Assignment 1   Roll No: 2022201024

I. Probability and Counting

① There are 3 ways in which the 5 digit number can be formed.

ⓐ No digits are repeated $= 9 \times 8 \times 7 \times 6 \times 5 = \underline{15120}$

ⓑ One digit appears twice $= {}^9C_1 \times {}^5C_2 \times 8 \times 7 \times 6 = \underline{30240}$

ⓒ 2 digits are repeated twice $= {}^9C_2 \times {}^5C_2 \times {}^3C_2 \times 7 = \underline{7560}$

$15120 + 30240 + 7560 = \underline{52920}$

② ${}^{12}C_3 \times {}^9C_4 \times {}^5C_5 = \underline{27,720}$ (Select 3 from 12 × Select 4 from rest 9 × Select 5 from rest of the 5)

③ ⓐ French & English seated together (Both French after English and English after French) $= \underline{9! \times 2}$

ⓑ French & English together along with USA and Russia together $= \underline{8! \times 2 \times 2}$ → For French after English, English after French, Russia after USA and USA after Russia.

Ans = ~~ⓑ~~ ⓐ − ⓑ $= 9! \times 2 - 8! \times 2 \times 2 = \underline{5,64,480}$

④ ⓐ Representation = { Red marble drawn first, then replaced and Red marble drawn again $= R_d R$ }

Sample Space = $\{R_dR, R_dG, R_dB, G_dR, G_dG, G_dB, B_dR, B_dG, B_dB\}$

ⓑ Sample Space = $\{R_dG, R_dB, G_dR, G_dB, B_dR, B_dG\}$

(5.) $E = \{(1,2), (1,4), (1,6), (2,1), (4,1), (6,1), (2,3), (3,2), (2,5), (5,2), (3,4), (4,3), (3,6), (6,3), (4,5), (5,4), (5,6), (6,5)\}$

$F = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (3,1), (4,1), (5,1), (6,1)\}$

$G = \{(1,4), (4,1), (2,3), (3,2)\}$

$EF (E \cap F) = \{(1,2), (1,4), (1,6), (2,1), (4,1), (6,1)\}$

$(E \cup F) = \{(1,2), (1,4), (1,6), (2,1), (4,1), (6,1), (1,1), (1,3), (3,1), (1,5), (5,1), (2,3), (3,2), (2,5), (5,2), (3,4), (4,3), (3,6), (6,3), (4,5), (5,4), (5,6), (6,5)\}$

$FG (F \cap G) = \{(1,4), (4,1)\}$

$EF^c (E \cap F^c) = \{(2,3), (3,2), (2,5), (5,2), (3,4), (4,3), (3,6), (6,3), (4,5), (5,4), (5,6), (6,5)\}$

$EFG (E \cap F \cap G) = \{(1,4), (4,1)\}$

(6.) 1. $2^5 = 32$

2.

	x_1	x_2	x_3	x_4	x_5
1.	1	1	0	0	0
2.	1	1	0	0	1
3.	1	1	0	1	0
4.	1	1	1	0	0
5.	1	1	0	1	1
6.	1	1	1	0	1
7.	1	1	1	1	0
8.	1	1	1	1	1
9.	0	0	1	1	0
10.	0	0	1	1	1
11.	0	1	1	1	0
12.	0	1	1	1	1

	x_1	x_2	x_3	x_4	x_5
13.	1	0	1	1	1
14.	1	0	1	1	0
15.	1	0	1	0	1

3. _ _ _ 0 0

$2^3 = 8$

7. $S = \{0g, 1g, 0f, 1f, 0s, 1s\}$ • $A = \{0s, 1s\}$ • $B = \{0g, 0f, 0s\}$

• $A \cup B^c = \{1g, 1f, 1s, 0s\}$

8. $P(A) = 0.3$ $P(B) = 0.5$

$P(A \cup B) = P(A) + P(B) = 0.8$

$P(A \cap B^c) = P(A) = 0.3$

$P(A \cap B) = 0$

9. $p(r) = 0.2$ $p(n) = 0.3$ $p(r^c \cap n^c) = 0.6 = \cancel{p(r \cap n)}$

$p(r \cup n) = p(r) + p(n) - p(r \cap n)$

$p(r^c \cap n^c) = 1 - p(r \cup n) = 0.6$

• $p(r \cup n) = \underline{0.4}$ (ring or a necklace)

• $p(r \cap n) = p(r) + p(n) - p(r \cup n)$

$= 0.2 + 0.3 - 0.4 = \underline{0.1}$ (ring & a necklace)

10. $P \cup M \cup C = 1000$ (given) $=$ (LHS) [P - Professionals, C - College graduates, M - Married]

$P \cup M \cup C = P + M + C - (P \cap M) - (P \cap C) - (M \cap C) + (M \cap P \cap C)$

$= 312 + 470 + 525 - 86 - 42 - 147 + 25 = \underline{\underline{1057}}$ (RHS)

LHS $\neq$ RHS

Thus the numbers reported in the study must be incorrect.

11. $P(5) = |\{(1,4), (4,1), (2,3), (3,2)\}| = \frac{4}{36} = \frac{1}{9}$

$P(7) = |\{(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)\}| = \frac{6}{36} = \frac{1}{6}$

$P((5 \cup 7)^c) = 1 - \left(\frac{1}{9} + \frac{1}{6}\right) = \frac{13}{18}$

$P(5 \text{ occurs first}) = \frac{1}{9} + \frac{13}{18} \times \frac{1}{9} + \left(\frac{13}{18}\right)^2 \frac{1}{9} + \left(\frac{13}{18}\right)^3 \frac{1}{9} + \cdots$

$= \frac{1}{9}\left(1 + \frac{13}{18} + \left(\frac{13}{18}\right)^2 + \left(\frac{13}{18}\right)^3 + \cdots\right) = \frac{1}{9} \cdot \frac{1}{1 - \frac{13}{18}} = \frac{2}{5}$

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12. P(dice lands on diff no:s) = 30/36 (Sample space = 30)
P(one lands on 6 | dice lands on diff nos) = 10/30 = $\underline{\underline{1/3}}$

13. The last person has only 2 possibilities, either the last person can sit on his last seat or, he can sit on the first seat. This can be proved by assuming the last person got to sit in seat no for eg: 63. But that means, 63rd person came before the last person and at that time also the 63rd seat was empty and the 63rd seat was not occupied by the 63rd person. But it should not happen like that.

So thus proven, the last person can either sit in his own seat or the first seat, when any of the displaced travellers decides to sit at the last seat.

When 1 sits in the wrong seat and any of the traveller ~~comes~~ whose seat is occupied by, he is displaced, for eg: if the traveller decides to sit in the 73rd seat, every other traveller until the 73rd traveller gets to sit in their own seats, and the 73rd traveller like any other displaced traveller has equal probability of occupying the last seat, first seat or any other unoccupied seats.

So ~~when the~~ since the probability of choosing the first seat and the last seat is same for any displaced customer,

So the probability that one seat (first or the last) is taken before the other is $\underline{\underline{1/2}}$

So if the first seat is taken before the last, it means, the last seat will be unoccupied when the last customer comes in & the probability of this = $\underline{\underline{1/2}}$

Thus Answer = $\underline{\underline{1/2}}$

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4. X Y Z . (Relating this to Monty-Hall problem, our chance now is X ?, and each of X, Y and Z has an equal chance of 1/3 to fail)

$P(F) = \frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

• Now we are shown Y's result. and Y passes. That is, Y is not the one who failed. So initially ~~P(F)~~ the probability that Y or Z will fail was $\left(\frac{1}{3}+\frac{1}{3}\right) = 2/3$. Now that we know Y passed, it means, that 2/3 probability got concentrated on Z, and the chance of Z failing increases from 1/3 to $\underline{\underline{2/3}}$.

• Therefore X is mistaken by his calculations. If Y passes, then X's probability of fail could not reduce, it could stay the same, so X's probability of pass also stays the same as $\underline{2/3}$. Only the probability of Z failing increases to $\underline{\underline{2/3}}$.

5. $\frac{r}{r+b}\left(\frac{r-1}{r+b-1}\right) = \frac{1}{2}$ let $\underline{\underline{r+b = x}}$

$\frac{r}{x}\left(\frac{r-1}{x-1}\right) = \frac{1}{2}$ (let $r = x-b$)

$2(x-b)(x-b-1) = x(x-1) \Rightarrow x^2 - 4bx - x + 2b^2 + 2b = 0$

$= x^2 - (4b+1)x + (2b^2+2b) = 0$

$$x = \frac{(4b+1) \pm \sqrt{(4b+1)^2 - 4(2b^2+2b^2)}}{2}$$

$$x = \frac{(4b+1) \pm \sqrt{8b^2+1}}{2}$$

Case at $b=1$ and $b=6$, $\sqrt{8b^2+1}$ becomes a perfect root, ~~at b=1~~ 1. at $\underline{\underline{b=1}}$ $x = \frac{(4+1) \pm \sqrt{9}}{2} = \frac{5+3}{2}$ or $\frac{5-3}{2}$

viable option $= 8/2 = 4$, $\therefore r = 3$ and $b = 1$ and $x = r+b = \underline{4}$

2. at $\underline{\underline{b=6}}$ $x = \frac{(24+1) \pm \sqrt{289}}{2}$, viable option $= \frac{25+17}{2} = 21$

so $b=6$, $r=15$ and $x = r+b = 21$

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