Oscillations and interactions of dark and dark–bright solitons in Bose–Einstein condensates

CHRISTOPH BECKER[1], SIMON STELLMER[1], PARVIS SOLTAN-PANAHI[1], SÖREN DÖRSCHER[1], MATHIS BAUMERT[1], EVA-MARIA RICHTER[1], JOCHEN KRONJÄGER[1], KAI BONGS[2] AND KLAUS SENGSTOCK[1]*

[1]Institut für Laser-Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany
[2]MUARC, School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK
*e-mail: sengstock@physnet.uni-hamburg.de

Published online: 4 May 2008; doi:10.1038/nphys962

Solitons are among the most distinguishing fundamental excitations in a wide range of nonlinear systems such as water in narrow channels, high-speed optical communication, molecular biology and astrophysics. Stabilized by a balance between spreading and focusing, solitons are wave packets that share some exceptional generic features such as form stability and particle-like properties. Ultracold quantum gases represent very pure and well-controlled nonlinear systems, therefore offering unique possibilities to study soliton dynamics. Here, we report on the observation of long-lived dark and dark–bright solitons with lifetimes of up to several seconds as well as their dynamics in highly stable optically trapped ^{87}Rb Bose–Einstein condensates. In particular, our detailed studies of dark and dark–bright soliton oscillations reveal the particle-like nature of these collective excitations for the first time. In addition, we discuss the collision between these two types of solitary excitation in Bose–Einstein condensates.

Nowadays, solitons are a very active field of research in many areas of science. They are characterized as localized solitary wave packets that maintain their shape and amplitude owing to a self-stabilization against dispersion through a nonlinear interaction. Although an early theoretical explanation of this non-dispersive wave phenomenon was given by Korteweg and de Vries in the late nineteenth century, it was not until after 1965 that numerical simulations of Zabusky and Kruskal theoretically proved that these solitary waves preserve their identity in collisions[1,2]. This revelation led to the term 'soliton' for this type of collective excitation.

Bose–Einstein condensates (BECs) of weakly interacting atoms build up a macroscopic wavefunction described by a nonlinear Schrödinger equation and therefore enable studies of solitons. In this respect, the existence and some fundamental properties of solitons have been deduced from a few experiments with BECs. Bright solitons, characterized as non-spreading matter-wave packets, have been observed in BECs with attractive interaction[3–5] where they represent the ground state of the system. In a repulsively interacting condensate confined in a periodic potential, bright gap solitons have been realized[6,7] by modelling a suitable anomalous dispersion.

BECs with repulsive interactions allow for dark-soliton solutions, characterized by a notch in the density distribution. In contrast to bright solitons, dark solitons are truly excited states with energies greater than the underlying BEC ground state. Dark solitons have been generated in a few pioneering experiments[8–10], boosting an immense theoretical interest in these nonlinear structures in BECs. Dynamical[10–17] and thermodynamical[18–20] instabilities as well as collisional properties[21,22] have been analysed theoretically in great detail. Moreover, the existence of bright solitons stabilized by the presence of dark solitons in another quantum state in repulsive BECs has been proposed[16] and confirmed in a proof-of-principle experiment[10]. The occurrence of undamped oscillations of solitons in axially harmonic traps comparable to those of a single particle has been predicted by mean field theory[16,19], see also ref. 23, and supported by numerical simulations[18] for dark solitons and appears as one of the paradigms of soliton physics in BECs as it clearly demonstrates the particle-like character of the soliton. Limited by very short lifetimes in previous experiments however, oscillations of dark solitons have not been observed yet.

Here, we show that very long-lived dark solitons can be generated in BECs, facilitating detailed studies of soliton oscillations as well as the creation of dark–bright solitons by filling the dark soliton with atoms in another hyperfine state. In addition, the reflection of a dark soliton off a filled soliton has been observed and is presented. In the following, we give a brief introduction to the description of solitons in BECs and characteristic properties of soliton dynamics.

DARK SOLITONS IN BECS

Close to absolute zero temperature, where thermal fluctuations can be neglected to first order, the condensate is well described within the framework of the nonlinear Gross–Pitaevskii equation[24] (GPE), which is known to support soliton solutions as it is closely related

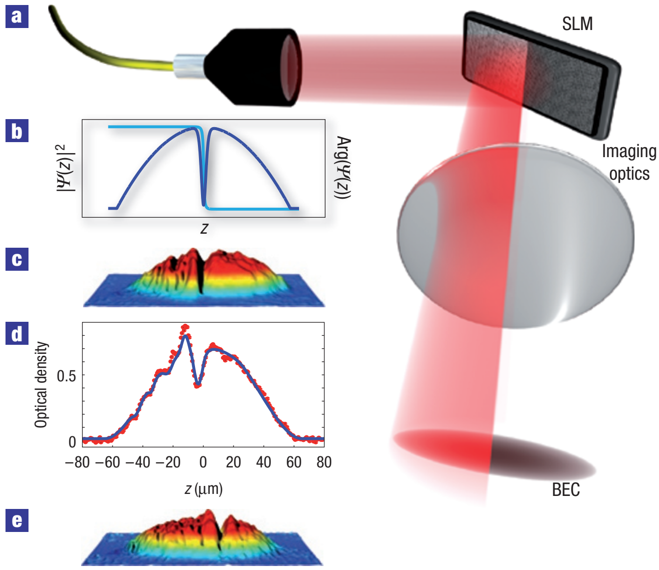


Figure 1 Principle of dark-soliton generation. a, Optical set-up. A spatial light modulator (SLM) is used to imprint a phase step by exposing part of the condensate to a far-detuned laser beam. **b**, Theoretical curve of a dark soliton's density $|\Psi|^2$ (dark blue line) and phase ϕ (light blue line), as described by equation (1). **c**, A typical absorption image of the condensate, taken directly after preparation of the soliton and a subsequent free expansion of 11 ms. Optical density is colour- and height-coded for better visibility. **d**, Integrated column density (red circles) of the data in **c** together with a fit to the data (dark blue line). **e**, Image of a soliton after an evolution time of 2.8 s.

to the cubic nonlinear equation vastly used in nonlinear optics to describe solitary wave propagation in optical fibres[25]

$$i\hbar\dot{\psi}(z,t) = -\frac{\hbar^2}{2m}\psi''(z,t) + \left[V_{\mathrm{ext}}(z) + g|\psi(z,t)|^2\right]\psi(z,t).$$

Here, ψ denotes the condensate wavefunction; $g = 2\hbar\omega_\perp a$ is a measure of the nonlinear atomic interaction in quasi-one-dimensional (1D) geometries with strong transverse confinement. g is determined by the s-wave scattering length a and the transverse trapping frequency $\omega_\perp$. V_{ext} describes a confining external potential. A dark-soliton solution to the GPE of a homogeneous BEC describing a density notch at position q propagating along z with a velocity $\dot{q}$ can be written as[26,27]

$$\psi_D(z,t) = \sqrt{n_0}\left\{ i\frac{\dot{q}}{\bar{c}_s} + \sqrt{1-\frac{\dot{q}^2}{\bar{c}_s^2}}\tanh\left[\kappa\,(z-q(t))\right]\right\} \mathrm{e}^{-ign_0t/\hbar}, \quad (1)$$

where the speed of sound in a quasi-1D condensate is given by $\bar{c}_s = \sqrt{n_0 g/2m}$ and n_0 is the peak density of the condensate. The inverse size of the soliton κ is determined by the healing length $\bar{\xi} = \hbar/m\bar{c}_s$ and the soliton speed $\dot{q}$ through $\kappa = \bar{\xi}^{-1}\times\sqrt{1-(\dot{q}/\bar{c}_s)^2}$. Note that in a quasi-1D condensate, the averaging of the density over the radial degrees of freedom effectively changes the speed of sound to $\bar{c}_s = c_s/\sqrt{2}$ and the healing length to $\bar{\xi} = \sqrt{2}\,\xi$ as compared with their 3D values c_s and ξ respectively. The phase and density distributions of a dark soliton are shown schematically in Fig. 1b. The phase only shows significant changes in the vicinity of the nodal plane of the soliton and is constant otherwise. Crossing the nodal plane of the soliton, the wavefunction accumulates a specific phase slip between 0 and π depending on the depth and speed of the soliton related by $n_s/n_0 = 1-(\dot{q}/\bar{c}_s)^2 = \sin^2(\phi/2)$, where n_s denotes the missing density at the position of the soliton. A phase jump of $\Delta\phi = \pi$ corresponds to a fully modulated soliton with zero velocity representing the only time-independent soliton solution of the GPE. As the phase difference diminishes the soliton velocity increases while it gets shallower and wider, ultimately vanishing at the speed of sound. The preceding considerations have led to the idea and experimental realization of creating dark solitons in BECs by optically imprinting a phase gradient of approximately π over a spatial region not larger than the healing length[8,9,28].

Whereas solitons in quasi 1D BECs are dynamically stable[12,19], experiments[9,10] and intensive theoretical studies suggest that in less constrained geometries, where the condition $\gamma = n_0 g/\hbar\omega_\perp \ll 1$ is not perfectly met, the growth of dynamically unstable modes will lead to a transfer of kinetic energy of the soliton to radial excitation modes of the condensate, mediated by the atomic interaction and resulting in a bending of the soliton plane[9], which may ultimately decay into a vortex ring as reported in ref. 10. As the energy of a dark soliton is always greater than the energy of the ground-state condensate, it is thermodynamically unstable in any case and shows fast decay even at reasonably low temperatures[18,20,29] as observed in experiment[8,9]. The dissipation accelerates the soliton according to its negative kinetic energy until it vanishes and smoothly transforms to the BEC ground state as it approaches the speed of sound[19]. This interesting aspect can be interpreted as an accelerating instability[16] and implies that a negative mass can be assigned to a dark soliton. Lifetimes of the order of 10 ms have been reported, preventing the observation of more complex soliton physics such as oscillations or collisions.

We have developed a reliable, robust method to produce elongated ^{87}Rb BECs at extremely low temperatures in an optical trapping potential overcoming former technical limitations. We produce a BEC composed of 5×10^4 ^{87}Rb atoms in the $5^2S_{1/2}$, $F=1$, $m_F=-1$ state in an optical dipole trap with trapping frequencies $\omega_z = 2\pi\times5.9$ Hz, $\omega_\perp^{\mathrm{ver}} = 2\pi\times85$ Hz and $\omega_\perp^{\mathrm{hor}} = 2\pi\times133$ Hz with no discernible thermal fraction. Trap frequencies have been cross-checked by the measurement of various collective oscillations. Typical atomic peak densities are 5.8×10^{13} cm^{-3}, implying a speed of sound of $\bar{c}_s = 1.0$ mm s^{-1}. The chemical potential is of the order of 20 nK.

Ultrastable laboratory conditions ensure exceptional reproducibility, enabling us to record time series of soliton dynamics with unprecedented precision. The low trap depth guarantees a slight but constant evaporative cooling, so that no heating can be detected for timescales as large as the lifetime of the condensate, which is greater than 10 s. Solitons are produced by optically imprinting a phase gradient as shown in Fig. 1a: a part of the condensate is exposed to the dipole potential U_{dip} of a laser beam detuned by some tens of gigahertz from atomic resonance. We image an optical mask pattern onto the BEC with diffraction-limited optical resolution of better than 2 μm. This results in a phase evolution of the masked relative to the unmasked part of the condensate of $\Delta\phi = U_{\mathrm{dip}}t/(i\hbar)$. The pattern is generated by a spatial light modulator (SLM) with an effective pixel size of 0.8 μm allowing for almost arbitrary optical potentials[30]. To imprint a phase slip of order π, we choose a pulse time $t_\pi = 40$ μs, much smaller than the correlation time $\tau_{\mathrm{corr}} = \bar{\xi}/\bar{c}_s = 700$ μs for our experimental parameters to avoid a simultaneous disturbance of the atomic density. This phase gradient leads to a local superfluid velocity of the condensate according to $v_{\mathrm{SF}} = \hbar/m\,\partial_z\phi$, which can also be interpreted as a local potential gradient transferring momentum to the BEC, thus assisting the formation of a density minimum[8]. As a dark soliton can be regarded as a hole rather than a particle, it moves in the direction opposite to the superfluid flow of the condensate. The appropriate equation of motion for small soliton velocities $\dot{q}$ in a BEC that can be described by the Thomas–Fermi approximation is given by[16,19]

$$m\,\ddot{q}(t) = -\frac{1}{2}\,\frac{\partial V(z)}{\partial z}. \qquad (2)$$

The ratio of the negative soliton mass to the likewise negative Thomas–Fermi density potential of the BEC, $M_s/V_{TF}(z)$, is precisely twice the ratio of atomic mass to external potential $m/V(z)$. Therefore, the soliton behaves like a classical particle with mass $2m$. This implies a soliton oscillation frequency of $\omega = \omega_z/\sqrt{2}$ for harmonic trapping. The same result is obtained in the Thomas–Fermi regime in a harmonic trap using a local density approximation where the speed of sound $\bar{c}$ is replaced by its local value $\bar{c}(z)$. Derived in this way, it has been shown that the equation of motion holds for almost arbitrary soliton velocities[23].

Figure 2a shows the time evolution of a dark soliton created by the aforementioned phase imprinting method. Absorption images were taken after a time-of-flight of 11.5 ms to enable the condensate and soliton to expand because the soliton size $l_s \approx \xi \approx 0.8\,\mu$m in the trap is beyond optical resolution. The soliton clearly propagates axially along the condensate with an initial velocity of $\dot{q} = 0.56\,\mathrm{mm\,s^{-1}} = 0.56\,\bar{c}_s$, indicating a relative soliton depth of $n_s = 0.68\,n_0$. We were able to detect nearly pure dark solitons after times as long as 2.8 s in single experimental realizations (Fig. 1e), surpassing lifetimes of dark solitons in any former experimental realization by more than a factor of 200. Fluctuations in the soliton position due to small preparation errors however prevent the observation of soliton oscillations for evolution times $\tau_{evol} \gg 250$ ms. The extraordinary long lifetimes facilitate the first observation of an oscillation of a dark soliton in a trapped BEC. An oscillation frequency of $\Omega = 2\pi \times (3.8 \pm 0.1)$ Hz has been recorded and could be followed for more than one period. Owing to the shallowness of our dipole trap, the atoms experience a full gaussian potential, which is less steep than harmonic leading to a larger amplitude-dependent oscillation period for the soliton. We have calculated the soliton oscillation frequency using equation (2) for a gaussian potential created by a laser beam with a waist of 125 μm with the observed soliton amplitude of $Z_s = 33\,\mu$m and find an oscillation frequency of $\Omega = 2\pi \times 4.0$ Hz. This is in good agreement with our experimental data. We also checked the validity of this model by calculating the frequency for shallower and therefore faster solitons and find very good agreement with experimental data. Furthermore, the observed amplitude enables a consistency check of the soliton depth. At the turning point of the soliton motion Z_s, the constant soliton depth equals the Thomas–Fermi density $n_{TF}(Z_s)$ of the condensate and interrupts the superfluid flow of atoms. At this point, the soliton starts to move in the opposite direction. Given the measured initial speed of the soliton and the observed density distribution of the condensate, Z_s can be calculated to be 36 μm and is in very good agreement with the measured value.

Another feature extracted from Fig. 2a is a density wave that travels in the opposite direction at a velocity equal to the speed of sound. The occurrence of such a density wave has been investigated theoretically and experimentally[8] and has been attributed to the method of soliton generation through phase imprinting while leaving the instantaneous density distribution unchanged. The density waves die out after approximately 50 ms, leaving a flat BEC with only one soliton excitation.

Calculating the dimensionality parameter $\gamma = n_0 g/\hbar\omega_\perp = 3.7$ and comparing this value with the critical ratio γ_c given by Muryshev *et al.*[20], we find our soliton to be right on the edge of the region of dynamical stability. This is confirmed regarding the observed soliton lifetimes.

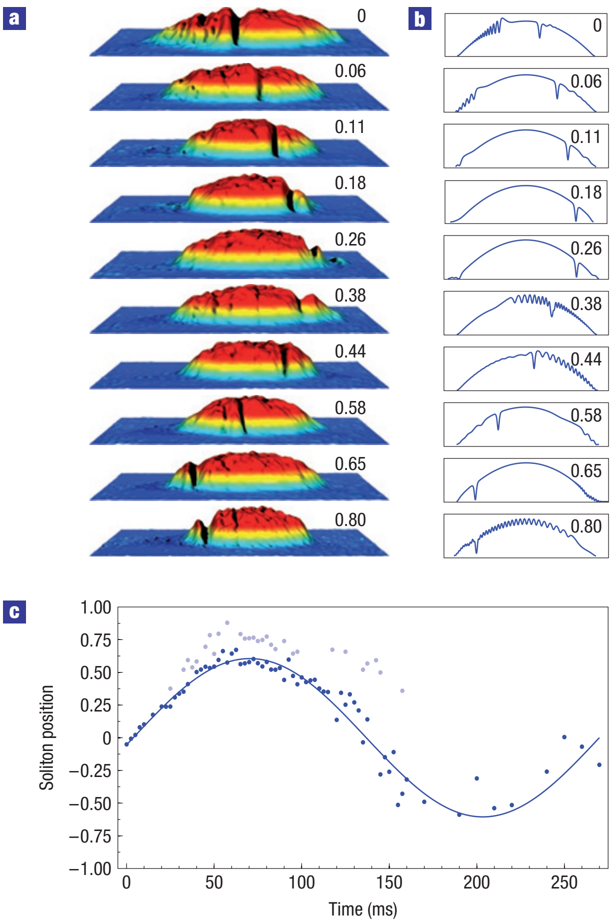
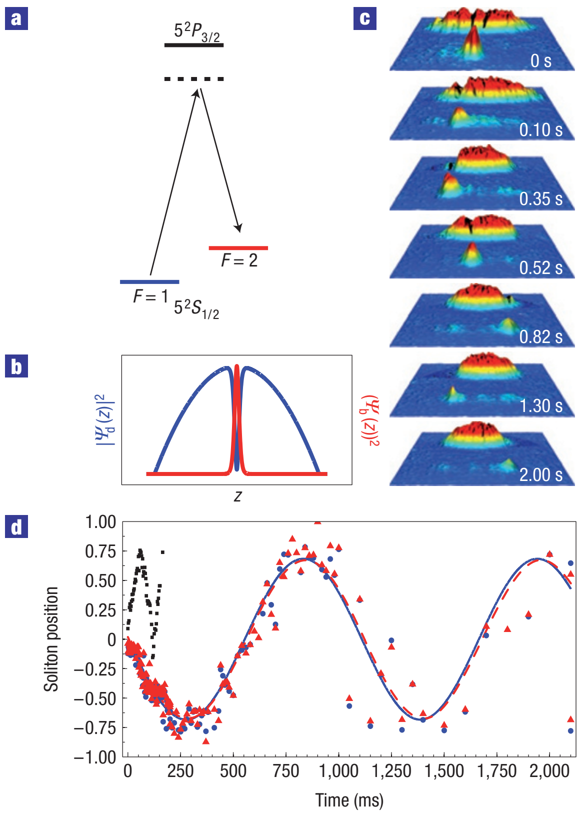


Figure 2 Dark-soliton oscillations in a trapped BEC. a, A set of absorption images showing the soliton position at various times after phase imprinting. The soliton propagates to the right and is reflected off the edge of the condensate after $t \approx 80$ ms. The corresponding evolution time for each image is given in units of the oscillation period T. **b**, Results of a numerical calculation solving the 1D Gross–Pitaevskii equation corresponding to our parameters in units of T. Experimentally observed features such as density modulations caused by a density wave on the left side of the condensate as well as the development of a tiny second soliton are reproduced. **c**, Axial positions of the soliton (dark blue dots) with respect to the centre of mass and normalized to the width of the condensate. The oscillation frequency is $\Omega = 2\pi \times (3.8 \pm 0.1)$ Hz. The position of a second tiny soliton (light blue dots) as well as a sinusoidal fit (blue line) to the position of the soliton are shown. Each data point was obtained from a different experimental run. The scatter is due to small fluctuations in the preparation process. Errors in extracting the soliton's position from the individual images are typically less than 0.02 and are therefore not plotted.

We have carried out numerical simulations of the 1D GPE showing that the phase imprinting method cannot create single perfect dark solitons but always creates density waves that carry away part of the imprinted phase gradient. Moreover, the occurrence of a second small soliton can be extracted from the simulations as shown in Fig. 2b.

The crucial feature to the observed long lifetimes of dark solitons seems to be the very low temperature of our samples. The critical temperature for Bose–Einstein condensation for our experimental parameters is (67 ± 5) nK. Estimating that a thermal fraction of at least 10% could have been detected in absorption imaging—which was not the case—an upper limit for



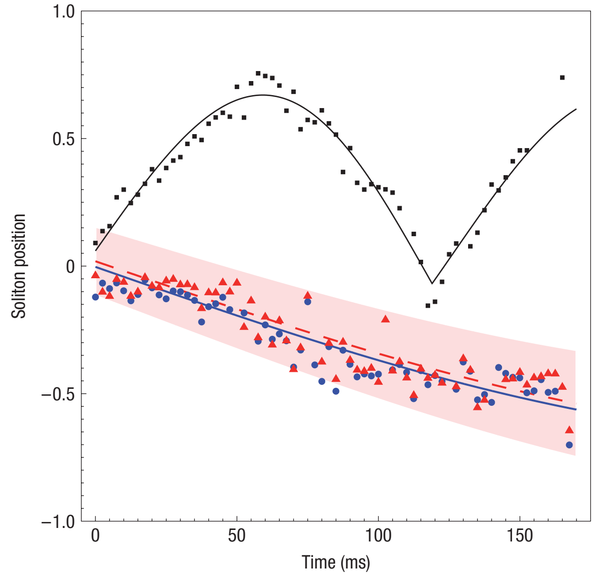


Figure 4 Collision of a dark and a dark–bright soliton. A detailed plot of the first 175 ms of Fig. 3 revealing the reflection of an extra dark soliton off the dark–bright soliton. The axial position of the extra dark soliton (filled squares) is plotted together with a fit to the data (lines). The mean e^{-2} width of the bright soliton is indicated (red shaded area). The reflection of the extra dark soliton is very close to that expected from a hard-wall reflection. The fit corresponds to a sine-function mirrored at the reflection time $t_r = 117$ ms.

Figure 3 Creation and oscillation of a dark–bright soliton. a,**b**, A local population transfer in the centre of the trapped BEC is achieved by a coherent two-photon Raman process between the two hyperfine states $F = 1$ and 2 (**a**) leading to the generation of a dark–bright soliton (**b**). **c**, A set of double-exposure absorption images showing the density distributions of the two components that undergo slow oscillations in the axial direction. **d**, Time series of the axial positions of the dark (blue circles) and bright (red triangles) components of the soliton in addition to corresponding sinusoidal fits to the position. Note that the timescale is different by almost an order of magnitude as compared with that in Fig. 2c. An extra dark soliton (black squares) is also observed. For details of the first 175 ms, see Fig. 4.

the temperature of $T \leq 0.5\,T_c = 30$ nK can be given, which is of the order of the chemical potential μ. We assume a significantly lower temperature, because temperatures of $T \approx 0.2\,T_c$ would already considerably limit the soliton's lifetime[18], which has not been observed in our experiment.

DARK–BRIGHT SOLITONS IN TWO-COMPONENT CONDENSATES

Despite the interesting physics that can be investigated using dark solitons, so-called dark–bright solitons appearing in multicomponent BECs show even more fascinating physical properties, such as enhanced dynamical stability and the possibility of bright-component particle exchange in soliton collisions. A dark–bright soliton is basically a dark soliton filled with atoms of a different species or in another internal state of an atomic matter wave[31,32] (see the Methods section). A dark–bright soliton can be generated in a ^{87}Rb BEC by imprinting a dark soliton in state $|F=1, m_F=0\rangle$ and filling the density dip with atoms in state $|F=2, m_F=0\rangle$, leading to the density distribution shown in Fig. 3b. Whereas dark solitons are unstable to transverse excitations with wavelengths greater than their extension $l_s \approx \bar{\xi}$, large dark–bright solitons are expected to overcome this restriction because their size can be much larger than $\bar{\xi}$ when the number of atoms in the other hyperfine state becomes very large. Dark–bright solitons should therefore be robust in trap geometries that are not truly 1D. We have used a method to simultaneously imprint the phase gradient and transfer atoms to the other hyperfine state through a coherent Raman pulse technique using a laser system phase-locked on the two-photon hyperfine resonance $|F=1, m_F=0\rangle \rightarrow |F=2, m_F=0\rangle$ (Fig. 3a). Applying a 2π pulse of duration 40 μs on one side of the condensate leaves the population effectively unchanged, but introduces a phase difference of π compared with the unperturbed part of the condensate. In a small region around the edge of the mask $l \approx \xi$, a transfer of the population to the $|F=2, m_F=0\rangle$ state occurs (Fig. 3b). By using a step-like intensity pattern rather than a simple edge, it is possible to vary the number of atoms transferred to the other hyperfine state by changing the width of the intermediate step. The mask pattern used in the experiment described here resulted in a bright-component population $N_B = 0.08 N_{tot}$, where N_{tot} is the total number of atoms. A time series of the propagation of such a dark–bright soliton is shown in Fig. 3c. After a short time-of-flight of 9 ms, the atoms are first exposed to a light pulse resonant only with $|F=2\rangle$. After another 2 ms, the $|F=1\rangle$ atoms are subsequently imaged. The dynamics of the dark–bright soliton could be followed for more than 2 s as seen in Fig. 3d. We observe an oscillation with a frequency of $\Omega_{db} = 2\pi \times (0.90 \pm 0.02)$ Hz $= 0.24 \times \Omega$, much smaller than the frequency of the corresponding dark soliton. An expression for the oscillation frequency can be given using the equation of motion for very strongly populated dark–bright solitons[32] and leads to

$$\Omega_{db} = \Omega \frac{\alpha_{z_{max}}}{\sqrt{2}} \left(\frac{4}{15} \frac{N_b R_z}{N_{tot} \bar{\xi}} \right)^{-1},$$

where R_z denotes the radius of the BEC along the axial direction and $\alpha_{z_{\max}}$ is an amplitude-dependent numerical factor that takes into account that the total potential experienced by the bright component depends on the inhomogeneous density of the dark component. For the observed values of N_b, R_z and N_{tot}, we calculate an oscillation frequency of $\Omega_{db} = 2\pi \times 1.27$ Hz. This is in qualitative agreement with the observed value taking into account the assumption of a purely 1D model.

Another spectacular feature that can be extracted from this measurement is the interaction of a dark soliton with the much slower dark–bright soliton. Owing to the method of initial-state preparation, an extra dark soliton is always generated in addition to the dark–bright soliton. As shown in Fig. 4, the dark soliton propagates in the opposite direction as compared with the dark–bright one and oscillates back with the same frequency as a dark soliton in an unperturbed experiment. After 120 ms, it thus approaches the position of the dark–bright soliton, which has moved only very little owing to its much smaller oscillation frequency. The dark soliton is reflected off the dark–bright one comparable to a hard-wall reflection and moves back. To our knowledge, this is the first observation of collisions of different types of matter-wave soliton.

In summary, we have realized long-lived solitons in ^{87}Rb BECs and observed soliton oscillations for the first time. Through a combination of a local Raman transfer and a phase imprinting method, we could realize two-component solitary excitations called dark–bright solitons that clearly exhibit very slow oscillatory dynamics in a trapped BEC. As a first striking example of soliton interaction, the reflection of a dark soliton bouncing off a filled soliton could be observed. These experiments pave the way for further studies on solitons and soliton dynamics in ultracold quantum gases.

Note added in proof. Since the submission of our manuscript, we learned about related experiments[33].

METHODS

CREATION OF SOLITONS

We create BECs of ^{87}Rb atoms through trapping of up to 5×10^9 atoms in a 3D magneto-optical trap, sub-doppler cooling of these atoms and subsequently transferring them into a magnetic trap. We evaporatively cool the atoms slightly above the critical temperature T_c for Bose–Einstein condensation within 20 s. Afterwards, we superimpose a crossed dipole trap realized by a Nd:YAG laser beam focused to a waist of 35 μm and a perpendicular Ti:Sa laser beam (830 nm) focused to a waist of 125 μm. The atoms are loaded in this dipole trap and further cooled evaporatively by smoothly lowering the optical power of the dipole trap beams until we end up with an almost pure BEC of 5–10×10^4 atoms. We reduce the dipole trap power as much as possible to ensure the lowest temperatures of the BEC. The trapping frequencies for this configuration have been determined by several independent approaches and are $2\pi\times(5.9, 85, 133)$ Hz. At this stage, the condensate consists of atoms in the $|F=1, m_F=-1\rangle$ state and can be transferred to any other state or superposition of states through radiofrequency- or microwave-pulse or -sweep techniques. The state preparation is carried out at a sufficiently large magnetic offset field to avoid undesired spin-mixing dynamics[34]. Moreover, the creation of dark–bright solitons demands a spatially selective transfer to the $|F=2, m_F=0\rangle$ state, which is accomplished by the use of a phase-locked Raman laser system with a relative phase error of not more than 0.44 rad. The optical transfer and phase imprinting is achieved by imaging a computer-generated pattern displayed on a spatial light modulator with a pixel size of 8 μm onto the BEC with an optical resolution of better than 2 μm. This is attained through a high-quality imaging optics with a magnification of 1/10. This optics is also used for the detection of BECs, yielding a magnification of 10. After creation of BECs in the optical dipole trap and the application of the phase imprinting light pattern via the SLM for a duration of 40 μs, we allow the BEC to evolve in the trap for a variable time τ_{evol}. Subsequently, we switch off the optical trapping potential within 1 μs. In the experiments presented here, we take an absorption image of the expanded atomic cloud after a time-of-flight of 11 ms. Note that this is a destructive detection technique. For the measurements presented here, each data point therefore corresponds to a new realization of Bose–Einstein condensation, phase imprinting, evolution time and detection.

COUPLED GPE DESCRIBING DARK–BRIGHT SOLITONS

Following ref. 32, a two-component BEC can be described by a set of coupled GPEs. After a renormalization of energy $E' = E/(\hbar\omega_\perp)$, length $x' = x/\sqrt{\hbar/(m\omega_\perp)}$ and wavefunction $\psi_i' = \psi_i\sqrt{g_{ii}/(\hbar\omega_\perp)}$, where g_{ij} are intra- and inter-species interaction parameters, these equations are given by

$$i\dot{\psi}_d = -\frac{1}{2}\psi_d'' + \left(V_d + |\psi_d|^2 + g_d|\psi_b|^2 - \mu\right)\psi_d$$

$$i\dot{\psi}_b = -\frac{1}{2}\psi_b'' + \left(V_b + |\psi_b|^2 + g_b|\psi_d|^2 - \mu - \Delta\right)\psi_b,$$

where ψ_d and ψ_b are the wavefunctions of the dark and bright components respectively, the V_i being external potentials and $\mu_b = \mu$ and $\mu_d = \mu + \Delta$ are the chemical potentials. The intra-species interaction parameters g_{ii} are normalized to unity, whereas the inter-species interaction parameters g_{ij} are very close to unity in the case of ^{87}Rb. The corresponding soliton solutions can be written as

$$\psi_d(z,t) = i\sqrt{\mu}\sin\alpha + \sqrt{\mu}\cos\alpha\tanh\{\kappa(x-q(t))\}$$

$$\psi_b(z,t) = \sqrt{\frac{N_b'\kappa}{2}}\operatorname{sech}\{\kappa(x-q(t))\}. \quad (3)$$

$\cos\alpha$ is the depth of the dark soliton and N_b' is the rescaled number of particles in the bright component. Note that a phase factor has been omitted in ψ_B that is relevant only for collisions of bright solitons, which is beyond the scope of this work. $\kappa = \sqrt{\mu\cos^2\alpha + (N_b'/4)^2} - N_b'/4$ is the inverse length of the dark–bright soliton, which is clearly expanded by the Thomas–Fermi-like repulsion of the bright component as compared with the unperturbed dark soliton. Rewriting this expression in SI units yields

$$\kappa\xi = \sqrt{\cos^2\alpha + \left(\frac{4}{15}\frac{N_b R_z}{N_{tot}\xi}\right)^2} - \frac{4}{15}\frac{N_b R_z}{N_{tot}\xi}.$$

For our experimental parameters, we get $\kappa^{-1} = 6.7$ μm.

DETERMINATION OF SOLITON PARAMETERS

We have determined soliton parameters such as position, width and amplitude from 2D fits to the absorption images. The function used consists of a Thomas–Fermi-like density distribution for the BEC modulated with individual solitons basically given by the square of equation (1). The bright component has been fitted using equation (3).

Received 18 December 2007; accepted 2 April 2008; published 4 May 2008.

References

1. Zabusky, N. J. & Kruskal, M. D. Interaction of solitons in a collisionless plasma and the recurrence of initial states. *Phys. Rev. Lett.* **15**, 240–243 (1967).
2. Zabusky, N. J. Solitons and bound states of the time-independent Schrödinger equation. *Phys. Rev.* **168**, 124–128 (1968).
3. Strecker, K. E., Partridge, G. B., Truscott, A. G. & Hulet, R. G. Formation and propagation of matter-wave soliton trains. *Nature* **417**, 150–153 (2002).
4. Khaykovich, L. *et al.* Formation of a matter-wave bright soliton. *Science* **296**, 1290–1293 (2002).
5. Cornish, S. L., Thompson, S. T. & Wieman, C. E. Formation of bright matter-wave solitons during the collapse of attractive Bose–Einstein condensates. *Phys. Rev. Lett.* **96**, 170401 (2006).
6. Eiermann, B. *et al.* Dispersion management for atomic matter waves. *Phys. Rev. Lett.* **91**, 060402 (2003).
7. Eiermann, B. *et al.* Bright Bose–Einstein gap solitons of atoms with repulsive interaction. *Phys. Rev. Lett.* **92**, 230401 (2004).
8. Burger, S. *et al.* Dark solitons in Bose–Einstein condensates. *Phys. Rev. Lett.* **83**, 5198–5201 (1999).
9. Denschlag, J. *et al.* Generating solitons by phase engineering of a Bose–Einstein condensate. *Science* **287**, 97–101 (2000).
10. Anderson, B. *et al.* Watching dark solitons decay into vortex rings in a Bose–Einstein condensate. *Phys. Rev. Lett.* **86**, 2926–2929 (2001).
11. Reinhardt, W. P. & Clark, C. W. Soliton dynamics in the collisions of Bose–Einstein condensates: An analogue of the Josephson effect. *J. Phys. B* **30**, L785–L789 (1997).
12. Jackson, B., Kavoulakis, G. M. & Pethick, C. J. Solitary waves in clouds of Bose–Einstein condensed atoms. *Phys. Rev. A* **58**, 2417–2422 (1998).
13. Feder, D. L., Pindzola, M. S., Collins, L. A., Schneider, B. I. & Clark, C. W. Dark-soliton states of Bose–Einstein condensates in anisotropic traps. *Phys. Rev. A* **62**, 053606 (2000).
14. Brand, J. & Reinhardt, W. P. Solitonic vortices and the fundamental modes of the snake instability: Possibility of observation in the gaseous Bose–Einstein condensate. *Phys. Rev. A* **65**, 043612 (2002).

15. Muryshev, A. E., van Linden van den Heuvell, H. B. & Shlyapnikov, G. V. Stability of standing matter waves in a trap. *Phys. Rev. A* **60**, R2665–R2668 (1999).
16. Busch, Th. & Anglin, J. R. Motion of dark solitons in trapped Bose–Einstein condensates. *Phys. Rev. Lett.* **84**, 2298–2301 (2000).
17. Theocharis, G., Kevrekidis, P. G., Oberthaler, M. K. & Frantzeskakis, D. J. Dark matter-wave solitons in the dimensionality crossover. *Phys. Rev. A* **76**, 045601 (2007).
18. Jackson, B., Proukakis, N. P. & Barenghi, C. F. Dark-soliton dynamics in Bose–Einstein condensates at finite temperature. *Phys. Rev. A* **75**, 051601 (2007).
19. Fedichev, P. O., Muryshev, A. E. & Shlyapnikov, G. V. Dissipative dynamics of a kink state in a Bose-condensed gas. *Phys. Rev. A* **60**, 3220–3224 (1999).
20. Muryshev, A., Shlyapnikov, G. V., Ertmer, W., Sengstock, K. & Lewenstein, M. Dynamics of dark solitons in elongated Bose–Einstein condensates. *Phys. Rev. Lett.* **89**, 110401 (2002).
21. Carr, L. D., Brand, J., Burger, S. & Sanpera, A. Dark-soliton creation in Bose–Einstein condensates. *Phys. Rev. A* **63**, 051601 (2001).
22. Burger, S., Carr, L. D., Öhberg, P., Sengstock, K. & Sanpera, A. Generation and interaction of solitons in Bose–Einstein condensates. *Phys. Rev. A* **65**, 043611 (2002).
23. Konotop, V. V. & Pitaevskii, L. Landau dynamics of a grey soliton in a trapped condensate. *Phys. Rev. Lett.* **93**, 240403 (2004).
24. Pitaevskii, L. & Stringari, S. *Bose–Einstein Condensation* (Oxford Science Publications, Oxford, 2003).
25. Kivshar, Y. S. & Luther-Davies, B. Dark optical solitons: Physics and applications. *Phys. Rep.* **298**, 81–197 (1998).
26. Tsuzuki, T. Nonlinear waves in the Pitaevskii-Gross equation. *J. Low Temp. Phys.* **4**, 441–457 (1971).
27. Zakharov, V. E. & Shabat, A. B. Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media. *Sov. Phys. JETP* **34**, 62 (1972).
28. Dobrek, Ł. *et al*. Optical generation of vortices in trapped Bose–Einstein condensates. *Phys. Rev. A* **60**, R3381–R3384 (1999).
29. Parker, N. G., Proukakis, N. P., Leadbeater, M. & Adams, C. S. Soliton-sound interactions in quasi-one-dimensional Bose–Einstein condensates. *Phys. Rev. Lett.* **90**, 220401 (2003).
30. Boyer, V. *et al*. Dynamic manipulation of Bose–Einstein condensates with a spatial light modulator. *Phys. Rev. A* **73**, 031402 (2006).
31. Dum, R., Cirac, J. I., Lewenstein, M. & Zoller, P. Creation of dark solitons and vortices in Bose–Einstein condensates. *Phys. Rev. Lett.* **80**, 2972–2975 (1998).
32. Busch, Th. & Anglin, J. R. Dark–bright solitons in inhomogeneous Bose–Einstein condensates. *Phys. Rev. Lett.* **87**, 010401 (2001).
33. Weller, A. *et al*. Experimental observation of oscillating and interacting matter wave dark solitons. Preprint at <http://arxiv.org/abs/0803.4352v1>.
34. Kronjäger, J., Becker, C., Navez, P., Bongs, K. & Sengstock, K. Magnetically tuned spin dynamics resonance. *Phys. Rev. Lett.* **97**, 110404 (2006).

Acknowledgements

We thank the Deutsche Forschungsgemeinschaft DFG for financial support within the Forschergruppe FOR801 and the GRK 1355. K.B. thanks the EPSRC for financial support through grant EP/E036473/1.

Author information

Reprints and permission information is available online at http://npg.nature.com/reprintsandpermissions. Correspondence and requests for materials should be addressed to K.S.