On the Effectiveness of Finite Traces in First-order Temporal Logic

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First-Order Temporal Logic on Finite Traces

Main joint work

[AMO19b] A. Artale, A. Mazzullo, and A. Ozaki. *Do You Need Infinite Time?*. In: IJCAI, 2019.

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[AMO19b] A. Artale, A. Mazzullo, and A. Ozaki. *Do You Need Infinite Time?*. In: IJCAI, 2019.

Related co-authored papers

[AMO18] A. Artale, A. Mazzullo, and A. Ozaki. *Temporal Description Logics over Finite Traces*. In: DL, 2018.

[AMO19a] A. Artale, A. Mazzullo, and A. Ozaki. *Temporal DL-Lite over Finite Traces (Preliminary Results)*. In: DL, 2019.

[AMO20] A. Artale, A. Mazzullo, and A. Ozaki. *Finite vs. Infinite Traces in Temporal Logics*. In: OVERLAY, 2020.

[AMOur] A. Artale, A. Mazzullo, A. Ozaki, *First-order Temporal Logic on Finite Traces: Semantic Properties, Decidable Fragments, and Applications*, submitted to *ACM Trans. Comput. Log.* (under review).

First-Order Temporal Logic on Finite Traces

Motivations

Renewed interest in finite traces applied to:

- verification (cf. e.g. Martin Leucker's talk)
- synthesis (cf. e.g. Giuseppe De Giacomo's & Luca Geatti's talk)
- planning (cf. e.g. Sheila McIlraith's talk)
- data-aware process modelling (cf. e.g. Marco Montali's talk)
- knowledge representation (cf. several talks)

First-Order Temporal Logic on Finite Traces

Goals

1. Semantic and syntactic conditions sufficient to preserve equivalences of FOTL formulas between finite and infinite traces
   - cf. e.g. Ben Greenman's talk/questionnaire
2. FOTL on finite traces in connection with related topics in AI
   - planning (insensitivity to infiniteness [DDM14] & f-FOLTL [BM06])
   - verification (safety [Sis94] & runtime verification maxims [BLS10])
3. Decidability and complexity results for FOTL fragments on finite traces, adapted from the infinite case [GKWZ03]
   - temporal formalisms for knowledge representation applications, e.g. temporal description logics ($\mathcal{ALC}$ & *DL-Lite*) on finite traces

First-Order Temporal Language

$T_{\mathcal{U}}\mathcal{QL}$ syntax

Predicates P (n-ary), terms τ (*constants* a, *variables* x), $\neg$, $\wedge$, $\exists$, $\mathcal{U}$ (until)

$$\varphi, \psi ::= P(\bar{\tau}) \mid \neg\varphi \mid \varphi \wedge \psi \mid \exists x \varphi \mid \varphi \,\mathcal{U}\, \psi$$

Abbreviations ($\vee$, $\rightarrow$, $\leftrightarrow$, $\bot$, $\top$, as usual)

- $\bigcirc\varphi := \bot\mathcal{U}\varphi$, $\Diamond\varphi := \top\mathcal{U}\varphi$, $\varphi\mathcal{U}^{+}\psi := \psi\vee(\varphi\wedge\varphi\mathcal{U}\psi)$, $\Diamond^{+}\varphi := \varphi\vee\Diamond\varphi$
- $\varphi\,\mathcal{R}\,\psi := \neg(\neg\varphi\,\mathcal{U}\,\neg\psi)$, $\bullet\varphi := \top\,\mathcal{R}\,\varphi$, $\Box\varphi := \bot\,\mathcal{R}\,\varphi$, last $:= \Box\bot$, $\varphi\,\mathcal{R}^{+}\psi := \psi \wedge (\varphi \vee \varphi\,\mathcal{R}\,\psi)$, $\Box^{+}\varphi := \varphi \wedge \Box\varphi$

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Fragments

- Two-variable monodic, $T_{\mathcal{U}}\mathcal{QL}^{2}_{\boxed{1}}$:
  ≤ 2 variables + temporal formulas ≤ 1 free variable
- Monadic, $T_{\mathcal{U}}\mathcal{QL}^{mo}$: predicates arity ≤ 1
- One-variable, $T_{\mathcal{U}}\mathcal{QL}_{1}$: ≤ 1 variables
- One-variable constant-free monadic, $T_{\mathcal{U}}\mathcal{QL}^{1,mo}_{\not{c}}$ ($\sim LTL_f \times S5$):
  predicates arity ≤ 1 + ≤ 1 variable $-$ constants

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Examples

- $T_{\mathcal{U}}\mathcal{QL}^2_{\boxed{1}}$:
  $\forall x\big(\mathsf{Reviewer}(x) \rightarrow \Box^+\forall y\big(\mathsf{Submission}(y) \wedge \mathsf{Reviews}(x,y) \rightarrow \Diamond^+\mathsf{Evaluated}(y)\big)\big)$
- $T_{\mathcal{U}}\mathcal{QL}^{mo}$:
  $\forall x\forall y\big(\mathsf{Reviewer}(x) \wedge \mathsf{Submission}(y) \rightarrow \Diamond^+(\mathsf{WritesReview}(x) \wedge \mathsf{Evaluated}(y)\big)$
- $T_{\mathcal{U}}\mathcal{QL}_1$: $\forall x\big(\mathsf{Reviewer}(x) \rightarrow \Box^+\mathsf{HasConflictWith}(x,x)\big)$
- $T_{\mathcal{U}}\mathcal{QL}^{1,mo}_{\not{c}}$: $\forall x\big(\mathsf{Reviewer}(x) \rightarrow \Diamond^+(\mathsf{WritesReview}(x)\big)$

Semantics on Finite and Infinite Traces

First-order temporal interpretation (trace)

$$\mathfrak{M} = (\Delta^{\mathfrak{M}}, (\mathcal{I}_n^{\mathfrak{M}})_{n \in \mathfrak{T}})$$

- $\mathfrak{T}$ interval $[0, \ell]$, $\ell \in \mathbb{N}$, or $[0, \infty)$
- $\mathcal{I}_n^{\mathfrak{M}}$ first-order interpretation with domain $\Delta^{\mathfrak{M}}$
  - constant domain assumption + rigid designators

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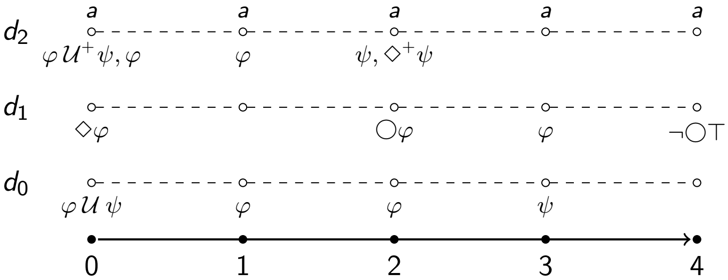
Satisfaction of formulas



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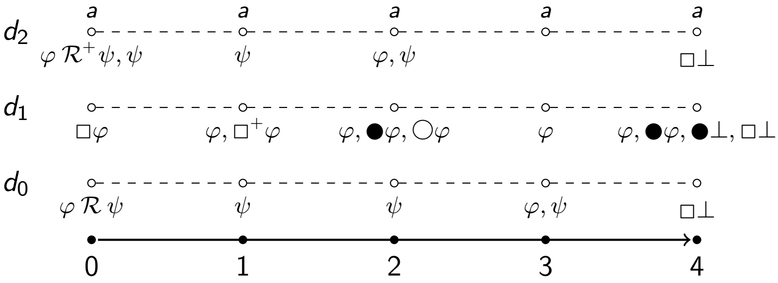

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Satisfaction of formulas

- φ entails ψ, $\varphi \models \psi$, iff for every $\mathfrak{M}$ and every $\mathfrak{a}$, if $\mathfrak{M}, 0 \models^{\mathfrak{a}} \varphi$, then $\mathfrak{M}, 0 \models^{\mathfrak{a}} \psi$
- φ and ψ are equivalent, $\varphi \equiv \psi$, iff $\varphi \models \psi$ and $\psi \models \varphi$
- infinite/finite traces entailment or equivalence: *i/f* subscript

Semantics on Finite and Infinite Traces

First-order temporal interpretation (trace)

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Satisfaction of formulas

- Finite trace, $\mathfrak{T} = [0, \ell] \rightsquigarrow \mathfrak{F} = (\Delta^{\mathfrak{F}}, (\mathcal{F}_n)_{n \in [0, \ell]})$
- Infinite trace, $\mathfrak{T} = [0, \infty) \rightsquigarrow \mathfrak{I} = (\Delta^{\mathfrak{I}}, (\mathcal{I}_n)_{n \in [0, \infty)})$
- Concatenation of $\mathfrak{F}$ with $\mathfrak{I} \rightsquigarrow \mathfrak{F} \cdot \mathfrak{I} = (\Delta^{\mathfrak{F} \cdot \mathfrak{I}}, (\mathcal{F} \cdot \mathcal{I}_n)_{n \in [0, \infty)})$

Finite and Infinite Traces Compared

Extensions of $\mathfrak{F}$

$$Ext(\mathfrak{F}) = \{\mathfrak{I} \mid \exists \mathfrak{I}' : \mathfrak{I} = \mathfrak{F} \cdot \mathfrak{I}'\}$$

Prefixes of $\mathfrak{I}$

$$Pre(\mathfrak{I}) = \{\mathfrak{F} \mid \exists \mathfrak{I}' : \mathfrak{I} = \mathfrak{F} \cdot \mathfrak{I}'\}$$

Semantic conditions

Given a $T_{\mathcal{U}}\mathcal{QL}$ formula φ and $\mathbb{Q} \in \{\exists, \forall\}$

$$\varphi \text{ is } \begin{cases} \mathsf{F}_{\mathbb{Q}} \text{ if for every } \mathfrak{F}, \mathfrak{a}\colon \mathfrak{F} \models^{\mathfrak{a}} \varphi \Leftrightarrow \mathbb{Q}\mathfrak{I} \in Ext(\mathfrak{F}).\mathfrak{I} \models^{\mathfrak{a}} \varphi \\ \mathsf{I}_{\mathbb{Q}} \text{ if for every } \mathfrak{I}, \mathfrak{a}\colon \mathfrak{I} \models^{\mathfrak{a}} \varphi \Leftrightarrow \mathbb{Q}\mathfrak{F} \in Pre(\mathfrak{I}).\mathfrak{F} \models^{\mathfrak{a}} \varphi \end{cases}$$

($\mathsf{F}_{\circ Q}$ / $\mathsf{I}_{\circ Q}$, $\circ \in \{\Rightarrow, \Leftarrow\}$: '$\Rightarrow$' /'$\Leftarrow$' directions of F_Q / I_Q)

Examples

Formulas satisfying exactly one of the corresponding conditions:

($\mathsf{F}_\exists$) $\Diamond^+ \mathsf{last} \lor \Diamond P(x)$ ($\mathsf{I}_\exists$) $\Box \bigcirc \top \lor \mathsf{last}$

($\mathsf{F}_\forall$) $\forall x \Diamond^+ P(x)$ ($\mathsf{I}_\forall$) $\Box^+ P(x) \lor \Diamond^+ (P(x) \land \mathsf{last})$

Finite and Infinite Traces Compared

Syntactic conditions

$$\alpha ::= P(\bar{\tau}) \mid \neg P(\bar{\tau})$$

$\mathcal{U}^+$-formulas

$$\alpha \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \exists x \varphi \mid \varphi \mathcal{U}^+ \psi$$

$\mathcal{R}^+$-formulas

$$\alpha \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \forall x \varphi \mid \varphi \mathcal{R}^+ \psi$$

$\mathcal{U}^+\forall$-formulas

$\mathcal{U}^+$-formulas $\mid \forall x \varphi$

$\mathcal{R}^+\exists$-formulas

$\mathcal{R}^+$-formulas $\mid \exists x \varphi$

$\mathcal{U}$-formulas

$$\alpha \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \exists x \varphi \mid \varphi \, \mathcal{U} \, \psi$$

$\mathcal{R}$-formulas

$$\alpha \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \forall x \varphi \mid \varphi \, \mathcal{R} \, \psi$$

$\mathcal{U}^+\mathcal{R}^+$-formulas

$$\alpha \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \exists x \varphi \mid \forall x \varphi \mid \varphi \, \mathcal{U}^+ \psi \mid \varphi \, \mathcal{R}^+ \psi.$$

Preservation of Equivalences & Satisfiability

	Properties	Equivalences
$\mathcal{U}^+\forall$	$\mathsf{F}_\forall$	$i \Rightarrow f$
$\mathcal{R}^+\exists$	$\mathsf{F}_\exists$	$i \Rightarrow f$
$\mathcal{U}$	$\mathsf{I}_\exists$	$f \Rightarrow i$
$\mathcal{R}$	$\mathsf{I}_\forall$	$f \Rightarrow i$
$\mathcal{U}^+$	$\mathsf{F}_\forall$, $\mathsf{I}_\exists$	$f \Leftrightarrow i$
$\mathcal{R}^+$	$\mathsf{F}_\exists$, $\mathsf{I}_\forall$	$f \Leftrightarrow i$

Table: FOTL fragments with corresponding semantic properties and preservation of equivalences

- $\mathcal{U}^+\mathcal{R}^+$-formulas: finite trace satisfiable $\Rightarrow$ infinite trace satisfiable
- Not vice versa: $\mathcal{U}^+\mathcal{R}^+$-formula satisfiable only on infinite traces

$$\Box^+\forall x\big((P(x) \wedge \neg Q(x)) \vee (Q(x) \wedge \neg P(x)\big) \wedge$$
$$\Box^+\forall x\big((P(x) \rightarrow \Diamond^+ Q(x)) \wedge (Q(x) \rightarrow \Diamond^+ P(x))\big)$$

Connections with Planning and Verification

Planning

- $T_{\mathcal{U}}\mathcal{QL}$ formula φ insensitive to infiniteness [DDM14]
- Extended prenex normal form f-FOLTL [BM06]

Verification

- *LTL* $\mathcal{R}$-($\mathcal{U}$)-formulas express (co-)safety properties [Sis94, AGGMM21]
- Runtime verification maxims [BLS10] $\begin{cases} \text{Impartiality: } \mathsf{F}_{\Rightarrow\forall}, \mathsf{F}_{\Leftarrow\exists} \\ \text{Anticipation: } \mathsf{F}_{\Leftarrow\forall}, \mathsf{F}_{\Rightarrow\exists} \end{cases}$

Complexity of Decidable Fragments

		Finite traces				Bounded traces			
$T_{\mathcal{U}}\mathcal{QL}^{1,mo}_{\not c}$	φ	ExpSpace				NExpTime			
$T_{\mathcal{U}}\mathcal{QL}^{1}$	φ	ExpSpace				NExpTime			
$T_{\mathcal{U}}\mathcal{QL}^{mo}_{\boxed{1}}$	φ	ExpSpace				NExpTime			
$T_{\mathcal{U}}\mathcal{QL}^{2}_{\boxed{1}}$	φ	ExpSpace				NExpTime			
$T_{\mathcal{U}}\mathcal{ALC}$	φ	ExpSpace				NExpTime			
	$\mathcal{K}$	?				ExpTime			
		bool	*horn*	*krom*	*core*	*bool*	*horn*	*krom*	*core*
$T_{\mathcal{U}}$*DL-Lite*$^{\mathcal{N}}_{\alpha}$	$\mathcal{K}$	PSpace	PSpace	PSpace	PSpace	PSpace	PSpace	PSpace	PSpace
$T_{\Box\bigcirc}$*DL-Lite*$^{\mathcal{N}}_{\alpha}$	$\mathcal{K}$	PSpace	PSpace	?	$\geq$ NP	PSpace	PSpace	?	$\geq$ NP

Table: Decidable FOTL fragments and temporal DLs complexity results, where

- φ, formula satisfiability
- $\mathcal{K}$, knowledge base (global) satisfiability
- $\alpha \in \{$*bool*, *horn*, *krom*, *core*$\}$

Future Work

FOTL safety and co-safety fragments on finite/infinite traces

- Complexity results have been recently established for (propositional) LTL safety and co-safety fragments on finite and infinite traces [AGGMM21]
- Lift this complexity analysis to FOTL safety and co-safety fragments on finite and infinite traces?

Proof theory of decidable FOTL fragments on finite traces

- $\mathcal{T}_{\mathcal{U}}\mathcal{QL}$ validities on finite and infinite traces are not r.e. [GKWZ03, CMP99]
- Monodic $\mathcal{T}_{\mathcal{U}}\mathcal{QL}_{\boxed{1}}$ validities on infinite traces are recursively axiomatisable [WZ02] and satisfiability is decidable with tableaux [KLWZ04]
- Study axiomatisability of $\mathcal{T}_{\mathcal{U}}\mathcal{QL}_{\boxed{1}}$ on finite traces, as well as tableaux algorithms (implementable in BLACK [GGM21]) for satisfiability?

Definite descriptions and non-rigid designators

- Definite descriptions ("the x such that φ") [AMOW21] are referring expressions behaving as non-rigid designators in temporal contexts
- Study FOTL fragments with definite descriptions but without rigid designators assumption on finite traces (undecidability behind the corner)?
  - cf. e.g. Sarah Winkler's and Nicola Gigante's talks

References I

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Thank You

I CAN'T SLEEP FOR WORRYING ABOUT WHAT THAT SPEAKER SAID, MARCIE! I'M SCARED!
WHAT IF THE WORLD COMES TO AN END TONIGHT, MARCIE?
I PROMISE THERE'LL BE A TOMORROW, SIR... IN FACT, IT'S ALREADY TOMORROW IN AUSTRALIA!
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GO TO SLEEP, SIR... THE SUN IS SHINING IN AUSTRALIA...