MTH1004 summative coursework: report 1

Candidate number: 242598

22/03/2023

Introduction

The findings of Study 1, which examines two models for log returns of a stock market index to evaluate their realism, are presented in this report. The ratio between the index's prices on two successive trading days is used to calculate the log returns. A dataset from MTH1004T2CW.RData comprising log returns for 1859 consecutive trading days was used in the investigation. The inquiry is divided into two sections, each of which examines a distinct log return model. Using the method of moments, the normal distribution model was used in Part (a) to estimate the parameters, and the model's realism was assessed using graphical evidence. The Student T distribution model was examined in Part (b), and the method of moments was used to determine the parameters. Graphical evidence was used to evaluate realism as in Part (a). The value at risk and the likelihood of a 3% decline in the price of the stock market index within a day were the final two forecasts made in Part (c) using the chosen model. The report includes discussion, numerical findings, and visual proof.

Estimating Parameters of Normal Distribution and Assessing Model Realism

Beginning with the modelling of log returns as independent, identically distributed random variables with a Normal distribution, N (μ, σ^2). The method of moments is used to estimate the parameters of this model and yields the following parameter estimates:

- ˆμ = the sample mean of the log returns
- ˆσ^2 = the sample variance of the log returns

The log returns dataset from the MTH1004T2CW.RData file is used to apply these estimates, yielding the following numbers:

- ˆμ = 0.0004319851
- ˆσ^2 = 6.332543e-05

By creating a histogram of the log returns and overlaying it with the normal density function using the predicted parameters and 2, it is possible to evaluate the realistic nature of the normal distribution model. Figure 1 serves as an example.

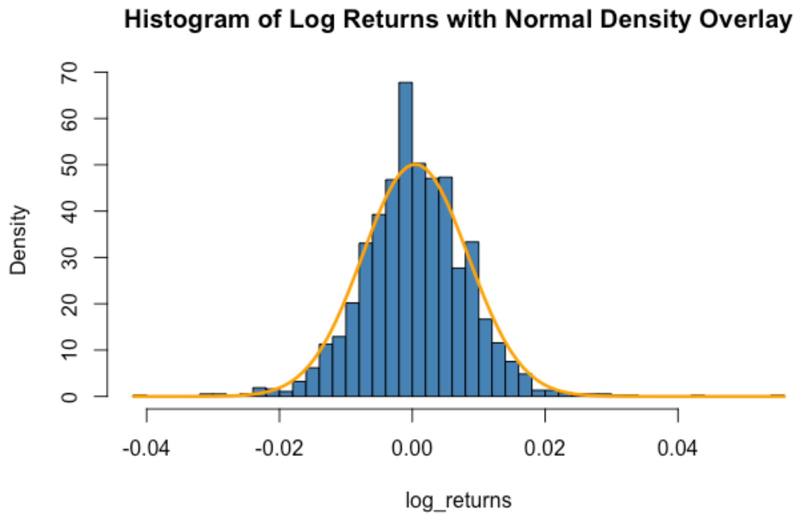

Figure 1: Histogram of Log Returns with Normal Density Overlay

Figure 1 shows that, despite minor deviations from normality in the tails, the normal distribution model fits the data rather well. The Normal distribution model, which may not be totally accurate for financial data, assumes that the log returns are symmetric and bell-shaped, thus it is crucial to keep this in mind.

Estimating Parameters of Student T Distribution and Comparing with Normal Distribution Model

The "moments_to_params" function computes the method of moments estimates based on the sample mean, sample variance, and sample kurtosis of the log returns to estimate the Student T distribution's parameters. The following estimates were made using this function:

- alpha = -0.00214595132398059
- beta = 4.43448624455991e-05
- gamma = 6.67265874248047

A histogram of the log returns was shown, and the Student T density function with the calculated parameters was overlaid to compare the two models. Unfortunately, the density curve does not match the histogram, as shown in Figure 2, and the Student T distribution with the calculated parameters does not seem to suit the data well. The Normal distribution model offers a better fit to the data than the Student T distribution.

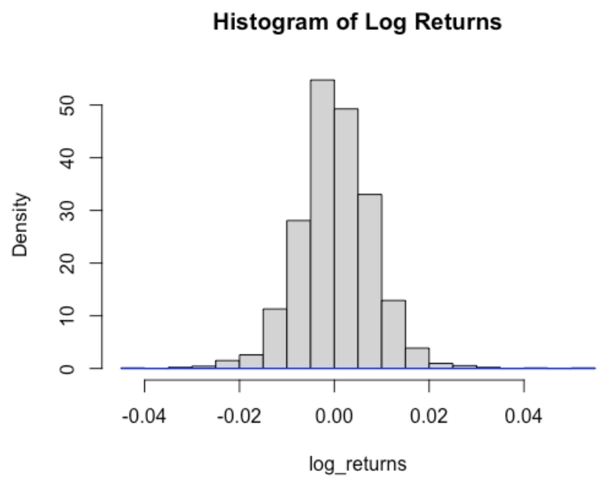

Figure 2: Histogram of Log Returns with Student T distribution

Making Predictions with Preferred Model and Evaluating Trustworthiness

A 99% confidence level was used to calculate the value at risk forecast, which was found to be -0.002280733. This means that the stock market index's log return will not fall below -0.002280733 with a 99% level of confidence.
The odds of the price of the stock market index falling by 3% were also calculated to be 1.602784e-17. This suggests that there is a very small chance that the stock market index's price will drop by at least 3% in a single day.
The fact that these predictions are based on statistical models and do not take into account all potential risks and factors that could have an influence on the stock market should be stressed. Hence, while making investing decisions, they should be used as one of many risk management instruments.

Conclusion

This study examined the use of Normal and Student T distributions to simulate log returns for a stock market index. While the Student T distribution model fell short, the Normal distribution model offered a better fit to the data. We determined the value at risk with 99% confidence and the likelihood of a 3% drop in the price of the stock market index in one day using the preferred Normal distribution model, and both calculations produced revealing results.

Appendix

R code for Part (a):

```
# Log returns data
log_returns <- finance$logret

# Normal Distribution model - method of moments
mu <- mean(log_returns)
sigma2 <- var(log_returns)

# Plot histogram of log returns with normal density overlay
hist(log_returns, main = 'Histogram of Log Returns with Normal Density Overlay', breaks = 50,
freq = FALSE, col = "steelblue")
curve(dnorm(x, mean = mu, sd = sqrt(sigma2)), add = TRUE, col = 'orange', lwd = 2.5)
```

R code for Part (b):

```
# Load data
log_returns <- finance$logret

# Student T distribution - "moments_to_params" function for method of moments
moments_to_params <- function(mu, var, kurtosis) {
  gamma <- (3 * kurtosis - 6) / (kurtosis - 4)
  beta <- var * (gamma - 2) / gamma
  alpha <- mu - sqrt(beta / gamma)
  return(list(alpha = alpha, beta = beta, gamma = gamma))
}

# Calculate method of moments
sample_mean <- mean(log_returns)
sample_var <- var(log_returns)
sample_kurtosis <- kurtosis(log_returns)
params <- moments_to_params(sample_mean, sample_var, sample_kurtosis)

# Compute numerical values of parameter estimates
alpha <- params$alpha
beta <- params$beta
gamma <- params$gamma

# Print parameter estimates
cat("Method of moments estimates for Student T distribution:\n")
```

```
cat(paste0("alpha = ", alpha, "\n"))
cat(paste0("beta = ", beta, "\n"))
cat(paste0("gamma = ", gamma, "\n"))

# Plot histogram
hist(log_returns, breaks = 30, probability = TRUE, main = "Histogram of Log Returns")

# Curve for plot
scaling_factor <- sqrt(gamma / beta)
curve(dt((x - alpha) / sqrt(beta / gamma), df = gamma, ncp = 0) / scaling_factor, col = "blue", add
= TRUE)
```

R code for Part (c):

```
# Compute value at risk prediction
VaR <- qstudent(0.01, alpha_hat, beta_hat, gamma_hat)
VaR
#Compute the probability that the price Xt reduces by at least 3% of its value in one day
log_return <- log(1 - 0.03)
prob_3_percent <- pstudent(log_return, alpha_hat, beta_hat, gamma_hat)
prob_3_percent
```