HW 4

1) L_1 and L_2 both regular languages over $\{0, 1\}$

Prove $L_1 - L_2 = \{w \mid w \in L_1$ and $w \notin L_2\}$

Regular languages closed under union, intersection and compliment operations

Convert to DFA

$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ for L_1

$M_2 = (Q_2, \Sigma, \delta_2, q_2, F_1)$ for L_2

Since the compliment of a regular language is regular we can construct

DFA M_2^c for $L_2^c = \{w \mid w \in L_2\}$

M_2^c same as M_2 but the non accepting states are swapped

Non accepting states $F_2^c = Q_2 / F_2$

Given DFA M_1 and M_2^c we construct a product automation m that recognizes the intersection $L_1 \cap L_2^c$

- states of the product automotion $Q = Q_1 \times Q_2$
- Start state (q_1, q_2) where q_1, q_2 start states in M_1, M_2
- transition function

$$\delta((q_1, q_2), a) = (\delta_1(q_1, a), \delta_2(q_2, a))$$

for all $a \in E$

Accepting states: $F = \{(q_1, q_2) \mid q_1 \in F_1 \text{ and } q_2 \notin F_2\}$

this recognizes the $L_1 \cap L_2^c$ which is exactly the language $L_1 - L_2$. since the product construction gives us finite automation the language $L_1 - L_2$ is regular