Zustandstransformation

$$\frac{d}{dt} i = \beta i (1 - i - r) - \gamma i \qquad i_d = 0{,}02$$

$$\frac{d}{dt} r = \gamma i - \delta r + u$$

$$\left| \begin{array}{ll} z_1 = \ln(i) - \ln(i_d) & ; \quad i = i_d \cdot e^{z_1} \\ z_2 = r & ; \quad r = z_2 \end{array} \right.$$

$$\frac{d}{dt} z_1 = \frac{1}{i} \frac{d}{dt} i - \frac{1}{i_d} \frac{d}{dt} i_d = \underline{\underline{\frac{1}{i} \frac{d}{dt} i}}$$

$$\frac{d}{dt} z_2 = \frac{d}{dt} r$$

Einsetzen

$$\frac{d}{dt} z_1 = \frac{1}{i} \left[\beta i (1 - i - r) - \gamma i \right] = \beta - \beta i - \beta r - \gamma = \beta - \beta i_d e^{z_1} - \beta z_2 - \gamma$$

$$= \beta \left(1 - i_d e^{z_1}\right) - \beta z_2 - \gamma$$

$$\frac{d}{dt} z_2 = \gamma i - \delta r + u = \gamma \cdot i_d \cdot e^{z_1} - \delta z_2 + u$$

Reglerentwurf

$$\frac{d}{dt} z_1 = \beta\left(1 - i_d e^{z_1}\right) - \beta z_2 - \tau \qquad z_2 = \alpha(z_1)$$

$$\frac{d}{dt} z_2 = \gamma i_d e^{z_1} - \delta z_2 + u$$

$$\alpha(z_1) = \left(1 - i_d e^{z_1}\right) - \frac{\tau}{\beta} + \frac{1}{\beta} c_1 z_1 \qquad ; c_1 > 0$$

$$\Rightarrow \frac{d}{dt} z_1 = \beta\left(\cancel{1 - i_d e^{z_1}}\right) - \beta\left(\cancel{1 - i_d e^{z_1}}\right) + \cancel{\tau} - c_1 z_1 - \cancel{\tau}$$

$$= -c_1 z_1$$

damit ist mit $V_1(z_1) = \frac{1}{2} z_1^2 > 0$ und $\dot{V}_1(z_1) = z_1 \cdot \dot{z}_1 = -c_1 z_1^2 < 0$

das erste Teilsystem stabil

$$V_a(z_1, z_2) = \frac{1}{2} z_1^2 + \frac{c}{2}\left(z_2 - \alpha(z_1)\right)^2 > 0 \qquad \text{mit } c \gg 1$$

$$\dot{V}_a(z_1, z_2) = z_1 \cdot \dot{z}_1 + c\left(z_2 - \alpha(z_1)\right)\left[\dot{z}_2 - \frac{\partial \alpha(z_1)}{\partial z_1} \dot{z}_1\right]$$

$$= z_1 \beta\left(1 - i_d e^{z_1}\right) - \beta z_1 z_2 - z_1 \tau + c\left(z_2 - \alpha(z_1)\right)\left[\dot{z}_2 - \frac{\partial \alpha(z_1)}{\partial z_1} \dot{z}_1\right]$$

$$= -c_1 z_1^2 + z_1 \beta\left[\left(1 - i_d e^{z_1}\right) - \frac{\tau}{\beta} + \frac{1}{\beta} c_1 z_1 - z_2\right] + c\left(z_2 - \alpha(z_1)\right)\left[\dot{z}_2 - \frac{\partial \alpha(z_1)}{\partial z_1} \dot{z}_1\right]$$

$$= -c_1 z_1^2 + c\left(z_2 - \alpha(z_1)\right)\left[\dot{z}_2 - \frac{\partial \alpha(z_1)}{\partial z_1} \dot{z}_1 - \frac{1}{c} \beta z_1\right]$$

$$= -c_1 z_1^2 + c\left(z_2 - \alpha(z_1)\right)\underbrace{\left[\gamma i_d e^{z_1} - \delta z_2 + u - \frac{\partial \alpha(z_1)}{\partial z_1} \dot{z}_1 - \frac{1}{c} \beta z_1\right]}_{\stackrel{!}{=} -c_2\left(z_2 - \alpha(z_1)\right)}$$

$$\Rightarrow -c_2\left(z_2 - \alpha(z_1)\right) = \gamma i_d e^{z_1} - \delta z_2 + u - \frac{\partial \alpha(z_1)}{\partial z_1} \dot{z}_1 - \frac{1}{c} \beta z_1$$

$$\boxed{\Rightarrow u = -c_2\left(z_2 - \alpha(z_1)\right) - \gamma i_d e^{z_1} + \delta z_2 + \frac{\partial \alpha(z_1)}{\partial z_1} \dot{z}_1 + \frac{1}{c} \beta z_1}$$