Modeling Summary

Stiffness Modeling

Theoretical Model:

$stiffnessFraction_{ij} = \alpha_0 + \alpha_1 freq + \alpha_2 amp + \alpha_3 I(chemIndex = 100) + \alpha_4 I(chemIndex = 121) + (\beta_0 + \beta_1 I(chemIndex = 100) + \beta_2 I(chemIndex = 121) + v_i) log(cycleNum) + u_i + \epsilon_{ij}$

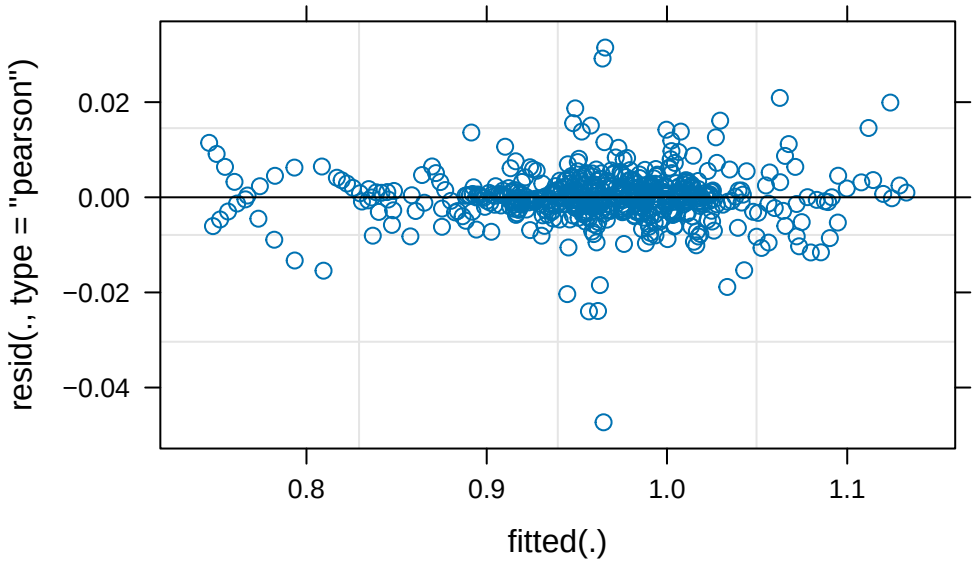
Code:

```
stiffModel <- lmer(stiffFrac~chemIndex*log(cycleNum)+freq + amp +
                     (log(cycleNum)|sampleNum),data = helmet)
summary(stiffModel)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: stiffFrac ~ chemIndex * log(cycleNum) + freq + amp + (log(cycleNum) |
    sampleNum)
   Data: helmet

REML criterion at convergence: -3665.6

Scaled residuals:
    Min      1Q  Median      3Q     Max
-7.7046 -0.3358  0.0023  0.3357  5.1265

Random effects:
 Groups    Name           Variance  Std.Dev. Corr
 sampleNum (Intercept)    0.0111929 0.10580
           log(cycleNum) 0.0002717 0.01648  -0.98
```

```
 Residual                 0.0000377 0.00614
Number of obs: 564, groups:  sampleNum, 46

Fixed effects:
                              Estimate Std. Error         df t value Pr(>|t|)
(Intercept)                  0.9854563  0.0248732 43.6480381  39.619  < 2e-16
chemIndex100                 0.0639227  0.0393397 38.9285916   1.625 0.112257
chemIndex121                 0.0765281  0.0375188 38.8223816   2.040 0.048224
log(cycleNum)                0.0072389  0.0037263 42.2833334   1.943 0.058739
freq                        -0.0018874  0.0005127 38.8581152  -3.681 0.000704
amp                         -1.4251037  0.2265351 36.8525769  -6.291 2.58e-07
chemIndex100:log(cycleNum) -0.0113351  0.0060772 42.0708233  -1.865 0.069140
chemIndex121:log(cycleNum) -0.0178860  0.0057993 42.0592256  -3.084 0.003599

(Intercept)                 ***
chemIndex100
chemIndex121                *
log(cycleNum)               .
freq                        ***
amp                         ***
chemIndex100:log(cycleNum) .
chemIndex121:log(cycleNum) **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
            (Intr) chI100 chI121 lg(cN) freq   amp    cI100:
chemIndx100 -0.588
chemIndx121 -0.620  0.395
log(cyclNm) -0.953  0.602  0.631
freq        -0.049 -0.028 -0.015  0.006
amp         -0.218 -0.015 -0.011  0.002 -0.244
chmI100:(N)  0.583 -0.981 -0.387 -0.613  0.000  0.002
chmI121:(N)  0.611 -0.387 -0.981 -0.643  0.002  0.001  0.394
```

```
plot(stiffModel)
```



Damping Modeling

Theoretical Model:

$DampingFraction_{ij} = \alpha_0 + \alpha_1 freq + \alpha_2 amp + \alpha_3 strainRate + \alpha_4 I(porosity = 81) + (\beta_0 + v_i) log(cycleNum) + u_i + \epsilon_{ij}$

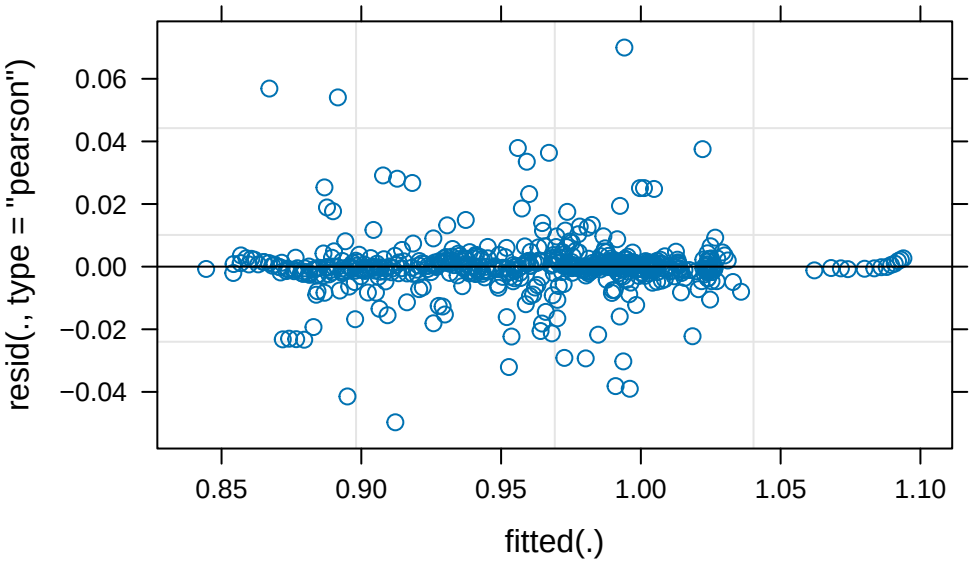
Code:

```
dampModel <- lmer(dampFrac ~ porosity + freq + amp +strainRate +
                    log(cycleNum)+(log(cycleNum)|sampleNum),data=helmet)
summary(dampModel)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: dampFrac ~ porosity + freq + amp + strainRate + log(cycleNum) +
    (log(cycleNum) | sampleNum)
   Data: helmet
```

```
REML criterion at convergence: -3300

Scaled residuals:
    Min      1Q  Median      3Q     Max
-5.0303 -0.1546  0.0042  0.1554  7.0802

Random effects:
 Groups    Name          Variance  Std.Dev. Corr
 sampleNum (Intercept)   1.431e-03 0.037835
           log(cycleNum) 6.748e-05 0.008215 -0.95
 Residual                9.767e-05 0.009883
Number of obs: 564, groups:  sampleNum, 46

Fixed effects:
               Estimate Std. Error        df t value Pr(>|t|)
(Intercept)    0.990215   0.020788 22.732507  47.633  < 2e-16 ***
porosity81     0.010344   0.005162 28.124324   2.004   0.0548 .
freq           0.004671   0.001629 18.331861   2.867   0.0101 *
amp            1.601876   0.759026 17.907646   2.110   0.0492 *
strainRate    -0.025427   0.015762 17.823774  -1.613   0.1243
log(cycleNum) -0.010519   0.001300 45.063139  -8.089 2.53e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
            (Intr) prst81 freq   amp    strnRt
porosity81  -0.237
freq        -0.900  0.158
amp         -0.926  0.140  0.935
strainRate   0.905 -0.171 -0.970 -0.976
log(cyclNm) -0.328 -0.001  0.023  0.014 -0.008
```

```
plot(dampModel)
```


resid(., type = "pearson")
0.06
0.04
0.02
0.00
−0.02
−0.04
0.85
0.90
0.95
1.00
1.05
1.10
fitted(.)