logic laws (logical equivalences)

equivalence	name
$p \wedge T \equiv p$ $p \vee F \equiv p$	identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	idempotent laws
$\neg(\neg p) \equiv p$	double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	associative laws

equivalence	name
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	absorption
$p \vee \neg p \equiv T$	tautology
$p \wedge \neg p \equiv F$	contradiction
$(p \rightarrow q) \equiv (\neg p \vee q)$	implication equivalence

rules of inference

rule	tautology	name
p $p \rightarrow q$ $\therefore q$	$[p \wedge (p \rightarrow q)] \rightarrow q$	modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	$[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$	modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \rightarrow q$	disjunctive syllogism
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	simplification

rule	tautology	name
p q $\therefore p \wedge q$	$[(p) \wedge (q)] \rightarrow (p \wedge q)$	conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$	resolution
$\forall x P(x)$ $\therefore P(c)$		universal instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$		universal generalization
$\exists x P(x)$ $\therefore P(c)$ for some c		existential instantiation
$P(c)$ for some c $\therefore \exists x P(x)$		existential generalization

fallacies

rule	contingency	name
$p \rightarrow q$ q $\therefore p$	$[(p \rightarrow q) \wedge q] \rightarrow p$	affirming the conclusion
$p \rightarrow q$ $\neg p$ $\therefore \neg q$	$[(p \rightarrow q) \wedge \neg p] \rightarrow \neg q$	denying the hypothesis

rule	contingency	name
p $\therefore p$	$p \rightarrow p$	begging the question

quantifiers

statement	when T (true)?	when F (false)?
$\forall x P(x)$	P(x) is T for every x	P(x) is F for at least one x
$\exists x P(x)$	P(x) is T for at least one x	P(x) is F for every x
$\forall x \forall y P(x,y)$ $\forall y \forall x P(x,y)$	$P(x,y)$ is T for every pair x, y	$P(x,y)$ is F for at least one pair x, y
$\forall x \exists y P(x,y)$	for every x, $P(x,y)$ is T for at least one y	there is at least one x such that $P(x,y)$ is F for every y
$\exists y \forall x P(x,y)$	for at least one x, $P(x,y)$ is T for every y	for every x, $P(x,y)$ is F for at least one y
$\exists x \exists y P(x,y)$ $\exists y \exists x P(x,y)$	$P(x,y)$ is T for at least one pair x, y	$P(x,y)$ is F for every pair x, y

types of proofs

type	description
direct	show that if p, then q must follow ($p \to q$)
contrapositive	show that if $\neg q$, then $\neg p$ must follow ($\neg q \to \neg p$)
vacuous	show that $\neg p$ always holds, making $p \to q$ a tautology
trivial	show that q always holds, making $p \to q$ a tautology
contradiction	show that $\neg p$ leads to a contradiction
cases	split the hypothesis into cases and show that $p_i \to q$ for each case
exhaustive	check a list of all possible cases, with each case representing a single example
existence (constructive)	find an element a (called the witness) such that P(a) is true
existence (nonconstructive)	show $\exists x P(x)$ is true without finding a witness (e.g., by showing $\neg \exists x P(x)$ leads to a contradiction)
uniqueness	show that there is exactly one element with a particular property (show existence and uniqueness)

definitions

term	definition
hypothesis/conclusion	in $p \to q$, p is the hypothesis and q is the conclusion
converse	the converse of $p \to q$ is $q \to p$
contrapositive	the contrapositive of $p \to q$ is $\neg q \to \neg p$
inverse	the inverse of $p \to q$ is $\neg p \to \neg q$
tautology	a proposition that is always true
contradiction	a proposition that is always false
contingency	a proposition that is neither a tautology nor a contradiction

term	definition
corollary	a theorem that can be established directly from a theorem that has been proved
even number	an integer that can be expressed in the form $n = 2k, k \in \mathbb{Z}$
odd number	an integer than can be expressed in the form $n = 2k+1, k \in \mathbb{Z}$
prime number	an integer that is only divisible by itself and 1; $\forall x((x \mid n) \to ((x = n) \vee (x = 1)))$
composite number	an integer that has an integer divisor other than itself and 1; $\exists x((x \mid n) \wedge (x \neq n) \wedge (x \neq 1))$