Hwk 6 review

1) 5-13 Ex 2 /

$$g(0, x) = x$$
$$g(n+1, x) = f(g(n, x))$$

5-13 Ex 3 /

a) Assume we are given a finite set $A = \{a_0, a_1, \ldots, a_n\}$.

Then: $\text{member}_A(x) = \begin{cases} 1 & \text{if } x = a_0 \\ 1 & \text{if } x = a_1 \\ \vdots & \\ 1 & \text{if } x = a_n \\ 0 & \text{o/w} \end{cases}$

~~$\text{member}_A(a) = 1$~~

b) Assume we are given a co-finite set $A = \mathbb{N} \setminus B$, for $B = \{b_0, b_1, \ldots, b_n\}$

Then A is PR,
by this definition:

$$\mathrm{member}_A(x) = \begin{cases} 0 & \text{if } x = b_0 \\ 0 & \text{if } x = b_1 \\ \vdots & \\ 0 & \text{if } x = b_n \\ 1 & \text{o/w} \end{cases}$$

5-13 Ex 4/

I'll type this out later.

2) Page 6-6 Ex1/

$$\#P = [\#(\text{if } X \neq 0 \text{ goto } E), \#(Y{+}{+})] - 1$$

$$= \dots$$

$$= 2^{766} \cdot 3^2 - 1$$

6-6 Ex 2/

$$P_0 = \boxed{}$$

$$P_{99} = \boxed{\begin{array}{l} Y{+}{+} \\ Y \leftarrow Y \\ Y{+}{+} \end{array}}$$

6-6 Ex 3/ Assume a G-computable function f is computed by G-program P. Then, it is also computed by P with any number of skip statements added to the end.

These other programs have different Gödel numbers.

Midterm review

Theorem 1.13 (c)

$\mathrm{PRED}(\mathbb{N})$ is uncountably infinite.

Pf/ Let $P = \{p_0, p_1, \dots\} \subseteq \mathrm{PRED}(\mathbb{N})$

written instead.

be any countable subset of $\mathrm{PRED}(\mathbb{N})$.

We will exhibit a witness

$$p \in \mathrm{PRED}(\mathbb{N}) \setminus P$$

i.e. a witness that $P \neq \mathrm{PRED}(\mathbb{N})$.

Define

$$p(x) = \neg p_x(x)$$

Then, $\forall n \cdot p(n) \neq p_n(n)$, so $p \neq p_n$. So $p \notin P$.

Page 5-12 Exercises

c) $perfsq(x)$

$$perfsq(x) = \exists\, i \leq x \cdot i * i = x$$

$$perfsq(x) = \ \ldots$$ There are other ways. None I can think of without quantifiers.

d) $sqrt(x)$

minimalization: $$sqrt(x) = \mu(y \leq x)\left[(y+1)(y+1) > x\right]$$

<u>or</u>

$$sqrt(0) = 0$$

primitive recursion: $$sqrt(n+1) = sqrt(n) + perfsq(n+1)$$

Note: unlike w/ perfsq above, sqrt(n+1) <u>does</u> clearly depend upon sqrt(n) in some way.
So we have both a version w/ quantifiers, and one w/o.