HW 4 | Arman Miri, CSCI 184, 0770000C039

1)

$$C = 1/m \sum_{i=1}^{m} (o^i - y^i)^2$$

$$\frac{\partial L}{\partial o^i} = 2(o^i - y^i)$$

Sigmoid $\rightarrow$ $o^i = \sigma(z_o^i) = \frac{1}{1+e^{-z_o^i}}$

$$\frac{\partial L}{\partial z_o^i} = \frac{\partial L}{\partial o^i} \cdot \frac{\partial o^i}{\partial z_o^i}$$

$$\frac{\partial o^i}{\partial z_o^i} = \sigma(z_o^i) \cdot (1 - \sigma(z_o^i))$$

$$= o^i \cdot 1 - o^i$$

$$\frac{\partial L}{\partial z_o^i} = 2(o^i - y^i) \cdot o^i \cdot (1 - o^i$$

$$\frac{\partial L}{\partial z_j^i} = \frac{\partial L}{\partial z_0^i} \cdot \frac{\partial z_0^i}{\partial h_j^i} \cdot \frac{\partial h_j^i}{\partial z_j^i}$$

$$\frac{\partial L}{\partial z_j^i} = \frac{\partial L}{\partial z_0^i} \cdot w_{j,0} \cdot h_j^i \cdot (1 - h_j^i)$$

$$\frac{\partial L}{\partial w_{1,2}} = \frac{\partial L}{\partial z_2^i} \cdot \frac{\partial z_2^i}{\partial w_{1,2}}$$

$$= \frac{\partial L}{\partial z_2^i} \cdot x_1^i$$

$$= \frac{\partial L}{\partial z_0^i} \cdot w_{2,0} \cdot h_2^i \cdot (1 - h_2^i)$$

$$= \left(\frac{\partial L}{\partial z_0^i} \cdot w_{2,0} \cdot h_2^i \cdot (1 - h_2^i) \right) \cdot x_1^i$$

$$w_{1,2} \leftarrow w_{1,2} - \alpha \cdot \frac{\partial L}{\partial w_{1,2}}$$

$$\boxed{w_{1,2} \leftarrow w_{1,2} - \alpha \left(\left(2(o^i - y^i) \cdot o^i \cdot (1-o^i) \cdot w_{2,0} \cdot h_2^i \cdot (1-h_2^i) \right) \cdot x_1^i \right)}$$

2a)

$i_1 = 2$ $\quad w_{11} = 1,\ w_{12} = -.5,\ w_{21} = .5,\ w_{22} = -1$

$i_2 = -1$ $\quad b_1 = .5,\ b_2 = .5,\ b_3 = -.1,\ b_4 = .5$

$w_{31} = .5,\ w_{32} = -1,\ w_{41} = -.5,\ w_{42} = 1$

$t_1 = 1,\ t_2 = .5$

$$h_1 = \sigma\left(w_{11} \cdot i_1 + w_{21} \cdot i_2 + b_1\right) \qquad \sigma = \frac{1}{1+e^{-x}}$$

$$h_2 = \sigma\left(w_{12} \cdot i_1 + w_{22} \cdot i_2 + b_2\right)$$

$$h_1 = \sigma\left(1 \cdot 2 + .5 \cdot -1 + .5\right) = \sigma(2) = 0.8808$$

$$h_2 = \sigma\left(-.5 \cdot 2 + 1 \cdot -1 - .5\right) = \sigma(-.5) = 0.3775$$

$$h_3 = \mathrm{ReLU}(h_1) = \max\{0, h_1\} = 0.8808$$

$$h_4 = \mathrm{ReLU}(h_2) = \max\{0, h_2\} = 0.3775$$

$$o_1 = \sigma(w_{31} h_3 + w_{32} h_4 + b_3)$$

$$o_2 = \sigma(w_{41} h_3 + w_{42} h_4 + b_4)$$

$$o_1 = \sigma(0.5 \cdot 0.8808 + (-.5)(.3775) + -1)$$

$$= \sigma(-0.74835)$$

$$= 0.3212$$

$$o_2 = \sigma(1 - (0.3775) + 1 \cdot .8808 + .5)$$

$$= \sigma(0.0033)$$

$$= 0.4992$$

$$\boxed{\sigma_1 = 0.3212, \quad \sigma_2 = 0.4992}$$

2b)

$$MSE = \left((o_1 - t_1)^2 + (o_2 - t_2)^2\right)$$

$$= (.3212 - 1)^2 + (.4992 - .5)^2$$

$$= .46077 + 0.000000064$$

$$\boxed{= 4608}$$

2c)

$$w_{21} \leftarrow w_{21} - \alpha \frac{\partial MSE}{\partial w_{21}}$$

$$\frac{\partial MSE}{\partial o_1} = o_1 - t_1 = .3212 - 1 = -.6788$$

$$\frac{\partial MSE}{\partial o_2} = o_2 - t_2 = .4992 - .5 = -.0008$$

h_1 only contributes from o_1 thus

$$\frac{\partial MSE}{\partial h_1} = \frac{\partial MSE}{\partial o_1} \cdot w_{31} = -.6788 \cdot .5 = -.3394$$

$$\frac{\partial MSE}{\partial w_{21}} = \frac{\partial MSE}{\partial h_1} \cdot \frac{\partial h_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{21}}$$

$$z_1 = w_{11} \cdot i_1 + w_{21} \cdot i_2 + b_2$$

$$\frac{\partial z_1}{\partial w_{21}} = i_2$$

$$\frac{\partial h_1}{\partial z_1} = h_1 \cdot (1 - h_1)$$

$i_2 = -1 \quad h_1 = .8808$

$$\frac{\partial h_1}{z_1} = .8808 \cdot (1 - .8808) = 0.1050$$

$$\frac{\partial MSE}{\partial w_{21}} = (-.3394)(.1050)(-1)$$

$$= 0.0356$$

$$w_{21} \leftarrow w_{21} - \alpha \cdot \frac{\partial MSE}{\partial w_{21}}$$

$$w_{21} \leftarrow 0.5 - 0.1(.0356)$$

$$\boxed{w_{21} \leftarrow 0.4964}$$

3a) state 4 = 10, 5 = 10, 6 = 60 $\gamma = .5$

$R_0 = 9$

$R_1 = 5$

$R_2 = 6$

$$G = R_0 + \gamma R_1 + \gamma^2 R_2$$

$$G = 10 + .5 \cdot 10 + .5^2 \cdot 60$$

$$G = 10 + 5 + 15$$

$$\boxed{G = 30}$$

3b) state 3 = 30, 4 = 10, 3 = 30, 2 = 20 $\gamma = .5$

$$G = R_0 + \gamma R_1 + \gamma^2 R_2 + \gamma^3 R_3 + \gamma^4 R_4$$

$R_0 = 3$

$R_1 = 4$

$R_2 = 3$

$R_3 = 2$

$$G = 30 + .5 \cdot 10 + .5^2 \cdot 30 + .5^3 \cdot 20 + .5^4 \cdot 100$$

$$G = 30 + 5 + 7.5 + 2.5 + 6.25$$

$$\boxed{G = 51.25}$$

3c)

$Q(s,a) = R(s) + \gamma \max_{a'} Q(s',a')$ ← this is V

$Q(3,L) \to R(3)=30,\ V(2),\ \pi(2)=L,\ R(2)=20$

$V(1) = R(1) = 100$

$V(2) = R(2) + \gamma V(1)$

$= 20 + .5 \cdot 100 = 70$

$Q(3,L) = R(3) + \gamma V(2)$

$= 30 + .5 \cdot 70 =$

$\boxed{= 65}$

$Q(3,R) \to R(4)=10,\ V(4),\ \pi(4)=L$

$V(3) = Q(3,L) = 65$

$V(4) = R(4) + \gamma V(3)$

$= 10 + .5 \cdot 65 = 42.5$

$Q(3,R) = R(3) + \gamma V(4)$

$= 30 + .5 \cdot 42.5$

$\boxed{= 51.25}$

$Q(4,R) \rightarrow R(5)=10,\ V(5) \quad \pi(5)=R$

$R(6)=60,\ V(6)=R(6)=60$ (no more)

$$V(5)=R(5)+\gamma V(6)$$
$$=10+.5\cdot 60=40$$

$$Q(4,R)=R(4)+\gamma V(5)$$
$$=10+.5\cdot 40$$
$$=30$$

$$\boxed{\begin{aligned} Q(3,L)&=65\\ Q(3,R)&=51.25\\ Q(4,R)&=30 \end{aligned}}$$

3d)

$Q(2,R) \to R(3) = 30,\ V(3) = Q(3,L) = 65$

$$Q(2,R) = R(2) + \gamma V(3)$$
$$= 20 + .5 \cdot 65$$
$$= 52.5$$

$Q(4,L) \to R(3) = 30,\ V(3) = Q(3,L) = 65$

$$Q(4,L) = R(4) + \gamma V(3)$$
$$= 10 + .5 \cdot 65$$
$$= 42.5$$

$$\boxed{\begin{aligned} Q(2,R) &= 52.5 \\ Q(4,L) &= 42.5 \end{aligned}}$$