Hwk 3 review

(1) If $A \approx B$ and A is finite, uncountable or countably infinite, then so is B.

Case 1: A is finite.

Pf/ $A \approx seg(n)$ ⟨for some n, by def. of finite.⟩

and $A \approx B$ ⟨by assumption⟩

Then by Cor. 1.3,

$B \approx seg(n)$,

So B is finite. $\square$

Case 2: A is countably infinite.

So $\mathbb{N} \approx A$, and $A \approx B$.

So $\mathbb{N} \approx B$. $\square$

Case 3/ A is uncountable.

So $\neg A \approx seg(n)$ for any n, and $\neg A \approx \mathbb{N}$.

By cases 1 & 2, if B were countably inf. or finite, then so would be A.

So B must be uncountable.

2) Prove $\mathrm{PRED}(\mathbb{N})$ is uncountable.

Recall $\mathrm{PRED}(\mathbb{N}) = \{ f \mid f : \mathbb{N} \to 2 \}$.

Let $P = \{ p_0, p_1, p_2, \ldots \}$ be any countable subset of $\mathrm{PRED}(\mathbb{N})$.

We will exhibit a predicate p s.t.

$$p \in \mathrm{PRED}(\mathbb{N}) \setminus P,$$

i.e., p will be a witness that

$$P \subsetneq \mathrm{PRED}(\mathbb{N}).$$

Define

$$p(n) = \neg\, p_n(n)$$

or, equivalently,

$$p(n) = 1 - p_n(n).$$

Then for all n, $p(n) \neq p_n(n)$, so $p \neq p_n$, hence $p \notin P$.

3) $\mathcal{G}$ - programs

(1) zero

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skip.
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or

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(empty program)

(2)

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A   Z++
    if Z≠0 goto A
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(3)

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A   Y ← 0 ; Y++
    if X ≠ 0 goto B
    goto E

B   Y ← 0 ;
    X --
    if X ≠ 0 goto C
    goto E

C   X--
    goto A
```

Hwk 4 hints

Use induction!

Show that for any $t \in \mathbb{N}$,

$f(t)$ is defined.

or

$f(\vec{x}, t)$