Morphological Image Processing: Gray-scale morphology

by Gleb V. Tcheslavski: gleb@lamar.edu

http://ee.lamar.edu/gleb/dip/index.htm

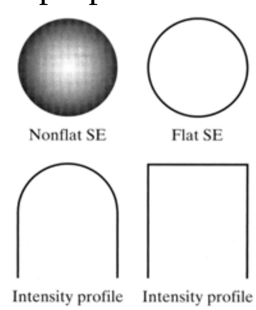
Preliminaries

We extend the basic operations of dilation, erosion, opening, and closing to gray-scale images.

Assume that $f(x, y)$ is a grey-scale image and $b(x, y)$ is a structuring element and both functions are discrete.

Similarly to binary morphology, the structuring elements are used to examine a given image for specific properties.

SEs belong to one of two categories: ***nonflat*** (continuous variation of intensity - rarely used) and ***flat***… The origin of SE must be specified. Unless mentioned otherwise, SEs are flat, symmetrical, of unit height, with the origin at the center.



Erosion and Dilation

The **erosion** of f by a ***flat*** structuring element b at any location (x, y) is defined as the ***minimum*** value of the image in the region coincident with b when the origin of b is at (x, y). Therefore, the erosion at (x, y) of an image f by a structuring element b is given by:

$$\left[f \ominus b \right](x, y) = \min_{(s,t) \in b} \left\{ f\left(x+s, y+t\right) \right\}$$

where, similarly to the correlation, x and y are incremented through all values required so that the origin of b visits every pixel in f. That is, to find the erosion of f by b, we place the origin of the structuring element at every pixel location in the image. The erosion is the minimum value of f from all values of f in the region of f coincident with b.

Erosion and Dilation

The **dilation** of f by a ***flat*** structuring element b at any location (x, y) is defined as the ***maximum*** value of the image in the window outlined by $\hat{b}$ when the origin of $\hat{b}$ is at (x, y). That is

$$\left[f \oplus b \right](x, y) = \max_{(s,t) \in b} \left\{ f\left(x-s, y-t\right) \right\}$$

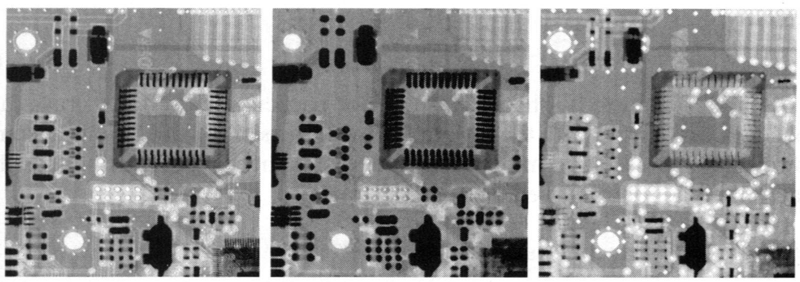
where we used that

$$\hat{b} = b(-x, -y)$$

The explanation is similar to one for erosion except for using maximum instead of minimum and that the structuring element is reflected about the origin.

Erosion and Dilation

Since gray-scale erosion with a flat SE computes the min intensity value of *f* in every neighborhood, the eroded gray-scale image should be darker (bright features are reduced, dark features are thickened, background is darker). The effects of dilation are opposite.



A 448 x 425 original erosion dilation
using a flat disk SE of unit height and a radius of 2 pixels

Erosion and Dilation

Nonflat SEs have gray-scale values that vary over their domain of definition. The **erosion** of image *f* by nonflat structuring element b_N is defined as

$$\left[f \ominus b_N \right](x, y) = \min_{(s,t)\in b_N} \left\{ f\left(x+s, y+t\right) - b_N\left(s,t\right)\right\}$$

Since we subtract values from *f*, the erosion with a nonflat SE is not bounded by the values of *f*, which may present a problem while interpreting results. Gray-scale SEs are rarely used.

Erosion and Dilation

Similarly, **dilation** of image *f* by nonflat SE b_N is defined as

$$\left[f \oplus b_N \right](x, y) = \max_{(s,t) \in b_N} \left\{ f(x-s, y-t) + b_N(s,t) \right\}$$

The same problem is attributed to dilation by nonflat SE; this operation is rarely performed also.

Erosion and dilation are duals with respect to function complementation and reflection:

$$(f \ominus b)^c (x, y) = \left(f^c \oplus \hat{b} \right)(x, y)$$

where

$$f^c = -f(x, y)$$

$$\hat{b} = b(-x, -y)$$

Erosion and Dilation

The same expression holds for nonflat structuring elements.

Usually, arguments are omitted for simplicity; therefore

$$(f \ominus b)^c = \left(f^c \oplus \hat{b} \right)$$

Similarly:

$$(f \oplus b)^c = \left(f^c \ominus \hat{b} \right)$$

Erosion and dilation by themselves are not very useful in gray-scale image processing. These operations become powerful when used in combination to develop high-level algorithms.

Opening and Closing

Same as for binary images, the **opening** of the image *f* by structuring element *b* is defined as the erosion of *f* by *b* followed by a dilation of the result with *b*:

$$f \circ b = \left(f \ominus b \right) \oplus b$$

Similarly, the closing of *f* by *b* is

$$f \bullet b = \left(f \oplus b \right) \ominus b$$

Opening and Closing

The opening and closing for gray-scale images are duals with respect to complementation and SE reflection:

$$\left(f \bullet b \right)^c = f^c \circ \hat{b}$$

$$\left(f \circ b \right)^c = f^c \bullet \hat{b}$$

Since $f^c = -f\left(x, y\right)$

then $-\left(f \bullet b \right) = \left(-f \circ \hat{b} \right)$

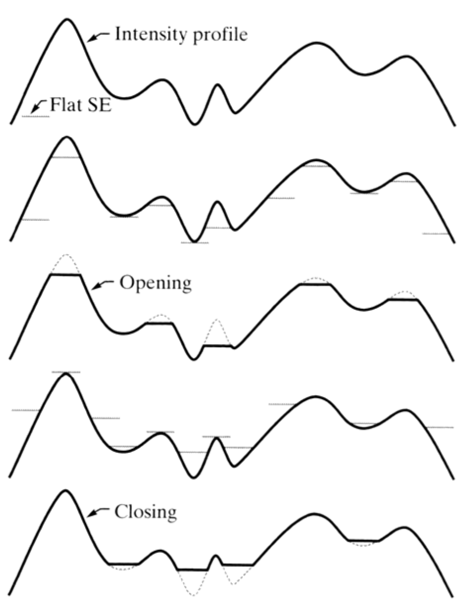
$$-\left(f \circ b \right) = \left(-f \bullet \hat{b} \right)$$

Opening and Closing

Opening and closing of images have a simple geometrical interpretation. Assume that the image $f(x,y)$ is viewed as a 3D surface – intensity values are interpreted as heights over the xy-plane. Then the opening of f by b can be interpreted as “pushing” the SE b up from below against the undersurface of f. At each location of the origin of b, the opening is the highest value reached by any part of b as it pushes against the undersurface of f. The complete opening is then the set of all such values obtained by having the origin of b visit every (x, y) coordinate of f.

Opening and Closing

For a 1D signal



Original 1D signal

Flat SE pushed up underneath the signal

Opening

Flat SE pushed down along the top of the signal

Closing

Opening and Closing

The gray-scale opening operation satisfies the following properties:

$$a)\quad f \circ b \dashv f$$

$$b)\quad If\ \ f_1 \dashv f_2,\ then\ \left(f_1 \circ b\right) \dashv \left(f_2 \circ b\right)$$

$$c)\quad \left(f \circ b\right) \circ b = f \circ b$$

$e \dashv r$ indicates that the domain of *e* is a subset of the domain of *r* and that $e(x,y) \le r(x,y)$ for any (x,y) in the domain of *e*.

Opening and Closing

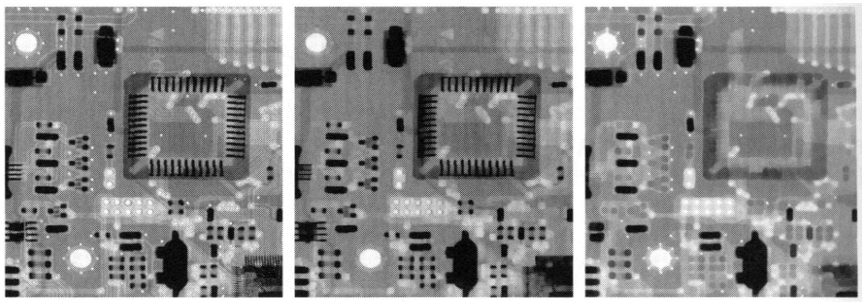
Similarly, the closing satisfies the following properties:

$$a)\quad f \bullet b \dashv f$$

$$b)\quad If\ \ f_1 \dashv f_2,\ then\ \left(f_1 \bullet b\right) \dashv \left(f_2 \bullet b\right)$$

$$c)\quad \left(f \bullet b\right) \bullet b = f \bullet b$$

Opening and Closing



A 448 x 425 original opening closing
using a flat disk SE of unit height and a radius of 3 and 5 pixels

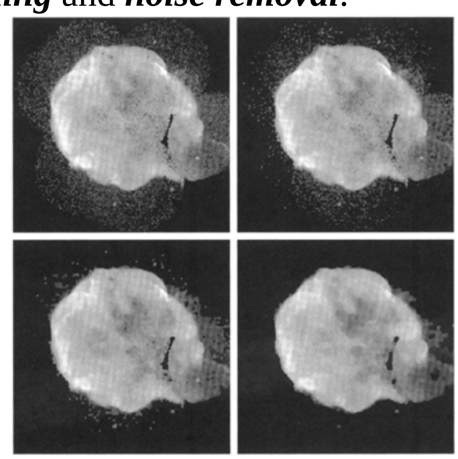
Opening: the intensity of all bright features decreased, depending on the sizes of features compared to the SE. unlike the erosion, opening has negligible effect on the dark features, and the effect on the background is negligible.
Closing: attenuated dark features, unaffected background.

Algorithms: Morphological Smoothing

Since opening suppresses bright details smaller than the specified SE, and closing suppresses dark details, they are often used in combination as morphological filters for ***image smoothing*** and ***noise removal***.

A 566 x 566 original image of Cygnus Loop supernova

A result of opening and closing with a disk SE of radius 1 pixel



A result of opening and closing with a disk SE of radius 3 pixels

A result of opening and closing with a disk SE of radius 5 pixels

Algorithms: Morphological Smoothing

As expected, more details are removed as the size of SE increases. In the last result, the object of interest is extracted almost completely (noise on the lower side could not be removed completely due to its density).

The technique shown before – an opening of the original image followed by closing the opening – is called sometimes an ***alternating sequential filtering***. This processing is used in automated image analysis, in which results at each step are compared to a specific metric.

Algorithms: Morphological Gradient

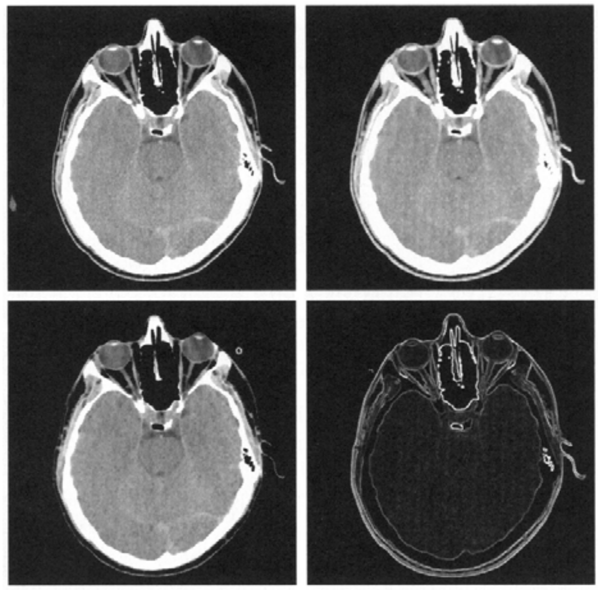
Dilation and erosion can be used in combination with image subtraction to obtain the morphological gradient *g* of an image as

$$g = \left(f \oplus b \right) - \left(f \ominus b \right)$$

The dilation thickens regions in an image and the erosion shrinks them. Therefore, their difference emphasizes the boundaries between regions. If the SE is relatively small, homogeneous areas will not be affected by dilation and erosion, so the subtraction tends to eliminate them. The net result is an image with the gradient-like effect.

Algorithms: Morphological Gradient

An original 512 x 512 CT scan



dilation

erosion

Morphological **gradient**: a difference between dilation and erosion

Algorithms: top-hat and bottom-hat transformations

Combining image subtraction with openings and closings results in **top-hat** and **bottom-hat** transformations.

The ***top-hat*** transformation of a gray-scale image *f* is defines as *f* minus its opening:

$$T_{hat}(f) = f - (f \circ b)$$

Similarly, the ***bottom-hat*** transformation of a gray-scale image *f* is defines as the closing of *f* minus *f*:

$$B_{hat}(f) = (f \bullet b) - f$$

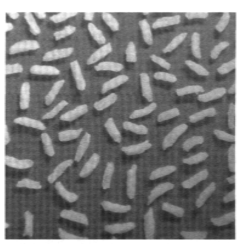
Algorithms: top-hat and bottom-hat transformations

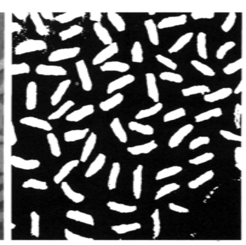
One principal application of these transforms is in removing objects from an image by using an SE in the opening and closing that does not fit the objects to be removed. The difference then yields an image with only the removed objects. The **top-hat** is used for *light objects on a dark background* and the **bottom-hat** – for *dark objects on a light background*.

An important use of top-hat transformation is in correcting the effects of non-uniform illumination.

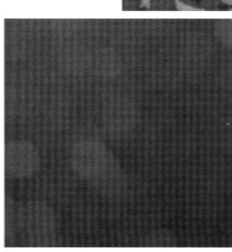
Algorithms: top-hat and bottom-hat transformations

A 600 x 600 original image of rice grains with non-uniform lighting (darker area in the bottom)

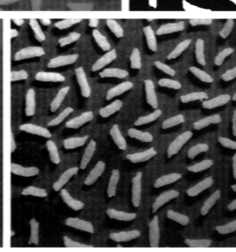




A result of an optimum thresholding (Otsu's method): several grains are not extracted from the background



Opening with a disk SE of a radius 40: eliminated objects; only background is left



Top-hat transformation: more uniform background

A result of optimum thresholding of the top-hat image

Algorithms: Granulometry

Granulometry (in image processing) is a field that deals with determining the size distribution of particles in an image. In practice, particles seldom are neatly separated, which makes particle counting by identifying individual particles a difficult task. Morphology can be used to estimate particle size distribution indirectly, without having to identify and measure every particle in the image.

The approach is simple. With particles of regular shapes that are lighter than the background, the method consists of applying openings with SEs of increasing size. The opening of a particular size should have the most effect on the particles of the same size…

Algorithms: Granulometry

Then, for each opening, the sum of the pixel values in the opening is computed. This sum (sometimes called the *surface area*) decreases as a function of increasing SE size since opening decreases the intensity of light features.

This procedure yields a 1D array of such numbers: the sum of pixels as a function of SE size. To emphasize changes between successive openings, the differences between successive openings can be computed and plotted.

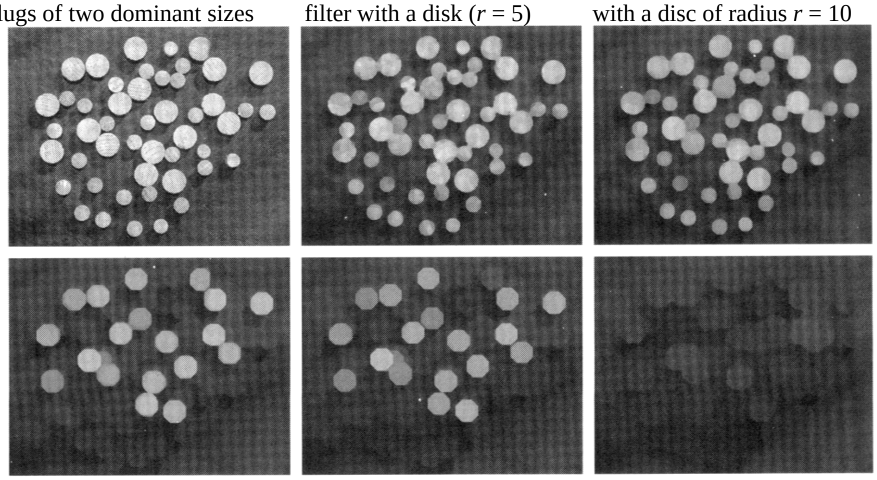
The peaks in the plot are an indication of the predominant size distributions of the particles in the image.

Algorithms: Granulometry

531x675 image of wood dowel plugs of two dominant sizes

Smoothed image by a morph filter with a disk (r = 5)

Opening of smoothed image with a disc of radius r = 10



Opening of smoothed image with a disc of radius r = 20

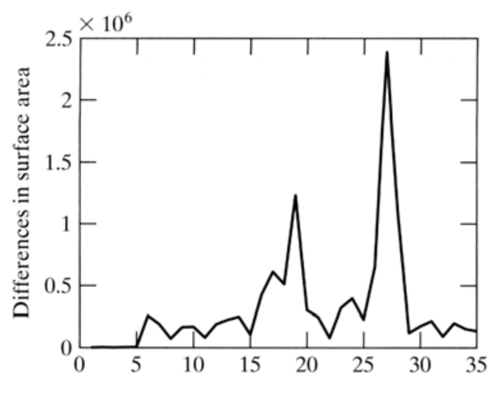
Opening of smoothed image with a disc of radius r = 25

Opening of smoothed image with a disc of radius r = 30

Algorithms: Granulometry

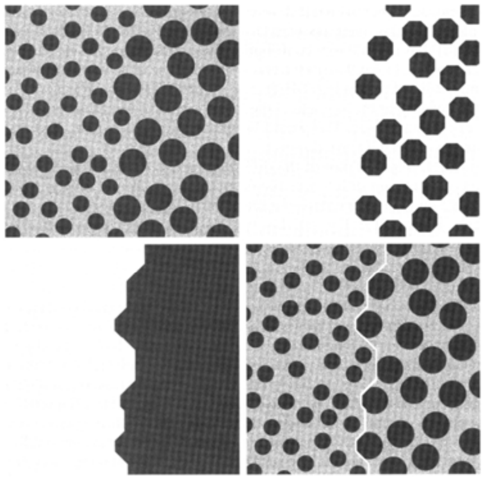
We notice that intensity contributions from small dowels is almost eliminated in "opening with r = 20". As SE size increases, intensity contributions for larger dowels reduces.

A difference array: two distinct peaks clearly identify the presence of two dominant object sizes in the image.



Algorithms: Textural segmentation

The objective is to find the boundary between the two regions based on their textural content.



A 600x600 noisy image of dark blobs on a light background with 2 regions composed of larger and smaller blobs.

Since the objects are darker than the background, closing with an SE (a disk of *r* = 30) larger than the small blobs removes the small blobs (of *r* about 25) .

Opening the image with an SE that is large relative to the separation between blobs (a disk of *r* = 60), the light patches between blobs are removed…

A morphological gradient with a 3x3 SE of ones superimposed on the original image: a region with large blobs is on the right from the gradient line.

Gray-scale morphological reconstruction

Gray-scale morphological reconstruction is defined identically as for binary images. Denote by *f* and *g* the ***marker*** and ***mask*** images and assume that both are gray-scale images of the same size and that $f \le g$. The **geodesic dilation** of size 1 of *f* with respect to *g* is defined as

$$D_g^{(1)}(f) = (f \oplus b) \wedge g$$

where $\wedge$ denotes the point-wise minimum operator.

Gray-scale morphological reconstruction

Therefore, the geodesic dilation of size 1 is obtained by computing the dilation of *f* by *b* and then selecting the minimum between the result and *g* at every point (*x*, *y*). The dilation expression depends on the type of SE (flat or nonflat).

The **geodesic dilation** of size *n* of *f* with respect to *g* is defined as

$$D_g^{(n)}(f) = D_g^{(1)}(f)\left[D_g^{(n-1)}(f)\right]$$

where $D_g^{(0)}(f) = f$

Gray-scale morphological reconstruction

Similarly, the **geodesic erosion** of size 1 of *f* with respect to *g* is defined as

$$E_g^{(1)}(f) = (f \ominus b) \vee g$$

where $\vee$ denotes the point-wise maximum operator.

The **geodesic erosion** of size *n* of *f* with respect to *g* is defined as

$$E_g^{(n)}(f) = E_g^{(1)}(f)\left[E_g^{(n-1)}(f)\right]$$

where $E_g^{(0)}(f) = f$

Gray-scale morphological reconstruction

The **morphological reconstruction by dilation** of a gray-scale image *g* by a gray-scale marker image *f* is defined as the geodesic dilation of *f* with respect to *g* repeated (iterated) until stability is reached:

$$R_g^D(f) = D_g^{(k)}(f)$$

with *k* such that $D_g^{(k)}(f) = D_g^{(k+1)}(f)$

Gray-scale morphological reconstruction

The **morphological reconstruction by erosion** of a gray-scale image *g* by a gray-scale marker image *f* is similarly defined as:

$$R_g^E(f) = E_g^{(k)}(f)$$

with *k* such that $E_g^{(k)}(f) = E_g^{(k+1)}(f)$

Gray-scale morphological reconstruction

As in the binary case, **opening by reconstruction** of a gray-scale images first erodes the input image and uses it as a marker. The **opening by reconstruction** of size *n* of an image *f* is defined as the reconstruction by dilation of *f* from the erosion of size *n* of *f*:

$$O_R^{(n)}(f) = R_f^D\left[\left(f \ominus nb\right)\right]$$

where $\left(f \ominus nb\right)$ denotes *n* erosions of *f* by *b*. The objective of opening by reconstruction is to preserve the shape of the image components that remain after erosion.

Gray-scale morphological reconstruction

Similarly, the **closing by reconstruction** of size *n* of an image *f* is defined as the reconstruction by erosion of *f* from the dilation of size *n* of *f*:

$$C_R^{(n)}(f) = R_f^E\left[\left(f \oplus nb\right)\right]$$

where $\left(f \oplus nb\right)$ denotes *n* dilations of *f* by *b*. Because of duality, the closing by reconstruction of an image can be obtained by complementing the image, obtaining the opening by reconstruction, and complementing the result.

Top-hat by reconstruction consists of subtracting from an image its opening by reconstruction.

Gray-scale morphological reconstruction

A gray-scale reconstruction to normalize the irregular background

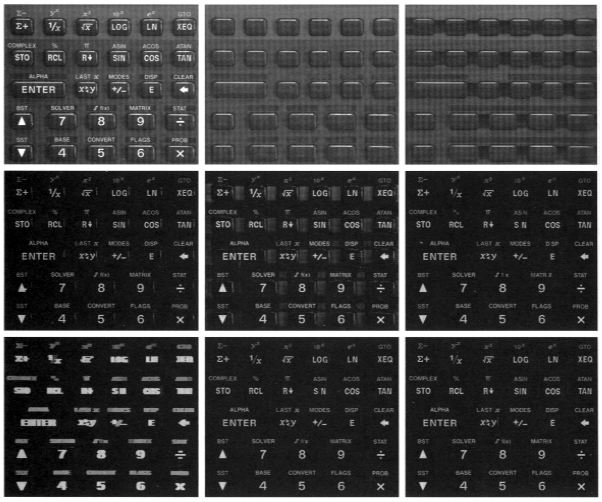
An 1134x1360 original

Opening by reconstruction; SE: horizontal line of l=71

Top-hat by reconstruction

Top-hat

Dilation of the previous; SE: horizontal line of length l = 21



Opening of the original; SE: horizontal line of l=71

Opening by reconstruction of t-h by rec; SE: horizontal line l = 11

Minimum of the previous and t-h by rec.

Final result

Gray-scale morphological reconstruction

Comparing the opening by reconstruction to the regular opening, we notice that opening by reconstruction yields more uniform background. Subtracting the opening by reconstruction from the original image suppresses variations in background. For the comparison, a top-hat transformation (subtracting the standard opening from the original) does not suppresses background variations.

Opening by reconstruction the result with a line SE of width approximately equal to the reflections (11 pixels) removes the vertical reflections from the edges of keys as well as thin vertical strokes of valid characters. Since the suppressed characters are very close to the other characters, dilating the remaining characters horizontally (SE of length 21) overlaps the are previously occupied by the suppressed characters.

Gray-scale morphological reconstruction

Evaluating the point-wise minimum between the dilated image and the top-hat by reconstruction restores most of the suppressed characters. The result is close to the objective but still non-perfect: for instance, “I” in “SIN” is still missing.

Finally, using the previous result as a marker image and the dilated image as the mask in gray-scale reconstruction, the final image was obtained. All characters are shown and the background in uniform.