Lemma 1.2 (c)

Show if g, f are surjective, so is $g \circ f$.

f is surjective $\equiv \forall\, b \in B \cdot \exists\, a \in A \cdot f(a) = b$

For any $c \in C$, we show $\exists\, a \in A$ s.t. $(g \circ f)(a) = c$.

pf

Consider any $c \in C$.

Then by g surjective,
$\exists\, b : B \cdot g(b) = c$.

Then, given that b,
by f surjective,
$\exists\, a : A \cdot f(a) = b$.

So, for any $c \in C$,
$\exists\, a : A \cdot g(f(a)) = c$, i.e., $(g \circ f)(a) = c$.

Lemma 1.2 (d)

From parts (b) and (c).

Cor. 1.3 (a)

Show $A \approx A$.

$\mathbb{1}_A$ is a bijection, i.e. $\mathbb{1}_A : A \approx A$

$\therefore$ $A \approx A$.

Cor. 1.3 (c)

Show if $A \underset{f}{\approx} B \underset{g}{\approx} C$, then $A \approx C$.

Take $g \circ f$. By an earlier result, since f, g are bijective, so is $g \circ f$.