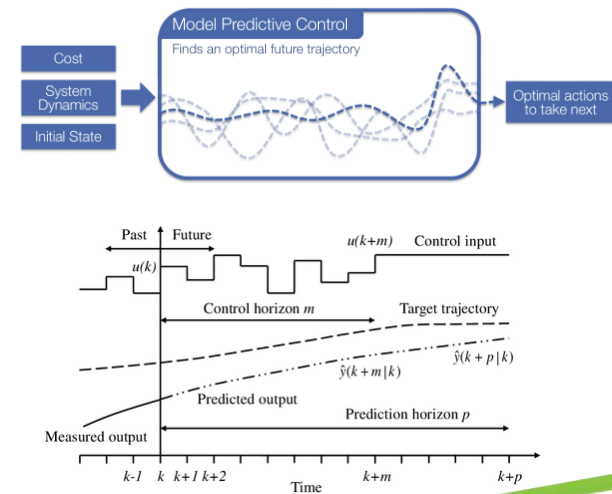
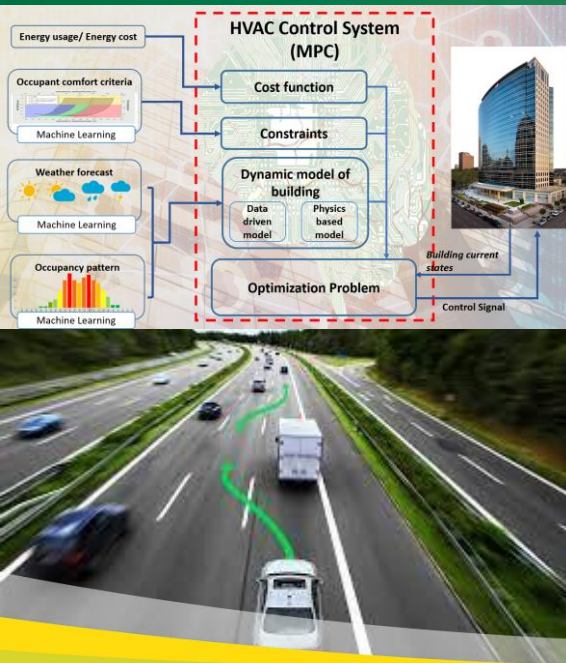


Mahdi Shahbakhti
Jun. 5, 2022



Model Predictive Control

Control technology survey results ranked by industry impact

Rank and Technology	High-Impact Ratings	Low- or No-Impact Ratings
PID control	100%	0%
Model predictive control	78%	9%
System identification	61%	9%
Process data analytics	61%	17%
Soft sensing	52%	22%
Fault detection and identification	50%	18%
Decentralized and/or coordinated control	48%	30%
Intelligent control	35%	30%
Discrete-event systems	23%	32%
Nonlinear control	22%	35%
Adaptive control	17%	43%
Robust control	13%	43%
Hybrid dynamical systems	13%	43%

Source: T. Samad. A survey on industry impact and challenges thereof, IEEE Control Systems Magazine, 2017.

1970 – 1990

- Developing MPC theory
- Implementing MPC in the petrochemical and process industry

1991 – 2000

- Nonlinear MPC
- Robust MPC

2001 – 2005

- Hybrid MPC
- Distributed MPC
- Stochastic MPC
- Explicit MPC

2006 – 2010

- Economic MPC
- Fast MPC (Online optimization)

2011 - 2021

- Real-time nonlinear/nonconvex MPC



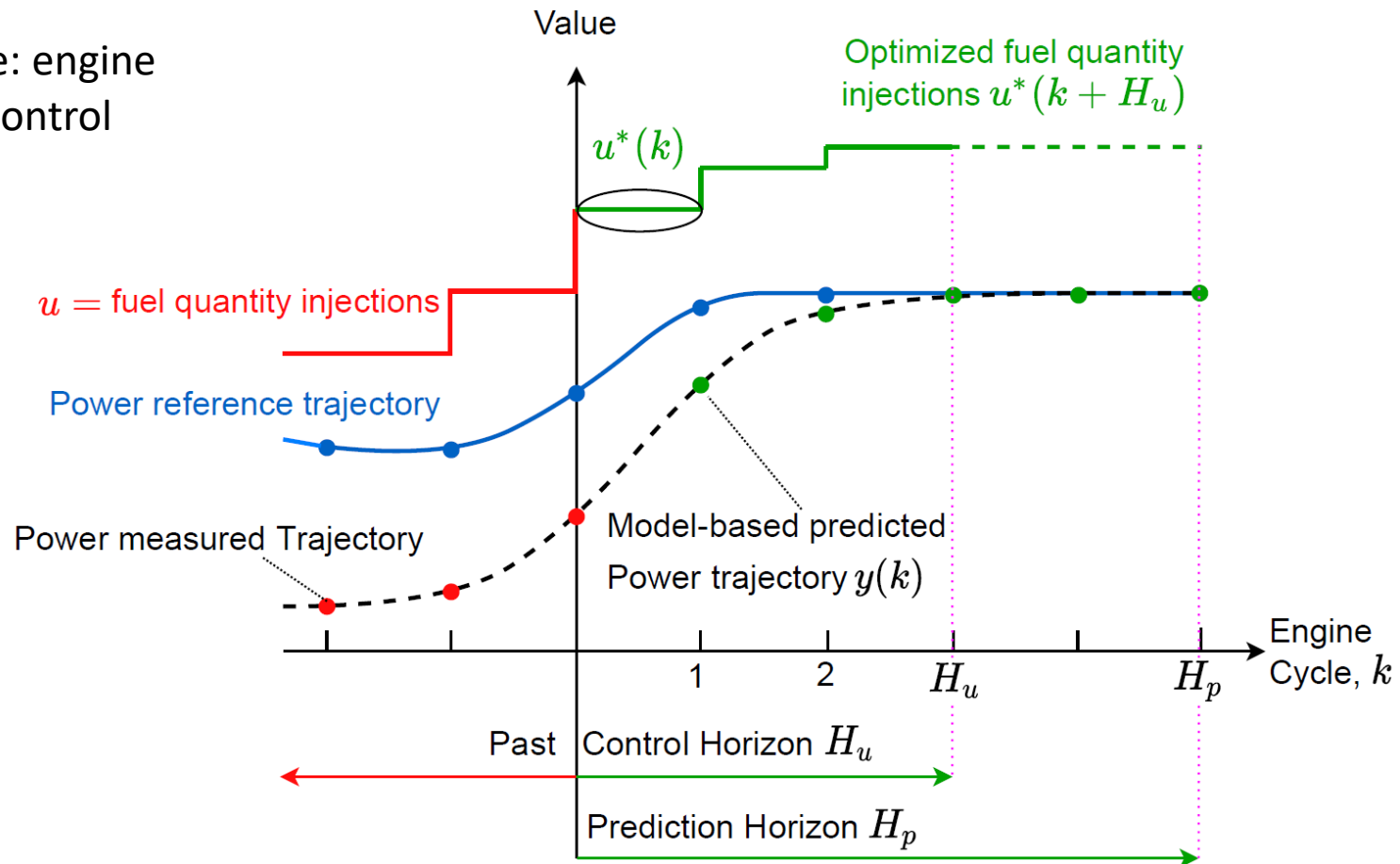
MPC Benefits

- Implicitly considers constraints on state, input and output variables;
- Provides closed loop control performance and stability for the optimal problem with constraints;
- Exploits the use of a future horizon while optimizing the current control law;
- Possibility of both offline and real-time implementation;
- Flexible to handling of delays, and non-linearity in the model.

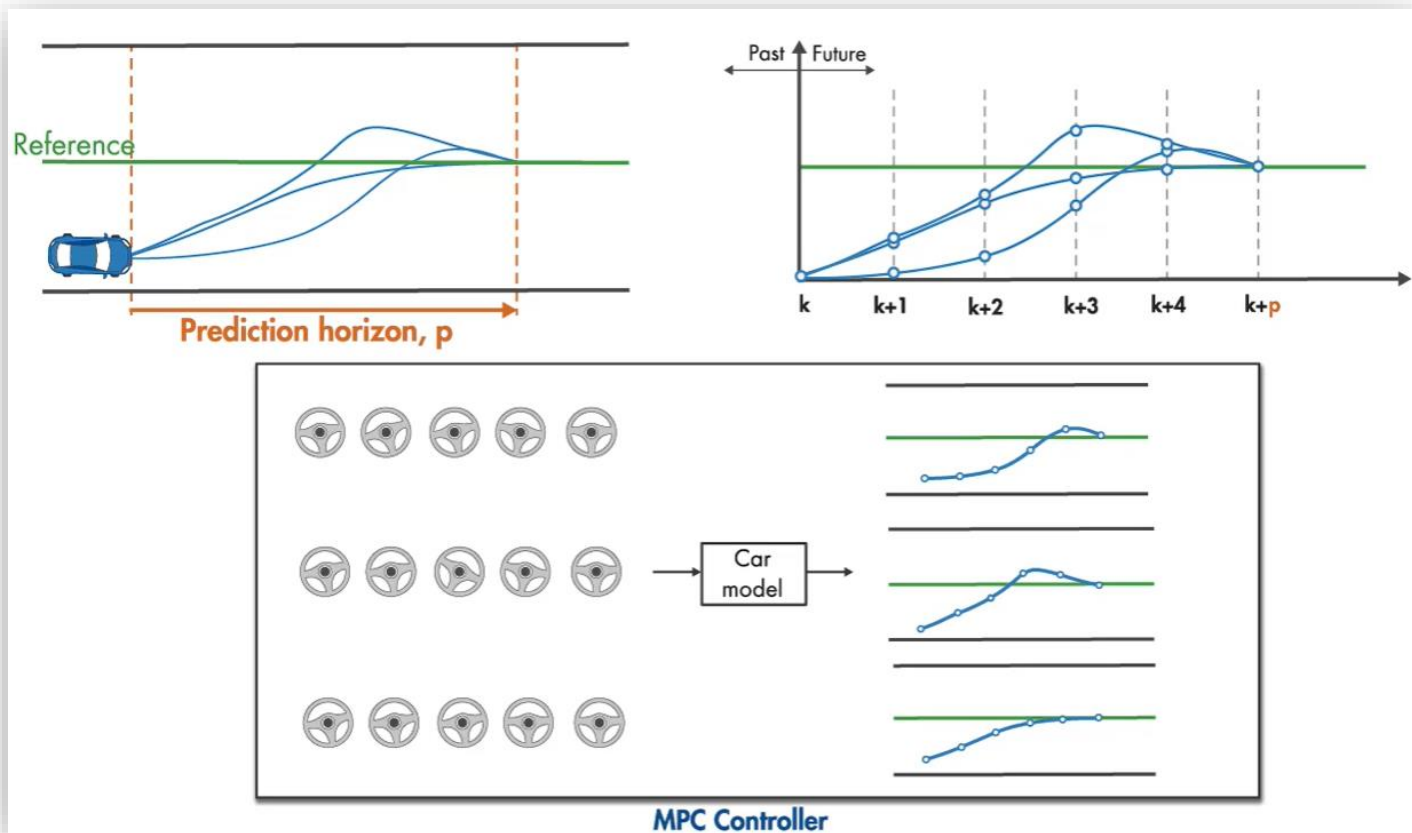
MPC Concept

Control horizon and prediction receding horizon

Example: engine power control



Concept Illustration



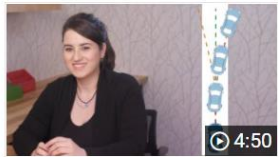
<https://www.mathworks.com/videos/understanding-model-predictive-control-part-2-what-is-mpc--1528106359076.html>

Interesting Videos

MATLAB® Tech Talks

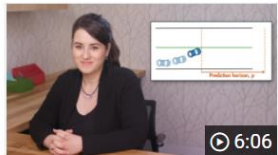
Taking you from learning to mastering

<https://www.mathworks.com/videos/series/understanding-model-predictive-control.html>



Part 1: Why Use MPC?

Learn about model predictive control (MPC). MPC handles input-output interactions, deals with constraints, and has been used in industries such as auto and aero.



Part 2: What Is MPC?

Learn how model predictive control (MPC) works. MPC makes predictions about future plant outputs. It solves each time step to find the optimal control action that gets the output to the desired reference as close as possible.



Part 3: MPC Design Parameters

To successfully control a system using MPC, you need design parameters. Learn how to select the controller sample time, prediction horizons, and constraints and weights.



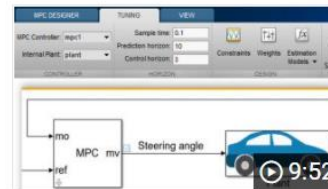
Part 4: Adaptive, Gain-Scheduled, and Nonlinear MPC

Learn about the type of MPC controller you can use based on constraints, and cost function. Options include the linear, gain-scheduled, and nonlinear MPC.



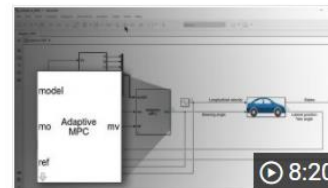
Part 5: How To Run MPC Faster

Learn which techniques you can use to speed up methods, such as explicit MPC and your applications with small sample times.



Part 6: How to Design an MPC Control Toolbox

Learn how to design an MPC controller using the Model Predictive Control Toolbox.



Part 7: Adaptive MPC Design with

Learn how to deal with changing parameters using an autonomous steering vehicle controller's design.

MPC

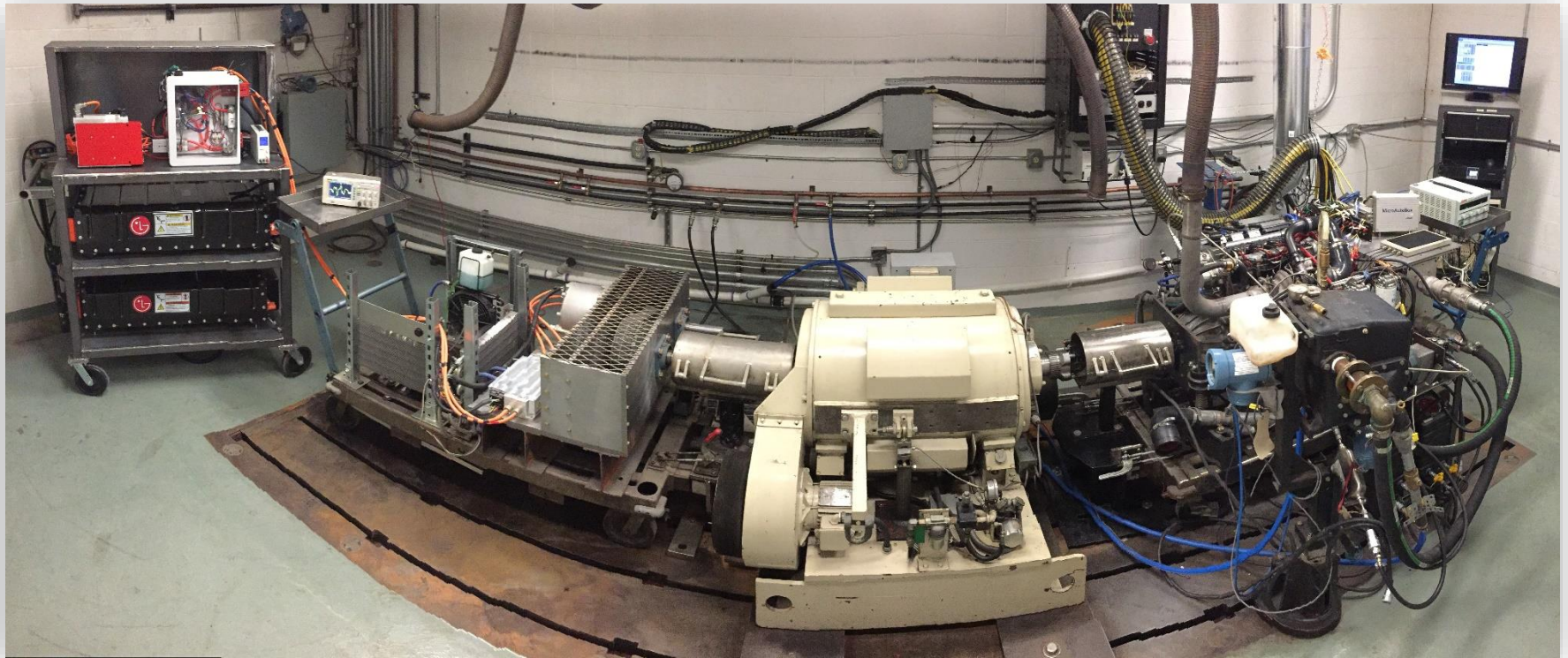
Formulation

$$\begin{aligned} & \min_{u_0, \dots, u_{N-1}} \quad \underbrace{J_f(x_N)}_{\text{Terminal cost}} + \underbrace{\sum_{k=0}^{N-1} J(x_k, y_k, r_k, u_k, s_k)}_{\text{Stage cost}} \\ & \text{s.t.} \quad x_{k+1} = f(x_k, u_k, d_k), \quad y_k = g(x_k, u_k, d_k) \quad k \in \mathbb{N}_0^{N-1} \\ & \quad \quad x_k \in \mathcal{X}, \quad u_k \in \mathcal{U} \quad k \in \mathbb{N}_0^{N-1} \\ & \quad \quad X_N \in \mathcal{X}_f \quad x_0 = x(t) \end{aligned}$$

- x, y, u, r, s, d are states, outputs, inputs, references, slack variables, disturbances;
- N is the prediction horizon;
- J, f, g , are the cost function, state function, output function;
- \mathcal{X}, \mathcal{U} , and \mathcal{X}_f are state constraint set, input constraint set, and terminal state constraint set.

MPC Example: HEV

Plant: Multi-Mode Powertrain for a Hybrid Electric Vehicle (HEV)



MPC Example: HEV

MPC Structure

Cost Function:

$$J(u(t)) = \int_0^T (\dot{m}_f(P_{bat}, t) + \Gamma.F_{p1} + \Lambda.m_{ij} + \Psi.F_{p2}) dt$$

Engine ON/OFF
Penalty

Mode-Switching
Penalty

Gear-Shifting
Penalty

Hard Constraints:

$$|SOC_f - SOC_0| \leq 0.01$$

$$0.3 \leq SOC(t) \leq 0.7$$

$$P_{bat,min} \leq P_{bat}(t) \leq P_{bat,max}$$

$$P_{eng,min}(\omega_{eng}) \leq P_{eng}(t, \omega_{eng}) \leq P_{eng,max}(\omega_{eng})$$

$$\omega_{eng,min} \leq \omega_{eng}(t) \leq \omega_{eng,max}$$

$$0 \leq P_{motor}(t) \leq 100 \text{ kW}$$

$$0 \leq \omega_{motor}(t) \leq 8000 \text{ RPM}$$

$$Temp_{exh}(\omega_{eng,min}, T_{eng,min}) \geq 300 \text{ } ^\circ C$$

$$\omega_{eng} \leq 1500 \text{ rpm}, \text{ if } V_{veh} \leq 40 \text{ km/h}$$

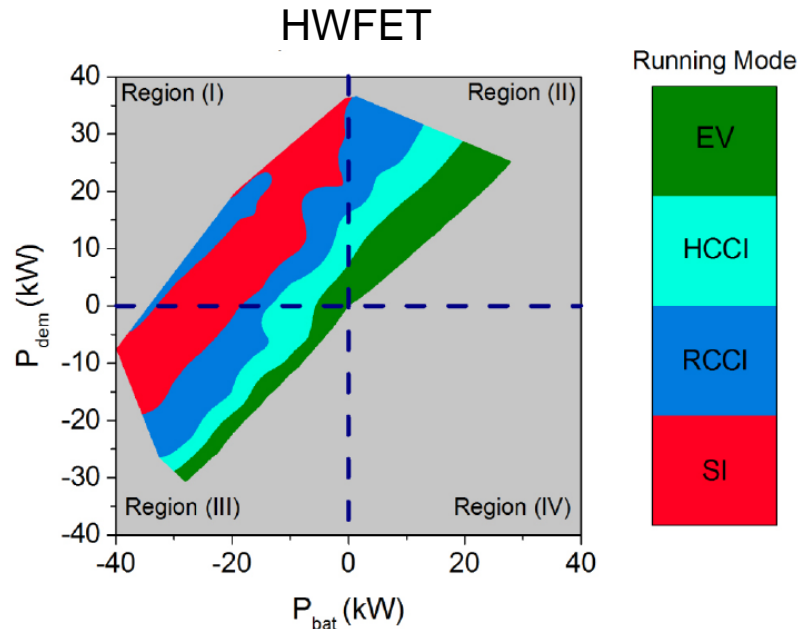
Catalyst light-off
constraint

NVH constraint

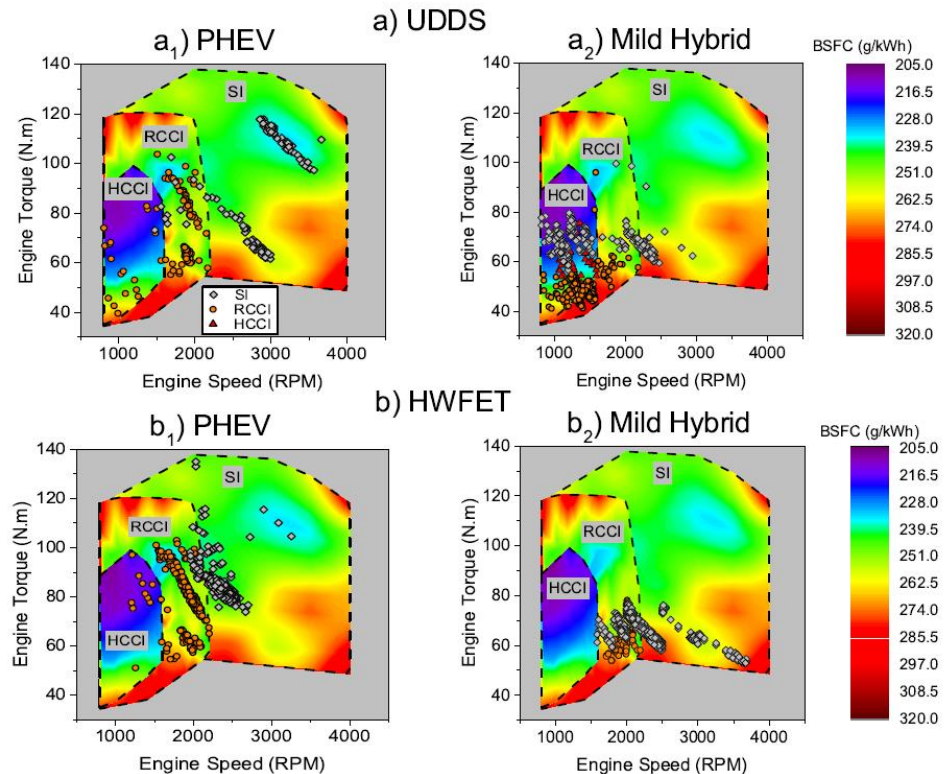
Source: A. Solouk, M. Shahbakhti, et. al., *Energy Conversion and Management*, 2018.

MPC Example: HEV

Results: Optimal Operating Mode during Vehicle Drive Cycles



Source: A. Solouk, M. Shahbakhti, et. al., *SAE Int. J. of Alternative Powertrains*, 2017.



Source: A. Solouk, M. Shahbakhti, et. al., *Energy Conversion and Management*, 2018.

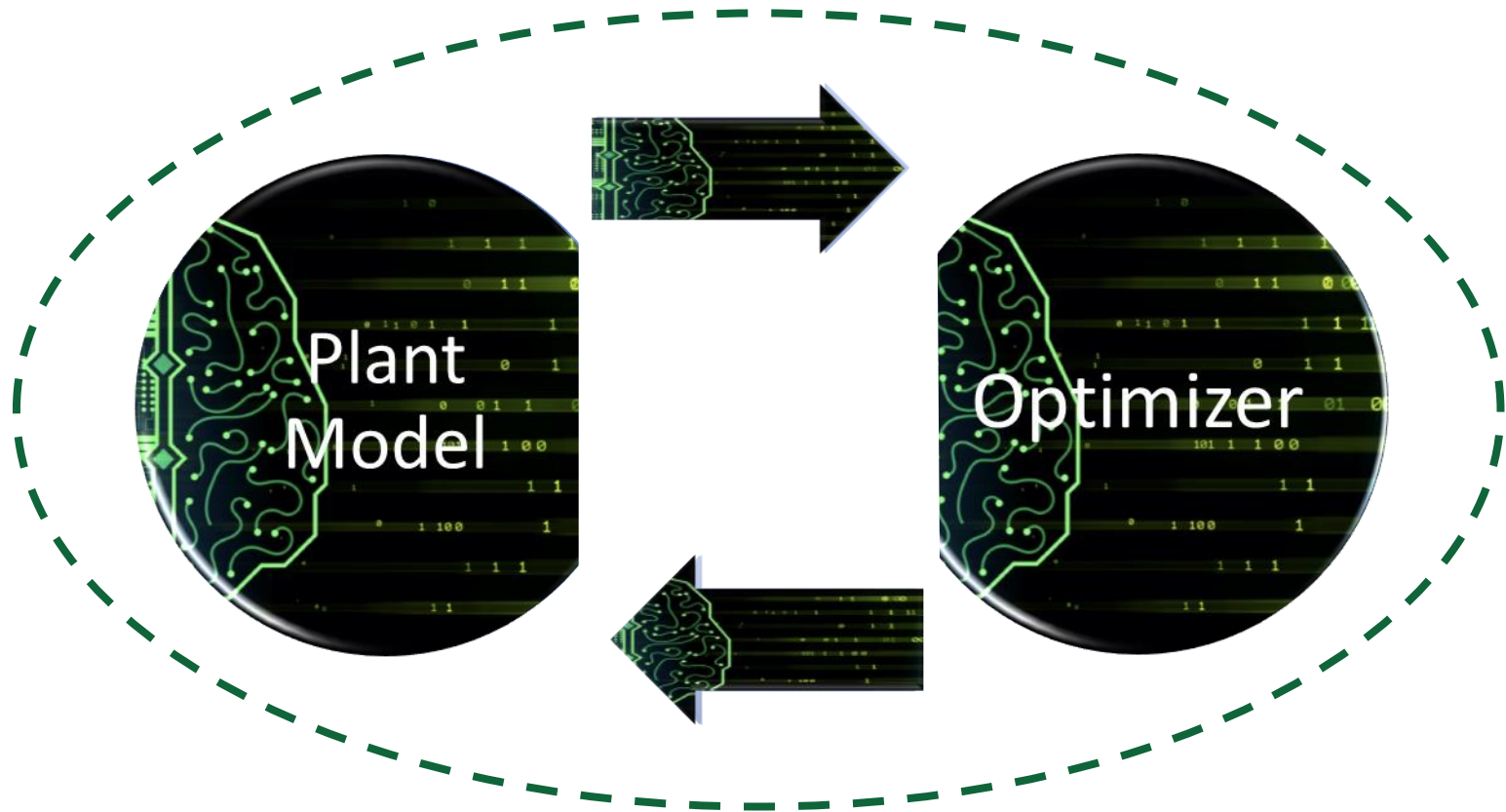
MPC Challenges

Limitations

- It is difficult and time consuming to develop proper accurate enough and “scalable” control oriented plant models for MPC;
- MPC performance is sensitive to plant and disturbance model uncertainties; robust MPC leads to conservative and less optimal operation;
- High computational cost and sometimes infeasibility of real-time operation on an economic microcontroller for control of systems with fast dynamics (e.g., μs to ms);
- Process of tuning the weights (e.g. in the cost function) in MPC structure,... can be time consuming and not fully optimal.

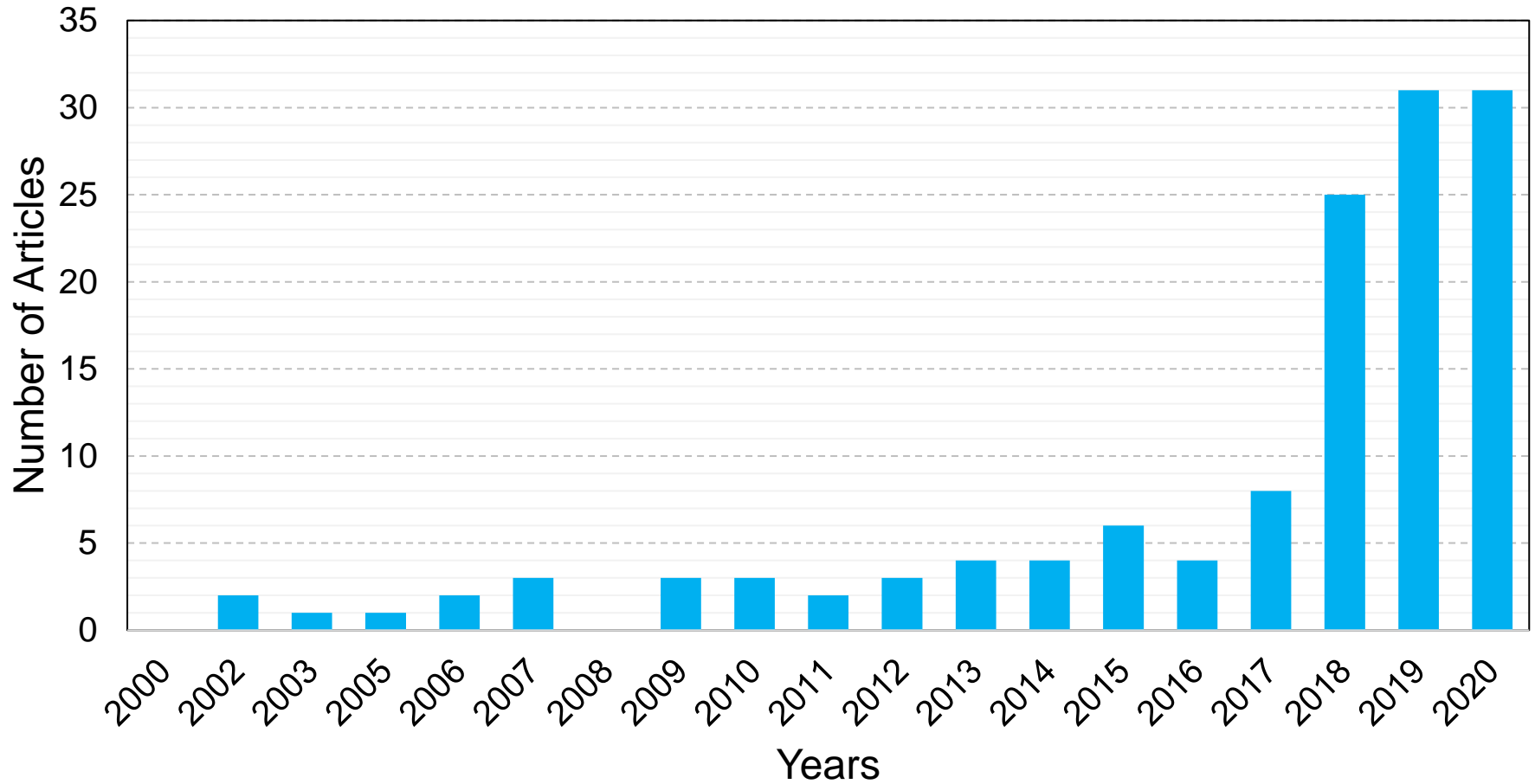
Machine learning can help to improve MPC performance in all these aspects!

MPC & ML Integration



Trend in Literature

Number of “AI+MPC” Article vs Years



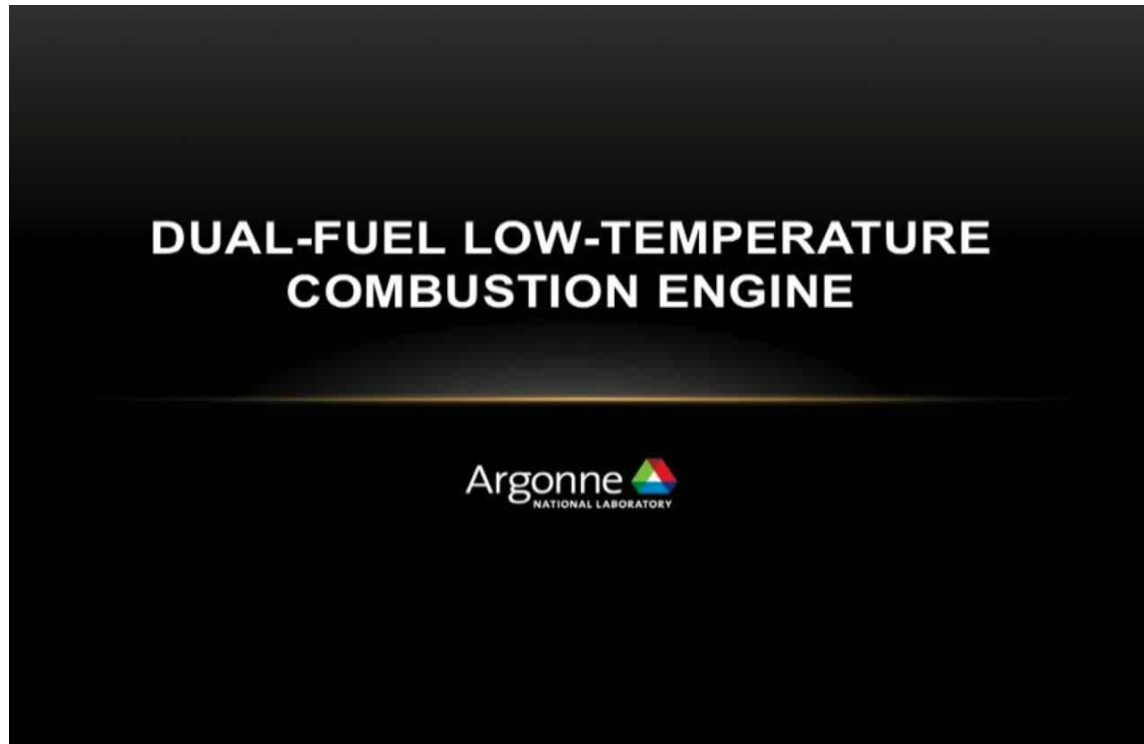
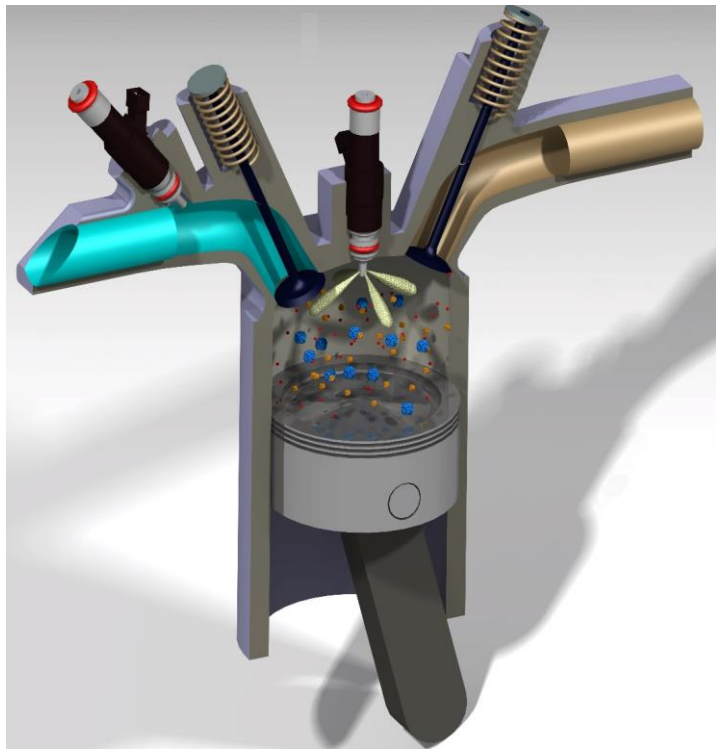
Trend in Literature

- Process control → **linear** MPC (some **nonlinear** too) 1970-2000
- Automotive control → **explicit, hybrid** MPC 2001-2010
- Aerospace systems and UAVs → **linear time-varying** MPC >2005
- Information and Communication Technologies (ICT)
(wireless nets, cloud) → **distributed/decentralized** MPC >2005
- Energy, finance, automotive, water → **stochastic** MPC >2010
- Industrial production → **embedded optimization** solvers for MPC >2010
- ➔ • Machine learning → **data-driven** MPC today

Source: http://cse.lab.imtlucca.it/~bemporad/teaching/mpc/imt/1-linear_mpc.pdf

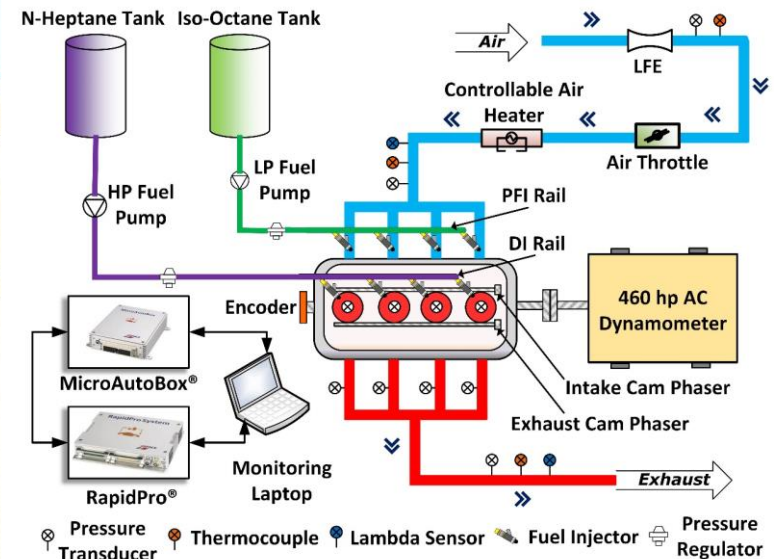
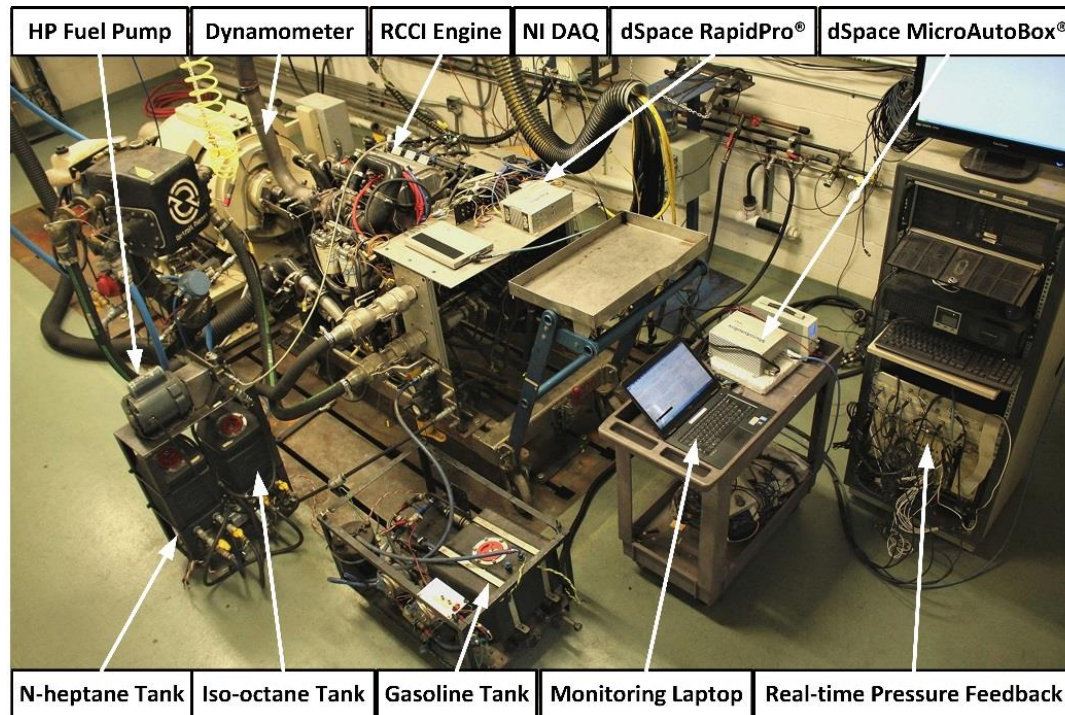
MPC example with ML-based plant model

Let's see an example: Model predictive combustion phasing control in an advanced dual fuel engine



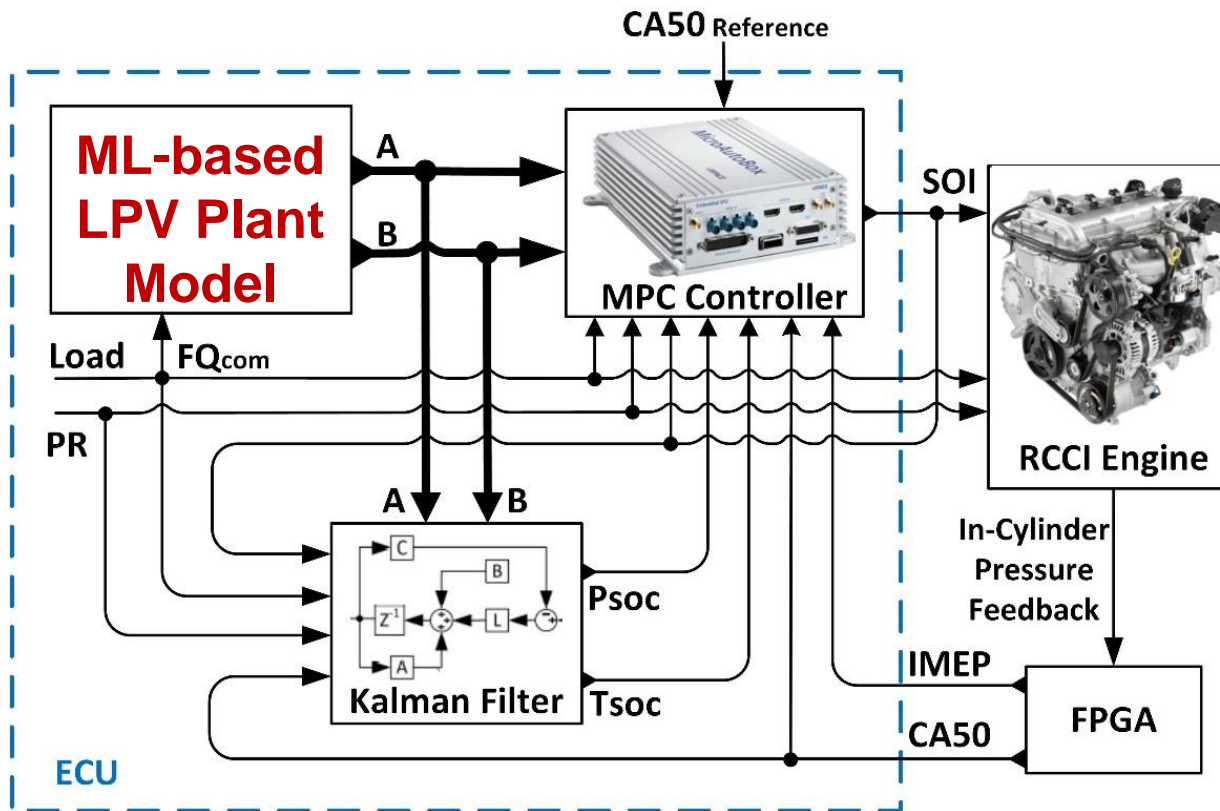
MPC example with offline ML-based plant model

Engine Experimental Setup



MPC example with offline ML-based plant model

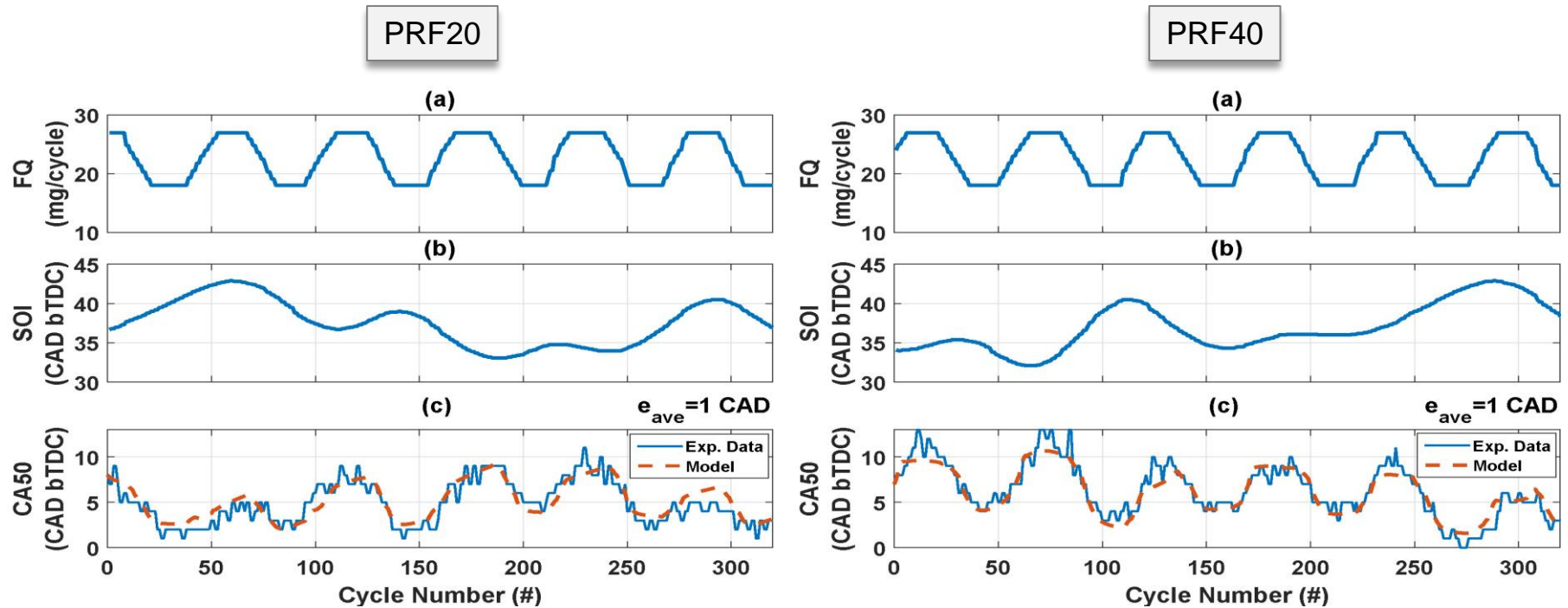
MPC Design using ML-based Linear Parameter Varying (LPV) Combustion Model



Source: B. Khoshbakht, M. Shahbakhti, et. al." Data-driven Modeling and Predictive Control of Combustion Phasing for RCCI Engines," American Control Conference, 2019.

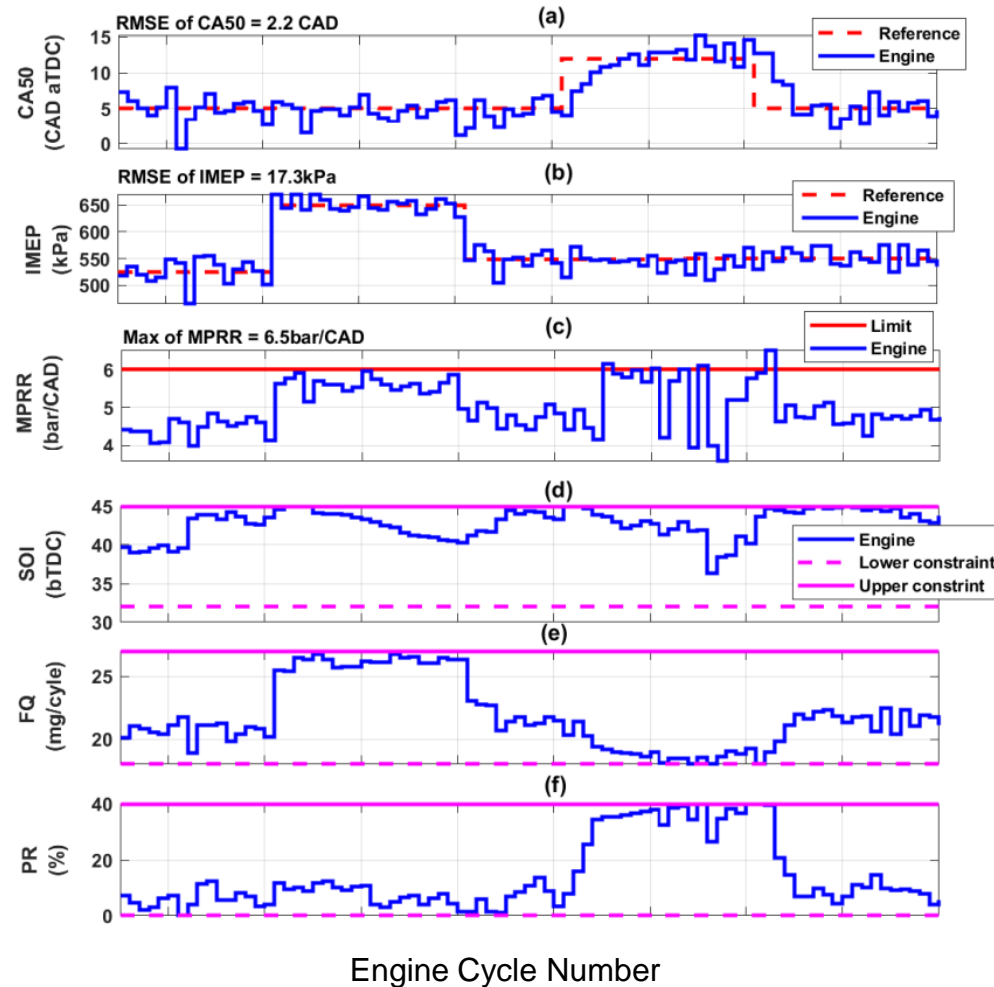
MPC example with offline ML-based plant model

LPV State Space Model Validation (sample results)



FQ: Fuel Quantity, SOI: Start of Injection, CA50: combustion phasing, PRF: premixed ratio of two fuels

Sample MIMO Control Results



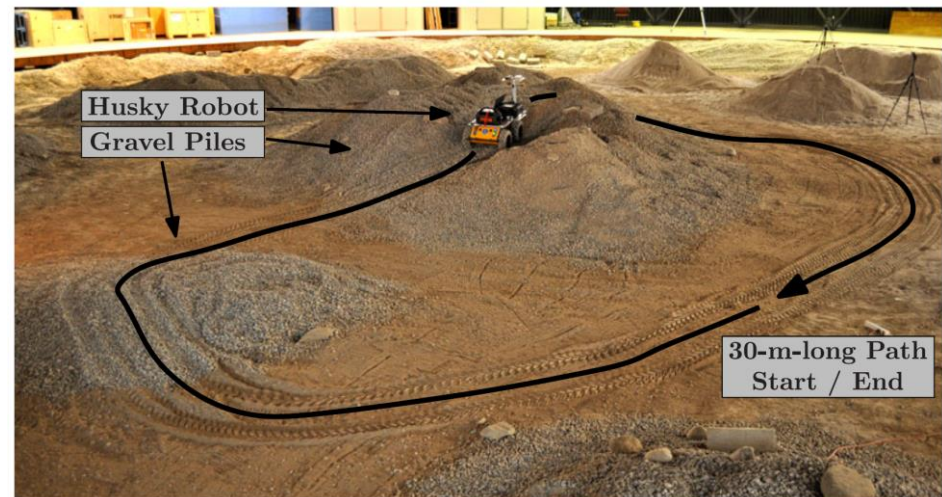
Source:
R. Sitaraman, M.
Shahbakhti, et al,
2022 MECC

CA50: combustion phasing, IMEP: Indicated Mean Effective Pressure, MPRR: Maximum Pressure Rise Rate,
PR: Premixed Ratio of two fuels, FQ: Fuel Quantity, SOI: Start of Injection

MPC example with online ML-based plant model

Example: Learning-based Nonlinear Model Predictive Control to Improve Vision-based Mobile Robot Path Tracking

Goal: Autonomous guidance, and control of three different mobile robots under “different paths with dirt, gravel, sand, grass, inclines, and side slopes”.



Source: C.J. Ostafew, et al, Journal of Field Robotics 33(1), 33–152 (2016)

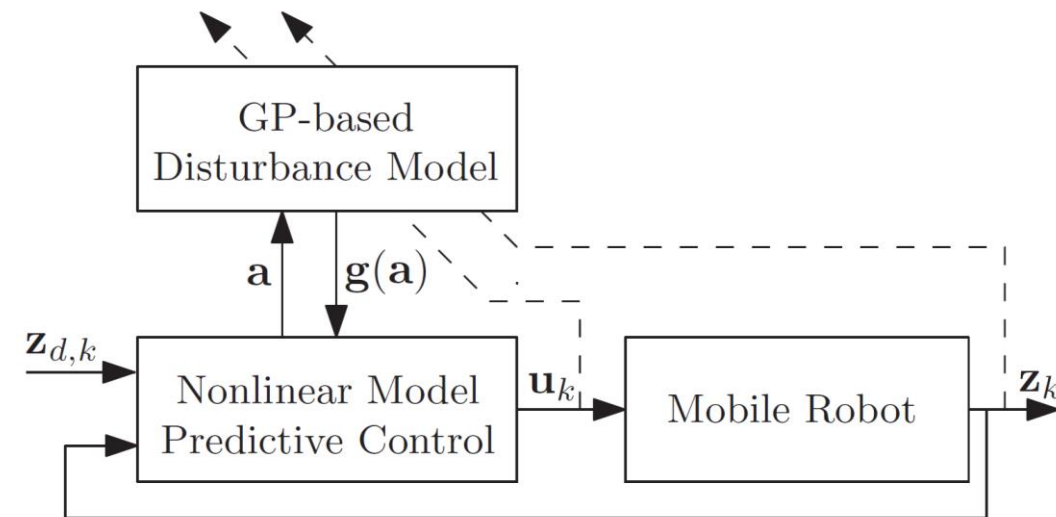
MPC example with online ML-based plant model

The disturbance model learns kinematics and dynamics of a significantly different mass and robot design:

a priori model learned disturbance model

$$\mathbf{z}_{k+1} = \overbrace{\mathbf{f}(\mathbf{z}_k, \mathbf{u}_k)}^{\text{a priori model}} + \overbrace{\mathbf{g}(\mathbf{z}_k, \mathbf{u}_k)}^{\text{learned disturbance model}}.$$

Model disturbances as a Gaussian process based on input-output data from previous trials.

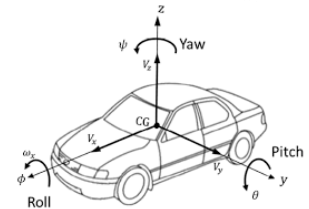
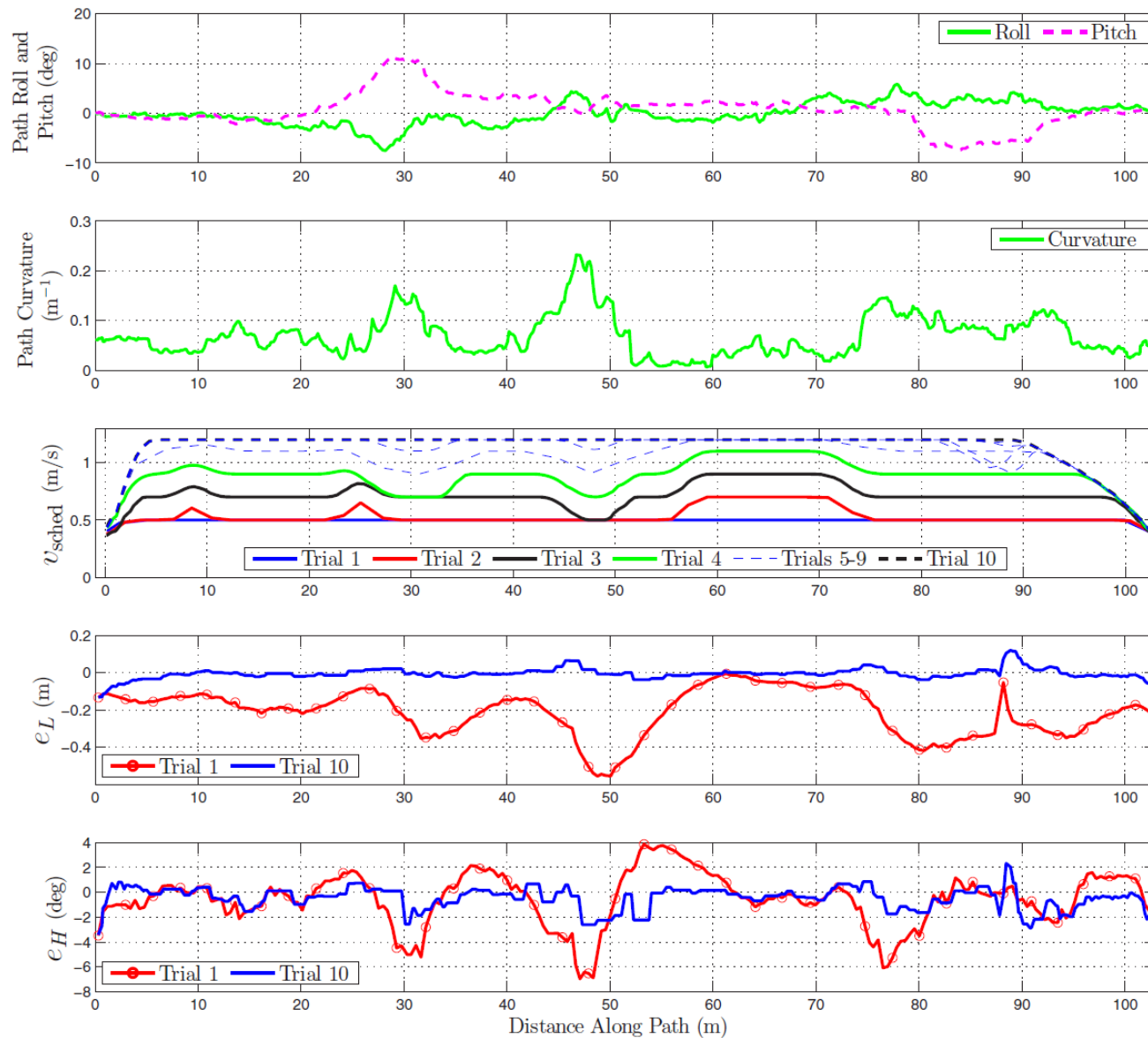


NMPC Cost function:

$$J(\mathbf{u}) = (\mathbf{z}_d - \mathbf{z})^T \mathbf{Q} (\mathbf{z}_d - \mathbf{z}) + \mathbf{u}^T \mathbf{R} \mathbf{u}$$

\mathbf{u} is a sequence of control inputs, \mathbf{z}_d is a sequence of desired states, \mathbf{z} is a sequence of predicted states

MPC example with online ML-based plant model



M. Kissai, et al, Machines, 2019

Source: C.J.
Ostafew, et al,
Journal of Field
Robotics 33(1),
33–152 (2016)