



Katholieke
Universiteit
Leuven

Department of
Computer Science

PROJECT

Advanced Programming Languages for A.I. (H02A8a)

DERUYTTERE-HALILOVIC

Thierry Deruyttere (r0660485)

Armin Halilovic (r0679689)

Academic year 2016-2017

Contents

Introduction	2
Task 1: Sudoku	3
1.1 Other Viewpoint	3
1.2 Criteria to judge if a viewpoint is good or not	4
1.3 Channeling	4
1.4 Experiments	6
1.4.1 ECLiPSe	6
1.4.2 Impact of the different search strategies and alldifferent	8
1.4.3 Analysis of differences	8
1.4.4 CHR	9
Task 2: Hashiwokakero	12
2.1 ECLiPSe	12
2.1.1 Basic solver	12
2.1.2 Improvements	13
2.1.3 Experiments	14
2.2 CHR	15
2.2.1 Data representation	15
2.2.2 Constraints representation	15
2.2.3 Propagation and search	17
2.2.4 Connectivity constraint	18
2.2.5 Improvements	20
2.2.6 Experiments	22
Conclusion	24
3.1 Weak points	24
3.2 Strong points	24
3.3 Lessons learned	24
Appendix	25

Introduction

In this report we discuss our different solutions we have created for Sudoku and Hashiwokakero. They are the result of a lot of work and a lot of back tracking on our previously done work. We often came in situations where we got stuck because of the limitations of the ECLiPSe and CHR systems but we also often had to back track on our work since we were often feeling that we were doing things in a non declarative way. We often tried to do things in a procedural way when we first started with Sudoku which means we lost quite some time here since we often had to rethink how we could write things in a more declarative way. For the Hashiwokakero part of the project things went a bit better but we lost here we tried to create a graph representation of the problem but we failed since ECLiPSe doesn't support constraints in conditionals. For this reason we had to retrack our steps back to the structure mentioned on the stackoverflow article.

In our solutions we decided to only use ECLiPSe and CHR. This was partially due to the fact that when we started this assignment we still hadn't seen Jess in class. Once we did have the class about Jess we found that since we were still novices at declarative programming languages, it would be a good exercise to continue using the more declarative systems to gain more experience with them since Jess can also be used to program in a more procedural way. Another reason for not using Jess was that the Jess syntax looked less appealing than the syntax CHR was offering us by all the parentheses used in its syntax.

Task 1: Sudoku

1.1 Other Viewpoint

For the other viewpoint in Sudoku we decided to use the following representation: each number has N positions, represented as tuples (X,Y) (with N the size from one side of the sudoku board). Each of these (X,Y) must be different from each other as well as each X must be different from any other X and Y must be different from all other Y as well. This representation has as a benefit that you know that each value from 1 to N must be in X , so we can already fill this in. The only thing left is to find the right Y .

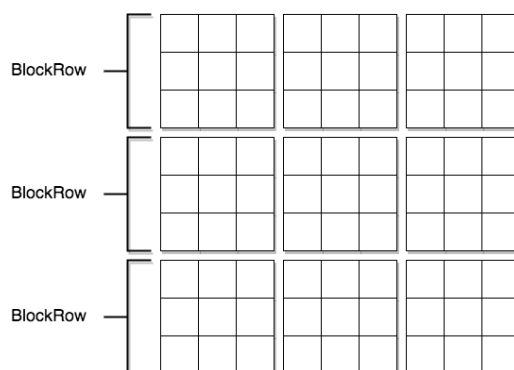


Figure 1.1: Illustration of the definition of a block row

The block constraints can be expressed with a bit of maths thanks to this representation. For a specific number we know each X of all the different positions, since we already filled them in, so we can check in which block row (see illustration) the position is. For example if you take a 9 x 9 board like in the illustration, you know that the rows 1,2,3 belong to block row 1, the rows 4,5,6 belongs to block row 2 and 7,8,9 belongs to block row 3. To express the block constraints we do the following:

1. Take all the positions of a number
2. Check which positions are in the same block row (there are exactly \sqrt{N} positions of a number in the same block row).
3. Express that each position in the same block row should be in a different block in that block row.

1.2 Criteria to judge if a viewpoint is good or not

This is really difficult in our opinion since we had some problems at first to find a good other viewpoint. We think that some good criterion's to judge if a viewpoint is good or not are

- how easy is it to express what you need with this viewpoint, obviously if you lose a lot of time with implementing an other viewpoint then it may be worth it to look for an other viewpoint
- How easy is it to use this different viewpoint? In other words, is it a logical representation for the problem?
- Lastly, think about the amount of constraints you need with this viewpoint. Is it really the lowest amount possible?

1.3 Channeling

The channeling constraint for our two viewpoints, the classical one and the one explained above, are in fact not that hard to express in words. If for the classical viewpoint you know which value there has to be at a certain (X,Y) position, you obviously immediately know it's (X,Y) values. For the other viewpoint, if you know the (X,Y) values of a number then obviously you can enter the number at that position. This is ofcourse trivial in words but programming it in ECLiPSe was not that easy.

ECLiPSe channeling constraints

```
1 channel(NumbersPositions, Board):-  
2   dim(Board, [N, N]),  
3   dim(NumbersPositions, [N, N, 2]),  
4   (multifor([Number, Position, Y], 1, N), param(NumbersPositions, Board, N) do  
5       #=(Board[Position, Y], Number, B),  
6       #=(NumbersPositions[Number, Position, 2], Y, B)  
7   ).
```

Listing 1.1: Channel constraints in ECLiPSe

The crucial bit of code for the channeling constraints can be found inside the multifor. There you can find two different constraints that links the two viewpoints together. The first one is the channeling constraint for the classical viewpoint. It lays a constraint on the value of Board[Position,Y]. The second constraint is the channeling constraint for our viewpoint. There you can see that we index with Number, Position and 2. The reason we're only interested in the second value is because the first value is already filled in as stated in section 1.1. Together these constraints expresses that if Y is know in our viewpoint then we can fill in the board with Number or if we know which number is at [Position,Y] in the classic viewpoint then we can fill in the Y of our viewpoint. This is done with the B at the end of both of the constraints. This B is crucial in the channeling constraints as it expresses that if one of the constraints is true, then the other should be true as well!

CHR channeling constraints

```
1 % Viewpoint 1 (thus board) is board(X,Y, BlockIndex, Value),
2 % Viewpoint 2 (thus board_other_viewpoint) is board(Value, X,Y, BlockIndex),
3
4 % The search variable for board is Value1
5 channel, board(X,Y, BlockIndex, Value)
6     , board_other_viewpoint(Value, X, Y2, B2) ==> number(Value), var(Y2), var(B2) |
7     Y2 is Y,
8     B2 is BlockIndex.
9
10 % The search variable for board_other_viewpoint is the Y index
11 channel, board_other_viewpoint(Value, X, Y, BlockIndex),
12     board(X,Y, BlockIndex, V2) ==> var(V2), number(Y), number(BlockIndex) |
13     V2 is Value.
```

Listing 1.2: Channel constraints in CHR

For the channeling constraints in CHR we decided to use an extra predicate `channel`. This is used just so that the system waits for the right moment to start setting up the channel constraints. Expressing the channeling constraints in CHR was a bit easier than doing it in ECLiPSe. Let's start with the first constraint. This expresses that if `Value` of `board` and `board_viewpoint2` are the same and `Value` is a number and `Y2` and `B2` are still variables then we can say that `Y2` is `Y` and `B2` is `BlockIndex`. The important part here was that `Y2` and `B2` are still variables since this means that we still don't know their values. Why do we use 'is' instead of just removing the old `board_viewpoint2` from the constraint store? If we would just remove the constraint then this would mean that other constraints would never be fired! That's why we use `is`. If `Y2` and `B2` gets their values this means that maybe other constraints can be solved too. If we would input a new fact into the constraint store we would break all these links!

For the second constraint we need to know that both `Y` and `BlockIndex` are numbers and `V2` is still a var. Since we know both `X`, `Y` and `BlockIndex` then we can just fill in the `Value` of that position in `board`. We use 'is' for the same reason as mentioned before. If `V2` finally get's a value then this means that there might be other propagations that can fire.

1.4 Experiments

1.4.1 ECLiPSe

input order as value heuristic, alldifferent from ic_global

Puzzle	Classical Viewpoint (input order)		Our Viewpoint (input order)		Channeling (input order)	
	ms	backtracks	ms	backtracks	ms	backtracks
lambda	0.01s	3	74.669s	78859	0.031s	2
hard17	0.01s	1	84.319s	130134	0.041s	0
eastermonster	0.21s	51	3.29s	3278	0.139s	24
tarek_052	0.34s	59	0.19s	166	0.03s	0
goldennugget	0.671s	104	12.79s	11171	0.63s	70
coloin	0.22s	88	6.469s	4717	1.071s	178
extra2	0.0s	0	691.25s	662099	0.05s	4
extra3	0.011s	3	70.23s	78859	0.03s	2
extra4	0.01s	4	174.23s	197306	0.03s	3
inkara2012	0.03s	3	4.23s	5273	0.19s	34
clue18	0.26s	69	162.28s	156554	0.17s	20
clue17	0.01s	0	180.65s	191972	0.01s	0
sudowiki_nb28	1.03s	413	39.82s	40477	3.36s	832
sudowiki_nb49	0.19s	48	9.86s	10771	0.2s	37

input order as value heuristic, alldifferent from ic

Puzzle	Classical Viewpoint (input order)		Our Viewpoint (input order)		Channeling (input order)	
	ms	backtracks	ms	backtracks	ms	backtracks
lambda	0.34s	4712	36.93s	99470	1.359s	1155
hard17	0.07s	873	53.289s	153383	0.781s	1187
eastermonster	0.01s	119	2.709s	4361	0.131s	85
tarek_052	0.02s	193	0.15s	201	0.01s	6
goldennugget	0.06s	520	8.81s	16938	0.42s	389
coloin	0.19s	2209	3.82s	6998	0.76s	676
extra2	0.18s	4652	477.78s	959808	6.57s	13979
extra3	0.33s	4712	34.6s	99470	1.21s	1155
extra4	0.95s	15116	89.85s	253873	1.19s	1540
inkara2012	0.0s	50	3.78s	8033	0.26s	196
clue18	0.14s	1838	118.8s	268974	3.19s	3604
clue17	0.33s	5520	98.43s	227622	0.15s	177
sudowiki_nb28	0.21s	2851	41.91s	76081	2.33s	2187
sudowiki_nb49	0.11s	1078	5.89s	14424	0.18s	232

first fail as value heuristic, alldifferent from ic_global

Puzzle	Classical Viewpoint (first fail)		Our Viewpoint (first fail)		Channeling (first fail)	
	ms	backtracks	ms	backtracks	ms	backtracks
lambda	0.02s	3	24.07s	32384	0.03s	2
hard17	0.01s	1	6.28s	9183	0.02s	0
eastermonster	0.12s	33	10.69s	11599	0.17s	25
tarek_052	0.16s	35	1.389s	1502	0.071s	6
goldennugget	0.29s	76	19.94s	19870	0.39s	59
coloin	0.04s	8	0.48s	527	0.32s	56
extra2	0.0s	0	0.68s	909	0.02s	0
extra3	0.01s	3	23.88s	32384	0.03s	2
extra4	0.01s	3	585.84s	858267	0.03s	3
inkara2012	0.08s	17	8.33s	10942	0.08s	15
clue18	0.04s	8	175.63s	221669	0.02s	0
clue17	0.01s	0	262.53s	336082	0.019s	0
sudowiki_nb28	0.549s	297	41.25s	46872	1.25s	344
sudowiki_nb49	0.21s	58	23.021s	24536	0.219s	38

first fail as value heuristic, alldifferent from ic

Puzzle	Classical Viewpoint (first fail)		Our Viewpoint (first fail)		Channeling (first fail)	
	ms	backtracks	ms	backtracks	ms	backtracks
lambda	0.091s	977	10.42s	36846	1.2s	1961
hard17	0.031s	419	7.06s	23036	0.139s	386
eastermonster	0.01s	101	6.71s	16122	0.11s	122
tarek_052	0.019s	130	1.11s	3178	0.05s	45
goldennugget	0.051s	358	9.609s	23741	0.261s	310
coloin	0.01s	83	0.29s	929	0.22s	227
extra2	0.49s	7690	0.3s	1044	0.0s	10
extra3	0.08s	977	10.89s	36846	1.34s	1961
extra4	0.19s	2097	289.22s	986497	0.32s	720
inkara2012	0.03s	273	5.36s	13509	0.09s	108
clue18	0.05s	439	88.58s	256741	0.06s	52
clue17	0.03s	270	140.96s	440342	0.01s	15
sudowiki_nb28	0.21s	2221	20.19s	60024	0.65s	864
sudowiki_nb49	0.07s	655	11.9s	32895	0.19s	241

1.4.2 Impact of the different search strategies and alldifferent

Input order vs first fail

When we compare the two tables for **input order** and **first fail** with the alldifferent from ic_global we see that actually for most of the puzzles in all the three columns, the **first fail** heuristic does a better job than **input order**! We can see that there are sometimes really big differences between the two tables, for example 'extra2' for our viewpoint in the **input order** table has 662k backtracks but when you look over at the **first fail** table you see that it only has 909 backtracks! Though, it is not always **first fail** that is faster. If we look at 'extra4' for yet again our viewpoint we see that **input order** is faster this time! 197306 backtracks **input order** for vs 858267 backtracks **first fail**. This is quite odd behavior since 'extra4' is just puzzle 'extra3' (where **first fail** outperforms **input order** by more than 50%) with an extra hint.

When we look at the tables where we use alldifferent from the ic, we see exactly the same behavior as mentioned before. We see that most of the times **first fail** is faster than **input order** but yet again we observe the same odd behavior for 'extra4'.

ic_global vs ic

When we compare the alldifferent from ic_global vs ic we see that the version from ic_global is faster/does less backtracking! For our viewpoint the increase is not that big but when you look at the channeling column and the classical viewpoint, then you see that the increases in backtracks here are quite big!

1.4.3 Analysis of differences

input order vs first fail

To be able to explain these behaviors we first have to know the exact difference between these two search strategies. **Input order** takes the first element of a domain and **first fail** starts to take elements from the smallest domain. We think that the difference for 'extra4' can be explained as follows: As you always take the smallest, if you make a wrong decision early on, you will need to do a lot of backtracks in a wrong branch of the tree. Since you always took the smallest ones first you get a tree branch that is very wide. If your first choice was a mistake you need to traverse this whole subtree first before you can backtrack to your first choice. We think that the extra hint resulted in creating just this problem. It has probably adapted an interval to something smaller which creates all these problems. **Input order** doesn't suffer from this since it just takes the first element of a domain. On the other hand sometimes taking the smallest domain gives you the best result because you have less choices to make here which would explain the behavior for 'extra2'.

alldifferent

The difference with the alldifferent is due to the fact that the alldifferent from ic_global has been programmed with stronger propagation behavior ¹ than the alldifferent from the ic library as explained in class. The ic_global implementation has a global view of the constraints and can thus make smarter decisions about it.

¹http://eclipseclp.org/doc/bips/lib/ic_global/alldifferent-1.html

1.4.4 CHR

For the CHR implementation of the two viewpoints we had a lot of troubles getting something performant. When we run the two viewpoints on harder boards it takes easily a couple of minutes. Obviously this isn't good so we tried to improve the performance of both our implementations.

Heuristics classical viewpoint

For the classical viewpoint we found that there was one obvious heuristic that we could use to try to improve the search. To demonstrate this let's take for example the 'lambda' board from the puzzles provided on toledo.

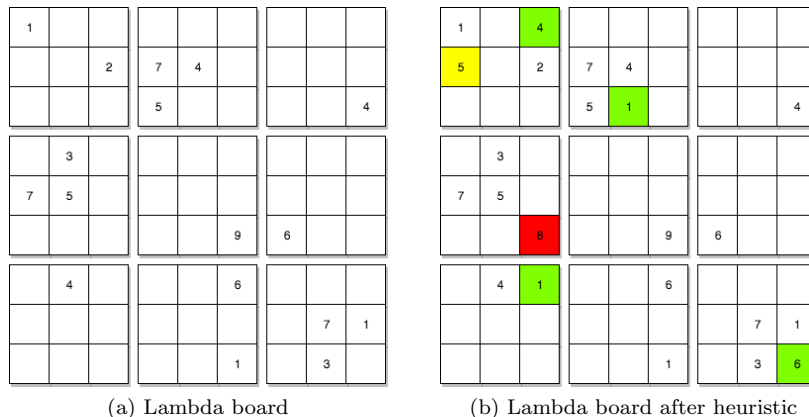


Figure 1.2: Lambda board pre vs post heuristic

When we look at 2.4(a) we for example see that in the bottom left block, at the position (9,9), the only possible number that we can play there is a 6 since it cannot be put anywhere else in the block because of the two sixes of the blocks next to us. This information is critical to be able to solve sudoku's in a smart way. This is what we tried to do with our likelihood heuristic. Our heuristic will try to let (9,9) know that it is very likely that 6 should be put there. In fact it is 100% 'likely' that 6 should be played there.

The way we do this is we check the domain of (9,9) and we compare it with the domains of all the other squares in the block. When doing this comparison we check for the numbers that is in our domain but not in the domain of the square we're comparing. If a number is not present in the domain of the comparing square, let's say 6 for example, then we know that the likelihood that this 6 is at our position increases. So we keep track of all the differences between the other squares and we count how many times we've seen a number. We then sort our domain according to this in a descending order based on the count. Obviously we want to try things that are very likely first and things that are less likely at a later moment. One important observation to this is that if the count of a number equals to 9 you know that we are the only position that can play this number else we would never see this number appear 8 times from the differences. We get to 9 because we use our domain as a starting point, so this means that each number in our domain immediately gets a count of 1. We do this because it is not certain that when checking all the differences with the other squares, you will get all the values from your domain. So if we would only use the numbers from the differences we might for example never see a 2 while actually 2 was the only answer that could be played at that position in the end.

When we look at figure 2.4(b), we can see the effect of our likelihood heuristic just before the start of the search. We see that already 5 different squares are filled in (the green and red ones). The green ones are some obvious plays but we were actually surprised when it played the red one. The other green ones are 'easy' because for example for the (9,9) example you see that this position is obviously the only place 6 can be played because of the other sixes. But for the red one we don't see a single other 8 on the board and yet it knew that it had to play an 8 there. We have to admit though that our system is a bit flawed since this heuristic cannot be used during the search since too many calculations are made. For example with everything we have filled in we could deduct that for the yellow square, 5 is the only possible play. But this is not feasible since it is way too slow to calculate this. This is why we ordered the domain according to likelihood. If we look at the domain of (2,1) sorted with likelihood, we see that the domain is [5, 3, 6, 8, 9]. So even though we're not fully sure that the 5 had to come there before we filled in the 4 in our block, our system captures the likelihood of a 5 at (2,1) still quite well.

Heuristics our viewpoint

To increase the search speed for our viewpoint we decided to do something about the search space itself. Consider the following example: Let's say we want to play a 3 at the

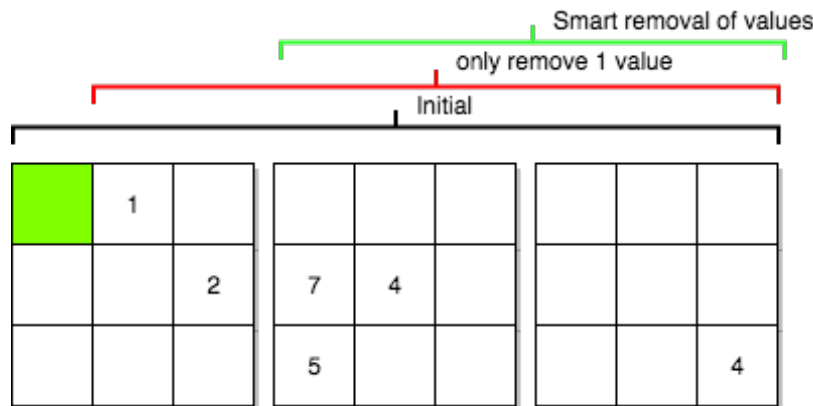


Figure 1.3: Smart domain changes

green square on figure 1.3. Our initial domain would be from 1 to 9. If we play the 3 in the green square then we know that for all our other 3's that we cannot put them in the first column anymore. So the other domains for playing 3 will remove the first column from their domain. But this play actually contains more information which we try to exploit with our system. Since we played in the first column, we know that for the other 3's in this block row they cannot be played anywhere in block 1 anymore! So for their domain, instead of just removing the first column, we remove all the columns from the first block from their domain! This means that from 9 values we go to 6 immediately and if we play another 3 in this block row then the last 3 only has a search space of 3 left! By doing this small change we effectively speed up our system quite a bit.

Experiments

When we compare both tables, we see that for our viewpoint we see that there is a good improvement. Although when we look for the classic viewpoint, we see that our heuristic is in most cases quite slower! Although in some cases it does increase it tremendously!

Puzzle	Classical Viewpoint ms	Our Viewpoint ms	Channeling ms
lambda	340.04s	118.72	218.44s
hard17	2003.50s	463.11s	413.61s
eastermonster	63.57s	235.10s	32.77s
tarek_052	13.58s	170.76s	26.12s
goldennugget	18.151s	155.49s	21.42s
coloin	132.72s	74.82s	69.82s
extra2	18.92s	2444.10s	14.14s
extra3	372.84s	134.93s	233.17s
extra4	141.55s	297.64s	115.41s
inkara2012	141.58s	1220.32s	116.46s
clue18	aborted after 30 min	aborted after 30 min	aborted after 30 min
clue17	11.29s	563.06s	8.56s
sudowiki_nb28	446.24s	45.05s	237.02s
sudowiki_nb49	73.02s	69.20s	68.67s

Table 1.1: Sudoku in CHR without heuristics

Puzzle	Classical Viewpoint ms	Our Viewpoint ms	Channeling ms
lambda	383.27s	8.07s	70.85s
hard17	282.39s	0.30s	1.61s
eastermonster	68.83s	14.74s	16.73s
tarek_052	1.70s	90.21s	1.70s
goldennugget	24.11s	43.1s	13.81s
coloin	90.63s	42.73s	17.73s
extra2	486.39s	54.98s	2.11s
extra3	385.02s	7.95s	70.63s
extra4	184.18s	10.60s	46.23s
inkara2012	10.75s	255.18s	5.50s
clue18	1068.80s	569.44s	249.05s
clue17	25.60s	1.01s	0.98s
sudowiki_nb28	1204.91s	4.58s	207.25s
sudowiki_nb49	130.41s	21.85s	70.93s

Table 1.2: Sudoku in CHR with heuristics

Task 2: Hashiwokakero

2.1 ECLiPSe

2.1.1 Basic solver

Constraints

The ECLiPSe Hashiwokakero solver is based on the [stackoverflow post²](https://stackoverflow.com/questions/20337029/hash-puzzle-representation-to-solve-all-solutions-with-prolog-restrictions) given in the assignment. We have used the proposed data structure and constraints in the post, and have added constraints for the connectivity of the islands. The resulting constraints are:

- For every position on the board it holds that the amount of bridges going in one direction (e.g. N) equals the amount of bridges going in the opposite direction (e.g. S) from the next position in the original direction (N in this example). If the position is on an edge of the board, the amount of bridges in the direction that would go outside of the board is zero.

```
1 ( X > 1 -> N #= Board[X-1, Y, 4] ; N = 0 ),
2 ( Y < YMax -> E #= Board[ X, Y+1, 5] ; E = 0 ),
3 ( X < XMax -> S #= Board[X+1, Y, 2] ; S = 0 ),
4 ( Y > 1 -> W #= Board[ X, Y-1, 3] ; W = 0 ),
```

- The sum of all bridges connected to an island must be equal to the number of the island.

```
1 [N, E, S, W] #:: 0..2,
2 N + E + S + W #= Amount
```

- The amount of bridges going in one direction equals the amount of bridges going the opposite direction and bridges cannot cross each other.

```
1 N = S, E = W,
2 (N #= 0) or (E #= 0)
```

- The connectivity constraint is implemented by checking that all islands can be visited starting from a certain island. This is done after the search. `fill_set_visit` is called with a certain starting island. Then, the `Visited` list is filled with the islands that can be reached from the starting island. Afterwards, the length of `Visited` must equal the amount of islands on the board. If the length of `Visited` does not equal the amount of islands on the board, this means that not all islands can be visited from a certain island, which then means that the bridges do not connect the islands into a single connected set.

²<https://stackoverflow.com/questions/20337029/hash-puzzle-representation-to-solve-all-solutions-with-prolog-restrictions>

```

1 board_connected_set(Board) :-
2     board_islands(Board, Islands),
3     length(Islands, N),
4     length(Visited, N),
5
6     % make the island be member of current set
7     nth1(1, Islands, [X, Y]),
8
9     % set position to visited
10    nth1(1, Visited, 1),
11
12    % travel to the neighbors of the current position and fill the Islands/Visited
13    set
14    fill_set_visit(Board, X, Y, Islands, Visited),
15
16    % if all free variables in Visited have been bound, then all islands form a
17    connected set
18    count_nonvars(Visited, N).

```

All the constraints are active, except for the connectivity constraints. The connectivity constraint does not directly make changes to domains of variables so it is a passive constraint. The other constraints do make changes to domains.

2.1.2 Improvements

When one thinks about improvements for solving Hashiwokakero there are a couple of obvious ones that pop up. For example, if there is a 4 in one of the four corners, the only way to get 4 bridges is by placing 2 bridges in the two possible directions. The same idea goes for a 6 at one of the edges of the board. Because one of its directions is blocked, the only way to get 6 bridges is by placing 2 bridges in all 3 of the possible directions. For an island with an 8, the only option is to place 2 bridges in every direction. These seem like good improvements, but actually they were already done implicitly by ECLiPSe's domain solving. Since these were not really improvements to our solver, we had to look for other improvements. The following improvements are inspired by <http://www.conceptispuzzles.com/index.aspx?uri=puzzle/hashi/techniques>.

1. The first improvement we made was stating that a 1 and another 1 cannot be connected directly to each other. If there was a bridge between them, we would immediately have an invalid solution, as all of the islands in the end result must form a single connected set. Another situation that corresponds with this idea is a 2 and another 2. These can never have 2 bridges between each other since then you would also create an isolated segment. By stating that there cannot be a bridge between islands with 1 and that there cannot be two bridges between islands with 2, we have added more knowledge before the search starts. Figure 2.4 shows the result of this improvement. It shows two boards before the search phase. The board with the improvement contains two more bridges before the search phase. This means that during the search less backtracking will occur in the worst case.
2. The second improvement builds further on the idea of avoiding isolated segments. Assume there is an island with a 2 that has three neighbors, two of which are islands with 1. If you were to place a bridge from the 2 to both of the 1's, then you would end up with an isolated segment. In this case, we know that the third neighbor will

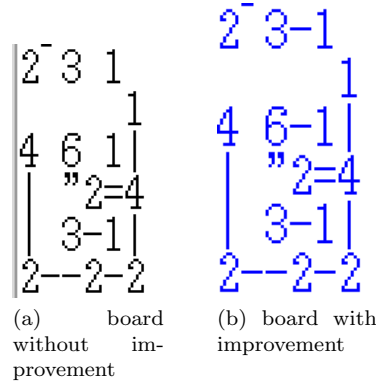


Figure 2.4: boards without and with improvement

have at least one bridge going to the 2. This is implemented by stating that the amount of bridges going from the 2 to the third neighbor cannot be zero. This adds more knowledge to start with before the search.

3. The third improvement avoids isolated segments for islands with a 3 that have 3 neighbors, among which one neighbor has a 1 and another has a two. If you connect one bridge with the 1 and 2 bridges with the 2, you get an isolated segment. In this case, we know for sure that the third neighbor of the island with 3 has at least one bridge going to the 3. This is implemented by stating that the amount of bridges going from the 3 to the third neighbor cannot be zero. Again, this adds more knowledge to start with before the search.

All three of the improvements have an impact on the connectivity constraint. Because the improvements avoid isolated segments before the search starts, the amount of backtracks (in the worst case) caused to satisfy the connectivity constraint will be reduced.

2.1.3 Experiments

Puzzle	Without improvements		With improvements	
	ms	backtracks	ms	backtracks
1	0	1	0	1
2	130	60	81	42
3	0	0	0	0
4	0	0	0	0
5	0	0	0	0
6	10	1	10	1
7	0	0	0	0
8	0	0	0	0
9	0	1	0	1
10	49	160	61	160
11	0	0	0	0
12	0	1	0	1
13	10	0	0	0
14	0	1	0	1
15	20	4	20	4
16	21	28	20	28
17	10	1	9	0
18	0	1	0	1
19	0	0	0	0
20	0	2	0	1
21	0	1	0	0
22	10	9	10	9

Table 2.3: ECLiPSe Hashiwokakero solutions using input order

2.2 CHR

2.2.1 Data representation

We used the same data representation as in the ECLiPSe solution. The data is stored in 'board/7' facts with variables X, Y, Am, N, E, S, W, where (X, Y) represents the position of the board, Am the amount of bridges that must be connected this position. N, E, S, and W represent the amount of bridges that go in a certain direction from position (X,Y).

2.2.2 Constraints representation

- The amount of bridges going in one direction equals the amount of bridges going in the opposite direction on the next position.

```

1 board_connected_set(Board) :-
2   bridge_constraints, board(X, Y, _, N, _, _, _),
3   board(X2, Y, _, _, _, S, _) ==> X > 1, X2 is X-1 |
4   N eq S.
5
6   bridge_constraints, board(X, Y, _, _, E, _, _),
7   board(X, Y2, _, _, _, W) ==> Y2 is Y+1 |
8   E eq W.
```

- There are no crossing bridges. In other words, if the amount of bridges going in one direction is greater than zero, then the amount of bridges going in the perpendicular

Puzzle	Without improvements		With improvements	
	ms	backtracks	ms	backtracks
1	0	1	0	1
2	170	61	20	61
3	0	0		0
4	0	0	0	0
5	0	0		0
6	10	1		1
7	0	0		0
8	0	0		0
9	0	1	0	1
10	110	152	69	152
11	0	0	0	0
12	0	1	10	1
13	0	0	0	0
14	10	1	0	1
15	80	4	90	4
16	50	28	61	28
17	0	1	10	0
18	0	1	0	1
19	0	0	0	0
20	0	2	0	1
21	0	1	0	0
22	10	9	10	9

Table 2.4: ECLiPSe Hashiwokakero solutions using first fail

position must be zero.

```

1 board(_, _, 0, N, E, _, _) ==> number(N), N > 0 | E = 0.
2 board(_, _, 0, N, E, _, _) ==> number(E), E > 0 | N = 0.

```

- Bridges cannot go outside of the board. The amount of bridges going in a direction that would leave the board are set to zero here.

```

1 bridge_constraints, board(1, _, _, N, _, _, _) ==> N = 0.
2 bridge_constraints, board(_, YMax, _, _, E, _, _), ymax(YMax) ==> E = 0.
3 bridge_constraints, board(XMax, _, _, _, S, _), xmax(XMax) ==> S = 0.
4 bridge_constraints, board(_, 1, _, _, _, W) ==> W = 0.

```

- If on a certain position there is no island, then the amount of bridges that come in from a certain direction equals the amount of bridges that leave in the opposite direction.

```

1 bridge_constraints, board(_, _, 0, N, E, S, W) ==> N = S, E = W.

```

- The amount of bridges connected to an island equals that island's amount. Three addition constraints are used here to represent

$$Amount = N + E + S + W$$

```

1 bridge_constraints, board(_, _, Amount, N, E, S, W) ==> Amount > 0 |
2   Sum in 0..4,
3   Sum2 in 0..4,
4   add(N, E, Sum),
5   add(S, W, Sum2),
6   add(Sum, Sum2, Amount).

```

2.2.3 Propagation and search

Constraint propagation is handled by the 'in/2', 'add/3' and 'eq/2' rules. The 'in/2' rule is used to set the domain of a variable. The domain of a variable is a list of integers represented by two integers A and B and is noted as 'A..B'. When the domain of a variable only contains one element (when A equals B), then the value of the variable is known. The 'add/3' and 'eq/2' rules are used to represent the rules for addition and equality in the program. additionally, their job is to reduce the domains of variables (see lines 10, 11, and 16 in the code below). This makes these constraints active constraints. An example of the domain reduction goes as follows. Assume the following facts are known. 'X in 0..4', 'Y in 0..4', 'Z in 0..2', add(X, Y, Z). We know that $X + Y$ must equal Z, and that Z must lie between 0 and 2. Therefore, we can reduce the domains of X and Y from '0..4' to '0..2'. This kind of behaviour is implemented by line 16 in the code excerpt below.

```

1 % remove duplicate indomain constraints
2 X in A..B \ X in A..B <=> var(X) | true.
3 % when a variable's domain gets reduced to 1 number, set the value of the variable
4 X in A..A <=> var(X) | X = A.
5
6 % equality constraint
7 X eq Y <=> number(X), number(Y) | X == Y.
8 % equality domain constraint solving
9 X eq Y \ X in A..B, Y in C..D <=> A \== C | L is max(A, C), X in L..B, Y in L..D.
10 X eq Y \ X in A..B, Y in C..D <=> B \== D | U is min(B, D), X in A..U, Y in C..U.
11
12 % addition constraint
13 add(X, Y, Z) <=> number(X), number(Y), number(Z) | Z is X + Y.
14 % addition domain constraint solving
15 add(X, Y, Z) \ X in A..B, Y in C..D, Z in E..F <=>
16   not( ( A >= E-D, B <= F-C, C >= E-B, D <= F-A, E >= A+C, F <= B+D ) ) |
17   NewA is max(A, E-D), NewB is min(B, F-C), X in NewA..NewB,
18   NewC is max(C, E-B), NewD is min(D, F-A), Y in NewC..NewD,
19   NewE is max(E, A+C), NewF is min(F, B+D), Z in NewE..NewF.

```

Listing 2.3: Constraint propagation

The actual search for values of variables is handled by the 'search/0' and 'enum/1' rules. When 'enum/1' is fired with a variable that has a domain constraint ('X in A..B'), then it will cause X to take a value in its domain. Values are assigned to X by Prolog's 'between(A, B, X)' procedure. Each time X is assigned a value by this procedure, other rules may be fired, depending on what is in the constraint store at that time. Among those other rules could lie equality and addition constraints which can now further reduce domains of other variables because the domain of X has been removed. If this causes backtracking to happen back up to the 'between' procedure, the next value in the domain will be assigned to X.

```

1 % assign values to variables X. X must lie between A and B

```

```

2 enum(X) <=> number(X) | true.
3 enum(X), X in A..B <=> between(A, B, X).
4
5 % search for constraint variables
6 search, X in ..._ ==> var(X) | enum(X).

```

Listing 2.4: Search

2.2.4 Connectivity constraint

The constraint that in the end result, the bridges must connect all islands into a single connected group, is handled in two different ways.

Version 1

The first version implements the connectivity constraint by checking whether or not all islands are reachable from a certain island. 'reachable/2' is used to represent which positions on the board are reachable. The reachable set can be expanded whenever a new bridge between two islands ('connection/2') is detected. The actual connectivity constraint is represented by lines 10 and 11 in the excerpt below. If, after the search phase is over, there is an 'island/3' fact without an accompanying 'reachable/2' fact, then the islands are not connected into a single group. This then causes backtracking to happen which causes search to try other values for variables.

```

1 % put first island in reachable set
2 island(X, Y, _) \ pick_first_island <=> reachable(X, Y).
3
4 % build up reachable set
5 reachable(X, Y) \ connected([X, Y], [A, B]) <=> reachable(A, B).
6 reachable(X, Y) \ connected([A, B], [X, Y]) <=> reachable(A, B).
7 reachable(X, Y) \ reachable(X, Y) <=> true.
8
9 % connectivity constraint: each island fact needs to have an accompanying reachable
  fact
10 connected \ island(X, Y, _), reachable(X, Y) <=> true.
11 connected, island(_, _, _) <=> false.

```

Listing 2.5: Connectivity constraint propagator 1

Version 2

The second connectivity constraint propagator follows a similar idea, but implements it by using a disjoint-set-like data structure. After the puzzle board is loaded, each island forms its own 'connected set' (or segment). A counter 'connected_sets_counter/1' represents the amount of connected sets that are on the board. 'connected_set/3' represents which island belongs to which connected set. After a bridge has been formed between two islands ('connected/2'), their connected sets are merged together (lines 1, 4, and 10 below) by changing the identifier of the smaller connected set into the identifier of the larger one. After the sets have been merged, the 'connected_sets_counter/1' is decremented. The actual connectivity constraint is represented by the last line in the code excerpt. If, after the search phase is over, there exists more than one connected set, then the solution is not valid, and backtracking is forced.

Compared to version 1, this connectivity constraint propagator is actually worse. This is because it is more costly to keep track of all of the connected sets during search than

to just check whether or not all island are reachable from a certain island. However, subsection 2.2.5 contains an improvement for this version which makes it perform better than the previous one.

```

1 connected_set(A, B, Set1), connected_set(C, D, Set2), connected_set_counter(Set1,
  Count1)
2 \ connected([A, B], [C, D]), connected_set_counter(Set2, Count2) <=> Set1 \==
  Set2, Count2 <= Count1 |
3   connected_sets_union(Set1, Set2).
4 connected_set(A, B, Set1), connected_set(C, D, Set2), connected_set_counter(Set1,
  Count1)
5 \ connected([C, D], [A, B]), connected_set_counter(Set2, Count2) <=> Set1 \==
  Set2, Count2 <= Count1 |
6   connected_sets_union(Set1, Set2).
7 connected_set(A, B, Set), connected_set(C, D, Set) \ connected([A, B], [C, D]) <=>
  true.
8 connected_set(A, B, Set), connected_set(C, D, Set) \ connected([C, D], [A, B]) <=>
  true.
9
10 connected_sets_union(Set1, Set2) \ connected_set(X, Y, Set2),
  connected_set_counter(Set1, Count) <=> NCount is Count + 1 |
11   connected_set(X, Y, Set1),
12   connected_set_counter(Set1, NCount).
13 connected_sets_union(_, _), connected_sets_counter(Count) <=> NCount is Count - 1 |
  connected_sets_counter(NCount).
14
15 connected, connected_sets_counter(C) ==> C > 1 | fail.

```

Listing 2.6: Connectivity constraint propagator 2

2.2.5 Improvements

Improvement 1

Our first improvement in the CHR solution follows the same logic as the first improvement in the ECLiPSe solution. Islands with value 1 cannot be connected to one another and islands with value 2 cannot be connected by 2 bridges (because then they would form isolated segments). This is not detected automatically by the domain solving rules of 'eq/2' and 'add/3', so we have added 4 rules which reduce the domains of variables before the search phase starts.

```

1 % improvement A: island with 1 cannot be connected to other island with 1, so set
  variable to 0
2 make_domains, board(X1, Y1, 1, N, _, _, _), neighbors(X1, Y1, 'N', X2, Y2),
  island(X2, Y2, 1) \ N in ..._ <=> var(N) | N = 0.
3 make_domains, board(X1, Y1, 1, _, E, _, _), neighbors(X1, Y1, 'E', X2, Y2),
  island(X2, Y2, 1) \ E in ..._ <=> var(E) | E = 0.
4 % improvement B: 2 cannot be connected to 2 by 2 bridges, so make domain A..1
5 make_domains, board(X1, Y1, 2, N, _, _, _), neighbors(X1, Y1, 'N', X2, Y2),
  island(X2, Y2, 2) \ N in A..2 <=> var(N) | N in A..1.
6 make_domains, board(X1, Y1, 2, _, E, _, _), neighbors(X1, Y1, 'E', X2, Y2),
  island(X2, Y2, 2) \ E in A..2 <=> var(E) | E in A..1.

```

Listing 2.7: Connectivity constraint propagator 2

This improvement removes a lot of the guess work done by 'between/3', because the reduction of some of the domains can cause a domino effect to occur in which a lot of domains can subsequently be reduced. On some boards there isn't even a need for the

search phase any more because the board can be solved by the domain solving of 'eq/2' and 'add/3'.

This improvement also has an impact on the connectivity constraint, as isolated sets like 1-1 and 2=2 cause the connectivity constraint to not be satisfied. This is particularly impactful for example on boards for which the solver would guess an isolated set like 1-1 right at the beginning of the search phase. In that case, backtracking would need to go over all choice points made after the variables included in the 1-1 set.

Improvement 2

The biggest flaw in the two connectivity constraint propagators was that checking the connectivity constraint only happens after a value has been assigned to all variables. The connected sets of the second connectivity constraint propagator allows us to detect isolated segments earlier during the search phase. First, 'no_isolated_segments/0' is added to the constraint store after a new value has been assigned to a variable. 'no_isolated_segments/0' is used to add 'connected_set_not_isolated/1' into the constraint store. This then makes sure that each connected set currently on the board is not isolated. A connected set is not isolated when one of the islands in the set can still create a bridge (and this is possible when one of the directions in the 'board/7' is still a variable). If no such island is found in a set, then backtracking is forced (last line in the excerpt below).

```

1 search, X in ... => var(X) | enum(X), no_isolated_segments.
2
3 ...
4
5 no_isolated_segments, connected_set_counter(Set, _) =>
6   connected_set_not_isolated(Set).
7 no_isolated_segments <=> true.
8
9 connected_sets_counter(1) \ connected_set_not_isolated(_) <=> true.
10 connected_set(X, Y, Set1), board(X, Y, _, N, _, _, _) \
11   connected_set_not_isolated(Set1) <=> var(N) | true.
12 connected_set(X, Y, Set1), board(X, Y, _, _, E, _, _) \
13   connected_set_not_isolated(Set1) <=> var(E) | true.
14 connected_set(X, Y, Set1), board(X, Y, _, _, _, S, _) \
15   connected_set_not_isolated(Set1) <=> var(S) | true.
16 connected_set(X, Y, Set1), board(X, Y, _, _, _, _, W) \
17   connected_set_not_isolated(Set1) <=> var(W) | true.
18 connected_set_not_isolated(_) <=> false.

```

Listing 2.8: Connectivity constraint propagator 2

This improvement has a large impact on the connectivity constraint, because now the connectivity constraint is checked during the search phase instead of after the search phase. If an isolated segment were to be formed early in the search phase, this improvement would detect it and cause backtracking to happen instantly, whereas without this improvement backtracking would need to go over all choice points made after the creation of the isolated set. The effect of this improvement is most visible on board 2; its solving time went from 1600s (check this / update table) with improvement 1 to 850s with improvement 1 and 2. This improvement did not have a great impact on most other boards. This is because most other boards had smaller and fewer (or zero in some cases) domains to chose from during the search phase because of the domain solving done by 'eq/2' and 'add/3' (CHECK THIS WITH UPDATED TABLE).

2.2.6 Experiments

board number	no improvements	isolation improvements
1	0.049s	0.044s
2	> 1h30m	497.33s (8.3m)
3	0.026s	0.03s
4	0.054s	0.057s
5	0.05s	0.049s
6	0.121s	0.114s
8	0.006s	0.004s
9	0.006s	0.007s
10	26.642s	1.344s
11	0.044s	0.049s
12	0.05s	0.05s
13	0.056s	0.056s
14	0.097s	0.066s
15	1.584s	1.527s
16	1.11s	1.113s
17	2.152s	0.222s
18	0.035s	0.051s
19	0.01s	0.011s
20	0.034s	0.024s
21	0.012s	0.013s
22	0.028s	0.027s

Table 2.5: Improvements on CHR solver with connectivity propagator 1

board number	no improvements	isolation improvements	connectivity constraint improvement	all improvements
1	0.05s	0.053s	0.054s	0.048s
2	unknown	515.92 (8.6m)	???	316.37s (5.3m)
3	0.028s	0.028s	0.028s	0.026s
4	0.065s	0.059s	0.057s	0.062s
5	0.058s	0.052s	0.051s	0.051s
6	0.125s	0.119s	0.127s	0.116s
8	0.007s	0.004s	0.006s	0.006s
9	0.007s	0.01s	0.007s	0.006s
10	27.415s	1.356s	50.548s	1.34s
11	0.046s	0.047s	0.048s	0.048s
12	0.052s	0.048s	0.053s	0.052s
13	0.059s	0.059s	0.06s	0.06s
14	0.093s	0.077s	0.093s	0.076s
15	1.587s	1.613s	1.604s	1.555s
16	1.127s	1.226s	1.132s	1.143s
17	2.221s	0.232s	2.585s	0.234s
18	0.047s	0.049s	0.06s	0.039s
19	0.01s	0.011s	0.012s	0.011s
20	0.024s	0.023s	0.024s	0.029s
21	0.012s	0.013s	0.014s	0.012s
22	0.03s	0.037s	0.024s	0.026s

Table 2.6: Improvements on CHR solver with connectivity propagator 2

Conclusion

3.1 Weak points

There are a couple of weak points in our report:

- We have to admit that the speed of our Sudoku implementations in CHR are not that great. We did try to improve it with our heuristics but we think this is still too slow. We think this was more due to the fact that we were still novices with how CHR worked as our hashiwokakero CHR implementation is quite fast.
- Our connectivity constraint in ECLiPSe for hashiwokakero is also rather a weak point. Ideally we should've implemented it as a connected set but we couldn't figure out how we had to do this.
- We think that implementing hashiwokakero as a graph is a more logical approach than the one we used now, but we had problems with ECLiPSe and how to express the no crossing bridge constraint.

3.2 Strong points

The strong points in our report are:

- Our hashiwokakero CHR implementation is quite fast thanks to our connected set.
- We think we have good heuristics for Sudoku in CHR. Certainly the heuristic for our viewpoint has made good improvements on the speed of the search.

3.3 Lessons learned

We have learned a lot of things during this project due to the fact we often had to re-track our steps. If we would now have to do an other project with these systems we think we would have a better idea for how to start and what to think about. We spend quite some time with both ECLiPSe and CHR so we now also know very well what the strong points are and the weak points are for each system.

Appendix

We started working on this project before the Easter holiday. In the beginning we often lost quite some time, since we didn't really know how the systems worked. During the second week of the Easter holiday, we continued to work on the project each evening and we finished the Sudoku task and the ECLiPSe part of hashiwokakero before the end of the holiday. We then had to halt our work for a while since we had a deadline for a ridiculously large project for another course. As the semester was coming to an end other deadlines and an exam were coming up so we had to manage those first. Thus it was only at the start of the study period that we could continue working. From the start of the study period we tried to spend around 6 hours of work each day for this project. The work was not really divided since we were doing pair programming most of the time. Sometimes someone made individual changes when they had time but most of our work was done online using Hangouts and its screen sharing functionality. We could argue that by doing pair programming we lost quite some time, which is true, but by doing this we worked very closely together and we learned quite a lot. We have each individually put more than 50 hours in this project.