Mid term review

1) Similar to diagonalisation for $2^{\mathbb{N}} = \mathrm{PRED}(\mathbb{N})$.

Witness: $g(x) = \begin{cases} 1 & \text{if } f_x(x) \neq 1 \\ 0 & \text{o/w} \end{cases}$

2) a) FN_0 is <u>countable</u>.

Let $FN_0^n = \{ f : \mathbb{N} \to \mathbb{N} \mid \forall k \geq n \ f(k) = 0 \}$

$FN_0^n \approx \mathbb{N}^n$; which is countable by Thm 1.14(6)

and $FN_0 = \bigcup_{n=0}^{\infty} FN_0^n$, which is countable by Thm 1.14(3).

2) b) $g \notin FN_0$

3) $\text{primepow}(n) = \cancel{\exists p \,\exists k\, [\text{prime}(p) \wedge n = p^k]}$

$$\exists p \le n\ \exists k \le n\ [\text{prime}(p) \wedge n = p^k \wedge k > 0]$$

4) $$\text{lcm}(x,y) = \begin{cases} \mu z \le x*y\,[\,x \mid z \wedge y \mid z \wedge z > 0\,] & \text{if } x > 0 \text{ and } y > 0 \\ 0 & \text{o/w} \end{cases}$$

5) $$\text{perf}(n) = \left(\sum_{k<n} (k \mid n) * k\right) = n \ \wedge\ n > 0$$

6) a)

$$root(0, 0) = 1$$

$$root(k, 0) = 0$$

$$\cancel{root(k, n+1) = \{ root(k,n) \text{ if } root(k,n)\ldots}$$

$$root(k, n+1) = \begin{cases} root(k,n) & \text{if } (root(k,n)+1)^k > n \\ root(k,n)+1 & \text{o/w} \end{cases}$$

b) $\cancel{root(k,n)}$

$$root(k, n) = \begin{cases} 1 & \text{if } k=0,\ n=0 \\ 0 & \text{if } k=0,\ n \neq 0 \\ \mu y \leq n \left[(y+1)^k > n \right] & \end{cases}$$

7) a) $0 \bmod y = 0$

$$(x+1) \bmod y = \begin{cases} (x \bmod y)+1 & \text{if } (x \bmod y)+1 < y \\ 0 & \text{o/w} \end{cases}$$

if $(x \bmod y)+1 > y$

b) $0 \text{ div } y = 0$

$$(x+1) \text{ div } y = \begin{cases} x \text{ div } y & \text{if } x+1 \bmod y \neq 0 \\ (x \text{ div } y)+1 & \text{o/w} \end{cases}$$