TEscuela Politecnica Nacional

Métodos Numericos-Tarea 7

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Tabla de Contenidos

Conjunto de ejercicios

1) Dados los puntos (0,1), (1,5), (2,3), determine el spline cúbico.

Para resolver este ejercicio, primero terminè de completar el codigo de la pregunta 3.

```
import sympy as sym
from IPython.display import display
def cubic_spline(xs: list[float], ys: list[float]) -> list[sym.Symbol]:
    points = sorted(zip(xs, ys), key=lambda x: x[0])
    xs = [x for x, _ in points]
    ys = [y for _, y in points]

    n = len(points) - 1
    h = [xs[i + 1] - xs[i] for i in range(n)]
```

```python
    # alpha = # completar
    alpha = [0] * (n + 1)
    for i in range(1, n):
        alpha[i] = 3 / h[i] * (ys[i + 1] - ys[i]) - 3 / h[i - 1] * (ys[i] - ys[i - 1])

    l = [1] * (n + 1)
    u = [0] * (n + 1)
    z = [0] * (n + 1)

    for i in range(1, n):
        l[i] = 2 * (xs[i + 1] - xs[i - 1]) - h[i - 1] * u[i - 1]
        u[i] = h[i] / l[i]
        z[i] = (alpha[i] - h[i - 1] * z[i - 1]) / l[i]

    l[n] = 1
    z[n] = 0
    c = [0] * (n + 1)

    x = sym.Symbol("x")
    splines = []

    for j in range(n - 1, -1, -1):
        c[j] = z[j] - u[j] * c[j + 1]
        b = (ys[j + 1] - ys[j]) / h[j] - h[j] * (c[j + 1] + 2 * c[j]) / 3
        d = (c[j + 1] - c[j]) / (3 * h[j])
        a = ys[j]
        S = a + b * (x - xs[j]) + c[j] * (x - xs[j])**2 + d * (x - xs[j])**3
        splines.append(S)

    splines.reverse()
    return splines
```

```python
# Para los puntos: (0,1), (1,5), (2,3)
xs = [0,1]
ys = [1,5]
splines = cubic_spline(xs, ys)
for i, spline in enumerate(splines):
    print(f"Spline {i}:")
    display(spline)
```

```
Spline 0:
```

$4.0x + 1$

2) Dados los puntos (−1,1), (1,3), determine el spline cúbico sabiendo que $f'(x_0) = 1$, $f'(x_n) = 2$.

```
def cubicspline(xs: list[float], ys: list[float], f_prime_x0: float, f_prime_xn: float) -> li
    points = sorted(zip(xs, ys), key=lambda x: x[0])
    xs = [x for x, _ in points]  # Modificación aquí
    ys = [y for _, y in points]

    n = len(points) - 1
    h = [xs[i + 1] - xs[i] for i in range(n)]

    alpha = [0] * (n + 1)
    for i in range(1, n):
        alpha[i] = 3 / h[i] * (ys[i + 1] - ys[i]) - 3 / h[i - 1] * (ys[i] - ys[i - 1])

    l = [1] * (n + 1)
    u = [0] * (n + 1)
    z = [0] * (n + 1)

    for i in range(1, n):
        l[i] = 2 * (xs[i + 1] - xs[i - 1]) - h[i - 1] * u[i - 1]
        u[i] = h[i] / l[i]
        z[i] = (alpha[i] - h[i - 1] * z[i - 1]) / l[i]

    l[n] = 1
    z[n] = 0
    c = [0] * (n + 1)

    x = sym.Symbol("x")
    splines = []

    for j in range(n - 1, -1, -1):
        c[j] = z[j] - u[j] * c[j + 1]

    b_0 = f_prime_x0
    b_n = f_prime_xn
    d_0 = (c[1] - c[0]) / h[0] / 3
    d_n = (-c[n]) / h[n-1] / 3
```

```
    for j in range(n):
        a = ys[j]
        b = (ys[j+1] - ys[j]) / h[j] - h[j] * (c[j+1] + 2*c[j]) / 3
        d = (c[j+1] - c[j]) / (3 * h[j])
        S = a + b * (x - xs[j]) + c[j] * (x - xs[j])**2 + d * (x - xs[j])**3
        splines.append(S)

    return splines

xs = [-1, 1]
ys = [1, 3]

f_prime_x0 = 1
f_prime_xn = 2

spline = cubicspline(xs, ys, f_prime_x0, f_prime_xn)
for S in spline:
    display(S)
```

$1.0x + 2.0$

3) Diríjase al pseudocódigo del spline cúbico con frontera natural provisto en clase, en base a ese pseudocódigo complete la siguiente función:

```
def cubic_spline(xs: list[float], ys: list[float]) -> list[sym.Symbol]:
    """
    Cubic spline interpolation ``S``. Every two points are interpolated by a cubic polynomial
    ``S_j`` of the form ``S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3.``

    xs must be different  but not necessarily ordered nor equally spaced.

    ## Parameters
    - xs, ys: points to be interpolated

    ## Return
    - List of symbolic expressions for the cubic spline interpolation.
    """

    points = sorted(zip(xs, ys), key=lambda x: x[0])  # sort points by x

    xs = [x for x, _ in points]
    ys = [y for _, y in points]

    n = len(points) - 1  # number of splines

    h = [xs[i + 1] - xs[i] for i in range(n)]  # distances between  contiguous xs

    # alpha = # completar
    for i in range(1, n):
        alpha[i] = 3 / h[i] * (ys[i + 1] - ys[i]) - 3 / h[i - 1] * (ys[i] - ys[i - 1])

    l = [1]
```

https://github.com/ztjona/EPN-numerical-analysis/blob/main/cubic_splines.ipynb

```
import sympy as sym
from IPython.display import display

def cubic_spline(xs: list[float], ys: list[float]) -> list[sym.Symbol]:
    """
    Cubic spline interpolation ``S``. Every two points are interpolated by a cubic polynomia
    ``S_j`` of the form ``S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3.``

    xs must be different but not necessarily ordered nor equally spaced.

    ## Parameters
    - xs, ys: points to be interpolated

    ## Return
    - List of symbolic expressions for the cubic spline interpolation.
```

```
    """

    points = sorted(zip(xs, ys), key=lambda x: x[0])  # sort points by x
    xs = [x for x, _ in points]
    ys = [y for _, y in points]

    n = len(points) - 1  # number of splines
    h = [xs[i + 1] - xs[i] for i in range(n)]  # distances between contiguous xs

    # alpha = # completar
    alpha = [0] * (n + 1)
    for i in range(1, n):
        alpha[i] = 3 / h[i] * (ys[i + 1] - ys[i]) - 3 / h[i - 1] * (ys[i] - ys[i - 1])

    l = [1] * (n + 1)
    u = [0] * (n + 1)
    z = [0] * (n + 1)

    for i in range(1, n):
        l[i] = 2 * (xs[i + 1] - xs[i - 1]) - h[i - 1] * u[i - 1]
        u[i] = h[i] / l[i]
        z[i] = (alpha[i] - h[i - 1] * z[i - 1]) / l[i]

    l[n] = 1
    z[n] = 0
    c = [0] * (n + 1)

    x = sym.Symbol("x")
    splines = []

    for j in range(n - 1, -1, -1):
        c[j] = z[j] - u[j] * c[j + 1]
        b = (ys[j + 1] - ys[j]) / h[j] - h[j] * (c[j + 1] + 2 * c[j]) / 3
        d = (c[j + 1] - c[j]) / (3 * h[j])
        a = ys[j]
        S = a + b * (x - xs[j]) + c[j] * (x - xs[j])**2 + d * (x - xs[j])**3
        splines.append(S)

    splines.reverse()
    return splines
```

4) Usando la función anterior, encuentre el spline cúbico para:

xs = [1, 2, 3]
ys = [2, 3, 5]

```
# Para los puntos:
# xs = [1, 2, 3]
# ys = [2, 3, 5]
xs = [1,2,3]
ys = [2,3,5]
splines = cubic_spline(xs, ys)
for i, spline in enumerate(splines):
    print(f"Spline {i}:")
    display(spline)
```

```
Spline 0:
Spline 1:
```

$$0.75x + 0.25\left(x-1\right)^3 + 1.25$$

$$1.5x - 0.25\left(x-2\right)^3 + 0.75\left(x-2\right)^2$$

5) Usando la función anterior, encuentre el spline cúbico para:

xs = [0, 1, 2, 3]
ys = [-1 ,1, 5, 2]

```
# Para los puntos:
# xs = [0,1,2,3]
# ys = [-1,1,5,2]
xs = [0,1,2,3]
ys = [-1,1,5,2]
splines = cubic_spline(xs, ys)
for i, spline in enumerate(splines):
    print(f"Spline {i}:")
    display(spline)
```

```
Spline 0:
Spline 1:
Spline 2:
```

$1.0x^3 + 1.0x - 1$

$4.0x - 3.0\,(x-1)^3 + 3.0\,(x-1)^2 - 3.0$

$1.0x + 2.0\,(x-2)^3 - 6.0\,(x-2)^2 + 3.0$

6) Use la función cubic_spline_clamped, provista en el enlace de Github, para graficar los datos de la siguiente tabla

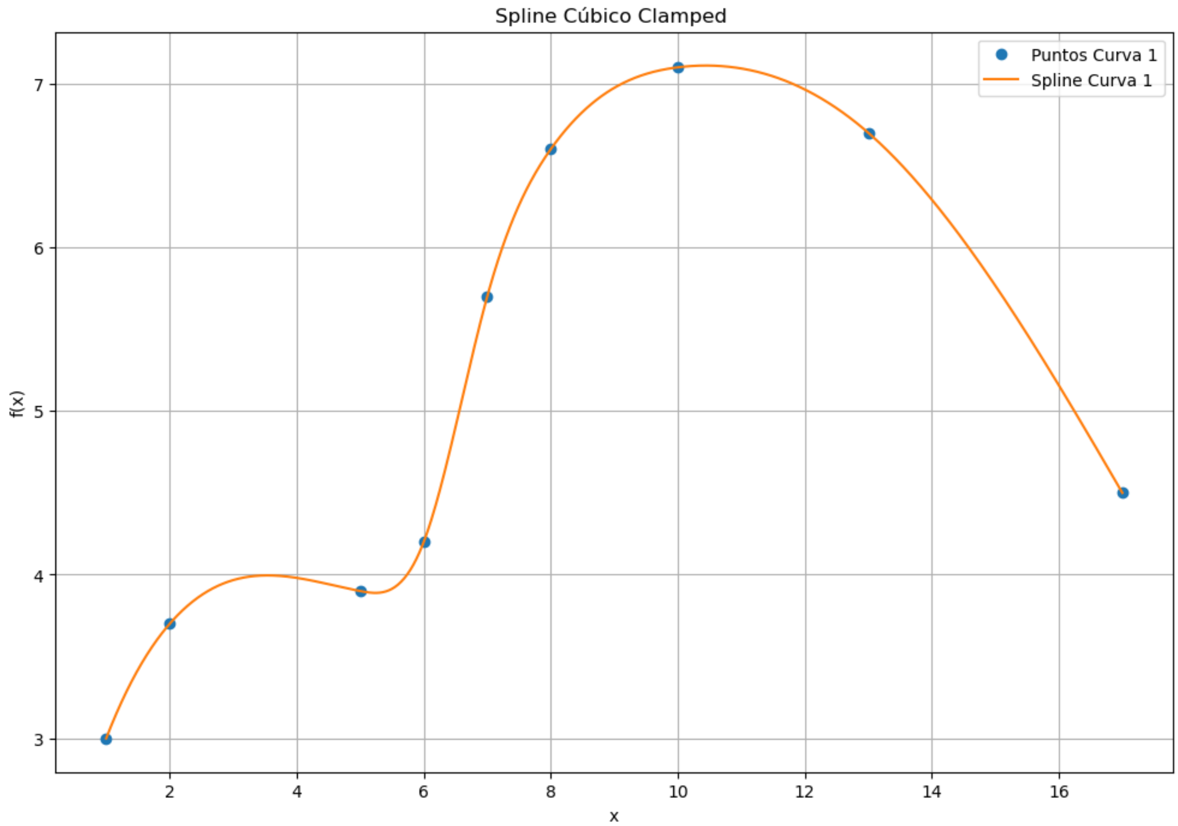
Curva 1				Curva 2				Curva 3			
i	x_i	$f(x_i)$	$f'(x_i)$	i	x_i	$f(x_i)$	$f'(x_i)$	i	x_i	$f(x_i)$	$f'(x_i)$
0	1	3.0	1.0	0	17	4.5	3.0	0	27.7	4.1	0.33
1	2	3.7		1	20	7.0		1	28	4.3	
2	5	3.9		2	23	6.1		2	29	4.1	
3	6	4.2		3	24	5.6		3	30	3.0	−1.5
4	7	5.7		4	25	5.8					
5	8	6.6		5	27	5.2					
6	10	7.1		6	27.7	4.1	−4.0				
7	13	6.7									
8	17	4.5	−0.67								

```python
import sympy as sym
import numpy as np
import matplotlib.pyplot as plt

# ####################################################################
def cubic_spline_clamped(
    xs: list[float], ys: list[float], d0: float, dn: float
) -> list[sym.Symbol]:

    points = sorted(zip(xs, ys), key=lambda x: x[0])  # sort points by x
    xs = [x for x, _ in points]
    ys = [y for _, y in points]
    n = len(points) - 1  # number of splines
    h = [xs[i + 1] - xs[i] for i in range(n)]  # distances between  contiguous xs

    alpha = [0] * (n + 1)  # prealloc
    alpha[0] = 3 / h[0] * (ys[1] - ys[0]) - 3 * d0
    alpha[-1] = 3 * dn - 3 / h[n - 1] * (ys[n] - ys[n - 1])

    for i in range(1, n):
```

```
        alpha[i] = 3 / h[i] * (ys[i + 1] - ys[i]) - 3 / h[i - 1] * (ys[i] - ys[i - 1])

    l = [2 * h[0]]
    u = [0.5]
    z = [alpha[0] / l[0]]

    for i in range(1, n):
        l += [2 * (xs[i + 1] - xs[i - 1]) - h[i - 1] * u[i - 1]]
        u += [h[i] / l[i]]
        z += [(alpha[i] - h[i - 1] * z[i - 1]) / l[i]]

    l.append(h[n - 1] * (2 - u[n - 1]))
    z.append((alpha[n] - h[n - 1] * z[n - 1]) / l[n])
    c = [0] * (n + 1)  # prealloc
    c[-1] = z[-1]

    x = sym.Symbol("x")
    splines = []
    for j in range(n - 1, -1, -1):
        c[j] = z[j] - u[j] * c[j + 1]
        b = (ys[j + 1] - ys[j]) / h[j] - h[j] * (c[j + 1] + 2 * c[j]) / 3
        d = (c[j + 1] - c[j]) / (3 * h[j])
        a = ys[j]
        print(j, a, b, c[j], d)
        S = a + b * (x - xs[j]) + c[j] * (x - xs[j]) ** 2 + d * (x - xs[j]) ** 3

        splines.append(S)
    splines.reverse()
    return splines
def evaluate_spline(splines, xs, x_vals):
    x = sym.Symbol("x")
    y_vals = []
    for x_val in x_vals:
        for i in range(len(xs) - 1):
            if xs[i] <= x_val <= xs[i + 1]:
                y_vals.append(splines[i].subs(x, x_val))
                break
    return y_vals
# Datos de la tabla
curvas = {
    "Curva 1": {
        "xs": [1, 2, 5, 6, 7, 8, 10, 13, 17],
```

```python
        "ys": [3.0, 3.7, 3.9, 4.2, 5.7, 6.6, 7.1, 6.7, 4.5],
        "d0": 1.0,
        "dn": -0.67
    }
}

# Graficar las curvas y sus splines cúbicos
plt.figure(figsize=(12, 8))

for curva, datos in curvas.items():
    xs = datos["xs"]
    ys = datos["ys"]
    d0 = datos["d0"]
    dn = datos["dn"]

    splines = cubic_spline_clamped(xs, ys, d0, dn)

    x_vals = np.linspace(xs[0], xs[-1], 1000)
    y_vals = evaluate_spline(splines, xs, x_vals)

    plt.plot(xs, ys, 'o', label=f'Puntos {curva}')
    plt.plot(x_vals, y_vals, label=f'Spline {curva}')

plt.title("Spline Cúbico Clamped ")
plt.xlabel("x")
plt.ylabel("f(x)")
plt.legend()
plt.grid(True)
plt.show()
```

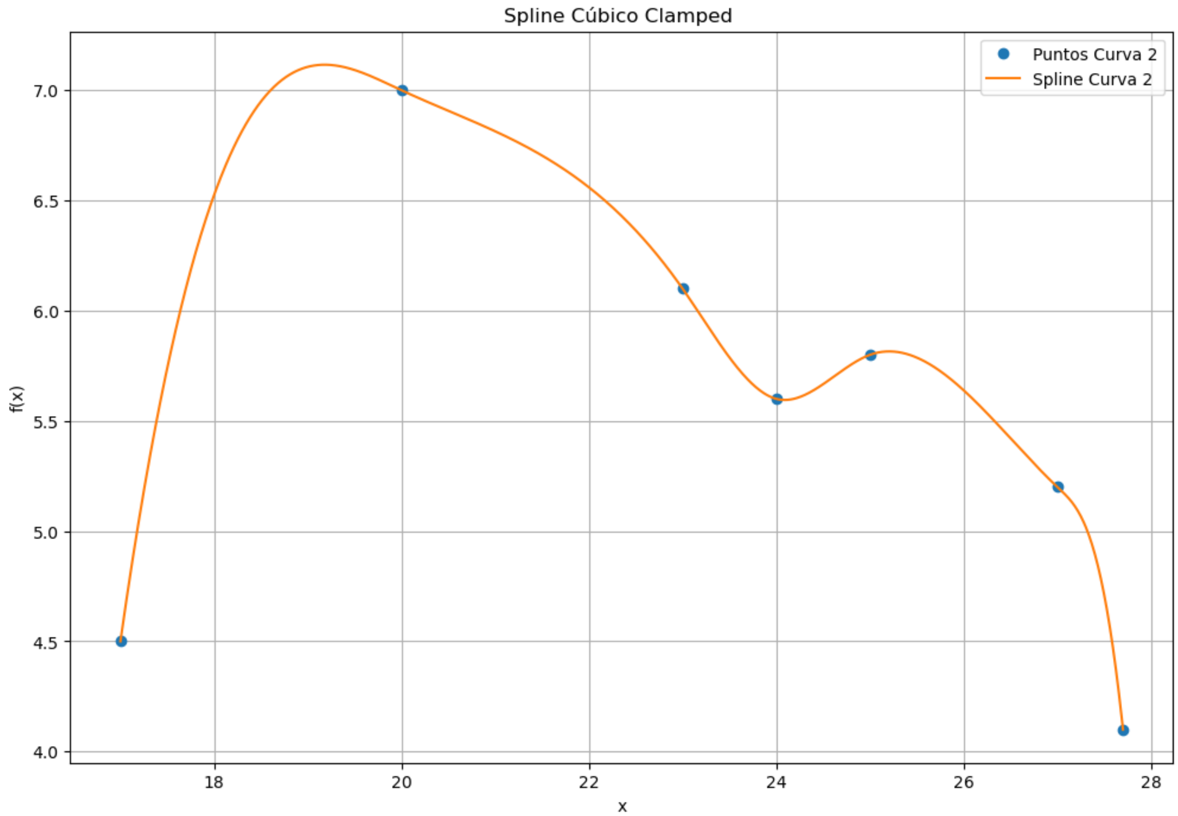
```
7 6.7 -0.3381314976116886 -0.07593425119415571 0.0057417813992694635
6 7.1 0.04846024164091059 -0.052929661890044014 -0.0025560654782346335
5 6.6 0.5472201929380908 -0.19645031375854607 0.023920108644750342
4 5.7 1.4091093003652708 -0.665438793668634 0.15632949330336265
3 4.2 1.0163426056008245 1.0582054884330803 -0.5745480940339047
2 3.9 -0.07447972276856785 0.03261683993631198 0.3418628828322561
1 3.7 0.4468099653460711 -0.2063800693078582 7 0.02655521213824114
0 3.0 1.0 -0.3468099653460706 0.046809965346070785
```




```
curvas = {
        "Curva 2": {
        "xs": [17, 20, 23, 24, 25, 27, 27.7],
        "ys": [4.5, 7.0, 6.1, 5.6, 5.8, 5.2, 4.1],
        "d0": 3.0,
        "dn": -4.0
    },

}
plt.figure(figsize=(12, 8))

for curva, datos in curvas.items():
    xs = datos["xs"]
    ys = datos["ys"]
    d0 = datos["d0"]
    dn = datos["dn"]

    splines = cubic_spline_clamped(xs, ys, d0, dn)
```

```python
    x_vals = np.linspace(xs[0], xs[-1], 1000)
    y_vals = evaluate_spline(splines, xs, x_vals)

    plt.plot(xs, ys, 'o', label=f'Puntos {curva}')
    plt.plot(x_vals, y_vals, label=f'Spline {curva}')

plt.title("Spline Cúbico Clamped ")
plt.xlabel("x")
plt.ylabel("f(x)")
plt.legend()
plt.grid(True)
plt.show()
```

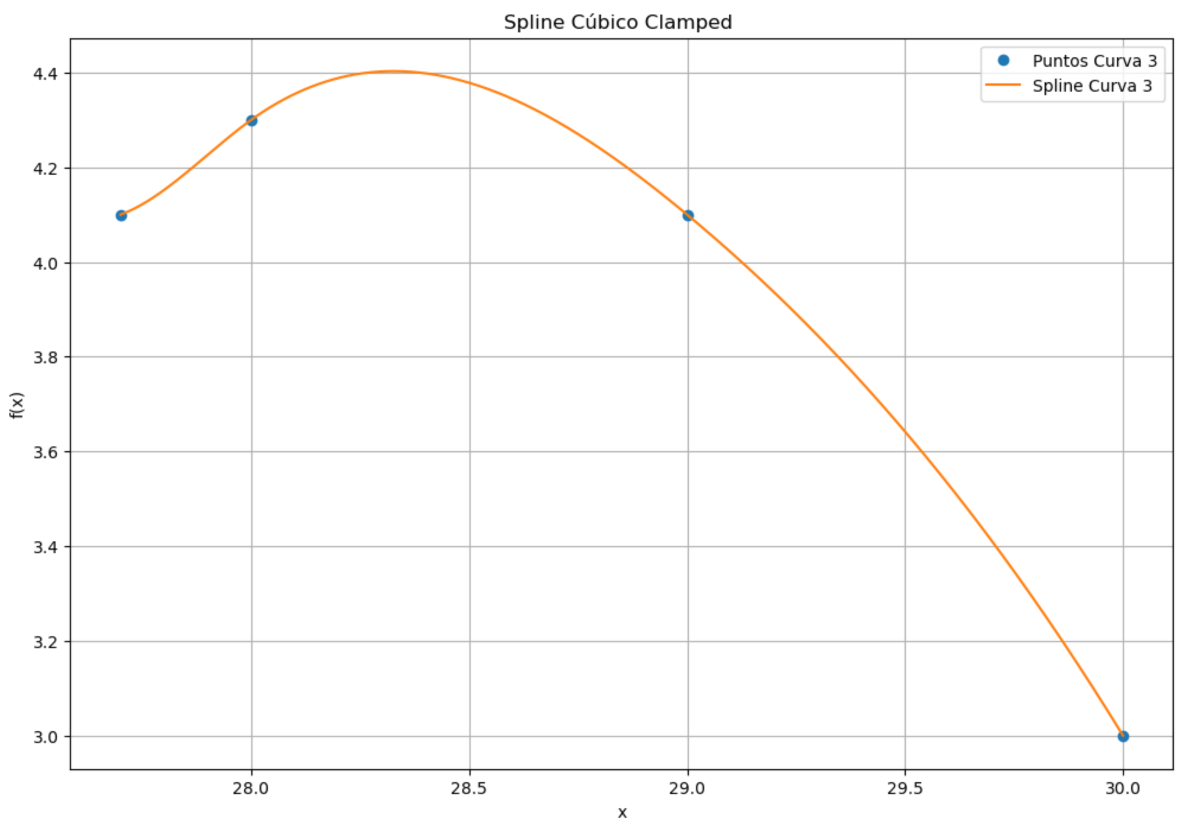
```
5 5.2 -0.4011781849199465 0.1258152222202451 -2.568002126658778
4 5.8 0.1539868142803838 -0.4033977218204103 0.08820215734010924
3 5.6 -0.11137135038117751 0.6687558864819717 -0.35738453610079396
2 6.1 -0.6085014127556733 -0.17162582410747595 0.2801272368631492
1 7.0 -0.19787464681108174 0.03475023545927881 -0.022930673285194974
0 4.5 3.0 -1.1007084510629728 0.12616207628025017
```




```
curvas = {
    "Curva 3": {
        "xs": [27.7, 28, 29, 30],
        "ys": [4.1, 4.3, 4.1, 3.0],
        "d0": 0.33,
        "dn": -1.5
    }
}
plt.figure(figsize=(12, 8))

for curva, datos in curvas.items():
    xs = datos["xs"]
    ys = datos["ys"]
    d0 = datos["d0"]
    dn = datos["dn"]

    splines = cubic_spline_clamped(xs, ys, d0, dn)
```

```python
    x_vals = np.linspace(xs[0], xs[-1], 1000)
    y_vals = evaluate_spline(splines, xs, x_vals)

    plt.plot(xs, ys, 'o', label=f'Puntos {curva}')
    plt.plot(x_vals, y_vals, label=f'Spline {curva}')

plt.title("Spline Cúbico Clamped ")
plt.xlabel("x")
plt.ylabel("f(x)")
plt.legend()
plt.grid(True)
plt.show()
```

```
2 4.1 -0.7653465346534649 -0.26930693069306927 -0.06534653465346556
1 4.3 0.6613861386138599 -1.1574257425742556 0.2960396039603954
0 4.1 0.3299999999999999 2.2620462046204524 -3.799413274660778
```



Link repositorio GitHub: https://github.com/armando-2002/Metodos_Numericos.git