Convex Optimization CHW2

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1.1

First we obtain log likelihood function.

$$log - likelihood = \sum_{t=1}^{24} -\lambda_t + N_t log\lambda_t - logk!$$

By deriving from the log likelihood function with respect to λ_t and solving for zero:

$$\lambda_t = N_t$$

1.2

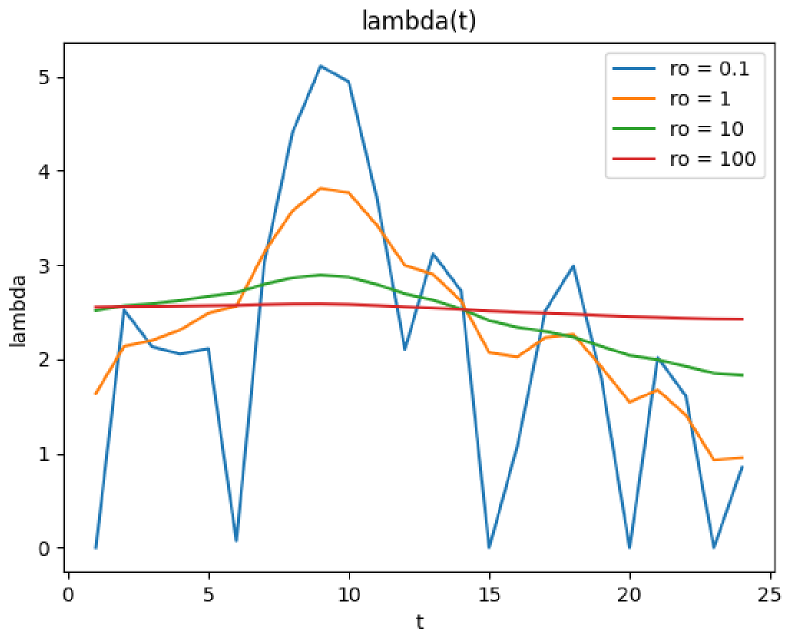
In order to findλ_t, we maximize

$$(\sum_{t=1}^{24} -\lambda_t + N_t log\lambda_t - logk!) - \rho(\sum_{t=1}^{23}(\lambda_{t+1} - \lambda_t)^2) + (\lambda_1 - \lambda_{24})^2)$$

1.3

When $\rho \to -\infty$ the regularization term gets a higher value proportional to ρ. So, we only need to minimize the regularization term since it has a high value. Therefore $\lambda_t = \lambda_{t+1}$ for $t = 1, ..., 23$ or in other words all λ_ts are equal.

1.4

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ro = 0.1:
Lamda t =  [3.10118243e-08 2.52592511e+00 2.13395930e+00 2.05587020e+00
 2.11366234e+00 7.47669468e-02 3.03586663e+00 4.40906064e+00
 5.11211464e+00 4.94675736e+00 3.71682217e+00 2.10594426e+00
 3.12083711e+00 2.72719261e+00 7.54227453e-08 1.07776130e+00
 2.51627749e+00 2.99360798e+00 1.79003898e+00 7.67748971e-08
 2.01957339e+00 1.61183688e+00 3.10294972e-08 8.54101858e-01]
ro = 1:
Lamda t =  [1.6373822  2.13738315 2.20166    2.3117339  2.48923212 2.56413528
 3.13903841 3.57680371 3.81562086 3.76819624 3.42463463 2.99706882
 2.90267307 2.61925716 2.07226617 2.02527578 2.23140601 2.26531408
 1.91634177 1.54554185 1.67474221 1.40828198 0.93173636 0.95519151]
ro = 10:
Lamda t =  [2.51820054 2.56820053 2.59032498 2.62384423 2.66925145 2.70846311
 2.79767476 2.8653984  2.89587409 2.87275403 2.79520448 2.69610376
 2.62845773 2.53472144 2.41208101 2.33944056 2.2954275  2.23606717
 2.13726416 2.04167243 1.99608069 1.92534184 1.85266431 1.8299868 ]
ro = 100:
Lamda t =  [2.55490394 2.55990394 2.56209115 2.56537531 2.56976141 2.57331039
 2.58185939 2.58766202 2.58880339 2.58335636 2.57129647 2.55645841
 2.54466452 2.53001105 2.51245246 2.49989389 2.49033525 2.47975333
 2.46610608 2.45340387 2.44570168 2.43686628 2.42892727 2.42598826]
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1.5

The best ρ is "1"

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ro = 0.1: log likelihood = -55.27402701647113
ro = 1: log likelihood = -7.264874579867907
ro = 10: log likelihood = -10.584015289918277
ro = 100: log likelihood = -12.942659360832444

ro best =  1
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2

2.1

The basic problem can be expressed as

$$\begin{gathered}\text{maximize } \sum_{i=1}^{n} r_j(x) \\ \text{subject to } x \geq 0 \\ Ax \leq c_{max}\end{gathered}$$

This is a convex optimization problem since the objective is concave and the constraints are a set of linear inequalities. To transform it to an equivalent LP, we first express the revenue functions as

$$r_j(x) = min\{p_j x_j, p_j q_j + p_j^{disc}(x_j - q_j)\}$$

which holds since r_j is concave. It follows that $r_j(x) \geq u_j$ if and only if

$$p_j q_j \geq uj, \qquad p_j q_j + p_j^{disc}(x_j - q_j) \geq u_j$$

We can form an LP as

$$\begin{gathered}\text{maximize } 1^T u \\ \text{subject to } x \geq 0 \\ Ax \leq c_m ax \\ p_j x_j \geq u_j, p_j q_j + p_j^{disc}(x_j - q_j) \geq u_j\end{gathered}$$

2.2

3

3.1

The fuel consumed over the ith segment is $(d_i/s_i)(s_i)$, so the total fuel used is $\sum_{i=1}^{n}(d_i/s_i)(si)$. The vehicle arrives at way point i at time ${}_i = \sum_{j=1}^{i}(d_j/s_j)$. Thus our problem is

$$\begin{gathered}\text{minimize} \sum_{i=1}^{n}(d_i/s_i)\Phi(s_i)\\ \text{subject to } s_i^{min} \leq s_i \leq s_i max \qquad i=1,...,n\\ T_i^{min} \leq \sum_{j=1}^{i}(d_j/s_j)_i^{max} \qquad i=1,...,n\end{gathered}$$

with variables $s_1, ..., s_n$. In this form, this is not a convex problem: the objective function need not be convex in s_i , and the inequalities $T_i^{min} \leq \sum_{j=1}^{i}(d_j/s_j)$ are not convex. However, we can formulate this as a convex problem by making a change of variables. We formulate the problem using the transit times of the segments, t_i , as the optimization variable, where $t_i = \frac{d_i}{s_i}$. Our problem can be written as

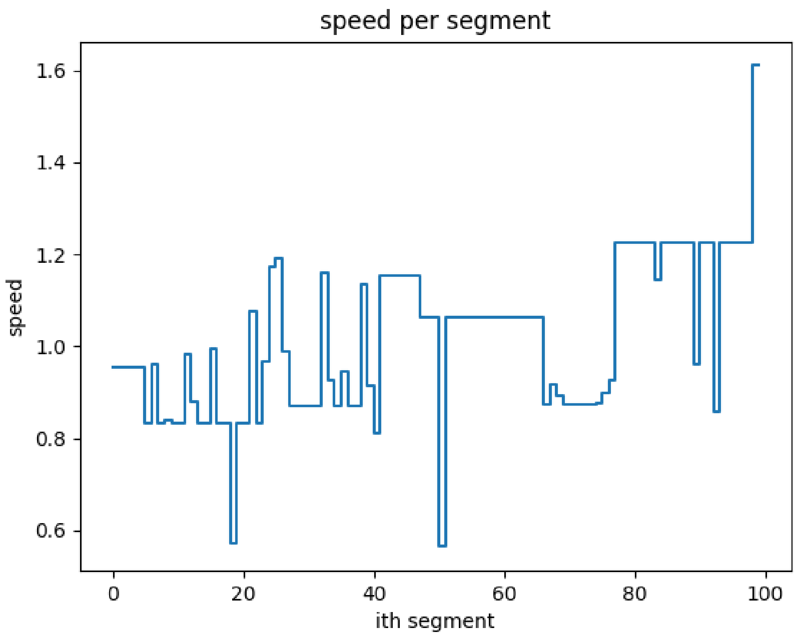
$$\begin{gathered}\text{minimize} \sum_{i=1}^{n} t_i\Phi(\tfrac{d_i}{t_i})\\ \text{subject to } \tfrac{d_i}{s_i^{max}} \leq t_i \leq \tfrac{d_i}{s_i^{min}}\\ T_i^{min} \leq \sum_{j=1}^{i} t_j \leq T_i^{max}\end{gathered}$$

This is a convex problem. The function $t_i\phi(\frac{d_i}{t_i})$, the perspective of $\phi, isconvexjointlyin \mathrm{d}_i$ and t_i . Therefore the objective function is convex, since it

is a positive weighted sum of convex functions. The constraints are all linear in t.

3.2

The optimal fuel consumption is 2617.825 kg



4

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[cp.inv_pos(x) + cp.inv_pos(y) <= 1, x >= 0, y >= 0]
[y >= cp.inv_pos(x), x >= 0, y >= 0]
[cp.quad_over_lin(x + y, cp.sqrt(y)) <= x - y + 5]
[x + z <= 1 + cp.geo_mean(cp.vstack([x - cp.quad_over_lin(z,y), y])), x >= 0, y >= 0]
```