CentraleSupélec

Tutorial 2
Interactive robotics

Dynamics and control

1 Introduction

We propose to study the dynamic modeling and control of a robot manipulator developed by the *Interactive Robotics Laboratory* of the *CEA List* (Fig. 1). Its geometric and kinematic models were studied in Tutorial 1.

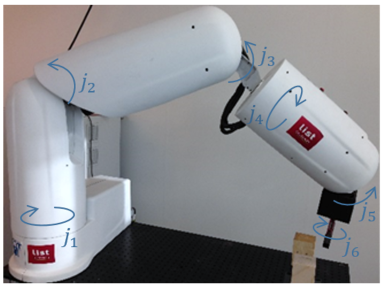

FIGURE 1 – Prototype of the robot manipulator of *CEA-LIST*.

This is a continuation of Tutorial 1. Solutions of questions from tutorial 1 are sometimes required for some of the questions of the present tutorial. The numerical values of the robot parameters, required for the completion of this tutorial, are specified in the table 1. The use of *Matlab*^{TM} software is required to perform the tutorial.

2 Dynamic model

The matrix form of the inverse dynamic model for rigid robot manipulator is recalled below :

$$A(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + G(q) + \Gamma_f(\dot{q}) = \Gamma$$

- $A(q) \in \mathbb{R}^{6\times 6}$ inertia matrix, symmetric and positive definite ;
- $C(q,\dot{q})\,\dot{q} \in \mathbb{R}^6$ vector of joint torques due to the Coriolis and centrifugal forces ;
- $G(q) \in \mathbb{R}^6$ vector of joint torques due to gravity ;
- $\Gamma_f(\dot{q}) = \begin{bmatrix}\tau_{f_1} & \dots & \tau_{f_6}\end{bmatrix}^t \in \mathbb{R}^6$ vector of joint friction torques.

The vectors of joint positions, velocities and accelerations are denoted respectively $q = [q_1, \dots, q_6]^t$, $\dot{q} = [\dot{q}_1, \dots, \dot{q}_6]^t$, $\ddot{q} = [\ddot{q}_1, \dots, \ddot{q}_6]^t$, and the vector of the joint torques is denoted $\Gamma = [\tau_1, \dots, \tau_6]^t$. The frames $\mathcal{R}_i$ attached to the links of the robot have been defined in Tutorial 1, $\boldsymbol{Q}1$.

Q12. The objective of this question is to determine the velocity ${}^{0}V_{G_i}$ of the center of mass G_i and the rotation speed ${}^{0}\omega_i$ of all the rigid bodies $\mathcal{C}_i$ in the frame $\mathcal{R}_0$. Write a function which returns the Jacobian matrices ${}^{0}J_{vG_i}$ and ${}^{0}J_{\omega_i}$ defined as follows :

$$ {}^{0}V_{G_i} = {}^{0}J_{vG_i}(q)\,\dot{q} \quad \text{and} \quad {}^{0}\omega_i = {}^{0}J_{\omega_i}(q)\,\dot{q}. $$

To this end, the position of the center of mass G_i expressed in the frame $\mathcal{R}_i$ of body $\mathcal{C}_i$ is given by :

$$ {}^{i}\overrightarrow{O_iG_i} = \begin{bmatrix} x_{G_i} & y_{G_i} & z_{G_i} \end{bmatrix}^t \quad \text{for} \quad i = 1, \ldots, 6 $$

where the coordinates x_{G_i}, y_{G_i}, and z_{G_i} are provided in Table 1.
To answer this question, are useful : the functions developed in question ***Q***3 and the function ComputeJac $(alpha, d, theta, r)$ developed in question ***Q***6, providing the Jacobian ${}^{0}J_{O_E}$ which is used in the computation of the velocity ${}^{0}\mathcal{V}_{0,E}$ of the end-effector O_E. The syntax for the function to be programmed will be[1] :
$\left[{}^{0}J_{vG_i}, {}^{0}J_{\omega_i}\right] = \texttt{ComputeJacGi}\left(alpha, d, theta, r, x_G, y_G, z_G\right)$.

Q13. Write a function $A = \texttt{ComputeMatInert}\,(q)$ returning the inertia matrix $A(q) \in \mathbb{R}^{6\times6}$ of the robot. To this end, the inertia tensors I_i expressed in their frames $\mathcal{R}_i$ (of origin O_i)[2] and the mass m_i of each body $\mathcal{C}_i$. Moreover, the actuator inertia contributions J_{m_i} ($i = 1, \ldots, 6$) taken after the joint level will be added the diagonal of $A(q)$ (reduction ratios r_{red_i} and inertia J_{m_i} are provided in Table 1).

Q14. Using the computation of the eigenvalues of $A(q)$, propose two scalar numbers $0 < \mu_1 < \mu_2$ for the lower and the upper bounds of the inertia matrix, i.e.

$$ \mu_1 \mathbb{I} \preceq A(q) \preceq \mu_2 \mathbb{I} $$

for joint angles comprised between limits q_{min} et q_{max} defined in question ***Q***10.

Q15. Write a function $G = \texttt{ComputeGravTorque}\,(q)$ returning the vector of joint torques due to the gravity $G(q) \in \mathbb{R}^6$. The analytical expression of the gradient of the potential energy $E_p(q) = g^t \left(\sum_{i=1}^{6} m_i {}^{0}p_{G_i}(q)\right)$ can be used, that is :

$$ G(q) = -\left({}^{0}J_{vG_1}^t m_1 g + \ldots + {}^{0}J_{vG_6}^t m_6 g\right) $$

where $g = \begin{bmatrix} 0 & 0 & -9.81 \end{bmatrix}^t$.

Q16. Propose an upper bound g_b of $\|G(q)\|_1$, such that :

$$ \forall q \in [q_{min}, q_{max}], \quad \|G(q)\|_1 \leq g_b $$

where $\|\bullet\|_1$ denotes the norm 1 of a vector.

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1. You can use the *Varignon* formula : $V_{G_i} = V_{O_E} + \omega_i \times \overrightarrow{O_EG_i}$, ie : ${}^{0}J_{G_i} = \begin{bmatrix} I_{3\times3} & -{}^{0}\widehat{\overrightarrow{O_EG_i}} \\ 0_{3\times3} & I_{3\times3} \end{bmatrix} {}^{0}J_{O_E}$
2. You will need to express the inertia tensors in the frame $\mathcal{R}_i$ (of origin G_i) using the *Huygens* theorem.

Q17. Design a simulation block of the robot using *Simulink*^{TM} by programming its *direct* dynamic model exploiting the previously obtained functions, a function $\Gamma_f =$ `ComputeFrictionTorque` $(\dot{q})$ to be programmed and the function $c =$ `ComputeCCTorques` $(q, \dot{q})$ which is given.

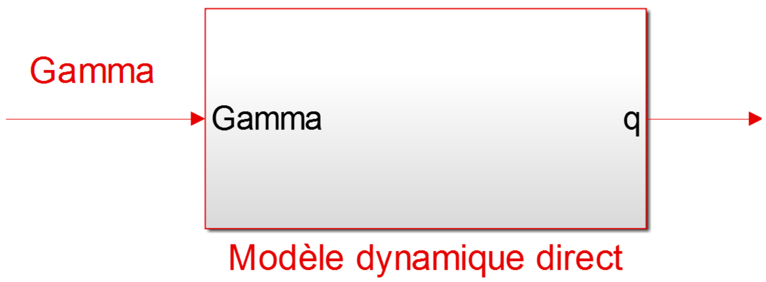


FIGURE 2 – Inputs and outputs of the robot simulation block in *Simulink*^{TM}

The function $\Gamma_f =$ `ComputeFrictionTorque` $(\dot{q})$ to be programmed should return the vector of joint torques produced by joint friction. In this tutorial, the following friction model will be considered :

$$\tau_{f_i}(\dot{q}_i) = diag(\dot{q}_i) F_{v_i} \quad \text{pour} \quad i = 1, \ldots, 6$$

where the parameters F_{v_i} are provided in the table 1.
Moreover, the function $c =$ `ComputeCCTorques` $(q, \dot{q})$, which is provided, returns the vector of joint torques du to the *Coriolis* and centrifugal effects (where the vector c corresponds to the quantity $C(q, \dot{q})\dot{q}$).

3 Trajectory generation in the joint space

Q18. We want to generate a polynomial trajectory of degree 5 to be followed in the joint space allowing to reach in minimal time t_f the desired final configuration q_{d_f} from the initial configuration q_{d_i}. This movement is performed at zero initial and final velocities and accelerations, and is sampled at a period $T_e = 1$ms. We give :

- $q_{d_i} = [-1.00, 0.00, -1.00, -1.00, -1.00, -1.00]^t$ rad,
- $q_{d_f} = [0.00, 1.00, 0.00, 0.00, 0.00, 0.00]^t$ rad.

What is the final minimum time t_{f_i} for each joint i taking into account only the vector k_a of maximum joint accelerations ? The terms k_{a_i} will be calculated based on the ratio of the maximum motor torques τ_{max_i} (given in table 1), the reduction ratios r_{red_i} and the maximal inertias seen by the joints (all assumed to be equal to the value μ_2 calculated in question ***Q***14).

Q19. Program a function $q_c =$ `GenTraj`(q_{d_i}, q_{d_f}, t) capable of generating the desired joint trajectory point $q_c(t)$ at time t of the previously requested trajectory. In this question, you will choose a minimum final global time $t_f = 0.5$ s to coordinate all the joints [3]. Plot the temporal evolution of the desired joint trajectories q_{c_i} (for $i = 1, \ldots, 6$) when t varies from 0 to t_f.

3. This minimum final global time is an upper bound of the t_{f_i} times that were calculated in question ***Q***18.

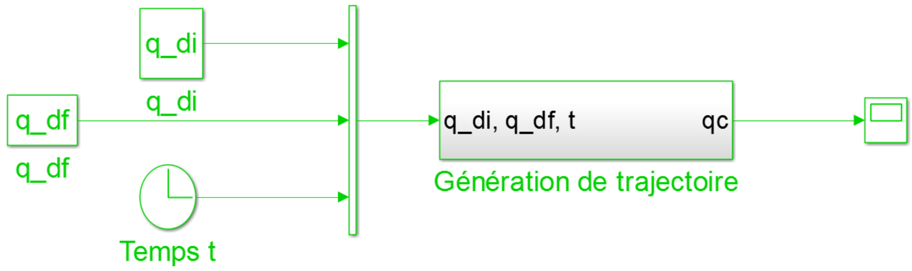


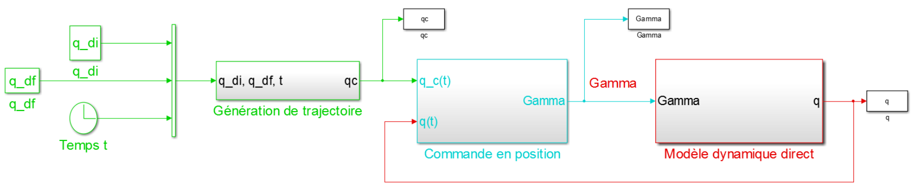
FIGURE 3 – Trajectory generation with *Simulink*^{TM}

4 Joint control law

Q20. The robot is position-controlled with a P.D. controller with gravity compensation :

$$\Gamma = K_p\left(q_d - q\right) + K_d\left(\dot{q}_d - \dot{q}\right) + \hat{G}\left(q\right)$$

Create the corresponding position-control block in *Simulink*^{TM}, then build the closed-loop control scheme integrating the previously defined blocks.



Propose a tuning of the joint gains K_{p_i} and K_{d_i} capable of ensuring a stable and damped temporal response of the closed-loop system, with a tracking error $e\left(t\right) = q_{c_i}\left(t\right) - q_i\left(t\right)$ no greater than 0.05 rad for each joint. In addition, your K_{p_i} and K_{d_i} gain settings must be acceptable in terms of the maximum allowable joint torques (calculated from the maximum motor torques τ_{max_i}).

Give the numerical values for your gain tuning.

Plot the temporal evolution of joint trajectories $q_i\left(t\right)$, as well as tracking errors $e\left(t\right)$.

Plot the temporal evolution of the control joint torques $\tau_i\left(t\right)$ corresponding to your gain tuning.

Annexe

Parameters	Numerical values	Type of parameter
x_{G_1}, y_{G_1}, z_{G_1}	0m, 0m, −0.25m	Coordinates of G_1 given in frame $\mathcal{R}_1$
x_{G_2}, y_{G_2}, z_{G_2}	0.35m, 0m, 0m	Coordinates of G_2 given in frame $\mathcal{R}_2$
x_{G_3}, y_{G_3}, z_{G_3}	0m, −0.1m, 0m	Coordinates of G_3 given in frame $\mathcal{R}_3$
x_{G_4}, y_{G_4}, z_{G_4}	0m, 0m, 0m	Coordinates of G_4 given in frame $\mathcal{R}_4$
x_{G_5}, y_{G_5}, z_{G_5}	0m, 0m, 0m	Coordinates of G_5 given in frame $\mathcal{R}_5$
x_{G_6}, y_{G_6}, z_{G_6}	0m, 0m, 0m	Coordinates of G_6 given in frame $\mathcal{R}_6$
m_1	15.0kg	Mass of the body 1
m_2	10.0kg	Mass of the body 2
m_3	1.0kg	Mass of the body 3
m_4	7.0kg	Mass of the body 4
m_5	1.0kg	Mass of the body 5
m_6	0.5kg	Mass of the body 6
I_1	$\begin{bmatrix} 0.80 & 0 & 0.05 \\ 0 & 0.80 & 0 \\ 0.05 & 0 & 0.10 \end{bmatrix}_{\mathcal{R}_{O_1}} kg.m^2$	Inertial tensor of the body 1
I_2	$\begin{bmatrix} 0.10 & 0 & 0.10 \\ 0 & 1.50 & 0 \\ 0.10 & 0 & 1.50 \end{bmatrix}_{\mathcal{R}_{O_2}} kg.m^2$	Inertial tensor of the body 2
I_3	$\begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}_{\mathcal{R}_{O_3}} kg.m^2$	Inertial tensor of the body 3
I_4	$\begin{bmatrix} 0.50 & 0 & 0 \\ 0 & 0.50 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}_{\mathcal{R}_{O_4}} kg.m^2$	Inertial tensor of the body 4
I_5	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}_{\mathcal{R}_{O_5}} kg.m^2$	Inertial tensor of the body 5
I_6	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}_{\mathcal{R}_{O_6}} kg.m^2$	Inertial tensor of the body 6
J_{m_i} $(i = 1, \ldots, 6)$	$10 \times 10^{-6} kg.m^2$	Moment of inertia of the actuator rotor
r_{red_i} $(i = 1, \ldots, 3)$	100	Reduction ratio
r_{red_i} $(i = 4, \ldots, 6)$	70	Reduction ratio
$F_{v_1}, \ldots, F_{v_6}$	$10 N.m.rad^{-1}.s$	Joint viscous frictions
τ_{max_i} $(i = 1, \ldots, 6)$	$5 N.m$	Maximal motor torques

TABLE 1 – Numerical values of the robot.