Extend <.,.>

Mittwoch, 3. Mai 2023 16:00

$$\phi(x,y) = \sum_{t=1}^{T} \phi(x,y;t)$$

$$\phi(x,y;t) = \begin{pmatrix} \vdots \\ \phi^{tt}_{r\sigma} = [\![y^t = \sigma]\!] \, \psi_r(x^t) \\ \vdots \\ \phi^{(t-1)t}_{\sigma\tau} = [\![y^{t-1} = \sigma \wedge y^t = \tau]\!] \\ \vdots \end{pmatrix} \begin{matrix} \} \; d \times |\Sigma| \\ \\ \} \; |\Sigma| \times |\Sigma| = |\Sigma|^2 \end{matrix}$$

$$\langle \phi(x,y), \phi(\bar{x},\bar{y}) \rangle$$

$$= \langle \sum_s \phi(x,y;s), \sum_t \phi(\bar{x},\bar{y};t) \rangle$$

$$= \sum_s \langle \phi(x,y;s), \sum_t \phi(\bar{x},\bar{y};t) \rangle$$

$$= \sum_{st} \langle \phi(x,y;s), \phi(\bar{x},\bar{y};t) \rangle$$

$$\langle \phi(x,y;s), \phi(\bar{x},\bar{y};t) \rangle = P_1 + P_2$$

$$P_1 = \sum_\sigma \sum_r [\![y^s = \sigma]\!] \psi_r(x^s) \cdot [\![\bar{y}^t = \sigma]\!] \psi_r(\bar{x}^t)$$

$$\overset{*}{=} [\![y^s = \bar{y}^t]\!] \sum_r \psi_r(x^s) \cdot \psi_r(\bar{x}^t)$$

$$= [\![y^s = \bar{y}^t]\!] \underbrace{\langle \psi(x^s), \psi(\bar{x}^t) \rangle}_{=:\, k(x^s, \bar{x}^t)}$$

[illegible] to

$$=: k(x^s, \bar{x}^t)$$

2x * ("analogous to homework")

$$P_2 = \sum_\sigma \sum_\tau [\![y^{s-1} = \sigma \wedge y^s = \tau]\!] [\![y^{t-1} = \sigma \wedge y^t = \tau]\!]$$

$$\overset{*}{=} [\![y^{s-1} = y^{t-1}]\!] [\![y^s = y^t]\!]$$

$$\Rightarrow \langle \phi(x, y), \phi(\bar{x}, \bar{y}) \rangle = \sum_{s,t} (P_1 + P_2)$$

$$= \sum_{s,t} P_1 + \sum_{s,t} P_2 \qquad \blacksquare$$