HW 3

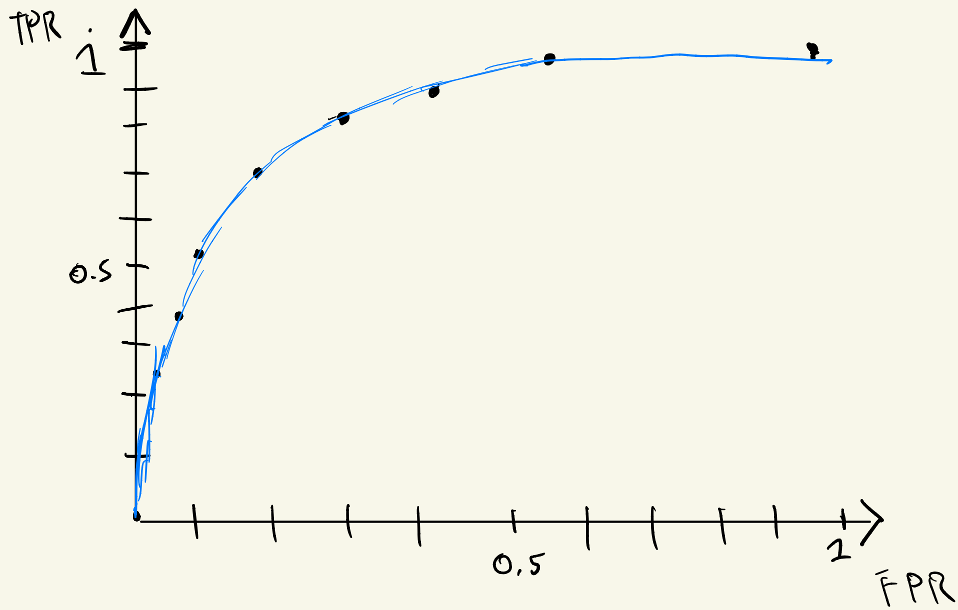
A)

Threshold	True Positives (TP)	False Positives (FP)	True Negatives (TN)	False Negatives (FN)	TPR	FPR
0.95	10	2	58	30	.25	.03333
0.85	15	4	56	25	.375	.06666
0.75	22	8	52	18	.55	.1333
0.65	28	12	48	12	.7	.2
0.55	32	18	42	8	.8	.3
0.45	35	25	35	5	.875	.4166
0.35	38	35	25	2	.95	.5833

) For each threshold: Calculate the True Positive Rate (TPR) and False Positive R

$$TPR = \frac{TP}{TP+FN} \qquad FPR = \frac{FP}{FP+TN}$$

B)



c) $Area = \sum \frac{1}{2}(FPR_{i+1} - FPR_i) \times (TPR_{i+1} + TPR_i)$

$A_1 = \frac{1}{2}(.0667 - .033) \times (.25 + .375) = 0.0104$

$A_2 = \frac{1}{2}(.133 - .667) \times (.375 + .55) = 0.0302$

$A_3 = \frac{1}{2}(.200 - .133) \times (.55 + .7) = 0.0417$

$A_4 = \frac{1}{2}(.3 - .2) \times (.7 + .8) = 0.075$

$A_5 = \frac{1}{2}(.4167 - .3) \times (.8 + .875) = 0.0927$

$A_6 = \frac{1}{2}(.5833 - .4167) \times (.875 + .95) = 0.1577$

$A_7 = \frac{1}{2}(.033 - 0) \times (.25 + 0) = .0042$

$A_8 = \frac{1}{2}(1 - .583) \times (.95 + 1) = .4115$

$AUC = \sum_{i=1}^{8} A$

$AUC = .0104 + .0302 + .0417 + .0750 +$
$.0927 + .1577 + .0042 + .04115$

$= 0.8234$

$\boxed{\approx .823}$

2)

A) $x(1,1,01,0,1) \rightarrow$ BCEF   bias $=1$

1 0 -1 1

$1 + (1 \cdot 1) + (0 \cdot 0) + (0 \cdot -1) + (1 \cdot 1) = 3$

$\text{sign}(3) = 1$ $\boxed{\text{so A is } 1}$

B) $x(0,1,0,1,0,1) \rightarrow$ CDEF

0 -3 1 2   Bias $=1$

$1 + (1 \cdot 0) + (1 \cdot -3) + (1 \cdot 0) + (1 \cdot 2) = 0$

$\text{sign}(0) = -1$ $\boxed{\text{so b is } -1}$

since sign $0 = -1$

3) Linear threshold → $w_0 \sum_{i=1}^{n} w_i x_i > 0$

positive class based off $n/2$ being false

Seperates linear hyperplane

$n \geq 4$ proves this as $n/2$ will create a non linear boundary because the sum of weights can't identify when $n/2$ inputs are false versus other

$n = 4$

Positive: $(0,0,1,1)$ $(0,1,0,1)$, $(0,1,1,0)$, etc
with 2 zeros

Negative: Any other combination

this shows you can't distinguish using a linear boundary because the false cases have fewer or more than 2 zeros. they cannot be seperated soley on a weighted sum.

Perceptrons cannot handle parity function, on an exact count being true or false

4)

x_1	x_2	x_3	y
0	0	0	1
0	0	1	-1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	-1
1	1	0	1
1	1	1	1

conflicting outputs

conflicting outputs

The data is not linearly separable since there is no single set of weights that can consistently separate positive and negative classes. It lacks linear separability in the feature space.

$$x_1, x_2, x_3 = 0, 0, 1 \to y = -1$$

$$x_1, x_2, x_3 = 0, 1, 1 \to y = 1$$

Positive Class $\to$ (0,0,0), (0,1,0), (0,1,1), (1,0,0), (1,1,0)
$y=1$ (1,1,1)

Negative Class $\to$ (0,0,1) (1,0,1)
$y=-1$

these cannot be linearly seperable as since $y=-1$ is scattered across $y=1$ or the positive class.

Examples

$(0,0,1)$ and $(0,0,0)$ adjacent but have different labels

Same with $(1,0,1)$ and $(1,0,0)$ which are adjacent but have diff labels

Since the data is not linearly seperable it cannot be represented with a linear threshold function