GPU Accelerated Support Vector Machines via Quadratic Programming

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Objective

Solve classification problems quickly

- Support Vector Machines
- ADMM Algorithm
- CPU and GPU Implementations
- Results

Paper from Oxford Control Group (part of osqp)

GPU Acceleration of ADMM for Large-Scale Quadratic Programming [2]

Michel Schubiger, Goran Banjac, and John Lygeros - 2019

Support Vector Machine

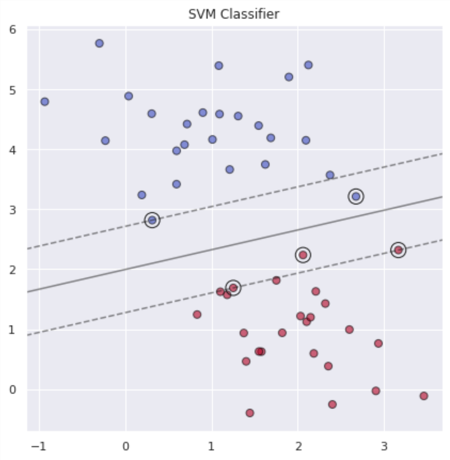


Figure 1: Cartoon of an SVM classification problem

Support Vector Machine

We can write the SVM problem as one that relies on convex optimization, specifically *quadratic programming*

$$\{x : f(x) = x^T\beta + 1 = 0\} \tag{1}$$

Support Vector Machine

We can write the SVM problem as one that relies on convex optimization, specifically *quadratic programming* [1]

$$L_P = \frac{1}{2}||\beta||^2 + C\sum_{i=1}^{N}\xi_i - \sum_{i=1}^{N}\alpha_i\left[y_i\left(x_i^T\beta + 1\right) - (1-\xi_i)\right] - \sum_{i=1}^{N}\mu_i\xi_i \quad (2)$$

$$L_D = \sum_{i=1}^{N}\alpha_i - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j y_i y_j x_i^T x_j \quad (3)$$

$$\alpha_i[y_i(x_i^T\beta + 1) - (1-\xi_i)] = 0 \quad (4)$$

$$mu_i\xi_i = 0 \quad (5)$$

$$y_i(x_i^T\beta + 1) - (1-\xi_i) \geq 0 \quad (6)$$

This allows us to solve the SVM problem with an algorithm called ADMM

ADMM

Alternating Direction Method of Multipliers

- A technique to minimize convex functions
- Intuition: Try to minimize a convex function by alternatively minimize Lagrangian in different directions

ADMM

Algorithm 1: ADMM algorithm as presented in [3]

given: x^0, z^0, y^0 *and parameters* $\rho > 0,\ \sigma > 0,\ \alpha \in [0, 2]$

while *not terminated* **do**

$$(\tilde{x}^{k+1}, v^{k+1}) \leftarrow \begin{bmatrix} P + \sigma I & A^T \\ A & -\rho^{-1} I \end{bmatrix} \begin{bmatrix} \tilde{x}^{k+1} \\ v^{k+1} \end{bmatrix} = \begin{bmatrix} \sigma x^k - q \\ z^k - \rho^{-1} y^k \end{bmatrix}$$

$$\tilde{z}^{k+1} \leftarrow z^k + \rho^{-1}(v^{k+1} - y^k)$$

$$x^{k+1} \leftarrow \alpha \tilde{x}^{k+1} + (1 - \alpha) x^k$$

$$z^{k+1} \leftarrow \prod \left(\alpha \tilde{z}^{k+1} + (1 - \alpha) z^k + \rho^{-1} y^k\right)$$

$$y^{k+1} \leftarrow y^k + \rho(\alpha \tilde{z}^{k+1} + (1 - \alpha) z^k - z^{k+1}$$

end

ADMM

Issue: solving this linear system is challenging

$$\begin{bmatrix} P + \sigma I & A^T \\ A & -\rho^{-1} I \end{bmatrix} \begin{bmatrix} \tilde{x}^{k+1} \\ v^{k+1} \end{bmatrix} = \begin{bmatrix} \sigma x^k - q \\ z^k - \rho^{-1} y^k \end{bmatrix} \tag{7}$$

ADMM

Issue: solving this linear system is challenging

$$\begin{bmatrix} P + \sigma I & A^T \\ A & -\rho^{-1} I \end{bmatrix} \begin{bmatrix} \tilde{x}^{k+1} \\ v^{k+1} \end{bmatrix} = \begin{bmatrix} \sigma x^k - q \\ z^k - \rho^{-1} y^k \end{bmatrix} \tag{8}$$

- Use LDL Factorization
- Use Preconditioned Conjugate Gradient (PCG)

PCG

Algorithm 2: PCG algorithm as presented in [2]

initialise: $r^0 = Kx^0 - b,\ y^0 = M^{-1}r^0,\ p^0 = -y^0,\ k = 0$

while $||r^k|| > \epsilon||b||$ **do**

$$\alpha^k \leftarrow -\frac{\left(r^k\right)^T y^k}{\left(p^k\right)^T K p^k}$$

$$x^{k+1} \leftarrow x^k + \alpha^k p^k$$

$$r^{k+1} \leftarrow r^k + \alpha^k K p^k$$

$$y^{k+1} \leftarrow M^{-1} r^{k+1}$$

$$\beta^{k+1} \leftarrow -\frac{\left(r^{k+1}\right)^T y^{k+1}}{\left(r^k\right)^T y^k}$$

$$p^{k+1} \leftarrow -y^{k+1} + \beta^{k+1} p^k$$

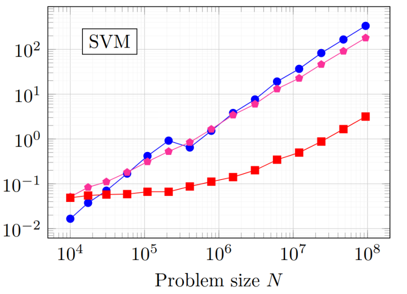
$$k \leftarrow k + 1$$

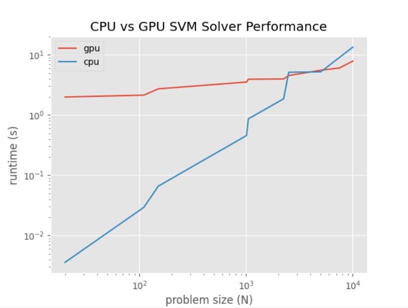
end

GPU Optimizations

- matrix representation - CSR
- `cuBLAS`
- `cuSPARSE`

Results


SVM
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{4}
10^{5}
10^{6}
10^{7}
10^{8}
Problem size N


CPU vs GPU SVM Solver Performance
gpu
cpu
runtime (s)
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{2}
10^{3}
10^{4}
problem size (N)

References

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