CO CHW1 99105129

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1 eigenvalues and eigenvectors

1. To calculate the eigenvalues we do the following steps:

```
Q, R = np.linalg.qr(A)
    V = Q.copy()
    for i in range(iterations):
        Q, R = np.linalg.qr(R @ Q)
        V = V @ Q
        D = V.T @ A @ V
```

$$A = Q_1 R_1$$
$$R_2 Q_2 = Q_1 R_1$$
$$V_1 = Q_1 Q_2$$
$$A_2 = V_1^T A V_1$$
$$A_2 = Q_2^T Q_1^T A Q_1 Q_2$$

by repeating the calculation:

$$A_k = Q_k^T ... Q_3^T Q_2^T Q_1^T A Q_1 Q_2 Q_3 ... Q_k$$

Since Q is orthonarmal, A and A_k have the same eigenvalues and since $R_k = Q_k^T A_k$ is upper-triangular, When k tends to infinity A_k tends to Λ.

Since $A = QR$ and Q is an orthonormal matrix, Both A and Q have equal eigenvalues.On the other hand, R is an upper-triangular matrix so

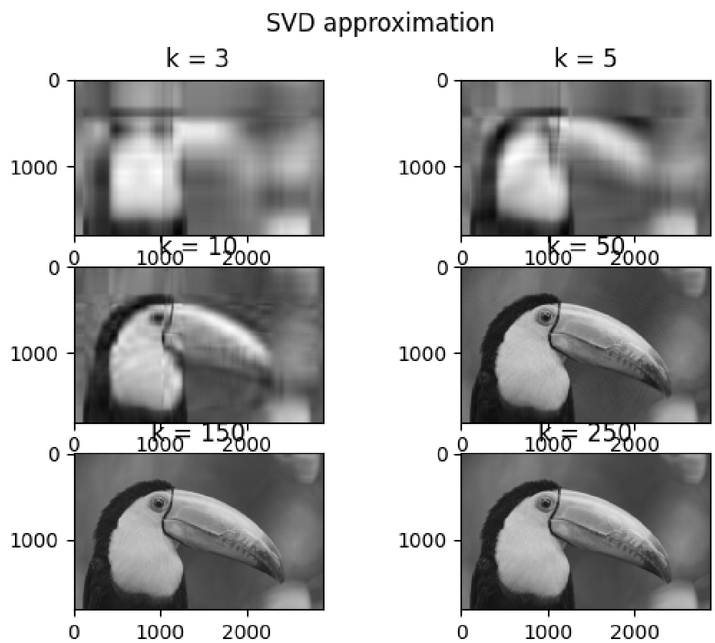
3. The overall complexity (number of floating points) of the algorithm is $O(n3)$, which we will see is not entirely trivial to obtain. The major limitation of the QR algorithm is that already the first stage generates usually complete fill-in in general sparse matrices.

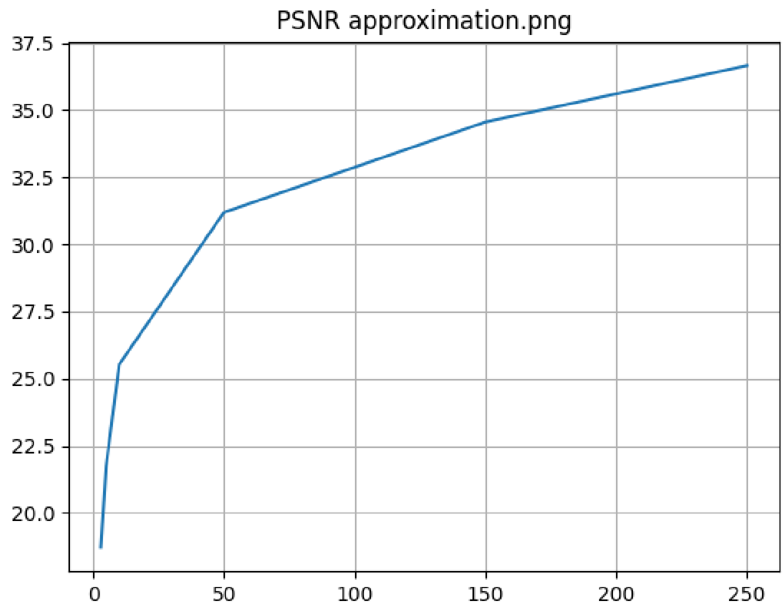
2 SVD and image processing

1.

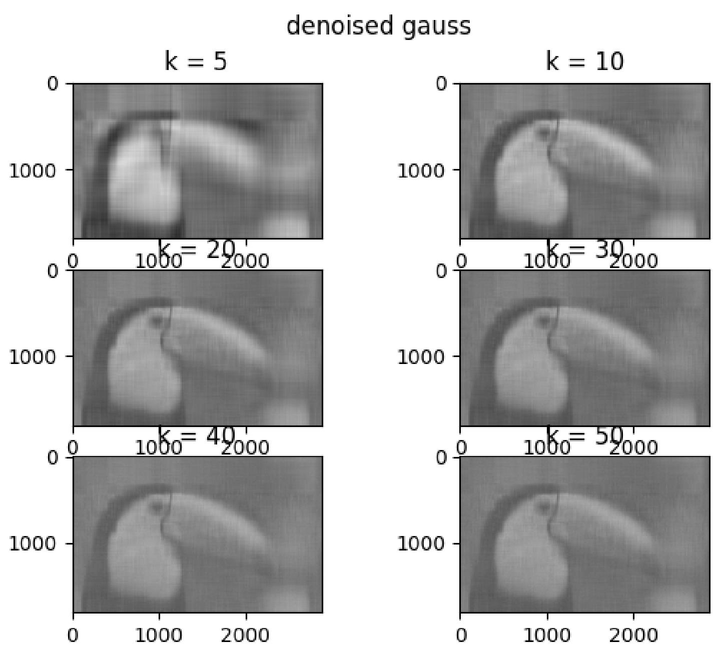
$$PSNR = 10log_{10}(\frac{MAX^2}{MSE})$$

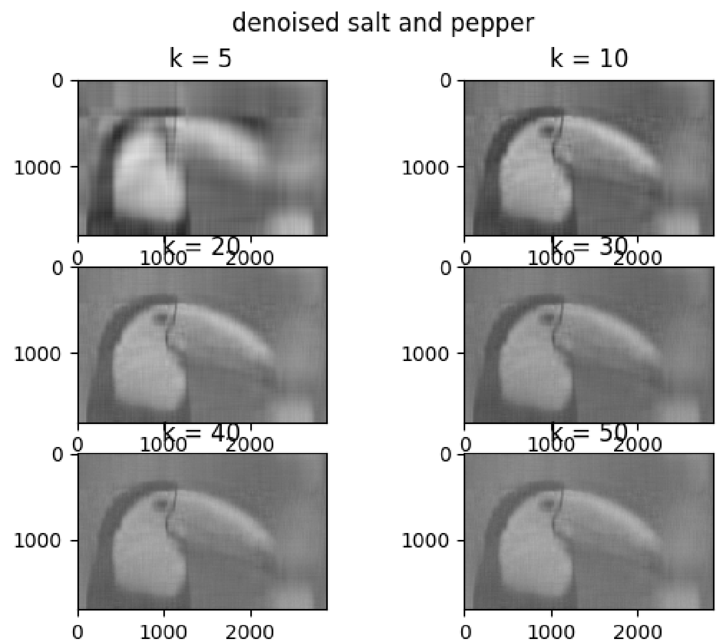
As we expected, By increasing r both quality of the reconstructed image and PSNR increases.



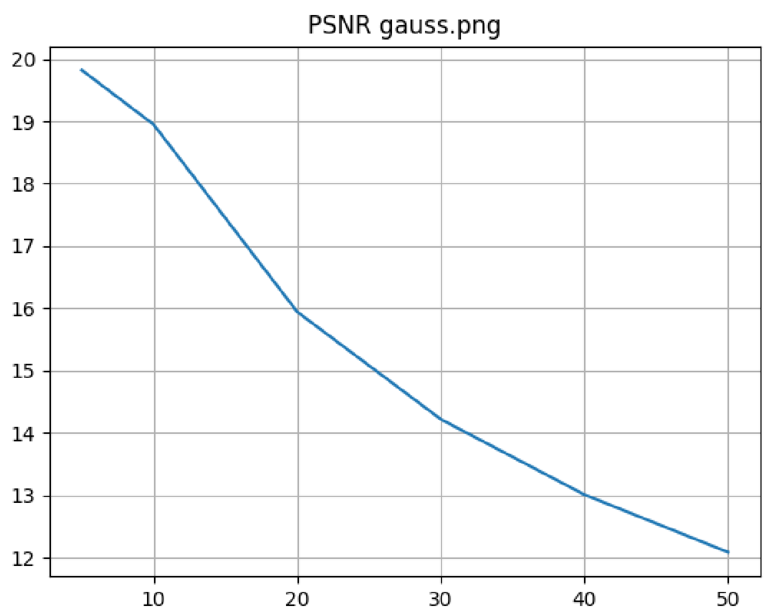

PSNR approximation.png
37.5
35.0
32.5
30.0
27.5
25.0
22.5
20.0
0
50
100
150
200
250

2.

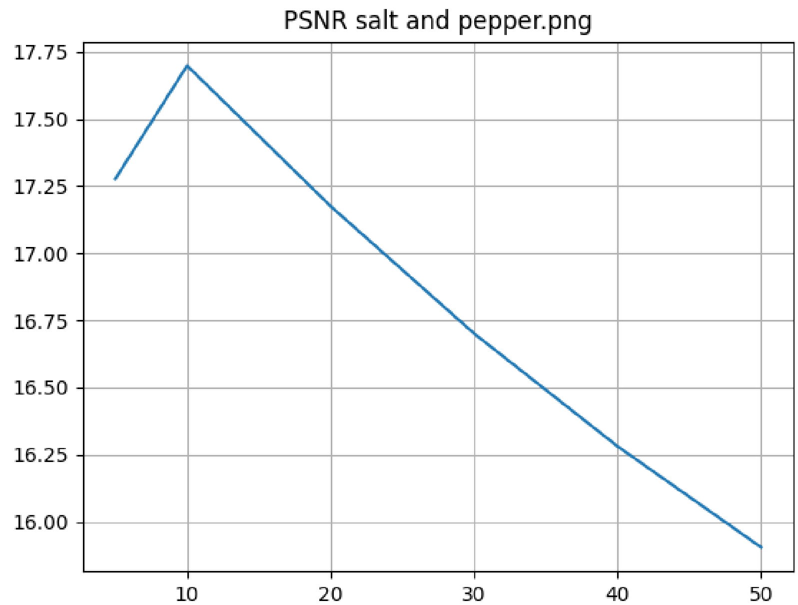

denoised gauss
k = 5
k = 10
k = 20
k = 30
k = 40
k = 50
0
1000
2000



denoised salt and pepper
k = 5
k = 10
k = 20
k = 30
k = 40
k = 50
0
1000
2000



PSNR gauss.png
20
19
18
17
16
15
14
13
12
10
20
30
40
50



3 PCA

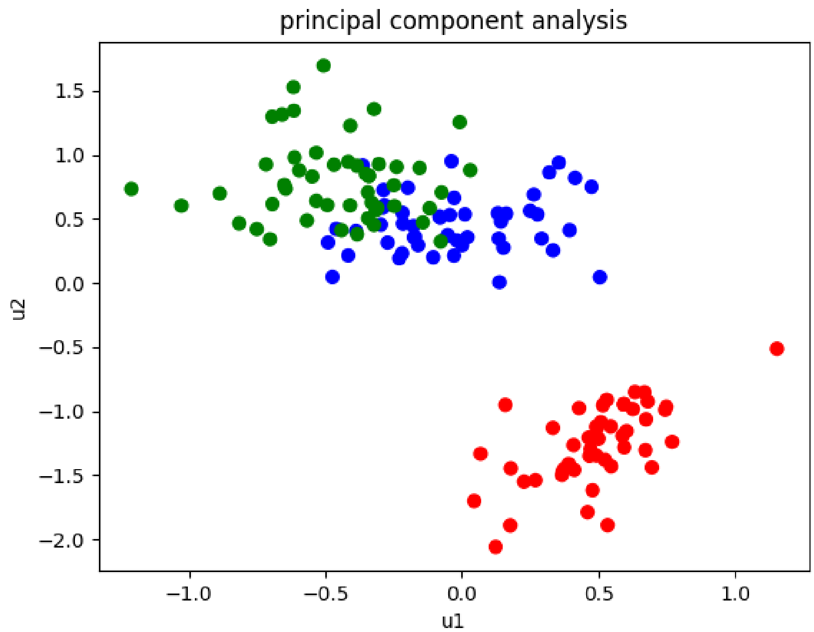
1.

$$Cov = \frac{1}{n-1}\tilde{X}\tilde{X^T} = \frac{1}{n-1}(U\Sigma V^T)(U\Sigma V^T)^T = \frac{1}{n-1}U\Sigma^2 U$$

2. It is clear that first column of U (u1) has direction of the maximum variation and second column of U (u2) has maximum variation perpendicular to u1 and so on. Hence, Columns of U are the main components. As we mentioned, The covariance is proportional to Σ^2.

3. In order to reduce dimensions from n to l, We should only consider the components with the greatest impacts, This means keeping only first l singular values and first l columns of U and V. Therefore we calculate the SVD decomposition and use first l columns of u as principal components.

5. As we expected, We were able to classify outputs using only 2 components,



principal component analysis
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
u2
−1.0
−0.5
0.0
0.5
1.0
u1