

Hwk 3 review

(1) If $A \approx B$ and A is finite, uncountable or countably infinite, then so is B .

Case 1: A is finite.

Pf/ $A \approx \text{seg}(n)$ (for some n ,
and $A \approx B$ (by def. of finite.)
(by assumption)

Then by Cor. 1.3,

$B \approx \text{seg}(n)$,

so B is finite. \square

Case 2: A is countably infinite.

So $\mathbb{N} \approx A$, and $A \approx B$.

So $\mathbb{N} \approx B$. \square

Case 3/ A is uncountable.

So $\rightarrow A \approx \text{seg}(n)$ for any n ,
and $\rightarrow A \approx \mathbb{N}$.

By cases 1 & 2, if B were
countable or finite, then

so would be A .

So B must be uncountable.

2) Prove $\text{PRED}(\mathbb{N})$ is uncountable.

Recall $\text{PRED}(\mathbb{N}) = \{f \mid f: \mathbb{N} \rightarrow \mathbb{Z}\}$.

Let $P = \{p_0, p_1, p_2, \dots\}$ be any countable subset of $\text{PRED}(\mathbb{N})$.

We will exhibit a predicate p s.t.

$$p \in \text{PRED}(\mathbb{N}) \setminus P,$$

i.e., p will be a witness that

$$P \subsetneq \text{PRED}(\mathbb{N}).$$

Define

$$p(n) = \neg p_n(n)$$

or, equivalently,

$$p(n) = 1 - p_n(n).$$

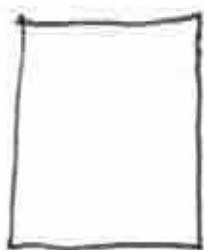
Then for all n , $p(n) \neq p_n(n)$, so $p \neq p_n$,
hence $p \notin P$.

3) G - programs

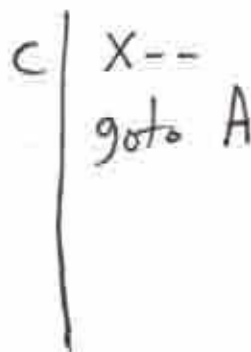
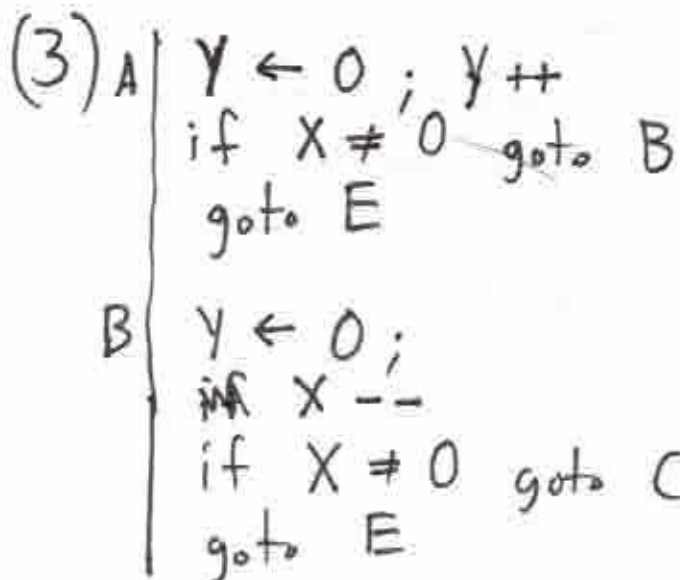
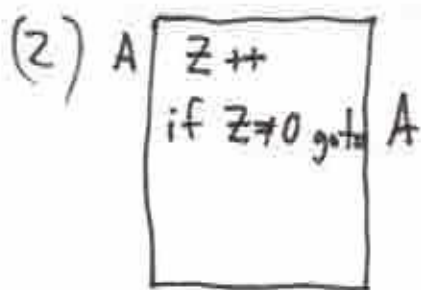
(1) zero



or



(empty program)



Hwlc 4 hints

Use induction!

Show that for any $t \in \mathbb{N}$,

$f(t)$ is defined.

or

$f(\vec{x}, t)$