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1- $\lim_{n\to\infty} \left(1+\frac{2}{n}\right)^{3n}$

$= \left[\lim_{n\to\infty} \left(1+\frac{2}{n}\right)^{n}\right]^{3} = (e^{2})^{3} = e^{6}$

2- $\lim_{n\to\infty} \left(1-\frac{4}{3n}\right)^{n-1}$

$= \lim_{n\to\infty} \left(1-\frac{\frac{4}{3}}{n}\right)^{n} \lim_{n\to\infty} \left(1-\frac{\frac{4}{3}}{n}\right)^{-1}$

$= e^{-\frac{4}{3}} \times 1 = e^{-\frac{4}{3}}$

3- $\lim_{n\to\infty} \left(\frac{n-3}{n+4}\right)^{2n-7} \xrightarrow{\div n} = \lim_{n\to\infty} \left(\frac{1-\frac{3}{n}}{1+\frac{4}{n}}\right)^{2n-7}$

$= \lim_{n\to\infty} \left(\frac{1-\frac{3}{n}}{1+\frac{4}{n}}\right)^{2n} \lim_{n\to\infty} \left(\frac{1-\frac{3}{n}}{1+\frac{4}{n}}\right)^{-7}$

$= \lim_{n\to\infty} \left(\frac{1-\frac{3}{n}}{1+\frac{4}{n}}\right)^{2n} \times 1$

$= \lim_{n\to\infty} \left(\frac{\left(1-\frac{3}{n}\right)^{n}}{\left(1+\frac{4}{n}\right)^{n}}\right)^{2}$

$= \left(\frac{e^{-3}}{e^{4}}\right)^{2} = e^{-14}$

4- $\lim_{x\to 0} \frac{1}{x} \ln \sqrt{\frac{1+x}{1-x}} = \lim_{x\to 0} \ln \left(\frac{1+x}{1-x}\right)^{\frac{1}{2x}} = \ln \left[\frac{\lim_{n\to 0} (1+x)^{\frac{1}{x}}}{\lim_{n\to 0} (1-x)^{\frac{1}{x}}}\right]^{\frac{1}{2}}$

$= \ln \left[\frac{e}{e^{-1}}\right]^{\frac{1}{2}} = \ln [e^{2}]^{\frac{1}{2}} = \ln e = 1$

5- $\cosh^2 x - \sinh^2 x = 1$

$$= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$= \frac{1}{4}\left(\left(\cancel{e^{2x}} + 2 + \cancel{e^{-2x}}\right) - \left(\cancel{e^{2x}} - 2 + \cancel{e^{-2x}}\right)\right)$$

$$\frac{1}{4} \quad 2 \quad - \quad -2$$

$$= \frac{1}{4}(4) = 1$$

★ 6- $\lim\limits_{x \to 0} \frac{\sin 3x}{x} = 3 \lim \frac{\sin 3x}{3x} = 3(1) = 3$

7- $2\cosh 2x + 10 \sinh 2x = 5$

$\cosh 2x = \frac{e^{2x} + e^{-2x}}{2}$ $\quad \sinh 2x = \frac{e^{2x} - e^{-2x}}{2}$

$$= \cancel{2}\left(\frac{e^{2x} + e^{-2x}}{\cancel{2}}\right) + \overset{5}{\cancel{10}}\left(\frac{e^{2x} - e^{-2x}}{\cancel{2}}\right) = 5$$

$$e^{2x} + e^{-2x} + 5e^{2x} - 5e^{-2x} = 5$$

$$6e^{2x} - 4e^{-2x} = 5 \longrightarrow 6e^{4x} - 4 = 5e^{2x}$$

$$6e^{4x} - 5e^{2x} - 4 = 0$$

$$(3e^{2x} - 4)(2e^{2x} + 1) = 0$$

$\therefore \cancel{e^{2x} = 4}$ $3e^{2x} - 4 = 0$ / $2e^{2x} + 1 = 0$

$e^{2x} = \frac{4}{3}$ or $e^{2x} = -\frac{1}{2}$

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8- $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

Let $\alpha = \sin^{-1}x \longrightarrow x = \sin\alpha$

$\beta = \sin^{-1}y \longrightarrow y = \sin\beta$

$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-x^2}$

$\cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-y^2}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\sin^{-1}x + \sin^{-1}y) = x\sqrt{1-y^2} + y\sqrt{1-x^2}$

$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

9- $F(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x \neq 2 \\ 4, & x = 2 \end{cases}$

$F(2) = 4$

$$\lim_{x\to 2} F(x) = \lim_{x\to 2}\frac{x^2-4}{x-2} = \lim_{x\to 2}\frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = \lim_{x\to 2}(x+2) = 2+2 = 4$$

$$F(2) = \lim_{x\to 2} F(x) = 4$$

10- $\lim_{x\to 0}\dfrac{\tan 3x}{\sin 9x}$

$$= \lim_{x\to 0}\frac{\frac{\tan 3x}{x}}{\frac{\sin 9x}{x}} = \frac{\lim \frac{\tan 3x}{x}}{\lim \frac{\sin 9x}{x}} = \frac{3}{9}$$

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$$11- \lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{h\to 0} \frac{1-\left(1-2\sin^2\frac{x}{2}\right)}{x^2}$$

$$\cancel{= \lim_{h\to 0} \frac{2\sin^2 x}{2}}$$

$$= \lim_{h\to 0} \frac{2\sin^2\frac{x}{2}}{x^2} = 2\left(\lim_{h\to 0} \frac{\sin\frac{x}{2}}{x}\right)^2$$

$$= 2\left(\frac{1}{2}\right)^2 = 2\times\frac{1}{4} = \frac{1}{2}$$

$$12- \lim_{x\to\infty} \frac{3x^5+4x^2+7}{4x^2+5x}$$

$$= \infty \qquad (m=5, n=2, m>n)$$

*لو $n > m$ هتبقى صفر

لو $n = m$ هتاخد الرقم الي جنب m, n وتبقى $\frac{m}{n}$

$$13- F(x) = \sqrt{9-4x}$$

$$F(x) = \lim_{h\to 0} \frac{F(x+h)-F(x)}{h} \longrightarrow F(x) = \lim_{h\to 0} \frac{\sqrt{9-4(x+h)}-\sqrt{9-4x}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{9-4(x+h)}-\sqrt{9-4x}}{h} \times \frac{\sqrt{9-4(x+h)}+\sqrt{9-4x}}{\sqrt{9-4(x+h)}+\sqrt{9-4x}}$$

$$= \lim_{h\to 0} \frac{9-4x-4h-(9-4x)}{h\sqrt{9-4(x+h)}+\sqrt{9-4x}} = \lim_{h\to 0} \frac{\cancel{9-x}\ -4h}{h\sqrt{9-4(x+h)}+\sqrt{9-4x}}$$

$$= -4 \lim_{h\to 0} \frac{1}{\sqrt{9-4(x+h)}+\sqrt{9-4x}}$$

$$= -4\times\frac{1}{2\sqrt{9-4x}} = -\frac{2}{\sqrt{9-4x}}$$

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Derivation

14- $x^3 + 3x^2y + 3xy^2 + y^3 = 0$

$= 3x^2 + 3x^2y' + 6xy + 3y^2 + 3y^2y' = 0$

$= (3x^2 + 6xy + 3y^2)\,y' + (3x^2 + 6xy + 3x^2) = 0$

$$y' = -\frac{(3x^2+6xy+3y^2)}{(3x^2+6xy+3y^2} = -1$$

15- Find $\frac{dy}{dx}$. $y = x^5 \sin x$

$$\frac{dy}{dx} = x^5 \cos x + \sin x \cdot x\,5x^4$$

$$= x^5 \cos x + 5x^4 \sin x$$

16- $\frac{d}{dx} \ln x = \frac{1}{x}$

$$F(x) = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h\to 0} \frac{\ln(x+h) - \ln x}{h}$$

$$= \lim_{h\to 0} \frac{1}{h} \ln\left(\frac{x+h}{x}\right) = \lim_{h\to 0} \ln\left(\frac{x+h}{h}\right)^{\frac{1}{h}}$$

$$= \ln \lim_{h\to 0} \left(1 + \frac{1}{x} h\right)^{\frac{1}{h}} = \ln e^{\frac{1}{x}} = \frac{1}{x} \ln e = \frac{1}{x}$$

17- $y = 5x^6 \cosh 3x$

$$y' = 5x^6 \cosh 3x \times 3 + 5 \cosh 3x \times 6x^5$$

$$= 15x^6 \cosh 3x + 30x^5 \cosh 3x$$

18- $\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{1+x^2}}$ = Let $y = \sinh^{-1} x \rightarrow x = \sinh y$

$$1 = \cosh y \frac{dy}{dx} \longrightarrow \frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{1+\sinh^2 x}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$$

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L'hopital Theorem

19- $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$

$$= \lim_{n\to 0} \frac{\sin x}{2x} = \lim_{x\to 0} \frac{\cos x}{2} = \frac{1}{2}$$

20- $\lim\limits_{x\to\infty} \dfrac{6x^3-5x^2+7}{2x^3-5x+2}$

$$= \lim_{x\to\infty} \frac{18x^2-10x}{6x^2-5}$$

$$= \lim_{x\to\infty} \frac{36x}{12x} = \lim_{n\to\infty} 3 = 3$$

21- $\lim\limits_{x\to 0} \left(\dfrac{1}{x} - \dfrac{1}{\sin x}\right)$

$$= \lim_{x\to 0} \frac{\sin x - x}{x\sin x}$$

$$= \lim_{x\to 0} \frac{\cos x - 1}{x\cos x + \sin x} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{\cdot\cos x}{-x\cos x + 5\sin x} = \frac{0}{2} = 0$$

22- $\lim\limits_{x\to 0} x\cot 2x$

$$= \lim_{n\to 0} \frac{x}{\tan 2x}$$

$$= \lim_{n\to 0} \frac{1}{2\sec^2 2x} = \frac{1}{2}$$

23- $\lim\limits_{x\to 0} x^x$

Let $L = \lim\limits_{x\to 0} x^x$

$$\ln L = \lim_{x\to 0} \ln x^x = \lim_{x\to 0} x\ln x = \lim_{x\to 0} \frac{\ln x}{\frac{1}{x}}$$

$$= \lim_{x\to 0} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = -\lim_{x\to 0} x = 0$$

$$L = \lim_{x\to 0} x^x = e^0 = 1$$

Integration

24- $\int [5x^3 - 10x^2 + 2x - 3]\,dx$

$5\int x^3\,dx - 10\int x^2\,dx + 2\int x\,dx - 3\int dx$

$= \frac{5}{4}x^4 - \frac{10}{3}x^3 + x^2 - 3x + c$

25- $\int \frac{x+1}{2x^2+4x+3}\,dx$

$= \frac{1}{4}\int \frac{4(x+1)}{2x^2+4x+3}\,dx$ (ضربنا في 4 جوه وقسمنا على 4 بره)

$= \frac{1}{4}\ln|2x^2+4x+3| + c$

26- $\int \frac{e^x}{\sqrt{1+e^x}}\,dx$

$= 2\sqrt{1+e^x} + c$

27- $\int e^{3x}\,dx$

$= \frac{1}{3}\int e^{3x}(3)\,dx$

$= \frac{1}{3}e^{3x} + c$

28- $\int e^{-\frac{1}{x} - 2\ln x}\,dx$

$= \int e^{-\frac{1}{x}} e^{-2\ln x}\,dx$

$= \int e^{-\frac{1}{x}}\left(\frac{1}{x^2}\right)dx \quad = e^{-\frac{1}{x}} + c$

29- $\int 2^x\,dx$

$= \frac{2^x}{\ln 2} + c$

30- $\int \tan x\,dx$

$= -\int \frac{-\sin x}{\cos x}\,dx = -\ln|\cos x| + c = \ln|\sec x| + c$

$31- \int x^2 \cos x^3 \, dx$

let $u = x^3 \longrightarrow du = 3x^2 dx$

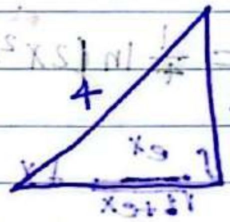
$= \frac{1}{3} \int \cos u \, du$

$= \frac{1}{3} \sin u \, du = \frac{1}{3} \sin x^3 + c$

$32- \int \sqrt{16-x^2} \, dx$

Let $x = 4\sin\theta \longrightarrow dx = 4\cos\theta \, d\theta$

$= \int \sqrt{16-16\sin^2\theta} \; 4\cos\theta \, d\theta$



$= \int 4\cos\theta \; 4\cos\theta \, d\theta$

$= 16 \int \cos^2\theta \, d\theta$

$= 8 \int (1+\cos 2\theta) \, d\theta$

$= 8[\theta + \frac{\sin 2\theta}{2}] + c$

$= 8[\theta + \sin\theta\cos\theta] + c$

$= 8\sin^{-1}(\frac{x}{4}) + \frac{1}{2} x \sqrt{16-x^2} + c$

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33- $\int x\cos x \, dx$

Let $u = x$ $\quad dv = \cos x \, dx$

$du = dx$ $\quad v = \sin x$

$= x\sin x + \int \sin x \, dx = x\sin x - \cos x + c$

34- $\int x^3 \sin x \, dx$

x^3	$+$	
$3x^2$	$-$	
$6x$	$+$	$-\sin x$
6	$-$	$\cos x$
0	$+$	$\sin x$

$\int x^3 \sin x \, dx = -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + c$

35- $\int \sin^3 x \, dx$

$= \int \sin^2 x \sin x \, dx$

$= \int (1-\cos^2 x)\sin x \, dx$

$= \int \sin x \, dx - \int \cos^2 x \sin x \, dx$

$= -\cos x + \frac{1}{3}\cos^3 x + c$

36- $\int \frac{x+8}{x^2+6x+8}\,dx$

$$\frac{x+8}{x^2+6x+8} = \frac{x+8}{(x+2)(x+4)} = \frac{A}{(x+2)} + \frac{B}{(x+4)}$$

$$x+8 = A(x+4) + B(x+2)$$

at $x = -2 \rightarrow 2A = 6 \rightarrow A = 3$

at $x = -4 \rightarrow -2B = 4 \rightarrow B = -2$

$$= \int \left[\frac{3}{(x+2)} - \frac{2}{(x+4)}\right] dx$$

$$= 3\ln|x+2| - 2\ln|x+4| + c$$