

Homework 1

(Due 22/02/2021, 17:00)

Instructions:

1. Prepare a report (including your answers/plots) to be uploaded on Moodle.
2. The report should be typeset (no handwriting allowed except for lengthy derivations, which may be scanned and embedded into the report).
3. Show all the steps of your work clearly.
4. Unclear presentation of results will be penalized heavily.
5. No partial credits to unjustified answers.
6. Use Matlab or Python for computations.
7. Return all Matlab/Python code that you wrote in a single file.
8. Code should be commented, code for different HW questions should be clearly separated.
9. The code file should NOT return an error during runtime.
10. If the code returns an error at any point, the remaining part of your code will not be evaluated (i.e., 0 points).

Question	Points	Your Score
Q1	50	
Q2	50	
TOTAL	100	

Question 1. [50 points] Assume that a neural population computes weighted linear combinations of its input x , characterized by a system of equations $Ax = b$. Here A is the transfer function and b is the output vector.

A single output measurement is recorded, given by
$$\begin{pmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & -1 & 5 \\ 3 & 3 & 0 & 9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix}.$$

Answer the questions below. Derive the results by hand first and then confirm them on the computer.

- a) Find all solutions x_n to $Ax = 0$.
- b) Find a particular solution x_p to $Ax = b$.
- c) Find all solutions to $Ax = b$.
- e) Find the pseudo-inverse of A .
- e) Find the sparsest solution to $Ax = b$ (the solution with the least number of non-zero entries).
- f) Find the least-norm solution to $Ax = b$ (the solution with the minimum Euclidean norm).

Question 2. [50 points]

‘Reverse inference’ is a common, albeit poorly exercised method in neuroscience. It refers to the practice of inferring that a cognitive process is engaged on the basis of activation in some brain area. For example, if Broca’s area was found to be activated in some task, researchers might infer that the subjects were using language. After a comprehensive search of the literature, we find that Broca’s area was reported to be activated in 103 out of 869 fMRI tasks involving engagement of language, but this area was also active in 199 out of 2353 tasks not involving language.

- a) Assume that the conditional probability of activation given language and activation given no language, each follow a Bernoulli distribution (i.e., active with some probability p , or not with probability $1 - p$). Compute the likelihoods of observed frequencies of activation in literature, as functions of the possible values of their respective Bernoulli probability parameters $p = x_l$ and $p = x_{nl}$. Compute these functions at the values $x = [0:.001:1]$ and plot them as separate bar charts.
- b) Find the values of x_l and x_{nl} that maximize their respective discretized likelihood functions.
- c) Using the likelihood functions computed for discrete x , compute and plot the discrete posterior distributions $P(X|data)$ and the associated cumulative distributions $P(X \leq x|data)$ for both processes (language and no language cases). To do this, assume a uniform prior $P(x) \propto 1$ and note that it will be necessary to compute (rather than ignore) the normalizing constant for Bayes’ rule. Use the cumulative distributions to compute (discrete approximations to) upper and lower 95% confidence bounds on each proportion ($x_{l,nl}$).
- d) Consider the joint posterior distribution $P(X_l, X_{nl}|data)$ over x_l and x_{nl} , the Bernoulli probability parameters for the language and non-language contrasts. Given that these two frequencies are independent, the (discrete) joint distribution is given by the outer product of the two marginals. Plot it (with `imagesc`). Compute (by summing the appropriate entries in the joint distribution) the posterior probabilities that $P(X_l > X_{nl}|data)$ and conversely that $P(X_l \leq X_{nl}|data)$.
- e) Using the estimates from **part b** as the relevant conditional probabilities, and assuming the prior that a contrast engages language, $P(language) = 0.5$, compute the probability $P(language|activation)$ that observing activation in this area implies engagement of language processes. Is the critique on ‘reverse inference’ correct? How confident should you be in implicating language if you observe activity in Broca’s area?