

Manual of `faz_ecc_avx2`

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Abstract

`faz_ecc_avx2` is an optimized mathematical software library for computing Edwards Digital Signature Algorithm (EdDSA) and the Diffie-Hellman functions X25519 and X448. This library was specially accelerated with the Intel Advanced Vector eXtensions version 2 (AVX2).

This library belongs to the research project titled:

“High Performance Implementation of Elliptic Curve
Cryptography using Vector Instructions”

authored by:

- Armando Faz Hernández (armfazh@ic.unicamp.br), University of Campinas, Brazil.
- Julio López, University of Campinas, Brazil.
- Ricardo Dahab, University of Campinas, Brazil.

This research work is part of a paper submission to the ACM Journal of Mathematical Software. Software will be publicly available after paper acceptance in the Collected Algorithms from ACM (CALGO) website and in the ACM Digital Library. During reviewing process, source code is available only to reviewers and journal editors through the ACM submission website.

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1 Installation

1.1 Installation Prerequisites

The following instructions are suitable to work in a Linux environment. Compilation steps may be different under Windows, MacOS, or Android operating systems.

Compilation. The tools required before to start compilation are the following:

- A file extractor, such as `tar`.
- A C compiler which supports the Intel Intrinsics and AVX2 instruction set. Additionally, compiler must be able to generate code for Haswell, Broadwell or Skylake micro-architectures.

We recommend the use of one of the following compilers:

- GNU C compiler¹ version 4.8.5 or later.
- Intel C Compiler² version 16 or later.
- Clang³ compiler version 3.8 or later.

- `make`. Tool for automated compilation.
- `cmake`.⁴ Compilation tool-kit version 2.8.4 or later.

Documentation. For source code documentation, you will need the following tools:

- `doxygen`⁵ tool.
- \LaTeX program.
- Graphviz. For generating diagrams.

See Section 5.4 for more details about generation of documentation.

1.2 Third-Party Supporting Code

Our library is self contained. It does not require any other software beyond than the tool-chain for compilation and the standard libraries.

1.3 Source Code Portability

Our code has been compiled with three different compilers: the GNU C Compiler, the Intel C Compiler, and the Clang compiler. In all the cases, code was compiled with the following compilation flags:

```
-O3 -Wall -Wextra -pedantic -std=c99 -mavx2 -march=native -mtune=native
```

Using these flags code is verified to follow the standard of C99 (ISO/IEC 9899:1999); thus, our code does not report any errors and passes all testing programs. See Section 2, for more details about testing driver.

¹gcc: <http://gcc.gnu.org>

²icc: <https://software.intel.com/en-us/c-compilers>

³clang: <https://clang.llvm.org/>

⁴cmake: <https://cmake.org/>

⁵doxygen: <http://www.stack.nl/~dimitri/doxygen/>

1.4 Machine Dependent Code

Our code is machine dependent since the main purpose is to accelerate the calculation of cryptographic algorithms using a special vector unit called AVX2, which stands for Advanced Vector eXtensions version 2. Thus, our library runs only on AVX2-ready processors.

The Haswell, Broadwell, Skylake, and Kaby Lake are code-names of Intel micro-architectures that support AVX2, and consequently, they are also able to execute our library. AMD's Ryzen processor also supports AVX2, however, we have not tested our code in this processor.

To verify if your machine supports AVX2 instructions, there are several ways to test it:

1. If you are in a Linux environment, execute:

```
1 $ cat /proc/cpuinfo
```

This command will report in the *flags* field the features that processor supports. It is mandatory that your processor has the features listed in the following table.

Micro-arch.	Flag	Feature Description
Haswell, Broadwell, Skylake, Kaby Lake.	sse2 ssse3 sse4_1 sse4_2	SIMD Streaming Extensions.
	avx	Advanced Vector eXtensions.
	avx2	Advanced Vector eXtensions version 2.
	bmi1 bmi2	Bit Manipulation Instructions version 1, and 2. The MULX integer multiplier belongs to this feature.
	cmov	Conditional Move Instruction.
	adx	ADOX/ADCX new instructions for addition with carry and addition with overflow.

2. A second way to verify the support for AVX2 instructions consists on running a companion program that verifies these flags. Follow the instructions of compilation (in Section 1.5), and then, run `cpu_id` program.

```
1 $ cd apps/samples
2 $ gcc -o cpu_id cpu_id.c
3 $ ./cpu_id
```

This program will report whether processor has or not the required features to run our library. For example: this is the output produced by our Haswell computer:

```
1 === Environment Information ===
2 Program compiled with: 4.8.5
3 bit_CMOV    : [Yes]
4 bit_SSE     : [Yes]
5 bit_SSE2    : [Yes]
6 bit_SSE3    : [Yes]
```

```

7 bit_SSSE3 : [Yes]
8 bit_SSE4_1 : [Yes]
9 bit_SSE4_2 : [Yes]
10 bit_AVX : [Yes]
11 bit_AVX2 : [Yes]
12 bit_BMI : [Yes]
13 bit_BMI2 : [Yes]
14 bit_ADX : [No]
15 Machine supports our library: Yes

```

and the following is the output produced by our Skylake processor.

```

1 === Environment Information ===
2 Program compiled with: 4.8.5
3 bit_CMOV : [Yes]
4 bit_SSE : [Yes]
5 bit_SSE2 : [Yes]
6 bit_SSE3 : [Yes]
7 bit_SSSE3 : [Yes]
8 bit_SSE4_1 : [Yes]
9 bit_SSE4_2 : [Yes]
10 bit_AVX : [Yes]
11 bit_AVX2 : [Yes]
12 bit_BMI : [Yes]
13 bit_BMI2 : [Yes]
14 bit_ADX : [Yes]
15 Machine supports our library: Yes

```

1.5 Compilation and Installation Process

Follow the steps for compiling and installing our library.

1. Once you have `faz_ecc_avx2.zip` file, extract it in an empty folder.

```
1 $ unzip faz_ecc_avx2.zip
```

2. This command will create a new folder containing a directory structure like this:

```

faz_ecc_avx2
├── apps
│   ├── bench
│   ├── tests
│   └── samples
├── doc
├── include
├── src
├── CMakeLists.txt
└── README.txt

```

3. After that, create an empty folder for building the library.

```
1 $ cd faz_ecc_avx2
2 $ mkdir build
3 $ cd build
```

4. Then execute the `cmake` command specifying the parent folder (or the path where the `CMakeLists.txt` file can be found). `cmake` will scan the compiler and headers required for compilation.

```
1 $ cmake ..
```

5. If everything is fine, `cmake` will create a `Makefile` file containing instructions for compilation. Then run `make` tool:

```
1 $ make all
```

6. After compilation, you can (optionally) install library in your system by typing:

```
1 $ make install
```

Beware that installation on operating system may require administrative permissions.

1.6 Customized Compilation

1.6.1 Setting a Compiler

If you want to switch to a determined compiler, you must indicate to `cmake` the path of the new compiler. For that end, you must set the environment variable `CC` as follows:

```
1 $ export CC=clang
2 $ cmake ..
```

Then, `cmake` will verify this compiler and will produce an output similar to this:

```
1 -- The C compiler identification is Clang 3.9.1
2 -- Check for working C compiler: /usr/bin/clang
3 -- Check for working C compiler: /usr/bin/clang -- works
4 ...
```

After that, compile library by running `make` command.

```
1 $ make all
```

1.6.2 Setting Installation Directory

By default, `cmake` will install library in `/usr/local` folder. However, if you want to install library in other path, you must specify this to `cmake` as follows:

```
1 $ cmake -DCMAKE_INSTALL_PREFIX:PATH=/opt ..
```

This command will tell `cmake` to install library in `/opt` folder. Then, run:

```
1 $ make install
```

After that, the /opt directory will have the following files installed in your system:

```
/opt
├── include
│   ├── faz_ecdh_avx2.h
│   ├── faz_eddsa_avx2.h
│   └── faz_fp_avx2.h
└── lib
    ├── libfaz_ecc_avx2_static.a
    └── libfaz_ecc_avx2_shared.so
```

2 Testing Library

faz_ecc_avx2 contains a set of unit tests called by a driver program for verifying the functionality of the library. To run those tests, perform the following instructions:

```
1 $ cd faz_ecc_avx2
2 $ mkdir build
3 $ cd build
4 $ cmake ..
5 $ make all
6 $ ./bin/tests
```

Program will run and produce an output similar to this:

```
1 === Testing Arith ===
2 === p=2^255-19 ===
3 ===== 1-way AVX2 =====
4 Test mul/sqr: 50000 OK
5 Test mul/inv: 50000 OK
6 ===== 1-way x64 =====
7 Test mul/sqr: 50000 OK
8 Test mul/inv: 50000 OK
9 ===== 2-way AVX2 =====
10 Test mul/sqr: 50000 OK
11 ...
```

Once program ends, it will report whether or not passed all the tests.

2.1 Our Testing Environment

We use three different computers for testing our library.

Haswell This is a desktop computer that has a Core-i7 4770 running at 3.4 GHz, this processor belongs to the Haswell micro-architecture family of Intel, and also known as the 4-th family of Core processors.

Yoga This is a laptop computer that has a Core-i3 4012Y running at 1.5 GHz, this processor belongs to the Haswell micro-architecture family of Intel, and also known as the 4-th family of Core processors.

Skylake This is a desktop computer that has a Core-i7 6700K running at 4.0 GHz, this processor belongs to the Skylake micro-architecture family of Intel, and also known as the 6-th family of Core processors.

All systems run Fedora 24 operating system.

3 Benchmarking Library

3.1 Previous Considerations

You must beware about the following issues when benchmarking library. Since we report timings in clock cycles, then it is recommended to turn off the following processor technologies to get accurate measurements.

- Intel Hyper-Threading. This technology makes that a physical processing core be shared by two or more logical processor units. Then, when measuring a code with Intel Hyper-Threading activated, the clock cycle counter could measure the time spend by other programs running in the same physical core.
- Intel Speed-Step. This technology makes that a processor varies the operating clock frequency. Then, when processor has a low workload, Intel Speed-Step reduces frequency with the aim to save energy. This can affect benchmark measurements.
- Intel Turbo Boost. This is the complement of the Intel Speed-Step, whenever a processor has a CPU-bound workload, then Intel Turbo Boost increases clock frequency beyond the limits of the nominal frequency, affecting also the clock cycle counter. Measuring timings with this technology activated can result on non-reproducible timings.

Additionally, we also recommend to perform benchmarking using the a minimal operating system. This mode can be activated by running the Level 3 of Linux operating system.

```
1 $ telinit 3
```

This command will enter in the level 3 of the system, which disables graphical user interface. To recover from Level 3 you can execute

```
1 $ telinit 5
```

to return to the default mode.

3.2 Benchmark Report

Our library has a companion program to perform the benchmark of the library. Compile this program using:

```
1 $ cd faz_ecc_avx2
2 $ mkdir build
3 $ cd build
4 $ cmake ..
5 $ make all
6 $ ./bin/bench
```

When you run the benchmark program, program will produce an output like the following:

```
1 === Benchmarking Arith ===
2 ===== p=2^255-19 =====
3 ===== 1-way AVX2 =====
4 addition      =      7 cc
5 subtraction   =      9 cc
```

```

6 multiplication =      56 cc
7 squaring       =      69 cc
8 coef_reduction =      22 cc
9 inversion      =    17140 cc
10 square_root   =    17593 cc
11 ...

```

Benchmark program takes a couple of minutes to finish (this mainly depends on the clock frequency of the benchmarking machine). The `bench` program reports the number of clock cycles taken for computing each operation used on the library. We measure the following primitives:

- Prime field arithmetic for \mathbb{F}_q , where $q = 2^{255} - 19$ and $q = 2^{448} - 2^{224} - 1$. Using both the single, 2-way, and 4-way operations.
- Elliptic curve arithmetic on twisted Edwards curves. Point additions and doublings.
- Miscellaneous. w -NAF recoding, signed digit regular recoding, and hash function evaluations.
- Diffie-Hellman computation. Time for computing key generation and shared secret using X25519 and X448.
- EdDSA operations. Time for computing key generation, signature generation, and signature verification using Ed25519 and Ed448.

For protocol operations, program reports timings measured in μ -seconds (10^{-6} seconds), the ratio of operations per second, and the clock cycles taken for completing an operation.

```

1 ===== Benchmarking DH =====
2 ===== X25519 AVX2 =====
3 ecdh->keygen   : 12  $\mu$ s, 81694.1 oper/s, 41522 cycles/op
4 ecdh->shared    : 37  $\mu$ s, 26859.3 oper/s, 126293 cycles/op
5 == Benchmarking EdDSA ==
6 ===== Ed25519 AVX2 =====
7 eddsa->keygen   : 12  $\mu$ s, 81365.1 oper/s, 41690 cycles/op
8 eddsa->sign     : 13  $\mu$ s, 72506.0 oper/s, 46784 cycles/op
9 eddsa->verify   : 46  $\mu$ s, 21541.9 oper/s, 157467 cycles/op
10 ...

```

4 Usage and Driver Examples

For this section, we assume that library was already compiled and installed in /opt folder.

4.1 Hello world.

This is a simple program to verify the correct linkage of the library. This program can be found in faz_ecc_avx2/apps/samples/hello.c path. Now, we list the hello.c file:

```
1  #include<faz_eddsa_avx2.h>
2  #include<stdio.h>
3
4  int main(void)
5  {
6      printf("Size of an Ed25519 key: %d bytes.\n",
7             ED25519_KEY_SIZE_BYTES_PARAM);
8      return 0;
9  }
```

To compile this program with the GNU C Compiler (gcc) execute the next steps:

```
1  $ cd apps/samples
2  $ gcc -o hello hello.c -I/opt/include -L/opt/lib -lfaz_ecc_avx2
3  $ LD_LIBRARY_PATH=/opt/lib ./hello
```

If compilation was successful, this will be the output after running the hello program.

```
1  Size of an Ed25519 key: 32 bytes.
```

4.2 Prime Field Arithmetic

The program faz_ecc_avx2/apps/samples/field_arith.c exemplifies how to use the Fp global object.

```
1  #include<faz_fp_avx2.h>
2  #include<stdio.h>
3
4  void identity(const struct _struct_Fp_1way * fp)
5  {
6      /* Initialize variables */
7      argElement_1w a      = fp->init(); argElement_1w b      = fp->init();
8      argElement_1w add_ab = fp->init(); argElement_1w sub_ab = fp->init();
9      argElement_1w left   = fp->init(); argElement_1w right  = fp->init();
10     argElement_1w prime  = fp->get_modulus();
11
12     printf("Testing the identity: (a+b)*(a-b) = (a^2-b^2) \n");
13     printf("Modulus: "); fp->print(prime);
14
15     /* Two random numbers */
16     fp->rand(a); printf("a: "); fp->print(a);
17     fp->rand(b); printf("b: "); fp->print(b);
18
19     /* Left-hand side */
20     fp->add(add_ab,a,b);          fp->cred(add_ab);
21     fp->sub(sub_ab,a,b);          fp->cred(sub_ab);
22     fp->mul(left,add_ab,sub_ab);  fp->cred(left);
```

```

23     printf("left: ");          fp->print(left);
24
25     /* Right-hand side */
26     fp->sqr(a);                 fp->cred(a);
27     fp->sqr(b);                 fp->cred(b);
28     fp->sub(right,a,b);         fp->cred(right);
29     printf("right: ");        fp->print(right);
30
31     /* Equality testing */
32     if( fp->cmp(left,right) == 0 )
33         printf("Test passed.\n\n");
34     else
35         printf("Test failed.\n\n");
36
37     /* Clearing variables */
38     fp->clear(a);               fp->clear(b);
39     fp->clear(add_ab);          fp->clear(sub_ab);
40     fp->clear(left);            fp->clear(right);
41     fp->clear(prime);
42 }
43
44 int main(void)
45 {
46     identity(&Fp.fp25519._1way_x64);
47     identity(&Fp.fp25519._1way);
48     identity(&Fp.fp448._1way);
49     return 0;
50 }

```

To compile this program with the Clang compiler (clang) execute the next steps:

```

1  $ cd apps/samples
2  $ clang -o field_arith field_arith.c -I/opt/include \
3      -L/opt/lib -lfaz_ecc_avx2
4  $ LD_LIBRARY_PATH=/opt/lib ./field_arith

```

If compilation was successful, this will be an output similar to the one produced by field_arith program.

```

1  Testing the identity: (a+b)*(a-b) = (a^2-b^2)
2  Prime: 0x7fffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffed
3  a: 0x267a08ce69569234f496b41c13d93342fae9df3145e04a6674bf1a5622c2c76f
4  b: 0x7155c0268e59e6b4fe135448fb1b49eba62f92f022f7a2c6ab03025616165dcd
5  left: 0x3715b9125633b48de8e5c097d3f1d536a7615c180ec60b4c2899a6f36ab73384
6  right: 0x3715b9125633b48de8e5c097d3f1d536a7615c180ec60b4c2899a6f36ab73384
7  Test passed.
8  ...

```

4.3 Elliptic Curve Diffie-Hellman

Now, we show an example of the calculation of a shared secret between Alice and Bob using the X25519 Diffie-Hellman function. File: faz_ecc_avx2/apps/samples/ecdh.c.

```

1  #include<faz_ecdh_avx2.h>
2  #include<faz_fp_avx2.h> /* For print_bytes and random_bytes */

```

```

3  #include<stdio.h>
4
5  int main(void)
6  {
7      size_t keysize = ECDH25519_KEY_SIZE_BYTES;
8      ECDH_X25519_KEY alice_private,alice_session,alice_shared;
9      ECDH_X25519_KEY bob_private,bob_session,bob_shared;
10
11     printf("=== X25519 DH Example ===\n");
12     /* Alice session key generation */
13     random_bytes(alice_private,keysize);
14     ECDHX.X25519.keygen(alice_session,alice_private);
15     printf("Alice private key:\n");    print_bytes(alice_private,keysize);
16     printf("Alice session key:\n");    print_bytes(alice_session,keysize);
17
18     /* Bob session key generation */
19     random_bytes(bob_private,keysize);
20     ECDHX.X25519.keygen(bob_session,bob_private);
21     printf("Bob private key:\n");      print_bytes(bob_private,keysize);
22     printf("Bob session key:\n");      print_bytes(bob_session,keysize);
23
24     /* Shared secret generation */
25     ECDHX.X25519.shared(alice_shared,bob_session,alice_private);
26     ECDHX.X25519.shared(bob_shared,alice_session,bob_private);
27     printf("Alice shared secret:\n");  print_bytes(alice_shared,keysize);
28     printf("Bob shared secret:\n");    print_bytes(bob_shared,keysize);
29
30     return 0;
31 }

```

To compile this program with the Clang compiler (clang) execute the next steps:

```

1  $ clang -o ecdh ecdh.c -I/opt/include \
2      -L/opt/lib -lfaz_ecc_avx2
3  $ LD_LIBRARY_PATH=/opt/lib ./ecdh

```

If compilation was successful, and after running the ecdh program, program will print an output similar to this:

```

1  === X25519 Example ===
2  Alice private key:
3  0x5a8ba93dbcf00cf2d7ff4e8421a6ff0c507189a50152bcfe11ace38c8cd31e761
4  Alice session key:
5  0x6283ceb1ae91013681cc1e0cdc2d2582e557a9b43f5ac3d821e7bd6c4d6e0f07
6  Bob private key:
7  0x28eb924f067e1d52c427eb04ba11e499022fff63ee76284d7de61cc188b6fc73
8  Bob session key:
9  0x45c13739af0b0e62c7e05e307d835b98a7846f1c092642f5e7cabff4a9e6eb6a
10 Alice shared secret:
11 0x2801ce8dd194a6bd93a90ef0590ea47bf41b4b22f4986464a61c07a7114261b9
12 Bob shared secret:
13 0x2801ce8dd194a6bd93a90ef0590ea47bf41b4b22f4986464a61c07a7114261b9

```

4.4 EdDSA signatures

The following script is an example about the use of Ed25519 signature scheme. You can find this file in `faz_ecc_avx2/apps/samples/eddsa.c`.

```
1  #include<faz_eddsa_avx2.h>
2  #include<faz_fp_avx2.h> /* For print_bytes and random_bytes */
3  #include<stdio.h>
4  #include<string.h>
5
6  int main(void)
7  {
8      size_t size_key = ED25519_KEY_SIZE_BYTES_PARAM;
9      size_t size_sig = ED25519_SIG_SIZE_BYTES_PARAM;
10     Ed25519_PublicKey pub_key;
11     Ed25519_PrivateKey pri_key;
12     Ed25519_Signature signature;
13     int response = 0;
14
15     uint8_t * message = (uint8_t *)"Keep Calm and Carry On";
16     const size_t size_msg = strlen((const char *)message);
17
18     printf("=== Ed25519 Example ===\n");
19     /* Key generation */
20     random_bytes(pri_key, size_key);
21     EdDSA.Ed25519.keygen(pub_key, pri_key);
22     printf("Alice's Private Key:\n"); print_bytes(pri_key, size_key);
23     printf("Alice's Public Key:\n"); print_bytes(pub_key, size_key);
24
25     printf("Message in ascii: %s\n", message);
26     printf("Message in hex: "); print_bytes(message, size_msg);
27
28     /* Signature generation */
29     EdDSA.Ed25519.sign(signature, message, size_msg, pub_key, pri_key);
30     printf("Ed25519 Signature:\n"); print_bytes(signature, size_sig);
31
32     /* Signature verification */
33     response = EdDSA.Ed25519.verify(message, size_msg, pub_key, signature);
34     if( response == EDDSA_VERIFICATION_OK )
35         printf("Valid signature.\n");
36     else
37         printf("Signature invalid.\n");
38
39     return 0;
40 }
```

To compile this program with the GNU C Compiler (`gcc`) execute the next steps:

```
1  $ gcc -o eddsa eddsa.c -I/opt/include \
2    -L/opt/lib -lfaz_ecc_avx2
3  $ LD_LIBRARY_PATH=/opt/lib ./eddsa
```

If compilation was successful, the following is a similar output to the one produced by `eddsa` program.

```
1  === Ed25519 Example ===
2  Alice's Private Key:
3  0x643cd6bc8245d0d04f83cfc189daaa6d08ebc18b7ded4a6ad51b8a118ff9073c
4  Alice's Public Key:
```

```
5 0xbbfd3c9726ec2d53bb3a36d2d9c315be0e3d943ac71d811df5bbbd9d29839fb3
6 Message in ascii:
7 Keep Calm and Carry On
8 Message in hex:
9 0x6e4f20797272614320646e61206d6c6143207065654b
10 Ed25519 Signature:
11 0x0fe79adaba74849b6728c9600d0ae212cc60a4deb9f016d84cfb3634235cd2a
12 a2c569caaeaf2d3811f731aec3701a846137d2bdac3714c0fa941b8ad6b991a3a
13 Valid signature.
```

5 Source Code Structure

Our library is split in three main modules or software layers: the prime field arithmetic, the elliptic curve Diffie-Hellman functions, and the Edwards Digital Signature functions.

5.1 Prime Field Arithmetic

This software layer provides the functions to compute arithmetic operations in the fields: \mathbb{F}_q , where $q = \{2^{255} - 19, 2^{448} - 2^{224} - 1\}$, which are the primes used in the implementation of the elliptic curve cryptographic protocols detailed below. All the functions contained in this layer can be accessed by including the header file:

```
1 #include<faz_fp_avx2.h>
```

5.1.1 Data Types

Single field elements, i.e. $A \in \mathbb{F}_q$, are represented by the following data types:

- `Element_1w_Fp25519`. A single element of \mathbb{F}_q , for $q = 2^{255} - 19$.
- `Element_1w_Fp448`. A single element of \mathbb{F}_q , for $q = 2^{448} - 2^{224} - 1$.
- `argElement_1w`. A pointer to a variable of type `Element_1w_Fp25519` or `Element_1w_Fp448`.

For the parallel implementation of prime field arithmetic, we define two-way and four-way variables as follows:

- `Element_2w_Fp25519`. Two elements $A, B \in \mathbb{F}_q$, for $q = 2^{255} - 19$. Following the notation of research paper, they are packed as $\langle A, B \rangle$.
- `Element_4w_Fp25519`. Four elements $A, B, C, D \in \mathbb{F}_q$, for $q = 2^{255} - 19$. Following the notation of research paper, they are packed as $\langle A, B, C, D \rangle$.
- `Element_2w_Fp448`. Two elements $A, B \in \mathbb{F}_q$, for $q = 2^{448} - 2^{224} - 1$. Following the notation of research paper, they are packed as $\langle A, B \rangle$.
- `Element_4w_Fp448`. Four elements $A, B, C, D \in \mathbb{F}_q$, for $q = 2^{448} - 2^{224} - 1$. Following the notation of research paper, they are packed as $\langle A, B, C, D \rangle$.

5.1.2 Operations

For each field and data types, we define the following operations:

<code>add</code>	Computes a square: $c = a + b$.	<code>cred</code>	Coefficient reduction of a variable.
<code>sub</code>	Computes a square: $c = a - b$.	<code>cmp</code>	Compare for equality two variables.
<code>mul</code>	Computes a square: $c = a \times b$.	<code>rand</code>	Set a random number in a variable.
<code>sqr</code>	Computes a square: $c = a^2$.	<code>print</code>	Print a variable in hexadecimal base.
<code>inv</code>	Computes a modular inverse: $c = \frac{1}{a}$.	<code>init</code>	Allocate dynamic memory for a variable.
<code>sqrt</code>	Computes a square-root: $c = \sqrt{a}$.		

clear Deallocate dynamic memory of a variable.
get_modulus Returns the prime q .

5.1.3 Usage of Prime Field Arithmetic

All of these combination of operations and data types are condensed into a single global variable called `Fp`. This variable is a data structure containing function pointers to operations defined for each prime field, as depicted in Figure 1.

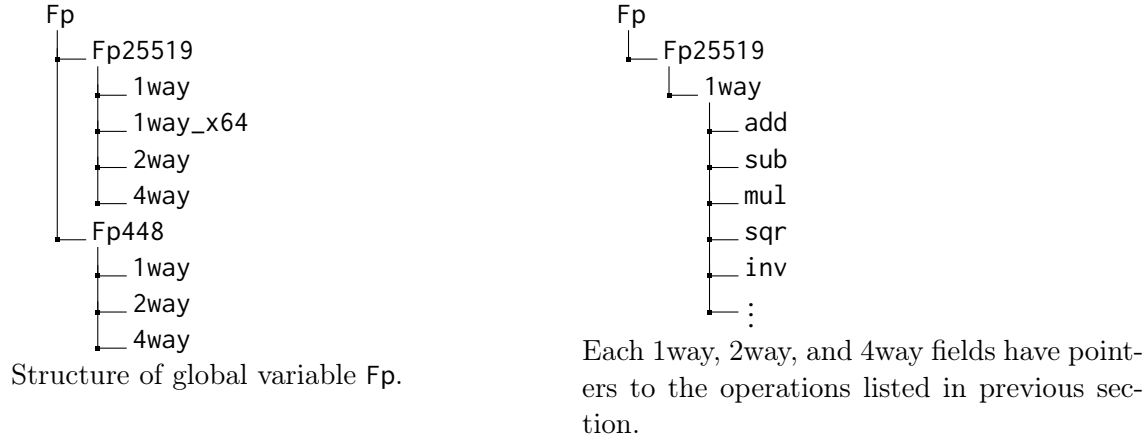


Figure 1: Global variable `Fp` containing pointer to functions.

Example 5.1. This is an example of the addition of $C = A +_2 B$; which denotes the following operation:

$$\begin{aligned} C_0 &= A_0 + B_0 \\ C_1 &= A_1 + B_1 \end{aligned}$$

where $A_0, A_1, B_0, B_1 \in \mathbb{F}_q$, and these elements are packed as $A = \langle A_0, A_1 \rangle$, and $B = \langle B_0, B_1 \rangle$. In this version, variables are statically allocated.

```
1 #include <faz_fp_avx2.h>
2
3 int main(void)
4 {
5     Element_2w_Fp25519 A,B,C;
6     Fp.fp25519._2way.add(C,A,B);
7     return 0;
8 }
```

5.1.4 Misc Functions

This layer defines some functions for manipulating string of bytes.

- `print_bytes`. Print a string of bytes of length n in hexadecimal base.
- `random_bytes`. Fill a string of bytes of length n with random data. **Warning!** This function **must be** rewritten to include a stronger pseudo-random number generator. Our implementation accesses to the Linux file `/dev/urandom` to generate random bytes.

- `compare_bytes`. Compare for equality two strings of bytes of length n . Outputs 0 whenever both strings match; otherwise, returns a non-zero value.

5.2 Elliptic Curve Diffie-Hellman

This section describes the second software layer of this library. To use the functions of this layer include the following header file:

```
1 #include<faz_ecdh_avx2.h>
```

5.2.1 Constants

- `ECDH25519_KEY_SIZE_BYTES`. This is the size (expressed in bytes) of the keys used to perform the Diffie-Hellman protocol using X25519. This value is set to 32 bytes.
- `ECDH448_KEY_SIZE_BYTES`. This is the size (expressed in bytes) of the keys used to perform the Diffie-Hellman protocol using X448. This value is set to 56 bytes.

5.2.2 Data Types

- `ECDH_X25519_KEY`. This is the data type used to store a X25519 key.
- `ECDH_X448_KEY`. This is the data type used to store a X448 key.
- `argECDHX_Key`. Pointer to an ECDH key.

5.2.3 Operations

The Diffie-Hellman protocol defines only two operations:

- `keygen`. Alice uses this function giving as an input her private key; then, `keygen` function will generate a session key for Alice.
- `shared`. Alice invokes this function giving as an input the session key of Bob and her private key; then, `shared` function produces a shared secret for Alice.

5.2.4 Usage of ECDH Functions

To access to the ECDH functions, we provide the global variable called `ECDHX` that contains pointers to the implementation of the operations listed before. Figure 2 shows the structure of the `ECDHX` variable.

Example 5.2. This is an example of the calculation of a shared secret for Alice using function `X448`.

```
1 #include<faz_ecdh_avx2.h>
2 #include<faz_fp_avx2.h> /* For print_bytes */
3 #include<stdio.h>
4
5 void alice_shared(
6     argECDHX alice_shared,
7     argECDHX bob_session,
8     argECDHX alice_private
```

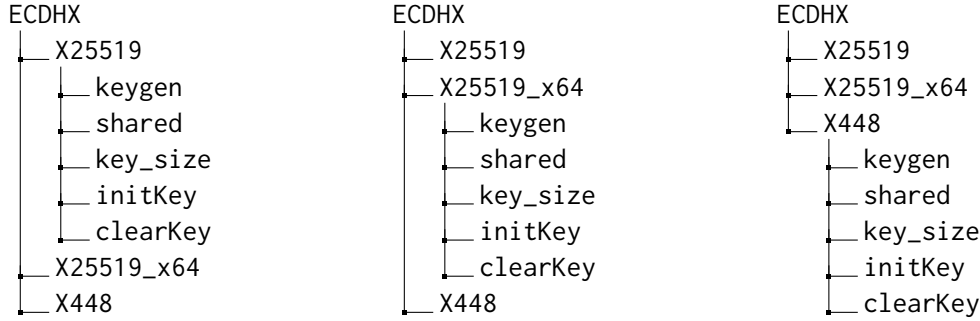


Figure 2: Global variable ECDHX containing pointer to functions X25519 and X448.

```

9  )
10 {
11     size_t keysize = ECDH448_KEY_SIZE_BYTES;
12     ECDHX.X448.shared(alice_shared, bob_session, alice_private);
13     printf("Alice shared secret: \n");
14     print_bytes(alice_shared, keysize);
15 }

```

5.3 EdDSA

This section describes the third software layer of the library; such layer corresponds to the signature generation using the EdDSA algorithm. To have access to the definitions provided by this layer include the header file:

```

1  #include <faz_eddsa_avx2.h>

```

5.3.1 Constants

These are some constants defined for EdDSA algorithm.

- ED25519_KEY_SIZE_BYTES_PARAM. This is the size of a public or private key on the Ed25519 signature scheme. This value is set to 32 bytes.
- ED25519_SIG_SIZE_BYTES_PARAM. This is the size of an Ed25519 signature. This value is set to 64 bytes.
- ED448_KEY_SIZE_BYTES_PARAM. This is the size of a public or private key on the Ed448 signature scheme. This value is set to 57 bytes.
- ED448_SIG_SIZE_BYTES_PARAM. This is the size of an Ed448 signature. This value is set to 114 bytes.
- EDDSA_NOCONTEXT. This flag is used as a parameter to indicate to signature the use of no-context, i.e. the use of a zero length context associated to a message.

5.3.2 Data Types

These are the data types provided by this software layer:

- `Ed25519_PrivateKey`. Ed25519 private key variable
- `Ed25519_PublicKey`. Ed25519 public key variable.
- `Ed25519_Signature`. Ed25519 signature variable.
- `Ed448_PrivateKey`. Ed448 private key variable.
- `Ed448_PublicKey`. Ed448 public key variable.
- `Ed448_Signature`. Ed448 signature variable.
- `argEdDSA_PrivateKey`. Pointer to a private key.
- `argEdDSA_PublicKey`. Pointer to a public key.
- `argEdDSA_Signature`. Pointer to a signature.

5.3.3 Operations

These are the functions to compute digital signatures.

- `keygen` . Creates a public key from a private key.
- `sign` . Computes the signature of a message.
- `verify` . Verifies whether the tuple (message, context, signature, public key) are valid.
- `key_size` . Returns the size of keys.
- `signature_size` . Returns the size of signatures.
- `initKey` . Allocates dynamic memory for storing a key.
- `initSignature` . Allocates dynamic memory for storing a signature.
- `clearKey` . Deallocates dynamic memory of a key.
- `clearSignature` . Deallocates dynamic memory of a signature.

The following are some error codes that the previous operations could return. These flags are enumerated in the `EDDSA_FLAGS` variable.

- `EDDSA_KEYGEN_OK`. Key generation was computed successfully.
- `EDDSA_SIGNATURE_OK`. Signature generation was computed successfully.
- `EDDSA_VERIFICATION_OK`. Signature verification indicates that signature is valid for that message.
- `EDDSA_INVALID_SIGNATURE`. Signature verification indicates that signature is invalid for that message.
- `EDDSA_ERROR_PUBLICKEY`. Signature verification indicates that public key is not in the appropriate format.
- `EDDSA_ERROR_CONTEXT`. Signature verification indicates that context was not provided in the appropriate format.
- `EDDSA_ERROR_PHFLAG`. Indicates that `phflag` is different from 0 or 1.

5.3.4 Usage of EdDSA

To access to the EdDSA functions, our library provides a global variable called `EdDSA`, which is a structure containing pointers to functions defined above. The Figure 3 shows the structure of this variable.

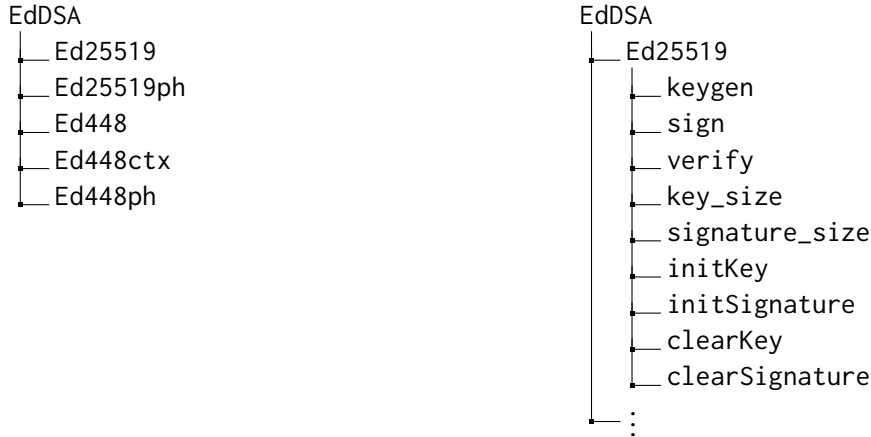


Figure 3: Global variable `EdDSA` containing pointer to functions `Ed25519` and `Ed448`.

Example 5.3. This is an example of the calculation of a signature of the message “Keep Calm and Carry On” using the `Ed25519` signature scheme.

```
1  #include<faz_eddsa_avx2.h>
2  #include<faz_fp_avx2.h> /* For printing keys */
3  #include<stdio.h>       /* For printf */
4  #include<string.h>      /* For strlen */
5
6  void signature(argECDHX alice_public, argECDHX alice_private)
7  {
8      size_t size_sig = ED25519_SIG_SIZE_BYTES_PARAM;
9      Ed25519_Signature signature;
10
11      uint8_t * message = (uint8_t *)"Keep Calm and Carry On";
12      const size_t size_msg = strlen((const char *)message);
13
14      EdDSA.Ed25519.sign(signature,
15                          message, size_msg,
16                          alice_public, alice_private);
17
18      printf("Signature: \n");
19      print_bytes(signature, size_sig);
20 }
```

5.4 Compiling Source Code Documentation

Inside the folder `faz_ecc_avx2/doc`, there is a configuration file for running `doxygen` toolkit; which is used to generate source code documentation.

```
1  $ cd faz_ecc_avx2/doc
2  $ doxygen gen_doc.doxyfile
```

Once command ends, a new folder will be created `faz_ecc_avx2/doc/html`, this is a HTML machine-readable documentation of the source code, and can be visualized on an HTML browser. For example on Firefox:

```
1  $ cd faz_ecc_avx2/doc
2  $ firefox html/index.html
```

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