



Tecnológico
de Monterrey

Visión Computacional para imágenes y video (Gpo 10)

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4. Image Convolution

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Importing Libraries

In [26]:

```
import cv2
import math
import matplotlib.pyplot as plt
import numpy as np
```

```
from PIL import Image
from scipy.ndimage.filters import median_filter
```

Simple Convolution

Definition

- **I**: Image to convolve.
- **H**: filter matrix to convolve the image with.
- **J**: Result of the convolution.

The following graphics shows exemplary the mathematical operations of the convolution. The filter matrix **H** is shifted over the input image **I**. The values 'under' the filter matrix are multiplicated with the corresponding values in **H**, summed up and written to the result **J**. The target position is usually the position under the center of **H**.



In order to implement the convolution with a block filter, we need two methods. The first one will create the block filter matrix **H** depending on the filter width/height **n**.

A block filter holds the value $\frac{1}{n \cdot n}$ at each position:

```
In [2]: def block_filter(n):
    H = np.ones((n, n)) / (n * n) # each element in H has the value 1/(n*n)
    return H
```

We will test the method by creating a filter with `n = 5`:

```
In [5]: H = block_filter(5)
print(H)

[[0.04 0.04 0.04 0.04 0.04]
 [0.04 0.04 0.04 0.04 0.04]
 [0.04 0.04 0.04 0.04 0.04]
 [0.04 0.04 0.04 0.04 0.04]
 [0.04 0.04 0.04 0.04 0.04]]
```

Next, we define the actual convolution operation. To prevent invalid indices at the border of the image, we introduce the padding **p**.

```
In [6]: def apply_filter(I, H):
    h, w = I.shape # image dimensions (height, width)
    n = H.shape[0] # filter size
    p = n // 2 # padding size
    J = np.zeros_like(I) # output image, initialized with zeros

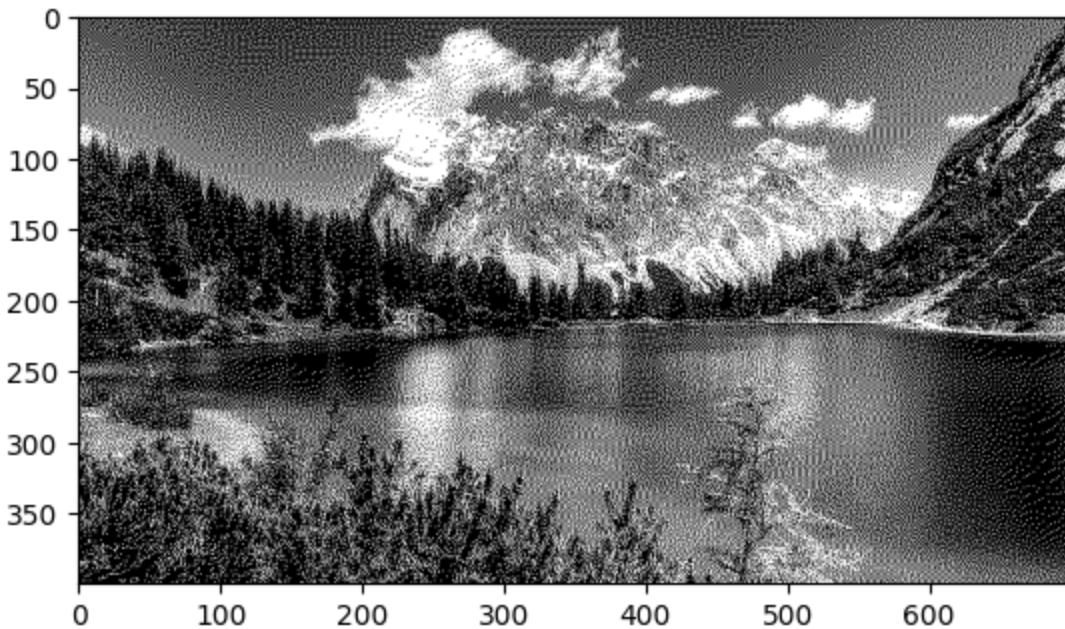
    for x in range(p, h-p):
        for y in range(p, w-p):
            J[x, y] = np.sum(I[x-p:x+n-p, y-p:y+n-p] * H)
    return J
```

```
In [7]: image = Image.open('data/image.jpg')
image = image.convert('1') # convert image to black and white

image = np.array(image)
```

```
# image = np.zeros((200, 200), dtype=np.float)
# for x in range(200):
#     for y in range(200):
#         d = ((x-100)**2+(y-100)**2)**0.5
#         image[x, y] = d % 8 < 4

plt.imshow(image, cmap='gray', vmin=0.0, vmax=1.0)
plt.show()
```

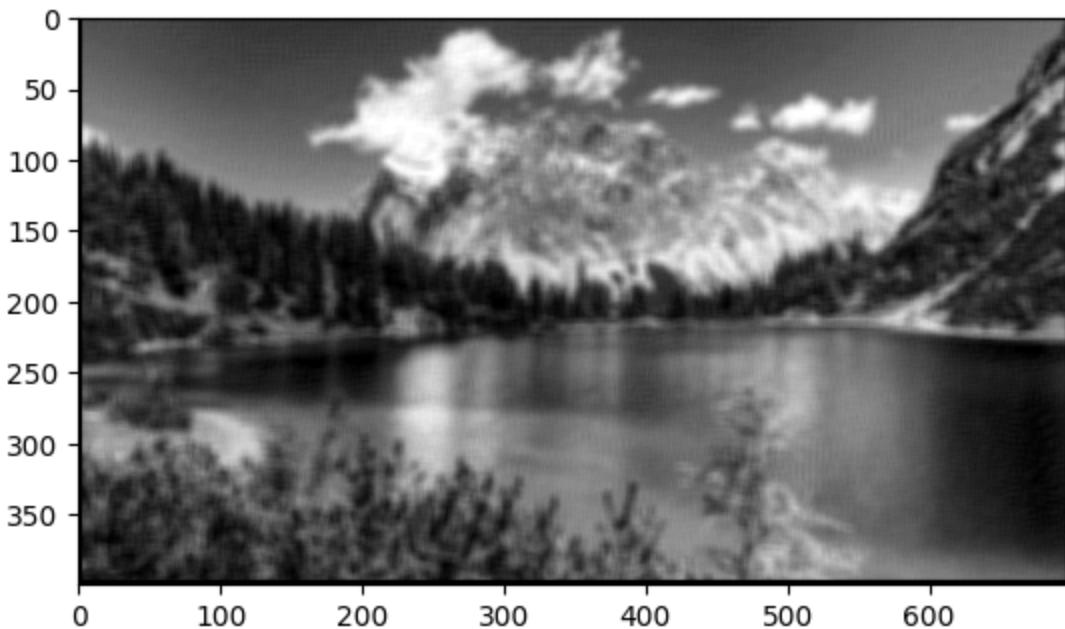


In [8]: `image = image.astype(float)`

Next we test our implementation and apply a block filter with size 7

```
n = 7
H = block_filter(n)
J = apply_filter(image, H)

plt.imshow(J, cmap='gray')
plt.show()
```



PyTorch Convolution

```
In [10]: from PIL import Image
```

```
img = Image.open('data/image.jpg')
img.thumbnail((256, 256), Image.ANTIALIAS) # Resize to half to reduce the size of this no
```

```
In [11]: img
```

```
Out[11]:
```



```
In [13]: import torch, torchvision
```

```
from torchvision import transforms
from torch import nn
```

```
In [14]: to_tensor = transforms.Compose([
    transforms.Grayscale(), # Convert image to grayscale.
    transforms.ToTensor() # Converts a PIL Image in the range [0, 255] to a torch.Floa
```

```
])
```

```
to_pil = transforms.Compose([
    transforms.ToPILImage()
])
```

```
In [15]: input = to_tensor(img)
input.shape
```

```
Out[15]: torch.Size([1, 146, 256])
```

```
In [16]: to_pil(input)
```

```
Out[16]:
```



2D convolution over an input image:

- `in_channels = 1`: an input is a grayscale image
- `out_channels = 1`: an output is a grayscale image
- `kernel_size = (3, 3)`: the kernel (filter) size is 3×3
- `stride = 1`: the stride for the cross-correlation is 1
- `padding = 1`: zero-paddings on both sides for 1 point for each dimension
- `bias = False`: no bias parameter (for simplicity)

```
In [17]: conv = nn.Conv2d(1, 1, (3, 3), stride=1, padding=1, bias=False)
```

```
In [18]: # The code below does not work because the convolution layer requires the dimension for conv(input)
```

```
Out[18]: tensor([[[[-0.0008,  0.0100,  0.0099, ...,  0.0122,  0.0371,  0.1503],  
[ 0.0017,  0.0805,  0.0813, ...,  0.1232,  0.1750,  0.1756],  
[ 0.0017,  0.0849,  0.0836, ...,  0.1757,  0.1461,  0.1435],  
...,  
[ 0.0097, -0.0294,  0.0751, ...,  0.0902,  0.0812,  0.2132],  
[-0.0401,  0.0466,  0.0898, ...,  0.0967,  0.0887,  0.2387],  
[-0.0362, -0.0026, -0.0170, ..., -0.0224, -0.0267,  0.0562]]],  
grad_fn=<SqueezeBackward1>)
```

We need to insert a dimension for a batch at dim=0.

```
In [19]: input = input.unsqueeze(0)  
input.shape
```

```
Out[19]: torch.Size([1, 1, 146, 256])
```

```
In [20]: output = conv(input)  
output.shape
```

```
Out[20]: torch.Size([1, 1, 146, 256])
```

Setting `padding=1` in the convolution layer, we obtain an image of the same size.

```
In [21]: output.shape
```

```
Out[21]: torch.Size([1, 1, 146, 256])
```

We need to remove the first dimension before converting to a PIL object.

```
In [22]: output.data.squeeze(dim=0).shape
```

```
Out[22]: torch.Size([146, 256])
```

Display the output from the convolution layer by converting `output` to a PIL object.

```
In [23]: to_pil(output.data.squeeze(dim=0))
```



Clip every value in the output tensor within the range of [0, 1].

```
In [24]: to_pil(torch.clamp(output, 0, 1).data.squeeze(dim=0))
```

```
Out[24]:
```



In [25]:

```
def display(img1, img2):
    im1 = to_pil(torch.clamp(img1, 0, 1).data.squeeze(dim=0))
    im2 = to_pil(torch.clamp(img2, 0, 1).data.squeeze(dim=0))
    dst = Image.new('RGB', (im1.width + im2.width, im1.height))
    dst.paste(im1, (0, 0))
    dst.paste(im2, (im1.width, 0))
    return dst
```

In [26]:

```
display(input, output)
```

Out[26]:



Identity

In [27]:

```
conv.weight.data = torch.tensor([[[[
    [0., 0., 0.],
    [0., 1, 0.],
    [0., 0., 0.],
]]]])  
  
output = conv(input)  
display(input, output)
```

Out[27]:



Brighten

In [30]:

```
conv.weight.data = torch.tensor([[[[
    [0., 0., 0.],
    [0., 1.5, 0.],
    [0., 0., 0.],
]]]])  
print(conv.weight.data)  
output = conv(input)  
display(input, output)  
  
tensor([[[[0.0000, 0.0000, 0.0000],
```

```
[0.0000, 1.5000, 0.0000],  
[0.0000, 0.0000, 0.0000]]]))
```

Out[30]:



Darken

```
In [31]: conv.weight.data = torch.tensor([[[  
    [0., 0., 0.],  
    [0., 0.5, 0.],  
    [0., 0., 0.],  
],]])  
print(conv.weight.data)  
output = conv(input)  
display(input, output)  
  
tensor([[[[0.0000, 0.0000, 0.0000],  
        [0.0000, 0.5000, 0.0000],  
        [0.0000, 0.0000, 0.0000]]]])
```

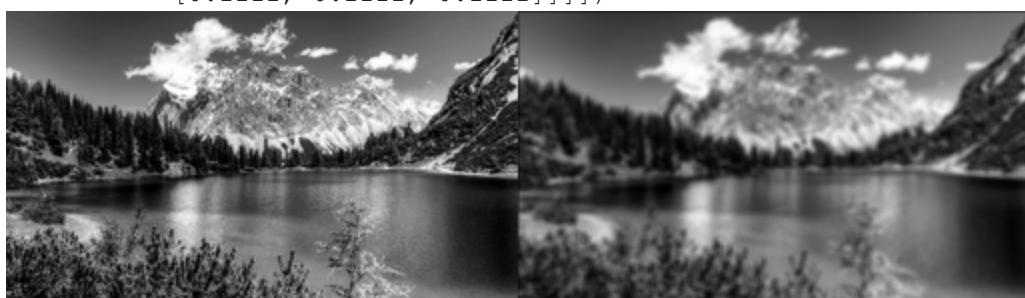
Out[31]:



Box blur

```
In [32]: conv.weight.data = torch.ones((1, 1, 3,3), dtype=torch.float) / 9.  
print(conv.weight.data)  
output = conv(input)  
display(input, output)  
  
tensor([[[[0.1111, 0.1111, 0.1111],  
        [0.1111, 0.1111, 0.1111],  
        [0.1111, 0.1111, 0.1111]]]])
```

Out[32]:



Gaussian blur

```
In [33]: conv.weight.data = torch.tensor([[[  
    [1., 2., 1.],  
    [2., 4., 2.],  
    [1., 2., 1.],  
],]])  
print(conv.weight.data)  
output = conv(input)  
display(input, output)
```

```
]]]) / 16.  
print(conv.weight.data)  
output = conv(input)  
display(input, output)  
  
tensor([[[[ 0.0625,  0.1250,  0.0625],  
         [ 0.1250,  0.2500,  0.1250],  
         [ 0.0625,  0.1250,  0.0625]]]])
```

Out[33]:



Sharpen

```
In [34]: conv.weight.data = torch.tensor([[[  
    [0., -1., 0.],  
    [-1., 5., -1.],  
    [0., -1., 0.],  
],]])  
print(conv.weight.data)  
output = conv(input)  
display(input, output)  
  
tensor([[[[ 0., -1.,  0.],  
         [-1.,  5., -1.],  
         [ 0., -1.,  0.]]]])
```

Out[34]:



```
In [35]: conv.weight.data = torch.tensor([[[  
    [0., -2., 0.],  
    [-2., 10., -2.],  
    [0., -2., 0.],  
],]])  
print(conv.weight.data)  
output = conv(input)  
display(input, output)  
  
tensor([[[[ 0., -2.,  0.],  
         [-2., 10., -2.],  
         [ 0., -2.,  0.]]]])
```

Out[35]:



Edge detection

In [36]:

```
conv.weight.data = torch.tensor([[[[  
    [0., 1., 0.],  
    [1., -4., 1.],  
    [0., 1., 0.],  
]]]])  
print(conv.weight.data)  
output = conv(input)  
display(input, output)
```

```
tensor([[[[ 0.,  1.,  0.],  
        [ 1., -4.,  1.],  
        [ 0.,  1.,  0.]]]])
```

Out[36]:



In [37]:

```
conv.weight.data = torch.tensor([[[[  
    [-1., -1., -1.],  
    [-1.,  8., -1.],  
    [-1., -1., -1.],  
]]]])  
output = conv(input)  
display(input, output)
```

```
tensor([[[[ -1., -1., -1.],  
        [-1.,  8., -1.],  
        [-1., -1., -1.]]]])
```

Out[37]:



Excercises

1. Edge detection algorithm implementation

In [3]:

```
# Function to plot images  
def plot_images(orig_img, trans_img):  
    plt.figure(figsize=(20, 20))  
    plt.subplot(121),plt.imshow(orig_img)  
    plt.title('Original Image'), plt.xticks([]), plt.yticks([])  
    plt.subplot(122),plt.imshow(trans_img)  
    plt.title('Transformed Image'), plt.xticks([]), plt.yticks([])  
  
# Function to plot original and grayscaled Image  
def plot_images_gray(orig_img, gray_img):  
    plt.figure(figsize=(20, 20))  
    plt.subplot(121),plt.imshow(orig_img)  
    plt.title('Original Image'), plt.xticks([]), plt.yticks([])  
    plt.subplot(122),plt.imshow(gray_img, cmap=plt.get_cmap('gray'))  
    plt.title('Grayscale Image'), plt.xticks([]), plt.yticks([])
```

```
# Function to plot original, grayscaled and Edge Detected Images
def plot_images_edgeDetection(orig_img, gray_img, edge_img):
    plt.figure(figsize=(40, 40))
    plt.subplot(131),plt.imshow(orig_img)
    plt.title('Original Image'), plt.xticks([]), plt.yticks([])
    plt.subplot(132),plt.imshow(gray_img, cmap=plt.get_cmap('gray'))
    plt.title('Grayscale Image'), plt.xticks([]), plt.yticks([])
    plt.subplot(133),plt.imshow(edge_img, cmap=plt.get_cmap('gray'))
    plt.title('Edge Detected Image'), plt.xticks([]), plt.yticks([])
```

Prewitt

```
In [66]: # 0. Load the original image:
img = Image.open('./data/tiger.jpeg')

# 1.0 Transform image to a numpy array:
img = np.array(img)

# 2.1. Extract the red, green, and blue channels:
r, g, b = img[:, :, 0], img[:, :, 1], img[:, :, 2]

# 2.2. Scalars to convert the image to grayscale:
grey_scalars = [0.2989, 0.5870, 0.1140]

# 3.0. We apply the scalar to the channels to create the grey-scale image (this is our s
greyimg = grey_scalars[0] * r + grey_scalars[0] * g + grey_scalars[0] * b

# 4.0. Let's define the kernels of Prewitt:
Gx = np.array([
    [-1, 0, 1],
    [-1, 0, 1],
    [-1, 0, 1]
])

Gy = np.array([
    [-1, -1, -1],
    [ 0, 0, 0],
    [ 1, 1, 1]
])

# 5.0. Let's go row by row and then col by col (nested loop),
# applying the mask of the prewitt kernel on X and Y over the original image to get the

# 5.1. First let's get the image dimensions:
rows, cols = greyimg.shape

# 5.1. Then let's create an empty image:
prewittimg = np.zeros((rows, cols))

# 5.2 Let's loop through the image rows:
for i in range(1, rows-1):
    # 5.3. And let's loop through the columns:
    for j in range(1, cols-1):
        # Apply the x kernel to the current pixel
        x_edge = np.sum(greyimg[i-1:i+2, j-1:j+2] * Gx)
        # Apply the y kernel to the current pixel
        y_edge = np.sum(greyimg[i-1:i+2, j-1:j+2] * Gy)
        # Compute the gradient magnitude
        prewittimg[i, j] = np.sqrt(x_edge**2 + y_edge**2)

# 6.0 voila!

## Output:
plot_images_edgeDetection(img, greyimg, prewittimg)
```



Sobel

```
In [16]: # First we read our image file and converted to gray scale.
img = Image.open('data/tiger.jpeg')

# Then we convert our image to a numpy array where the height, width and colors are separated
img = np.array(img)

# Then we separate the RGB colors on individual variables.
redCh, greenCh, blueCh = img[:, :, 0], img[:, :, 1], img[:, :, 2]

# As we are not allowed to use libraries, we apply a Gamma filter to convert our image to grayscale
# If we recall the gamma correction will let us play with the luminance and contrast of the image
# By elevating our array to a specific value and multiplying it by the specified gamma
# A gamma > 1 is called decoding and with help us to change the color of our image to grayscale
gamma_val = 1.400

# We also need to add specific weights for each RGB channel, these values are the ones below
# We found these values through research as the ideal for this conversion to an optimized grayscale
redConst, greenConst, blueConst = 0.2126, 0.7152, 0.0722
grayimg = redConst * redCh ** gamma_val + greenConst * greenCh ** gamma_val + blueConst
```

```
In [23]: """
Now that we got our image in grayscale mode, we need to apply the Sobel Filter to detect the edges. As we can remember from the lectures, a Sobel Operator is a specific type of 2D derivative operator that detects the edges of an image. When using this 2D spatial gradient measurement of an image, the pixels which correspond to edges.
```

Sobel operator works by applying the operator that consists of a pair of 3x3 convolution kernels. One kernel is simply the other rotated by 90°. One of the kernels is applied to the X (Horizontal) direction and the other is applied to the Y (Vertical) direction.

As an example we could define the following:

$$G_x = \begin{bmatrix} -1.0 & 0.0 & 1.0 \\ -2.0 & 0.0 & 2.0 \\ -1.0 & 0.0 & 1.0 \end{bmatrix} \quad \text{and} \quad G_y = \begin{bmatrix} -1.0 & -2.0 & -1.0 \\ 0.0 & 0.0 & 0.0 \\ 1.0 & 2.0 & 1.0 \end{bmatrix}$$

```
# Naturally the first step will be to define our kernels onto 2D arrays
kernX = np.array([[-1.0, 0.0, 1.0], [-2.0, 0.0, 2.0], [-1.0, 0.0, 1.0]])
kernY = np.array([[-1.0, -2.0, -1.0], [0.0, 0.0, 0.0], [1.0, 2.0, 1.0]])

# Then our next step is to obtain the shape of our grayscale image, which will help us on how to build the Sobel filter
[r, col] = np.shape(grayimg)
sobelImg = np.zeros(shape=(r, col))

# Next we go through the x and y values of our image
```

```
# then we store the computed value of the sobel transformation on the corresponding position
for i in range(r - 2):
    for j in range(col - 2):
        # To find the Gradient on each direction, we multiply each of our kernels to the
        # We apply this operation separately to produce a measure of the gradient component
        gx = np.sum(np.multiply(kernX, grayimg[i:i + 3, j:j + 3]))
        gy = np.sum(np.multiply(kernY, grayimg[i:i + 3, j:j + 3]))
        # Finally we find the absolute magnitude of the gradient at each point, which is
        sobelImg[i + 1, j + 1] = np.sqrt(gx ** 2 + gy ** 2)

plot_images_edgeDetection(img, grayimg, sobelImg)
```



Laplacian

```
In [68]: def laplacian_filter(original_image):
    #We first need to get the original image dimensions
    w, h = image.size
    #After we define the Laplacian kernel
    kernel = [[0, 1, 0], [1, -4, 1], [0, 1, 0]]
    #Then we proceed to create a new image with the same size and mode as the original image
    new_filtered_image = Image.new("L", (w, h))
    pixels = new_filtered_image.load()

    for y in range(1, h - 1):
        for x in range(1, w - 1):
            # We now proceed to apply the Laplacian filter using convolution
            convolution = 0
            for i in range(-1, 2):
                for j in range(-1, 2):
                    convolution += kernel[i + 1][j + 1] * image.getpixel((x + j, y + i))
            pixels[x, y] = convolution

    return new_filtered_image

img_path = 'data/tiger.jpeg'
original_image = Image.open(img_path)
image_for_filter = original_image.convert('L')
laplacian_image = laplacian_filter(image_for_filter)

plot_images_edgeDetection(original_image, image_for_filter, laplacian_image.convert('P'))
```



Which is more efficient?

Sobel and Prewitt same complexity, they are the same algorithm with different mask [1], they have the same

time complexity of $O(n)$, n is defined as the number of the pixels in the image. The operation of the convolution has a constant time of $O(1)$ therefore the time complexity is determined by the size of the image, that means, correlated with the number of the pixels in the image. Meanwhile, Laplacian has a similar complexity that Sobel and Prewitt but due that its operator can be applied in a different manners, for example by using different kernels, its complexity will be determined by the used operators and the size of the kernel. However, Laplacian sometimes demonstrates an advantage for edge detections [2]

2. Image enhancement and image masking

```
In [21]: # Function to plot original and grayscaled Image
def plot_images_enhanced(orig_img, gray_img, enhanced_img):
    orig_img = cv2.cvtColor(orig_img, cv2.COLOR_BGR2RGB)
    plt.figure(figsize=(40, 40))
    plt.subplot(131), plt.imshow(orig_img, cmap=plt.get_cmap('gray'))
    plt.title('Original Image'), plt.xticks([]), plt.yticks([])
    plt.subplot(132), plt.imshow(gray_img, cmap=plt.get_cmap('gray'))
    plt.title('Grayscale Image'), plt.xticks([]), plt.yticks([])
    plt.subplot(133), plt.imshow(enhanced_img, cmap=plt.get_cmap('gray'))
    plt.title('Enhanced Image'), plt.xticks([]), plt.yticks([])

In [24]: # Convert image from color to grayscale, then applies a median filter and soften image,
# canny to apply the mask with the soften and finally image is sharpened with the alpha
def line_enhancement(img, alpha=3):
    gray = cv2.cvtColor(img, cv2.COLOR_BGR2GRAY)
    kernel = np.ones((7,7),np.float32)/5
    soften = cv2.filter2D(gray,-1,kernel)
    edges = cv2.Canny(soften, 80, 200)
    mask = cv2.subtract(soften, edges)
    enhanced = cv2.addWeighted(gray, alpha, mask, 1-alpha, 0)
    plot_images_enhanced(img, gray, enhanced)

    img = cv2.imread("data/lena.jpg")
    line_enhancement(img, 1)
```



3. Median filtering

```
In [29]: # Function to plot original, grayscaled and Edge Detected Images
def plot_images_medianFilter(orig_img, blurred_image, median_image):
    plt.figure(figsize=(40, 40))
    plt.subplot(131), plt.imshow(orig_img)
    plt.title('Original Image'), plt.xticks([]), plt.yticks([])
    plt.subplot(132), plt.imshow(blurred_image, cmap=plt.get_cmap('gray'))
    plt.title('Blurred Image'), plt.xticks([]), plt.yticks([])
    plt.subplot(133), plt.imshow(median_image, cmap=plt.get_cmap('gray'))
    plt.title('Median Filtered Image'), plt.xticks([]), plt.yticks([])
```

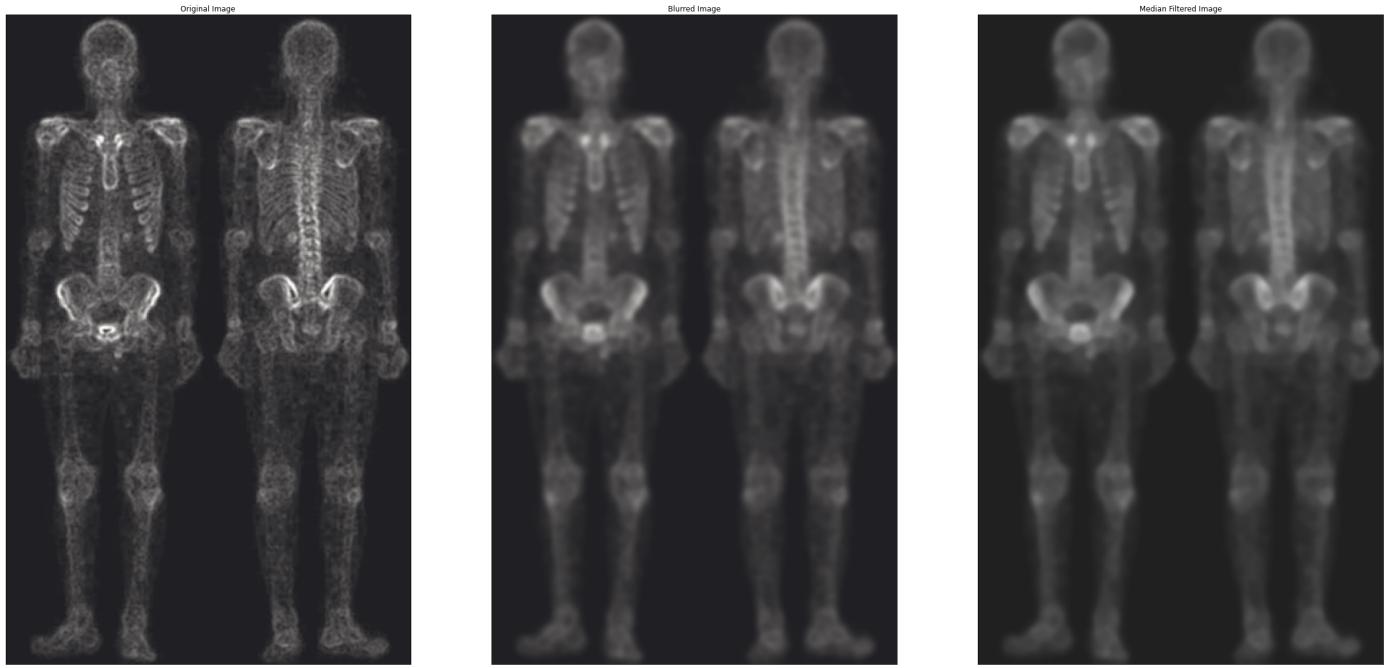
```
In [8]: # Create the gaussian kernel function with size and sigma values
def gaussian_kernel(size, sigma):
    size = int(size) // 2
    x, y = np.mgrid[-size:size+1, -size:size+1]
    normal = 1 / (2.0 * np.pi * sigma**2)
    g = np.exp(-(x**2 + y**2) / (2.0*sigma**2)) * normal
    return g
```

```
In [33]: # load original image
image = cv2.imread("data/PET.png")

# define kernel and apply gaussian filter to image
kernel = gaussian_kernel(17, 3)
blurred_image = cv2.filter2D(image, -1, kernel)

# apply median filter
median_image = median_filter(blurred_image, 7)

# Plot the images
plot_images_medianFilter(image, blurred_image, median_image)
```



References

- [1] Ahmed, A. S. (2018). Comparative study among Sobel, Prewitt and Canny edge detection operators used in image processing. J. Theor. Appl. Inf. Technol., 96(19), 6517-6525.
- [2] Asst. lecturer Aadel I. Khalil, I. S. A. A.-K. "COMPARATIVE STUDY FOR EDGE DETECTION OF NOISY IMAGE USING SOBEL AND LAPLACE OPERATORS". Journal of the College of Education for Women, vol. 23, no. 2, Feb. 2019, <https://jcoeduw.uobaghdad.edu.iq/index.php/journal/article/view/853>.