Social Network Analysis and Mining (2018) 8:13
https://doi.org/10.1007/s13278-018-0493-2

REVIEW ARTICLE

Study on centrality measures in social networks: a survey

Kousik Das[1] · Sovan Samanta[2] · Madhumangal Pal[1]

Received: 29 September 2017 / Revised: 19 February 2018 / Accepted: 19 February 2018
© Springer-Verlag GmbH Austria, part of Springer Nature 2018

Abstract
Social networks are absolutely a useful and important place for connecting people within the world. A basic issue in a social network is to identify the key persons within it. This is why different centrality measures have been found over the years. In this survey paper, we present past and present research works on measures of centrality in social network. For this plan, we discuss mathematical definitions and different developed centrality measures. We also present some applications of centrality measures in biology, research, security, traffic, transportation, drug, class room. At last, our future research work on centrality measure is given.

Keywords Social networks · Centrality · Degree · Neighbor · k-shell

1 Introduction

Social network is a connection of individuals or groups connected by some relations. It may be offline or online mode. Members of a family form a network (family network), some farmers in a village form a farmer network, some businessmen form a business network and these are common offline networks. A million of people are using smart phones and they are very comfort to use social app to connect each other and share information to each other. Recently online social networks like Twitter, Facebook and LinkedIn have grown extremely popular in human life. Individuals, colleagues, organizations, etc. are attached to each other within the social network. So, it is a platform of marketing product, spreading news, etc. To furnish these types of works in social network, identifying the important or central or influential node is a fundamental task. Centrality indicates the most important node or central node or influential node within a network. Hence, measuring centrality is a very essential task in social network. Suppose there are 100 members in a club. President of the club is assuming to be central. In a country, there are many branches of a bank. The headquarters of the bank is assuming to be central. Monitor of a class is assuming as central student in the classroom. Principal of a college among the teachers is assuming as central.

In practical, sometimes large numbers of data arise in network having high time complexity and hard to measure centrality. Centrality measures also play an important role in finding a critical and powerful element for a large amount of data. Identifying influential node is common issue in these types of network. Virus spreading, disease spreading, information or news broad casting, etc. are many activities involved in online or offline networks. In the network analysis, it is very useful to identify important vertices within network. Every centrality measure is not suitable for every application. Also time complexity is an important part in the network analysis. Based on the varying notions of importance of vertices or edges, different centrality measures were developed over the years and applied them in suitable area. We reviewed information of existing centrality measures for proper selection of applications area and also presented some important centrality measures with some of their applications.

The paper is organized as follows. In Sect. 1, we introduce the survey paper by giving some motivating examples in real life. In Sect. 2, we describe development of centrality

✉ Sovan Samanta
ssamantavu@gmail.com

Kousik Das
kousikmath@gmail.com

Madhumangal Pal
mmpalvu@gmail.com

1 Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore 721102, India

2 Department of Mathematics, Tamralipta Mahavidyalaya, Tamluk 721636, India

Published online: 28 February 2018

measures from the beginning. In Sect. 3, we describe some basic concepts required to develop the results. In Sect. 4, we present fourteen centrality measures with category of centrality measures and gist of limitations and application areas. In Sect. 5, we discuss some specific applications. At last in Sect. 6, we discuss conclusion and plan of our future works.

2 Development of centrality measures

Day to day, many centrality measures were introduced and hence developed to apply proper problems in real life. Bavelas (1950) first defined measure of centrality for connected graphs and proposed its application to the study of communication network. Shimbel (1953) introduced stress centrality based on shortest path to measure amount of communication. Katz (1953) introduced Katz centrality to measure the relative degree of influence of a node in the network. Beauchamp (1965) discussed about meaning and also limitations against the Bavelas's (1948) measure of centrality and proposed an improvement of centrality index containing points and also graphs in order to extend its analytical utility. Sabidussi (1966) claimed about the Beauchamp's (1965) improvement of centrality index and gave a precise definition and tested extent of known indices for satisfying the requirements of that definition. Nieminen (1974) modified some axioms of Sabidussi (1966) and introduced centrality index based on degrees of vertices for undirected graph.

Freeman (1978) developed three type measures of centrality for each notion, first one absolute and second one comparative measure of centrality for locations for a network and third one presenting degree of centralization for entire network. These measures were examined for the experimental culture of small groups. Stephenson and Zelen (1989) introduced information centrality based on information that can be transmitted between any two vertices in a connected network. A new measure introduced by Freeman et al. (1991) to find centrality by the idea of network flows. This was like Freeman's measure but different from original. White and Borgatti (1994) generalized geodesic centrality measures of Freeman (1978) for betweenness for undirected graph to the more directed graphs. Everett and Borgatti (1999) extended the standard three centrality measures to apply for groups and classes and also as individuals.

The centrality measures started by Freeman's (1978) were only on binary networks. There were huge contributions to generalize Freeman's (1978) centrality measures to weighted networks. Brandes (2001) introduced a faster algorithm on betweenness centrality which shortens time and also space of comparative analysis. Costenbader and Valente (2003) discussed the stability of all centrality measures when network was sampled. To get over the limitations of Costenbader and Valente (2003) and Borgatti (2006), a computational approach was taken and robustness was examined for large sample of graphs. Estrada and Rodriguez-Velazquez (2005) introduced subgraph centrality to measure the number of times of a vertex involve in the different connected subgraph of the network. Rodriguez et al. (2006) generalized the subgraph centrality as functional centrality based on the idea that closed walks are appropriately weighted such that their influence on the centrality decreases as the order of the walk increases. Bonacich (2007) discussed some unique properties for eigenvector centrality. Opsahl et al. (2010) improved the measures of centrality for weighted network. Joyce et al. (2010) introduced a new centrality called leverage centrality and applied it to analyze human brain network.

Kitsak et al. (2010) introduced k-shell decomposition and proposed that the influential spreaders are staying within core of network. Zeng and Zhang (2013) proposed a method called mixed degree decomposition by taking residual degree and also exhausted degree. Liu et al. (2014) presented an improved method to display more distinguishable in the ranking list. This method considers k-shell values and the shortest distance from a target node to the network core which is defined as the set of nodes with the highest k-shell values. Taking together the degree and also coreness of node and its neighbor simultaneously, Bae and Kim (2014) introduced neighborhood coreness centrality that ranks all the nodes of a network. Liu et al. (2015) proposed an nth step neighborhood centrality to find influential node in complex network. Hence, Wang et al. (2017) proposed weighted neighborhood centrality to find more accurate ranking list of influential nodes.

Different developments over the years can be shown as a flowchart. Hence, the gist of contributions of authors is given in the flowchart depicted in Fig. 1.

A list of notations used in the paper is given in Table 1.

3 Basic concepts

Every network can be described by a graph $G(N, E)$ where N represents the set of vertices and E represents the set of edges.

Let $N = \{1, 2, 3, \ldots, n\}$ be a finite set of nodes. By $g_{ij} \in \{0, 1\}$, a relationship is denoted between nodes i and j, where

$$g_{ij} = \begin{cases} 1, & \text{if there is a link between } i \text{ and } j \\ 0, & \text{otherwise} \end{cases}$$

A network G is defined by a set containing nodes N and also links among these nodes.

The *neighbor* of a node i in a network G is defined by $N_i(G) = \left\{ j \in N : g_{ij} = 1 \right\}$.

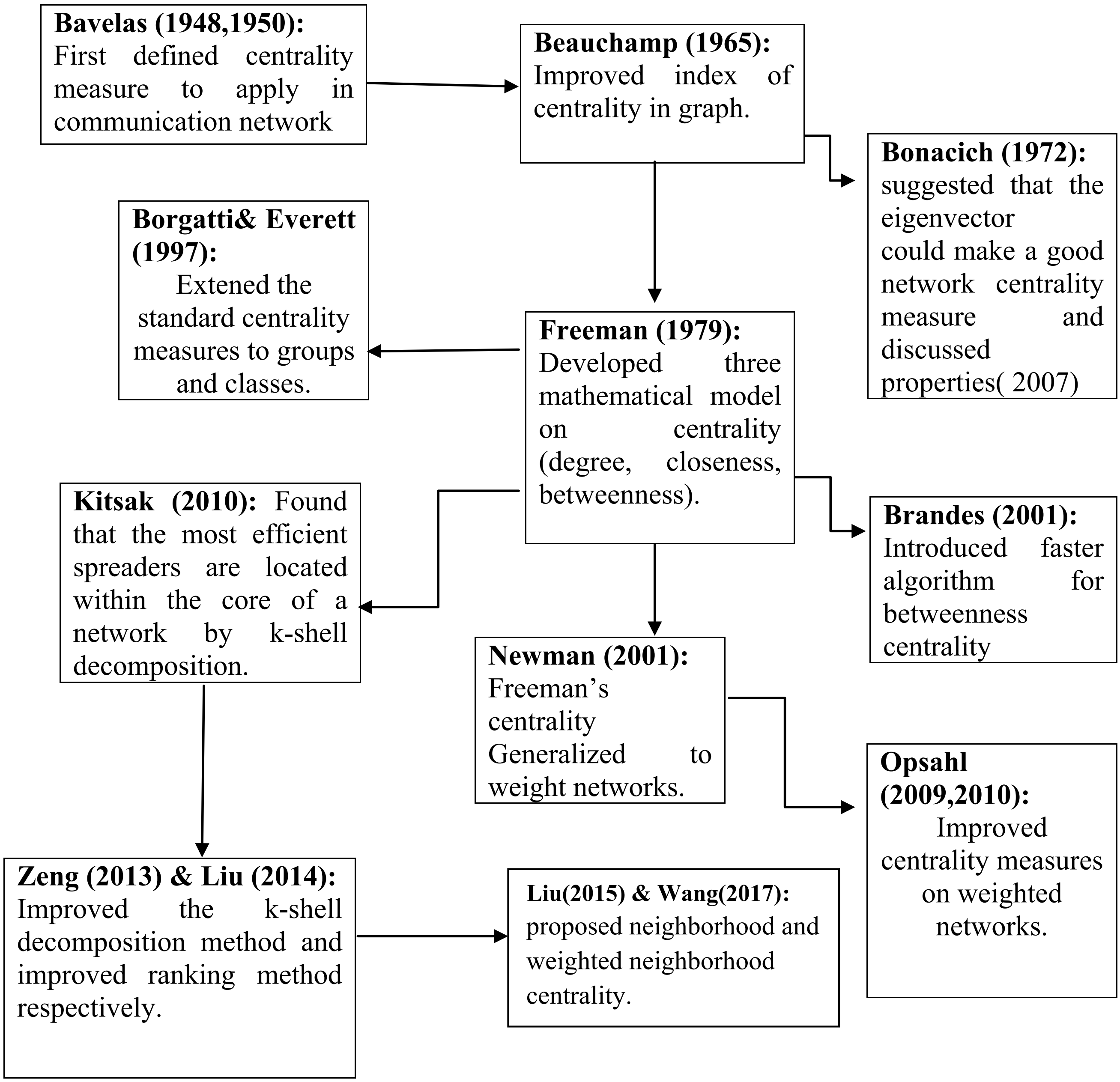


Fig. 1 Development of works contributed by the distinguished authors

In a network G, degree $d_i(G)$ of the node i is defined by the number of node adjacent to i in G, i.e., $d_i(G) = |N_i(G)|$.

A succession of nodes where two nodes are connected by a link and also a node or a link may occur more than once is called a *walk*. If all links are distinct in a walk, then it is called *trial.* A path is called a trial in which all nodes are distinct.

The *geodesic distance* $d(i, j)$ from the node i to the node j is given by $d(i, j)$ = the number of links in a shortest path to node j from node i, if path exists.

Path among every two nodes within a network, if exists, then the network is called *connected network*.

A network is called *directed network* if the nodes connected by links having direction associated to each link.

A network is called *weighted network* if the edges among the nodes have weights.

A network is called *social network* if nodes are persons or groups or organizations and there are relations or links or connections between them.

Table 1 Notations

Notations	Meaning
N	Set of nodes
E	Set of edges
n	Cardinality of V
m	Cardinality of E
d_i	Degree of a node i
$d(i, j)$	Geodesic distance from node i to node j
σ_{st}	Number of shortest paths from node s to node t
$\sigma_{st}(v)$	Number of shortest paths from node s to node t through node v
C_{D}	Degree centrality
C_{C}	Closeness centrality
C_{EC}	Eccentricity centrality
C_{I}	Information centrality
C_{S}	Stress centrality
C_{B}	Betweenness centrality
C_{K}	Katz centrality
C_{SG}	Subgraph centrality
C_{F}	Functional centrality
C_{L}	Leverage centrality
C_{NC}	Neighborhood coreness centrality
C_{N}	Neighborhood centrality

In Fig. 2, there are eight nodes and nine links form a social network.

4 Different kind of centrality measures

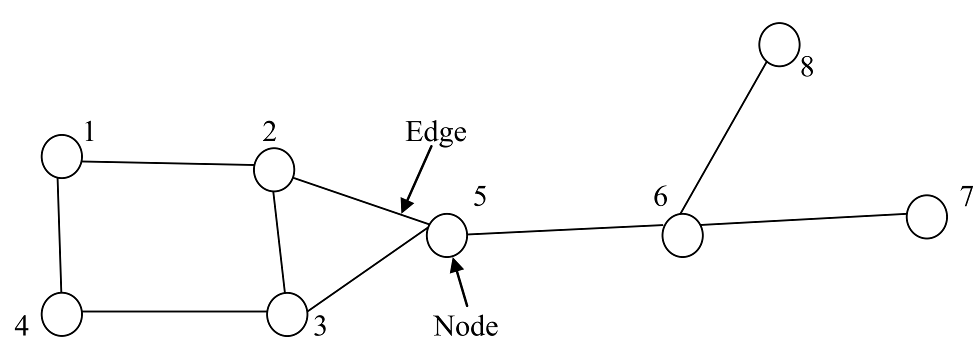
Centrality measures can be remarked as a mathematical tool applied in the social network analysis to identify important element in the network. Over the years, many centrality measures have been introduced and used to several topics within social networks. Before main discussion, centrality measures are categorized (Koschutzki et al. 2005) below:

- ***Reachability*** This category indicates whether two actors (nodes) are connected direct or indirect by any pathway. Centrality measures of this category are used to find the ability of vertices to reach all others vertices. Degree centrality, closeness centrality, eccentricity centrality, etc., are reachability-based centrality.
- ***Shortest path*** There are different paths between any two vertices in the network. Centrality measures of this category are used to find shortest path from source (starting vertex) to sink (end vertex). Stress centrality, betweenness centrality, etc., are shortest path based centrality.
- ***Feedback*** A node will be important if its neighbor is important. Centrality measure of a node depends on the measure of all nodes. Eigenvector centrality, subgraph centrality, functional centrality, etc., are feedback-based centrality.
- ***Current Flow*** Sometime flow of information follows the behavior of current network. Current flow betweenness centrality, current flow closeness centrality, etc., are current flow-based centrality.
- ***Random process*** In random walk, a vertex chooses any edges randomly to reach another vertex when calculating centrality. Random degree centrality, random closeness centrality, random betweenness centrality, etc., are random process-based centrality.
- ***Vitality*** Take an arbitrary function on the graph and the difference of the quantities of the function on the graph with and without vertex evaluate the vitality. Closeness vitality, flow betweenness vitality, etc., are vitality-based centrality.

Other than these categories, we add one other category based on k-shell. Details are below:

- ***K-shell*** A network is decomposed into many shells to investigate influential nodes that lying in the inner shells. Some newer centrality is invested like coreness centrality, neighborhood centrality.

Fig. 2 Structure of a network



4.1 Degree centrality

Shaw (1954) first introduced degree centrality as an index of important vertex and hence defined by Nieminen (1974). Freeman (1978) first developed a mathematical model for centrality which is based on the links connected to a vertex. Bonacich (1987) introduced a generalized concept of degree centrality to degree-based power and centrality. Barrat et al. (2004), Newman (2004) and Opsahl and Panzarasa (2009) generalized and extended the degree of node by considering sum of weights in place of number ties. Opsahl et al. (2010) generalized the previous measure of degree centrality taking degree and also strength of node.

Degree centrality of a node is a number that indicates the number of nodes directly linked to this node. Mathematically, degree centrality $C_{\mathrm{D}}(x)$ is defined by $C_{\mathrm{D}}(x) = d_x$ where d_x is the degree of a node x.

For normalization, degree centrality $C_{\mathrm{D}}'(x) = \frac{d_x}{n-1}$ where n is the size of the network.

Time complexity of this measure for unweighted network is $\mathrm{O}(m)$ where m is the number of edges. The major limitation of degree-based centrality is that it gives only local information of a vertex in the network. That is, this measure does not consider global structure. It is clear that this measure is less complex and useful in many applications. Using minimax criteria, this measure is used to find the position of emergency facility (facility location problem) like hospital, police station, military camp and also other facility like school, petrol pump, market, restaurant, hotel.

4.2 Closeness centrality

The idea of closeness centrality measure was introduced first by Bavelas (1948) and defined by Sabidussi (1966) as inverse of sum of geodesic distances to every other vertices from each vertex within the network. Freeman's (1979) second mathematical measure is on closeness centrality. Newman (2001) generalized closeness to weighted networks using the algorithm for shortest paths of Dijkstra (1959).

The centrality $C_{\mathrm{C}}(x)$ according to closeness of a vertex x is given by

$$C_{\mathrm{C}}(x) = \frac{1}{\sum_{y \in N} d(x, y)}$$

where N = set of vertices in the network.

$d(x, y)$ is geodesic distance between the vertices x and y.

For normalization, closeness centrality $C_{\mathrm{C}}'(x) = \frac{n-1}{\sum_{y \in N} d(x,y)}$

where n is the size of the network.

Time complexity of this measure is O(mn) for all vertices by the algorithm Brandes (2001). This method is preferable than degree centrality, as it does not count only direct connection among vertices but also indirect links. If a network has a disconnected component, then this measure could not be applied since finite distance cannot exist for any two nodes that lie in different components. This measure is used to determine a location for service facility like shopping market (facility location problem).

4.3 Eccentricity centrality

This centrality is closely connected to closeness centrality. Hage and Harary (1995) discussed about eccentricity in the networks. The eccentricity is the inverse of maximum geodesic distance between a vertex and any vertex of the network. Mathematically, the eccentricity $C_{\mathrm{EC}}(x)$ for a vertex x is given by

$$C_{\mathrm{EC}}(x) = \frac{1}{\max_{y \in N} d(x, y)}.$$

This measure has time complexity O(mn) for all vertices by Brandes (2001, 2008). This measure determines a location that can spread information fast. It is global measure that considers whole network. But one limitation is that it takes only maximum distance between nodes.

4.4 Information centrality

This measure was first proposed by Stephenson and Zelen (1989). It is related to closeness centrality and based on information which is containing in all possible paths among pairs of nodes.

Let A be the adjacency matrix of a network. Then, information centrality $C_{\mathrm{I}}(k)$ of a node k is given by

$$C_{\mathrm{I}}(k) = \frac{1}{b_{kk} + (T - 2R)/n},$$

where $T = \sum_{j=1}^{n} b_{jj}, R = \sum_{j=1}^{n} b_{ij}$ for any fixed i,

$$B = (D - A + U)^{-1},$$

D = diagonal matrix with degree values,

U = matrix with unitary elements.

Time complexity of this centrality is $\mathrm{O}(\mathrm{n}^3)$ for all vertices. This centrality measure does not consider only geodesic distances, also all other path distances. This measure is used in many situations where the flow of information along the network is considered.

4.5 Stress centrality

This measure was first introduced by Shimbel (1953). It is the first centrality index based on shortest path.

Mathematically, stress centrality $C_s(v)$ of a node v is given by

$$C_S(v) = \sum_{s \notin N} \sum_{t \notin N} \sigma_{st}(v)$$

where $\sigma_{st}(v)$ denotes the number of shortest paths between s and t containing v.

The same definition can be applied for edges given by $C_S(e) = \sum_{s \in N} \sum_{t \in N} \sigma_{st}(e)$ where $\sigma_{st}(e)$ denotes the number of shortest paths between s and t containing e.

It is obvious that communication or transportation will follow different paths in social network. This measure calculates the amount of communication passes through a node or an edge.

4.6 Betweenness centrality

The distance from one vertex to other vertices is not only important, more important is that which vertices staying on which shortest paths among pairs of others vertices in communication networks. Betweenness centrality was introduced first by Shaw (1954) as a measure for the control of human network in communication among other humans in social networks by Freeman (1977, 1978).

Betweenness centrality $C_B(x)$ of a vertex x in the network is given by

$$C_B(x) = \sum_{y \neq z \in N} \frac{\sigma_{st}(x)}{\sigma_{st}}$$

where $\sigma_{st}(x)$ denotes the number of shortest paths between s and t containing x, and σ_{st} denotes the number of all shortest paths between s and t in the network.

This is a global centrality measure and very complex in large network. Time complexity for this method is O(mn) by Brandes (2001). This measure determines the actor that controls information among others nodes via connecting path.

4.7 Eigenvector centrality

Bonacich (1972) proposed a new measure which was based on eigenvector of the greatest eigenvalue of the adjacency matrix. An alternative measure was suggested by Bonacich and Lloyd (2001) that gives meaningful comparable results. Bonacich (2007) proposed some unique theory of eigenvector centrality.

Let A be the adjacency matrix such that $a_{ij} = 1$ if node i is connected to node j and $a_{ij} = 0$ if not. Then, eigenvector centrality for node i is given by

$$Ax = \lambda x, \lambda x_i = \sum_{j=1}^{n} a_{ij} x_j \quad i = 1, 2 \ldots n, \quad \lambda \text{ is a constant.}$$

This measure is based on the idea that a node is important if neighbor is important. Time complexity is O(n^2) for all vertices. This measure is applicable in neural network (Fletcher and Wennekers 2017).

4.8 Katz centrality

It was first introduced by Katz (1953). It measures influence by calculating the total number of walks between a pair of nodes within a network.

Let A be the adjacency matrix of a network. Elements (a_{ij}) of A are given by

$$a_{ij} = \begin{cases} 1, & \text{if there is a link between } i \text{ and } j \\ 0, & \text{otherwise.} \end{cases}$$

Mathematically, Katz centrality $C_K(i)$ of a node i is given by

$$C_K(i) = \sum_{k=1}^{\infty} \sum_{j=1}^{n} \alpha^k \left(A^k\right)_{ji}$$

where α is smaller than the reciprocal of the absolute value of the largest eigenvalue of A. The powers of A mean the presence of links among two nodes through neighbors.

This measure is generalization of eigenvector centrality and based on the centrality of its neighbors. This measure is applicable in WWW (Newman 2010).

4.9 Subgraph centrality

It was first proposed by Estrada and Rodriguez-Velazquez (2005) by calculating the number of closed walks. The number of closed walks of length k starting and ending on the node i is given by the local spectral moments $\mu_k(i) = \left(A^k\right)_{ii}$, ith diagonal element of kth power of the adjacency matrix A of a network.

Mathematically, the subgraph centrality $C_{SG}(i)$ of a node i is given by

$$C_{SG}(i) = \sum_{k=0}^{\infty} \frac{\mu_k(i)}{k!}.$$

Closed walks are weighted such that their influence on the centrality decreases as the order of the walks increases, which gives more importance to smaller walks than to longer walks. Time complexity is same as eigenvector centrality. This is applicable in molecular connection of spectral structure in chemistry.

4.10 Functional centrality

It was first introduced by Rodriguez et al. (2006). Mathematically, the functional centrality $C_F(i)$ of a node i is given by

$$C_{\mathrm{F}}(i) = \sum_{j=0}^{\infty} a_j \mu_j(i).$$

The number of closed walks of length l is weighted by a_l $.\mu_k(i)$ are the local spectral moments obtained by the number of closed walks of length k starting and ending on the node i.

This measure is a generalization of subgraph centrality. It is applicable in detecting lethality of proteins in protein interaction networks (Tew et al. 2007).

4.11 Leverage centrality

This measure was first introduced by Joyce et al. (2010). Mathematically, leverage centrality $C_{\mathrm{L}}(i)$ of a node i is given by

$$C_{\mathrm{L}}(i) = \frac{1}{k_i} \sum_{N_i} \frac{k_i - k_j}{k_i + k_j}.$$

This measure combines the degree k_i of a node and the degree k_j of each of its neighbor's averaged over all neighbors N_i.

This measure is applicable in human brain network (Joyce et al. 2010). It is seen from definition that this measure is unique from existing measures and counts not only degree of given node but also degree of neighbors.

4.12 K-shell centrality

It is important to find influential spreaders for controlling spreading process on social networks. k-shell decomposition can identify most influential spreader in network. Kitsak et al. (2010) first proposed k-shell decomposition. Hence, it was improved by others researchers Zeng and Zhang (2013), Liu et al. (2014). Also for weighted network, a k-shell decomposition method was introduced by Garas et al. (2012).

In this method, a network is split into k-shell structure. First, we remove all nodes of degree $k = 1$ recursively from network and we assign $k_s = 1$. This procedure is succeeding iteratively till the nodes with degree $k \geq 2$ left the network. Subsequently, we remove all nodes with degree $k = 2$ and assign to them the integer value $k_s = 2$. Again, this procedure is succeeded continuously until there are only nodes with degree $k > 3$ left the network, and so on. This process is applied till all nodes in the network have identified to any one of k-shells. Core nodes (inner nodes) refer to more importance of node and hence more centrality than non-core nodes (outer nodes).

All the nodes in the network are identified by k-shell value. K-shell centrality of a node is equal to the k-shell value of the node. High k-shell value of nodes indicates the more central node.

In Fig. 3, the nodes 1, 2, 3 and 4 are lying in the k_3 shell, the nodes 5, 6 and 7 are lying in the k_2 shell, and nodes 8, 9, 10, 11, 12, 13 are lying in the k_1 shell. The nodes that lain in the k_3 shell are most influential spreaders, and the corresponding nodes are called the most core nodes.

It differentiates among core nodes and non-core nodes, but it takes all nodes as same priority. Calculation of this centrality is very easy and considers global structure. It is applicable to find super-spreader node in the network.

4.13 Neighborhood coreness centrality

This measure was first introduced by Bae and Kim (2014). The basic consideration to this centrality is that a node with more links to the neighbors located in the core of the network is more powerful. The neighborhood coreness C_{NC} of a node v is defined as follows.

$$C_{\mathrm{NC}}(v) = \sum_{w \in N(v)} ks(w)$$

where $N(v)$ is the set of the neighbors adjacent to node v and $ks(w)$ its neighbor node w. The extended neighborhood coreness of node v is defined as:

$$C_{\mathrm{NC+}}(v) = \sum_{w \in N(v)} C_{\mathrm{NC}}(w)$$

where $C_{\mathrm{nc}}(w)$ is the neighborhood coreness of neighbor w of node v.

This centrality combines degree and coreness of a node. It is easy to compute and consider local structure. This measure may be applied to find capability of spreading infection, spreading pollution, etc.

4.14 Neighborhood centrality

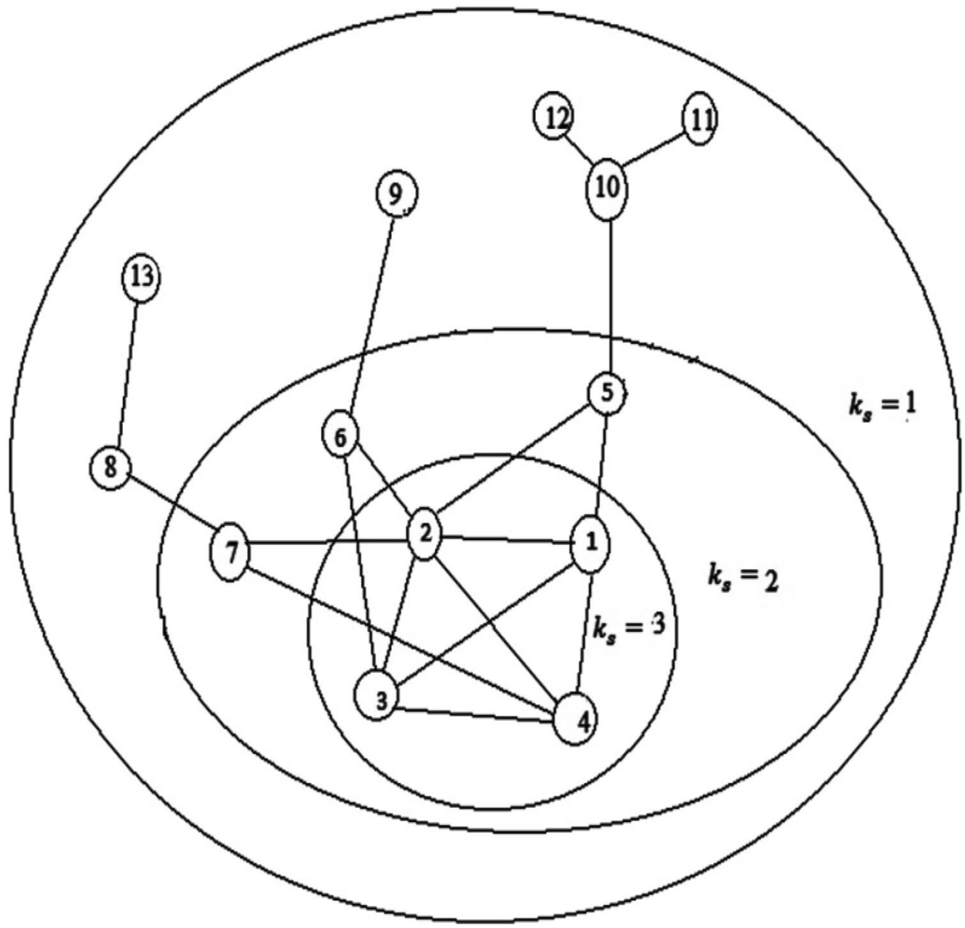
This measure was proposed by Liu et al. (2015) to identify influential spreaders. It considers the importance of n-steps neighbors. Longer distance from a node has a smaller effect to that node. Neighbourhood centrality $C_{\mathrm{N}}(i)$ of a node i is given by

$$C_{\mathrm{N}}(i) = d_i + p \sum_{j \in N(i)} d_j + p^2 \sum_{k \in N(j) \setminus i} d_k + p^3 \sum_{l \in N(k) \setminus j} d_l + \cdots + p^n \sum_{s \in N(l) \setminus x} d_s$$

where d_i is degree or k-shell value of node i, n is the step of neighbors, and $p \in [0, 1]$ is an adjustable parameter.

This centrality is hard to compute and consider global structure. This measure is applicable to measure spread of smoking, alcohol consumption, happiness, health screening.

Fig. 3 K-shell decomposition of a network



We present the gist of time complexity, limitations and applications area of some standard centrality measures in social networks (Table 2).

5 Applications

There are many applications of centrality measures in sociology, psychology, economics, anthropology, biology, terrorist, traffic, neural, business, etc.

In biological networks, nodes are gene, protein and metabolites and edges are interaction or cooperation between them. Degree centrality was used by Jeong et al. (2001) to correlate the degree of a protein in the network with lethality of its removal. Wuchty and Stadler (2003) applied closeness centrality to many biological networks and show the correspondence with the service facility location problem. Koschützki and Schreiber (2004) applied five centrality measures (degree, closeness, eccentricity, betweenness and eigenvector) to PPI network and gene regulatory network (Koschützki and Schreiber 2008) and show a strong correlation between eigenvector and closeness.

In co-authorship network, nodes are researcher or scientist and edges are co-author between them. Liu et al. (2005) applied centrality measures to co-authorship of JCDL research community and compared three measure of centrality and showed that betweenness centrality performed best among them. Betweenness centrality was used by Chen (2006) in the scientific literature and applied to weighted co-authorship network by Liu et al. (2007). Centrality measures were applied to impact analysis to co-authorship network analyzed by Yan and Ding (2009).

Security is an important part of people in online or offline network. Cybercrimes are using internet and mobile phones to do financial theft. In criminology, centrality measures detect the leaders of the criminal network. Degree, closeness, betweenness was applied by Sparrow (1991) in criminal intelligence. Coles (2001) also analyzed crime groups as social network. Social network analysis has been widely used in the field of national security as well as organized crime (Borgatti et al. 2009).

Roadways and junctions between roadways form a road network. Traffic congestion (Holme 2003) is a big problem of our daily life in the road network. Everyone wants low cost shortest path when traveling. Centrality

Table 2 A comparison of some standard centrality measures

Centrality measures	Time complexity	Limitations	Applications area
Degree centrality	O(m)	This measure counts only direct connections but does not take indirect connections	Finding a person who influences directly others
Closeness centrality	O(n^3)	This measure counts the distance from one node to others, but it does not depend on the path among nodes	Determining a location that can spread information fast
Eccentricity centrality	O(mn)	This measure takes only maximum distance between nodes	Determining a location that can spread information fast
Information centrality	O(n^3)	This measure does not consider shortest path but considers all paths	Many situations where flow of information along network is considered important
Stress centrality	O(mn)	This measure considers all shortest path between any two vertices	Determining shortest path in transportation problem
Betweenness centrality	O(n^3) or O(mn)	This measure counts only the distance between nodes, but it does not differentiate among the nodes of the network	Determining the node that controls the information among other nodes via connecting paths
Eigenvector centrality	O(n^2)	This measure counts only important links	Determining the location for emergency facility
K-shell centrality	O(m)	It differentiates among core nodes and non-core nodes, but it takes all the nodes as same priority	Finding super-spreader node in a network

measures (degree, closeness, betweenness, and eigenvector) are useful to identify the traffic congestion in the road network (Jayaweera et al. 2017).

Transportation network is very useful for transportation, telecommunication, business. Road, railways, urban street, airline, water path are different category of this network. Centrality measures (degree, closeness, betweenness) are used to analyze urban transport network (Wang and Xiufen 2017). Betweenness centrality applied to analyze global structure of worldwide air transportation network (Guimera et al. 2005).

In school or college, interactions or relationships between students of a class form a student network. Students improve their character, knowledge, etc. The social network analysis explains the behavior of relationship among students. The degree centrality and betweenness centrality measures analyze relationship among students (Grunspan et al. 2014). Position of student in communication and interaction networks is related to the student's performance (Bruun and Brewe 2013).

Drug abuse is a common problem increasing day to day among students for major depression. A network analysis of symptoms consisting symptoms and connection between symptoms gives a clear idea that which symptoms are central. Three measures of symptoms centrality (Boccaletti et al. 2006) are computed for both the cross-substance class network and the individual substance class networks (Rhemtulla et al. 2016).

6 Conclusion and future works

Centrality measures are very useful for network analysis. But their proper information, selection and application are also needful. Our main research work has to present information about centrality measures that can help to select proper application. There are creating and developing many centrality measures because of different new and large networks and different applications. Most researchers have focused to show their centrality measures are unique and better from others. To make difference and apply them in proper area is still an open issue. We have tried to present this.

In general, social network is a set of nodes or actors (person, organization, etc.) and a relation or connection or links between actors. All the persons may not have same character or behavior, and their relations may not have same strength in the network. So, there is vagueness in social network. This type of network where actors assume as fuzzy and their relations assume as fuzzy relations is called fuzzy social network. The fuzzy logic can make result better because of the better definitions and concept related to nodes and links in the social networks. Investigation on more about fuzzy social network analysis is also an open issue.

In this survey paper, we first discussed some basic concepts and reviewed the measure of centralities and compared them in the table. Our purpose is to study measures of centralities, and our aim for future is to improve the existing measure of centralities for better acceptance and is to research on measures of centrality on more acceptable fuzzy social networks.

References

Bae J, Kim S (2014) Identifying and ranking influential spreaders in complex networks by neighborhood coreness. Phys A 395:549–559

Barrat A et al (2004) The architecture of complex weighted networks. Proc Natl Acad Sci 101(11):3747–3752

Bavelas A (1948) A mathematical model for group structures. Appl Anthropol 7:16–30

Bavelas A (1950) Communication patterns in task oriented groups. J Acoust Soc Am 22:725–730

Beauchamp MA (1965) An improved index of centrality. Behav Sci 10:161–163

Boccaletti S et al (2006) Complex networks: structure and dynamics. Phys Rep 424:175–308

Bonacich P (1972) Factoring and weighing approaches to status scores and clique identification. J Math Sociol 2(1):113–120

Bonacich P (1987) Power and centrality: a family of measures. Am J Sociol 92(5):1170–1182

Bonacich P (2007) Some unique properties of eigenvector centrality. Soc Netw 29:555–564

Bonacich P, Lloyd P (2001) Eigenvector-like measures of centrality for asymmetric relations. Soc Netw 23(3):191–201

Borgatti SP (2006) Identifying sets of key players in a social network. Comput Math Organ Theory 12:21–34

Borgatti SP et al (2009) Network analysis in the social sciences. Sci New Ser 323(5916):892–895

Brandes U (2001) A faster algorithm for betweenness centrality. J Math Sociol 25(2):163–177

Brandes U (2008) On variants of shortest-path betweenness centrality and their generic computation. Soc Netw 302:136–145

Bruun J, Brewe E (2013) Talking and learning physics: predicting future grades from network measures and Force Concept Inventory pretest scores. Phys Rev Phys Educ Res 9:020109

Chen CM (2006) Cite space II: detecting and visualizing emerging trends and transient patterns in scientific literature. J Am Soc Inform Sci Technol 57(3):359377

Coles N (2001) Analyzing serious crime groups as social network. Br J Criminol 41:580–594

Costenbader E, Valente TW (2003) The stability of centrality measures when networks are sampled. Soc Netw 25:283–307

Dijkstra EW (1959) A note on two problems in connexion with graphs. Numer Math 1:269–271

Estrada E, Rodriguez-Velazquez JA (2005) Subgraph centrality in complex networks. Phys Rev 71:056103

Everett MG, Borgatti SP (1999) The centrality of groups and classes. J Math Sociol 23(3):181–201

Fletcher JM, Wennekers T (2017) From structure to activity: using centrality measures to predict neuronal activity. Int J Neural Syst 27:1750013

Freeman LC (1977) A set of measures of centrality based on betweenness. Sociometry 40(1):35–41

Freeman LC (1978) Centrality in social networks conceptual clarification. Soc Netw 1:215–239

Freeman LC, Borgatti SP, White DR (1991) Centrality in valued graphs: a measure of betweenness based on network flow. Soc Netw 13(2):141–154

Garas A, Schweitzer F, Havlin S (2012) A k-shell decomposition method for weighted networks. New J Phys 14:083030

Grunspan DZ, Wiggins BL, Goodreau SM (2014) Understanding classrooms through social network analysis: a primer for social network analysis in education research. CBE Life Sci Educ 13:167–178

Guimera R et al (2005) The worldwide air transportation network: anomalous centrality, community structure, and cities global roles. Proc Natl Acad Sci 102(22):7794–7799

Hage P, Harary F (1995) Eccentricity and centrality in networks. Soc Netw 17:57–63

Holme P (2003) Congestion and centrality in traffic flow on complex networks. Adv Complex Syst 6(2):163–176

Jayaweera IMLN, Perera KKKR, Munasinghe J (2017) Centrality measures to identify traffic congestion on road networks: a case study of Sri Lanka. IOSR J Math 13(2):13–19

Jeong H et al (2001) Lethality and centrality in protein networks. Nature 411(6833):41–42

Joyce KE et al (2010) A new measure of centrality for brain networks. PLoS ONE 5(8):12200

Katz L (1953) A new status index derived from sociometric analysis. Psychometrika 18(1):39–43

Kitsak M et al (2010) Identification of influential spreaders in complex networks. Nat Phys 6(11):888–893

Koschutzki D et al. (2005) Centrality indices. In: Brandes U, Erlebach T (eds.) Network analysis: methodological foundations 3418:16–61

Koschutzki D, Schreiber F (2008) Centrality analysis methods for biological networks and their application to gene regulatory networks. Gene Regul Syst Biol 2:193–201

Koschützki D, Schreiber F (2004) Comparison of centralities for biological networks. German Conf Bioinf 53:199–206

Liu X et al (2005) Co-authorship networks in the digital library research community. Inf Process Manage 41:1462–1480

Liu LG et al (2007) Weighted network properties of Chinese nature science basic research. Phys A Stat Mech Appl 377(1):302–314

Liu JG, Ren ZM, Guo Q (2014) Ranking the spreading influence in complex networks. Phys A 392(18):4154–4159

Liu Y et al (2015) Identify influential spreaders in complex networks: the role of neighborhood. Phys A 452:289–298

Newman MEJ (2001) Scientific collaboration networks I. Network construction and fundamental results. Phys Rev E 64:016131

Newman MEJ (2004) Analysis of weighted networks. Phys Rev E 389:2134–2142

Newman MEJ (2010) Networks: an introduction. Oxford University Press, New York

Nieminen J (1974) On the centrality in a graph. Scand J Psychol 15:322–336

Opsahl T, Panzarasa P (2009) Clustering in weighted networks. Soc Netw 31:155–163

Opsahl T, Agneessens F, Skvoretz J (2010) Node centrality in weighted networks. Generalizing degree and shortest paths. Soc Netw 32(3):245–251

Rhemtulla M et al (2016) Network analysis of substance abuse and dependence symptoms. Drug Alcohol Depend 161:230–237

Rodriguez JA, Estrada E, Gutierrez A (2006) Functional centrality in graphs. Linear Multilinear Algebra 55:293–302

Sabidussi G (1966) The centrality index of a graph. Psychometrika 31(4):581–603

Shaw ME (1954) Group structure and the behavior of individuals in small groups. J Psychol 38:139–149

Shimbel A (1953) Structural parameters of communication networks. Bull Math Biophys 15(4):501–507

Sparrow MK (1991) The application of network analysis to criminal intelligence: an assessment of the prospects. Soc Netw 13(3):251–274

Stephenson K, Zelen M (1989) Rethinking centrality: methods and examples. Soc Netw 11:1–37

Tew KL, Li XL, Tan SH (2007) Functional centrality: detecting lethality of proteins in protein interaction networks. Genome Inform 19:166–177

Wang K and Xiufen F (2017) Research on centrality of urban transport network nodes, AIP Conference Proceedings 1839, 020181

Wang J et al (2017) A novel weight neighborhood centrality algorithm for identifying influential spreaders in complex networks. Phys A S0378–4371(17):30121–30128

White DR, Borgatti SP (1994) Betweenness centrality measures for directed graphs. Soc Netw 16:335–346

Wuchty S, Stadler PF (2003) Centers of complex networks. J Theor Biol 223(1):45–53

Yan E, Ding Y (2009) Applying centrality measures to impact analysis: a co-authorship network analysis. J Am Soc Inform Sci Technol 60(10):2107–2118

Zeng A, Zhang CJ (2013) Ranking spreaders by decomposing complex networks. Phys Lett A 377(14):1031–1035