gamma/(\gamma-1)}} = \frac{8}{(1150/985.23)^4} = \frac{8}{1.856}$$

i.e., $p_{03} = 4.31$ bar

Also from isentropic relation

$$p_3 = \frac{p_{03}}{\left(T'_{03}/T_3\right)^{\gamma/(\gamma-1)}} = \frac{4.31}{(985.23/938.697)^4} = \frac{4.31}{1.214} = 3.55\ \text{bar}$$

$$\rho_3 = \frac{p_3}{RT_3} = \frac{(3.55)(100)}{(0.287)(938.697)} = 1.32\ \text{kg/m}^3$$

$$A_3 = \frac{m}{\rho_3 Ca_3} = \frac{34}{(1.32)(390)} = 0.066\ \text{m}^2$$

Finally,

$$h = \frac{A_3}{2\pi r_m} = \frac{0.066}{(2\pi)(0.199)} = 0.053\,\text{m}$$

Design Example 7.7 The following data refer to a single-stage axial flow gas turbine with convergent nozzle:

Inlet stagnation temperature, T_{01}	1100 K
Inlet stagnation pressure, p_{01}	4 bar
Pressure ratio, p_{01}/p_{03}	1.9
Stagnation temperature drop	145 K
Mean blade speed	345 m/s
Mass flow, m	24 kg/s
Rotational speed	14, 500 rpm
Flow coefficient, Φ	0.75
Angle of gas leaving the stage	12°

$$C_{pg} = 1147\,\text{J/kg K}, \quad \gamma = 1.333, \quad \lambda_N = 0.05$$

Assuming the axial velocity remains constant and the gas velocity at inlet and outlet are the same, determine the following quantities at the mean radius:

(1) The blade loading coefficient and degree of reaction
(2) The gas angles
(3) The nozzle throat area

Solution

(1) $$\Psi = \frac{C_{pg}(T_{01} - T_{03})}{U^2} = \frac{(1147)(145)}{345^2} = 1.4$$

Using velocity diagram

$$U/Ca = \tan\beta_3 - \tan\alpha_3$$

or $$\tan\beta_3 = \frac{1}{\Phi} + \tan\alpha_3$$

$$= \frac{1}{0.75} + \tan 12°$$

$$\beta_3 = 57.1°$$

From Equations (7.14) and (7.15), we have

$$\Psi = \Phi\left(\tan\beta_2 + \tan\beta_3\right)$$

and

$$\Lambda = \frac{\Phi}{2}\left(\tan\beta_3 - \tan\beta_2\right)$$

From which

$$\tan\beta_3 = \frac{1}{2\Phi}(\Psi + 2\Lambda)$$

Therefore

$$\tan 57.1^\circ = \frac{1}{2\times 0.75}(1.4 + 2\Lambda)$$

Hence

$$\Lambda = 0.4595$$

(2)
$$\tan\beta_2 = \frac{1}{2\Phi}(\Psi - 2\Lambda)$$

$$= \frac{1}{2\times 0.75}(1.4 - [2][0.459])$$

$$\beta_2 = 17.8^\circ$$

$$\tan\alpha_2 = \tan\beta_2 + \frac{1}{\Phi}$$

$$= \tan 17.8^\circ + \frac{1}{0.75} = 0.321 + 1.33 = 1.654$$

$$\alpha_2 = 58.8^\circ$$

(3)
$$Ca_1 = U\Phi$$

$$= (345)(0.75) = 258.75 \text{ m/s}$$

$$C_2 = \frac{Ca_1}{\cos\alpha_2} = \frac{258.75}{\cos 58.8^\circ} = 499.49 \text{ m/s}$$

$$T_{02} - T_2 = \frac{C_2^2}{2C_p} = \frac{499.49^2}{(2)(1147)} = 108.76\,\text{K}$$

$$T_2 - T_{2s} = \frac{(T_N)(499.49^2)}{(2)(1147)} = \frac{(0.05)(499.49^2)}{(2)(1147)} = 5.438\,\text{K}$$

$$T_{2s} = T_2 - 5.438$$

$$T_2 = 1100 - 108.76 = 991.24\,\text{K}$$

$$T_{2s} = 991.24 - 5.438 = 985.8\,\text{K}$$

$$\frac{p_{01}}{p_2} = \left(\frac{T_{01}}{T_{2s}}\right)^{\gamma/(\gamma-1)}$$

$$p_2 = 4 \times \left(\frac{985.8}{1100}\right)^4 = 2.58$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{(2.58)(100)}{(0.287)(991.24)} = 0.911\ \text{kg/m}^3$$

(4) Nozzle throat area $= \dfrac{m}{\rho_1 C_1} = \dfrac{24}{(0.907)(499.49)} = 0.053\ \text{m}^2$

$$A_1 = \frac{m}{\rho_1 Ca_1} = \frac{24}{(0.907)(258.75)} = 0.102\ \text{m}^2$$

Design Example 7.8 A single-stage axial flow gas turbine with equal stage inlet and outlet velocities has the following design data based on the mean diameter:

Mass flow	20 kg/s
Inlet temperature, T_{01}	1150K
Inlet pressure	4 bar
Axial flow velocity constant through the stage	255 m/s
Blade speed, U	345 m/s
Nozzle efflux angle, α_2	60°
Gas-stage exit angle	12°

Calculate (1) the rotor-blade gas angles, (2) the degree of reaction, blade-loading coefficient, and power output and (3) the total nozzle throat area if the throat is situated at the nozzle outlet and the nozzle loss coefficient is 0.05.

Solution

(1) From the velocity triangles

$$C_{w2} = Ca \tan \alpha_2$$

$$= 255 \tan 60° = 441.67\ \text{m/s}$$

$$C_{w3} = Ca \tan \alpha_3 = 255 \tan 12° = 55.2\ \text{m/s}$$

$$V_{w2} = C_{w2} - U = 441.67 - 345 = 96.67\ \text{m/s}$$

$$\beta_2 = \tan^{-1} \frac{V_{w2}}{Ca} = \tan^{-1} \frac{96.67}{255} = 20.8°$$

Also $V_{w3} = C_{w3} + U = 345 + 55.2 = 400.2\ \text{m/s}$

$$\therefore\ \beta_3 = \tan^{-1} \frac{V_{w3}}{Ca} = \tan^{-1} \frac{400.2}{255} = 57.5°$$

(2) $$\Lambda = \frac{\Phi}{2}\left(\tan\beta_3 - \tan\beta_2\right)$$

$$= \frac{255}{2\times 345}(\tan 57.5^\circ - \tan 20.8^\circ) = 0.44$$

$$\Psi = \frac{Ca}{U}\left(\tan\beta_2 + \tan\beta_3\right)$$

$$= \frac{255}{345}(\tan 20.8^\circ + \tan 57.5^\circ) = 1.44$$

Power $W = mU(C_{w2} + C_{w3})$

$$= (20)(345)(441.67 + 54.2) = 3421.5\,\text{kW}$$

(3) $$\lambda_N = \frac{C_p\left(T_2 - T_2'\right)}{\frac{1}{2}C_2^2},\ C_2 = Ca\ \sec\alpha_2 = 255\sec 60^\circ = 510\ \text{m/s}$$

or $$T_2 - T_2' = \frac{(0.05)(0.5)(510^2)}{1147} = 5.67$$

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 1150 - \frac{510^2}{(2)(1147)} = 1036.6\,\text{K}$$

$$T_2' = 1036.6 - 5.67 = 1030.93\,\text{K}$$

$$\frac{p_{01}}{p_2} = \left(\frac{T_{01}}{T_2}\right)^{\gamma/(\gamma-1)} = \left(\frac{1150}{1030.93}\right)^4 = 1.548$$

$$p_2 = \frac{4}{1.548} = 2.584\,\text{bar}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{2.584\times 100}{0.287\times 1036.6} = 0.869\,\text{kg/m}^3$$

$$m = \rho_2 A_2 C_2$$

$$A_2 = \frac{20}{0.869\times 510} = 0.045\,\text{m}^2$$

Illustrative Example 7.9 A single-stage axial flow gas turbine has the following data

Mean blade speed	340 m/s
Nozzle exit angle	15°
Axial velocity (constant)	105 m/s
Turbine inlet temperature	900°C
Turbine outlet temperature	670°C
Degree of reaction	50%

Calculate the enthalpy drop per stage and number of stages required.

Solution

At 50%,

$$\alpha_2 = \beta_3$$

$$\alpha_3 = \beta_2$$

$$C_2 = \frac{U}{\cos 15°} = \frac{340}{\cos 15°} = 351.99 \text{ m/s}$$

$$\text{Heat drop in blade moving row} = \frac{C_2^2 - C_3^2}{2C_p} = \frac{(351.99)^2 - (105)^2}{(2)(1147)}$$

$$= \frac{123896.96 - 11025}{(2)(1147)}$$

$$= 49.2 \text{ K}$$

$$\text{Therefore heat drop in a stage} = (2)(49.2) = 98.41 \text{ K}$$

$$\text{Number of stages} = \frac{1173 - 943}{98.41} = \frac{230}{98.4} = 2$$

Design Example 7.10 The following particulars relate to a single-stage turbine of free vortex design:

Inlet temperature, T_{01}	1100K
Inlet pressure, p_{01}	4 bar
Mass flow	20 kg/s
Axial velocity at nozzle exit	250 m/s
Blade speed at mean diameter	300 m/s
Nozzle angle at mean diameter	25°
Ratio of tip to root radius	1.4

The gas leaves the stage in an axial direction, find:

(1) The total throat area of the nozzle.
(2) The nozzle efflux angle at root and tip.
(3) The work done on the turbine blades.

Take

$$C_{pg} = 1.147\,\text{kJ/kg K}, \ \gamma = 1.33$$

Solution

For no loss up to throat

$$\frac{p^*}{p_{01}} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} = \left(\frac{2}{2.33}\right)^4 = 0.543$$

$$p^* = 4 \times 0.543 = 2.172\,\text{bar}$$

Also $T^* = 1100\left(\frac{2}{2.33}\right)^4 = 944\,\text{K}$

$$T_{01} = T^* + \frac{C^2}{2C_{pg}}$$

$$C^* = \sqrt{2C_{pg}\left(T_{01} - T^*\right)}$$

$$= \sqrt{(2)(1147)(1100 - 944)} = 598 \ \text{m/s}$$

$$\rho^* = \frac{p^*}{RT^*} = \frac{(2.172)(100)}{(0.287)(944)} = 0.802\,\text{kg/m}^3$$

(1) Throat area

$$A = \frac{m}{\rho C^*} = \frac{20}{(0.802)(598)} = 0.042\,\text{m}^2$$

(2) Angle α_1, at any radius r and $\alpha_{1\text{m}}$ at the design radius r_m are related by the equation

$$\tan\alpha_1 = \frac{r_\text{m}}{r_1}\tan\alpha_{1\text{m}}$$

Given

$$\frac{\text{Tip radius}}{\text{Root radius}} = \frac{r_t}{r_r} = 1.4$$

$$\therefore \quad \frac{\text{Mean radius}}{\text{Root radius}} = 1.2$$

$$\alpha_{1m} = 25°$$

$$\tan \alpha_{1r} = \frac{r_{\text{mean}}}{r_{\text{root}}} \times \tan \alpha_{1m}$$

$$= 1.2 \times \tan 25° = 0.5596$$

$$\therefore \quad \alpha_{1r} = 29.23°$$

$$\tan \alpha_{1t} = \frac{r_r}{r_t} \times \tan \alpha_{1r} = \left(\frac{1}{1.4}\right)(0.5596) = 0.3997$$

$$\therefore \quad \alpha_{1t} = 21.79°$$

(3) $$C_{w2} = \frac{r_m}{r_r} x C_{w2m} = \frac{r_m}{r_r} \frac{Ca_2}{\tan \alpha_{2m}} = 1.2x \frac{250}{\tan 25°} = 643 \text{ m/s}$$

$$W = mUC_{w2} = \frac{(20)(300)(643)}{1000} = 3858 \text{ kW}$$

7.8 RADIAL FLOW TURBINE

In Sec. 7.1 "Introduction to Axial Flow Turbines", it was pointed out that in axial flow turbines the fluid moves essentially in the axial direction through the rotor. In the radial type the fluid motion is mostly radial. The mixed flow machine is characterized by a combination of axial and radial motion of the fluid relative to the rotor. The choice of turbine depends on the application, though it is not always clear that any one type is superior. For small mass flows, the radial machine can be made more efficient than the axial one. The radial turbine is capable of a high-pressure ratio per stage than the axial one. However, multistaging is very much easier to arrange with the axial turbine, so that large overall pressure ratios are not difficult to obtain with axial turbines. The radial flow turbines are used in turbochargers for commercial (diesel) engines and fire pumps. They are very compact, the maximum diameter being about 0.2 m, and run at very high speeds. In inward flow radial turbine, gas enters in the radial direction and leaves axially at outlet. The rotor, which is usually manufactured of

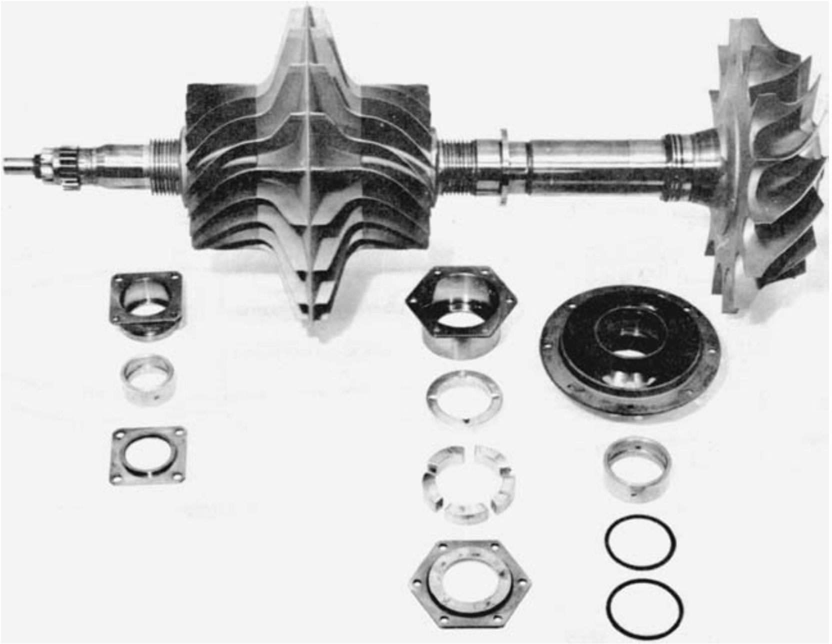


Figure 7.8 Radial turbine photograph of the rotor on the right.

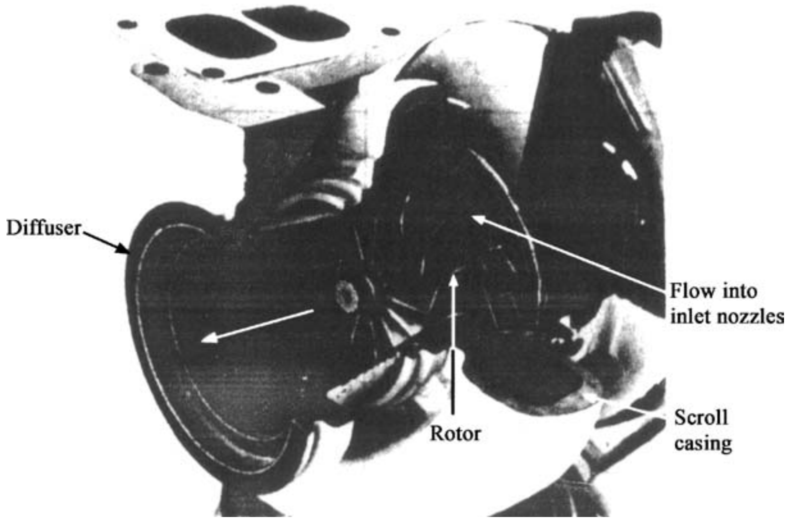


Figure 7.9 Elements of a 90° inward flow radial gas turbine with inlet nozzle ring.

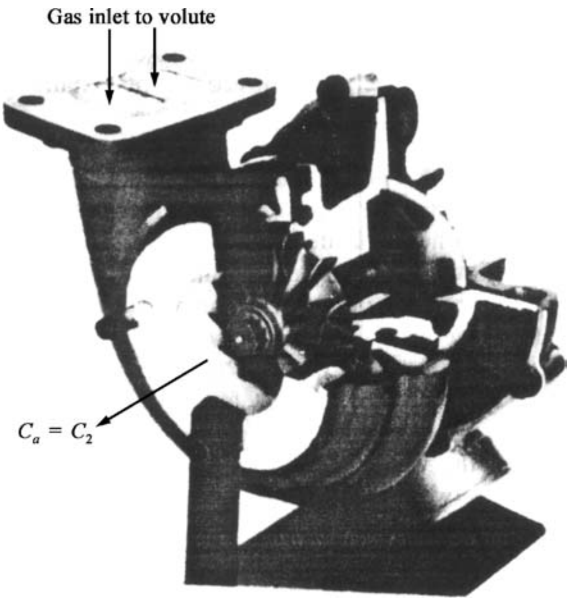


Figure 7.10 A 90° inward flow radial gas turbine without nozzle ring.

cast nickel alloy, has blades that are curved to change the flow from the radial to the axial direction. Note that this turbine is like a single-faced centrifugal compressor with reverse flow. Figures 7.8–7.10 show photographs of the radial turbine and its essential parts.

7.9 VELOCITY DIAGRAMS AND THERMODYNAMIC ANALYSIS

Figure 7.11 shows the velocity triangles for this turbine. The same nomenclature that we used for axial flow turbines, will be used here. Figure 7.12 shows the Mollier diagram for a 90° flow radial turbine and diffuser.

As no work is done in the nozzle, we have $h_{01} = h_{02}$. The stagnation pressure drops from p_{01} to p_1 due to irreversibilities. The work done per unit mass flow is given by Euler's turbine equation

$$W_t = (U_2C_{w2} - U_3C_{w3}) \tag{7.36}$$

If the whirl velocity is zero at exit then

$$W_t = U_2C_{w2} \tag{7.37}$$

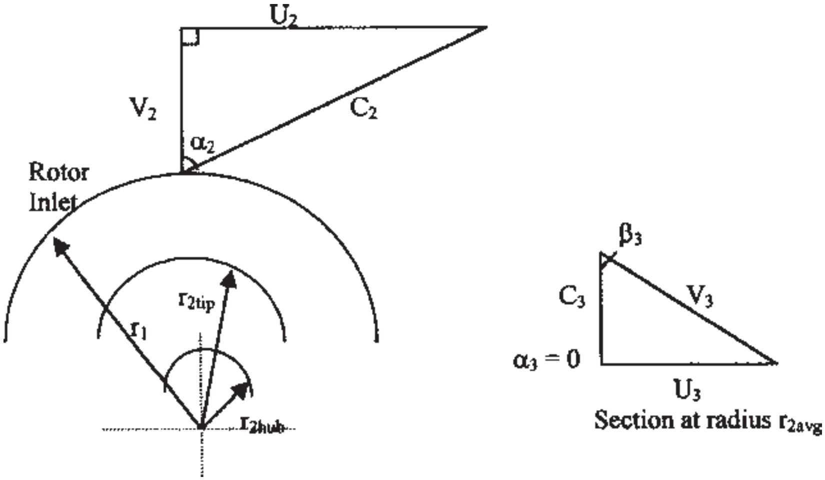


Figure 7.11 Velocity triangles for the 90° inward flow radial gas turbine.

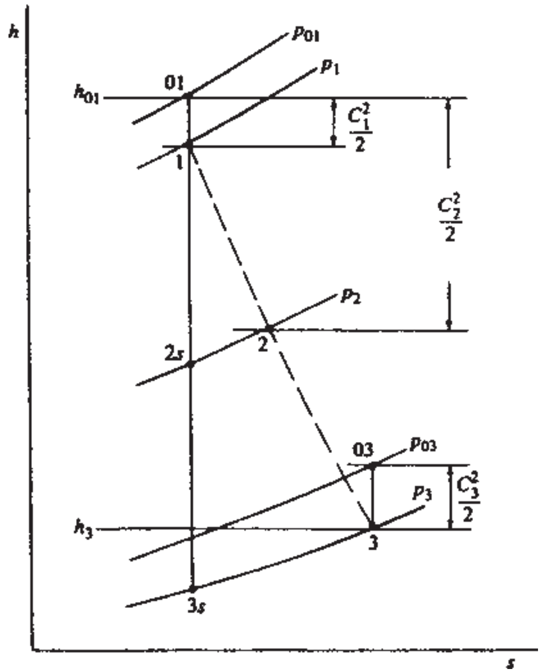


Figure 7.12 Mollier chart for expansion in a 90° inward flow radial gas turbine.

For radial relative velocity at inlet

$$W_t = U_2^2 \tag{7.38}$$

In terms of enthalpy drop

$$h_{02} - h_{03} = U_2 C_{w2} - U_3 C_{w3}$$

Using total-to-total efficiency

$$\eta_{tt} = \frac{T_{01} - T_{03}}{T_{01} - T_{03\,ss}},$$

efficiency being in the region of 80–90%

7.10 SPOUTING VELOCITY

It is that velocity, which has an associated kinetic energy equal to the isentropic enthalpy drop from turbine inlet stagnation pressure p_{01} to the final exhaust pressure. Spouting velocities may be defined depending upon whether total or static conditions are used in the related efficiency definition and upon whether or not a diffuser is included with the turbine. Thus, when no diffuser is used, using subscript 0 for spouting velocity.

$$\frac{1}{2} C_0^2 = h_{01} - h_{03\,ss} \tag{7.39}$$

or

$$\frac{1}{2} C_0^2 = h_{01} - h_{3\,ss} \tag{7.40}$$

for the total and static cases, respectively.

Now for isentropic flow throughout work done per unit mass flow

$$W = U_2^2 = C_0^2/2 \tag{7.41}$$

or

$$U_2/C_0 = 0.707 \tag{7.42}$$

In practice, U_2/C_0 lies in the range $0.68 < \frac{U_2}{C_0} < 0.71$.

7.11 TURBINE EFFICIENCY

Referring to Fig. 7.12, the total-to-static efficiency, without diffuser, is defined as

$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3\,ss}}$$

$$= \frac{W}{W + \frac{1}{2}C_3^2 + (h_3 - h_{3ss}) + (h_{3s} - h_{3\,ss})} \tag{7.43}$$

Nozzle loss coefficient, ξ_n, is defined as

$$\xi_n = \frac{\text{Enthalpy loss in nozzle}}{\text{Kinetic energy at nozzle exit}}$$

$$= \frac{h_{3s} - h_{3\,ss}}{0.5C_2^2(T_3/T_2)} \tag{7.44}$$

Rotor loss coefficient, ξ_r, is defined as

$$\xi_r = \frac{h_3 - h_{3s}}{0.5V_3^2} \tag{7.45}$$

But for constant pressure process,

$$T\,\mathrm{d}s = \mathrm{d}h,$$

and, therefore

$$h_{3s} - h_{3ss} = (h - h_{2s})(T_3/T_2)$$

Substituting in Eq. (7.43)

$$\eta_{ts} = \left[1 + \frac{1}{2}\left(C_3^2 + V_3^2\xi_r + C_2\xi_n T_3/T_2\right)W\right]^{-1} \tag{7.46}$$

Using velocity triangles

$$C_2 = U_2 \operatorname{cosec}\alpha_2, V_3 = U_3 \operatorname{cosec}\beta_3, C_3 = U_3 \cot\beta_3, W = U_2^2$$

Substituting all those values in Eq. (7.44) and noting that $U_3 = U_2\, r_3/r_2$, then

$$\eta_{ts} = \left[1 + \frac{1}{2}\left\{\xi_n \frac{T_3}{T_2}\operatorname{cosec}^2\alpha_2 + \left(\frac{r_3}{r_2}\right)^2\left(\xi_r \operatorname{cosec}^2\beta_3 + \cot^2\beta_3\right)\right\}\right]^{-1} \tag{7.47}$$

Taking mean radius, that is,

$$r_3 = \frac{1}{2}(r_{3t} + r_{3h})$$

Using thermodynamic relation for T_3/T_2, we get

$$\frac{T_3}{T_2} = 1 - \frac{1}{2}(\gamma - 1)\left(\frac{U_2}{a_2}\right)^2\left[1 - \cot^2\alpha_2 + \left(\frac{r_3}{r_2}\right)^2\cot^2\beta_3\right]$$

But the above value of T_3/T_2 is very small, and therefore usually neglected. Thus

$$\eta_{ts} = \left[1 + \frac{1}{2}\left\{\xi_n \operatorname{cosec}^2\alpha_2 + \left(\frac{r_{3\,av}}{r_2}\right)^2\left(\xi_r \operatorname{cosec}^2\beta_{3\,av} + \cot^2\beta_{3\,av}\right)\right\}\right]^{-1} \tag{7.48}$$

Equation (7.46) is normally used to determine total-to-static efficiency. The η_{ts} can also be found by rewriting Eq. (7.43) as

$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3\,ss}} = \frac{(h_{01} - h_{3\,ss}) - (h_{03} - h_3) - (h_3 - h_{3s}) - (h_{3s} - h_{3\,ss})}{(h_{01} - h_{3ss})}$$

$$= 1 - \left(C_3^2 + \xi_n C_2^2 + \xi_r V_3^2\right)/C_0^2 \tag{7.49}$$

where spouting velocity C_0 is given by

$$h_{01} - h_{3\,ss} = \frac{1}{2}C_0^2 = C_p T_{01}\left[1 - \left(p_3/p_{01}\right)^{\gamma - 1/\gamma}\right] \tag{7.50}$$

The relationship between η_{ts} and η_{tt} can be obtained as follows:

$$W = U_2^2 = \eta_{ts} W_{ts} = \eta_{ts}(h_{01} - h_{3ss}), \text{ then}$$

$$\eta_{tt} = \frac{W}{W_{ts} - \frac{1}{2}C_3^2} = \frac{1}{\frac{1}{\eta_{ts}} - \frac{C_3^2}{2W}}$$

$$\therefore \frac{1}{\eta_{tt}} = \frac{1}{\eta_{ts}} - \frac{C_3^2}{2W} = \frac{1}{\eta_{ts}} - \frac{1}{2}\left(\frac{r_{3\,av}}{r_2} - \cot\beta_{3\,av}\right)^2 \tag{7.51}$$

Loss coefficients usually lie in the following range for 90° inward flow turbines

$$\xi_n = 0.063\text{–}0.235$$

and

$$\xi_r = 0.384\text{–}0.777$$

7.12 APPLICATION OF SPECIFIC SPEED

We have already discussed the concept of specific speed N_s in Chapter 1 and some applications of it have been made already. The concept of specific speed was applied almost exclusively to incompressible flow machines as an important parameter in the selection of the optimum type and size of unit. The volume flow rate through hydraulic machines remains constant. But in radial flow gas turbine, volume flow rate changes significantly, and this change must be taken into account. According to Balje, one suggested value of volume flow rate is that at the outlet Q_3.

Using nondimensional form of specific speed

$$N_s = \frac{NQ_3^{1/2}}{(\Delta h_0')^{3/4}} \tag{7.52}$$

where N is in rev/s, Q_3 is in m^3/s and isentropic total-to-total enthalpy drop (from turbine inlet to outlet) is in J/kg. For the 90° inward flow radial turbine,

$U_2 = \pi N D_2$ and $\Delta h_{0s} = \frac{1}{2}C_0^2$, factorizing the Eq. (7.52)

$$N_s = \frac{Q_3^{1/2}}{\left(\frac{1}{2}C_0^2\right)^{3/4}}\left(\frac{U_2}{\pi D_2}\right)\left(\frac{U_2}{\pi N D_2}\right)^{1/2}$$

$$= \left(\frac{\sqrt{2}}{\pi}\right)^{3/2}\left(\frac{U_2}{C_0}\right)^{3/2}\left(\frac{Q_3}{N D_2^3}\right)^{1/2} \tag{7.53}$$

For 90° inward flow radial turbine, $U_2/C_0 = \frac{1}{\sqrt{2}} = 0.707$, substituting this value in Eq. (7.53),

$$N_s = 0.18\left(\frac{Q_3}{N D_2^3}\right)^{1/2}, \quad \text{rev} \tag{7.54}$$

Equation (7.54) shows that specific speed is directly proportional to the square root of the volumetric flow coefficient. Assuming a uniform axial velocity at rotor exit C_3, so that $Q_3 = A_3 C_3$, rotor disc area $A_d = \pi D_2^2/4$, then

$$N = U_2/(\pi D_2) = \frac{C_0\sqrt{2}}{2\pi D_2}$$

$$\frac{Q_3}{N D_2^3} = \frac{A_3 C_3 2\pi D_2}{\sqrt{2} C_0 D_2^2} = \frac{A_3}{A_d}\frac{C_3}{C_0}\frac{\pi^2}{2\sqrt{2}}$$

Therefore,

$$N_s = 0.336\left(\frac{C_3}{C_0}\right)^{\frac{1}{2}}\left(\frac{A_3}{A_d}\right)^{\frac{1}{2}}, \quad \text{rev} \tag{7.55}$$

$$= 2.11\left(\frac{C_3}{C_0}\right)^{\frac{1}{2}}\left(\frac{A_3}{A_d}\right)^{\frac{1}{2}}, \quad \text{rad} \tag{7.56}$$

Suggested values for C_3/C_o and A_3/A_d are as follows:

$$0.04 < C_3/C_0 < 0.3$$

$$0.1 < A_3/A_d < 0.5$$

Then $\quad 0.3 < N_s < 1.1, \quad \text{rad}$

Thus the N_s range is very small and Fig. 7.13 shows the variation of efficiency with N_s, where it is seen to match the axial flow gas turbine over the limited range of N_s.

Design Example 7.11 A small inward radial flow gas turbine operates at its design point with a total-to-total efficiency of 0.90. The stagnation pressure and temperature of the gas at nozzle inlet are 310 kPa and 1145K respectively. The flow leaving the turbine is diffused to a pressure of 100 kPa and the velocity of

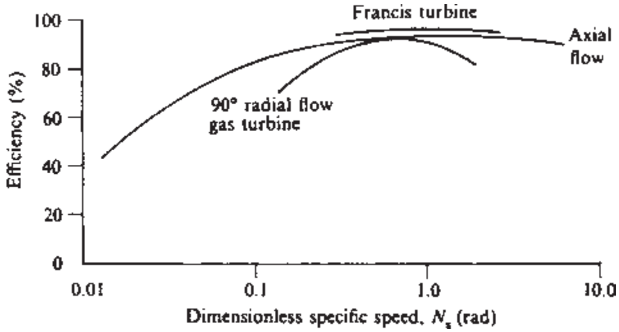


Figure 7.13 Variation of efficiency with dimensionless specific speed.

flow is negligible at that point. Given that the Mach number at exit from the nozzles is 0.9, find the impeller tip speed and the flow angle at the nozzle exit. Assume that the gas enters the impeller radially and there is no whirl at the impeller exit. Take

$$C_p = 1.147 \quad \text{kJ/kg K}, \quad \gamma = 1.333.$$

Solution

The overall efficiency of turbine from nozzle inlet to diffuser outlet is given by

$$\eta_{tt} = \frac{T_{01} - T_{03}}{T_{01} - T_{03\,ss}}$$

Turbine work per unit mass flow

$$W = U_2^2 = C_p(T_{01} - T_{03}), \quad (C_{w3} = 0)$$

Now using isentropic p–T relation

$$T_{01}\left(1 - \frac{T_{03ss}}{T_{01}}\right) = T_{01}\left[1 - \left(\frac{p_{03}}{p_{01}}\right)^{\gamma-1/\gamma}\right]$$

Therefore

$$U_2^2 = \eta_{tt} C_p T_{01}\left[1 - \left(\frac{p_{03}}{p_{01}}\right)^{\gamma-1/\gamma}\right]$$

$$= 0.9 \times 1147 \times 1145\left[1 - \left(\frac{100}{310}\right)^{0.2498}\right]$$

$\therefore$ Impeller tip speed, $U_2 = 539.45$ m/s

The Mach number of the absolute flow velocity at nozzle exit is given by

$$M = \frac{C_1}{a_1} = \frac{U_1}{a_1 \sin \alpha_1}$$

Since the flow is adiabatic across the nozzle, we have

$$T_{01} = T_{02} = T_2 + \frac{C_2^2}{2C_p} = T_2 + \frac{U_2^2}{2C_p \sin^2 \alpha_2}$$

or $$\frac{T_2}{T_{01}} = 1 - \frac{U_2^2}{2C_p T_{01} \sin^2 \alpha_2}, \text{ but } C_p = \frac{\gamma R}{\gamma - 1}$$

$$\therefore \frac{T_2}{T_{01}} = 1 - \frac{U_2^2(\gamma - 1)}{2\gamma R T_{01} \sin^2 \alpha_2} = 1 - \frac{U_2^2(\gamma - 1)}{2a_{01}^2 \sin^2 \alpha_2}$$

But $$\left(\frac{T_2}{T_{01}}\right)^2 = \frac{a_2}{a_{01}} = \frac{a_2}{a_{02}} \qquad \text{since } T_{01} = T_{02}$$

and $$\frac{a_2}{a_{02}} = \frac{U_2}{M_2 a_{02} \sin \alpha_2}$$

$$\therefore \left(\frac{U_2}{M_2 a_{02} \sin \alpha_2}\right)^2 = 1 - \frac{U_2^2(\gamma - 1)}{2a_{02}^2 \sin^2 \alpha_2}$$

and $$1 = \left(\frac{U_2}{a_{02} \sin \alpha_2}\right)^2 \left(\frac{(\gamma - 1)}{2} + \frac{1}{M_2^2}\right)$$

or $$\sin^2 \alpha_2 = \left(\frac{U_2}{a_{02}}\right)^2 \left(\frac{(\gamma - 1)}{2} + \frac{1}{M_2^2}\right)$$

But $$a_{02}^2 = \gamma R T_{02} = (1.333)(287)(1145) = 438043 \text{ m}^2/\text{s}^2$$

$$\therefore \sin^2 \alpha_2 = \frac{539.45^2}{438043} \left(\frac{0.333}{2} + \frac{1}{0.9^2}\right) = 0.9311$$

Therefore nozzle angle $\alpha_2 = 75°$

Illustrative Example 7.12 The following particulars relate to a small inward flow radial gas turbine.

Rotor inlet tip diameter	92 mm
Rotor outlet tip diameter	64 mm
Rotor outlet hub diameter	26 mm
Ratio C_3/C_0	0.447

Ratio U_2/C_0 (ideal)	0.707
Blade rotational speed	30,500 rpm
Density at impeller exit	1.75 kg/m^3

Determine

(1) The dimensionless specific speed of the turbine.
(2) The volume flow rate at impeller outlet.
(3) The power developed by the turbine.

Solution

(1) Dimensionless specific speed is

$$N_s = 0.336\left(\frac{C_3}{C_0}\right)^{\frac{1}{2}}\left(\frac{A_3}{A_d}\right)^{\frac{1}{2}}, \text{ rev}$$

Now

$$A_3 = \frac{\pi\left(D_{3t}^2 - D_{3h}^2\right)}{4}$$

$$= \frac{\pi\left(0.064^2 - 0.026^2\right)}{4} = (2.73)(10^{-3})\ \mathrm{m}^2$$

$$A_d = \frac{\pi D_2^2}{4} = \left(\frac{\pi}{4}\right)(0.092^2) = (6.65)(10^{-3})\ \mathrm{m}^2$$

Dimensionless specific speed

$$N_s = 0.336\left(\frac{[0.447][2.73]}{6.65}\right)^{\frac{1}{2}}$$

$$= 0.144 \text{ rev}$$

$$= 0.904 \text{ rad}$$

(2) The flow rate at outlet for the ideal turbine is given by Eq. (7.54).

$$N_s = 0.18\left(\frac{Q_3}{ND_2^3}\right)^{1/2}$$

$$0.144 = 0.18\left(\frac{[Q_3][60]}{[30,500][0.092^3]}\right)^{1/2}$$

Hence

$$Q_3 = 0.253\,\mathrm{m}^3/\mathrm{s}$$

(3) The power developed by the turbine is given by

$$\begin{aligned} W_t &= \dot{m}U_3^2 \\ &= \rho_3 Q_3 U_3^2 \\ &= 1.75 \times 0.253 \times \left(\frac{\pi N D_2}{60}\right)^2 \\ &= 1.75 \times 0.253 \times \left(\frac{[\pi][30,500][0.092]}{60}\right)^2 \\ &= 9.565 \text{ kW} \end{aligned}$$

PROBLEMS

7.1 A single-stage axial flow gas turbine has the following data:

Inlet stagnation temperature	1100K
The ratio of static pressure at the nozzle exit to the stagnation pressure at the nozzle inlet	0.53
Nozzle efficiency	0.93
Nozzle angle	20°
Mean blade velocity	454 m/s
Rotor efficiency	0.90
Degree of reaction	50%

$$C_{pg} = 1.147 \quad \text{kJ/kgK}, \quad \gamma = 1.33$$

Find (1) the work output per kg/s of air flow, (2) the ratio of the static pressure at the rotor exit to the stagnation pressure at the nozzle inlet, and (3) the total-to-total stage efficiency.

(282 kW, 0.214, 83.78%)

7.2 Derive an equation for the degree of reaction for a single-stage axial flow turbine and show that for 50% reaction blading $\alpha_2 = \beta_3$ and $\alpha_3 = \beta_2$.

7.3 For a free-vortex turbine blade with an impulse hub show that degree of reaction

$$\Lambda = 1 - \left(\frac{r_h}{r}\right)^2$$

where r_h is the hub radius and r is any radius.

7.4 A 50% reaction axial flow gas turbine has a total enthalpy drop of 288 kJ/kg. The nozzle exit angle is 70°. The inlet angle to the rotating blade row is inclined at 20° with the axial direction. The axial velocity is constant through the stage. Calculate the enthalpy drop per row of moving blades and the number of stages required when mean blade speed is 310 m/s. Take $C_{pg} = 1.147$ kJ/kgK, $\gamma = 1.33$.

(5 stages)

7.5 Show that for zero degree of reaction, blade-loading coefficient, $\Psi = 2$.

7.6 The inlet stagnation temperature and pressure for an axial flow gas turbine are 1000K and 8 bar, respectively. The exhaust gas pressure is 1.2 bar and isentropic efficiency of turbine is 85%. Assume gas is air, find the exhaust stagnation temperature and entropy change of the gas.

(644K, -0.044 kJ/kgK)

7.7 The performance date from inward radial flow exhaust gas turbine are as follows:

Stagnation pressure at inlet to nozzles, p_{01}	705 kPa
Stagnation temperature at inlet to nozzles, T_{01}	1080K
Static pressure at exit from nozzles, p_2	515 kPa
Static temperature at exit from nozzles, T_2	1000K
Static pressure at exit from rotor, p_3	360 kPa
Static temperature at exit from rotor, T_3	923K
Stagnation temperature at exit from rotor, T_{03}	925K
Ratio $\frac{r_{2\,\text{av}}}{r_2}$	0.5
Rotational speed, N	25, 500 rpm

The flow into the rotor is radial and at exit the flow is axial at all radii. Calculate (1) the total-to-static efficiency of the turbine, (2) the impeller tip diameter, (3) the enthalpy loss coefficient for the nozzle and rotor rows, (4) the blade outlet angle at the mean diameter, and (5) the total-to-total efficiency of the turbine.

[(1) 93%, (2) 0.32 m, (3) 0.019, 0.399, (4) 72.2°, (5) 94%]

NOTATION

A	area
C	absolute velocity

C_0	spouting velocity
h	enthalpy, blade height
N	rotation speed
N_s	specific speed
P	pressure
r_m	mean radius
T	temperature
U	rotor speed
V	relative velocity
Y_N	nozzle loss coefficient in terms of pressure
α	angle with absolute velocity
β	angle with relative velocity
ΔT_{0s}	stagnation temperature drop in the stage
ΔT_s	static temperature drop in the stage
ε_n	nozzle loss coefficient in radial flow turbine
ε_r	rotor loss coefficient in radial flow turbine
ϕ	flow coefficient
η_s	isentropic efficiency of stage
Λ	degree of reaction

8

Cavitation in Hydraulic Machinery

8.1 INTRODUCTION

Cavitation is caused by local vaporization of the fluid, when the local static pressure of a liquid falls below the vapor pressure of the liquid. Small bubbles or cavities filled with vapor are formed, which suddenly collapse on moving forward with the flow into regions of high pressure. These bubbles collapse with tremendous force, giving rise to as high a pressure as 3500 atm. In a centrifugal pump, these low-pressure zones are generally at the impeller inlet, where the fluid is locally accelerated over the vane surfaces. In turbines, cavitation is most likely to occur at the downstream outlet end of a blade on the low-pressure leading face. When cavitation occurs, it causes the following undesirable effects:

1. Local pitting of the impeller and erosion of the metal surface.
2. Serious damage can occur from prolonged cavitation erosion.
3. Vibration of machine; noise is also generated in the form of sharp cracking sounds when cavitation takes place.
4. A drop in efficiency due to vapor formation, which reduces the effective flow areas.

The avoidance of cavitation in conventionally designed machines can be regarded as one of the essential tasks of both pump and turbine designers. This cavitation imposes limitations on the rate of discharge and speed of rotation of the pump.

8.2 STAGES AND TYPES OF CAVITATION

The term *incipient stage* describes cavitation that is just barely detectable. The discernible bubbles of incipient cavitation are small, and the zone over which cavitation occurs is limited. With changes in conditions (pressure, velocity, temperature) toward promoting increased vaporization rates, cavitation grows; the succeeding stages are distinguished from the incipient stage by the term *developed*.

Traveling cavitation is a type composed of individual transient cavities or bubbles, which form in the liquid, as they expand, shrink, and then collapse. Such traveling transient bubbles may appear at the low-pressure points along a solid boundary or in the liquid interior either at the cores of moving vortices or in the high-turbulence region in a turbulent shear field.

The term *fixed cavitation* refers to the situation that sometimes develops after inception, in which the liquid flow detaches from the rigid boundary of an immersed body or a flow passage to form a pocket or cavity attached to the boundary. The attached or fixed cavity is stable in a quasi-steady sense. Fixed cavities sometimes have the appearance of a highly turbulent boiling surface.

In *vortex cavitation*, the cavities are found in the cores of vortices that form in zones of high shear. The cavitation may appear as traveling cavities or as a fixed cavity. Vortex cavitation is one of the earliest observed types, as it often occurs on the blade tips of ships' propellers. In fact, this type of cavitation is often referred to as "tip" cavitation. Tip cavitation occurs not only in open propellers but also in ducted propellers such as those found in propeller pumps at hydrofoil tips.

8.2.1 Cavitation on Moving Bodies

There is no essential difference between cavitation in a flowing stream and that on a body moving through a stationary liquid. In both cases, the important factors are the relative velocities and the absolute pressures. When these are similar, the same types of cavitation are found. One noticeable difference is that the turbulence level in the stationary liquid is lower. Many cases of cavitation in a flowing stream occur in relatively long flow passages in which the turbulence is fully developed before the liquid reaches the cavitation zone. Hydraulic machinery furnishes a typical example of a combination of the two conditions. In the casing, the moving liquid flows past stationary guide surfaces; in the runner, the liquid and the guide surfaces are both in motion.

8.2.2 Cavitation Without Major Flow—Vibratory Cavitation

The types of cavitation previously described have one major feature in common. It is that a particular liquid element passes through the cavitation zone only once. *Vibratory cavitation* is another important type of cavitation, which does not have this characteristic. Although it is accompanied sometimes by continuous flow, the velocity is so low that a given element of liquid is exposed to many cycles of cavitation (in a time period of the order of milliseconds) rather than only one. In vibratory cavitation, the forces causing the cavities to form and collapse are due to a continuous series of high-amplitude, high-frequency pressure pulsations in the liquid. These pressure pulsations are generated by a submerged surface, which vibrates normal to its face and sets up pressure waves in the liquid. No cavities will be formed unless the amplitude of the pressure variation is great enough to cause the pressure to drop to or below the vapor pressure of the liquid. As the vibratory pressure field is characteristic of this type of cavitation, the name "vibratory cavitation" follows.

8.3 EFFECTS AND IMPORTANCE OF CAVITATION

Cavitation is important as a consequence of its effects. These may be classified into three general categories:

1. Effects that modify the hydrodynamics of the flow of the liquid
2. Effects that produce damage on the solid-boundary surfaces of the flow
3. Extraneous effects that may or may not be accompanied by significant hydrodynamic flow modifications or damage to solid boundaries

Unfortunately for the field of applied hydrodynamics, the effects of cavitation, with very few exceptions, are undesirable. Uncontrolled cavitation can produce serious and even catastrophic results. The necessity of avoiding or controlling cavitation imposes serious limitations on the design of many types of hydraulic equipment. The simple enumeration of some types of equipment, structures, or flow systems, whose performance may be seriously affected by the presence of cavitation, will serve to emphasize the wide occurrence and the relative importance of this phenomenon.

In the field of hydraulic machinery, it has been found that all types of turbines, which form a low-specific-speed Francis to the high-specific-speed Kaplan, are susceptible to cavitation. Centrifugal and axial-flow pumps suffer from its effects, and even the various types of positive-displacement pumps may be troubled by it. Although cavitation may be aggravated by poor design, it may occur in even the best-designed equipment when the latter is operated under unfavorable condition.

8.4 CAVITATION PARAMETER FOR DYNAMIC SIMILARITY

The main variables that affect the inception and subsequent character of cavitation in flowing liquids are the boundary geometry, the flow variables of absolute pressure and velocity, and the critical pressure p_{crit} at which a bubble can be formed or a cavity maintained. Other variables may cause significant variations in the relation between geometry, pressure, and velocity and in the value of the critical pressure. These include the properties of the liquid (such as viscosity, surface tension, and vaporization characteristics), any solid, or gaseous contaminants that may be entrained or dissolved in the liquid, and the condition of the boundary surfaces, including cleanliness and existence of crevices, which might host undissolved gases. In addition to dynamic effects, the pressure gradients due to gravity are important for large cavities whether they be traveling or attached types. Finally, the physical size of the boundary geometry may be important, not only in affecting cavity dimensions but also in modifying the effects of some of the fluid and boundary flow properties.

Let us consider a simple liquid having constant properties and develop the basic cavitation parameter. A relative flow between an immersed object and the surrounding liquid results in a variation in pressure at a point on the object, and the pressure in the undisturbed liquid at some distance from the object is proportional to the square of the relative velocity. This can be written as the negative of the usual pressure coefficient C_p, namely,

$$-C_p = \frac{(p_0 - p)_d}{\rho V_0^2/2} \tag{8.1}$$

where ρ is the density of liquid, V_0 the velocity of undisturbed liquid relative to body, p_0 the pressure of undisturbed liquid, p the pressure at a point on object, and $(p_0 - p)_d$ the pressure differential due to *dynamic effects* of fluid motion.

This is equivalent to omitting gravity. However, when necessary, gravity effects can be included.

At some location on the object, p will be a minimum, p_{min}, so that

$$(-C_p)_{min} = \frac{p_0 - p_{min}}{\rho V_0^2/2} \tag{8.2}$$

In the absence of cavitation (and if Reynolds-number effects are neglected), this value will depend only on the shape of the object. It is easy to create a set of conditions such that p_{min} drops to some value at which cavitation exists. This can be accomplished by increasing the relative velocity V_0 for a fixed value of the pressure p_0 or by continuously lowering p_0 with V_0 held constant. Either procedure will result in lowering of the absolute values of all the local pressures on the surface of the object. If surface tension is ignored, the pressure p_{min} will be the pressure of the contents of the cavitation cavity. Denoting this as a bubble pressure p_b, we can define a *cavitation parameter* by replacing p_{min}; thus

$$K_b = \frac{p_0 - p_b}{\rho V_0^2/2} \tag{8.3}$$

or, in terms of pressure head (in feet of the liquid),

$$K_{\mathrm{b}} = \frac{(p_0 - p_{\mathrm{b}})/\gamma}{V_0^2/2g} \tag{8.4}$$

where p_0 is the absolute-static pressure at some reference locality, V_0 the reference velocity, p_{b} the absolute pressure in cavity or bubble, and γ the specific weight of liquid.

If we now assume that cavitation will occur when the normal stresses at a point in the liquid are reduced to zero, p_{b} will equal the vapor pressure p_{v}.

Then, we write

$$K_{\mathrm{b}} = \frac{p_0 - p_{\mathrm{v}}}{\rho V_0^2/2} \tag{8.5}$$

The value of K at which cavitation inception occurs is designated as K_{i}. A theoretical value of K_{i} is the magnitude $|(-C_{\mathrm{p}})_{\min}|$ for any particular body.

The initiation of cavitation by vaporization of the liquid may require that a negative stress exist because of surface tension and other effects. However, the presence of such things as undissolved gas particles, boundary layers, and turbulence will modify and often mask a departure of the critical pressure p_{crit} from p_{v}. As a consequence, Eq. (8.5) has been universally adopted as the parameter for comparison of vaporous cavitation events.

The beginning of cavitation means the appearance of tiny cavities at or near the place on the object where the minimum pressure is obtained. Continual increase in V_0 (or decrease in p_0) means that the pressure at other points along the surface of the object will drop to the critical pressure. Thus, the zone of cavitation will spread from the location of original inception. In considering the behavior of the cavitation parameter during this process, we again note that if Reynolds-number effects are neglected the pressure coefficient $(-C_{\mathrm{p}})_{\min}$ depends only on the object's shape and is constant prior to inception. After inception, the value decreases as $p_{\min}$ becomes the cavity pressure, which tends to remain constant, whereas either V_0 increases or p_0 decreases. Thus, the cavitation parameter assumes a definite value at each stage of development or "degree" of cavitation on a particular body. For inception, $K = K_{\mathrm{i}}$; for advanced stages of cavitation, $K < K_{\mathrm{i}}$. K_{i} and values of K at subsequent stages of cavitation depend primarily on the shape of the immersed object past which the liquid flows.

We should note here that for flow past immersed objects and curved boundaries, K_{i} will always be finite. For the limiting case of parallel flow of an ideal fluid, K_{i} will be zero since the pressure p_0 in the main stream will be the same as the wall pressure (again with gravity omitted and the assumption that cavitation occurs when the normal stresses are zero).

8.4.1 The Cavitation Parameter as a Flow Index

The parameter K_b or K can be used to relate the conditions of flow to the possibility of cavitation occurring as well as to the degree of postinception stages of cavitation. For any system where the existing or potential bubble pressure (p_b or p_v) is fixed, the parameter (K_b or K) can be computed for the full range of values of the reference velocity V_0 and reference pressure p_0. On the other hand, as previously noted, for any degree of cavitation from inception to advanced stages, the parameter has a characteristic value. By adjusting the flow conditions so that K is greater than, equal to, or less than K_i, the full range of possibilities, from no cavitation to advanced stages of cavitation, can be established.

8.4.2 The Cavitation Parameter in Gravity Fields

As the pressure differences in the preceding relations are due to dynamic effects, the cavitation parameter is defined independently of the gravity field. For large bodies going through changes in elevation as they move, the relation between dynamic pressure difference $(p_0 - p_{\min})_d$ and the actual pressure difference $(p_0 - p_{\min})_{\text{actual}}$ is

$$(p_0 - p_{\min})_d = (p_0 - p_{\min})_{\text{actual}} + \gamma(h_0 - h_{\min})$$

where γ is the liquid's specific weight and h is elevation. Then, in terms of actual pressures, we have, instead of Eq. (8.5),

$$K = \frac{(p_0 + \gamma h_0) - (p_v + \gamma h_{\min})}{\rho V_0/2} \tag{8.6}$$

For $h_0 = h_{\min}$, Eq. (8.6) reduces to Eq. (8.5).

8.5 PHYSICAL SIGNIFICANCE AND USES OF THE CAVITATION PARAMETER

A simple physical interpretation follows directly when we consider a cavitation cavity that is being formed and then swept from a low-pressure to a high-pressure region. Then the numerator is related to the net pressure or head, which tends to collapse the cavity. The denominator is the velocity pressure or head of the flow. The variations in pressure, which take place on the surface of the body or on any type of guide passage, are basically due to changes in the velocity of the flow. Thus, the velocity head may be considered to be a measure of the pressure reductions that may occur to cause a cavity to form or expand.

The basic importance of cavitation parameter stems from the fact that it is an index of dynamic similarity of flow conditions under which cavitation occurs. Its use, therefore, is subject to a number of limitations. Full dynamic similarity between flows in two systems requires that the effects of all physical conditions be reproduced according to unique relations. Thus, even if identical thermodynamics and chemical properties and identical boundary geometry are assumed, the variable effects of contaminants in the liquid-omitted dynamic similarity require that the effects of viscosity, gravity, and surface tension be in unique relationship at each cavitation condition. In other words, a particular cavitation condition is accurately reproduced only if Reynolds number, Froude number, Weber number, etc. as well as the cavitation parameter K have particular values according to a unique relation among themselves.

8.6 THE RAYLEIGH ANALYSIS OF A SPHERICAL CAVITY IN AN INVISCID INCOMPRESSIBLE LIQUID AT REST AT INFINITY

The mathematical analysis of the formation and collapse of spherical cavities, which are the idealized form of the traveling transient cavities, has proved interesting to many workers in the field. Furthermore, it appears that as more experimental evidence is obtained on the detailed mechanics of the cavitation process, the role played by traveling cavities grows in importance. This is especially true with respect to the process by which cavitation produces physical damage on the guiding surfaces.

Rayleigh first set up an expression for the velocity u, at any radial distance r, where r is greater than R, the radius of the cavity wall. U is the cavity-wall velocity at time t. For spherical symmetry, the radial flow is irrotational with velocity potential, and velocity is given by

$$\phi = \frac{UR^2}{r} \quad \text{and} \quad \frac{u}{U} = \frac{R^2}{r^2} \tag{8.7}$$

Next, the expression for the kinetic energy of the entire body of liquid at time t is developed by integrating kinetic energy of a concentric fluid shell of thickness dr and density ρ. The result is

$$(\mathrm{KE})_{\mathrm{liq}} = \frac{\rho}{2}\int_R^\infty u^2 4\pi r^2 \mathrm{d}r = 2\pi\rho U^2 R^3 \tag{8.8}$$

The work done on the entire body of fluid as the cavity is collapsing from the initial radius R_0 to R is a product of the pressure p_∞ at infinity and the change in volume of the cavity as no work is done at the cavity wall where the pressure is assumed to be zero, i.e.,

$$\frac{4\pi p_\infty}{3}\left(R_0^3 - R^3\right) \tag{8.9}$$

If the fluid is inviscid as well as incompressible, the work done appears as kinetic energy. Therefore, Eq. (8.8) can be equated to Eq. (8.9), which gives

$$U^2 = \frac{2p_\infty}{3\rho}\left(\frac{R_0^3}{R^3} - 1\right) \tag{8.10}$$

An expression for the time t required for a cavity to collapse from R_0 to R can be obtained from Eq. (8.10) by substituting for the velocity U of the boundary, its equivalent $\mathrm{d}R/\mathrm{d}t$ and performing the necessary integration.

This gives

$$t = \sqrt{\frac{3\rho}{2p_\infty}} \int_R^{R_0} \frac{R^{3/2}\mathrm{d}R}{\left(R_0^3 - R^3\right)^{1/2}} = R_0\sqrt{\frac{3\rho}{2p_\infty}} \int_\beta^1 \frac{\beta^{3/2}\mathrm{d}\beta}{(1-\beta^3)^{1/2}} \tag{8.11}$$

The new symbol β is R/R_0. The time τ of complete collapse is obtained if Eq. (8.11) is evaluated for $\beta = 0$. For this special case, the integration may be performed by means of functions with the result that τ becomes

$$\tau = R_0\sqrt{\frac{\rho}{6p_\infty}} \times \frac{\Gamma\left(\frac{3}{6}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{4}{3}\right)} = 0.91468 R_0\sqrt{\frac{\rho}{p_\infty}} \tag{8.12}$$

Rayleigh did not integrate Eq. (8.11) for any other value of β. In the detailed study of the time history of the collapse of a cavitation bubble, it is convenient to have a solution for all values of β between 0 and 1.0. Table 8.1 gives values of the dimensionless time $t = t/R_0\sqrt{\rho/p_\infty}$ over this range as obtained from a numerical solution of a power series expansion of the integral in Eq. (8.11).

Equation (8.10) shows that as R decreases to 0, the velocity U increases to infinity. In order to avoid this, Rayleigh calculated what would happen if, instead of having zero or constant pressure within the cavity, the cavity is filled with a gas, which is compressed isothermally. In such a case, the external work done on the system as given by Eq. (8.9) is equated to the sum of the kinetic energy of the liquid given by Eq. (8.8) and the work of compression of the gas, which is $4\pi Q R_0^3 \ln(R_0/R)$, where Q is the initial pressure of the gas. Thus Eq. (8.10) is replaced by

$$U^2 = \frac{2p_\infty}{3\rho}\left(\frac{R_0^3}{R^3} - 1\right) - \frac{2Q}{\rho} \times \frac{R_0^3}{R^3} \ln_0 \frac{R_0}{R} \tag{8.13}$$

For any real (i.e., positive) value of Q, the cavity will not collapse completely, but U will come to 0 for a finite value of R. If Q is greater than p_∞, the first movement of the boundary is outward. The limiting size of the cavity can be obtained by setting $U = 0$ in Eq. (8.13), which gives

$$p_\infty \frac{z-1}{z} - Q \ln z = 0 \tag{8.14}$$

Table 8.1 Values of the Dimensionless Time $t' = t/R_0\sqrt{\rho/p_\infty}$ from Eq. (8.11) (Error Less Than 10^{-6} for $0 \le \beta \le 0.96$)

β	$t\frac{\sqrt{p_\infty/\rho}}{R_0}$	β	$t\frac{\sqrt{p_\infty/\rho}}{R_0}$	β	$t\frac{\sqrt{p_\infty/\rho}}{R_0}$
0.99	0.016145	0.64	0.733436	0.29	0.892245
0.98	0.079522	0.63	0.741436	0.28	0.894153
0.97	0.130400	0.62	0.749154	0.27	0.895956
0.96	0.174063	0.61	0.756599	0.26	0.897658
0.95	0.212764	0.60	0.763782	0.25	0.899262
0.94	0.247733	0.59	0.770712	0.24	0.900769
0.93	0.279736	0.58	0.777398	0.23	0.902182
0.92	0.309297	0.57	0.783847	0.22	0.903505
0.91	0.336793	0.56	0.790068	0.21	0.904738
0.90	0.362507	0.55	0.796068	0.20	0.905885
0.89	0.386662	0.54	0.801854	0.19	0.906947
0.88	0.409433	0.53	0.807433	0.18	0.907928
0.87	0.430965	0.52	0.812810	0.17	0.908829
0.86	0.451377	0.51	0.817993	0.16	0.909654
0.85	0.470770	0.50	0.822988	0.15	0.910404
0.84	0.489229	0.49	0.827798	0.14	0.911083
0.83	0.506830	0.48	0.832431	0.13	0.911692
0.82	0.523635	0.47	0.836890	0.12	0.912234
0.81	0.539701	0.46	0.841181	0.11	0.912713
0.80	0.555078	0.45	0.845308	0.10	0.913130
0.79	0.569810	0.44	0.849277	0.09	0.913489
0.78	0.583937	0.43	0.853090	0.08	0.913793
0.77	0.597495	0.42	0.856752	0.07	0.914045
0.76	0.610515	0.41	0.860268	0.06	0.914248
0.75	0.623027	0.40	0.863640	0.05	0.914406
0.74	0.635059	0.39	0.866872	0.04	0.914523
0.73	0.646633	0.38	0.869969	0.03	0.914604
0.72	0.657773	0.37	0.872933	0.02	0.914652
0.71	0.668498	0.36	0.875768	0.01	0.914675
0.70	0.678830	0.35	0.878477	0.00	0.914680
0.69	0.688784	0.34	0.887062		
0.68	0.698377	0.33	0.883528		
0.67	0.707625	0.32	0.885876		
0.66	0.716542	0.31	0.222110		
0.65	0.725142	0.30	0.890232		

in which z denotes the ratio of the volume R_0^3/R^3. Equation (8.14) indicates that the radius oscillates between the initial value R_0 and another, which is determined by the ratio p_∞/Q from this equation. If $p_\infty/Q > 1$, the limiting size is a minimum. Although Rayleigh presented this example only for isothermal compression, it is obvious that any other thermodynamic process may be assumed for the gas in the cavity, and equations analogous to Eq. (8.13) may be formulated.

As another interesting aspect of the bubble collapse, Rayleigh calculated the pressure field in the liquid surrounding the bubble reverting to the empty cavity of zero pressure. He set up the radial acceleration as the total differential of the liquid velocity u, at radius r, with respect to time, equated this to the radial pressure gradient, and integrated to get the pressure at any point in the liquid. Hence,

$$a_r = -\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{\partial u}{\partial t} - u\frac{\partial u}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial r} \tag{8.15}$$

Expressions for $\partial u/\partial t$ and $u(\partial u/\partial r)$ as functions of R and r are obtained from Eqs. (8.7) and (8.10), the partial differential of Eq. (8.7) being taken with respect to r and t, and the partial differential of Eq. (8.7) with respect to t. Substituting these expressions in Eq. (8.15) yields:

$$\frac{1}{p_\infty}\frac{\partial p}{\partial r} = \frac{R}{3r^2}\left[\frac{(4z-4)R^3}{r^3} - (z-4)\right] \tag{8.16}$$

in which $z = * (R_0/R)^3$ and $r \leq R$ always. By integration, this becomes

$$\frac{1}{p_\infty}\int_{p_\infty}^{p} \mathrm{d}p = \frac{R}{3}\left[(4z-4)R^3\int_\infty^r \frac{\mathrm{d}r}{r^5} - (z-4)\int_\infty^r \frac{\mathrm{d}r}{r^2}\right] \tag{8.17}$$

which gives

$$\frac{p}{p_\infty} - 1 = \frac{R}{3r}(z-4) - \frac{R^4}{3r^4}(z-1) \tag{8.18}$$

The pressure distribution in the liquid at the instant of release is obtained by substituting $R = R_0$ in Eq. (8.18), which gives

$$p = p_\infty\left(1 - \frac{R_0}{r}\right) \tag{8.19}$$

In Eq. (8.18), $z = 1$ at the initiation of the collapse and increases as collapse proceeds. Figure 8.1 shows the distribution of the pressure in the liquid according to Eq. (8.18). It is seen that for $1 < z < 4$, $p_{\max} = p_\infty$ and occurs at $R/r = 0$, where $r \longrightarrow \infty$. For $4 < z < \infty$, $p_{\max} > p_\infty$ and occurs at finite r/R. This location moves toward the bubble with increasing z and approaches $r/R = 1.59$ as z approaches infinity. The location r_{m} of the maximum pressure in the liquid may be found by setting $\mathrm{d}p/\mathrm{d}r$ equal to zero in Eq. (8.16). This gives a maximum value for p when

$$\frac{r_{\mathrm{m}}^3}{R^3} = \frac{4z-4}{z-4} \tag{8.20}$$

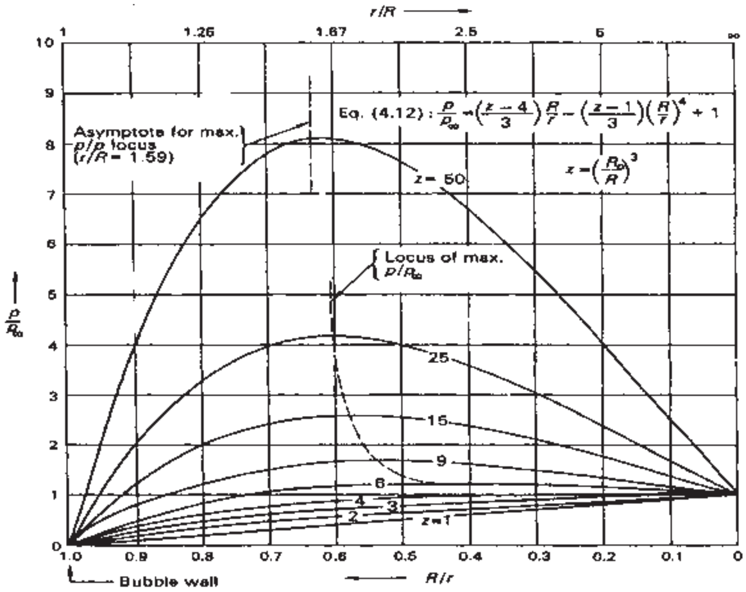


Figure 8.1 Rayleigh analysis: pressure profile near a collapsing bubble.

When $r_{\rm m}$ is substituted for r in Eq. (8.18), the maximum value of p is obtained as

$$\frac{p_{\max}}{p_\infty} = 1 + \frac{(z-4)R}{4r_{\rm m}} = 1 + \frac{(z-4)^{4/3}}{4^{4/3}(z-1)^{1/3}} \tag{8.21}$$

As cavity approaches complete collapse, z becomes great, and Eqs. (8.20) and (8.21) may be approximated by

$$r_{\rm m} = 4^{1/3}R = 1.587R \tag{8.22}$$

and

$$\frac{p_{\max}}{p_\infty} = \frac{z}{4^{4/3}} = \frac{R_0^3}{4^{4/3}R^3} \tag{8.23}$$

Equations (8.22) and (8.23) taken together show that as the cavity becomes very small, the pressure in the liquid near the boundary becomes very great in spite of the fact that the pressure at the boundary is always zero. This would suggest the possibility that in compressing the liquid some energy can be stored, which would add an additional term to Eq. (8.10). This would invalidate the assumption of incompressibility. Rayleigh himself abandoned this assumption in considering what happens if the cavity collapses on an absolute rigid sphere of radius R. In this treatment, the assumption of incompressibility is abandoned only at the instant that the cavity wall comes in contact with the rigid sphere. From that instant, it is assumed that the kinetic energy of deformation of the same particle is determined by the bulk modulus of elasticity of the fluid, as is common in water-hammer calculations. On this basis, it is found that

$$\frac{(P')^2}{2E}\frac{1}{2} = \rho U^2 = \frac{p_\infty}{3}\left(\frac{R_0^3}{R^3} - 1\right) = \frac{p_\infty}{3}(z - 1) \tag{8.24}$$

where P' is the instantaneous pressure on the surface of the rigid sphere and E is the bulk modulus of elasticity. Both must be expressed in the same units.

It is instructive to compare the collapse of the cavity with the predicted collapse based on this simple theory. Figure 8.2 shows this comparison.

This similarity is very striking, especially when it is remembered that there was some variation of pressure p_∞ during collapse of the actual cavity. It will be noted that the actual collapse time is greater than that predicted by Eq. (8.12).

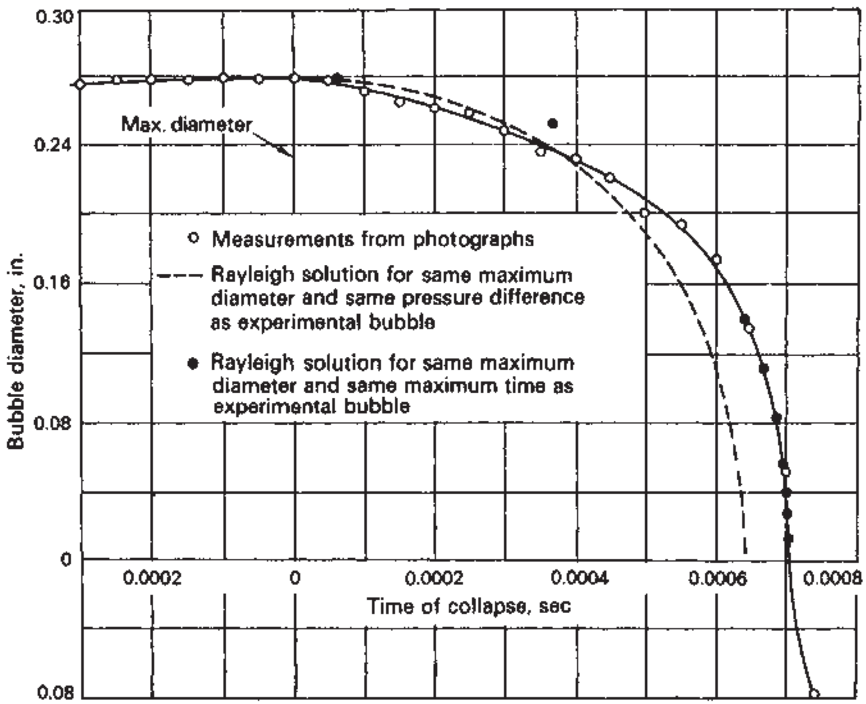


Figure 8.2 Comparison of measured bubble size with the Rayleigh solution for an empty cavity in an incompressible liquid with a constant pressure field.

8.7 CAVITATION EFFECTS ON PERFORMANCE OF HYDRAULIC MACHINES

8.7.1 Basic Nature of Cavitation Effects on Performance

The effects of cavitation on hydraulic performance are many and varied. They depend upon the type of equipment or structure under consideration and the purpose it is designed to fulfill. However, the basic elements, which together make up these effects on performance, are stated as follows:

1. The presence of a cavitation zone can change the friction losses in a fluid flow system, both by altering the skin friction and by varying the form resistance. In general, the effect is to increase the resistance, although this is not always true.
2. The presence of a cavitation zone may result in a change in the local direction of the flow due to a change in the lateral force, which a given element of guiding surface can exert on the flow as it becomes covered by cavitation.
3. With well-developed cavitation the decrease in the effective cross-section of the liquid-flow passages may become great enough to cause partial or complete breakdown of the normal flow.

The development of cavitation may seriously affect the operation of all types of hydraulic structures and machines. For example, it may change the rate of discharge of gates or spillways, or it may lead to undesirable or destructive pulsating flows. It may distort the action of control valves and other similar flow devices. However, the most trouble from cavitation effects has been experienced in rotating machinery; hence, more is known about the details of these manifestations. Study of these details not only leads to a better understanding of the phenomenon in this class of equipment but also sheds considerable light on the reason behind the observed effects of cavitation in many types of equipment for which no such studies have been made. Figures 8.3 and 8.4 display the occurrence of cavitation and its effect on the performance of a centrifugal pump.

8.8 THOMA'S SIGMA AND CAVITATION TESTS

8.8.1 Thoma's Sigma

Early in the study of the effects of cavitation on performance of hydraulic machines, a need developed for a satisfactory way of defining the operating conditions with respect to cavitation. For example, for the same machine operating under different heads and at different speeds, it was found desirable

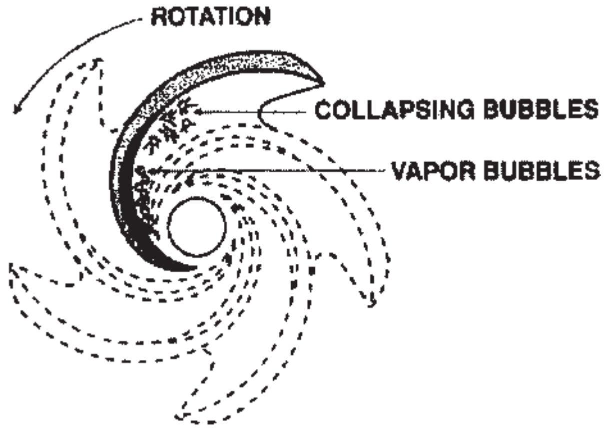


Figure 8.3 Cavitation occurs when vapor bubbles form and then subsequently collapse as they move along the flow path on an impeller.

to specify the conditions under which the degree of cavitation would be similar. It is sometimes necessary to specify similarity of cavitation conditions between two machines of the same design but of different sizes, e.g., between model and prototype. The cavitation parameter commonly accepted for this purpose was

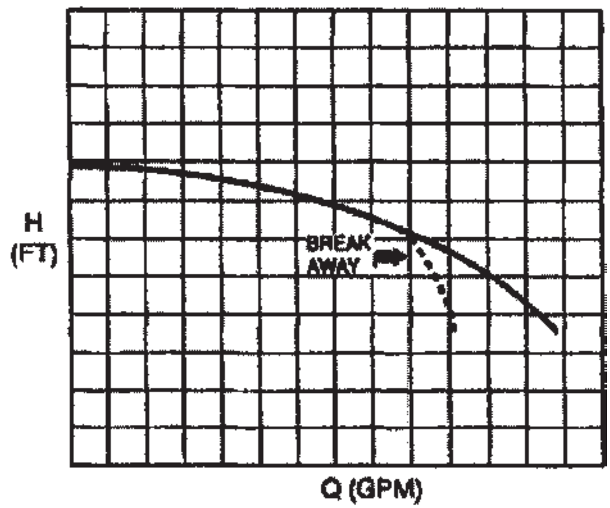


Figure 8.4 Effect of cavitation on the performance of a centrifugal pump.

proposed by Thoma and is now commonly known as the Thoma sigma, σ_T. For general use with pumps or turbines, we define sigma as

$$\sigma_{sv} = \frac{H_{sv}}{H} \tag{8.25}$$

where H_{sv}, the *net positive suction head* at some location = total absolute head less vapor-pressure head $= [(p_{atm}/\gamma) + (p/\gamma) + (V^2/2g) - (p_v/\gamma)]$, H is the head produced (pump) or absorbed (turbine), and γ is the specific weight of fluid.

For turbines with negative static head on the runner,

$$H_{sv} = H_a - H_s - H_v + \frac{V_e^2}{2g} + H_f \tag{8.26}$$

where H_a is the barometric-pressure head, H_s the static draft head defined as elevation of runner discharge above surface of tail water, H_v the vapor-pressure head, V_e the draft-tube exit average velocity (tailrace velocity), and H_f the draft-tube friction loss.

If we neglect the draft-tube friction loss and exit-velocity head, we get sigma in Thoma's original form:

$$\sigma_T = \frac{H_a - H_s - H_v}{H} \tag{8.27}$$

Thus

$$\sigma_T = \sigma_{sv} - \frac{V_e^2/2g + H_f}{H} \tag{8.28}$$

Sigma (σ_{sv} or σ_T) has a definite value for each installation, known as the plant sigma. Every machine will cavitate at some critical sigma (σ_{sv_c} or σ_{T_c}). Clearly, cavitation will be avoided only if the plant sigma is greater than the critical sigma.

The cavitation parameter for the flow passage at the turbine runner discharge is, say,

$$K_d = \frac{H_d - H_v}{V_d^2/2g} \tag{8.29}$$

where H_d is the absolute-pressure head at the runner discharge and V_d the average velocity at the runner discharge. Equation (8.29) is similar in form to Eq. (8.25) but they do not have exactly the same significance. The numerator of K_d is the actual cavitation-suppression pressure head of the liquid as it discharges from the runner. (This assumes the same pressure to exist at the critical location for cavitation inception.) Its relation to the numerator of σ_T is

$$H_d - H_v = H_{sv} - \frac{V_d^2}{2g} \tag{8.30}$$

For a particular machine operating at a particular combination of the operating variables, flow rate, head speed, and wicket-gate setting,

$$\frac{V_{\mathrm{d}}^2}{2g} = C_1 H \tag{8.31}$$

Using the previous relations, it can be shown that Eq. (8.29) may be written as

$$K_{\mathrm{d}} = \frac{\sigma_{\mathrm{T}}}{C_1} - \left(1 - \frac{H_{\mathrm{f}}}{C_1 H}\right) + \frac{V_{\mathrm{e}}^2/2g}{V_{\mathrm{d}}^2/2g}$$

The term in parenthesis is the efficiency of the draft tube, η_{dt}, as the converter of the entering velocity head to pressure head. Thus the final expression is

$$K_{\mathrm{d}} = \frac{\sigma_{\mathrm{T}}}{C_1} - \eta_{\mathrm{dt}} + \frac{V_{\mathrm{e}}^2}{V_{\mathrm{d}}^2} \tag{8.32}$$

C_1 is a function of both design of the machine and the setting of the guide vane; η_{dt} is a function of the design of the draft tube but is also affected by the guide-vane setting. If a given machine is tested at constant guide-vane setting and operating specific speed, both C_1 and η_{dt} tend to be constant; hence σ_{T} and K_{d} have a linear relationship. However, different designs usually have different values of C_1 even for the same specific speed and vane setting, and certainly for different specific speeds. K_{d}, however, is a direct measure of the tendency of the flow to produce cavitation, so that if two different machines of different designs cavitated at the same value of K_{d} it would mean that their guiding surfaces in this region had the dame value of K_{i}. However, sigma values could be quite different. From this point of view, sigma is not a satisfactory parameter for the comparison of machines of different designs. On the other hand, although the determination of the value of K_{d} for which cavitation is incipient is a good measure of the excellence of the *shape* of the passages in the discharge region, it sheds no light on whether or not the cross-section is an optimum as well. In this respect, sigma is more informative as it characterizes the discharge conditions by the total head rather than the velocity head alone.

Both K_{d} and sigma implicitly contain one assumption, which should be borne in mind because at times it may be rather misleading. The critical cavitation zone of the turbine runner is in the discharge passage just downstream from the turbine runner. Although this is usually the minimum-pressure point in the system, it is not necessarily the cross-section that may limit the cavitation performance of the machine. The critical cross-section may occur further upstream on the runner blades and may frequently be at the entering edges rather than trailing edges. However, these very real limitations and differences do not alter the fact that K_{d} and σ_{T} are both cavitation parameters and in many respects, they can be used in the same manner. Thus K_{d} (or K evaluated at any location in the machine) can be used to measure the tendency of the flow to cavitate, the conditions of the flow at which cavitation first begins (K_{i}), or the conditions of the flow corresponding to a certain degree of development of cavitation.

Likewise, σ_T can be used to characterize the tendency of the flow through a machine to cause cavitation, the point of inception of cavitation, the point at which cavitation first affects the performance, or the conditions for complete breakdown of performance.

K_i is a very general figure of merit, as its numerical value gives directly the resistance of a given guiding surface to the development of cavitation. Thoma's sigma can serve the same purpose for the entire machine, but in a much more limited sense. Thus, for example, σ_T can be used directly to compare the cavitation resistance of a series of different machines, all designed to operate under the same total head. However, the numerical value of σ_T, which characterizes a very good machine, for one given head may indicate completely unacceptable performance for another. Naturally, there have been empirical relations developed through experience, which show how the σ_T for acceptable performance varies with the design conditions. Figure 8.5 shows such a relationship.

Here, the specific speed has been taken as the characteristic that describes the design type. It is defined for turbines as

$$N_s = \frac{N\sqrt{\text{hp}}}{H^{5/4}} \tag{8.33}$$

where N is the rotating speed, hp the power output, and H the turbine head.

The ordinate is plant sigma ($\sigma_T = \sigma_{\text{plant}}$). Both sigma and specific speed are based on rated capacity at the design head.

In the use of such diagrams, it is always necessary to understand clearly the basis for their construction. Thus, in Fig. 8.5, the solid lines show the minimum-plant sigma for each specific speed at which a turbine can reasonably be expected to perform satisfactorily; i.e., cavitation will be absent or so limited as not to cause efficiency loss, output loss, undesirable vibration, unstable flow, or excessive pitting. Another criterion of satisfactory operation might be that cavitation damage should not exceed a specific amount, measured in pounds of metal removed per year. Different bases may be established to meet other needs. A sigma curve might be related to hydraulic performance by showing the limits of operation for a given drop in efficiency or for a specific loss in power output.

Although the parameter sigma was developed to characterize the performance of hydraulic turbines, it is equally useful with pumps. For pumps, it is used in the form of Eq. (8.25). In current practice, the evaluation of H_{sv} varies slightly depending on whether the pump is supplied directly from a forebay with a free surface or forms a part of a closed system. In the former case, H_{sv} is calculated by neglecting forebay velocity and the friction loss between the forebay and the inlet, just as the tailrace velocity and friction loss between the turbine-runner discharge and tail water are neglected. In the latter case, H_{sv} is calculated from the pressure measured at the inlet. Velocity is assumed to be the average velocity, Q/A. Because of this difference in meaning, if the same

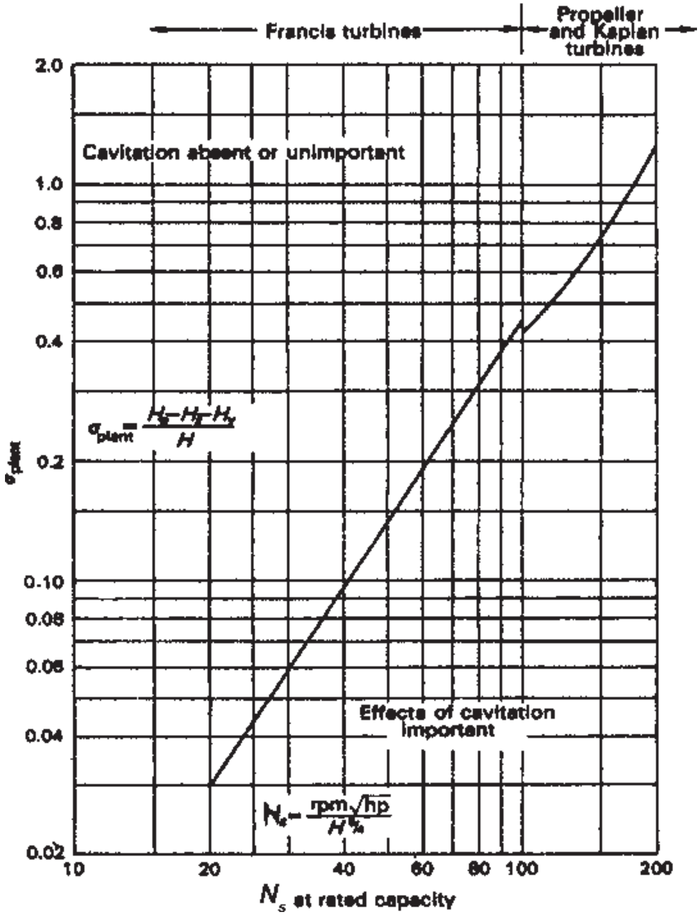


Figure 8.5 Experience limits of plant sigma vs. specific speed for hydraulic turbines.

machine was tested under both types of installation, the results would apparently show a slightly poor cavitation performance with the forebay.

8.8.2 Sigma Tests

Most of the detailed knowledge of the effect of cavitation on the performance of hydraulic machines has been obtained in the laboratory, because of the difficulty encountered in nearly all field installations in varying the operating conditions over a wide enough range. In the laboratory, the normal procedure is to obtain data for the plotting of a group of σ_T curves. Turbine cavitation tests are best run by operating the machine at fixed values of turbine head, speed, and guide-vane setting. The absolute-pressure level of the test system is the independent variable, and this is decreased until changes are observed in the machine performance. For a turbine, these changes will appear in the flow rate, the power output, and the efficiency. In some laboratories, however, turbine cavitation tests are made by operating at different heads and speeds, but at the same unit head and unit speed. The results are then shown as changes in unit power, unit flow rate, and efficiency.

If the machine is a pump, cavitation tests can be made in two ways. One method is to keep the speed and suction head constant and to increase the discharge up to a cutoff value at which it will no longer pump. The preferable method is to maintain constant speed and flow rate and observe the effect of suction pressure on head, power (or torque), and efficiency as the suction pressure is lowered. In such cases, continual small adjustments in flow rate may be necessary to maintain it at constant value.

Figure 8.6 shows curves for a turbine, obtained by operating at constant head, speed, and gate. Figure 8.7 shows curves for a pump, obtained from tests at constant speed and flow rate. These curves are typical in that each performance characteristic shows little or no deviation from its normal value

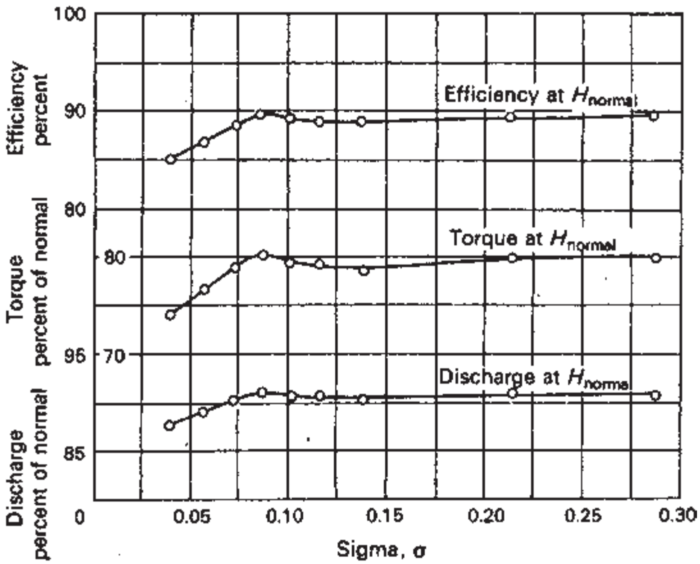


Figure 8.6 Sigma curves for a hydraulic turbine under constant head, speed, and gate opening. (Normal torque, head, and discharge are the values at best efficiency and high sigma.)

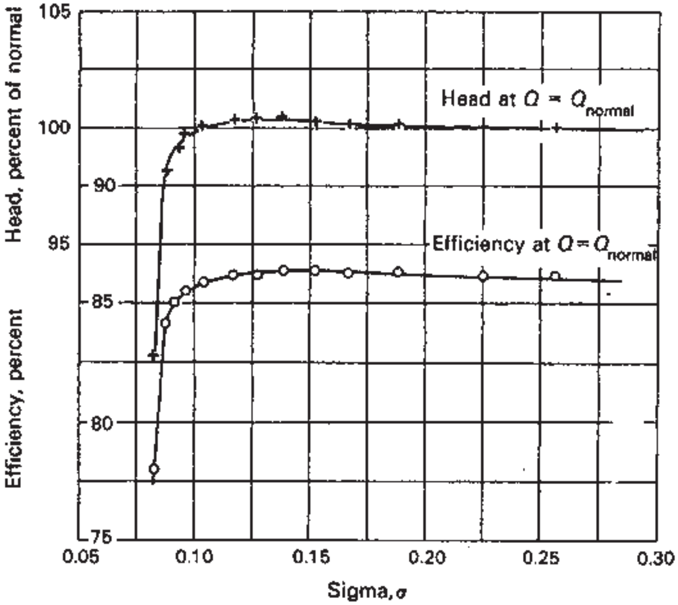


Figure 8.7 Sigma curves for a centrifugal pump at constant speed and discharge. (Normal head and discharge are the values at best efficiency and high sigma.)

(at high submergence) until low sigmas are reached. Then deviations appear, which may be gradual or abrupt.

In nearly all cases, the pressure head across a pump or turbine is so small in comparison with the bulk modulus of the liquid such that change in system pressure during a sigma test produces no measurable change in the density of the liquid. Thus, in principle, until inception is reached, all quantities should remain constant and the σ curves horizontal.

Figure 8.8 shows some of the experimental sigma curves obtained from tests of different pumps. It will be noted that the first deviation of head H observed for machines A and C is downward but that for machine B is upward. In each case, the total deviation is considerably in excess of the limits of accuracy of measurements. Furthermore, only machine A shows no sign of change in head until a sharp break is reached. The only acceptable conclusion is, therefore, that the inception point occurs at much higher value of sigma than might be assumed and the effects of cavitation on the performance develop very slowly until a certain degree of cavitation has been reached.

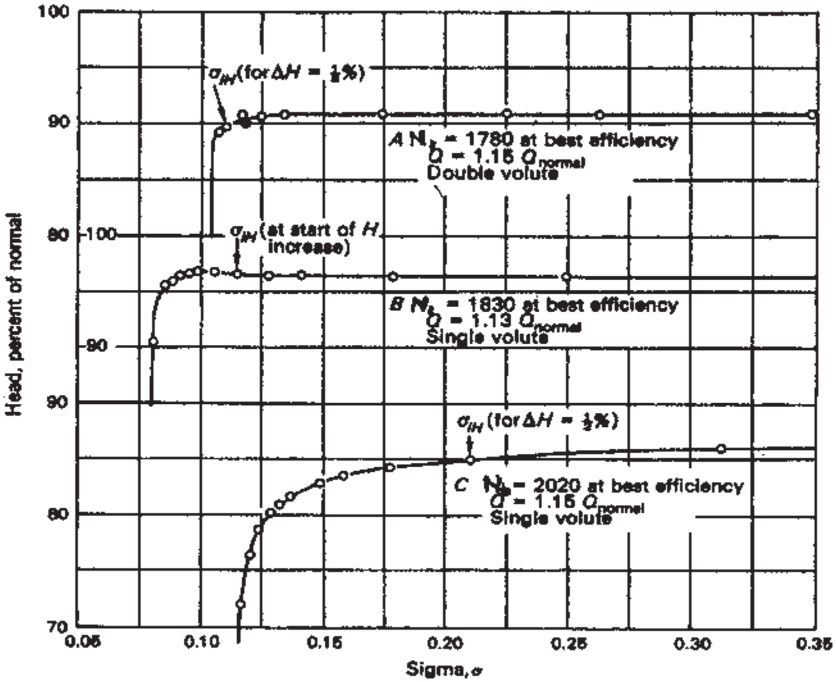


Figure 8.8 Comparison of sigma curves for different centrifugal pumps at constant speed and discharge. (Normal head and discharge are the values at best efficiency and high sigma.)

8.8.3 Interpretation of Sigma Tests

The sigma tests described are only one specialized use of the parameter. For example, as already noted, sigma may be used as a coordinate to plot the results of several different types of experience concerning the effect of cavitation of machines. Even though sigma tests are not reliable in indicating the actual inception of cavitation, attempts have often been made to use them for this purpose on the erroneous assumption that the first departure from the noncavitating value of any of the pertinent parameters marks the inception of cavitation. The result of this assumption has frequently been that serious cavitation damage has been observed in machines whose operation had always been limited to the horizontal portion of the sigma curve.

Considering strictly from the *effect* of cavitation *on the operating characteristics*, the point where the sigma curve departs from the horizontal may be designed as the inception of the effect. For convenience in operation, points could be designated as σ_{iP}, σ_i, σ_{iH}, or σ_{iQ}, which would indicate the values of σ_i for the specified performance characteristics. In Fig. 8.8, such points are marked in each curve. For pumps A and C, the indicated σ_{iH} is at the point where the head has decreased by 0.5% from its high sigma value. For pump B, σ_{iH} is shown where the head begins to increase from its high sigma value.

The curves of Fig. 8.8 show that at some lower limiting sigma, the curve of performance finally becomes nearly vertical. The knee of this curve, where the drop becomes very great, is called the breakdown point. There is remarkable similarity between these sigma curves and the lift curves of hydrofoil cascades. It is interesting to note that the knee of the curve for the cascade corresponds roughly to the development of a cavitation zone over about 10% of the length of the profile and the conditions for heavy vibrations do not generally develop until after the knee has been passed.

8.8.4 Suction Specific Speed

It is unfortunate that sigma varies not only with the conditions that affect cavitation but also with the specific speed of the unit. The suction specific speed represents an attempt to find a parameter, which is sensitive only to the factors that affect cavitation.

Specific speed as used for pumps is defined as

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}} \tag{8.34}$$

where N is the rotating speed, Q the volume rate of flow, and H the head differential produced by pump.

Suction specific speed is defined as

$$S = \frac{N\sqrt{Q}}{H_{sv}^{3/4}} \tag{8.35}$$

where H_{sv} is the total head above vapor at pump inlet. Runners in which cavitation depends only on the geometry and flow in the suction region will develop cavitation at the same value of S. Presumably, for changes in the outlet diameter and head produced by a Francis-type pump runner, the cavitation behavior would be characterized by S. The name “suction specific speed” follows from this concept. The parameter is widely used for pumping machinery but has not usually been applied to turbines. It should be equally applicable to pumps and turbines where cavitation depends only on the suction region of the runner. This is more likely to be the case in low-specific-speed Francis turbines. The following relation between specific speed (as used for pumps), suction specific speed, and sigma is obtained from Eqs. (8.34) and (8.35).

$$\frac{N_{s-\text{pump}}}{S} = \left(\frac{H_{sv}}{H}\right)^{3/4} = \sigma_{sv}^{3/4} \tag{8.36}$$

A corresponding relation between specific speed as used for turbines, suction specific speed, and sigma can be obtained from Eqs. (8.33) and (8.35) together with the expression

$$\text{hp} = \frac{\eta_t \gamma Q H}{550}$$

where η_t is the turbine efficiency.

Then

$$\frac{N_{s-\text{turb}}}{S} = \sigma_{sv}^{3/4}\left(\frac{\eta_t \gamma}{550}\right)^{1/2} \tag{8.37}$$

It is possible to obtain empirical evidence to show whether or not S actually possesses the desirable characteristic for which it was developed, i.e., to offer a cavitation parameter that varies only with the factors that affect the cavitation performance of hydraulic machines and is independent of other design characteristics such as total head and specific speed. For example, Fig. 8.9

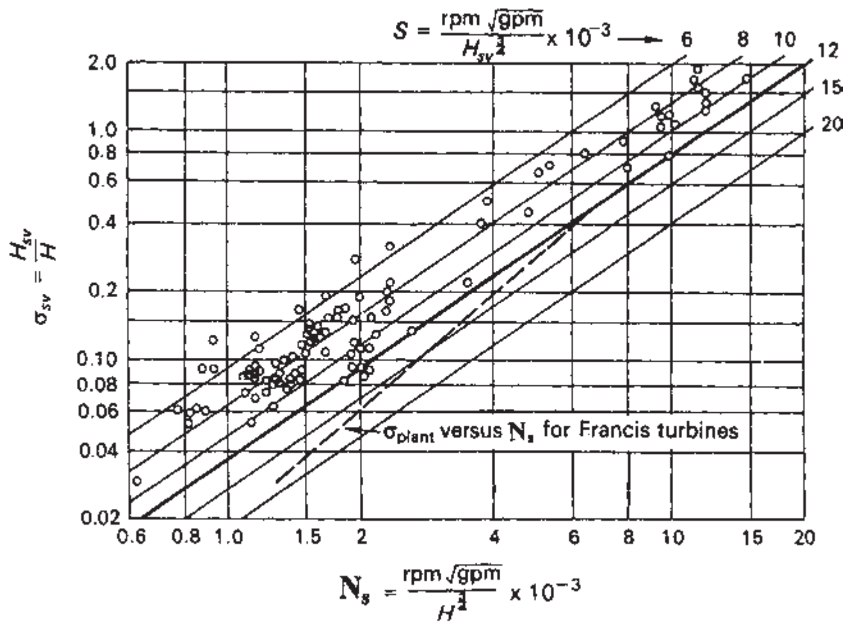


Figure 8.9 Sigma vs. specific speed for centrifugal, mixed-flow, and propeller pumps.

shows a logarithmic diagram of sigma vs. specific speed on which are plotted points showing cavitation limits of individual centrifugal, mixed-flow, and propeller pumps. In the same diagram, straight lines of constant S are shown, each with a slope of $(\log \sigma_{sv})/(\log N_s) = 3/4$ [Eq. (8.36)]. It should be noted that σ_{sv} and S vary in the opposite direction as the severity of the cavitation condition changes, i.e., as the tendency to cavitate increases, σ_{sv} decreases, but S increases.

If it is assumed that as the type of machine and therefore the specific speed change, all the best designs represent an equally close approach to the ideal design to resist cavitation, then a curve passing through the lowest point for each given specific speed should be a curve of constant cavitation performance. Currently, the limit for essentially cavitation-free operation is approximately $S = 12{,}000$ for standard pumps in general industrial use. With special designs, pumps having critical S values in the range of 18,000–20,000 are fairly common. For cavitating inducers and other special services, cavitation is expected and allowed. In cases where the velocities are relatively low (such as condensate pumps), several satisfactory designs have been reported for S in the 20,000–35,000 range.

As was explained, Fig. 8.5 shows limits that can be expected for satisfactory performance of turbines. It is based on experience with installed units and presumably represents good average practice rather than the optimum. The line for Francis turbines has been added to Fig. 8.9 for comparison with pump experience. Note that allowable S values for turbines operating with little or no cavitation tend to be higher than those for pumps when compared at their respective design conditions. Note also that the trend of limiting sigma for turbines is at a steeper slope than the constant S lines. This difference of slope can be taken to indicate that either the parameter S is affected by factors other than those involved in cavitation performance or the different specific-speed designs are not equally close to the optimum as regards cavitation. The latter leads to the conclusion that it is easier to obtain a good design from the cavitation point of view for the lower specific speeds.

NOTATION

a	Acceleration
A	Area
C_p	Pressure coefficient
E	Modulus of elasticity
h	Elevation
H	Head
K	Cavitation parameter
KE	Kinetic energy
N	Rotation speed

N_s	Specific speed
p	Pressure
Q	Flow rate
r	Radial distance
r_m	Mean radius
R	Radius of cavity wall
S	Suction specific speed
t	Time
u	Velocity
V	Velocity
Z	Dimensionless volume of bubble
ρ	Density
η_t	Turbine efficiency
γ	Specific weight
t'	Dimensionless time
σ	Cavitation parameter

SUFFIXES

0	Undisturbed fluid properties
atm	Atmospheric values
d	Dynamic effects
e	Exit
f	Friction
i	Inception properties
min	Minimum
r	Radial
s	Static
v	vapor