part 3: Maximum Likelihood Estimate

1\. a)
$$\hat{\theta}_{MLE} = \arg\max \prod_{i=1}^{n} P(y_i \mid x_i ; h_\theta)$$

if observations y_is are <u>i.i.d</u>

b) $P(y_i \mid x_i ; \theta)$: $y_i - h_\theta(x_i) \sim N(0, \sigma^2)$

$$\Rightarrow P(y_i - h_\theta(x_i)) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{y_i - h_\theta(x_i)}{\sigma}\right)^2}$$

c)
$$\hat{\theta}_{MLE} = \arg\max \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} e^{-1/2\left(\frac{y_i - h_\theta(x_i)}{\sigma}\right)^2}$$

$\Rightarrow$ Since log is a monotonically increasing function $\Rightarrow$ $\hat{\theta}_{MLE}$ of log is the same as $\hat{\theta}_{MLE}$ of normal likelihood.

$$\Rightarrow \hat{\theta}_{MLE} = \arg\max \log \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{y_i - h_\theta(x_i)}{\sigma}\right)^2}$$

$$= \sum_{i=1}^{n} \left[-\frac{1}{2}\ln(2\pi) - \ln\sigma - \frac{1}{2}\left(\frac{y_i - h_\theta(x_i)}{\sigma}\right)^2 \right]$$

$$= \arg\max \; -\frac{n}{2}\ln(2\pi) - n\ln\sigma - \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y_i - h_\theta(x_i)\right)^2$$

n

$$\Rightarrow \quad = \arg\min \sum_{i=1}^{n} \left(y_i - h_\theta(x_i)\right)^2$$

$\Rightarrow$ if residuals are normally dist

$\Rightarrow$ least square cost = MLE