Hwk 4 Review

1) Lemma 3.6

Define a unary function f by

$$f(0) = k$$
$$f(t+1) = h(t, f(t))$$

Then, if h is total, so is f.

Pf/ By induction on $t \in \mathbb{N}$.

Base case: $t = 0$.

Then: $f(t) = f(0) = k$.

$\therefore \neg f(0)\uparrow$.

$f(0)\downarrow k$.

Induction step: assume $f(t)\downarrow c$, for some c.

Then: $f(t+1) = h(t, f(t))$
$= h(t, c)$

And since h is total,

$$h(t, c) \downarrow c', \text{ for some } c'.$$

$$\therefore f(t+1) \downarrow c', \text{ for some } c'.$$

End induction step.

$\therefore$, since $f(t) \downarrow$ for all t,

f is total.

2) Lemma 3.8, very similar.

An "algorithm" for square root

In hwk 5, we ask for a PR defn of integer square root, sqrt(x).

So:

sqrt(0) = 0
sqrt(1) = 1
sqrt(2) = 1
sqrt(3) = 1
sqrt(4) = 2
sqrt(5) = 2
⋮
sqrt(8) = 2
sqrt(9) = 3
⋮

Idea for an algorithm:

Consider when sqrt(x+1) increases from sqrt(x).