HW 6

1)

$$V_{0,2} \to \epsilon V_{1,1} \epsilon$$

$$V_{1,1} \to 1 V_{1,1} 0$$

$$\to 0 V_{1,1} 1$$

$$\to V_{1,1} V_{1,1}$$

$$\to \epsilon$$

2) $\text{Prefix}(L) = \{\epsilon, 0, 1, 10, 01, 011\}$

Let L be the grammer $L = \langle V, T, S, P \rangle$

L is context free so it can have a CFG in CNF of

$$V \to a$$
$$V \to XY$$

the CFG being L' can represent $\text{Prefix}(L)$ where $L' = \langle V', T, S', P' \rangle$ with new production rules

$$V' \to \epsilon \mid a$$
$$V' \to \epsilon \mid X' \mid Y'$$

where these rules represent and encompass all prefixes of L.

Therefore since $\text{Prefix}(L)$ is represented by the CFG shown, we have shown that if L is context free $\text{Prefix}(L)$ is also context free

3) using a pumping lemma

Assume L is context-free

therefore there is a string z with a pumping constant p such $z \in L$ and $|z| \geq p$, then z can be split $z = uvwxy$ where $|vwx| \leq p$, $|vx| > 0$, $uv^i wx^i y \in L$ when $i \geq 0$

Choosing the string $z = 0^{p^3}$ $z \in L$, $|z| > p$

Since $|vwx| \leq p$, and $|vx| > 0$, we know that $0 < |vx| \leq p$

Applying the pumping lemma principle

$$p^3 < uv^i wx^i y \leq p^3 + (i-1)p \text{ as } |vx| \leq p$$

Choosing to pump $i = 2$ to math $uvvwxxy$

the conditions become

$$p^3 < uv^i wx^i y < p^3 + (2-1)p = p^3 + p$$

following p^3 in L the next string should be $0^{(p+1)^3}$, However since $p^3+p < (p+1)^3$ pumping uv^2wx^2y will not produce enough 0's to achieve $0^{(p+1)^3}$

Thus, uv^2wx^2y is not the language $L = (0^{n^3} : n \geq 0)$, thus a contradiction and $(0^{n^3} : n \geq 0)$ is not context free