Exploring and Extending the Differential Co-Evolution of Trading Strategies

Armand Mihai Cismaru
MEng Computer Science
University of Bristol
Bristol, United Kingdom
fz19792@bristol.ac.uk

***Abstract*—This paper reports results from experiments using and extending a new adaptive trading strategy called PRDE (Parameterised Response Differential Evolution). PRDE is a newly introduced variant of the PRZI trading strategy which uses Differential Evolution (DE) to pick its elite strategy *s* at the time *t*. However, the performance of PRDE depends on the values of its parameters, *k* and *F*. In this paper, we conduct a thorough analysis of the relationship between the values of k and F and the performance of the trading strategy compared to its proposed form and extending PRDE to use JADE [12], an improved form of DE. The experiments have been conducted using the Bristol Stock-Exchange (BSE) simulation [1], [2] running a Limit Order Book (LOB) to conduct a financial market simulation. The performance impact of various new combinations of *k* and *F* parameters are explored. Relevant discussion and evidence alongside visualisations are provided. The source code of BSE is freely available on GitHub as open-source for potential replication and extension of the results.**

***Index Terms*—Automated Trading, Adaptive Trader-Agents, Financial Market Simulations, Co-evolution, Optimization, Differential Evolution**

I. Introduction

The rise of automated trading has greatly impacted the financial market, enabling speed and efficiency never seen before. The developments in computational capacity and artificial intelligence have led to significant advances in trading strategies and the development of sophisticated adaptive trading agents. These agents are able to make rapid, data-driven decisions in adjusting their strategy to be as profitable as possible. Each adaptive trading agent competes in a market filled with other strategies that are continuously evolving to maintain and maximize their profitability, therefore modern financial markets are predominantly *co-evolutionary systems* [3]. Parameterized Response Zero Intelligence (PRZI) Traders have been introduced by Cliff in 2021 as a new trader agent intended to be used to study the dynamics of simulated continuous double auction markets (CDA) [4]. Parameterised Response Differential Evolution Traders (PRDE) are a form of PRZI agents that use DE to dynamically improve their strategy.

One of the scopes of this paper is to explore and experiment different choices of the *k* and *F* parameters that affect how PRDE behaves in CDA based simulated markets. The desire is to find a combination or combinations of parameters that yield statistically significant better performance. In a CDA, buyers and sellers are able to place bids and offers simultaneously and continuously, with the goal of reaching a mutually agreed-upon price for a particular good or security. This type of auction is commonly used in financial markets, where buyers and sellers can use it to trade stocks, bonds, and other securities in real time. Vernon Smith has pioneered the field of experimental economics and found that human traders reach CDA equilibrium fast even with small amounts of traders. The prices at which the transactions take place are dictated by the bids and offers of the participants and their limit order prices. CDAs can be conducted electronically, where each trader is a robot with a pre-defined strategy intended to maximize profits [5].

The Bristol Stock Exchange (BSE) [1], [2] is used to run the market simulations. BSE is an open-source high-fidelity simulation that runs a Limit Order Book (LOB) to conduct the CDA. It allows the user to populate the market with a range of trader agent strategies like 'ZIP', 'ZIC', 'PRZI', 'PRSH' and 'PRDE'. The traders interact by placing bids and offers on the LOB. Their behaviour can be linear or adaptive given the current conditions of the market. BSE supports both static and dynamic supply and demand schedules, as well as partially random order price generation.

Adaptive trading agent are a new addition to the simulation, 2021 seeing the novel 'PRZI' and 'PRSH' being added. PRZI behaviour is governed by its parameter $s \in [-1, 1] \subseteq \mathbb{R}$ [6] that determines its probability mass function (PMF). The value s can be thought of as the *'urgency'* of the trade, with an $s = -1$ basically doing the trade at a dumping price with no profit and $s = 1$ behaving like a sniper algorithm, fully maximizing its utility. When approaching $s = 0$, the PMF of PRZI behaves just like the ZIC algorithm [7].

PRZI with Differential Evolution (PRDE) does not rely on a singular choice of *s*, but instead learns and adapts its strategy based on its own evaluations in the market. To achieve this, it uses Differential Evolution (DE) to pick from a metapopulation of *k* values of *s* and evaluate its profit.

Section II talks about the background of this work by expanding the current research on PRDE and how DE works. Section III explores the motivation for the choice of experimental design, market conditions and evaluation methods. The full technical aspects of the setup will be discussed in Section IV while the results of the experiments are presented in Section V. The extension of PRDE with JADE [12], an improved form of DE, is presented in Section VI and final conclusions and potential future work are included in section VII.

II. Background

A. About Differential Evolution

Differential evolution [8] is a form of metaheuristic evolutionary computation to aims to iteratively optimize a candidate solution by evaluating its fitness to a specific problem. The method is a 'genetic' algorithm as it is using a combination of *crossover*, *mutation* and *selection*, inspired by the evolution of organisms found in nature to evolve and generate better populations. Its *NP* parameter, the population size, *k* as we are referring to it, represents the size of the population. The *CR* parameter is not used as our solution vector is one-dimensional. DE can optimize candidate solutions, its 'genome', in the form of vectors of integers or strings, in our case a 1-D vector for the parameter $s \in [-1, 1] \subseteq \mathbb{R}$. The other parameter of DE that concerns us is *F*, a factor that determines the strength of the *mutation* operation, namely the magnitude of mixing two candidate solutions with a third vector.

B. PRZI with differential evolution

The creation of PRDE arose from the context of better adaptive trading strategies. It will be introduced at the IEEE Symposium Series on Computational Intelligence, in Singapore in December 2022 as a successor version of PRZI and PRSH [3]. It distinguishes from the aforementioned by using a simple, but efficient version of DE, proven to be 100% more profitable than PRSH. As detailed by Cliff, PRDE keeps a population of size $k = 4$ of strategy-values to choose from. As it only needs the value of *s*, its 'genome' relies only on the *base vector*. After the method evaluates one strategy $s_{i,x}$, another one is chosen at random from the population $s_{i,a}, s_{i,b}, s_{i,c}$. A new candidate solution $s_{i,y}$ is created, and if it is better than $s_{i,x}$ then it will replace it. The parameter $F = 0.8$ is used when creating the new candidate solution, being the trader's differential weight distribution. This paper extends on the proposed further explorations of PRDE by experimenting new values of *k* and *F*.

III. Experiment Design Motivation

The design of the experiments were chosen with fairness and balance in mind. Drawing inspiration from the works of Tesauro and Das [9] a few options have been considered: homogeneous population, balanced-group and one-to-many tests. All the tests provide valuable insight and metrics to draw conclusions from, but given the time-frame available for conducting the research, balanced-group tests have been chosen.

The choice of balanced-group tests provide benefits to our research, namely:

- It is a stochastic-controlled trial method that helps reduce bias sources and improve the internal validity of the study. We want to make sure that the differences observed come from differences between trading strategies, not noise.

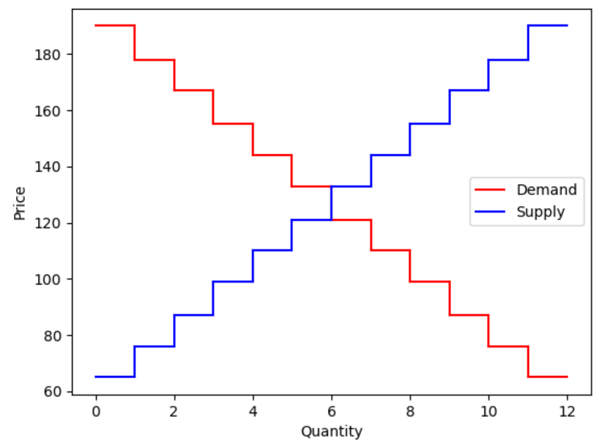

Fig. 1. Supply and demand market curves used outside the shock. Inside the shock, the curves take the same form as in the figure. Each step is of a fixed size, and the equilibrium price appears to be a inside a interval $\in [1.2, 1.32]$.

- The tests allows full control of the experiment conditions. The time frame of the simulation, the supply and demand schedules and order interval can all be controlled to isolate the differences between the chosen strategies.
- This type of design allows for easy and robust replication of experiment runs in order to validate the reliability and significance of results.

This type of test equally balances both trading strategies A and B compared, giving them fair chances to trade and make profit in the BSE simulated market.

BSE simulates the financial market at a sub-second rate, thus any reliable experiments on sufficiently large time-frames and populations of traders generate thousands of trades and need large amounts of real execution time. The time Δ_E required for the DE to evaluate one strategy *s* is $\Delta_E = 7200s$, meaning that for the reference $k = 4$ it would take 8 simulated hours to evaluate all *s*. The simulations used to validate different PRDE specifications have been run for 7 continuous simulated market days on a population number of traders $N_T = 24$, equally split between the sellers and buyers and complementary amounts of traders with the new (k, F) pair and the reference values from [3], $(k, F) = (4, 0.8)$.

Dynamic market schedules were chosen in order to increase the realism of the market simulations. It is common practice to use stress scenarios in the financial industry when simulating markets, with models such as Monte Carlo [10] or the 2008 Financial Crisis being well known. BSE enables simpler schedules, but with significant impact. The markets we are simulating run with equal supply and demand schedules but have introduced 2 shocks at 1/3 and 2/3 of the time $t = 7$ (days). The orders are given to traders by a fixed time step. For up to $t = 1/3$ the traders are selling and buying with no less than 0.65\$ and no more than 1.9\$ and reach market equilibrium at [\$1.2, \$1.32] per shown in Figure 1. A shock is then introduced and agents trade with prices between \$2 and

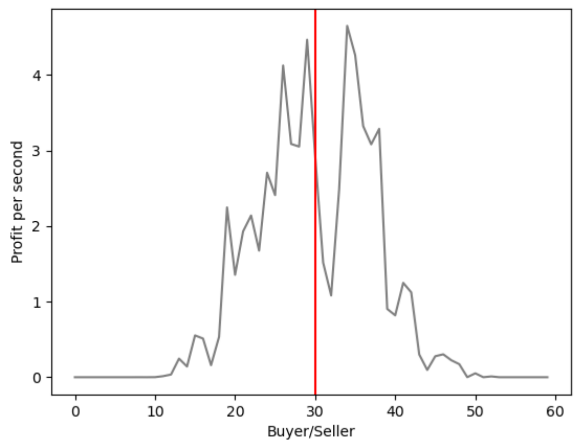

Fig. 2. Plot of profit per second for the last state of a 7 days simulation a $N_T = 60$ trader population. The simulation was run using the reference parameters. The left side corresponds to the buyers last PPS recorded, analogous with the sellers on the right of the line.

$2.7 with an equilibrium price of [$2.3, $2.38]. The second shock at $t = 2/3$ brings the market schedules back to the initial values.

The performance of different pairs of traders were measured by their Profit Per Second (PPS). This unit of measure is the ratio between the profit obtained in a specific time frame and its duration (in seconds). BSE output files provide an update of the PPS every 3600s seconds.

When each counterparty agrees to a transaction, their supply of the abstract commodity that is being traded on the BSE is depleted. BSE only allows a quantity of 1 to be traded in each transaction. After a random interval (around 5s) each trader is individually resupplied with a new order. Due to the equal market supply and demand, there are traders not able to find profitable trades given their quote prices. This was taken into consideration when assigning the different types of traders to their role as buyer or seller in our experiments. The preliminary experiments that were run on $N_T = 30$ homogeneous populations have shown that the most profitable traders of tend to cluster in the 'middle', between traders $B_i, i \in [15, 29]$ and $S_j, j \in [0, 15]$. Thus, each PRDE with both the new and reference configuration get an equal amount of profitable/non-profitable traders.

As the experiment methodology chosen is to compare PRDE agents with the reference configuration against a new one, much thought has been given to how to find a combination of *k* and *F* values that might yield better performance. A set of preliminary experiment market sessions have been run to gain sense how a change in the parameters might affect the trader populations. First intuition is that extreme values of F ($F \in [0, 2]$) do not have a large impact, but they actually result in small changes of performance. Given execution time constraints, the choice was made to search through all the values of F with a $\Delta = 0.1$ between each trial.

The DE form that PRDE uses evaluates each strategy of the *k* population in 2 hours of simulated market time, meaning that 8 hours are required to evaluate the whole set and pick the elite strategy. *k* values that are too high will impact the DE's capacity to efficiently iterate through its set of strategies. This constraint tights up the choice of *k* and the time-frame chosen for the entire simulation. The trials in this paper use $k = \{4, 5, , 6, 7\}$.

The experiments were run as a kind of Grid Search. One balanced-set test was run for each $F \in [0, 2]$ and $k \in 4, 5, 6, 7$ combination, resulting in a number of 84 total simulations. The overall profitability of a specific PRDE configuration was computed by summing the PPS across all time-frames. The ratio between the new and the reference PRDE configurations was computed and used to generate a percentage difference between their profits, acting as a comparison metric across all experiments.

IV. BSE Experimental setup

The setup was organised in such a way that allows replication and easy access to essential parameters and configurations. The setup allows easy tracking of results and multiple trials for each experiment, useful for dividing the simulations in batches across multiple machines.

The trials use 12 agents for each type of PRDE trader in the market. The market runs for 7 simulation days, totalling 601200 seconds and 168 entries of 3600s time-frames. The supply/demand schedules take a single tuple of identical ranges of order prices, with (65, 190) for the *'normal'* market and (200, 270) for the market *shocks* as well as fixed order generation schedules. The traders receive a continuous drip-feed resupply with quotes at a random interval of ≈ 5 seconds, generated from a uniform continuous distribution.

Each trial ID contains the *k* and *F* values, as well as the number of days and trial count. The output is set to be verbose, but *dump_all* is set to 'false' to keep the generated output to a necessary minimum. The code can be run with useful command line flags like $--F{-}value$, $--k{-}value$, $--n{-}days$ for running the simulations in batches. The design is intended to enable switching the trader agents configurations for other types of experiments like one-to-many and homogeneous by changing the $--experiment-type$ flag. The traders specs for our balanced-group tests are obtained by interpolating new vs reference agents one by one for both buyers and sellers. Results are aggregated using separate scripts and written to a separate *.csv* file for further analysis.

The trials were run using a 2020 Apple MacBook Pro with a 1.4 GHz Quad-Core Intel Core i5 silicon and AWS m5.large EC2 instances with 2vCPUs.

V. Results

A. Holistic approach of the parameter exploration

The first part of this section contains the discussion over the results of our GridSearch like approach to the optimal parameter lookup. The average PPS for each configuration in the trials was used as a quantifying metric for its result.

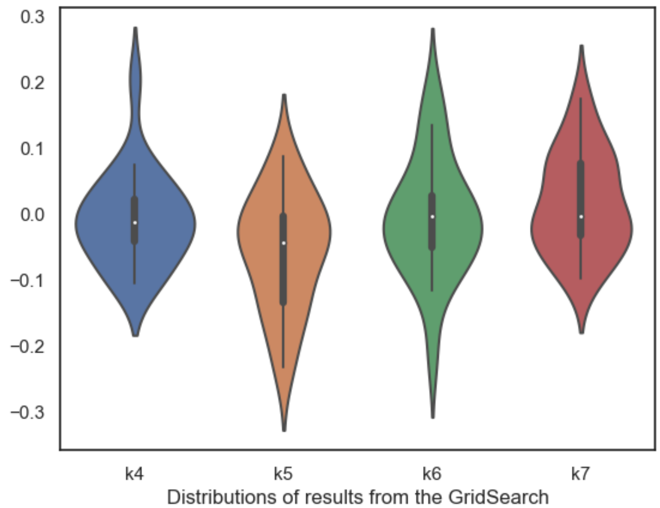

Fig. 3. Distributions of the performance difference resulted from our GridSearch style trials. Each individual distribution corresponds to 21 trials with a specific k value and all the $F \in [0, 2]$ values.

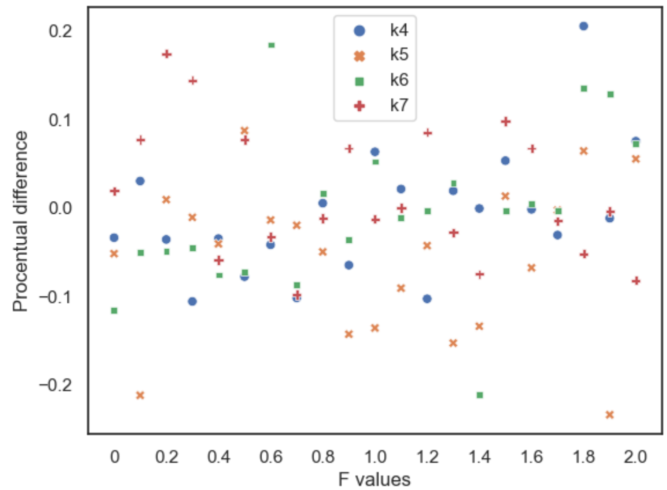

Fig. 5. Plot of trial results. Each point on the x-axis represents a trial for a specific value of F and each point in the plot on the y-axis is the percentage difference between agents. Each color corresponds to one of the 4 k values.

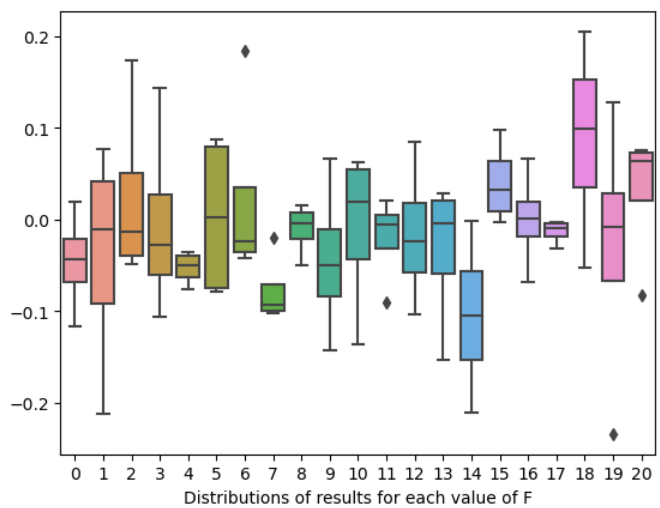

Fig. 4. Distributions of the performance difference resulted from our GridSearch style trials. Each individual distribution corresponds to 4 trials for each $F \in [0, 2]$ value.

Each simulation resulted in a pair of dependent variables, namely the *'new'* and *'reference'* total PPS. As detailed above, their percentage difference was used as a single metric used to differentiate between all the trials. Given time constraints, every simulation was run once and the ones with a positive improvement of more than 10% were chosen for an additional 6 trials each.

Figure 3 shows how the results are distributed across the 4 tested values of k. Shapiro-Wilk [11] tests were run to test the null hypothesis that the data is drawn from a normal distribution for each k, which was found to be true. The k4 distribution shows clustering of small percentage differences between the new and reference trader agents, with a long but thin tail in the direction of significant improvement. An interesting observation is how the k7 distribution has a short low-value tail, compared to k5 and k6.

Looking at individual F value distributions in Figure 4, it looks that the biggest improvements lay on the outer values of the interval. By combining this with the information provided by Figure 5 we can see that the bigger improvements on lower values of F happen on trials with bigger values of k, mostly k7. This might be explained by the fact that a larger value of k requires more time for the strategies to be evaluated by the DE while the low value of F speeds up the convergence of the optimization, while improving stability and robustness. On the opposite side, the bigger improvements for higher values of F happen in conjunction with smaller ks, most notably k4. This correlation can be caused by the fact that bigger choices of F prevent premature convergence of the optimization, thus balancing the choice of a smaller set of individual s candidate strategies. Replication of these experiments with an increased number of trials might help mitigate market randomness and noise.

B. *Individual strategy comparison and validation*

This part looks at the evaluation of the best performing new strategies. As we saw in Figures 5 and 4, the best performing strategies ($>$ 10% profit increase) are around F values of $0.2, 0.3$ with k7 and Fs at 1.8 with k4. To test for statistical significance of the results between a new PRDE strategy and the reference one, the following methodology was employed:

- The total profits after 10 additional BSE simulations per each type of configuration were organised as two sets of data.
- Kolmogorov-Smirnov tests are applied on both to determine the goodness of fit.
- If the datasets are informative, then a paired t-test is performed. This statistical test is used to compare the means of two related or paired samples, in our case,

the total PPS for the two trading strategies. The values in each pair are related, as they come from the same experiment. The null hypothesis for this test is that there is no difference between the means of the two samples.
- Based on the p-value of the test, the null hypothesis is rejected or accepted. A p-value < 0.5 rejects the null-hypothesis and indicates that there is a statistically significant difference between the means of the two samples.

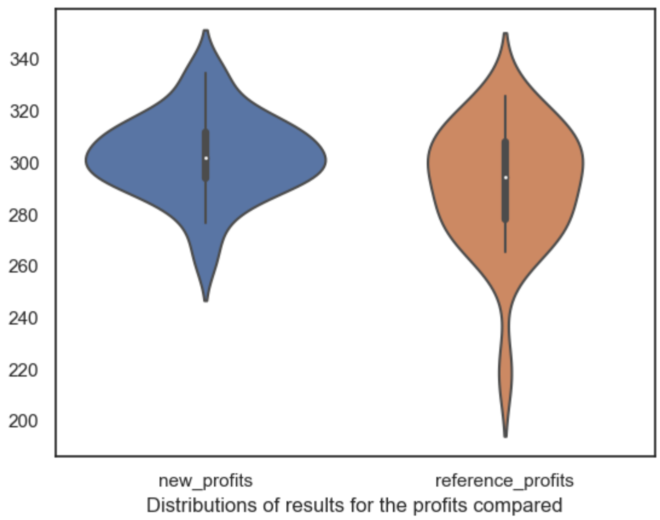

Fig. 6. Plot of the result distributions for $(k, F) = (4, 1.8)$ after the 25 trials. Each distribution corresponds to the profits obtained by the competing traders.

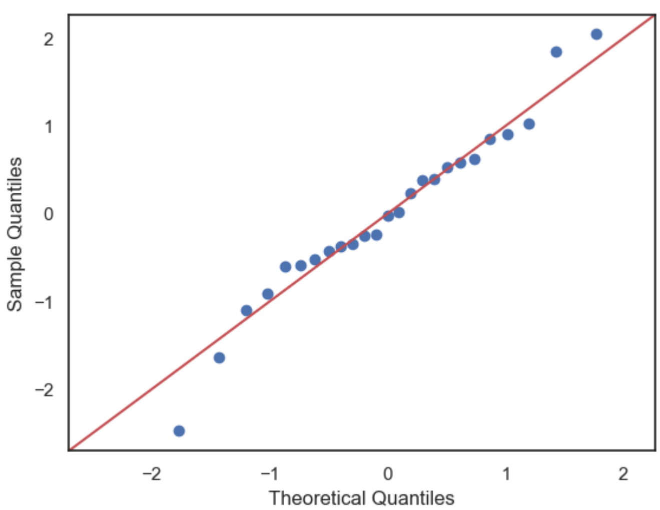

Fig. 7. QQ plot of the profits obtained in the 25 trials for the new configuration $(k, F) = (4, 1.8)$, compared with the normal cumulative distribution.

This methodology has been applied for each individual parameter pair $(k, F) \in \{(4, 1.8), (4, 1.9), (6, 0.6), (7, 0.2), (7, 0.3)\}$. Both k7 pairs and the k6 pair were found to have inconsequential goodness of fit on the average PPS observations. The pair (7, 0.2) had goodness of fit only on the reference profits, while (6, 0.6) had informative distribution only on the new configuration profits. (7,0.3) did not have a $p > 0.05$ for both the new and the reference traders. Visually inspecting the dataset distributions supported this result. The subsequent paired t-tests found that the trials did not produce any statistical significant results.

The Kolmogorov-Smirnov tests have instead found that both k4 pairs present have goodness of fit. Their visuals seemed to support this result. Given the promising preliminary results, to reduce bias and increase accuracy, 15 additional balanced-group simulations were run for both configurations. The combined previous and new trial data accounts for 25 observations for each pair. Before proceeding, the Kolmogorov-Smirnov tests have re-confirmed the goodness of fit, this time for the extended results. The paired t-tests have produced a $pval = 0.024$ for $(k, F) = (4, 1.8)$ and $pval = 0.139$ for $(k, F) = (4, 1.9)$. The null hypothesis that there is no difference between the means of the two samples was rejected for the first result and accepted for the second one. The results produced by the pair $(k, F) = (4, 1.8)$ are statistically significant from the reference parameters $(k, F) = (4, 0.8)$ with an average increase of $\approx 4\%$.

VI. Extending PRDE

A. Detailing JADE

This section is detailing the work that has been done to extend PRDE, using an improved form of DE, known as JADE [12]. Introduced by Zhang and Sanderson in 2009, it updates its control parameters in an adaptive way, reducing the need for manual trial-and-error parameter optimization. It uses a novel mutation strategy *"DE/current-to-pbest"*, with an optional archive that stores evolution progress. With JADE, the F parameter of PRDE evolves in an adaptive manner throughout the simulation. The implementation described in this paper includes the optional archive, which is claimed to have proved increased performance by the authors of the original paper. JADE introduces two new parameters p and c, whose role will be discussed in the following paragraphs.

Instead of picking 3 random strategies from the current population, like basic DE, JADE uses a more complex method to construct the mutation vector, as shown in (1):

$$s_{new} = s_i + F_i \cdot (s^p_{best} - s_i) + F_i \cdot (s_{r1} - \tilde{s}_{r2}) \qquad (1)$$

x^p_{best} is a random chosen strategy from the set of the top $100p\%$, $p \in (0, 1]$ best performing strategies, i.e. if $p = 0.10$ this would mean that the choice would be made from the best 10% s_j, $j = \{1, 2, .., k\}$. The other two strategies, s_{r1} and $\tilde{s}_{r2}$ are picked randomly from the current population, and the union of the archive of less performing strategies and current population, respectively.

Instead of having a fixed F, JADE uses a new F_i for each individual s_i strategy, which was initialised with 0.8 for the initial population. The factor is generated from a Cauchy distribution, using the location parameter μ_F. The parameter is initialised as $\mu = 0.5$ and gets updated after the whole population is evaluated. The other new parameter, c, controls the rate of parameter adaptation. The Lehmer mean, $mean_L$, is used to give the successful factors, S_F a higher weighting on the new F_i. Thus, the location parameter is updated as follows:

$$\mu_F = (1 - c) \cdot \mu_F + c \cdot mean_L(S_F) \tag{2}$$

The optional archive that is proposed with JADE stores the unsuccessful strategy values throughout the market simulation and, after one population is evaluated, it randomly removes elements so that its size does not exceed k. It helps store information about the strategy progress and diversify the population.

B. Implementing and experimenting JADE

JADE was implemented as a new trader agent in BSE, *'PRJADE'*, falling in the same category of *'PRZI'* traders. The design of experiments conducted with JADE follow the same pattern as what has been detailed in the parameter exploration in this paper, as follows:

- The traders are split into two balanced groups in the same simulation with 12 traders each. This choice was deemed to be the fairest way of comparing the traders, giving all equal chances in the same market. $k = 4$ throughout the whole experiment, and $F = 0.8$ for the basic PRDE. The F initialisation is the same for PRJADE, but only as starting point for the first population.
- 27 market simulations are run for 7 simulated days each. For simplicity, we used a perfectly elastic market demand, with sellers not trading below \$60 and buyers having a limit price of \$140. The orders are resupplied with a random ($\approx 5s$) time interval.
- The average PPS for each trader is used as a quantifying meter. The results are then checked for normality using the Shapiro-Will [11] test, visualised and tested whether they are significant or not using paired t-tests. The results are dependent, thus the choice of paired t-tests.

The normality tests for the profits of both traders have indicated that we can accept the null-hypothesis that the data comes from an approximately normal distribution, with p-values of 0.339 and 0.068, greater than the 0.05 confidence level. As seen in Figure 8, the violin plots depicting their distributions indicate approximately the same result. Figure 9 is an example of one of the most interesting trials, with a clear improvement in profits for JADE.

The null hypothesis that there is no significant difference between the two means of the profits was rejected by the paired t-test with a $p-value = 0.0336$. This result indicates that the performance increase of $\approx 3.7\%$ is statistically significant. We can therefore conclude that using JADE is slightly *more profitable* than the existing implementation of PRDE.

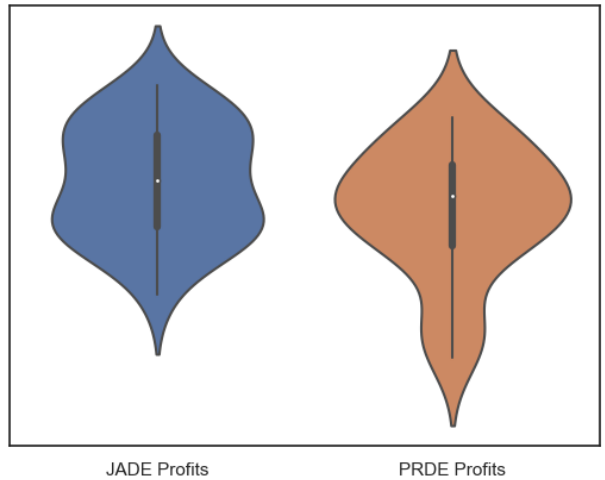

Fig. 8. Violin plot of the distributions of profits for PRJADE and PRDE after 27 trials.

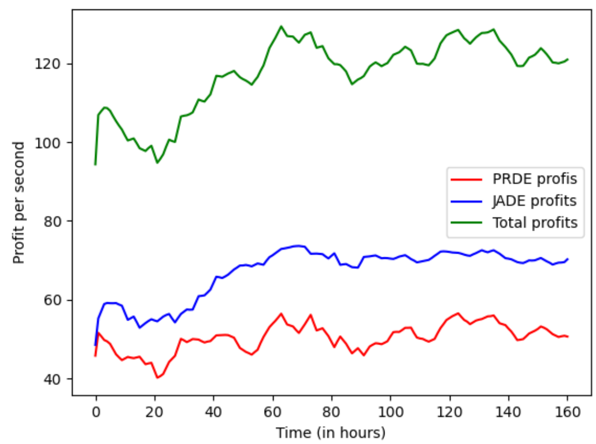

Fig. 9. Plot of profitability data for JADE and PRDE traders for one market simulation. Horizontal axis is time, measured in hours, vertical axis is simple moving average of PPS over the preceding 8 hours. The lines show total PPS per unit of time for the new and reference configurations. The green line is the sum of their profits.

VII. Conclusions

The experiments conducted in this paper extend novel work on co-adaptive trading agents introduced by Cliff in *"Metapopulation differential co-evolution of trading strategies in a model financial market"* in 2022. PRDE uses the Differential Evolution to optimize its strategy throughout the duration of the market while competing with other traders with identical, or as we experimented, different specifications of DE. The trader agent populations used in BSE can be thought of as *metapopulations*, as suggested in [3], populations of traders with independent subpopulations, namely each trader's k set of strategies used in the optimization process.

The conditions used in the simulations aim to replicate a real financial-market in which co-adaptive strategies compete against each other. In our work, different forms of PRDE hyperparameter optimizations compete against the simplest form of DE, *DE/rand/1* with the conventional choice of $k = 4$ and $F = 4$. The market curves chosen aim to simulate real life stress scenarios, with 2 shocks bringing the market prices sharply up and then down. This has proved to prevent any new strategy from having categorical constant improvements over the other. Our experiments have demonstrated a statistically significant improvement for the higher value of F on the default k4, but with a modest $\approx 4\%$. As seen in Figures 6 and 7, the *'winner'* parameter combination's profits come from a normal distribution. The QQ plot compares the results with the normal cumulative distribution, with points clustering along the red line.

The shocks in the market force the traders to change and adapt their strategies quickly, so any upper hand that one strategy might have developed until the shock might be completely gone once the market orders change dramatically. It is true that the 7 day-long market simulation might have not been sufficient for the agents to fully explore their k-strategies and adapt after a shock. An interesting line of further research would be to run longer sessions and take a modular approach and evaluate the performances between the shocks vs the holistic approach.

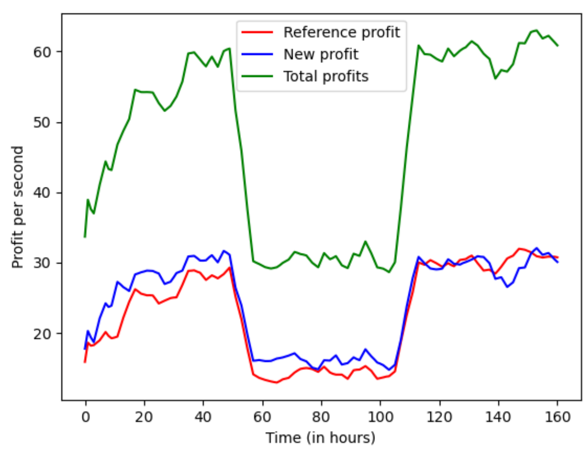

Fig. 10. Plot of profitability data for both traders for one market simulation over 7 days on the 'winner' $(k, F) = (4, 1.8)$ and reference $(k, F) = (4, 0.8)$ configuration. Horizontal axis is time, measured in hours, vertical axis is simple moving average of PPS over the preceding 8 hours. The lines show total PPS per unit of time for the new and reference configurations. The green line is the sum of their profits.

The plots in Figure 10 support this claim. The new strategy is clearly outpacing the reference, but the shock impacts both. Profits plunge as it takes time for the DE to re-evaluate k strategies so as to come with an elite set, adjusted for new market conditions. Following the last shock, the agents seem to go back to the initial profit levels with the new strategy struggling to get past the reference.

The findings in this paper have shown interesting results in exploring the parameters k and F. Extending PRDE to use JADE has proved that it is more profitable than the simple DE presented. This field of research is continuously and rapidly improving, and subsequent work will bring us closer to the real life financial markets where significant numbers of different types of agents evolve together while competing against each other for profits. Further research may look into different optimization techniques, as well as more fine-tuning of the aforementioned parameters on longer market simulations and different conditions.

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