Physics-I

[Ph-1101, PHYS- 1014, PHY 103, Phy 105]

Waves & Oscillations; Optics; Electricity &Magnetism; Properties of Matter

By

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Marking System:

- Class Participation (Quiz, Assignments, etc.) - 30%
- Midterm Examination - 30%
- Final Examination - 40%

Class Attendance

Attendance	Marks
90% and above	10
85% to less than 90%	09
80% to less than 85%	08
75% to less than 80%	07
70% to less than 75%	06
65% to less than 70%	05
60% to less than 65%	04

Grading System:

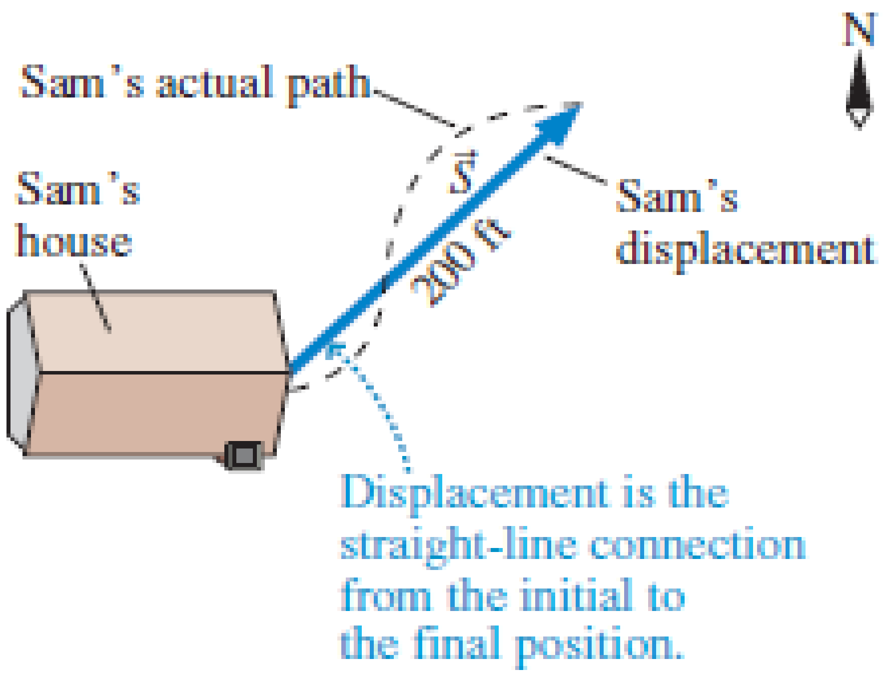
Numerical Grade	Letter Grade	Grade Point
80% and above	A+	4.00
75% to less than 80%	A	3.75
70% to less than 75%	A-	3.50
65% to less than 70%	B+	3.25
60% to less than 65%	B	3.00
55% to less than 60%	B-	2.75
50% to less than 55%	C+	2.50
45% to less than 50%	C	2.25
40% to less than 45%	D	2.00
Less than 40%	F	0.00
	F* Failure I** Incomplete W*** Withdrawal R**** Repeat Y***** Audit	

Vector and Scalar Quantities

- A **scalar quantity** is completely specified by a single value with an appropriate unit and has no direction.
- A **vector quantity** is completely specified by a number with an appropriate unit (the *magnitude* of the vector) plus a direction.

Suppose Sam starts from his front door, walks across the street, and ends up 200 ft to the northeast of where he started. Sam's displacement,

$$\vec{S} = (200\text{ ft, northeast})$$

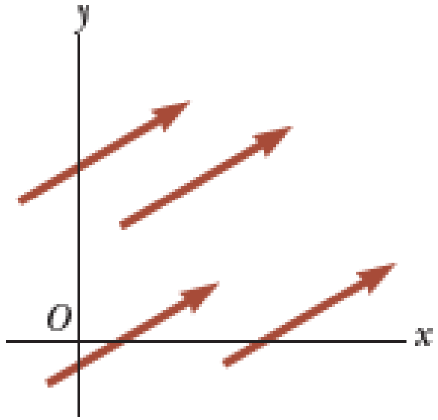


Sam's displacement is a vector quantity.
But, Sam's actual path is a Scalar quantity.

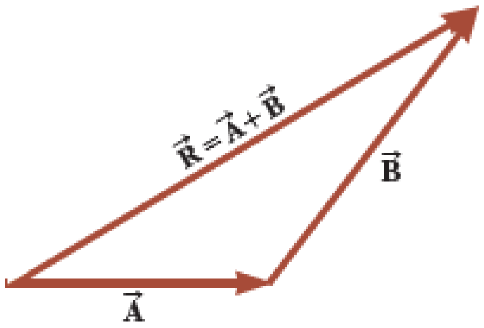
Some Properties of Vectors

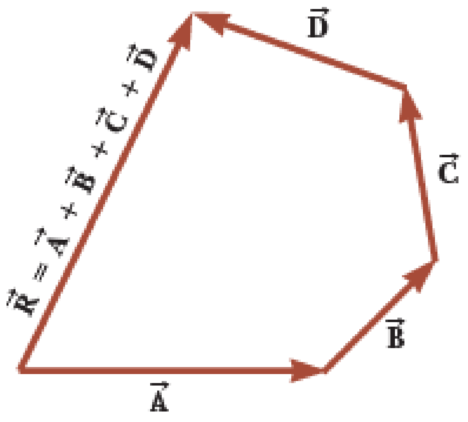
- **Equality of Two Vectors**

These four vectors are equal because they have equal lengths and point in the same direction.

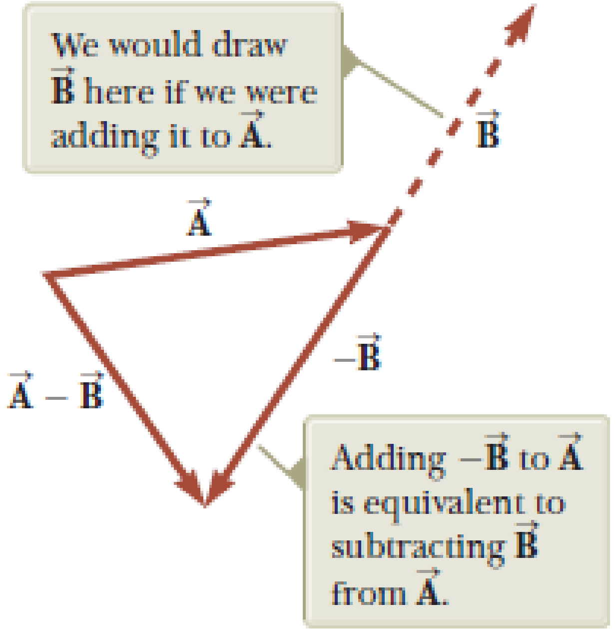


- **Adding Vectors**





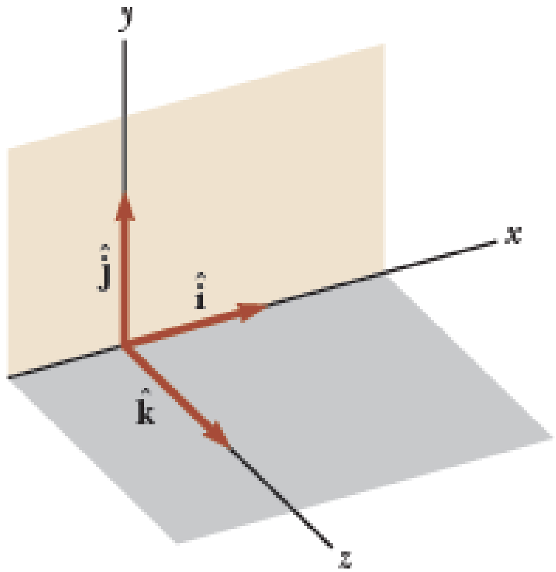
- **Subtracting Vectors**

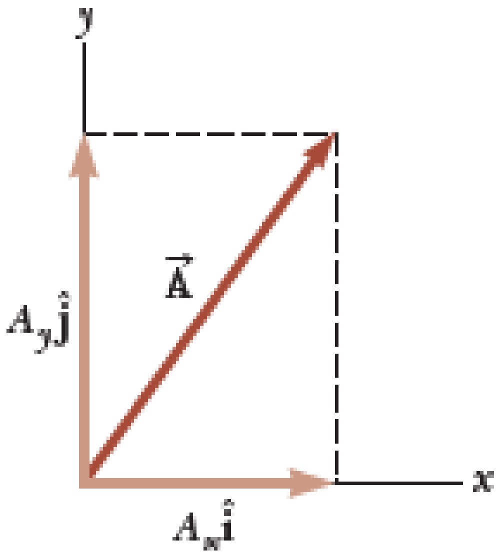


Unit Vectors

A **unit vector** is a dimensionless vector having a magnitude of exactly 1. Unit vectors are used to specify a given direction and have no other physical significance.

We shall use the symbols $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ to represent unit vectors pointing in the positive *x*, *y* and *z* directions, respectively.





$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$$

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

The sum of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is

$$\vec{\mathbf{R}} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} + (A_z + B_z)\hat{\mathbf{k}}$$

Find the sum of two displacement vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ lying in the xy plane and given by

$$\vec{\mathbf{A}} = (2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\text{ m} \quad \text{and} \quad \vec{\mathbf{B}} = (2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}})\text{ m}$$

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (2.0 + 2.0)\hat{\mathbf{i}}\text{ m} + (2.0 - 4.0)\hat{\mathbf{j}}\text{ m} \qquad R_x = 4.0\text{ m} \qquad R_y = -2.0\text{ m}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0\text{ m})^2 + (-2.0\text{ m})^2} = \sqrt{20}\text{ m} = \boxed{4.5\text{ m}}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2.0\text{ m}}{4.0\text{ m}} = -0.50 \qquad \theta = \boxed{333^\circ}$$

A particle undergoes three consecutive displacements: $\Delta\vec{\mathbf{r}}_1 = (15\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$ cm, $\Delta\vec{\mathbf{r}}_2 = (23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}})$ cm, and $\Delta\vec{\mathbf{r}}_3 = (-13\hat{\mathbf{i}} + 15\hat{\mathbf{j}})$ cm. Find unit-vector notation for the resultant displacement and its magnitude.

$$\Delta\vec{\mathbf{r}} = \Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_3 = (15 + 23 - 13)\hat{\mathbf{i}}\text{ cm} + (30 - 14 + 15)\hat{\mathbf{j}}\text{ cm} + (12 - 5.0 + 0)\hat{\mathbf{k}}\text{ cm}$$

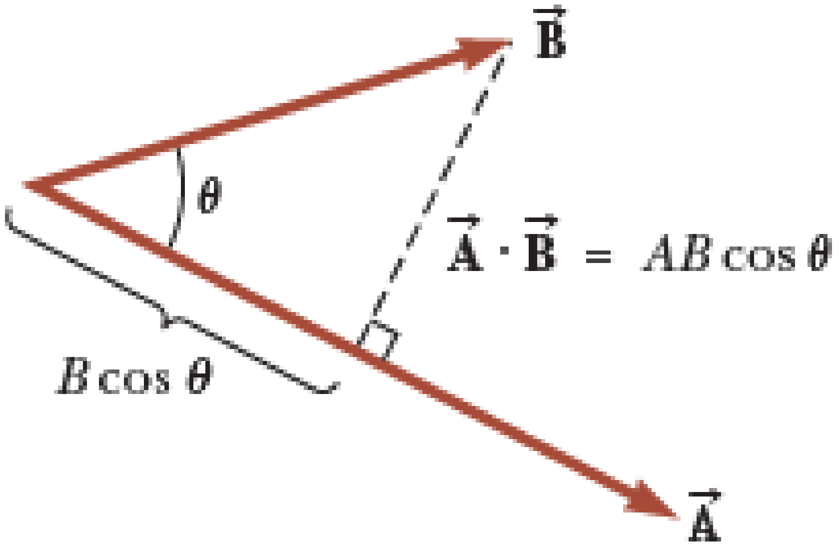
$$= \boxed{(25\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 7.0\hat{\mathbf{k}})\text{ cm}}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(25\text{ cm})^2 + (31\text{ cm})^2 + (7.0\text{ cm})^2} = \boxed{40\text{ cm}}$$

The Scalar Product of Two Vectors

The scalar product of any two vectors $\vec{A}$ and $\vec{B}$ is defined as a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle u between them:

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv AB\cos\theta$$



$$W = F\Delta r\cos\theta = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$$

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{A}} = A_x^2 + A_y^2 + A_z^2 = A^2$$

The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are given by $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$.

(A) Determine the scalar product $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$.

$$\begin{aligned}\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} &= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \\ &= -2\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} + 2\hat{\mathbf{i}} \cdot 2\hat{\mathbf{j}} - 3\hat{\mathbf{j}} \cdot \hat{\mathbf{i}} + 3\hat{\mathbf{j}} \cdot 2\hat{\mathbf{j}} \\ &= -2(1) + 4(0) - 3(0) + 6(1) = -2 + 6 = \boxed{4}\end{aligned}$$

(B) Find the angle θ between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

$$\cos\theta = \frac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{65}} = \boxed{60.3^\circ}$$

A particle moving in the xy plane undergoes a displacement given by $\Delta\vec{\mathbf{r}} = (2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})$ m as a constant force $\vec{\mathbf{F}} = (5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})$ N acts on the particle. Calculate the work done by $\vec{\mathbf{F}}$ on the particle.

The Vector Product and Torque

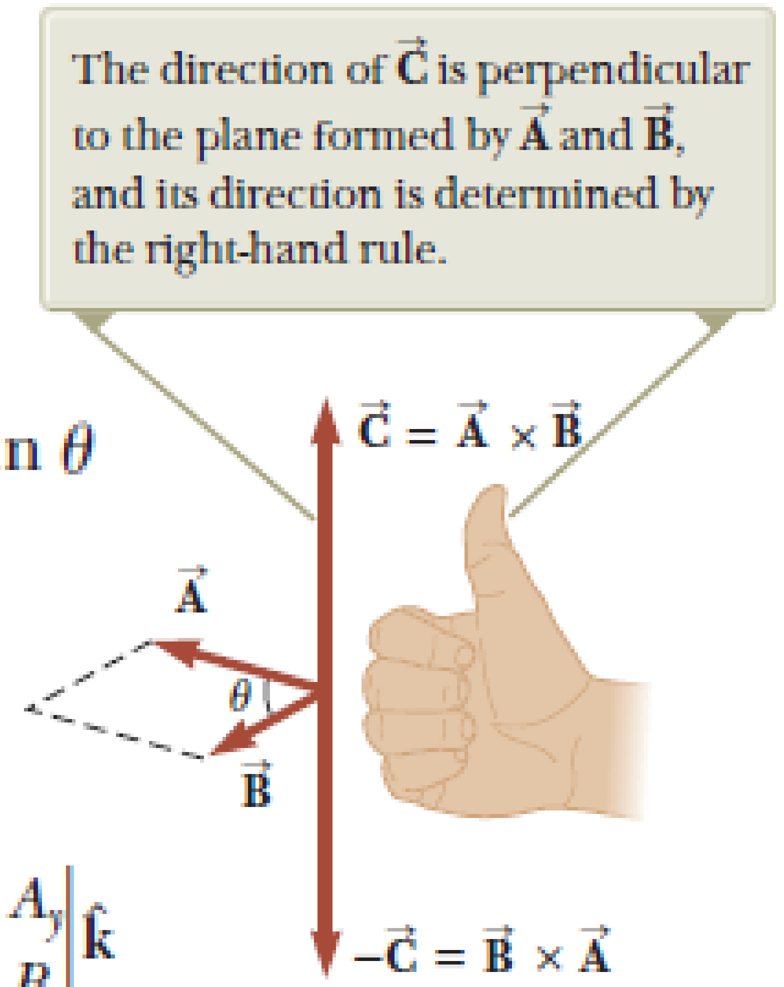
$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$

$$C = AB \sin\theta$$



$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{\mathbf{i}} + \begin{vmatrix} A_z & A_x \\ B_z & B_x \end{vmatrix} \hat{\mathbf{j}} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{\mathbf{k}}$$

Two vectors lying in the xy plane are given by the equations $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$. Find $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ and verify that $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = -\vec{\mathbf{B}} \times \vec{\mathbf{A}}$.

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) = 2\hat{\mathbf{i}} \times (-\hat{\mathbf{i}}) + 2\hat{\mathbf{i}} \times 2\hat{\mathbf{j}} + 3\hat{\mathbf{j}} \times (-\hat{\mathbf{i}}) + 3\hat{\mathbf{j}} \times 2\hat{\mathbf{j}}$$

$$= 0 + 4\hat{\mathbf{k}} + 3\hat{\mathbf{k}} + 0 = 7\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} \times \vec{\mathbf{A}} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \times (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) = (-\hat{\mathbf{i}}) \times 2\hat{\mathbf{i}} + (-\hat{\mathbf{i}}) \times 3\hat{\mathbf{j}} + 2\hat{\mathbf{j}} \times 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} \times 3\hat{\mathbf{j}}$$

$$= 0 - 3\hat{\mathbf{k}} - 4\hat{\mathbf{k}} + 0 = -7\hat{\mathbf{k}}$$

A force of $\vec{\mathbf{F}} = (2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}})$ N is applied to an object that is pivoted about a fixed axis aligned along the z coordinate axis. The force is applied at a point located at $\vec{\mathbf{r}} = (4.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})$ m. Find the torque $\vec{\boldsymbol{\tau}}$ applied to the object.

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = [(4.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})\text{ m}] \times [(2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}})\text{ N}] = [(4.00)(2.00)\hat{\mathbf{i}} \times \hat{\mathbf{i}} + (4.00)(3.00)\hat{\mathbf{i}} \times \hat{\mathbf{j}}$$

$$+ (5.00)(2.00)\hat{\mathbf{j}} \times \hat{\mathbf{i}} + (5.00)(3.00)\hat{\mathbf{j}} \times \hat{\mathbf{j}}]\text{ N}\cdot\text{m}$$

$$\vec{\boldsymbol{\tau}} = [0 + 12.0\hat{\mathbf{k}} - 10.0\hat{\mathbf{k}} + 0]\text{ N}\cdot\text{m} = 2.0\hat{\mathbf{k}}\text{ N}\cdot\text{m}$$

Analysis Model: Particle in Simple Harmonic Motion

When the block is displaced to a position *x*, the spring exerts on the block a force that is proportional to the position and given by **Hooke's law:**

$$F_S = -kx$$

$$\sum F_x = ma_x \rightarrow -kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x \quad where, \omega^2 = k/m$$

The solution is, $x(t) = A\cos(\omega t + \phi)$

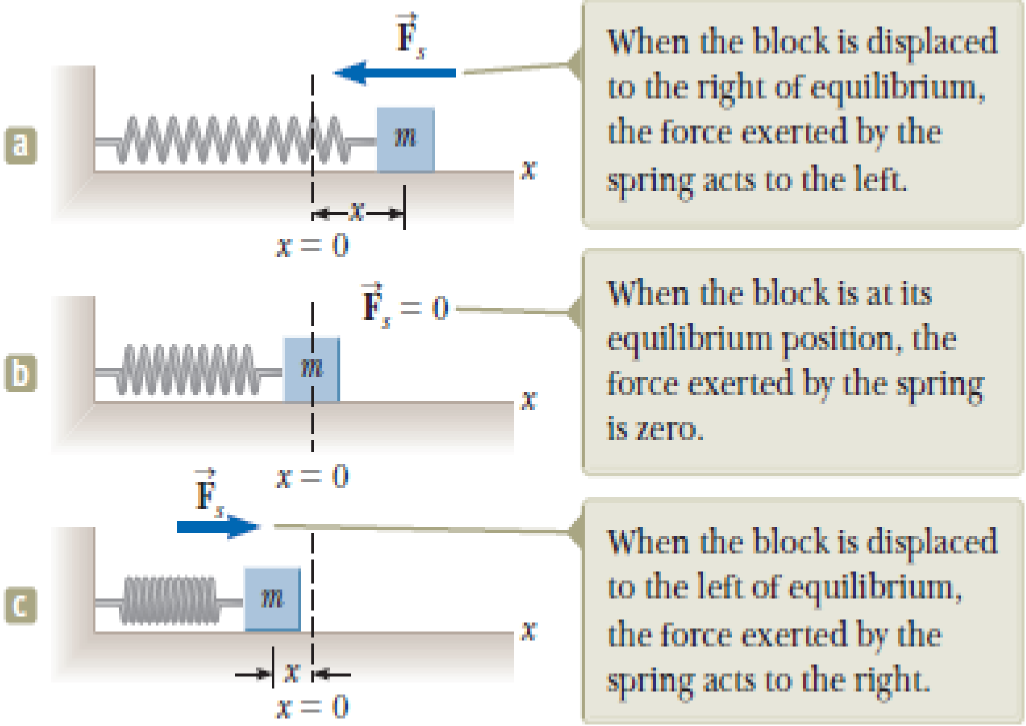


Figure 1 A block attached to a spring moving on a frictionless surface.

Where A, ω and ϕ are constants. This is position versus time for a particle in simple harmonic motion.

Further investigation of the mathematical description of simple harmonic motion.

$x(t) = A\cos(\omega t + \phi)$

The **angular frequency,** $\boldsymbol{\omega = \sqrt{\frac{k}{m}}}$ and the constant angle ϕ is called the **phase constant** (or initial phase angle) and, along with the amplitude A.

Simplifying this expression gives $\omega T = 2\pi$ or $T = 2\pi/\omega$.

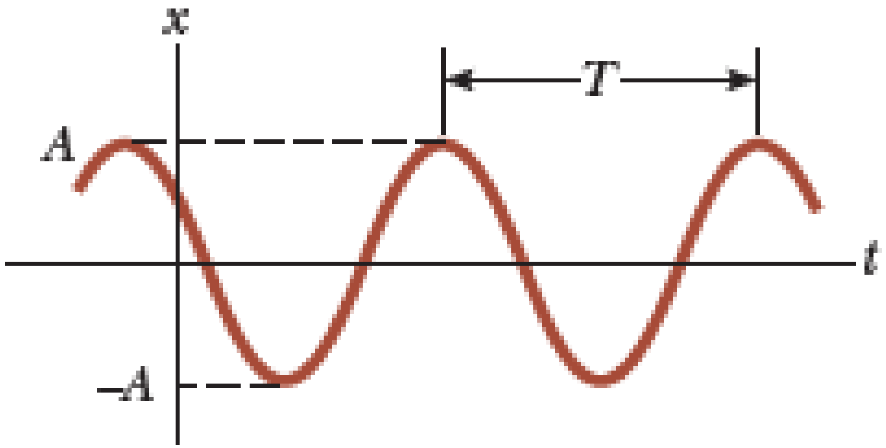
The inverse of the period is called the **frequency** f of the motion.

$$f = 1/T = \omega/2\pi \quad or \qquad \omega = 2\pi f = 2\pi/T$$

$$\therefore The\ Period,$$

$$T = 2\pi/\omega = 2\pi\sqrt{m/k}$$

$$Frequency, f = 1/T = 1/2\pi\sqrt{k/m}$$



$$\boldsymbol{The\ velocity,}$$

$$\boldsymbol{v = \frac{dx}{dt} = -\omega A sin(\omega t + \phi)}$$

$$\boldsymbol{The\ acceleration,}$$

$$\boldsymbol{a = \frac{d^2x}{dt^2} = -\omega^2 A cos(\omega t + \phi)}$$

An example

A 200-g block connected to a light spring for which the force constant is 5.00 N/m is free to oscillate on a frictionless, horizontal surface. The block is displaced 5.00 cm from equilibrium and released from rest as in Figure 15.6.

(A) Find the period of its motion.

Solution:

the angular frequency of the

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00\ \mathrm{N/m}}{200 \times 10^{-3}\ \mathrm{kg}}} = 5.00\ \mathrm{rad/s}$$

1 the period of the system:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00\ \mathrm{rad/s}} = \boxed{1.26\ \mathrm{s}}$$

Energy of the Simple Harmonic Oscillator

The kinetic energy of the block as,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

The potential energy of the block as,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

The total mechanical energy of the simple harmonic oscillator as

$$E = K + U =$$
$$1/2kA^2[\sin^2(\omega t + \phi) + \sin^2(\omega t + \phi)]$$
$$= 1/2kA^2$$

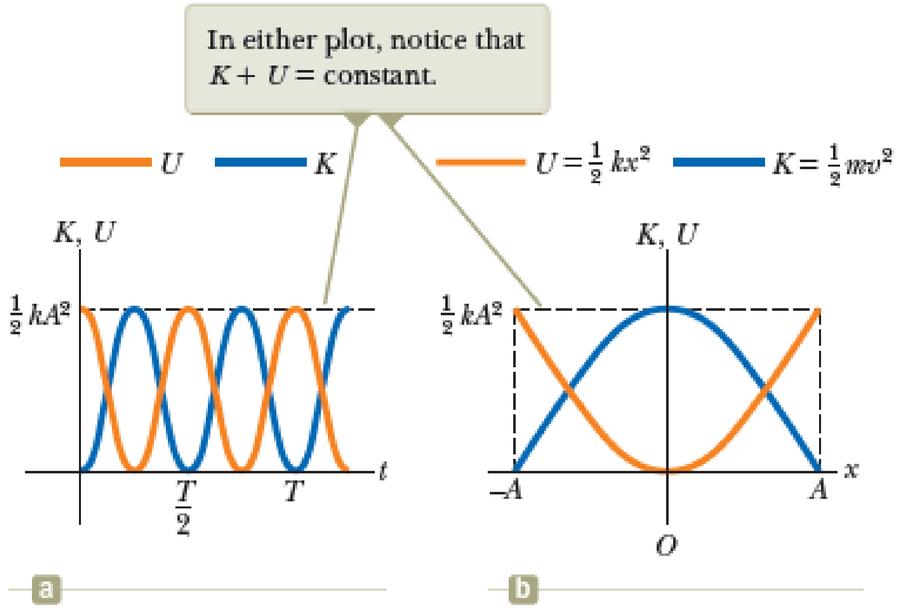


Figure 2 (a) Kinetic energy and potential energy versus time for a simple harmonic oscillator with $\phi = 0$. (b) Kinetic energy and potential energy versus position for a simple harmonic oscillator.

A 0.500-kg cart connected to a light spring for which the force constant is 20.0 N/m oscillates on a frictionless, horizontal air track.

(A) Calculate the maximum speed of the cart if the amplitude of the motion is 3.00 cm.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\text{max}}^2$$

$$v_{\text{max}} = \sqrt{\frac{k}{m}}\,A = \sqrt{\frac{20.0\ \text{N/m}}{0.500\ \text{kg}}}(0.030\,0\ \text{m}) = \boxed{0.190\ \text{m/s}}$$

The simple Pendulum

It consists of a particle-like bob of mass *m* suspended by a light string of length *L* that is fixed at the upper end as shown in Fig. Newton's second law for motion:

$$F_t = ma_t \rightarrow \quad -mg\sin\theta = m\frac{d^2s}{dt^2}$$

Because $s = L\theta$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

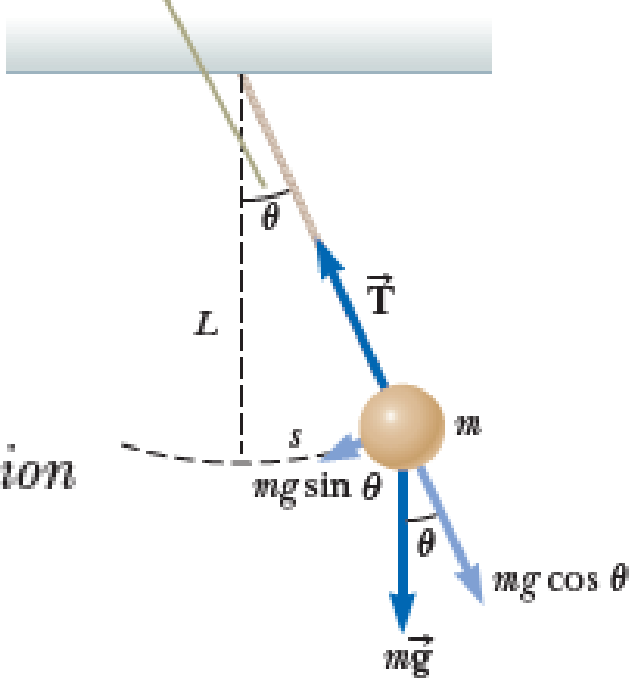
Or, $$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$ (for small values of θ)

$\theta = \theta_{max}\cos(\omega t + \phi)$, θ_{max} is the *maximum angular position*

and the angular frequency ω is $$\omega = \sqrt{\frac{g}{L}}$$

The period of the motion is $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

When θ is small, a simple pendulum's motion can be modeled as simple harmonic motion about the equilibrium position $\theta = 0$.



Christian Huygens (1629–1695), the greatest clockmaker in history, suggested that an international unit of length could be defined as the length of a simple pendulum having a period of exactly 1 s. How much shorter would our length unit be if his suggestion had been followed?

$$L = \frac{T^2 g}{4\pi^2} = \frac{(1.00\ \mathrm{s})^2(9.80\ \mathrm{m/s^2})}{4\pi^2} = \boxed{0.248\ \mathrm{m}}$$

WHAT IF? What if Huygens had been born on another planet? What would the value for g have to be on that planet such that the meter based on Huygens's pendulum would have the same value as our meter?

$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(1.00\ \mathrm{m})}{(1.00\ \mathrm{s})^2} = 4\pi^2\ \mathrm{m/s^2} = 39.5\ \mathrm{m/s^2}$$

No planet in our solar system has an acceleration due to gravity that large.

Forced Oscillations

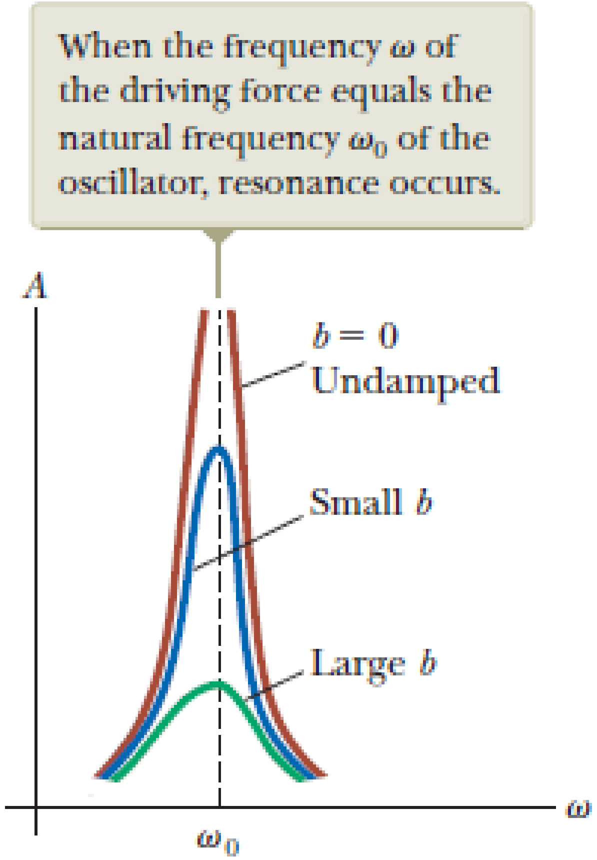
A common example of a forced oscillator is a damped oscillator driven by an external force that varies periodically, $F(t) = F_0 \sin \omega t$, where F_0 is a constant and ω is the angular frequency of the driving force.

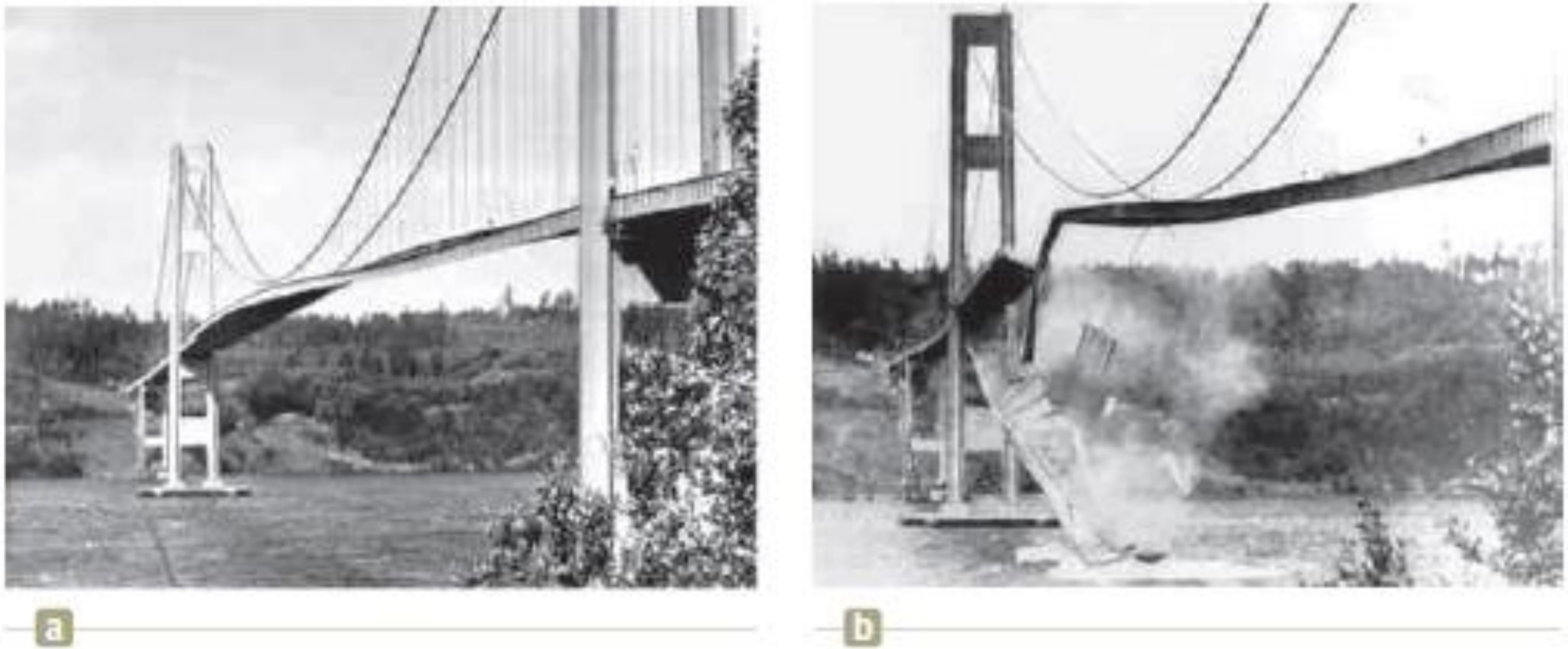
The solution of this equation is rather lengthy and will not be presented.

$$x = A \cos (\omega t + \phi)$$

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$ where $\omega_0 = \sqrt{k/m}$

The dramatic increase in amplitude near the natural frequency is called **resonance,** and the natural frequency ω_0 is also called the **resonance frequency** of the system.





(a) In 1940, turbulent winds set up torsional vibrations in the Tacoma Narrows Bridge, causing it to oscillate at a frequency near one of the natural frequencies of the bridge structure. (b) Once established, this resonance condition led to the bridge’s collapse. (Mathematicians and physicists are currently challenging some aspects of this interpretation.)

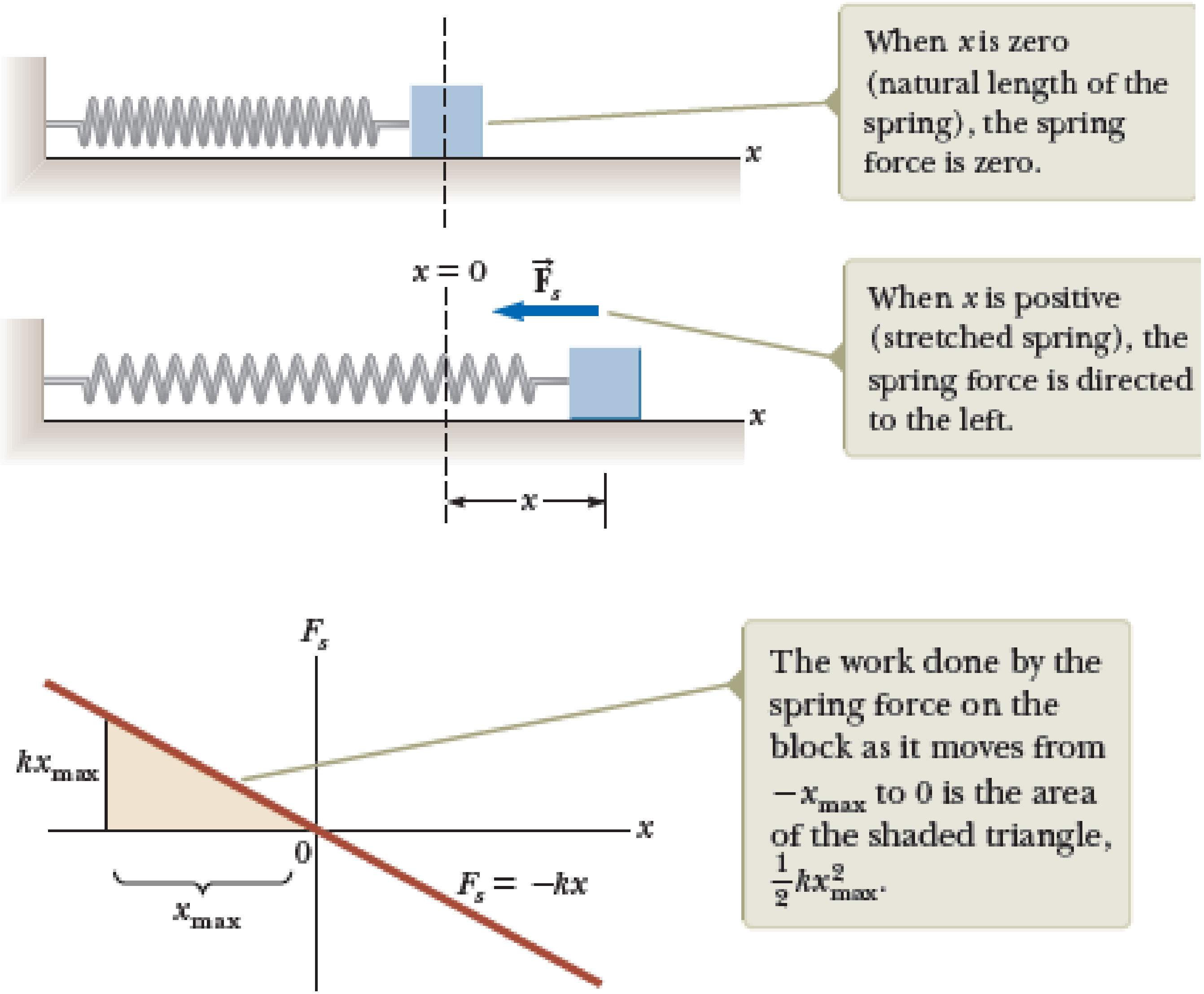

Elasticity

Work Done by a Spring

For many springs, if the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force that can be mathematically modeled as

$$F_s = -kx$$

This force law for springs is known as **Hooke's law.**



Elastic Properties of Solids

We shall discuss the deformation of solids in terms of the concepts of *stress* and *strain*.

- **Stress** is a quantity that is proportional to the force causing a deformation; more specifically, stress is the external force acting on an object per unit cross-sectional area. The result of a stress is **strain,** which is a measure of the degree of deformation.
- It is found that, for sufficiently small stresses, stress is proportional to strain; the constant of proportionality depends on the material being deformed and on the nature of the deformation. We call this proportionality constant the **elastic modulus.** The elastic modulus is therefore defined as the ratio of the stress to the resulting strain:

$$\boldsymbol{Elastic\ modulus \equiv \frac{stress}{strain}}$$

The elastic modulus

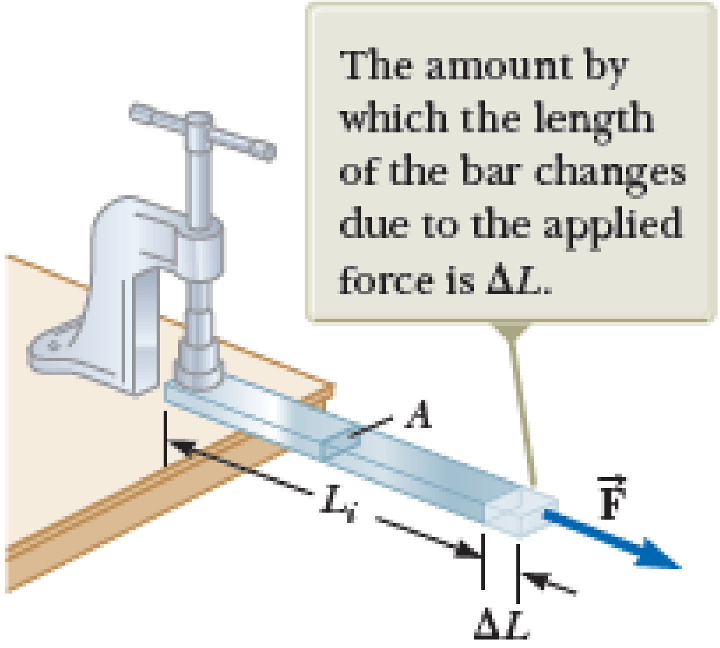
We consider three types of deformation and define an elastic modulus for each:

- **Young's modulus** measures the resistance of a solid to a change in its length.
- **Shear modulus** measures the resistance to motion of the planes within a solid parallel to each other.
- **Bulk modulus** measures the resistance of solids or liquids to changes in their volume.

Young's Modulus: Elasticity in Length

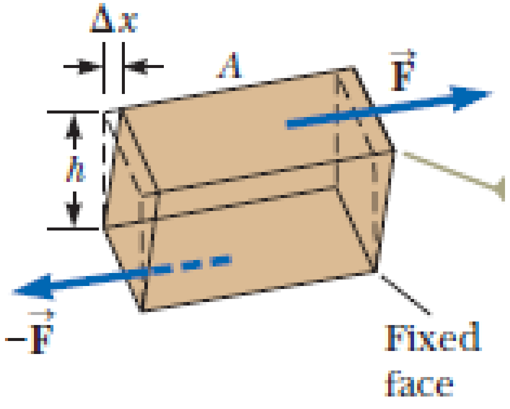
$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i}$$

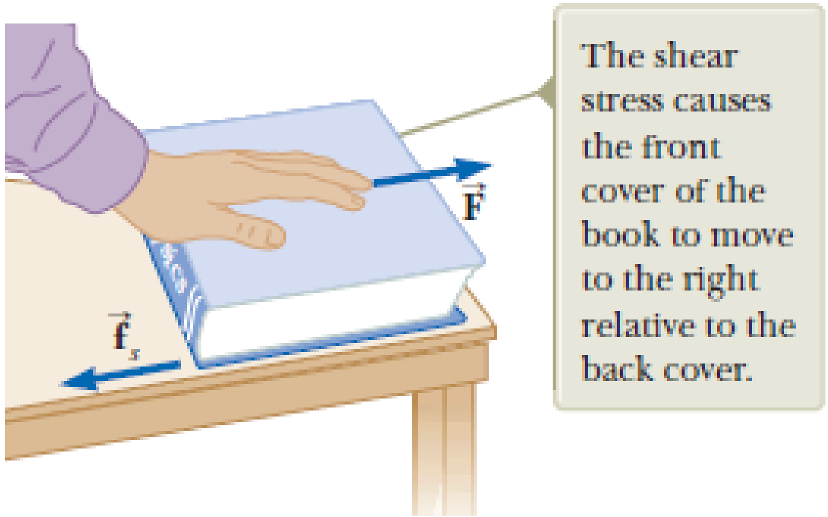
Because strain is a dimensionless quantity, *Y* has units of force per unit area.



Shear Modulus: Elasticity of Shape

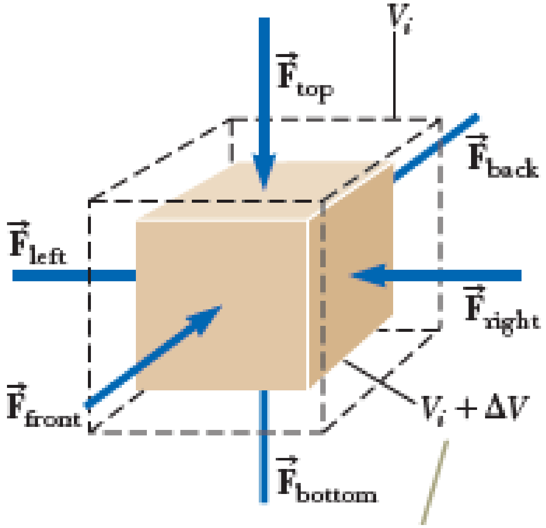
$$S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h}$$





Bulk Modulus: Volume Elasticity

$$B \equiv \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F/A}{\Delta V/V_i} = -\frac{\Delta P}{\Delta V/V_i}$$



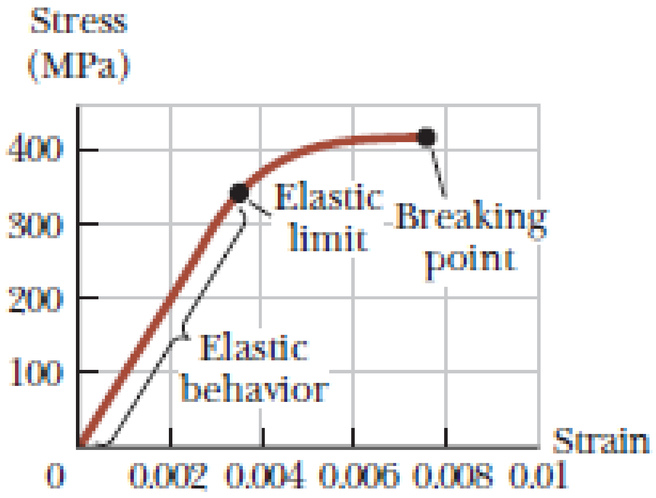


Table 12.1 Typical Values for Elastic Moduli

Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Examples: Now suppose the tension in the cable is 940 N as the actor reaches the lowest point. What diameter should a 10-m-long steel cable have if we do not want it to stretch more than 0.50 cm under these conditions?

$$A = \frac{FL_i}{Y\Delta L}$$

Assuming the cross section is circular, find the diameter of the cable from $d = 2r$ and $A = \pi r^2$:

$$d = 2r = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{FL_i}{\pi Y \Delta L}} = 2\sqrt{\frac{(940\ \text{N})(10\ \text{m})}{\pi(20 \times 10^{10}\ \text{N/m}^2)(0.005\,0\ \text{m})}}$$

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is $1.0 \times 10^5\ N/m^2$(normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is $2.0 \times 10^7\ N/m^2$. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

$$\Delta V = -\frac{V_i \Delta P}{B} = -\frac{(0.50\ \text{m}^3)(2.0 \times 10^7\ \text{N/m}^2 - 1.0 \times 10^5\ \text{N/m}^2)}{6.1 \times 10^{10}\ \text{N/m}^2}$$

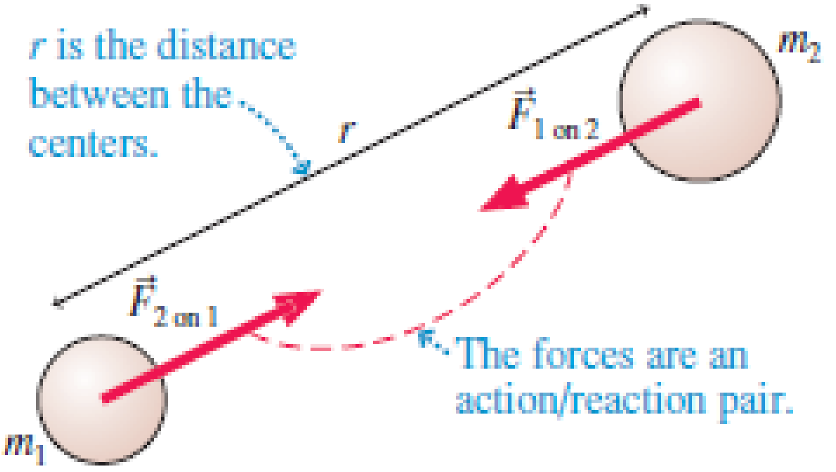
$$= \boxed{-1.6 \times 10^{-4}\ \text{m}^3}$$

Newton's Law of Universal Gravitation

Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

If the particles have masses m_1 and m_2 and are separated by a distance r, the magnitude of this gravitational force is

$$F_g = G\frac{m_1 m_2}{r^2}$$



where G is a constant, called the *universal gravitational constant.* Its value in SI units is $G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$

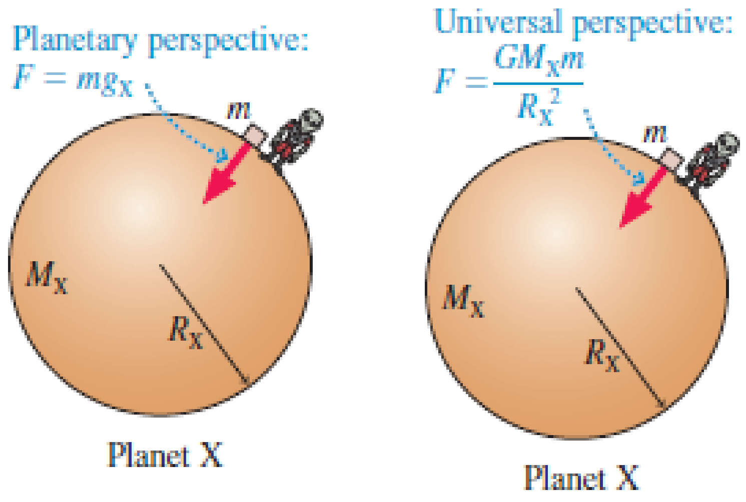
Free-Fall Acceleration and the Gravitational Force

$$mg = G\frac{M_E m}{R_E^{\,2}}$$

$$g = G\frac{M_E}{R_E^{\,2}}$$

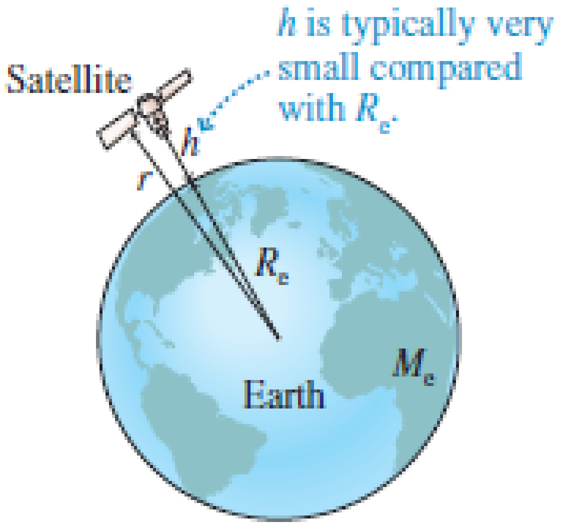
$$\boxed{F_g = mg}$$

$$F_g = G\frac{M_E m}{R_E^{\,2}}$$



Now consider an object of mass *m* located a distance *h* above the Earth's surface or a distance *r* from the Earth's center, where $r = R_E + h$. The magnitude of the gravitational force acting on this object is

$$F_g = G\frac{M_E m}{r^2} = G\frac{M_E m}{(R_E + h)^2}$$



Using the known radius of the Earth and that $\boldsymbol{g = 9.80\ m/s^2}$ at the Earth's surface, find the average density of the Earth.

$$M_E = \frac{gR_E^2}{G}$$

$$\rho_E = \frac{M_E}{V_E} = \frac{gR_E^2/G}{\frac{4}{3}\pi R_E^3} = \frac{3}{4}\frac{g}{\pi G R_E}$$

$$= \frac{3}{4}\frac{9.80\ \mathrm{m/s^2}}{\pi(6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(6.37 \times 10^6\ \mathrm{m})}$$

$$= \boxed{5.50 \times 10^3\ \mathrm{kg/m^3}}$$

Free-Fall Acceleration *g* at Various Altitudes Above the Earth's Surface

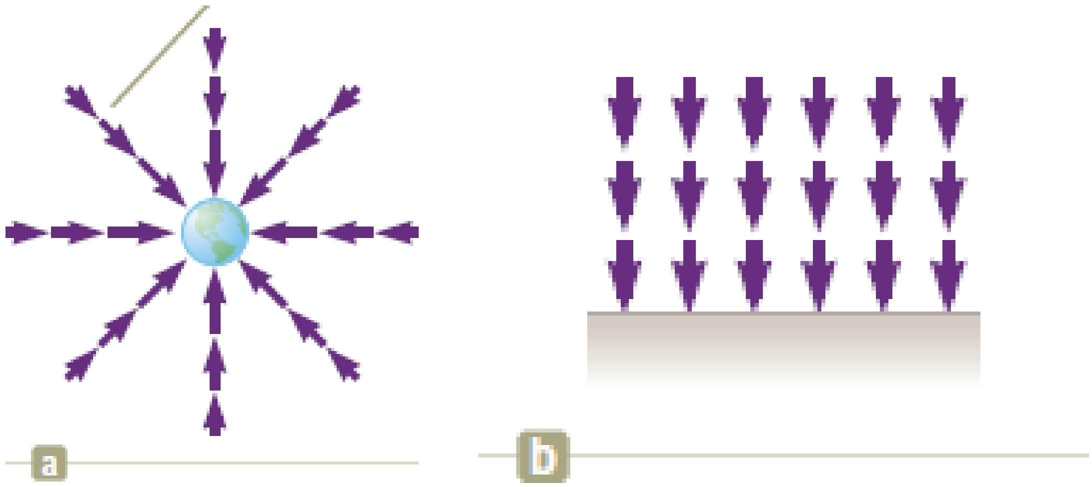
Altitude h (km)	g (m/s²)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13
∞	0

Analysis Model: Particle in a Field (Gravitational)

Gravitational field ▶

$$\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}_g}{m} = -\frac{GM_E}{r^2}\hat{\mathbf{r}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing radially outward from the Earth and the negative sign indicates that the field points toward the center of the Earth as illustrated in Figure a.



(a) The gravitational field vectors in the vicinity of a uniform spherical mass such as the Earth vary in both direction and magnitude. (b) The gravitational field vectors in a small region near the Earth's surface are uniform in both direction and magnitude.

The International Space Station operates at an altitude of 350 km. Plans for the final construction show that material of weight 4.22×10^6 N, measured at the Earth's surface, will have been lifted off the surface by various spacecraft during the construction process. What is the weight of the space station when in orbit?

$$m = \frac{F_g}{g} = \frac{4.22 \times 10^6 \text{ N}}{9.80 \text{ m/s}^2} = 4.31 \times 10^5 \text{ kg}$$

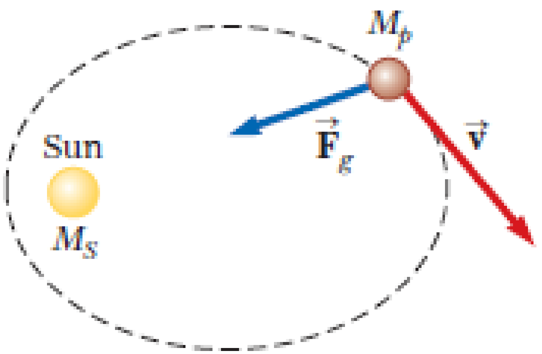
$$g = \frac{GM_E}{(R_E + h)^2}$$

$$= \frac{(6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})}{(6.37 \times 10^6 \text{ m} + 0.350 \times 10^6 \text{ m})^2} = 8.82 \text{ m/s}^2$$

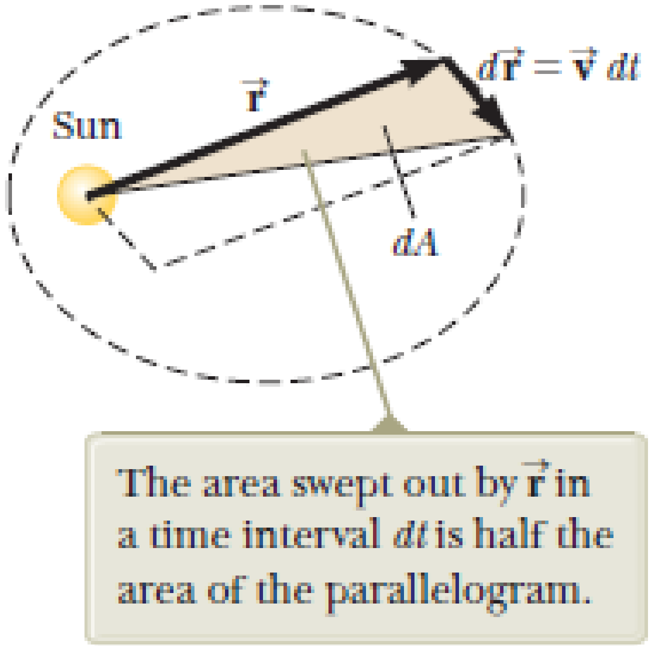
$$F_g = mg = (4.31 \times 10^5 \text{ kg})(8.82 \text{ m/s}^2) = \boxed{3.80 \times 10^6 \text{ N}}$$

Kepler's Laws and the Motion of Planets

Kepler's First Law: All planets move in elliptical orbits with the Sun at one focus.



Kepler's Second Law: The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals.



Evaluating $\vec{\mathbf{L}}$ for the planet,

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = M_p \vec{\mathbf{r}} \times \vec{\mathbf{v}} \rightarrow L = M_p |\vec{\mathbf{r}} \times \vec{\mathbf{v}}|$$

$$dA = \tfrac{1}{2}|\vec{\mathbf{r}} \times d\vec{\mathbf{r}}| = \tfrac{1}{2}|\vec{\mathbf{r}} \times \vec{\mathbf{v}}\,dt| = \tfrac{1}{2}|\vec{\mathbf{r}} \times \vec{\mathbf{v}}|\,dt$$

$$dA = \tfrac{1}{2}\left(\frac{L}{M_p}\right)dt$$

$$\frac{dA}{dt} = \frac{L}{2M_p}$$ where L and M_p are both constants.

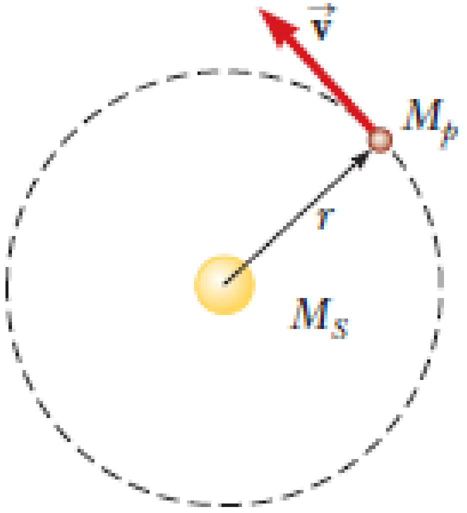
Kepler's Third Law: The square of the orbital period of any planet is proportional to the cube of the semimajor axis of the elliptical orbit.

$$F_g = M_p a \quad \rightarrow \quad \frac{GM_S M_p}{r^2} = M_p\left(\frac{v^2}{r}\right) \qquad \frac{GM_S}{r^2} = \frac{(2\pi r/T)^2}{r}$$

$$T^2 = \left(\frac{4\pi^2}{GM_S}\right)r^3 = K_S r^3$$

where K_S is a constant given by

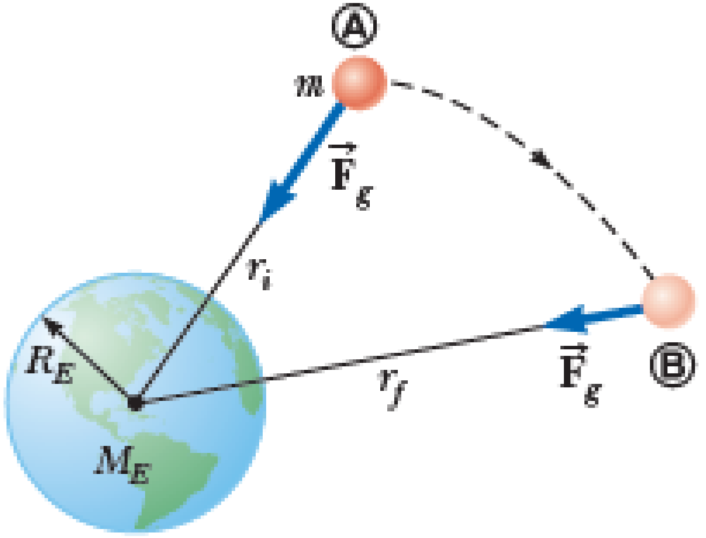
$$K_S = \frac{4\pi^2}{GM_S} = 2.97 \times 10^{-19}\ \mathrm{s^2/m^3}$$



Calculate the mass of the Sun, noting that the period of the Earth's orbit around the Sun is 3.156×10^7 s and its distance from the Sun is 1.496×10^{11} m.

$$M_S = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(1.496 \times 10^{11}\ \mathrm{m})^3}{(6.674 \times 10^{-11}\ \mathrm{N\cdot m^2/kg^2})(3.156 \times 10^7\ \mathrm{s})^2} = \boxed{1.99 \times 10^{30}\ \mathrm{kg}}$$

As a particle of mass *m* moves from A to B above the Earth's surface, the gravitational potential energy of the particle–Earth system changes according to Equation:



$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)\, dr \qquad F(r) = -\frac{GM_E m}{r^2}$$

$$U_f - U_i = GM_E m \int_{r_i}^{r_f} \frac{dr}{r^2} = GM_E m \left[-\frac{1}{r}\right]_{r_i}^{r_f}$$

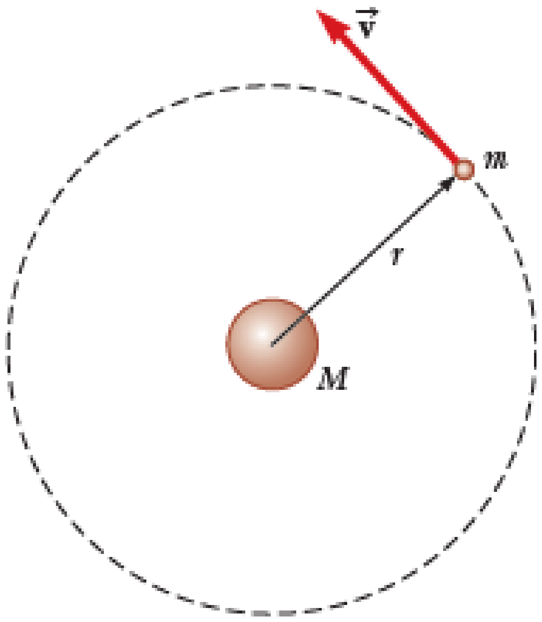
$$U_f - U_i = -GM_E m \left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

Taking $U_i = 0$ at $r_i = \infty$, we obtain the important result

$$\boxed{U(r) = -\frac{GM_E m}{r}}$$

Energy Considerations in Planetary and Satellite Motion

$$E = K + U \qquad E = \tfrac{1}{2}mv^2 - \frac{GMm}{r}$$



$$F_g = ma \quad \rightarrow \quad \frac{GMm}{r^2} = \frac{mv^2}{r}$$

Multiplying both sides by *r* and dividing by 2 gives

$$\tfrac{1}{2}mv^2 = \frac{GMm}{2r}$$ Substituting $$E = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$E = -\frac{GMm}{2r} \quad \text{(circular orbits)}$$

A space transportation vehicle releases a 470-kg communications satellite while in an orbit 280 km above the surface of the Earth. A rocket engine on the satellite boosts it into a geosynchronous orbit. How much energy does the engine have to provide?

$$r_i = R_E + 280 \text{ km} = 6.65 \times 10^6 \text{ m}$$

$$\Delta E = E_f - E_i = -\frac{GM_E m}{2r_f} - \left(-\frac{GM_E m}{2r_i}\right) = -\frac{GM_E m}{2}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

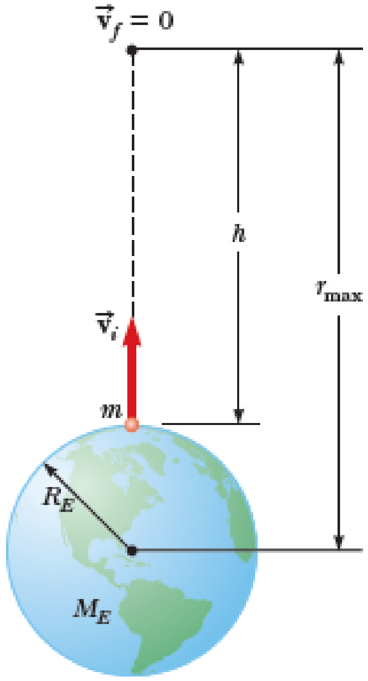
$$\Delta E = -\frac{(6.674 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})(470 \text{ kg})}{2} \times \left(\frac{1}{4.22 \times 10^7 \text{ m}} - \frac{1}{6.65 \times 10^6 \text{ m}}\right) = \boxed{1.19 \times 10^{10} \text{ J}}$$

Escape Speed

As the object is projected upward from the surface of the Earth, $v = v_i$ and $r = r_i = R_E$. When the object reaches its maximum altitude, $v = v_f = 0$ and $r = r_f = r_{max}$.

$$\frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{\max}}$$ Letting $r_{\max} \to \infty$

$$v_{\text{esc}} = \sqrt{\frac{2GM_E}{R_E}}$$



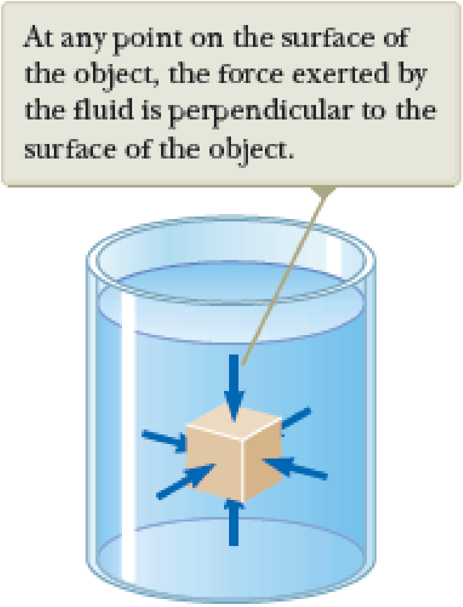
Calculate the escape speed from the Earth for a 5 000-kg spacecraft and determine the kinetic energy it must have at the Earth's surface to move infinitely far away from the Earth.

$$v_{\text{esc}} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2(6.674 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})}{6.37 \times 10^6\ \text{m}}} = \boxed{1.12 \times 10^4\ \text{m/s}}$$

$$K = \frac{1}{2}mv_{\text{esc}}^2 = \frac{1}{2}(5.00 \times 10^3\ \text{kg})(1.12 \times 10^4\ \text{m/s})^2 = \boxed{3.13 \times 10^{11}\ \text{J}}$$

Fluid Mechanics :Pressure

If F is the magnitude of the force exerted on the piston and A is the surface area of the piston, the **pressure** P of the fluid at the level to which the device has been submerged is defined as the ratio of the force to the area: $P \equiv \frac{F}{A}$



The SI unit of pressure is the **pascal** (Pa): $1 \text{ Pa} \equiv 1 \text{ N/m}^2$

Example: The mattress of a water bed is 2.00 m long by 2.00 m wide and 30.0 cm deep. **(A)** Find the weight of the water in the mattress. **(B)** Find the pressure exerted by the water bed on the floor.

Solution: (A) $V = (2.00 \text{ m})(2.00 \text{ m})(0.300 \text{ m}) = 1.20 \text{ m}^3$

$Mg = (1.20 \times 10^3 \text{ kg})(9.80 \text{ m/s}^2) = 1.18 \times 10^4 \text{ N}$ $\quad M = \rho V = (1\,000 \text{ kg/m}^3)(1.20 \text{ m}^3) = 1.20 \times 10^3 \text{ kg}$

(B)

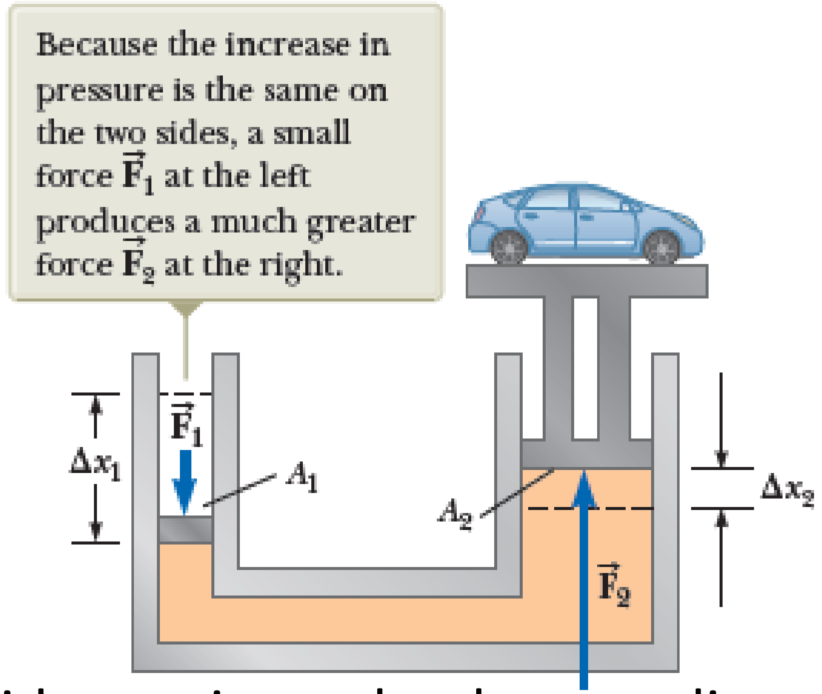
$$P = \frac{1.18 \times 10^4 \text{ N}}{4.00 \text{ m}^2} = 2.94 \times 10^3 \text{ Pa}$$

Pascal's law:

a change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

$$F_1 \Delta x_1 = F_2 \Delta x_2.$$

the work done by $\vec{F}_1$ on the input piston equals the work done by $\vec{F}_2$ on the output piston.



Example: In a car lift, compressed air exerts a force on a small piston that has a circular cross section of radius 5.00 cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0 cm. **(A)** What force must the compressed air exert to lift a car weighing 13 300 N? **(B)** What air pressure produces this force?

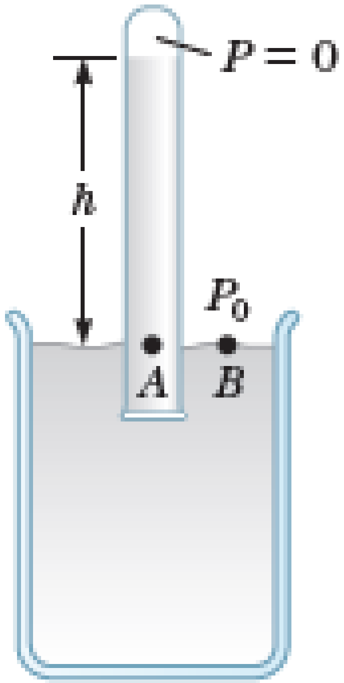
(A)

$$F_1 = \left(\frac{A_1}{A_2}\right)F_2 = \frac{\pi(5.00 \times 10^{-2}\ \mathrm{m})^2}{\pi(15.0 \times 10^{-2}\ \mathrm{m})^2}(1.33 \times 10^4\ \mathrm{N}) = \boxed{1.48 \times 10^3\ \mathrm{N}}$$

(B)

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3\ \mathrm{N}}{\pi(5.00 \times 10^{-2}\ \mathrm{m})^2} = \boxed{1.88 \times 10^5\ \mathrm{Pa}}$$

Pressure Measurements: Torricelli experiment



A long tube closed at one end is filled with mercury and then inverted into a dish of mercury (Fig.). The closed end of the tube is nearly a vacuum, so the pressure at the top of the mercury column can be taken as zero. In Figure, the pressure at point *A*, due to the column of mercury, must equal the pressure at point *B*, due to the atmosphere. If that were not the case, there would be a net force that would move mercury from one point to the other until equilibrium is established. Therefore, $P_0 = h\rho_{Hg}g$, where ρ_{Hg} is the density of the mercury and *h* is the height of the mercury column.

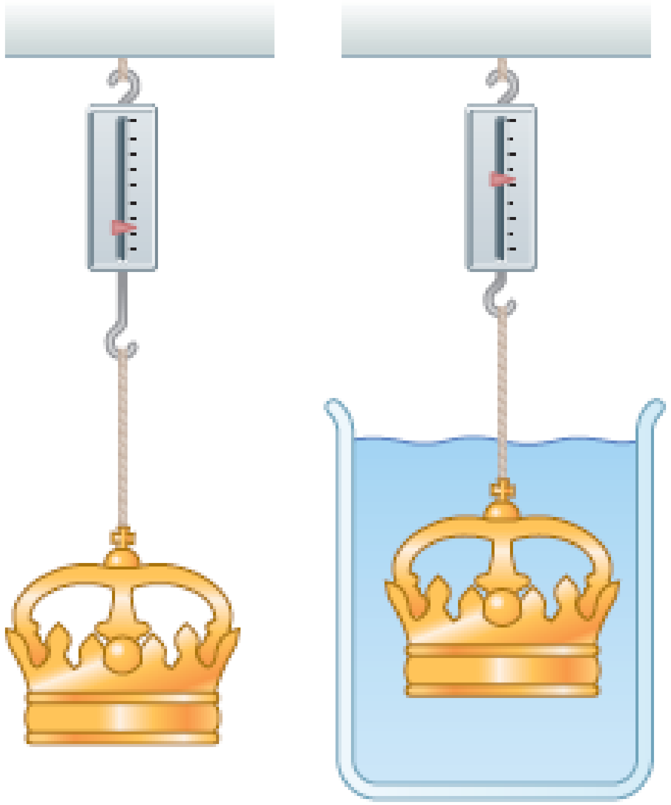
Let us determine the height of a mercury column for one atmosphere of pressure

$$P_0 = \rho_{\mathrm{Hg}} g h \quad \rightarrow \quad h = \frac{P_0}{\rho_{\mathrm{Hg}} g} = \frac{1.013 \times 10^5 \ \mathrm{Pa}}{(13.6 \times 10^3 \ \mathrm{kg/m^3})(9.80 \ \mathrm{m/s^2})} = 0.760 \ \mathrm{m}$$

Buoyant force and Archimedes's Principle

The upward force exerted by a fluid on any immersed object is called a **buoyant force.**

The magnitude of the buoyant force on an object always equals the weight of the fluid displaced by the object. This statement is known as **Archimedes's principle.**



Fluid Dynamics:

Because the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our simplification model of **ideal fluid flow,** we make the following four assumptions:

1. The fluid is nonviscous. In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.

2. The flow is steady. In steady (laminar) flow, all particles passing through a point have the same velocity.

3. The fluid is incompressible. The density of an incompressible fluid is constant.

4. The flow is irrotational. In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, the flow is irrotational.

Equation of Continuity for Fluids:

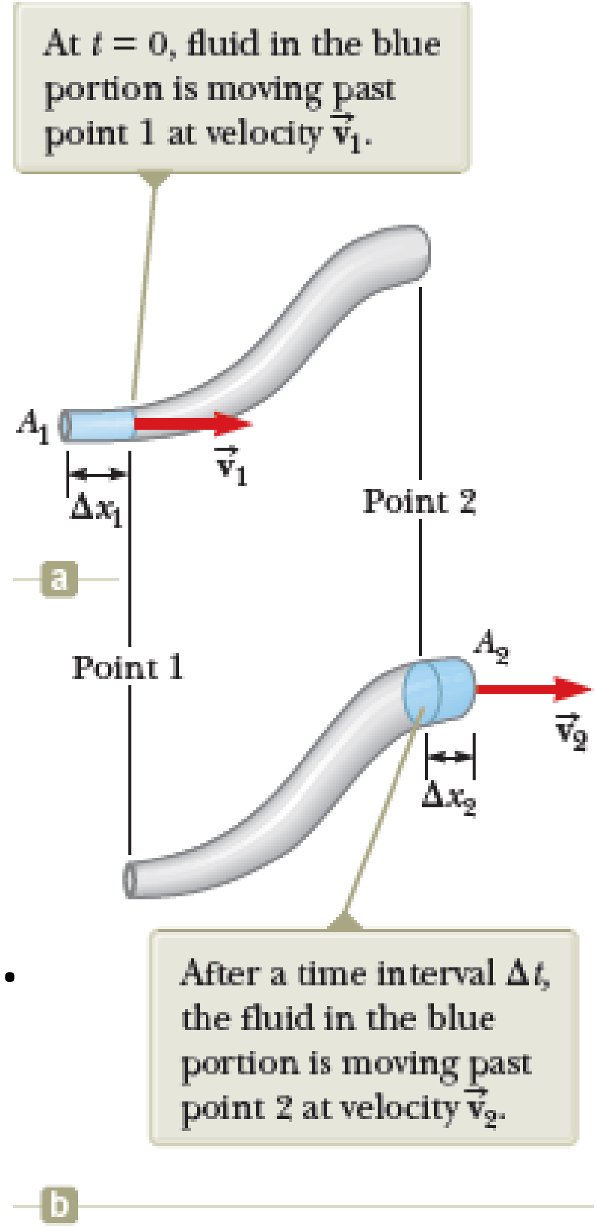
A fluid moving with steady flow through a pipe of varying cross-sectional area. (a) At t =0, the small blue-colored portion of the fluid at the left is moving through area A_1. (b) After a time interval Δt, the blue-colored portion shown here is that fluid that has moved through area A_2.

$$m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t, \qquad m_2 = \rho A_2 \Delta x_2 = \rho A_2 v_2 \Delta t.$$

$$m_1 = m_2 \text{ or } \rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t,$$

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

This expression is called the **equation of continuity for fluids.**



Bernoulli's Equation:

The equation of continuity is one of two important relationships for ideal fluids. The other is a statement of energy conservation.

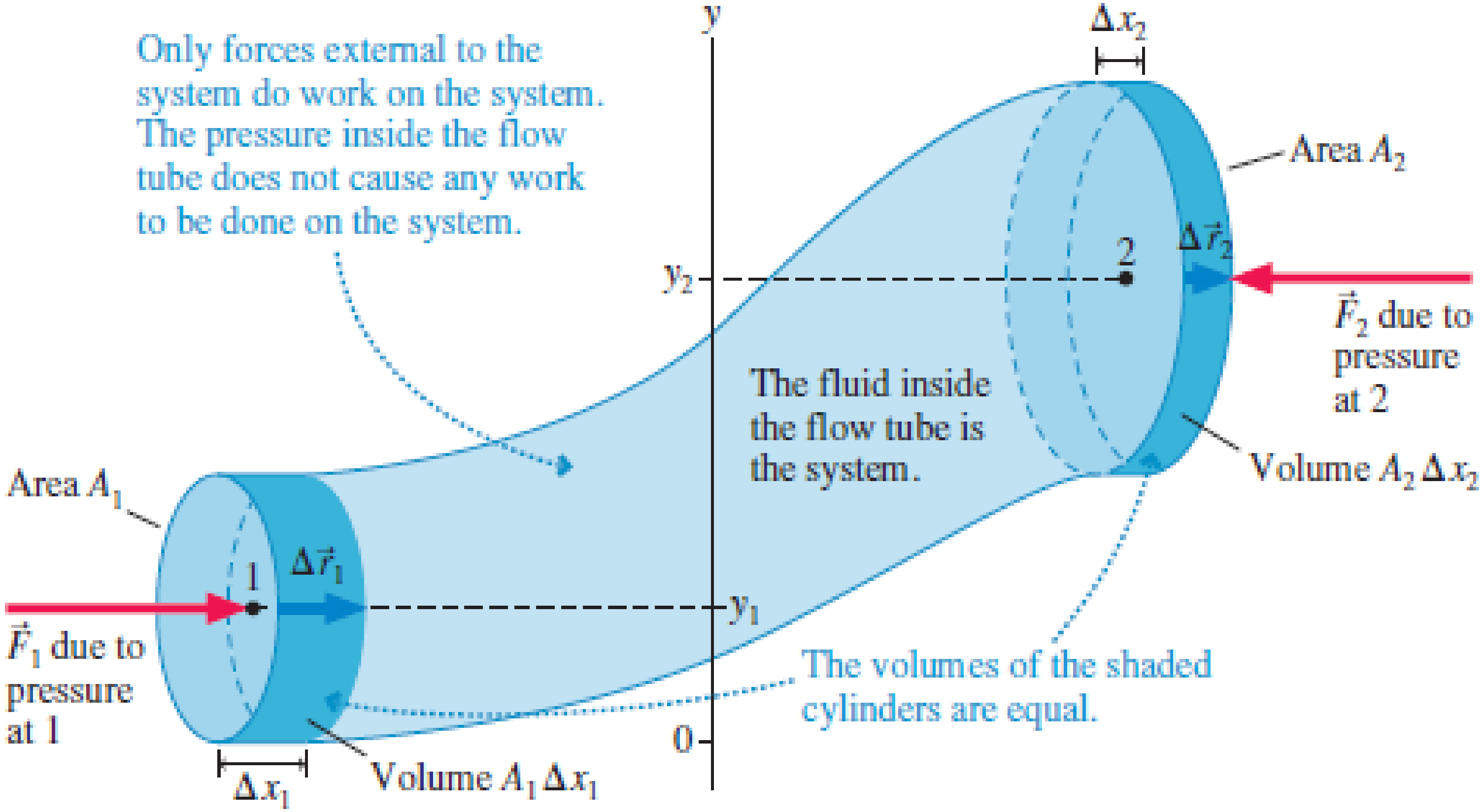
$\Delta K + \Delta U = W_{ext}$ where W_{ext} is the work done by any external forces.

$$W_1 = \vec{F}_1 \cdot \Delta\vec{r}_1 = F_1 \Delta r_1$$
$$= (p_1 A_1)\Delta x_1 = p_1 V$$

$$W_2 = \vec{F}_2 \cdot \Delta\vec{r}_2 = -F_2 \Delta r_2$$
$$= -(p_2 A_2)\Delta x_2 = -p_2 V$$

$$W_{ext} = W_1 + W_2 = p_1 V - p_2 V$$

$$\Delta U = mgy_2 - mgy_1$$
$$= \rho V g y_2 - \rho V g y_1$$



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where ρ is the fluid density. Similarly, the change in kinetic energy is

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}\rho V v_2^2 - \frac{1}{2}\rho V v_1^2$$

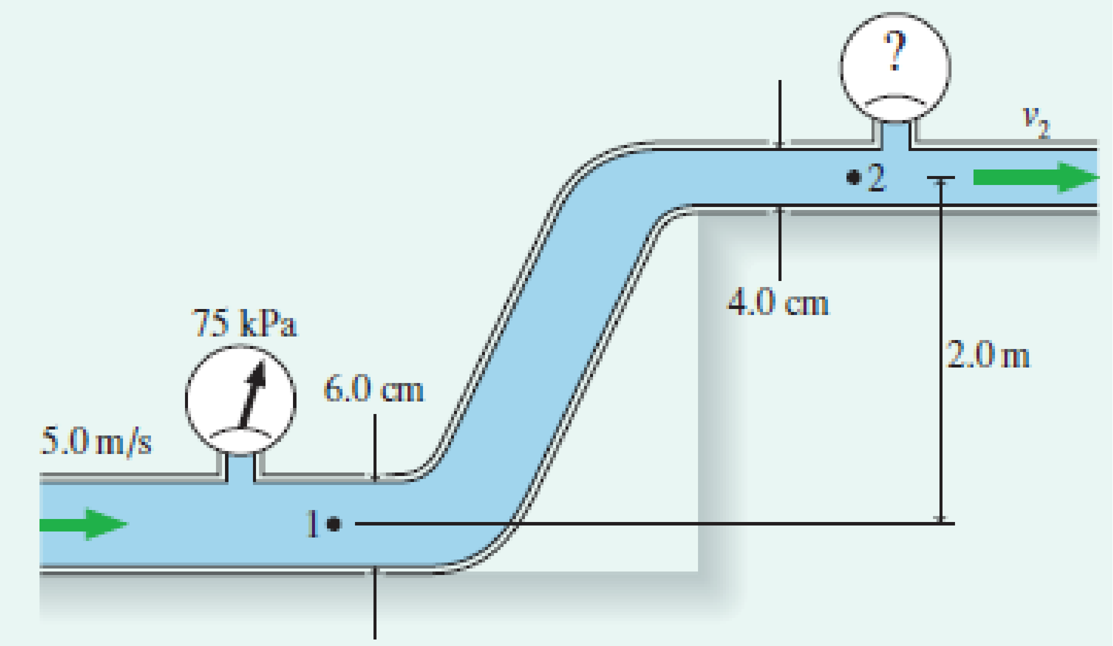
$$\frac{1}{2}\rho V v_2^2 - \frac{1}{2}\rho V v_1^2 + \rho V g y_2 - \rho V g y_1 = p_1 V - p_2 V$$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

This equation is called **Bernoulli's equation.** It is sometimes useful to express Bernoulli's equation in the alternative form

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Water flows through the pipes shown in FIGURE 15.32. The water's speed through the lower pipe is 5.0 m/s and a pressure gauge reads 75 kPa. What is the reading of the pressure gauge on the upper pipe?



Bernoulli's equation

$$p_2 = p_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 + \rho g y_1 - \rho g y_2$$
$$= p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2)$$

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{A_1}{A_2}v_1 = \frac{r_1^2}{r_2^2}v_1 = \frac{(0.030\text{ m})^2}{(0.020\text{ m})^2}(5.0\text{ m/s}) = 11.25\text{ m/s}$$

The pressure at point 1 is $p_1 = 75\text{ kPa} + 1\text{ atm} = 176{,}300\text{ Pa}.$

We can now use the above expression for $p2$ to calculate:

$p_2 = 105{,}900\text{ Pa}.$ $\quad$ $p_2 = 105{,}900\text{ Pa} - 1\text{ atm} = 4.6\text{ kPa}$

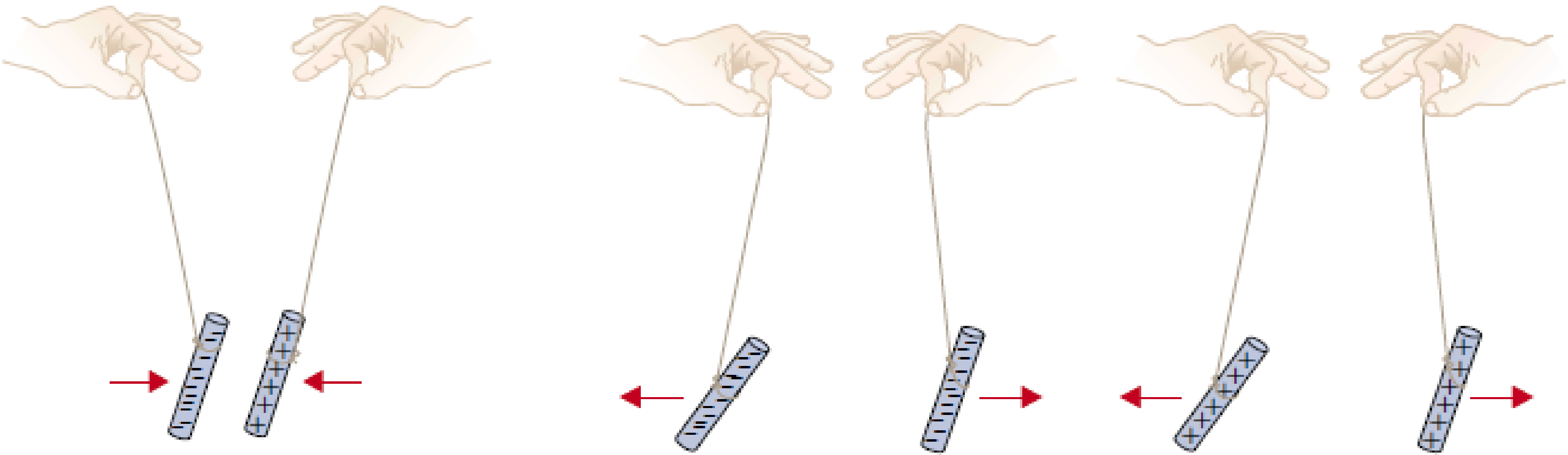
Ph-1101 Physics-I

Electricity & Magnetism

Electricity & Magnetism

- Electric Charge,
- Coulomb's law,
- Electric Field,
- Calculation of the Electric Field Strength,
- A dipole in an Electric Field,
- electric Flux and Gauss's Law.
- Electric Potential (V),
- Relation between E & V,
- Electric Potential Energy,
- Capacitor and Capacitance.

Concepts of Electric Charge:

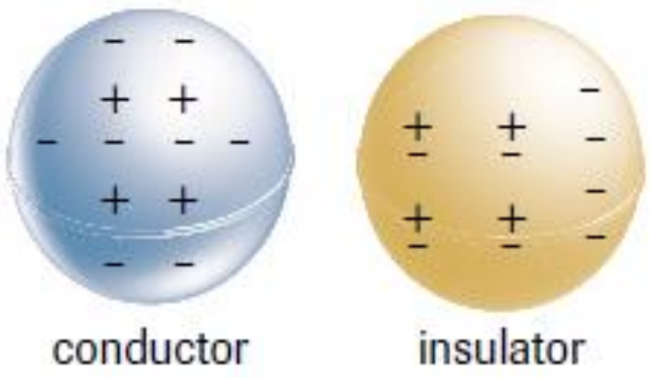


The Laws of Electric Charges:

- Opposite electric charges attract each other.
- Similar electric charges repel each other.
- Charged objects attract some neutral objects.

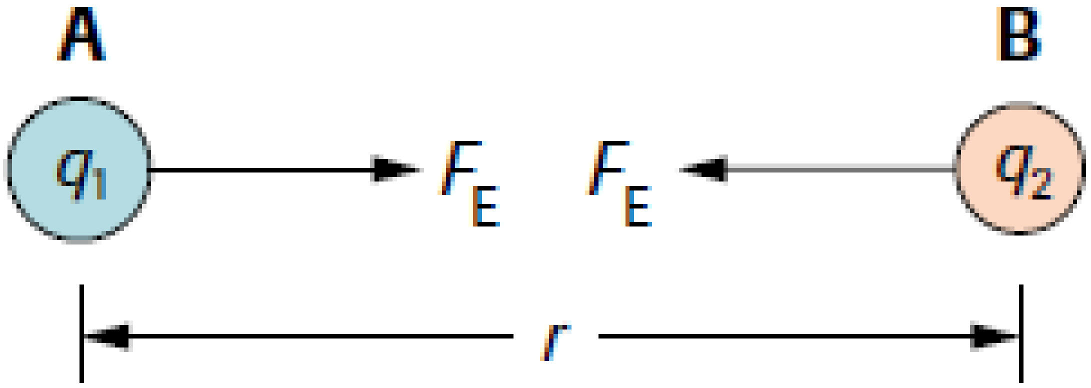
Conductors & insulators

On a spherical conductor charge spreads out evenly. On an insulator the charge remains in the spot where it was introduced.



- An electric conductor is a solid in which electrons are able to move easily from one atom to another. Most metals, such as silver, gold, copper, and aluminum, are conductors. Certain liquids are also conductors of electricity.
- An insulator is a solid in which the electrons are not free to move about easily from atom to atom. Plastic, cork, glass, wood, and rubber are insulators.
- Pure water contains essentially only neutral molecules and is therefore an insulator. However, when a chemical such as table salt (or copper sulphate, potassium nitrate, hydrochloric acid, chlorine, etc.) is added to the water, the solution becomes a conductor.

Electric Forces: Coulomb's Law



$$F_E \propto \frac{1}{r^2} \qquad F_E \propto q_1 q_2$$

Combining, $$F_E \propto \frac{q_1 q_2}{r^2}$$

$$F_E = \frac{k q_1 q_2}{r^2}$$

where *k* is known as Coulomb's constant. $k = 9.0 \times 10^9\ N.m^2/C^2$

Coulomb's Law

The force between two point charges is inversely proportional to the square of the distance between the charges and directly proportional to the product of the charges. Coulomb (C) the SI unit of electric charge.

A Sample Problem:

What is the magnitude of the force of repulsion between two small spheres 1.0 m apart, if each has a charge of 1.0×10^{-12} C?

Solution

$q_1 = q_2 = 1.0 \times 10^{-12}$ C

$r = 1.0$ m

$F_E = ?$

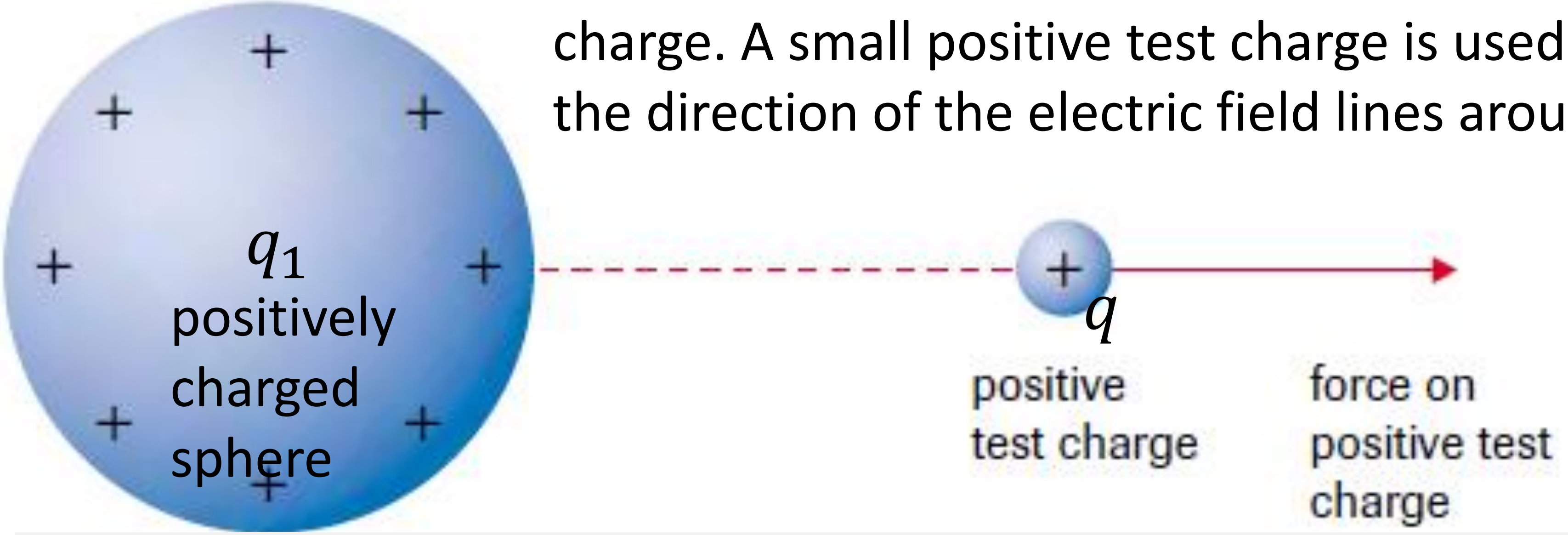
$$F_E = \frac{kq_1q_2}{r^2}$$

$$= \frac{(9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0 \times 10^{-12}\ \text{C})^2}{(1.0\ \text{m})^2}$$

$$F_E = 9.0 \times 10^{-15}\ \text{N}$$

The magnitude of the force of repulsion is 9.0×10^{-15} N, a very small force.

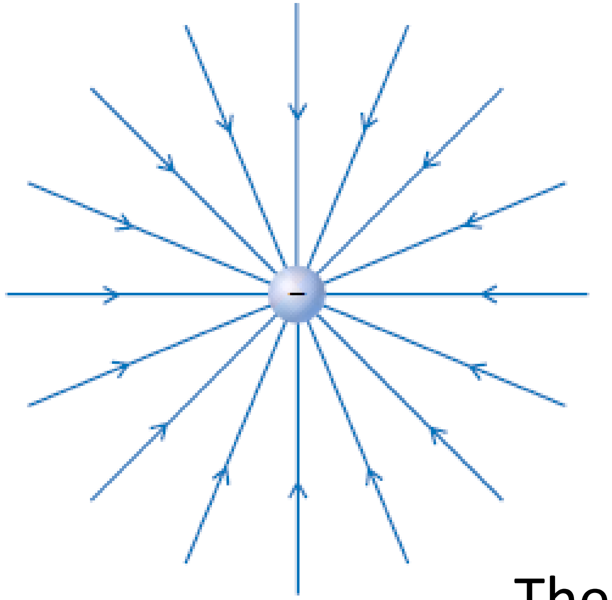
Electric Fields: The region in which a force is exerted on an electric charge. A small positive test charge is used to determine the direction of the electric field lines around a charge.



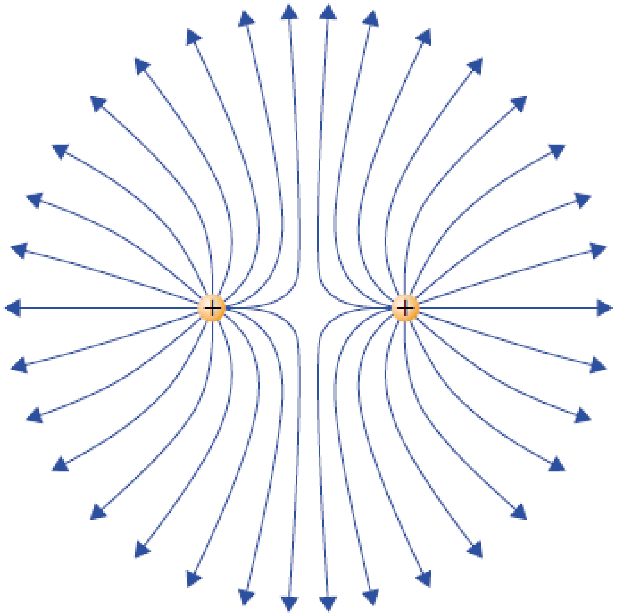
The electric field $\vec{E}$ at any point is defined as the electric force per unit positive charge and is a vector quantity: $\vec{E} = \frac{\vec{F}_E}{q}$, where the units are newtons per coulomb (N/C) in SI. So, we get,

$$E = \frac{F_E}{q} = \frac{kq_1q}{r^2q} = \frac{kq_1}{r^2}$$

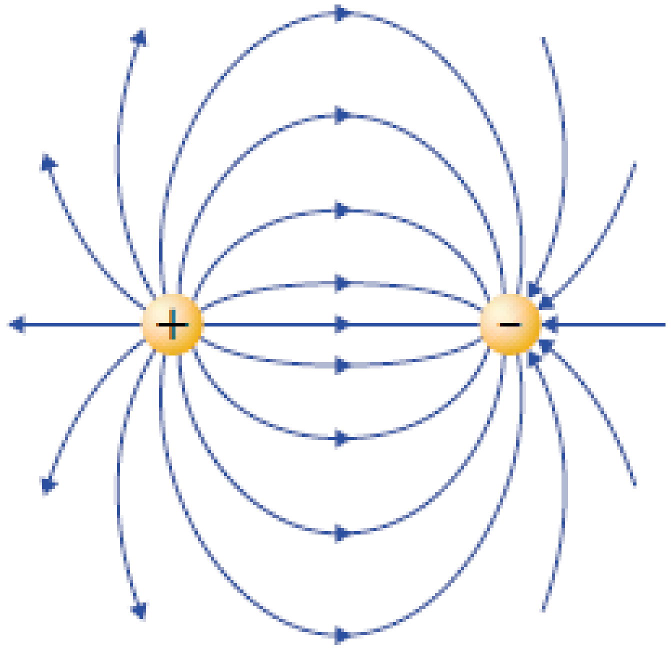
Drawing Electric Fields



Negatively charged sphere



The electric field lines of two equal positive charges. Notice the electric field is zero at the midpoint of the two charges.



The electric field of two equal but opposite charges (dipole).

A sample problem

What is the electric field 0.60 m away from a small sphere with a positive charge of 1.2×10^{-8} C?

Solution:

$q = 1.2 \times 10^{-8}\text{C}$

$$r = 0.60\ m$$

$$E = ?$$

$$E = \frac{kq}{r^2} = \frac{\left(9.0 \times 10^9 N.\frac{m^2}{C^2}\right)(1.2 \times 10^{-8} C)}{(0.60\ m)^2}$$

$$= 3.0 \times 10^2\ N/C$$

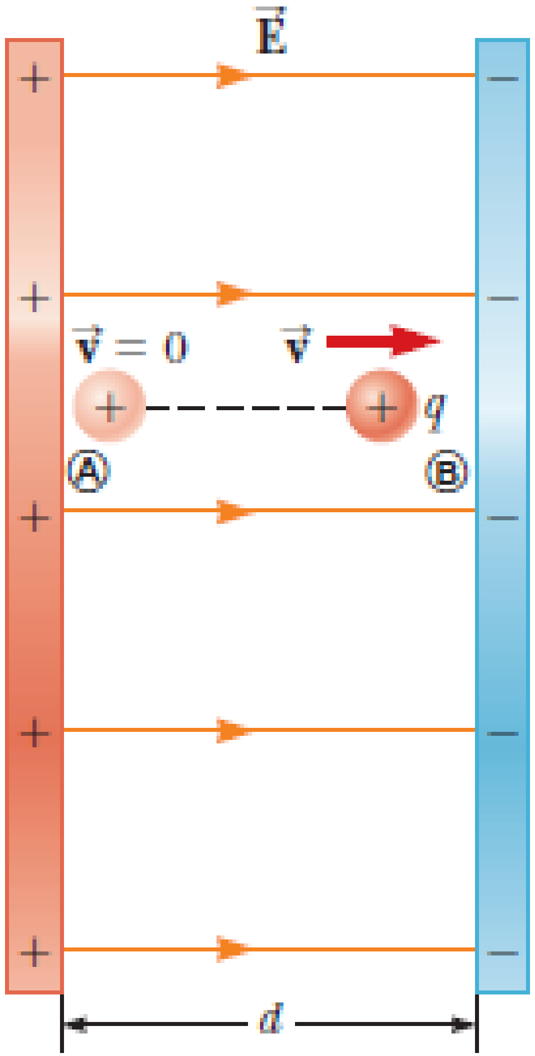
$\vec{E} = 3.0 \times 10^2 N/C$ [radially outward]

Motion of a Charged Particle in a Uniform Electric Field

When a particle of charge q and mass m is placed in an electric field $\vec{\boldsymbol{E}}$, the electric force exerted on the charge is $q\vec{\boldsymbol{E}}$. If that is the only force exerted on the particle, it must be the net force, and it causes the particle to accelerate according to the particle under a net force model ($\vec{F} = m\vec{a}$). Therefore,

$$\vec{F}_E = q\vec{E} = m\vec{a}$$

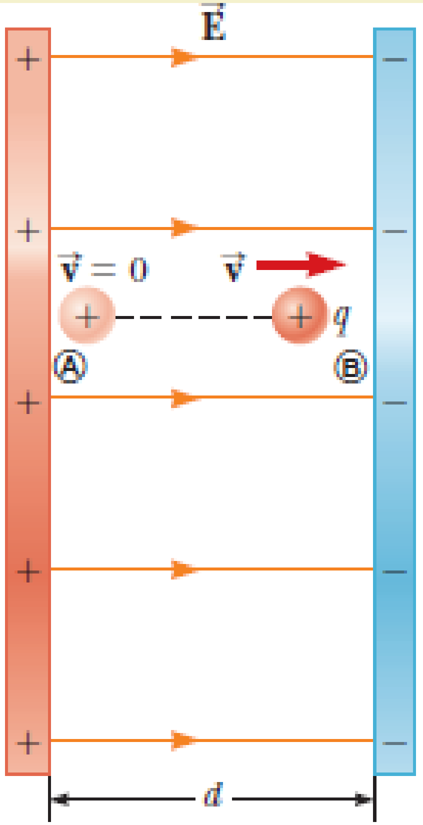
$$\vec{a} = q\vec{E}/m$$



Example

A uniform electric field $\vec{\mathbf{E}}$ is directed along the x axis between parallel plates of charge separated by a distance d as shown in Figure 23.23. A positive point charge q of mass m is released from rest at a point Ⓐ next to the positive plate and accelerates to a point Ⓑ next to the negative plate.

(A) Find the speed of the particle at Ⓑ by modeling it as a particle under constant acceleration.



Solution:

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0 + 2a(d - 0) = 2ad$$

$$v_f = \sqrt{2ad} = \sqrt{2\left(\frac{qE}{m}\right)d} = \boxed{\sqrt{\frac{2qEd}{m}}}$$

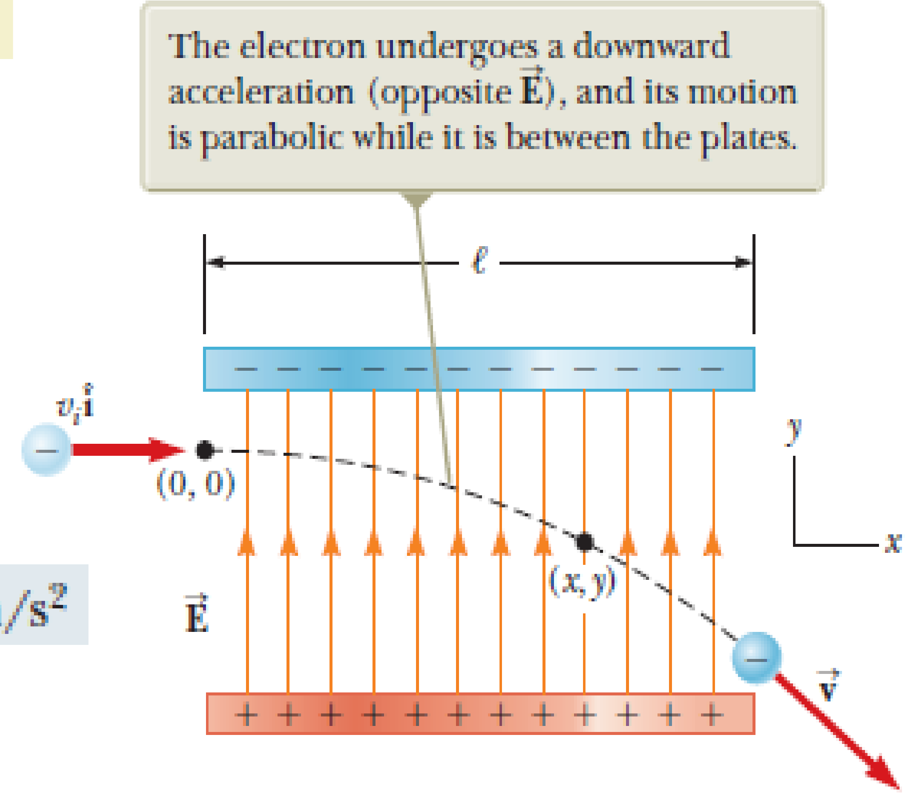
Example

An electron enters the region of a uniform electric field as shown in Figure 23.24, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m.

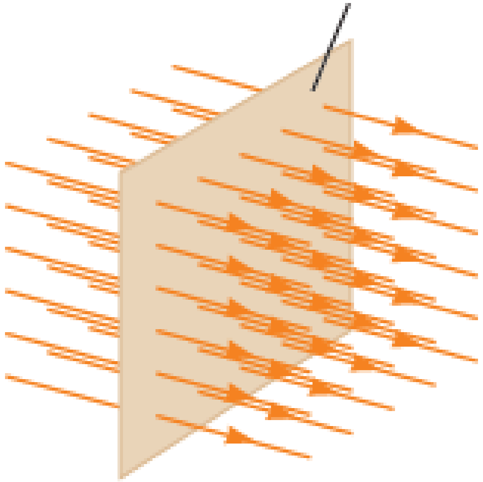
(A) Find the acceleration of the electron while it is in the electric field.

$$a_y = -\frac{eE}{m_e}$$

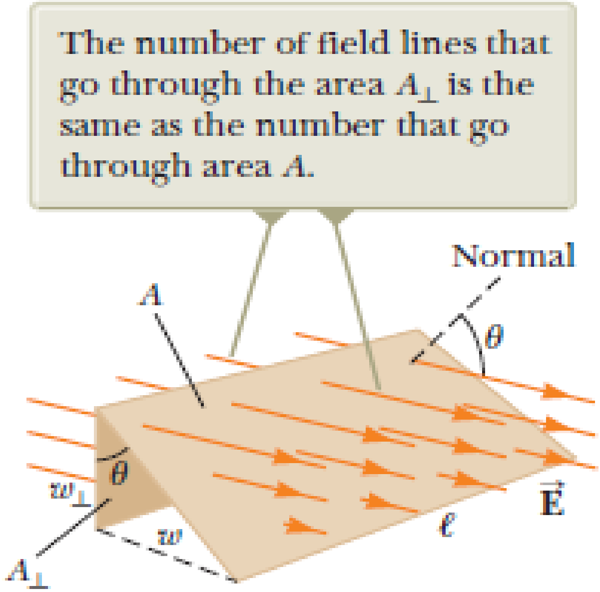
$$a_y = -\frac{(1.60 \times 10^{-19}\ \text{C})(200\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}} = \boxed{-3.51 \times 10^{13}\ \text{m/s}^2}$$



Electric Flux:



Uniform electric field penetrating a plane of area perpendicular to the field.



Uniform electric field penetrating an area *A* whose normal is at an angle θ to the field.

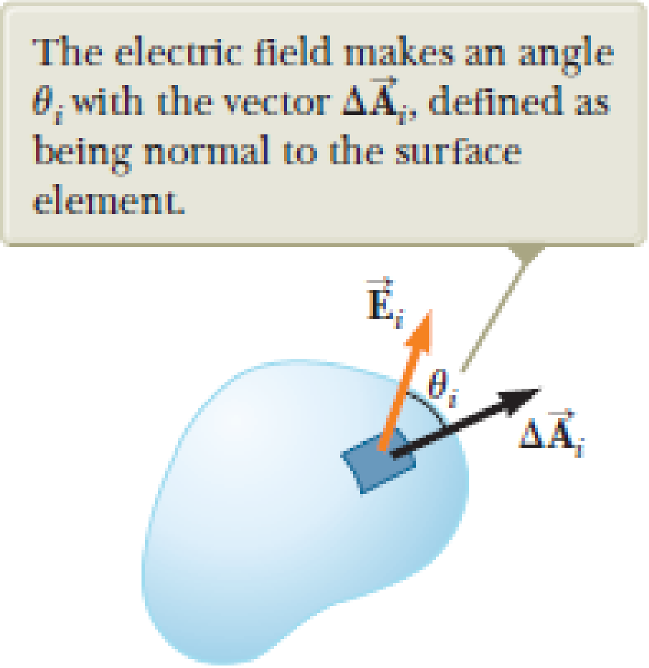
The total number of lines penetrating the surface is proportional to the product *EA.* This product of the magnitude of the electric field and surface area perpendicular to the field is called the **electric flux**:

$$\Phi_E = EA_\perp = EA\cos\theta$$

This is true for a small element of area over which the field is constant.

Electric Flux (Cont.):

Consider a general surface divided into a large number of small elements, each of area $\Delta\vec{A}_i$, whose magnitude represents the area of the *ith*element of the large surface and whose direction is defined to be *perpendicular* to the surface element as shown in Fig.



The electric flux: $\Phi_{E,i} = E_i \Delta A_i \cos\theta_i = \vec{E}_i . \Delta\vec{A}_i$

Summing all elements: $\Phi_E \approx \sum \vec{E}_i . \Delta\vec{A}_i$

If the area of each element approaches zero, the number of elements approaches infinity and the sum is replaced by an integral. Therefore, the electric flux is

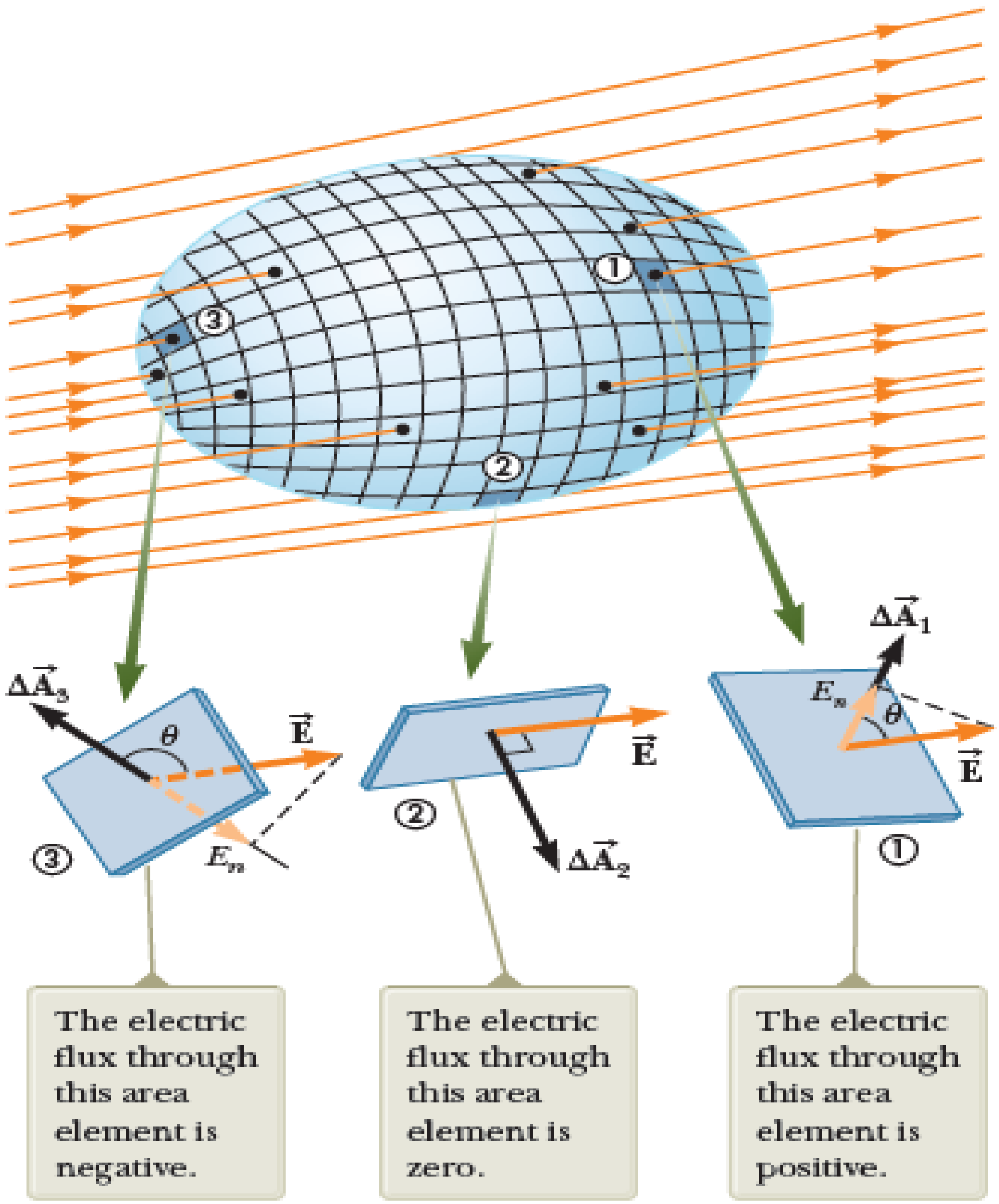
$$\Phi_E \equiv \int_{Surface} \vec{E} . d\vec{A}$$

Flux through a *closed surface*

The *net* flux through the surface is proportional to the net number of lines leaving the surface, where the net number means *the number of lines leaving the surface minus the number of lines entering the surface.* If more lines are leaving than entering, the net flux is positive. If more lines are entering than leaving, the net flux is negative.

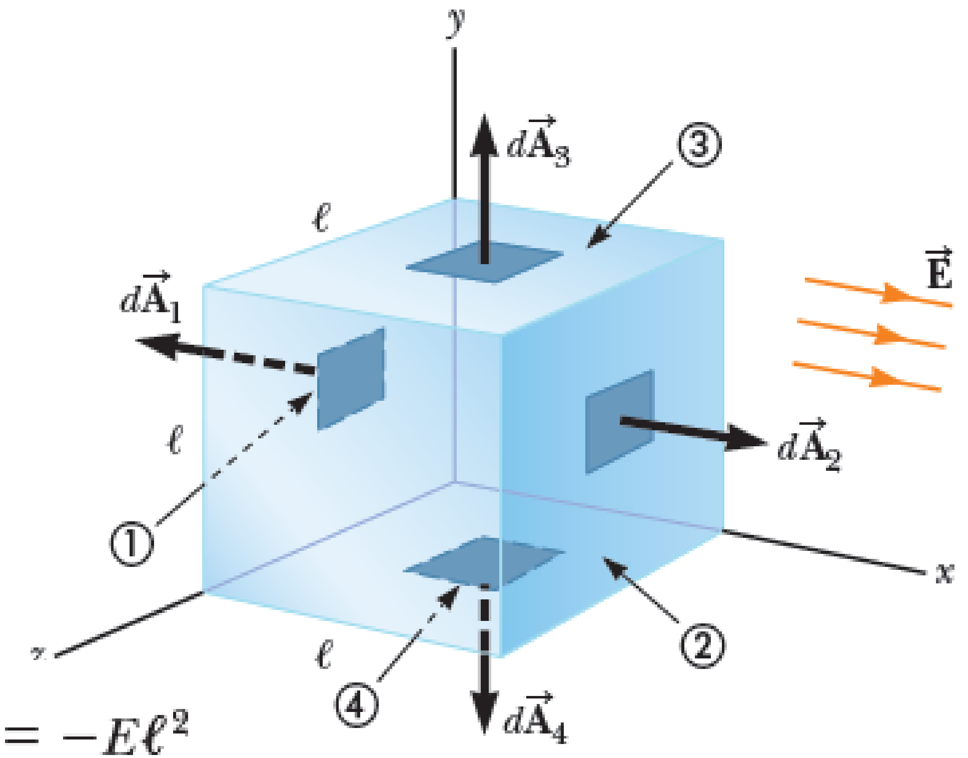
$$\Phi_E = \oint \vec{E}.d\vec{A} = \oint E_n dA$$

where E_n is the component normal to the surface.



Example Flux Through a Cube

Consider a uniform electric field $\vec{E}$ oriented in the *x* direction in empty space. A cube of edge length l, is placed in the field, oriented as shown in Fig. Find the net electric flux through the surface of the cube.



$$\Phi_E = \int_1 \vec{E} \cdot d\vec{A} + \int_2 \vec{E} \cdot d\vec{A}$$

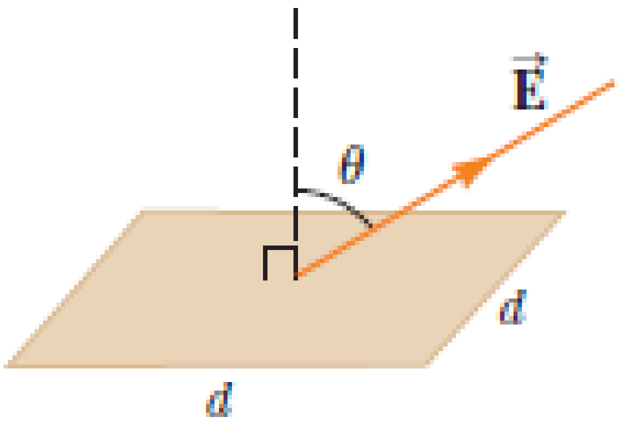
$$\int_1 \vec{E} \cdot d\vec{A} = \int_1 E(\cos 180°)\, dA = -E\int_1 dA = -EA = -E\ell^2$$

$$\int_2 \vec{E} \cdot d\vec{A} = \int_2 E(\cos 0°)\, dA = E\int_2 dA = +EA = E\ell^2$$

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = \boxed{0}$$

Find the net flux by adding the flux over all six faces:

Assignment:



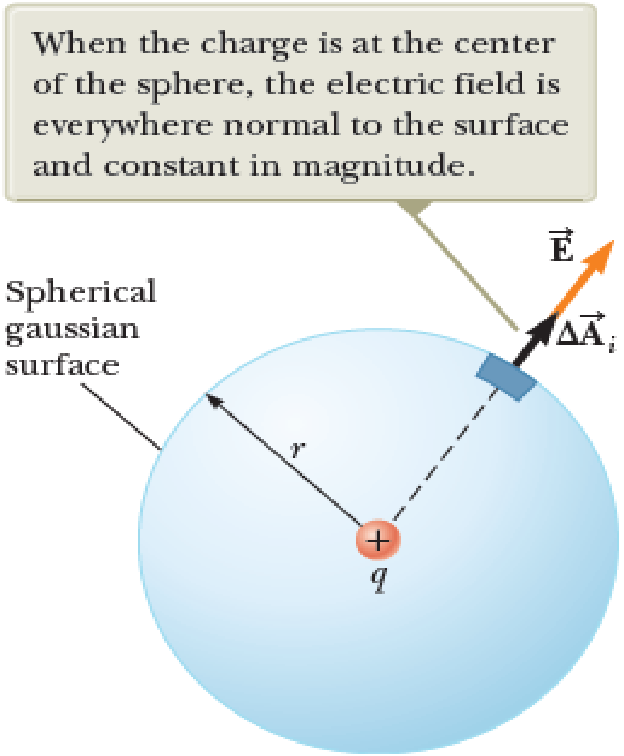
1. Consider a plane surface in a uniform electric field as in Fig., where $d = 15.0$ cm and $\theta = 70.0^0$. If the net flux through the surface is $6.00\ N.m^2/C$, find the magnitude of the electric field.
2. Find the electric flux through the plane surface shown in Fig. if $\theta = 60^0$, $E = 350$ N/C, and $d = 5.00$ cm. The electric field is uniform over the entire area of the surface.

Gauss's Law

A general relationship between the net electric flux through a closed surface and the charge enclosed by the surface is known as *Gauss's law.*

The magnitude of the electric field everywhere on the surface of the sphere is $E = k_e q/r^2$.

The net flux through the gaussian surface is



$$\Phi_E = \oint \vec{E}.d\vec{A} = \oint EdA = E\oint dA = \left(\frac{k_e q}{r^2}\right)(4\pi r^2)$$

$$= 4\pi k_e q = \frac{q}{\epsilon_0}$$

Gauss's law says that the net electric flux Φ_E through any closed gaussian surface is equal to the *net* charge q_{in} inside the surface divided by ϵ_0:

$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

Example A Cylindrically Symmetric Charge Distribution

Find the electric field a distance *r* from a line of positive charge of infinite length and constant charge per unit length λ.

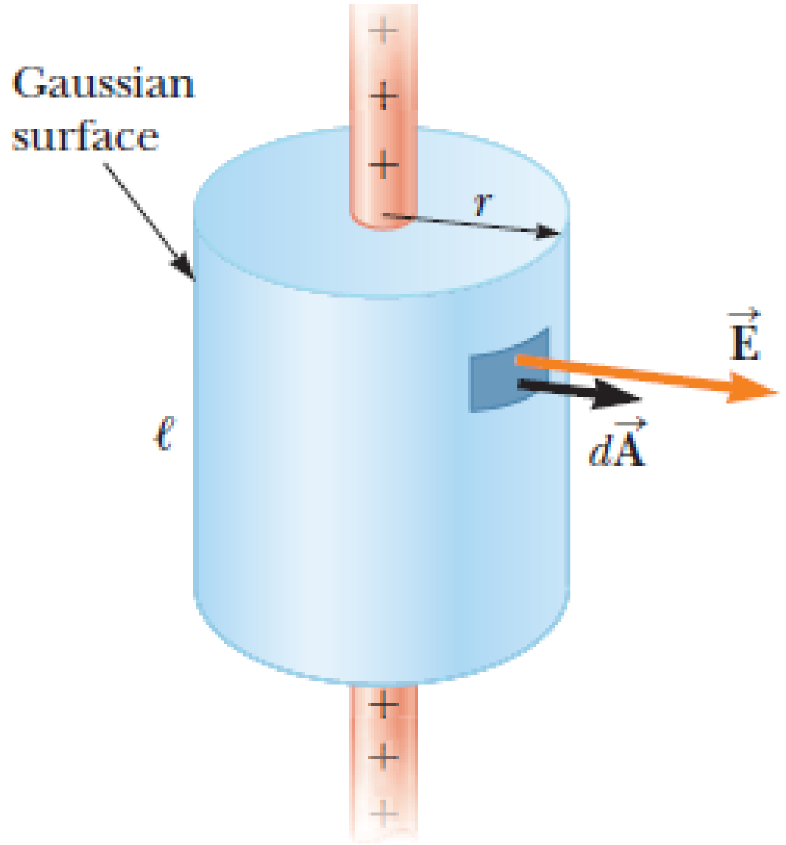
Solution:

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E \oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0}$$

Substitute the area $A = 2\pi r \ell$

$$E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0}$$

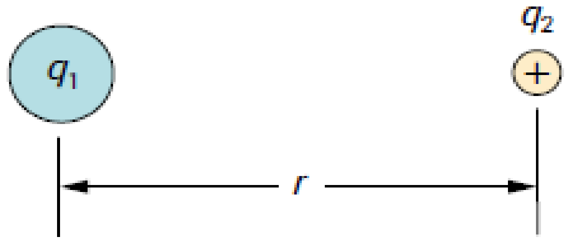
$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \boxed{2k_e \frac{\lambda}{r}}$$



Electric Potential Energy

When a charge q is placed in an electric field $\vec{E}$, then there is an electric force $q\vec{E}$ acting on the charge. For an infinitesimal displacement $d\vec{s}$ of a point charge q in an electric field, the work done by the electric field on the charge is $W = \vec{F}.d\vec{s} = q\vec{E}.d\vec{s}$. This work done is equal to the negative of the change in the potential energy of the system: $W = -\Delta U$. Therefore, as the charge q is displaced, the electric potential energy is changed by an amount $dU = -W = -q\vec{E}.d\vec{s}$. For a finite displacement of the charge from some point A in space to some other point B, the change in electric potential energy of the system is

$$\Delta U = -q\int_A^B \vec{E}\,.\,d\vec{s}$$



The potential energy of **two point charges** separated by distance *r* is

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

Electric Potential

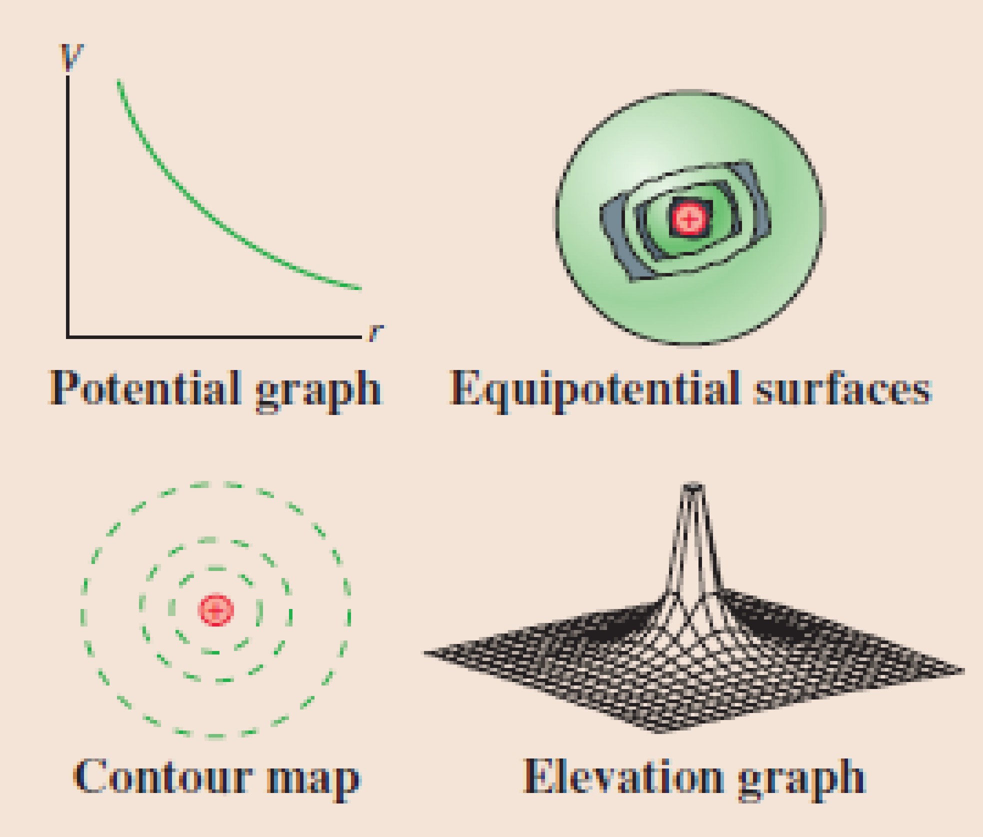
Dividing the potential energy by the charge is called the **electric potential** (or simply the **potential**)

$$V = \frac{U}{q} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

The **potential difference f**or a finite displacement of a unit charge from some point A in space to some other point B,

$$\Delta \boldsymbol{V} = \frac{\Delta \boldsymbol{U}}{\boldsymbol{q}} = -\int_{A}^{B} \vec{\boldsymbol{E}}.\boldsymbol{d}\vec{\boldsymbol{s}}$$

Graphical representations of the potential:



Units

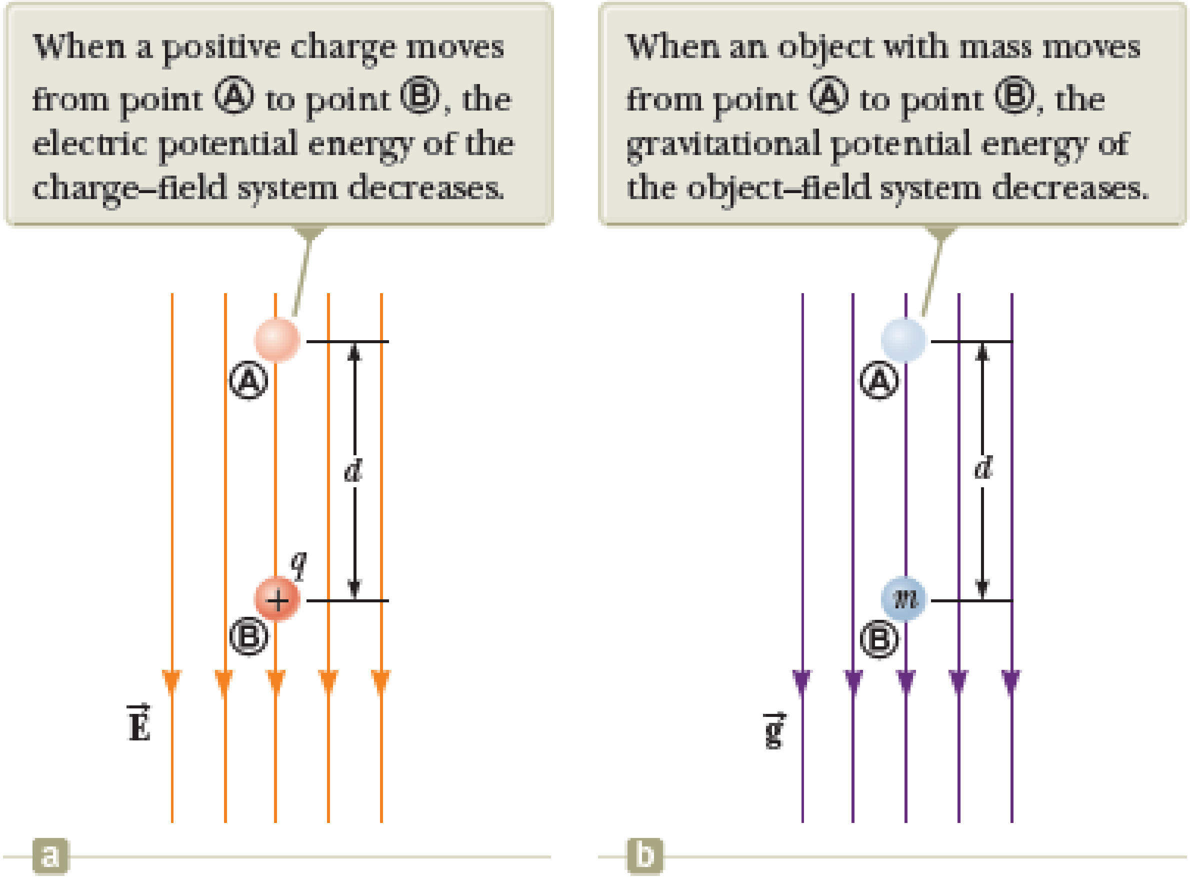
Electric potential: 1 V = 1 J/C

Potential Difference in a Uniform Electric Field

(a) When the electric field $\vec{E}$ is directed downward, point B is at a lower electric potential than point A. (b) A gravitational analog to the situation in (a).

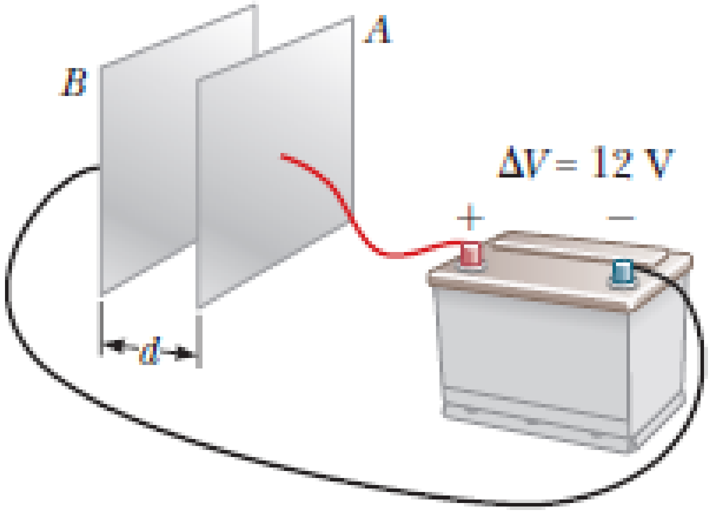
- Potential difference between two points in a uniform electric field
  $$V_B - V_A = \Delta V = -Ed$$

- The potential energy of the charge–field system
  $$\Delta U = q\Delta V = -qEd$$



Example The Electric Field Between Two Parallel Plates of Opposite Charge

A battery has a specified potential difference ΔV between its terminals and establishes that potential difference between conductors attached to the terminals. A 12-V battery is connected between two parallel plates as shown in Fig. The separation between the plates is d = 0.30 cm, and we assume the electric field between the plates to be uniform. Find the magnitude of the electric field between the plates.

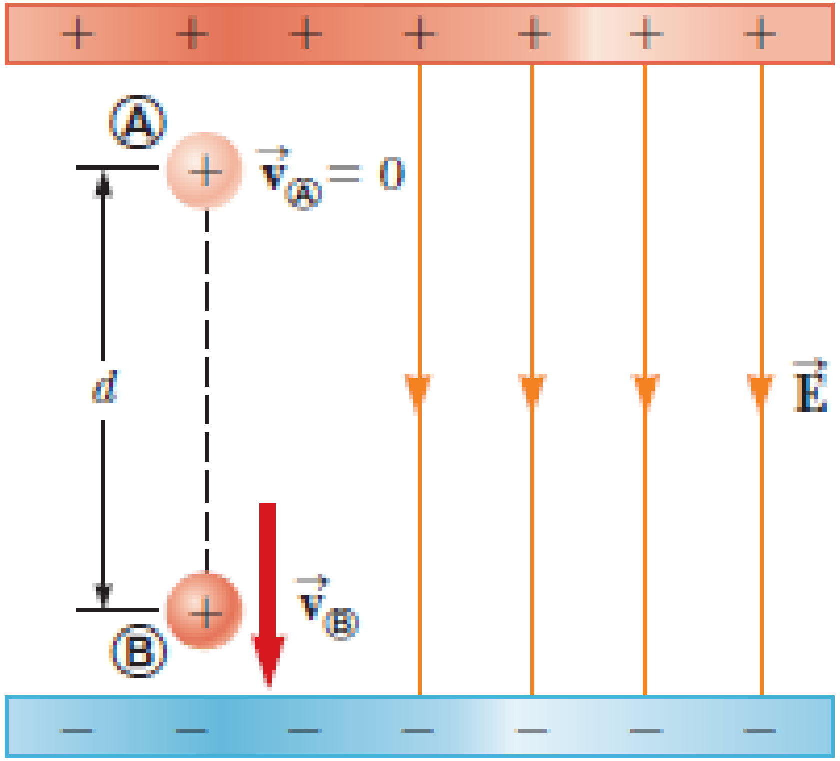


Solution:

$$E = \frac{|V_B - V_A|}{d} = \frac{12\ \text{V}}{0.30 \times 10^{-2}\ \text{m}} = \boxed{4.0 \times 10^{3}\ \text{V/m}}$$

Example Motion of a Proton in a Uniform Electric Field

A proton is released from rest at point A in a uniform electric field that has a magnitude of 8.0×10^4 V/m. The proton undergoes a displacement of magnitude $d = 0.50$ m to point B in the direction of $\vec{E}$. Find the speed of the proton after completing the displacement. **Solution:**

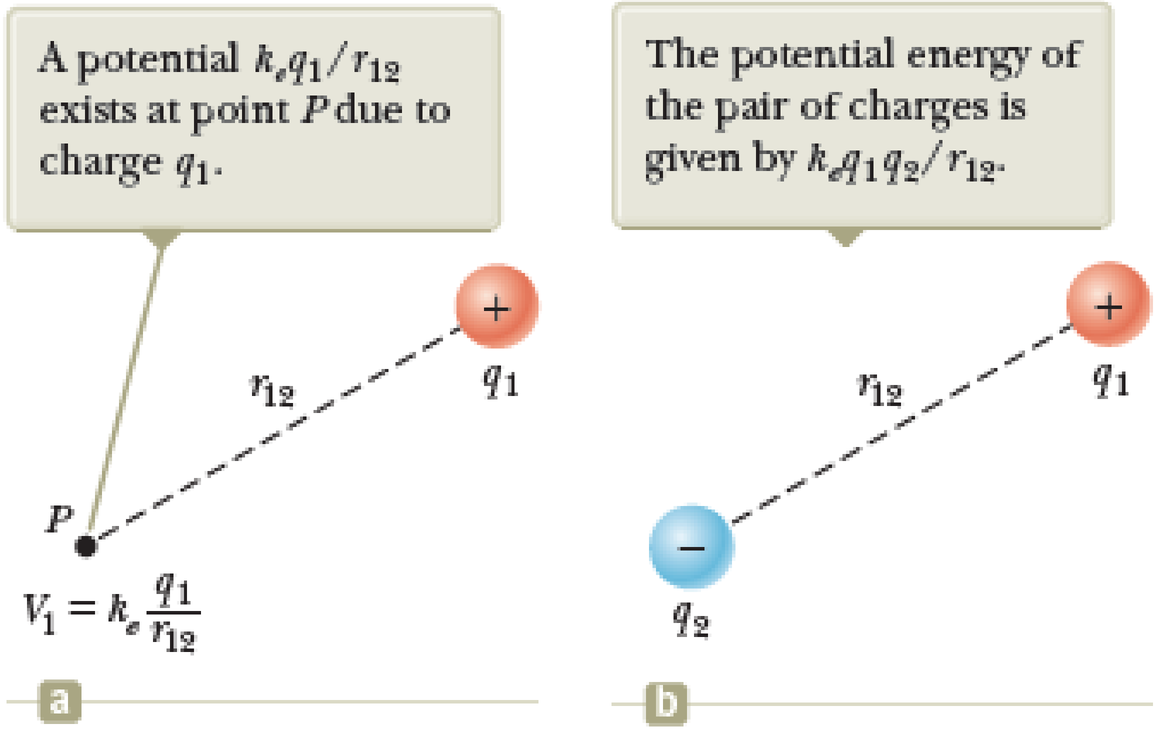


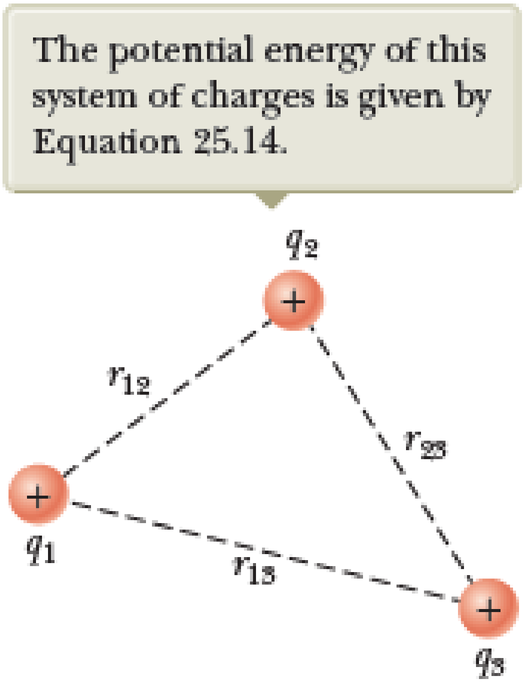
$$\Delta K + \Delta U = 0 \quad \text{or} \quad \left(\tfrac{1}{2}mv^2 - 0\right) + e\,\Delta V = 0$$

$$v = \sqrt{\frac{-2e\,\Delta V}{m}} = \sqrt{\frac{-2e(-Ed)}{m}} = \sqrt{\frac{2eEd}{m}}$$

$$v = \sqrt{\frac{2(1.6 \times 10^{-19}\ \mathrm{C})(8.0 \times 10^4\ \mathrm{V})(0.50\ \mathrm{m})}{1.67 \times 10^{-27}\ \mathrm{kg}}} = 2.8 \times 10^6\ \mathrm{m/s}$$

Electric potential & Electric potential energy: (a) Charge q_1 establishes an electric potential V_1 at point *P*. (b) Charge q_2 is brought from infinity to point *P*.





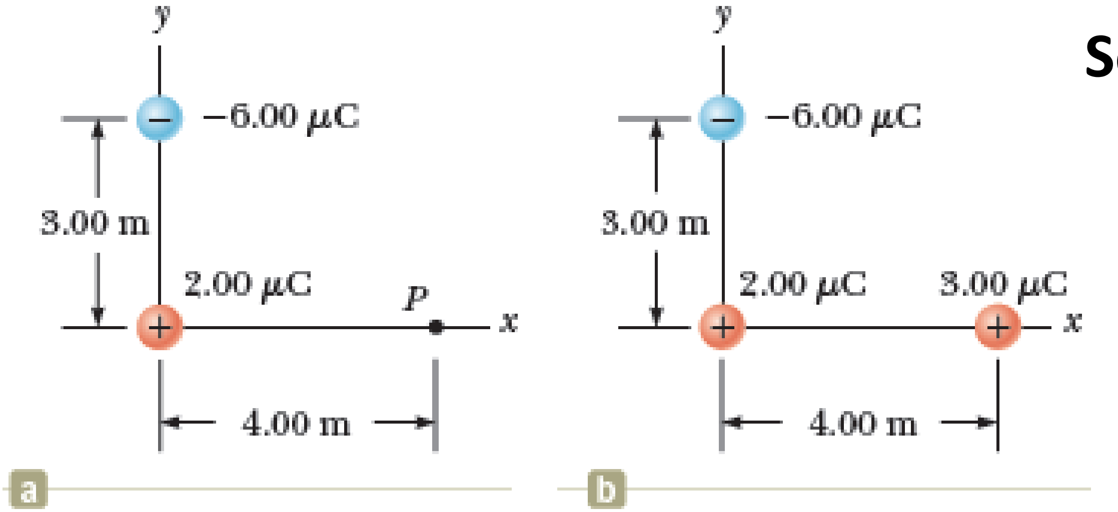
$$V = k_e \sum_i \frac{q_i}{r_i}$$

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

The total potential energy of the system of three charges shown

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

Example As shown in Fig.a, a charge $q_1 = 2.00\ \mu$C is located at the origin and a charge $q_2 = -6.00\ \mu$C is located at (0, 3.00) m. (A) Find the total electric potential due to these charges at the point *P*, whose coordinates are (4.00, 0) m. **(B)** Find the change in potential energy of the system of two charges plus a third charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point *P* (Fig.b).



Solution: (a)

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2})\left(\frac{2.00 \times 10^{-6}\ \mathrm{C}}{4.00\ \mathrm{m}} + \frac{-6.00 \times 10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right) = \boxed{-6.29 \times 10^3\ \mathrm{V}}$$

Solution: (b)

$$U_f = q_3 V_P$$

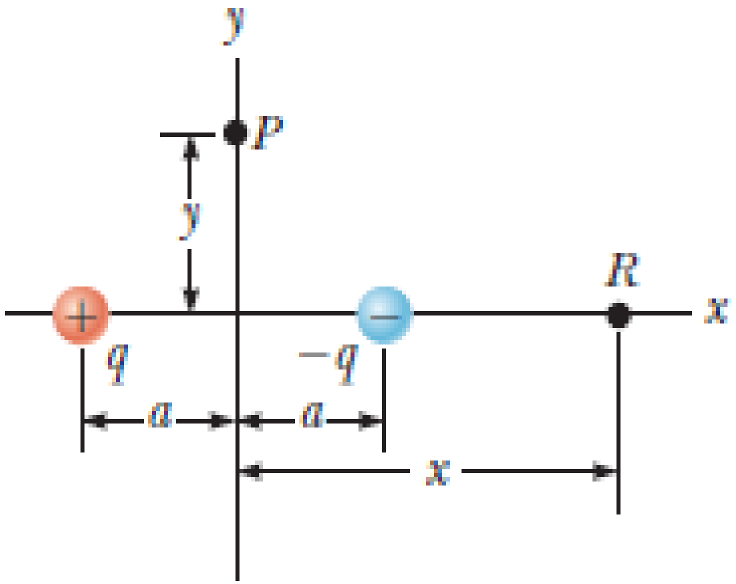
$$\Delta U = U_f - U_i = q_3 V_P - 0$$

$$= (3.00 \times 10^{-6}\ \mathrm{C})(-6.29 \times 10^3\ \mathrm{V})$$

$$= \boxed{-1.89 \times 10^{-2}\ \mathrm{J}}$$

Example The Electric Potential Due to a Dipole

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance 2*a* as shown in Fig. The dipole is along the *x* axis and is centered at the origin. **(A)** Calculate the electric potential at point *P* on the *y* axis.



$$V_P = k_e \sum_i \frac{q_i}{r_i} = k_e \left(\frac{q}{\sqrt{a^2 + y^2}} + \frac{-q}{\sqrt{a^2 + y^2}} \right) = \boxed{0}$$

(B) Calculate the electric potential at point *R* on the positive *x* axis.

$$V_R = k_e \sum_i \frac{q_i}{r_i} = k_e \left(\frac{-q}{x - a} + \frac{q}{x + a} \right) = \boxed{-\frac{2k_e qa}{x^2 - a^2}}$$

(C) Calculate *V* and E_x at a point on the *x* axis far from the dipole.

$$V_R = \lim_{x >> a} \left(-\frac{2k_e qa}{x^2 - a^2} \right) \approx \boxed{-\frac{2k_e qa}{x^2} \quad (x >> a)} \qquad E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e qa}{x^2} \right)$$

Definition of Capacitor and Capacitance

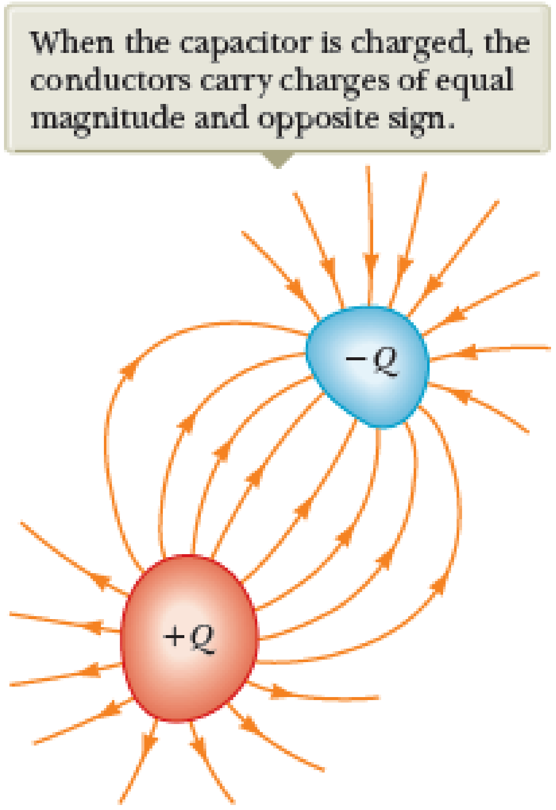
Consider two conductors as shown in Fig. Such a combination of two conductors is called a **capacitor.** The conductors are called *plates.* If the conductors carry charges of equal magnitude and opposite sign, a potential difference ΔV exists between them.

Experiments show that the quantity of charge Q on a capacitor is linearly proportional to the potential difference between the conductors; that is,

$$Q \propto \Delta V$$

$$Q = C\Delta V \qquad \therefore C = \frac{Q}{\Delta V}$$

Where, the proportionality constant depends on the shape and separation of the conductors and *is* **capacitance.** The SI unit of capacitance is the **farad** (F): 1 F = 1 C/V



A capacitor consists of two conductors.

Calculating Capacitance

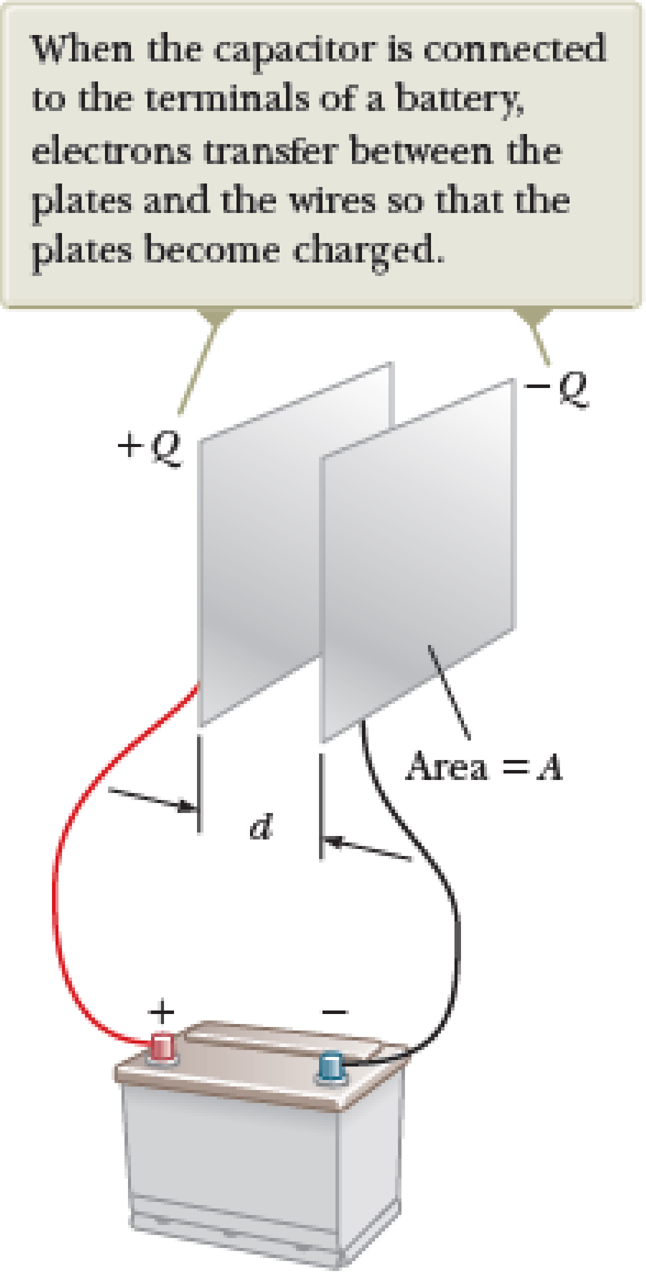
The calculation is relatively easy if the geometry of the capacitor is simple.

Parallel-Plate Capacitors

Two parallel, metallic plates of equal area *A* are separated by a distance *d* as shown in Fig. One plate carries a charge +*Q* , and the other carries a charge -*Q*. The value of the electric field between the plates is $E = \frac{Q}{\epsilon_0 A}$. The magnitude of the potential difference between the plates = *Ed.*

$$\therefore \Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A} = \frac{\epsilon_0 A}{d}$$



Combinations of Capacitors: Parallel Combination

From Fig.

$$\Delta V_1 = \Delta V_2 = \Delta V$$

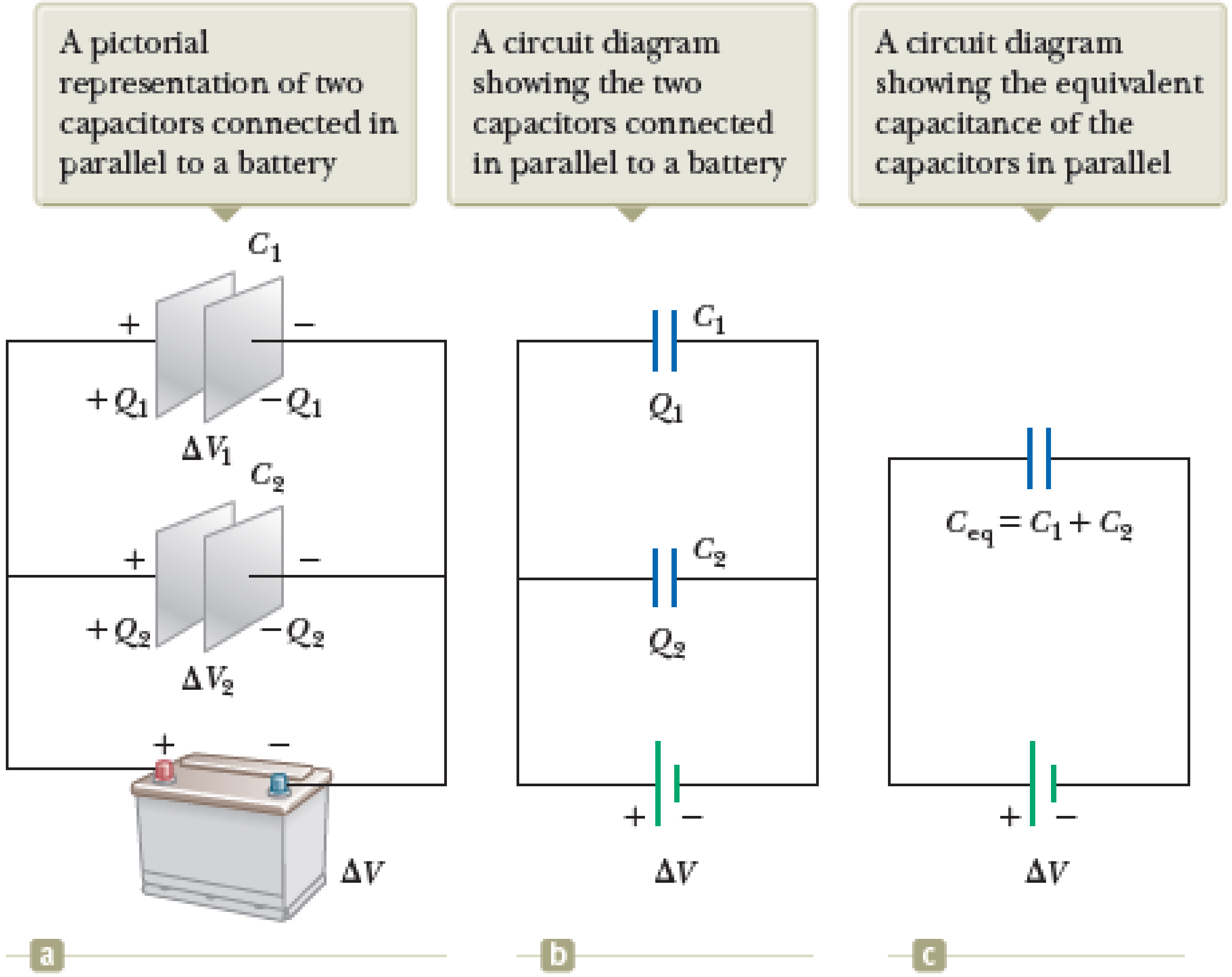
$$Q_{total} = Q_1 + Q_2$$
$$= C_1\Delta V_1 + C_2\Delta V_2$$

The equivalent capacitor

$$Q_{total} = C_{eq}\Delta V$$
$$\therefore\ C_{eq}\Delta V = C_1\Delta V_1 + C_2\Delta V_2$$
$$\Rightarrow C_{eq}\Delta V = C_1\Delta V + C_2\Delta V$$
$$C_{eq} = C_1 + C_2$$

For parallel:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$



Combinations of Capacitors: Series Combination

Series combination of capacitors: From Fig.

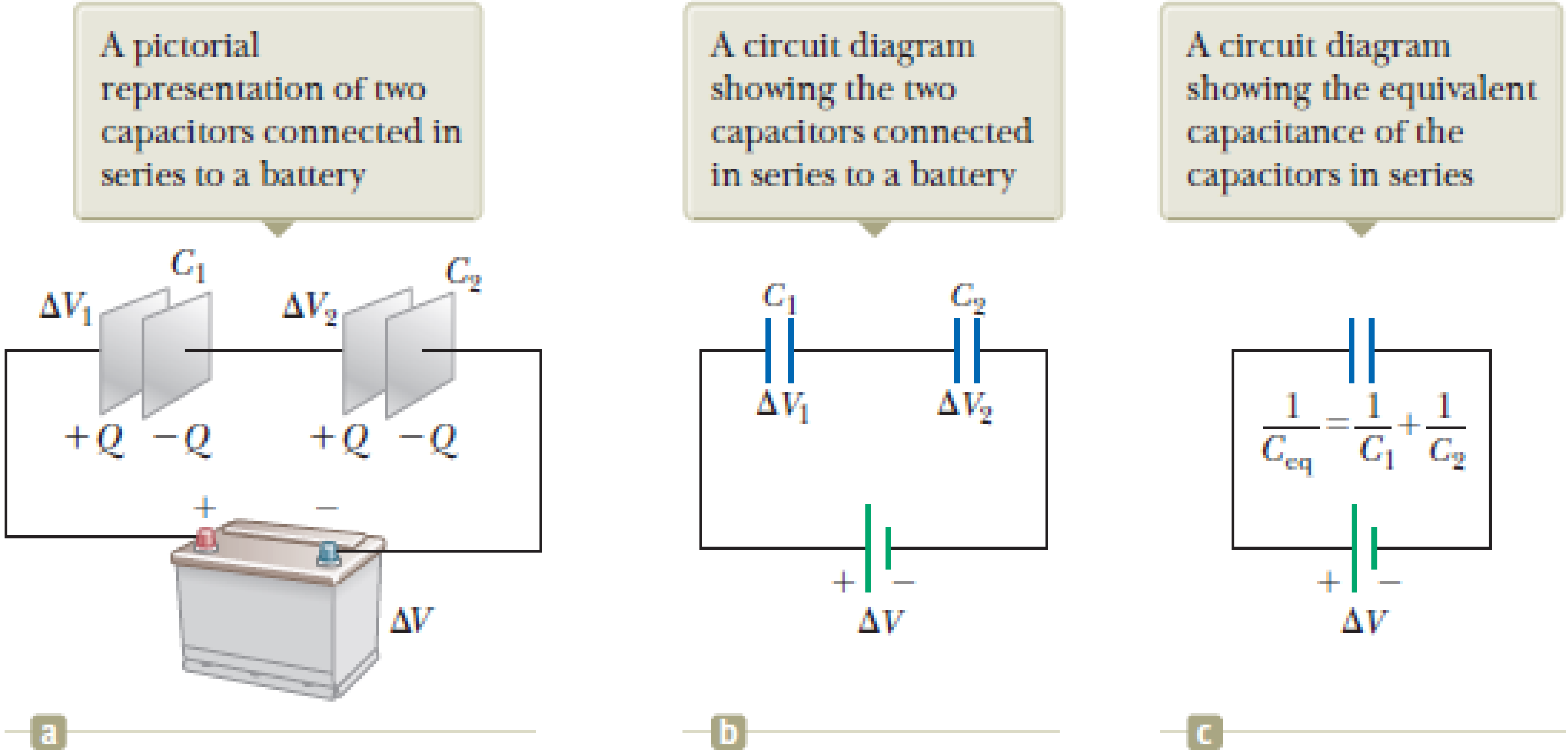
$$Q_1 = Q_2 = Q$$

We know

$$\Delta V_{Tot} = \Delta V_1 + \Delta V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

We know

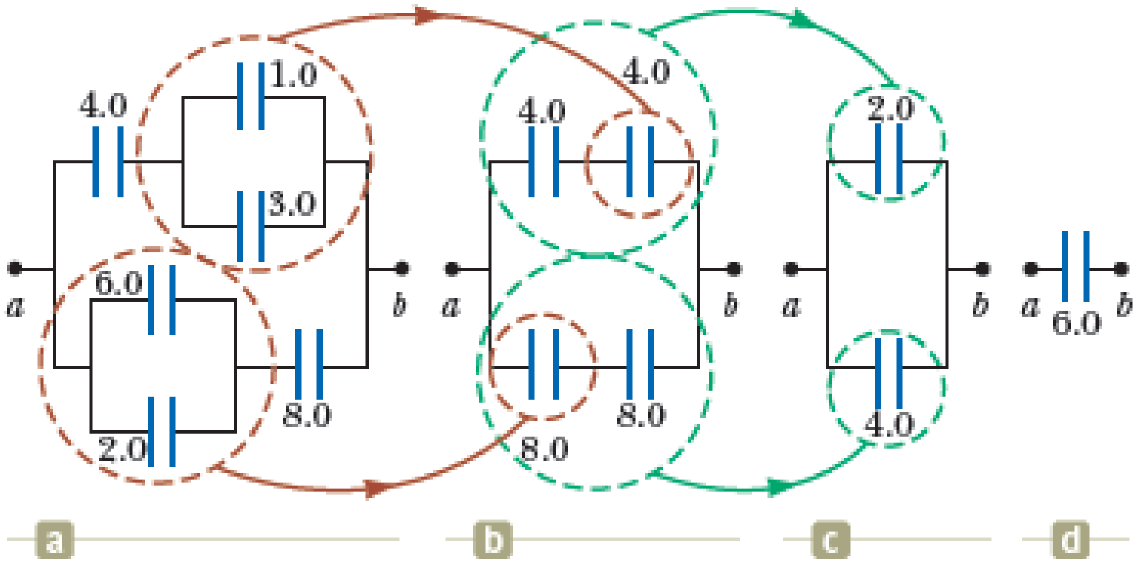
$$\Delta V_{Tot} = \frac{Q}{C_{eq}}$$



$$\therefore \frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \quad \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

Example

Find the equivalent capacitance between *a* and *b* for the combination of capacitors shown in Fig. All capacitances are in microfarads.



$$C_{eq} = C_1 + C_2 = 4.0\ \mu\text{F}$$

$$C_{eq} = C_1 + C_2 = 8.0\ \mu\text{F}$$

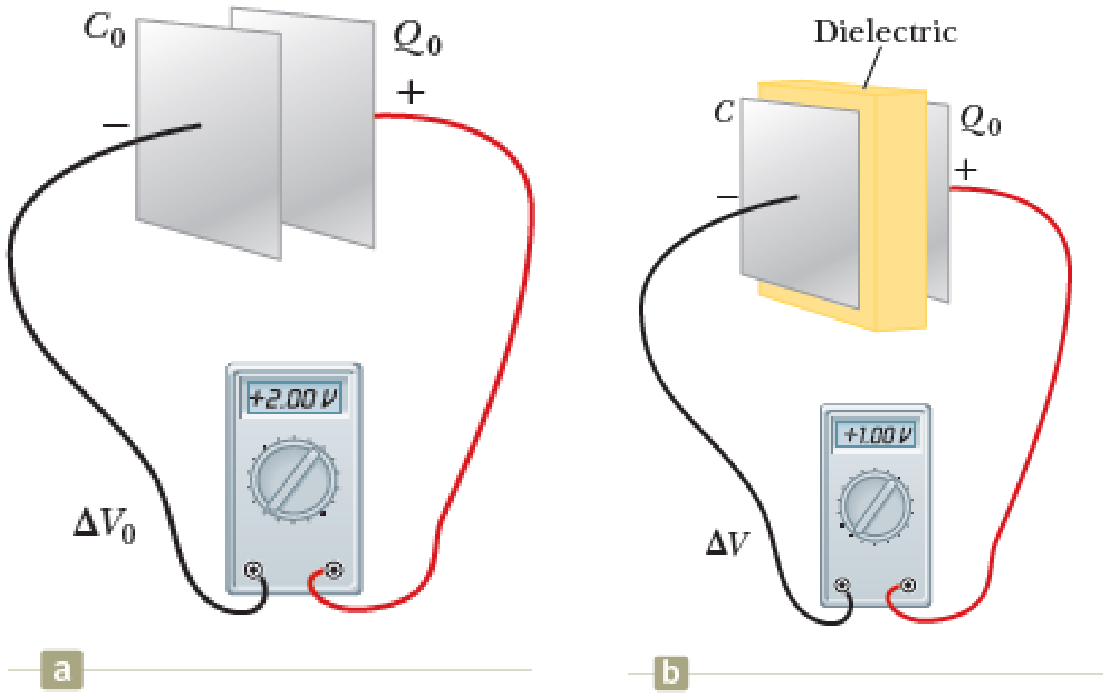
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu\text{F}} + \frac{1}{4.0\ \mu\text{F}} = \frac{1}{2.0\ \mu\text{F}} \qquad C_{eq} = 2.0\ \mu\text{F}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{8.0\ \mu\text{F}} + \frac{1}{8.0\ \mu\text{F}} = \frac{1}{4.0\ \mu\text{F}} \qquad C_{eq} = 4.0\ \mu\text{F}$$

$$C_{eq} = C_1 + C_2 = \boxed{6.0\ \mu\text{F}}$$

Capacitors with Dielectrics

A **dielectric** is a nonconducting material such as rubber, glass, or waxed paper. Consider a parallel-plate capacitor that without a dielectric has charge Q_0 and a capacitance C_0. The potential difference across the capacitor is $\Delta V_0 = \frac{Q_0}{c_0}$ or $C_0 = \frac{Q_0}{V_0}$.



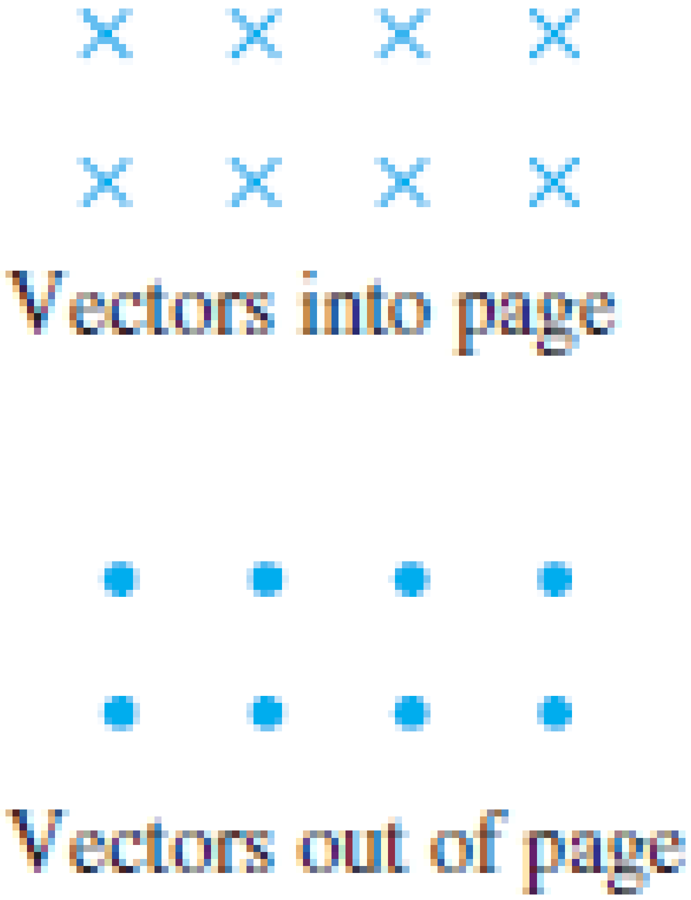
If a dielectric is now inserted between the plates as in Fig.b, the voltmeter indicates that the voltage between the plates decreases to a value ΔV. The voltages with and without the dielectric are related by a factor k as follows: $\Delta V = \Delta V_0/k$ where the dielectric constant $k > 1$. The new capacitance:

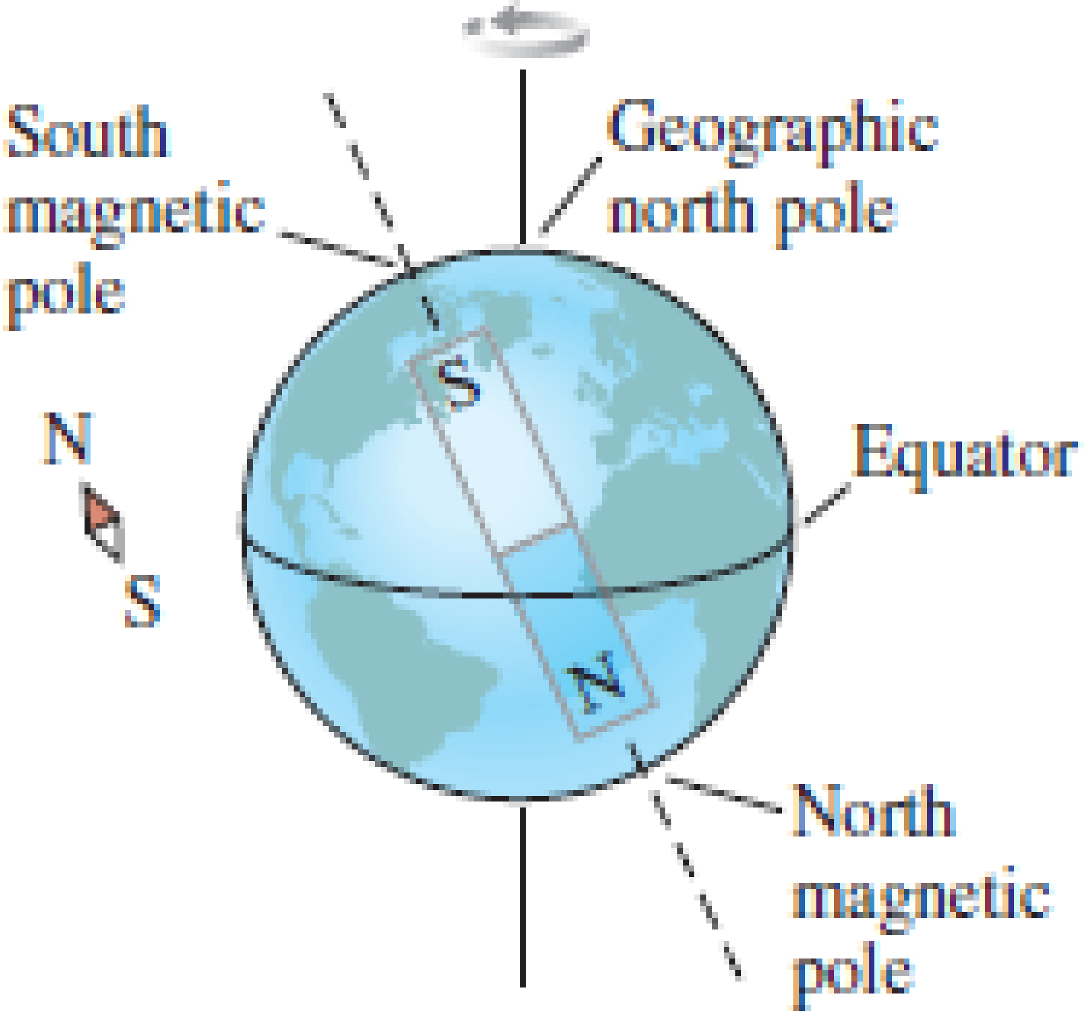
$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/k} = k\frac{Q_0}{\Delta V_0} = kC_0 = k\frac{\epsilon_0 A}{d} \qquad \text{[as } C_0 = \frac{\epsilon_0 A}{d}\text{]}$$

Table 26.1 Approximate Dielectric Constants

Material	Dielectric Constant κ
Air (dry)	1.000 59
Bakelite	4.9
Fused quartz	3.78
Mylar	3.2
Neoprene rubber	6.7
Nylon	3.4
Paper	3.7
Paraffin-impregnated paper	3.5
Polystyrene	2.56
Polyvinyl chloride	3.4
Porcelain	6
Pyrex glass	5.6
Silicone oil	2.5
Strontium titanate	233
Teflon	2.1
Vacuum	1.000 00
Water	80

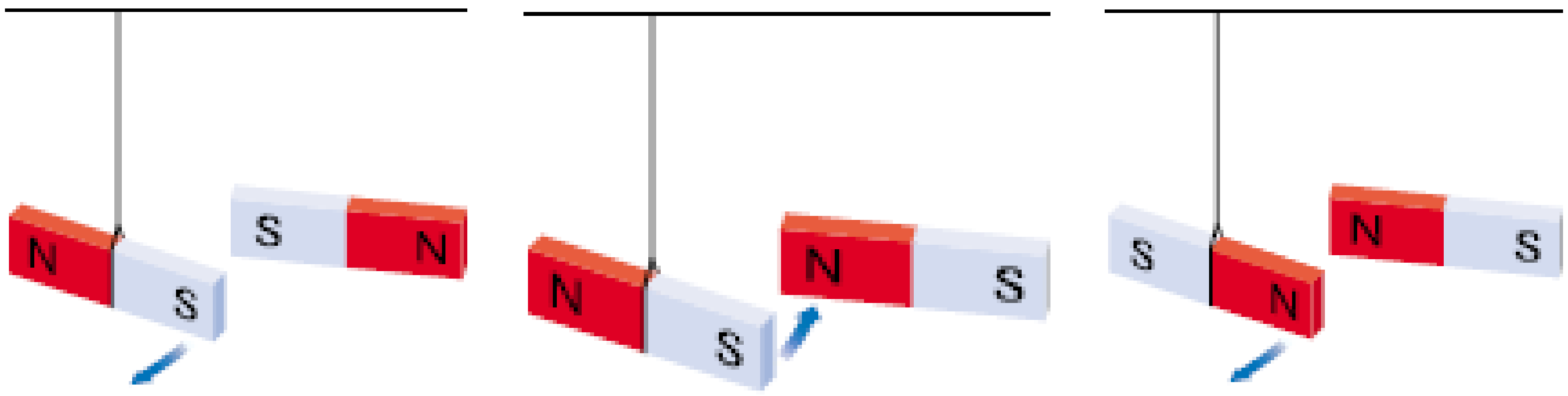
Magnet :The earth is a large magnet.

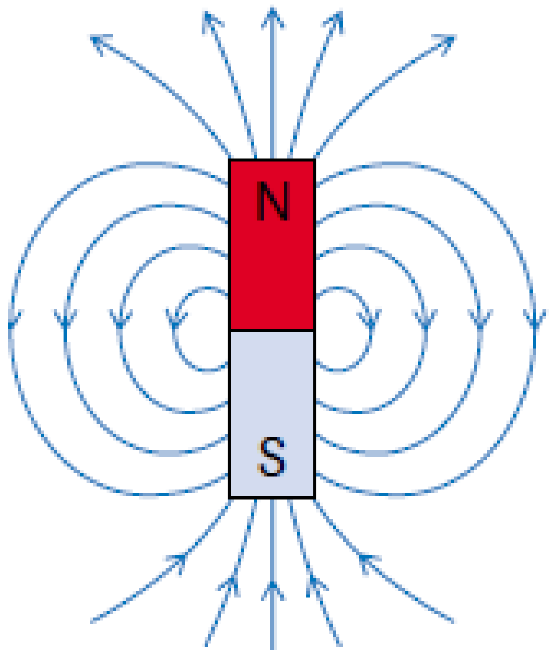




Magnets & Magnetic Fields

Law of Magnetic Poles: Opposite magnetic poles attract. Similar magnetic poles repel.



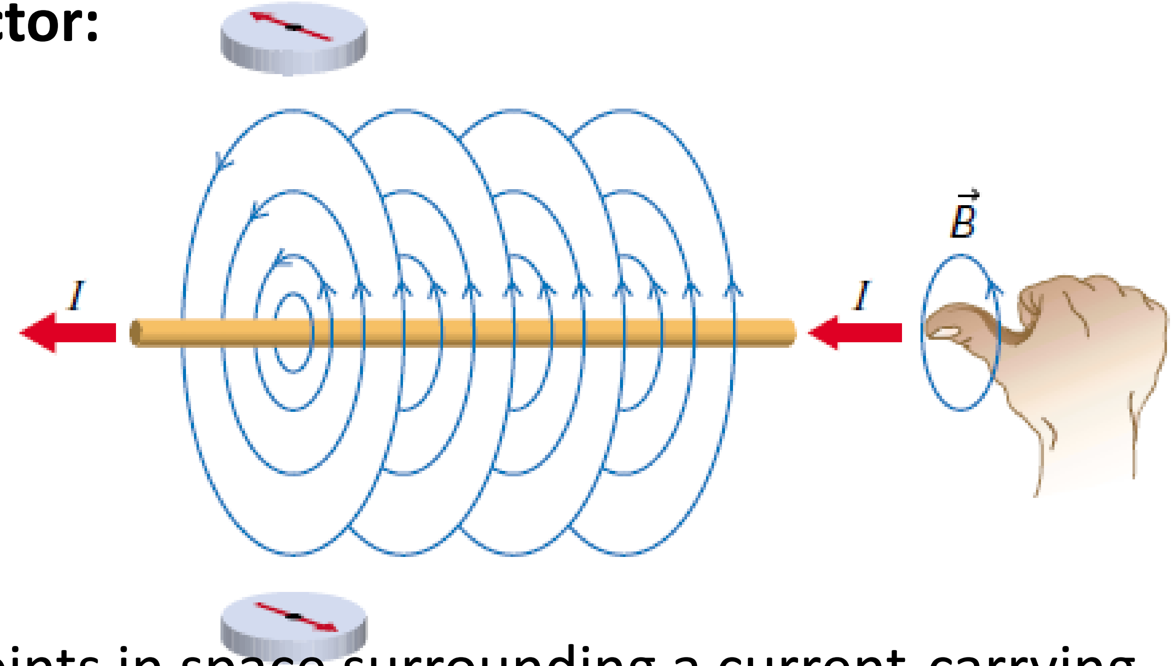


Magnetic force field the area around a magnet in which magnetic forces are exerted

Principle of Electromagnetism: Moving electric charges produce a magnetic field.

Right-Hand Rule for a Straight Conductor:

If a conductor is grasped in the right hand, with the thumb pointing in the direction of the current, the curled fingers point in the direction of the magnetic field lines.

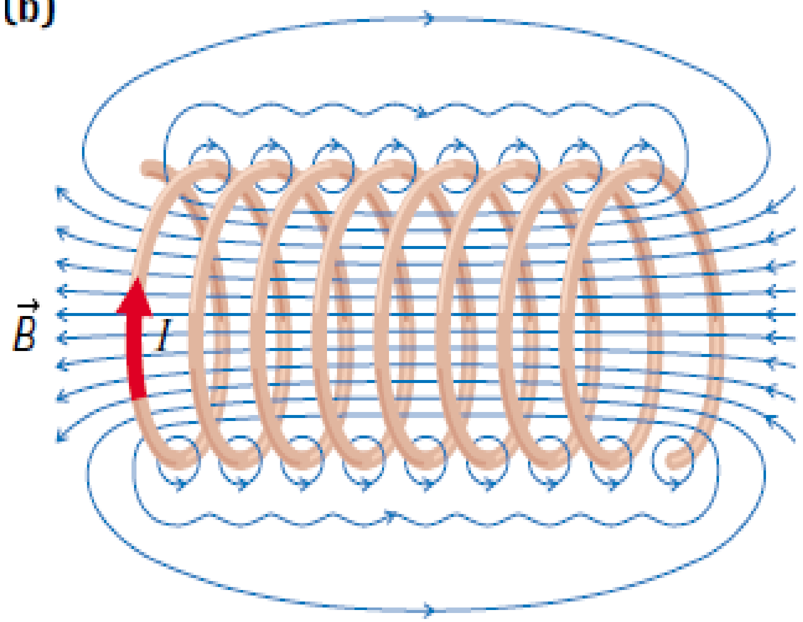


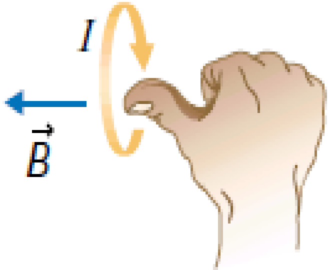
Let us define the **magnetic field** $\vec{B}$ as having the following properties:

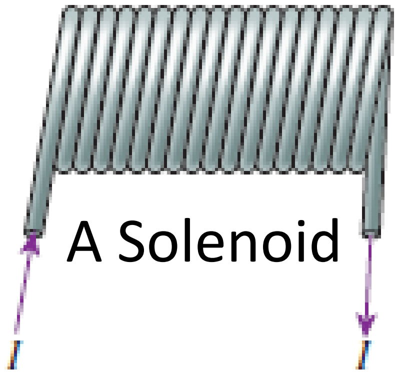
1. A magnetic field is created at *all* points in space surrounding a current-carrying wire.
2. The magnetic field at each point is a vector. It has both a magnitude, which we call the *magnetic field strength B,* and a direction.
3. The magnetic field exerts forces on magnetic poles. The force on a north pole is parallel to $\vec{B}$; the force on a south pole is opposite $\vec{B}$.

Right-Hand Rule for a Solenoid

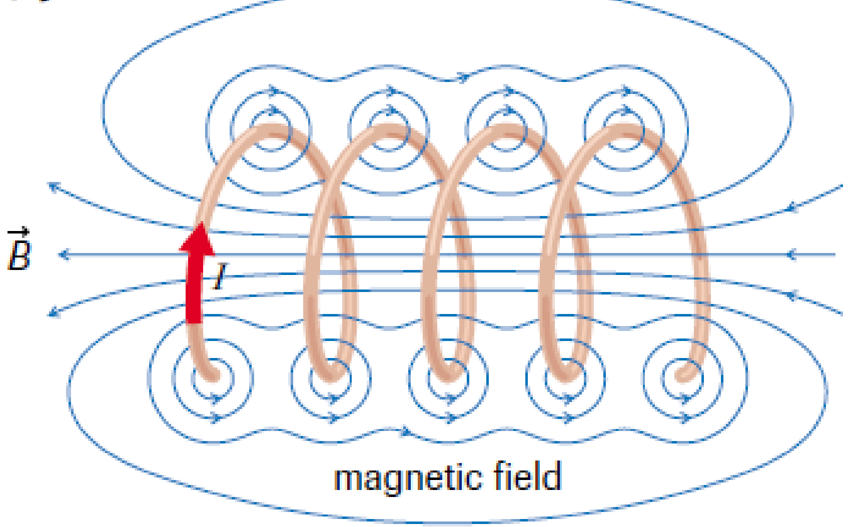
(b)







(a)

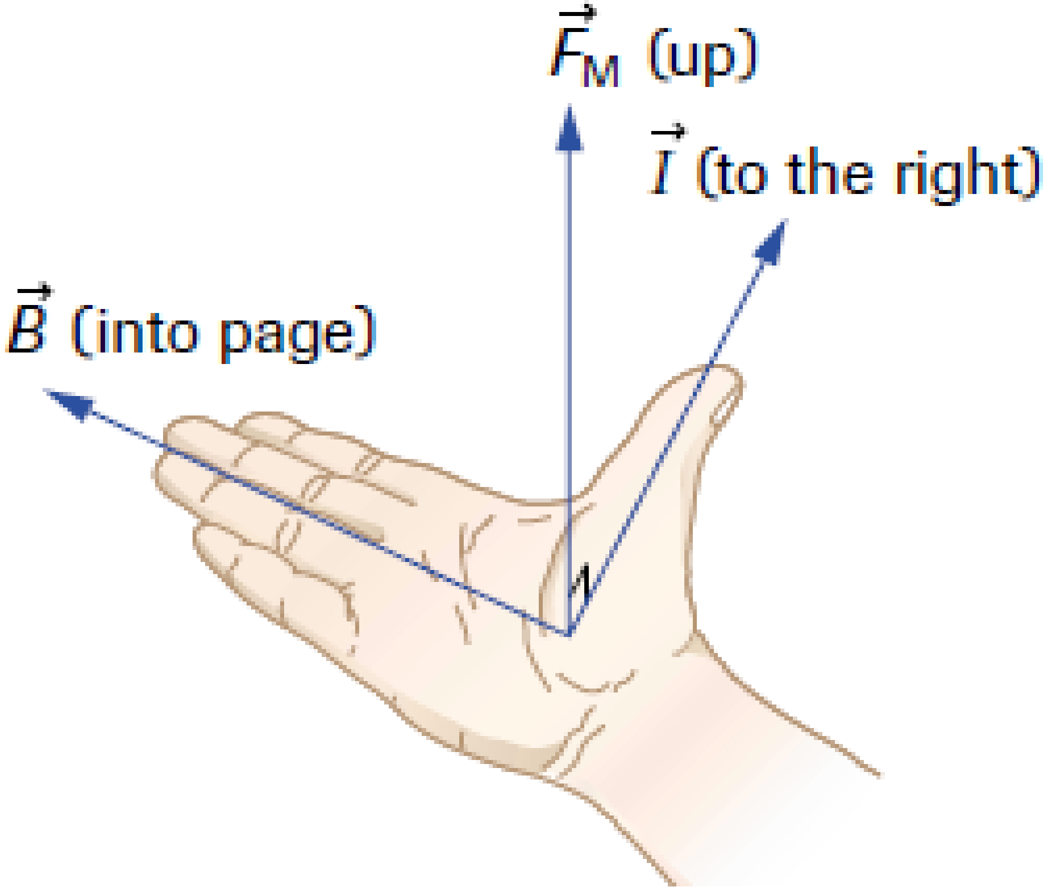


If a solenoid is grasped in the right hand, with the fingers curled in the direction of the electric current, the thumb points in the direction of the magnetic field lines in its core.

Magnetic Force on Moving Charges

The magnitude of the magnetic force $\vec{F}_M$ on a charged particle

- is directly proportional to the magnitude of the magnetic field $\vec{B}$, the velocity $\vec{v}$, and the charge q of the particle.
- depends on the angle θ between the magnetic field $\vec{B}$ and the velocity $\vec{v}$



Combining these factors gives

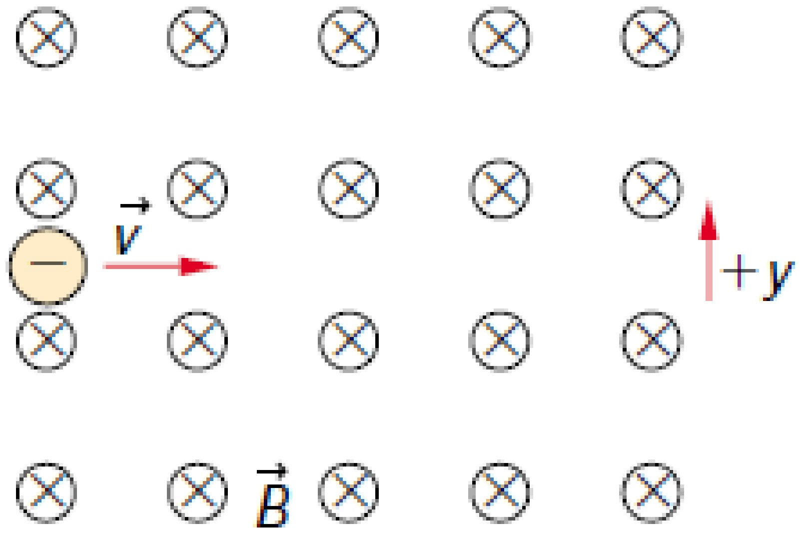
$$F_M = qvB\sin\theta$$

Example

An electron accelerates from rest in a horizontally directed electric field through a potential difference of 46 V. The electron then leaves the electric field, entering a magnetic field of magnitude 0.20 T directed into the page

(a) Calculate the initial speed of the electron upon entering the magnetic field.

(b) Calculate the magnitude and direction of the magnetic force on the electron.



Solution

$\Delta V = 46\,\text{V}$ $\quad v = ?$

$B = 0.20\,\text{T} = 0.20\,\text{kg/C}\cdot\text{s}$ $\quad F_M = ?$

$m_e = 9.11 \times 10^{-31}$ kg (from Appendix C)

$q = 1.6 \times 10^{-19}$ C

$$-\Delta E_E = \Delta E_K$$

$$q\Delta V = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

(b) $\quad F_M = qvB\sin\theta$

Direction?

Assignment

An electron moving through a uniform magnetic field with a velocity of 2.0×10^6 m/s [up] experiences a maximum magnetic force of 5.1×10^{-14}N [left]. Calculate the magnitude and direction of the magnetic field.

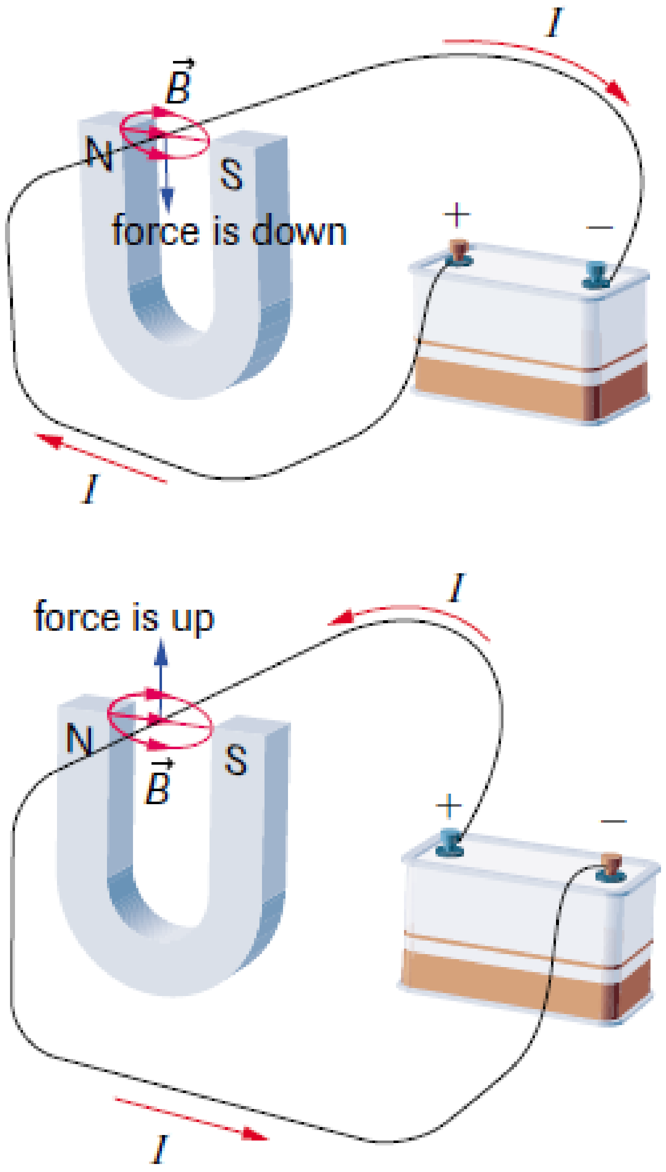
Magnetic Force on a Conductor

Consider a conductor with a current *I*, placed in a magnetic field of magnitude *B*. The force F on the conductor is directly proportional to the magnitude of the magnetic field B, to the current in the conductor I, and to the length of the conductor ℓ. In addition, if the angle between the conductor (or current) and the magnetic field lines is θ, the magnitude of the magnetic force is directly proportional to $\sin\theta$. Combining these relationships: $F \propto I\ell B \sin\theta$.

$F = kI\ell B \sin\theta$, where the constant k=1.

$$F = I\ell B \sin\theta$$

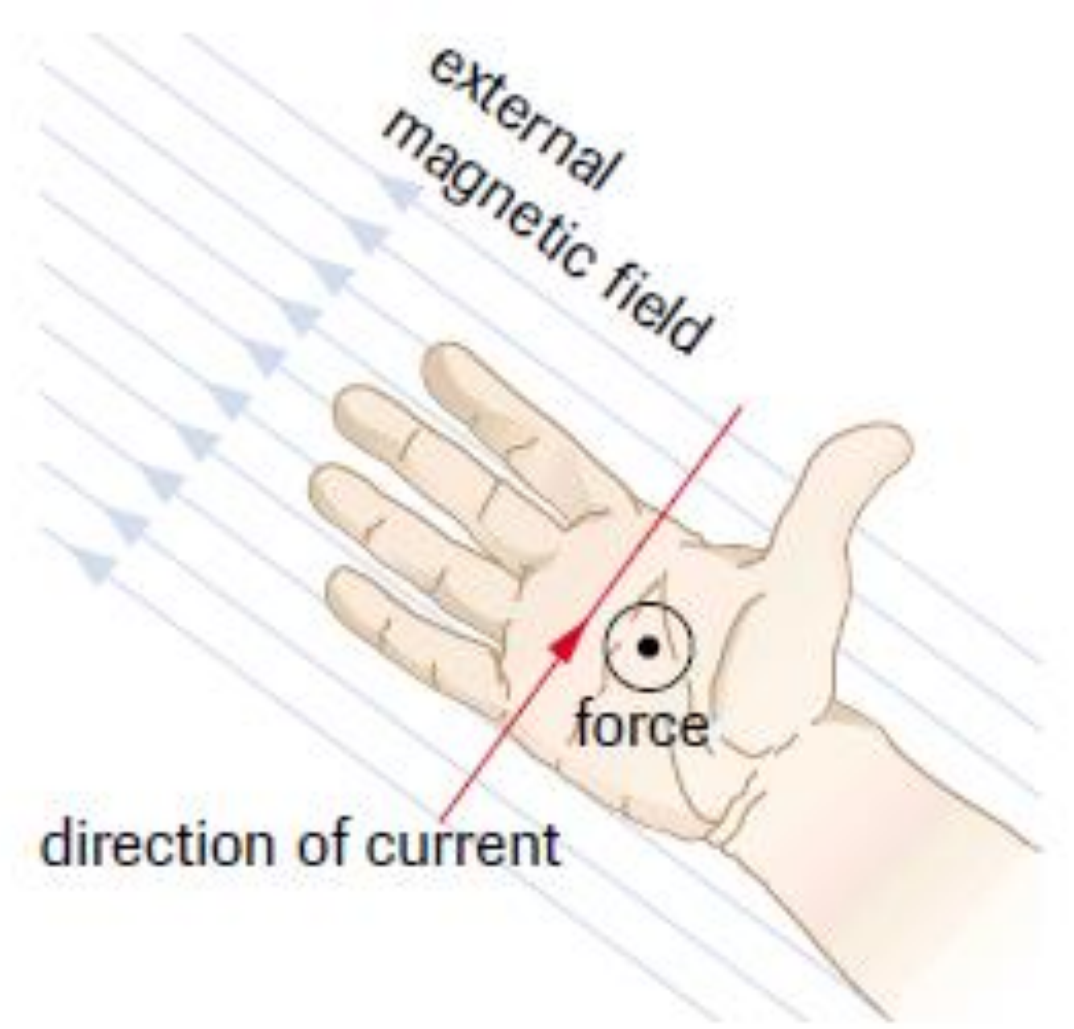
B is the magnitude of the magnetic field strength, in teslas (T).



Right-Hand Rule for the Motor Principle

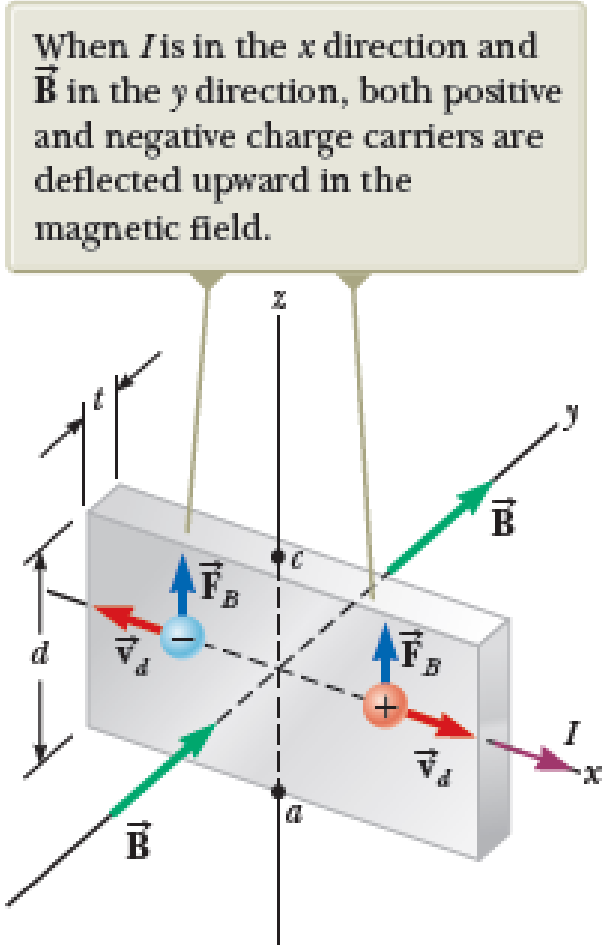
Another simple right-hand rule, equivalent to the one for charges moving in a magnetic field, can be used to determine the relative directions of $\vec{F}$, I, and $\vec{B}$:

- If the right thumb points in the direction of the current (flow of positive charge), and the extended fingers point in the direction of the magnetic field, the force is in the direction in which the right palm pushes.

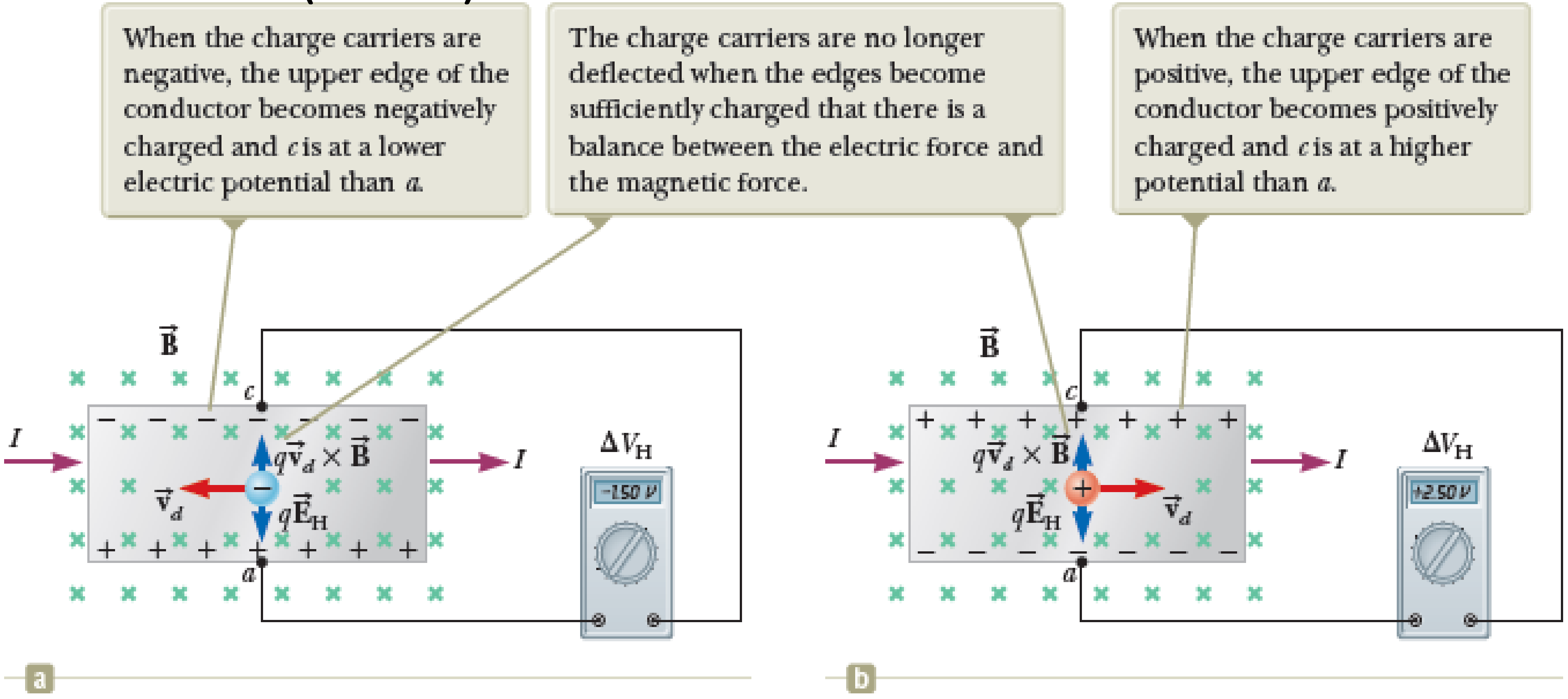


The Hall Effect

When a current-carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field. This phenomenon is known as the *Hall effect. A* flat conductor carrying a current I in the x direction is shown in Fig. A uniform magnetic field $\vec{\boldsymbol{B}}$ is applied in the y direction. If the charge carriers are electrons moving in the negative x direction with a drift velocity $\vec{\boldsymbol{v}}_{\boldsymbol{d}}$, they experience an upward magnetic force $\vec{\boldsymbol{F}}_{\boldsymbol{B}} = \boldsymbol{q}\vec{\boldsymbol{v}}_{\boldsymbol{d}} \times \vec{\boldsymbol{B}}$, are deflected upward, and accumulate at the upper edge of the flat conductor, leaving an excess of positive charge at the lower edge (Fig. a in next page). This accumulation of charge at the edges establishes an electric field in the conductor and increases until the electric force on carriers remaining in the bulk of the conductor balances the magnetic force acting on the carriers.



The Hall Effect (Contd.)



A sensitive voltmeter connected across the sample as shown in Fig. (a & b) can measure the potential difference, known as the **Hall voltage**, generated across the conductor.

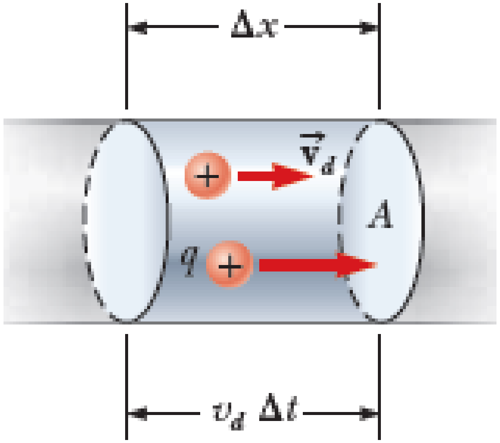
Hall voltage

The magnetic force exerted on the carriers has magnitude qv_dB. In equilibrium, this force is balanced by the electric force qE_H, where E_H is the magnitude of the electric field due to the charge separation (sometimes referred to as the *Hall field*). Therefore,

$$qv_dB = qE_H \qquad \Rightarrow E_H = v_dB$$

If *d* is the width of the conductor, the Hall voltage is

$$\Delta V_H = E_Hd = v_dBd$$



The total charge ΔQin this segment is, $\Delta Q = (nA\,\Delta x)q = (nAv_d\Delta t)q$, where *n* = the charge carrier density, *q* is the charge on each carrier.

$$\therefore I = \frac{\Delta Q}{\Delta t} = (nAv_d)q \qquad \Rightarrow v_d = I/nAq$$

$$\Delta V_H = \frac{IBd}{nAq} = \frac{IB\cancel{d}}{n(t\cancel{d})q} = \frac{IB}{ntq}$$ Where, *t* is the thickness of the conductor.

Assignment:

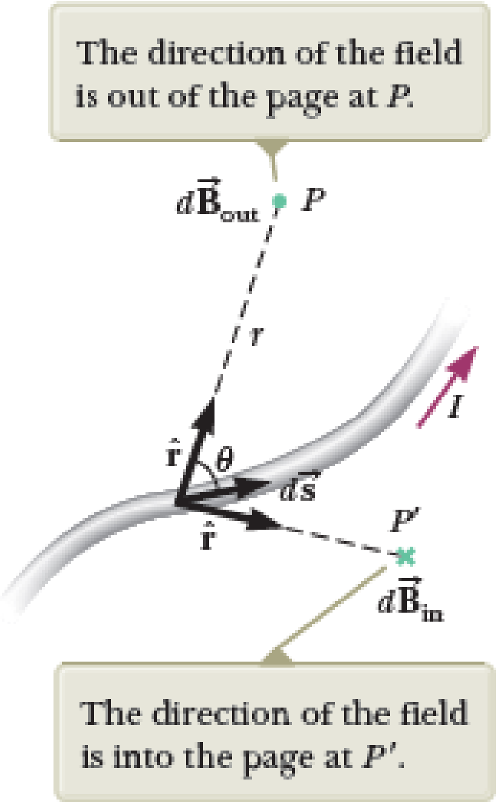
The accompanying table shows measurements of the Hall voltage and corresponding magnetic field for a probe used to measure magnetic fields. (a) Plot these data and deduce a relationship between the two variables. (b) If the measurements were taken with a current of 0.200 A and the sample is made from a material having a charge-carrier density of 1.00×10^{26} carriers/m^3, what is the thickness of the sample?

ΔV_H (μV)	B (T)
0	0.00
11	0.10
19	0.20
28	0.30
42	0.40
50	0.50
61	0.60
68	0.70
79	0.80
90	0.90
102	1.00

The Biot–Savart Law

For the magnetic field $d\vec{B}$ at a point *P* associated with a length element $d\vec{s}$ of a wire carrying a steady current I (Fig.):

- The vector $d\vec{B}$ is perpendicular both to $d\vec{s}$ (which points in the direction of the current) and to the unit vector $\hat{\boldsymbol{r}}$ directed from $d\vec{s}$ toward *P.*
- The magnitude of $d\vec{B}$ is inversely proportional to r^2, where *r* is the distance from $d\vec{s}$ to *P.*
- The magnitude of $d\vec{B}$ is proportional to the current $I$and to the magnitude *ds* of the length element $d\vec{s}$.
- The magnitude of $d\vec{B}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\vec{s}$ and $\hat{\boldsymbol{r}}$.



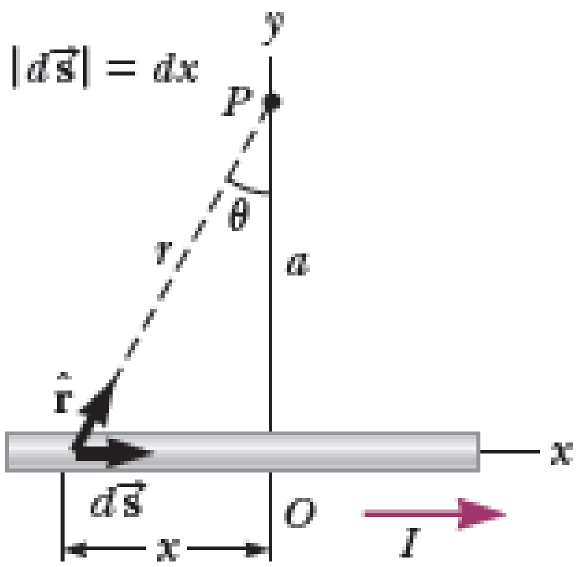
These observations is known as the **Biot–Savart law:**

$$\boldsymbol{d\vec{B}} = \frac{\boldsymbol{\mu_0}}{\boldsymbol{4\pi}} \frac{\boldsymbol{Id\vec{s}} \times \hat{\boldsymbol{r}}}{\boldsymbol{r^2}}$$

where $\mu_0 = 4\pi \times 10^{-7}$T.m/A, is the **permeability of free space.**

Example:

Consider a thin, straight wire of finite length carrying a constant current *I* and placed along the *x* axis as shown in Fig. Determine the magnitude and direction of the magnetic field at point *P* due to this current.

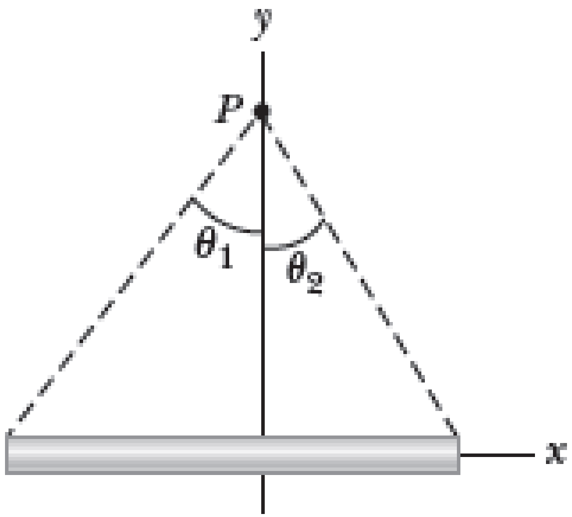


$$d\vec{s} \times \hat{r} = |d\vec{s} \times \hat{r}|\hat{k} = \left[dx \sin\left(\frac{\pi}{2} - \theta\right)\right]\hat{k} = (dx\cos\theta)\hat{k}$$

$$(1) \quad d\vec{B} = (dB)\hat{k} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{k} \qquad (2) \quad r = \frac{a}{\cos\theta}$$

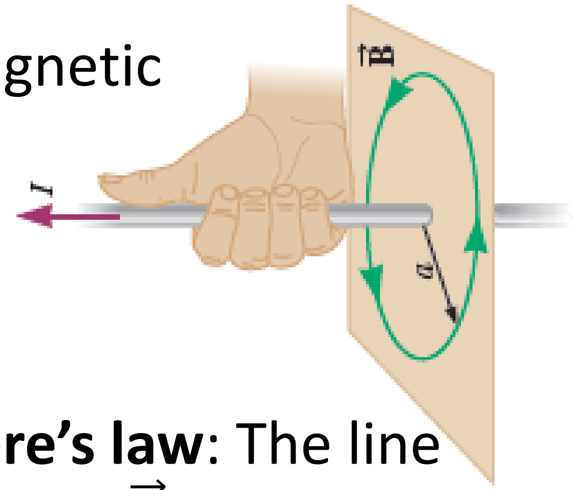
$$x = -a\tan\theta \qquad (3) \quad dx = -a\sec^2\theta\, d\theta = -\frac{a\, d\theta}{\cos^2\theta}$$

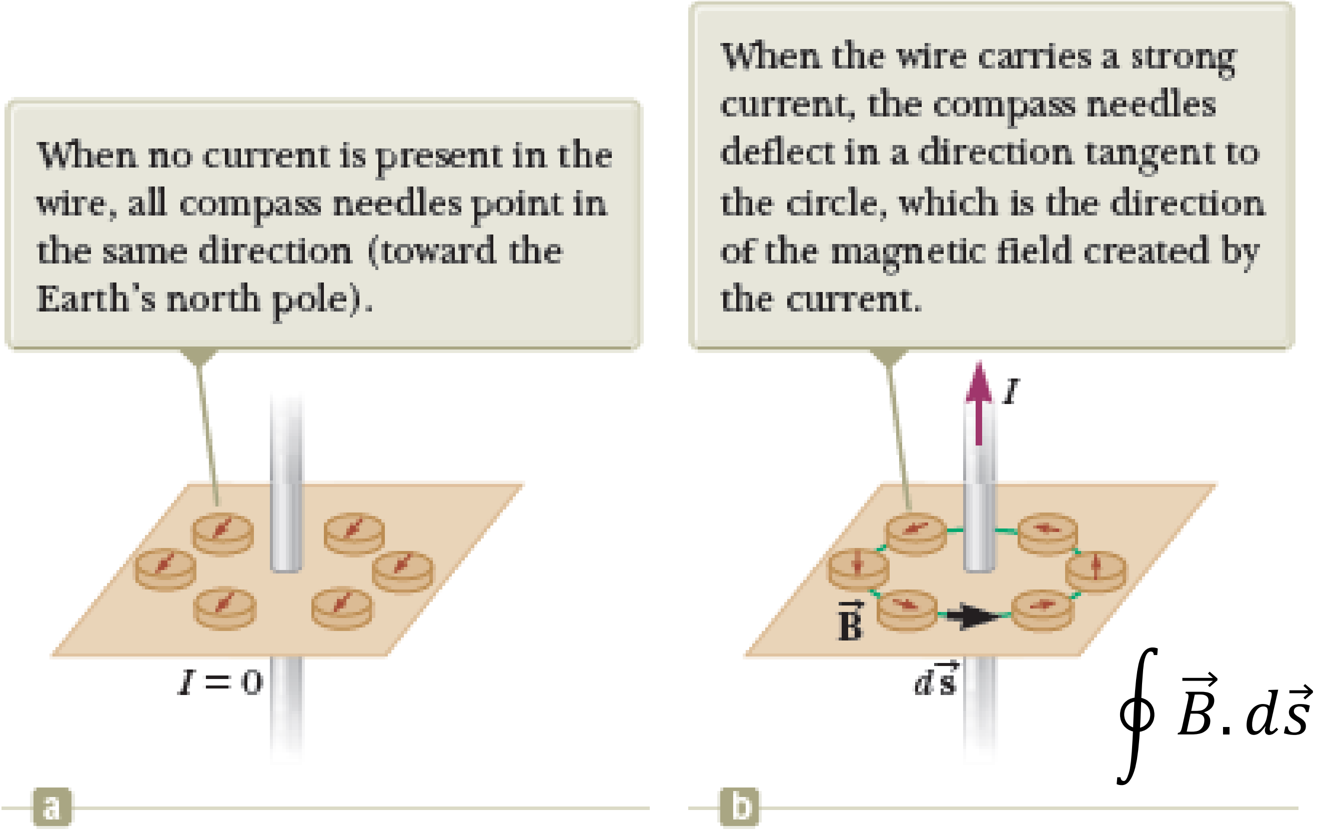
$$(4) \quad dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\, d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta = -\frac{\mu_0 I}{4\pi a}\cos\theta\, d\theta$$



$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta = \boxed{\frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)}$$

The right-hand rule for determining the direction of the magnetic field surrounding a long, straight wire carrying a current.





Ampère's law: The line integral of $\vec{B}.d\vec{s}$ around any closed path equals $\mu_0 I$, where *I* is the total steady current passing through any surface bounded by the closed path:

$$\oint \vec{B}.d\vec{s} = B\oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

The Magnetic Field of a Solenoid

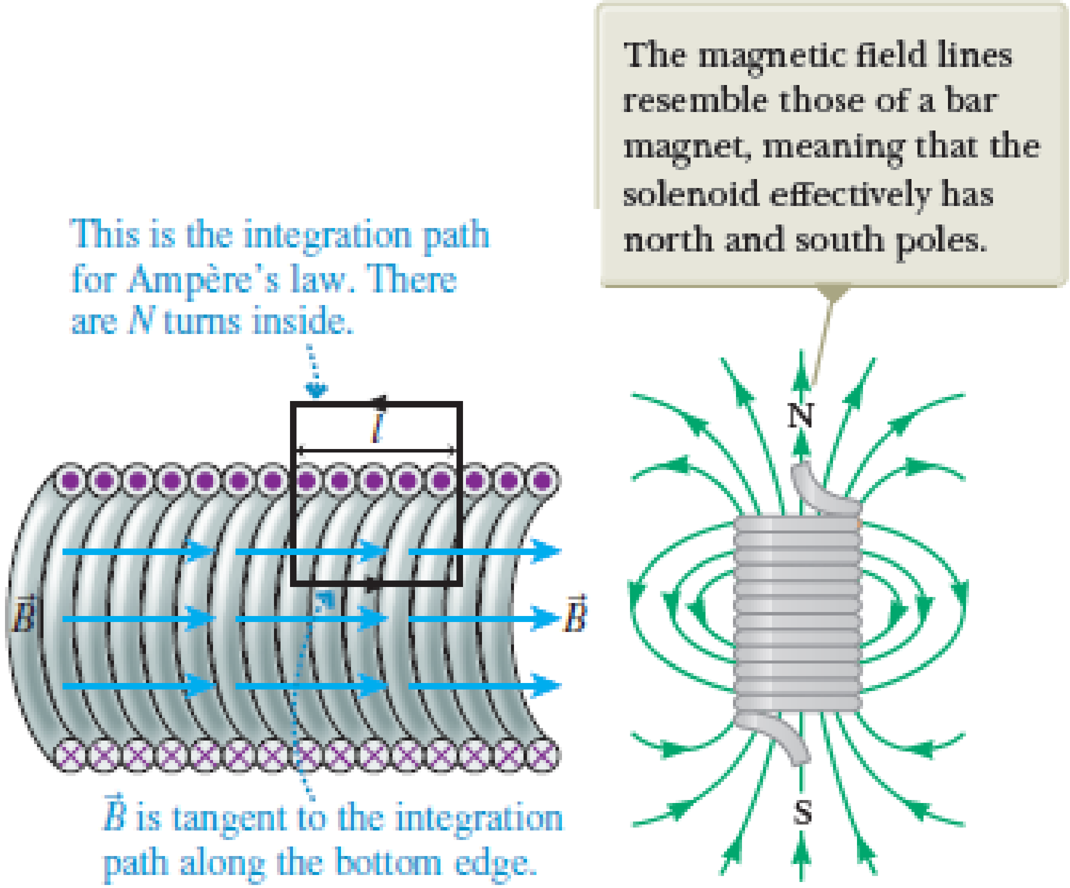
If N is the number of turns in the length ℓ, the total current through the rectangle is NI. Therefore, Ampère's law applied to this path gives

$$\oint \vec{\boldsymbol{B}}.d\vec{\boldsymbol{s}} = \mathrm{B}\ell = \mu_0 NI$$

The strength of the uniform magnetic field inside a solenoid is

$$B_{solenoid} = \frac{\mu_0 NI}{\ell} = \mu_0 nI$$

where $n = N/\ell$ is the number of turns per unit length.



Measurements that need a uniform magnetic field are often conducted inside a solenoid, which can be built quite large.

Example: Generating an MRI magnetic field

A 1.0-m-long MRI solenoid generates a 1.2 T magnetic field. To produce such a large field, the solenoid is wrapped with superconducting wire that can carry a 100 A current. How many turns of wire does the solenoid need?

Solution:

$$N = \frac{lB}{\mu_0 I} = \frac{(1.0\ \text{m})(1.2\ \text{T})}{(4\pi \times 10^{-7}\ \text{T m/A})(100\ \text{A})} = 9500\ \text{turns}$$

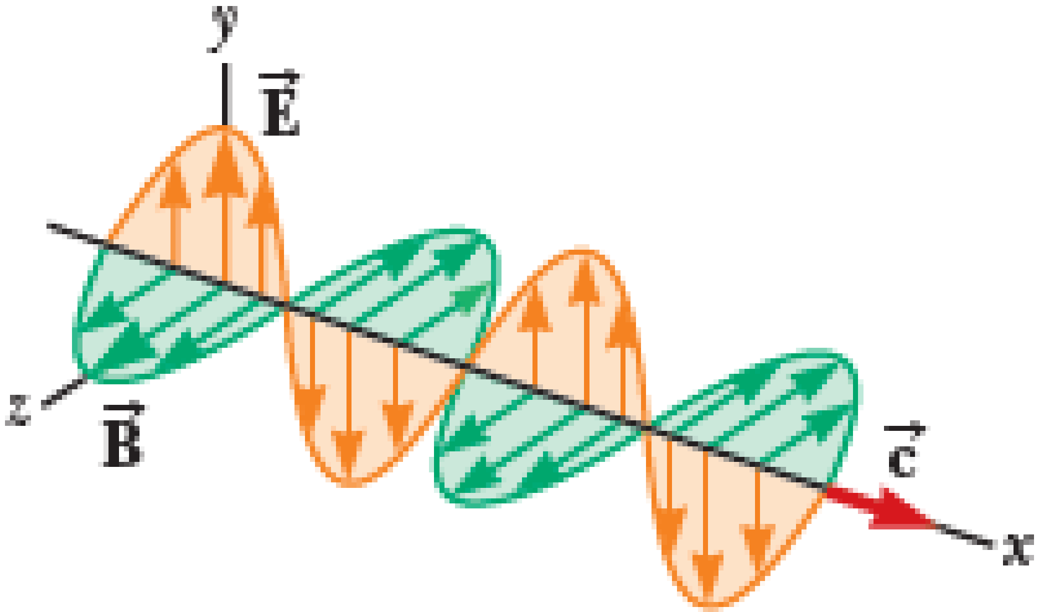
Quick Quiz Consider a solenoid that is very long compared with its radius. Of the following choices, what is the most effective way to increase the magnetic field in the interior of the solenoid? **(a)** double its length, keeping the number of turns per unit length constant **(b)** reduce its radius by half, keeping the number of turns per unit length constant **(c)** overwrap the entire solenoid with an additional layer of current-carrying wire

PHY 103 Physics II

Optics

Physical Optics:

- Theories of light,
- Hugen’s principles & construction,
- Interference of light,
- Young’s double slit experiment,
- Newton’s rings.
- Interferometers,
- Diffraction of light: Fresnel and Fraunhoffer diffraction,
- Diffraction by single slit,
- Diffraction by double slit
- Diffraction gratings
- Polarization: production &analysis of polarized light
- Optical activity



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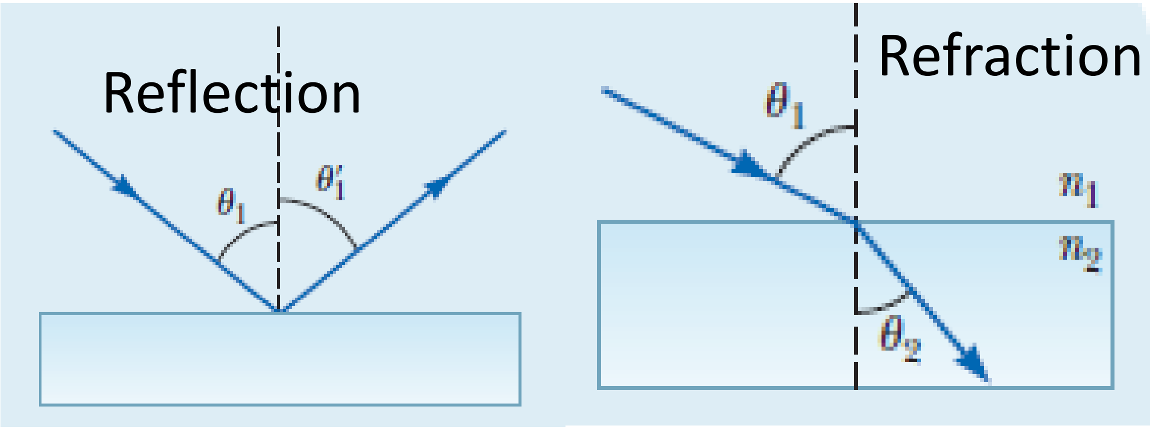
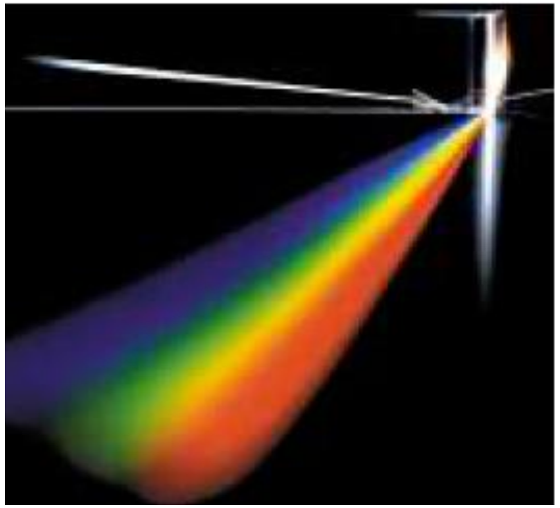
Light: Wave or Particle?

- Newton’s particle theory provided a satisfactory explanation for four properties of light: rectilinear propagation, reflection, refraction, and dispersion. The theory was weak in its explanations of diffraction and partial reflection–partial refraction.
- Huygens’ wave theory considered every point on a wave front as a point source of tiny secondary wavelets, spreading out in front of the wave at the same speed as the wave itself. The surface envelope, tangent to all the wavelets, constitutes the new wave front.
- Huygens’ version of the wave theory explained many of the properties of light, including reflection, refraction, partial reflection–partial refraction, diffraction, and rectilinear propagation.

Newton’s Particle Theory: light consisted of streams of tiny particles, which he called “corpuscles,” shooting out like bullets from a light source.

- Rectilinear propagation of light the term used to describe light travelling in straight lines.
- Diffraction: the beam of light bent slightly outward at the edges of the aperature, a phenomenon named diffraction.





Dispersion

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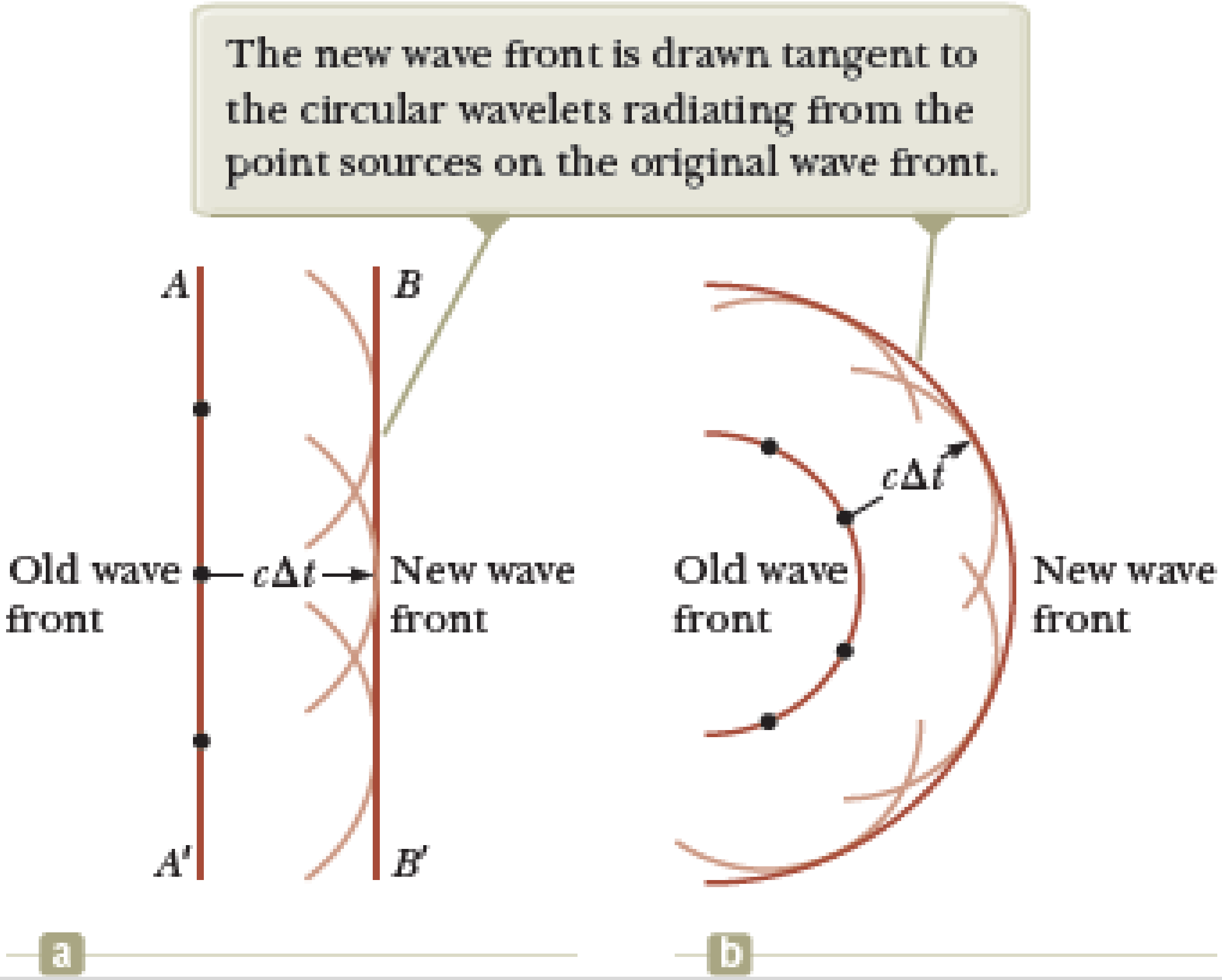
Huygens's Principle:

All points on a given wave front are taken as point sources for the production of spherical secondary waves, called wavelets, that propagate outward through a medium with speeds characteristic of waves in that medium. After some time interval has passed, the new position of the wave front is the surface tangent to the wavelets.



Huygens's construction for (a) a plane wave propagating to the right and (b) a spherical wave propagating to the right.

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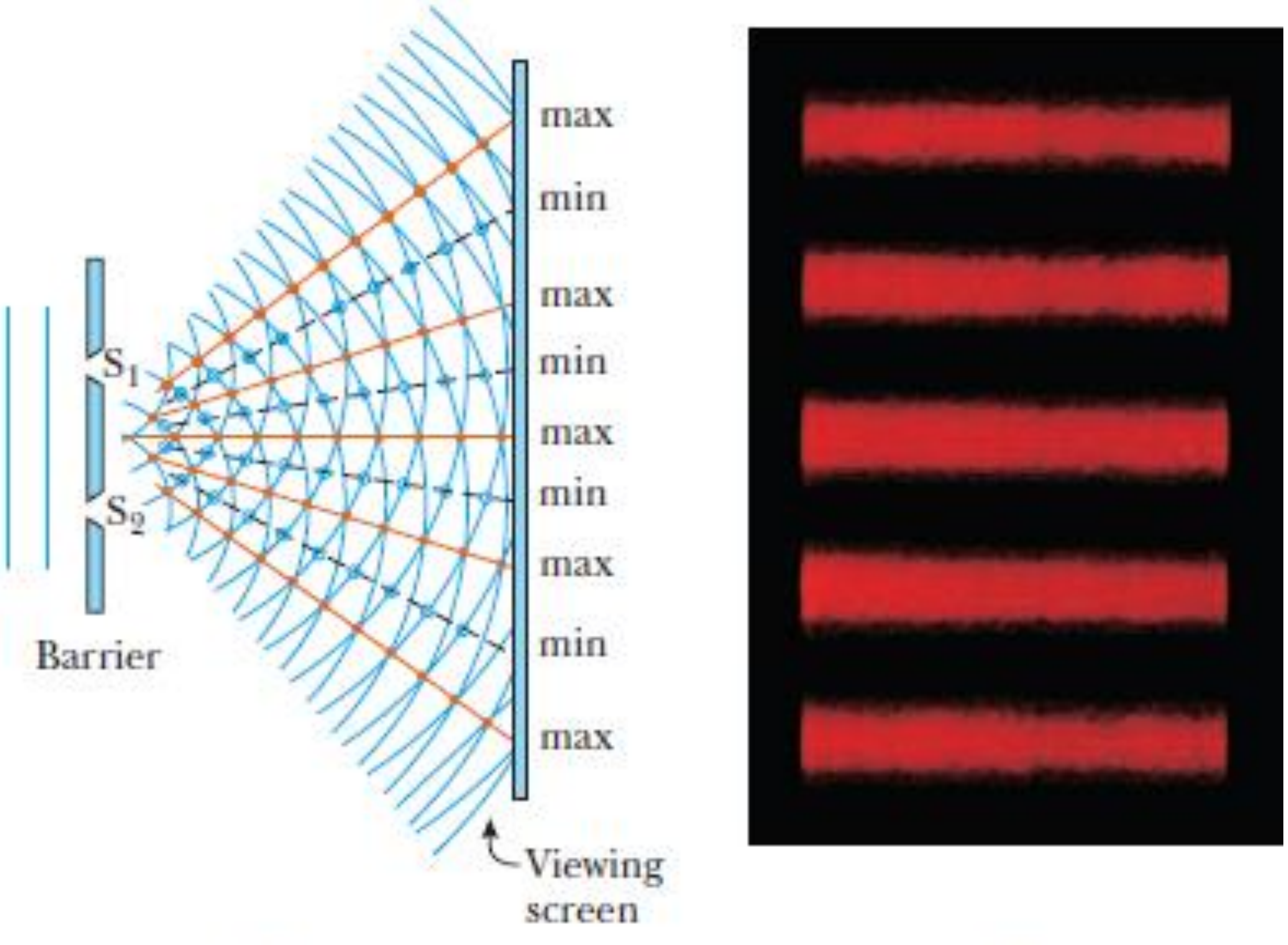
Young's Double-Slit Experiment:

Interference in Young's experiment was a crucial test for the wave theory of light.

When the light from S1 and that from S2 both arrive at a point on the screen such that constructive interference occurs at that location, a bright fringe appears. When the light from the two slits combines destructively at any location on the screen, a dark fringe results.



(a) (b)

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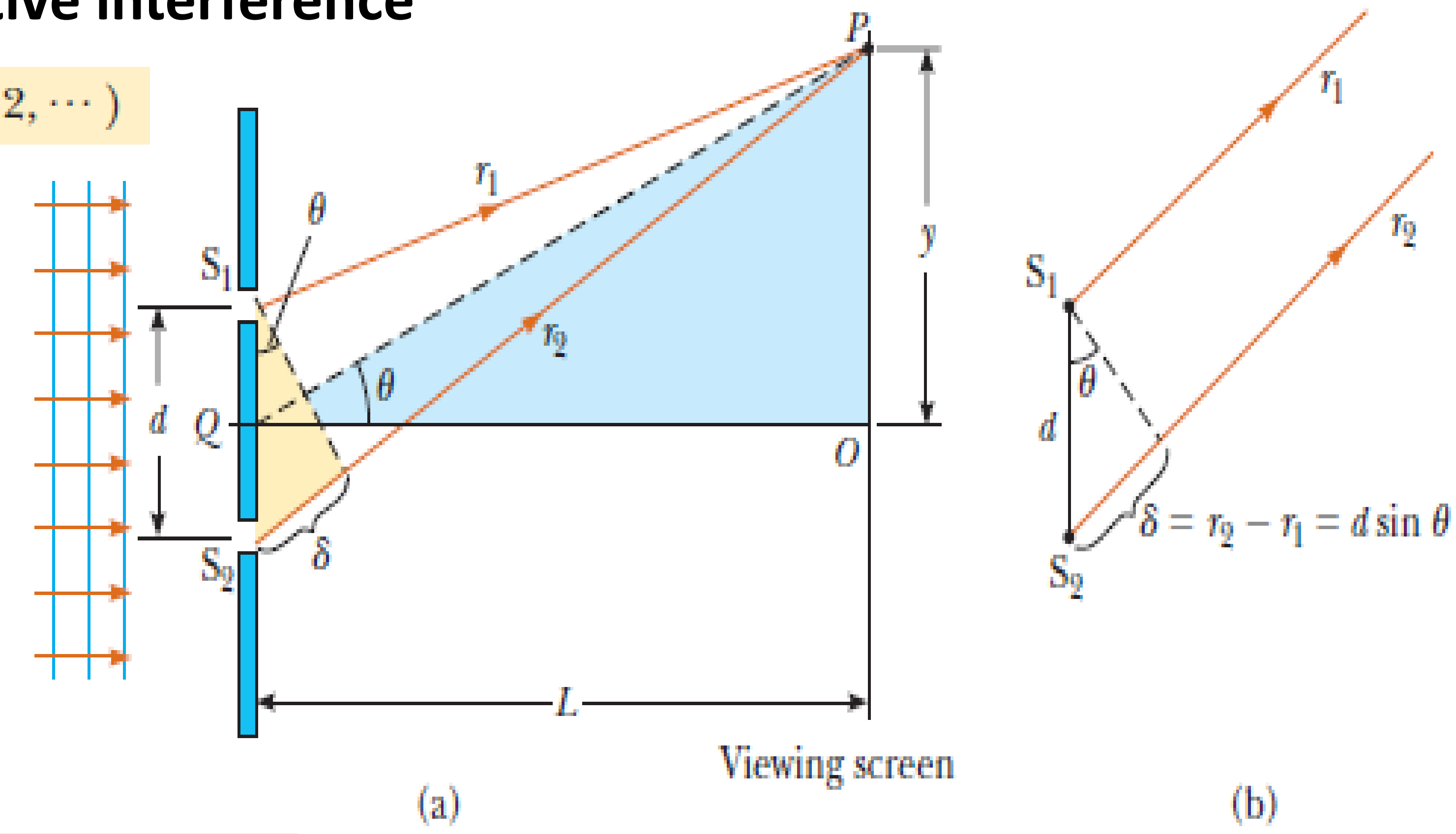
Young's Double-Slit Experiment:

bright fringes, or **constructive interference**

$$d \sin \theta_{\text{bright}} = m\lambda \quad (m = 0, \pm 1, \pm 2, \cdots)$$

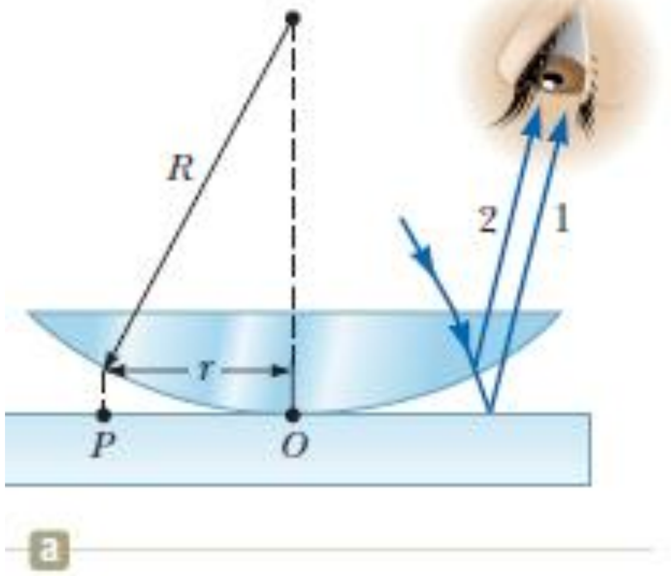


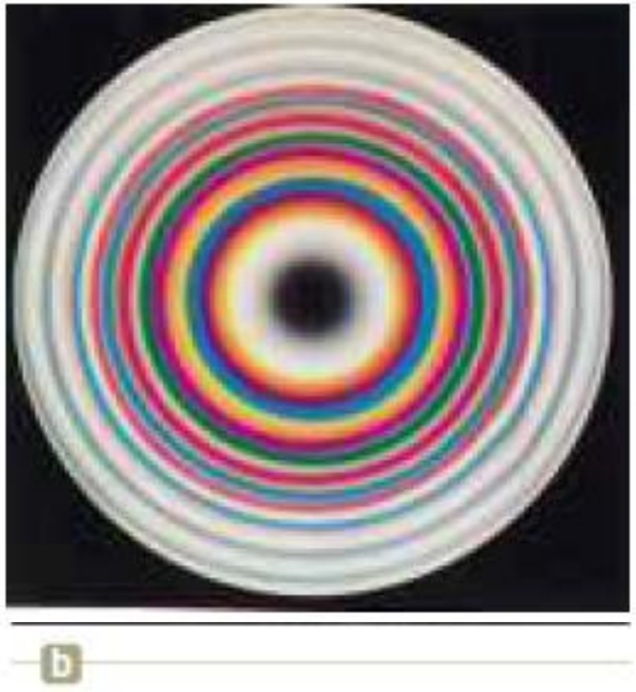
Dark fringes, or
destructive interference

$$d \sin \theta_{\text{dark}} = (m + \tfrac{1}{2})\lambda \quad (m = 0, \pm 1, \pm 2, \cdots)$$

Newton's Rings

Another method for observing interference in light waves is to place a plano-convex lens on top of a flat glass surface as shown in Fig a.

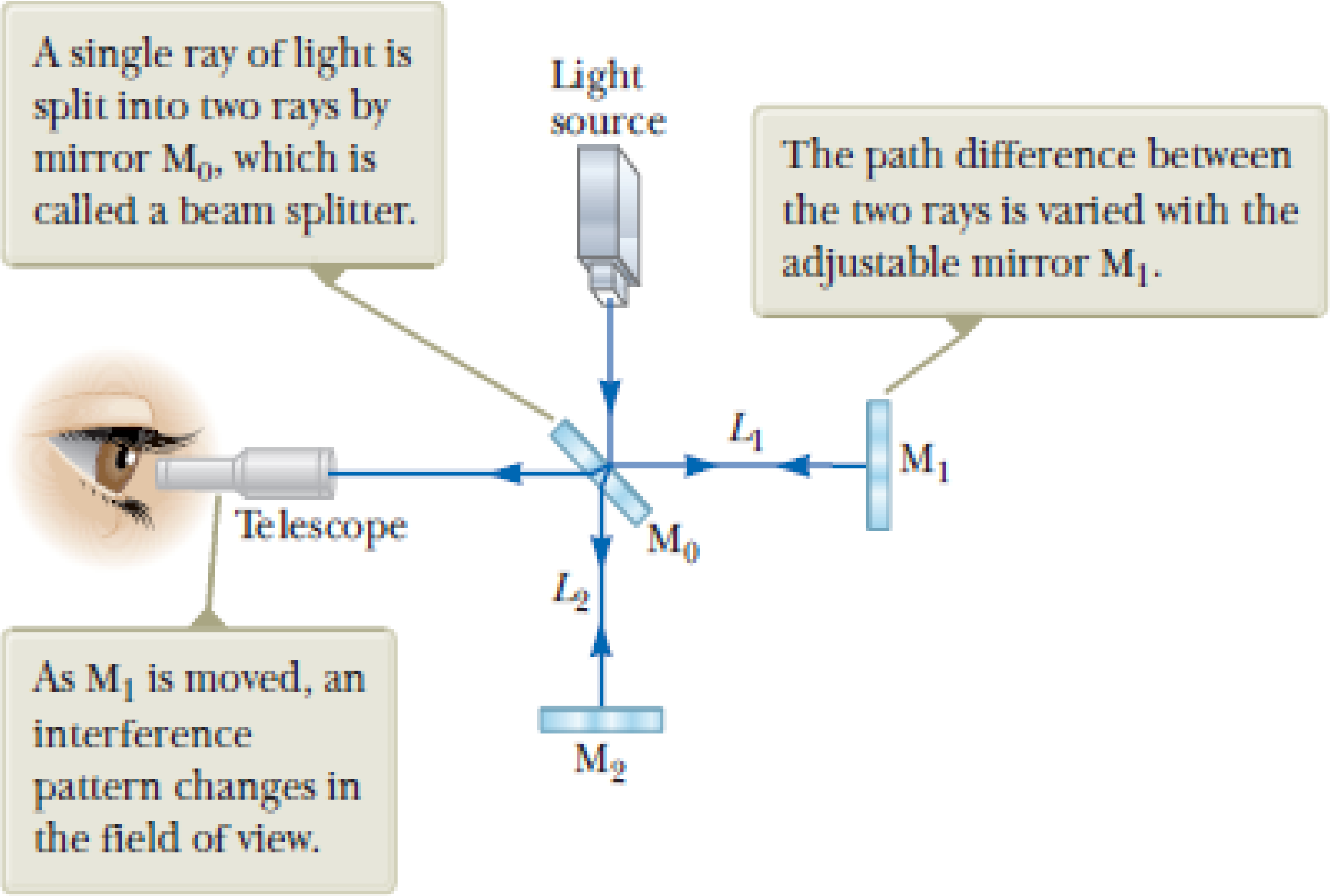




With this arrangement, the air film between the glass surfaces varies in thickness from zero at the point of contact to some nonzero value at point *P.* If the radius of curvature *R* of the lens is much greater than the distance *r* and the system is viewed from above, a pattern of light and dark rings is observed as shown in Fig b. These circular fringes, discovered by Newton, are called **Newton's rings.**

The Michelson Interferometer

M_1 is moved a distance $\lambda/4$ toward M_0, the path difference changes by $\lambda/2$. As M_1 is moved an additional distance $\lambda/4$ toward M_0, the bright circle becomes a dark circle again. Therefore, the fringe pattern shifts by one-half fringe each time M_1 is moved a distance $\lambda/4$.



The wavelength of light is then measured by counting the number of fringe shifts for a given displacement of M_1. Using this we can measure the speed of light.

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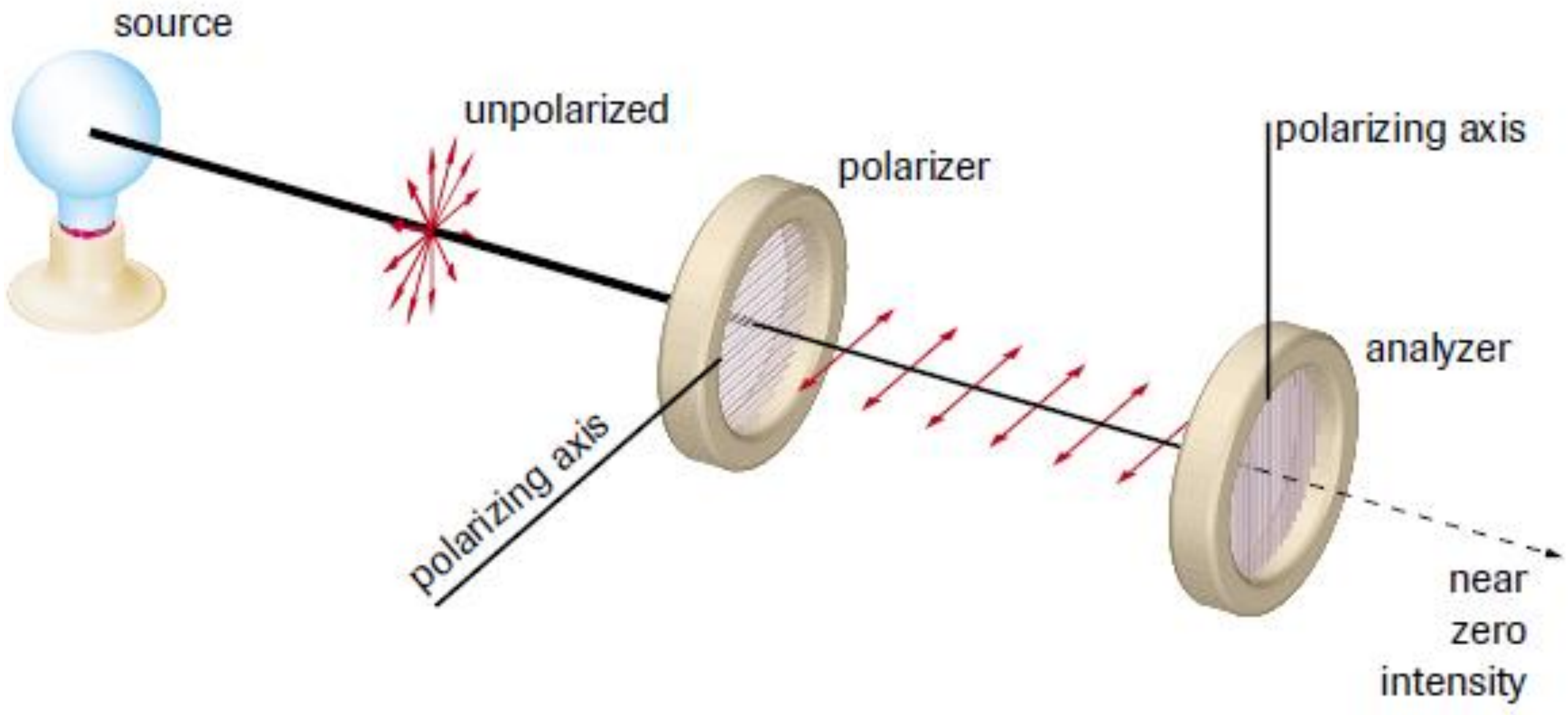
Polarization of Light

Polarization provided the proof that light is a transverse wave.



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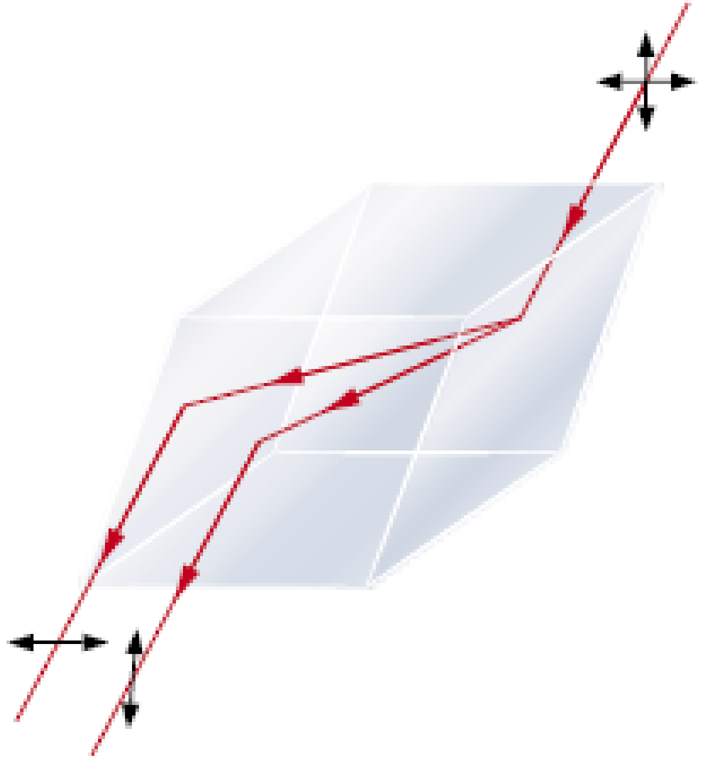
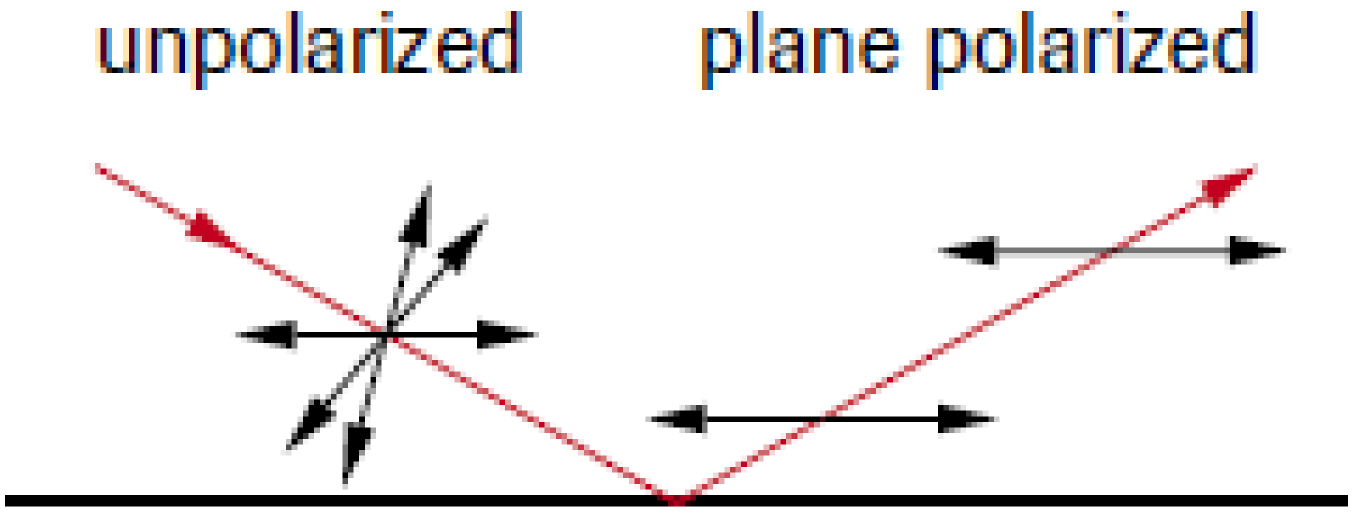
Polarization of Light

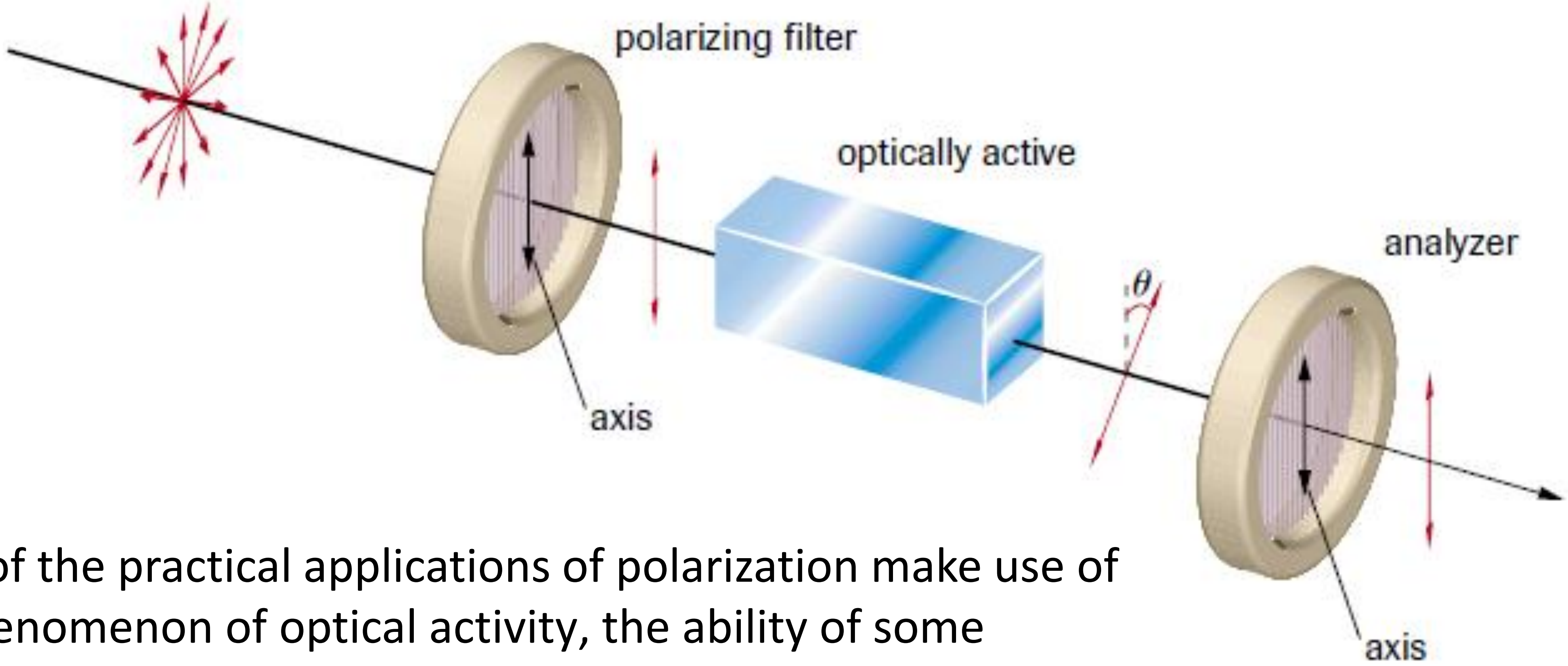


Double refraction



Light reflected by a non-metallic surface is partially polarized in the horizontal plane.

An analyzer reveals the ability of an optically active substance to rotate the plane of polarization.



Many of the practical applications of polarization make use of the phenomenon of optical activity, the ability of some substances, such as sugar, turpentine, and insulin, to rotate the plane of polarization of a beam of light.

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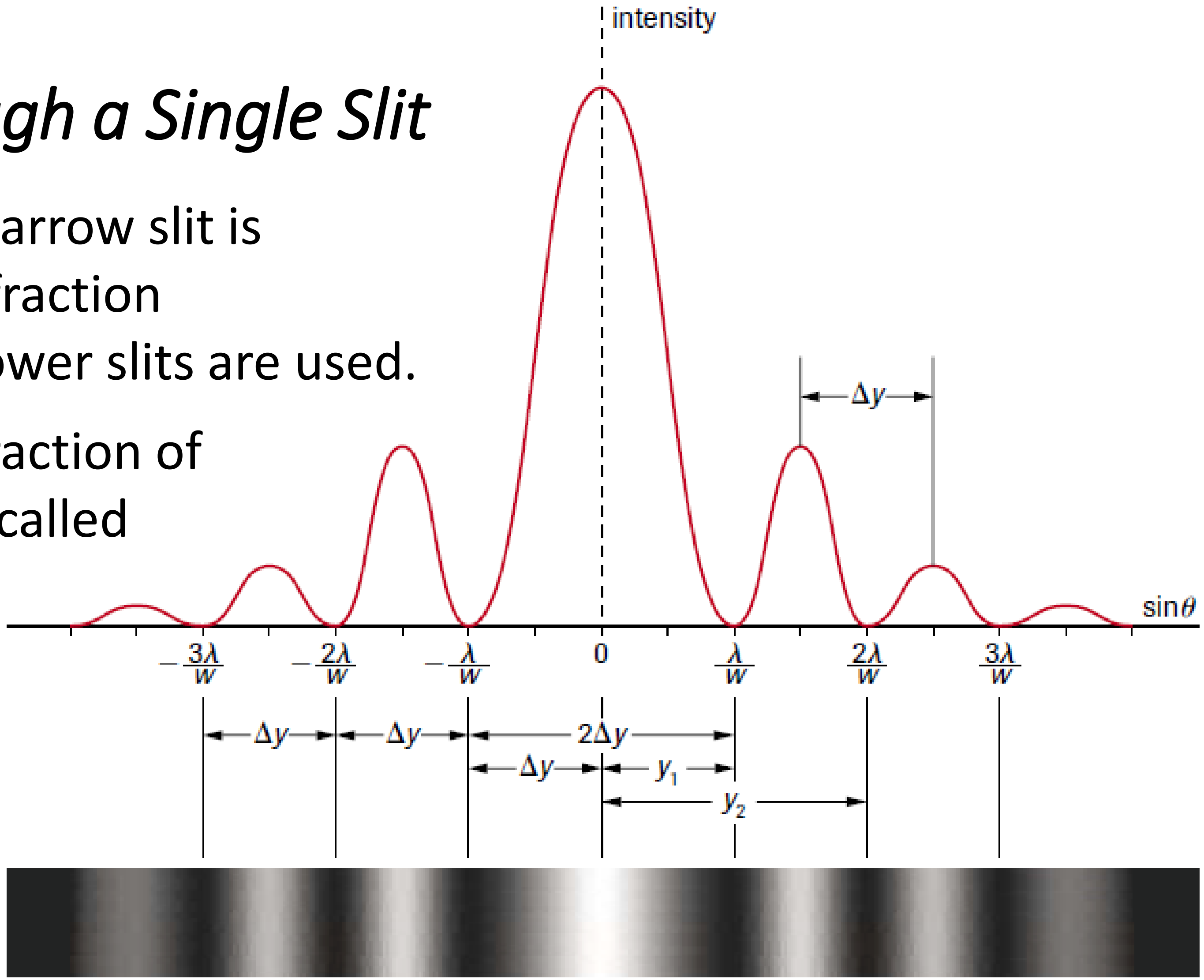
Diffraction of Light through a Single Slit

Light passing through a single, narrow slit is diffracted. The extent of the diffraction increases as narrower and narrower slits are used.

Fraunhofer Diffraction: The diffraction of light through a single slit is also called Fraunhofer diffraction.

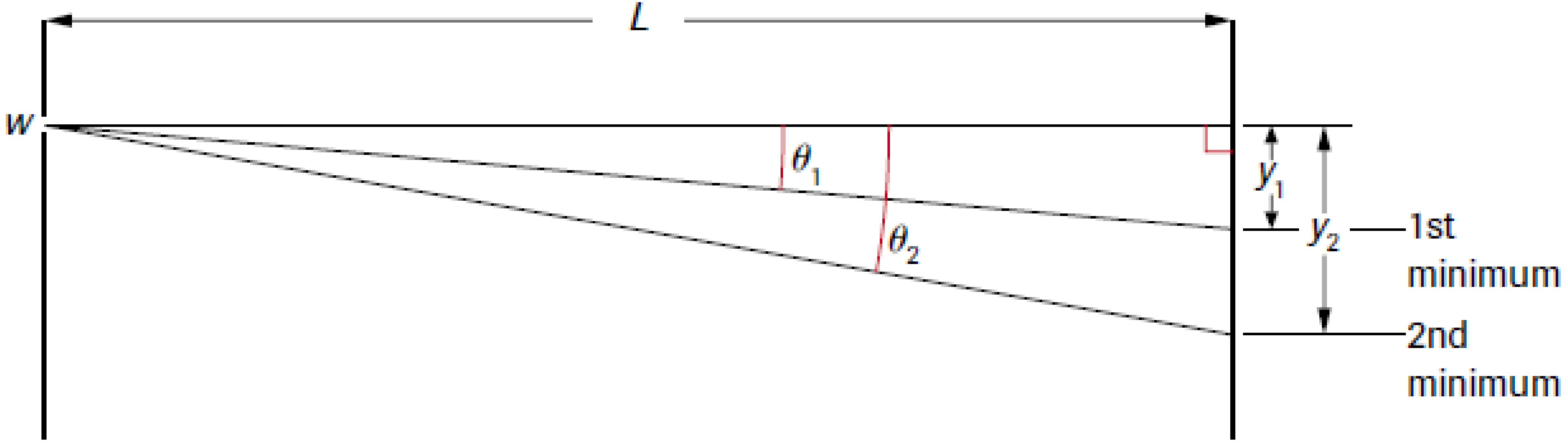


$$\sin\theta_n = \frac{n\lambda}{w} \quad \text{(minima)}$$

$$\sin\theta_m = \frac{\left(m + \frac{1}{2}\right)\lambda}{w} \quad \text{(maxima)}$$

where θ is the direction of the fringe, λ is the wavelength, *w* is the width of the slit, and $m = 1, 2, 3, \ldots$ is order of the maxima/minima beyond the central maximum.

Diffraction of Light through a Single Slit (Contd.)



$$\lambda = \frac{wy_1}{L}$$

Light from a laser pointer, with a wavelength of 6.70×10^2 nm, passes through a slit with a width of 12 μm. A screen is placed 30 cm away.

(a) How wide is the central maximum (i) in degrees and (ii) in centimetres?

(b) What is the separation of adjacent minima (excluding the pair on either side of the central maximum)?

Solution

(a) $\lambda = 6.70 \times 10^2 \text{ nm} = 6.70 \times 10^{-7} \text{ m}$ $\quad n = 1$

$w = 12 \ \mu\text{m} = 1.2 \times 10^{-5} \text{ m}$ $\quad \theta = ?$

$L = 0.30 \text{ cm}$

(i) On either side of the central line, the width of the central maximum is defined by the first-order dark fringes. Thus,

$$\sin\theta_1 = \frac{\lambda}{w}$$

$$= \frac{6.70 \times 10^{-7} \text{ m}}{1.2 \times 10^{-5} \text{ m}}$$

$$\sin\theta_1 = 5.58 \times 10^{-2}$$

$$\theta_1 = 3.2^\circ$$

The angular width of the central maximum is $2 \times 3.2^\circ = 6.4^\circ$.

Contd.

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Contd.

(ii)

$$\sin\theta_1 = \frac{y_1}{L}$$

$$y_1 = L\sin\theta_1$$

$$= (0.30\ \text{m})\sin 3.2°$$

$$y_1 = 1.67\times10^{-2}\ \text{m, or } 1.7\ \text{cm}$$

The width of the central maximum is $2y_1$, or 2×1.67 cm $= 3.3$ cm.

(b)

$$\lambda = \frac{w\Delta y}{L}$$

$$\Delta y = \frac{L\lambda}{w}$$

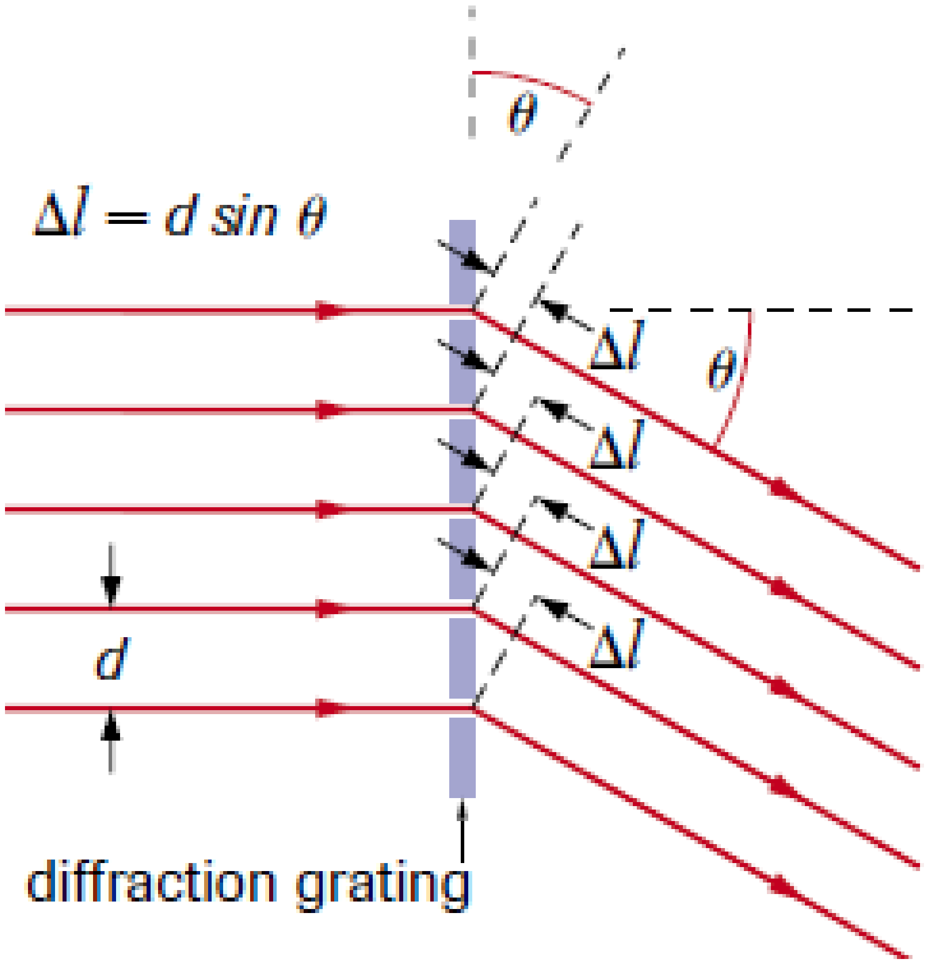
$$= \frac{(0.30\ \text{m})(6.70\times10^{-7}\ \text{m})}{1.2\times10^{-5}\ \text{m}}$$

$$\Delta y = 1.7\times10^{-2}\ \text{m, or } 1.7\ \text{cm}$$

The separation of adjacent minima is 1.7 cm. The central maximum is exactly twice the width of the separation of other adjacent dark fringes.

Diffraction Gratings:

A diffraction grating, a device used for wave analysis, has a large number of equally spaced parallel slits, which act as individual line sources of light. The wave analysis of the pattern produced by these openings resembles the analysis we considered for the double slit. The waves passing through the slits interfere constructively on the viewing screen when $\sin\theta_m = \frac{m\lambda}{d}$, where $m = 0, 1, 2, 3$ ... is the order of the bright line or maximum. This condition for constructive interference is derivable from the path difference $\Delta l = d\sin\theta$ between successive pairs of slits in the grating.



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At what angle will 638-nm light produce a second-order maximum when passing through a grating of 900 lines/cm?

Solution

$\lambda = 638 \text{ nm} = 6.38 \times 10^{-7} \text{ m}$ $\qquad d = \dfrac{1}{900 \text{ lines/cm}} = 1.11 \times 10^{-3} \text{ cm} = 1.11 \times 10^{-5} \text{ m}$

$m = 2$ $\qquad \theta = ?$

$$\sin\theta_m = \frac{m\lambda}{d} \quad \text{(for bright maxima)}$$

$$\sin\theta_2 = \frac{2(6.38 \times 10^{-7} \text{ m})}{1.11 \times 10^{-5} \text{ m}}$$

$$\sin\theta_2 = 1.15 \times 10^{-1}$$

$$\theta_2 = 6.60°$$

The angle to the second maximum is 6.60°.

Diffraction Gratings:

- The surface of a diffraction grating consists of a large number of closely spaced, parallel slits.
- Diffraction gratings deliver brighter interference patterns than typical double-slit setups, with maxima that are narrower and more widely separated.
- Diffraction gratings are governed by the relationship $\sin\theta_m = \frac{m\lambda}{d}$, where d is the distance between adjacent gratings, and m is the order of the maxima.

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