Ex 3, Task 2: F = F_1 + F_2

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Show that $F(x,y) = F_1(x,y) + F_2(x,y)$

with $F(x,y) = \sum_i \sum_{\bar{y}} \alpha_i(\bar{y}) \underbrace{\langle \phi(x_i, \bar{y}), \phi(x,y) \rangle}_{=: \mathbb{P}}$

$$\mathbb{P} = \sum_{s,t} [\![\bar{y}^{s-1} = y^{t-1} \wedge \bar{y}^s = y^t]\!] + \sum_{s,t} [\![\bar{y}^s = y^t]\!] k(x_i^s, x^t)$$

$$F_1(x,y) = \sum_{\sigma,\tau} \left(\sum_{i,\bar{y}} \alpha_i(\bar{y}) \sum_t [\![\bar{y}^{t-1} = \sigma \wedge \bar{y}^t = \tau]\!] \right) \cdot \sum_s [\![y^{s-1} = \sigma \wedge y^s = \tau]\!]$$

$$F_2(x,y) = \sum_{s,\sigma} [\![y^s = \sigma]\!] \sum_{i,t} \left(\sum_y [\![y^t = \sigma]\!] \alpha_i(y) \right) \cdot k(x^s, x_i^t)$$

with $\sigma, \tau \in \Sigma$, $y^t \in \Sigma$

where Σ (Sigma): "label set"

$\Longrightarrow$ more testing than anything else...

useful properties:

(1) $\sum a_{ij} := \sum \left(\sum a_{ij} \right)$ with

(1) $\sum_{i,j} a_{ij} := \sum_i \left(\sum_j a_{ij}\right)$ with (

$$\sum_{i,j} a_{ij} = \sum_{j,i} a_{ij}$$

$$\sum_j c \cdot \alpha_j = c \sum \alpha_j$$

(2) $[\cdot]$ Iverson bracket, where

$$[P] = \begin{cases} 1, & \text{if } P \text{ is true;} \\ 0, & \text{otherwise.} \end{cases}$$

special cases:

$\delta_{ij} = [i=j]$ kronecker delta notation

$$I_A(x) = [x \in A]$$

with $[P \wedge Q] = [P][Q]$

= let's begin:

$F(x,y) =$

$$\sum_{i,\bar{y}} \alpha_i(\bar{y}) \sum_{s,t} [\bar{y}^{s-1} = y^{t-1} \wedge \bar{y}^s = y^t]$$

$$+ \sum_{i,\bar{y}} \alpha_i(\bar{y}) \sum_{s,t} [\bar{y}^s = y^t]\, k(x_i^s, x^t)$$

$$+ \sum_{i,\bar{y}} \alpha_i(y) \sum_{s,t} [\![y' = y]\!] k(x_i', x^t)$$

since $\alpha_i(\bar{y})$ is constant wrt s,t ...

$$= \underbrace{\sum_{i,\bar{y},s,t} \alpha_i(\bar{y}) [\![\underbrace{\bar{y}^{s-1} = y^{t-1} \wedge \bar{y}^s = y^t}_{=:A}]\!]}_{=:F_1^c}$$

$$+ \underbrace{\sum_{i,\bar{y},s,t} \alpha_i(\bar{y}) [\![\underbrace{\bar{y}^s = y^t}_{=:B}]\!] k(x_i^s, x^t)}_{=:F_2^c}$$

$\Rightarrow$ it suffices to show, that $F_1^c = F_1$ and $F_2^c = F_2$

$$B = [\![\bar{y}^s = y^t]\!] \overset{*}{=} \sum_\sigma [\![\bar{y}^s = \sigma \wedge y^t = \sigma]\!]$$

$$= \sum_\sigma [\![\bar{y}^s = \sigma]\!] [\![y^t = \sigma]\!]$$

$$* = \sum_\sigma [\![\bar{y}^s = \sigma]\!] [\![\bar{y}^s = y^t]\!]$$

$$= \sum_\sigma [\![\bar{y}^s = y^t \wedge \bar{y}^s = \sigma]\!]$$

$$= \sum_\sigma [\![\bar{y}^s = \sigma]\!] [\![y^t = \sigma]\!]$$

$$\Rightarrow F_2^c = \sum_{i,\bar{y},s,t} \alpha_i(\bar{y}) \Big(\sum_\sigma [\![\bar{y}^s = \sigma]\!] [\![y^t = \sigma]\!] \Big) k(x_i^s, x^t)$$

$$= \sum_{i,\bar{y},s,t,\sigma} \alpha_i(\bar{y}) [\![\bar{y}^s = \sigma]\!] [\![y^t = \sigma]\!] k(x_i^s, x^t)$$

$$= \sum_{t,\sigma} [\![y^t = \sigma]\!] \sum_{i,\bar{y},s} [\bar{y}^s = \sigma] \alpha_i(\bar{y})\, k(x_i^s, x^t)$$

$$= \sum_{t,\sigma} [\![y^t = \sigma]\!] \sum_{i,s} \Big(\sum_{\bar{y}} [\![\bar{y}^s = \sigma]\!] \alpha_i(\bar{y}) \Big) k(x_i^s, x^t) = F_2$$

exchanging indices s and t and using variable "y" instead of "$\bar{y}$" to for sum we discussed that in class ...

$$A = [\![\bar{y}^{s-1} = y^{t-1}]\!] [\![\bar{y}^s = y^t]\!]$$

analogous to our treatment of B

$$= \sum_{\sigma} [\![\bar{y}^{s-1} = \sigma]\!] [\![y^{t-1} = \sigma]\!] \Big(\sum_{\tau} [\![\bar{y}^s = \tau]\!] [\![y^t = \tau]\!] \Big)$$

$$= \sum_{\sigma,\tau} [\![\bar{y}^{s-1} = \sigma \wedge \bar{y}^s = \tau]\!] [\![y^{t-1} = \sigma \wedge y^t = \tau]\!]$$

$$\Rightarrow F_1^c = \sum_{i,\bar{y},s,t} \alpha_i(\bar{y}) \Big(\sum_{\sigma,\tau} [\![\bar{y}^{s-1} = \sigma \wedge \bar{y}^s = \tau]\!] [\![y^{t-1} = \sigma \wedge y^t = \tau]\!] \Big)$$

$$= \sum_{\sigma,\tau} \Big(\sum_{i,\bar{y},s} \alpha_i(\bar{y}) [\![\bar{y}^{s-1} = \sigma \wedge \bar{y}^s = \tau]\!] \Big) \sum_{t} [\![y^{t-1} = \sigma \wedge y^t = \tau]\!]$$

again exchanging indices s and t

$$= F_1$$

$$\Rightarrow F = F_1^c + F_2^c = F_1 + F_2 \quad \blacksquare$$