Forecast and Simulation **Summer Term 2023**
http://ml3.leuphana.de/lectures/summer23/FS/
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Exercise 1

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Task 1

A local scientist at Lüneburg, called Alice, needs to recover the weather condition on each weekday (Monday to Friday) from previous week.[†] Alice uses an unorthodoxy method to perform this task: she asks his friend Bob about his activities on each of these five days. Alice knows that Bob performs only one activity per day, and that he decides his activity based on the weather condition on each day. Alice believes that the relationship between weather condition and activity is governed by the hidden Markov model (HMM) given in Figure 1. Then, based on the sequence of activities performed by Bob throughout the week, she decides what is the most probable sequence of weather conditions over the five days.

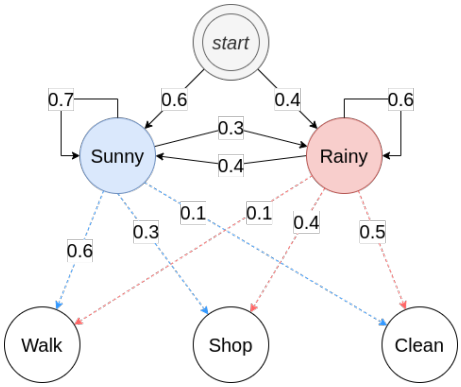

Figure 1: HMM regulating the weather-activity relationship.

From the figure, we can see that Bob is interested on only three activities: walking, shopping, and cleaning his flat. Additionally, there are only two weather conditions: sun or rain. Alice can only *observe* the report about Bob's activities over the five days in question which is given by $X = (x_1, x_2, \ldots, x_5)$, where $X_t \in \{\mathsf{Walk}, \mathsf{Shop}, \mathsf{Clean}\}$, X_1 is his activity on Monday, X_2 is his activity on Tuesday, and so on. The weather condition on each day is *hidden* from her.

Considering the HMM in Figure 1, implement the Viterbi algorithm and apply it to find the most probable sequence of weather conditions $Y = (y_1, y_2, \ldots, y_5)$ given the following sequence of activities reported by Bob $X = (\mathsf{Walk}, \mathsf{Shop}, \mathsf{Clean}, \mathsf{Shop}, \mathsf{Walk})$.

[†]Adapted from https://en.wikipedia.org/wiki/Hidden_Markov_model