Hwk 5 review

1) Exercise 2, page 5-6.

Show f is PR by induction on n.

Base case: "definition by 2 cases is PR"

Let $n=2$. Then

$$f = \begin{cases} g_1(\vec{x}) & \text{if } P_1(\vec{x}) \\ g_2(\vec{x}) & \text{o/w} \end{cases}$$

This is PR by Lemma 5.4,

with $g = g_1$, $h = g_2$, $P = P_1$.

Induction step: "assume definition by n many cases is PR. Prove definition by n+1 many cases is PR".

Assume

$$h(\vec{x}) \simeq \begin{cases} g_2(\vec{x}) & \text{if } P_2(\vec{x}) \\ g_3(\vec{x}) & \text{if } \neg P_2(\vec{x}) \wedge P_3(\vec{x}) \\ \vdots & \\ g_{n-1}(\vec{x}) & \text{if } \ldots \\ g_n(\vec{x}) & \text{o/w} \end{cases}$$

is PR in $g_2, \ldots, g_n, P_2, \ldots P_{n-1}$.

Then f is PR in $g_1, \ldots, g_n, P_1, \ldots, P_{n-1}$ by the PR definition

$$f(\vec{x}) = \begin{cases} g_1(\vec{x}) & \text{if } P_1(\vec{x}) \\ h(\vec{x}) & \text{o/w} \end{cases}$$

2) <u>Corollary 5.9</u>

Define g'', h'' by primitive recursion on y:

$$g''(0, \vec{x}) = 0$$
$$g''(y+1, \vec{x}) = g''(y, \vec{x}) + f(\underline{y+1}, \vec{x})$$

h'' is similar.

3) <u>Corollaries 5.11, 5.12</u>

5.11/ Define Q', R' using Q, R.

$$Q'(y, \vec{x}) = Q(y+1, \vec{x})$$
$$R'(y, \vec{x}) = R(y+1, \vec{x}).$$

5.12/ Define Q'', R'' as

$$Q''(y, \vec{x}) = Q(f(y, \vec{x}), \vec{x})$$

$$R''(y, \vec{x}) = R(f(y, \vec{x}), \vec{x})$$

Is unbounded minimalisation in PR(P) ?

Answer: no.

Why? Because $f(\vec{x})$ may diverge even if $P(y, \vec{x})$ does not diverge for any y.

5) a) $\begin{cases} even(0) = 1 \\ even(t+1) = 1 \dot{-} even(t) \end{cases}$

b) $max(x,y) = \begin{cases} x & \text{if } x \leq y \\ y & o/w \end{cases}$

c) $perfsq(x) = sqrt(x) * sqrt(x) = x$

d) $sqrt(x) = (\mu z \leq x)[(z+1)*(z+1) > x]$

e) $gcd(x,y) =$

Hwk 6 hints

1) Exercises 2, 3, 4 p. 5-13.

(2)
$$f^0(x) = x = id(x)$$
$$f^1(x) = f(x)$$
$$f^2(x) = (f \circ f)(x)$$
$$f^3(x) = (f \circ f \circ f)(x)$$
$$\vdots$$

(3) a) ~~A~~ If a set $A \subset \mathbb{N}$ is finite, we can write $A = \{a_0, \ldots, a_n\}$.

b) A set $A \subseteq \mathbb{N}$ is co-finite if there is a $B \subset \mathbb{N}$ s.t. $\mathbb{N} = A \cup B$, and $A = \mathbb{N} - B$.

4) Review long division.

~~2) 1) Exercise 3, page 6-9: note we say G-computable fn~~

MT "hints"

- Section 6 is not explicitely covered.
  - But you can use PR definition techniques from it, e.g., simultaneous primitive recursion.
- Primitive recursive definitions ~~feature~~ are heavily featured.
- Hints about notes 1 content to come...