6 Laminated Composite Columns

The purpose of this chapter is to provide information and guidance for analyzing laminated composite columns. Topics include column stability, column crippling, beam-column analysis and torsional instability. Note that since composites exhibit very little or no plasticity plastic deformation of columns is not considered.

6.1 References

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6-2. Staff, *LTV Composite Structures Analysis Manual*, LTV Aircraft Products Group, Grand Prairie, TX, August, 1984.
6-3. Reeve, S.R., *Crippling Methodology for Composite Structure, Memo*, #E-73C2-0039-92, 1992.

6-4. Renieri, M.P. and Garrett, R.A., *Investigation of the Local Buckling Postbuckling and Crippling Behavior of Graphite/Epoxy Short Thin-Walled Compression Members*, July 1981.
6-5. FZM 10126, Crippling Design Curves for Composites, 2013.
6-6. Fogarty, J.H., *Crippling of Titanium Matrix Composite Stiffeners*, Journal of Composite Materials, Vol. 26. No. 7, 1992.
6-7. Spier, E.E., *Crippling/Column Buckling Analysis and Test of Graphite/Epoxy-Stiffened Panels*, AIAA Paper 75-753, May 1975.
6-8. Kassapolgou, C., *Design and Analysis of Composite Structures with Application to Aerospace Structures*, John Wiley & Sons, Ltd., 2010.
6-9. Rivello, R.M., *Theory and Analysis of Flight Structures*, McGraw-Hill, Inc, 1969.
6-10. Timoshenko, S.P. and Gere, J.M., *Theory of Elastic Stability*, McGraw-Hill, Inc, 1961.
6-11. *Mil-HDBK 17-3F*[1], Chapter 5.
6-12. Anon, *Stiffener Crippling Stresses*, SM110, Oct 1998.

Table 6.1-1 Symbols and Nomenclature

Symbol	Description	Units
[A]	Laminate in-plane extensional stiffness matrix	lb/in
[a]	Inverse of the [A] matrix: $[A]^{-1}$	in/lb
[B]	Laminate in-plane coupling stiffness matrix	lb
A_i	Area of cross-section	in^2
A_{skin}	Area of cross-section of skin	in^2
A_{stiff}	Area of cross-section of stiffener	in^2
b	Element effective crippling width	in
b_e	Effective width	in
b_i	Space between stiffeners	in
Branch/Segment/Element	A cross-section shell segment between two junctions or between a junction and an end point.	
c, c_i	End fixity coefficient	
d_i	Distance between two points/vertex	in
[D]	Laminate bending stiffness matrix	In-lb
DUL	Design Ultimate Load	lb
E_{11}, E_x	Axial Young's modulus in 1, X direction	psi
E_{22}	Axial Young's modulus in 2 direction	psi
E_{skin}	Young's modulus of skin in the direction of load	psi
E_{stiff}	Young's modulus of stiffener in the direction of load	psi

E_{xc}	Compressive Young's Modulus in X direction	psi
E_{yc}	Compressive Young's Modulus in Y direction	psi
$[(E_{xc}A)_{min}]_i$	Modulus weighted area of the cross section of the element i	lb
$[(E_{xc}I)_{min}]_i$	Modulus weighted minimum moment of inertia of the element i	lb-in^2
F_{allw}	Allowable stress	psi
F_c	Laminate allowable axial compressive strength (Fcu or FHC or OHC)	psi
$F^{cc}_{skin/stif}$	Crippling stress of skin/stiffener combination	psi
F_{cc}, F_{cci}	Crippling strength of the cross section	psi
F_{cl}	Crippling allowable corrected for length	psi
$F_{cl\text{-}EOD}$	Crippling allowable corrected for length and Effects-of-Defects	psi
F_{col}	Column critical buckling stress	psi
F_{cr}	Buckling stress	psi
F_{cu}	Unnotched compressive strength allowable	psi
FEA	Finite Element Analysis	
FHC	Filled hole compression	psi
F_{ir}	Inter-rivet buckling allowable	psi
F_{max}	Maximum stress acting on effective area of the skin	psi
G_{xz}	Transverse shear modulus	psi
G_x	In-plane shear modulus	psi
$(G_{xz}A)$	Product of the transverse shear modulus and area of the section	lb
$(G_{xz}A)_f$	Product of transverse shear modulus and area of the flange	lb
$(G_{xz}A)_w$	Product of transverse shear modulus and area of the web	lb
H	Total height of the stiffener	in
H/W	Hot-wet environmental conditioning	
I_{min}	Minimum moment of Inertia	in^4
I_{yy},I_{zz}	Moment of inertia about y or x axis	in^4

Junction	Intersections between branch segments	
K	Axial or rotational stiffness	lb/in
K_{EOD}	Effects of defects reduction factor	
L'	Effective length of Column	in
L	Total length of the column	in
M	Applied moment, Beam bending moment	in-lb
M_o	Beam bending moment without beam-column magnification	in-lb
n	Number of elements	
NEF	No edge free	
n_s	Shape factor	
OEF	One edge free	
OHC	Open hole compression	psi
P, P'	Applied axial force	lb
P_c	Column critical buckling load	lb
$P^{cc}_{skin+stif}$	Crippling load of skin/stiffener combination	
P_{cl}	Allowable crippling load	lb
P_{cr}	Buckling load	lb
P_{cu}	Unnotched compressive load allowable	lb
q	Applied running shear load	lb/in
q_{avg}	Applied average running shear load	lb/in
R_i	Inner Radius	in
R_o	Outer radius	in
s	Fastener pitch	in
s_{cavg}	Average compressive stress	psi
s_{cmax}	Maximum compressive stress	psi
s_{cmin}	Minimum compressive stress	psi
t, t_i	Thickness	in

t_{skin}	Thickness of skin	in
t_{stiff}	Thickness of stiffener	in
V	Beam shear force	lb
V_o	Beam shear without beam-column magnification	lb
X	Defines X axis of the crippling design curve	
Y	Defines the Y axis of the crippling design curve	
Z_c	Modulus weighted centroid of section	in
Z_{NA}	Z coordinate of neutral axis	in
α	Angle between web and bottom skin	radians
β	Relative stiffness of the elastic end support K to the rotational support of the column	
δ	Deflection	in
ϕ	Slope of the beam	radians
ν_{xy}, ν_{yx}	Poisson's ratio	
θ	Angle subtended by curved cap at the center of radius	radians
ρ	Radius of gyration	in
σ_{ceq}	Equivalent compressive stress	psi

6.2 Definitions

Columns are structural members whose cross sectional dimensions are small compared to the length. Columns are primarily loaded in compression although sometimes they may have direct or induced bending stresses present. To facilitate definitions of the cross section terminology for composite beams and columns some common laminated composite beam and column cross section shapes which are shown in Figure 5.6-1 is reproduced in Figure 6.2-1for convenience.

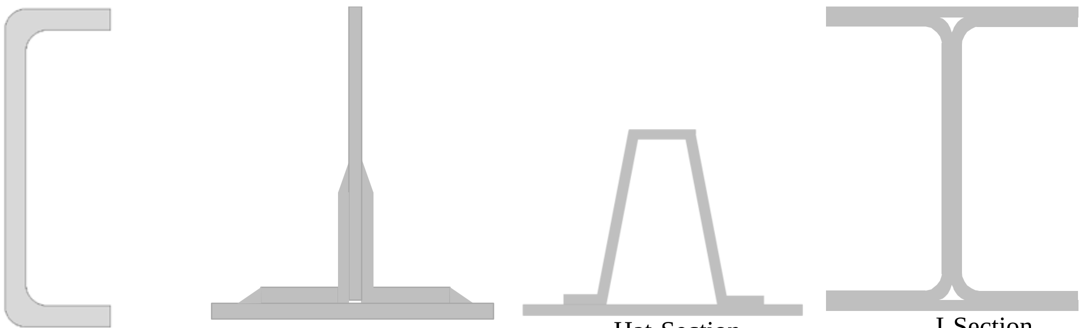


Figure 6.2-1 Common Laminated Beam Sections

The cross section terminologies that are used in discussions below are shown in Figure 6.2-2 and described in Table 6.2-1.

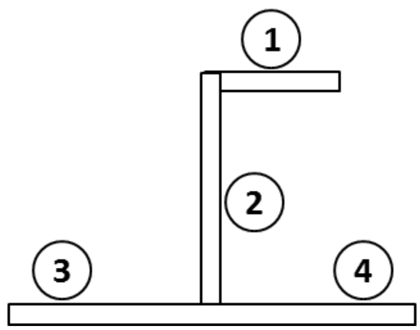


Figure 6.2-2 Beam Cross Section Terminology

Table 6.2-1 Description of Cross Section Terminology

Number	Terminology
1,3,4	Flange
2	Web
1,2,3,4	Element, Segment, Branch

6.3 General Design Guidelines

General design guidelines in Section 2.3 must be considered in addition to the guidelines given below. The analysis methods discussed in this section are specific to symmetric and balanced laminates. For a composite laminated column, it is possible that each branch of the cross section could be tailored to resist a certain type of load that is dominant in that region which results in unique guidelines for that specific region. The top flange that is the furthest from the skin and the bottom flange closest to the skin should have sufficient percentage of 0^o plies aligned along the axis of the column to increase the moment of inertia of the section and thereby increase the buckling and crippling strength. The web primarily resists shear and therefore should have sufficient $\pm 45^o$ plies to increase the shear strength. The outer ply in the cap region should be a glass scrim ply and/or 45^o ply for improved impact resistance by protecting the lower 0º plies. However, the ability to transition plies and manufacture the part are also important considerations and so some judgment should be used in tailoring the part ply layup.

At the termination of the flange/skin interface there is an abrupt change in the load path that results in peak interlaminar stresses that tend to debond the flange from the skin in situations where the flange is bonded to the skin. The peak stresses can be minimized by reducing the stiffness mismatch between the skin and the flange and by transitioning the load from the flange into the skin smoothly by providing a taper on the flange. A 20-30 degree taper from horizontal is recommended to allow for the cross-sectional centroid to shift towards the skin.

To provide better continuity of load path from the web to the skin a portion of the web plies should continue and become part of the flange attached to the skin. This results in a curved portion that encloses a weak resin rich insert region in the I, Pi and the Hat sections. It is recommended that the insert region be reinforced with uniaxial fibers. The effect of this stiffened insert region is to increase the crippling strength (Reference 6-11). However this beneficial effect is ignored in crippling calculations. For other types of loading such as pull-off and bending loads the insert region is prone to delamination. A program of testing at the element and component level may be necessary to develop allowables to ensure that stress/strain in the web and flanges are sufficiently low to prevent delamination in the insert region. The insert region may also have to be considered for better correlating numerical solutions such as FEA with test data.

The skin may be both bonded and fastened to the flange and good design practice dictates that provisions for fastening be included in design for a wider range of repair options. In this case the margins for the fastener hole would be determined using filled-hole assumptions for fasteners with a bolt to hole clearance of less than 0.004 in. If the fastener is a loose fit with a bolt to hole clearance greater than 0.004 in then open-hole assumptions should be used. Program guidance may specify other criteria for repair considerations.

6.3.1 Loads

All loads including skin loads, flange loads, web loads plus loads such as membrane effects which are not included in Finite Element Analysis should be obtained by free-body analysis and distributed across the section on an EA basis. System interface loads should also be included in the analysis. In addition other loads, such as handling loads, should be considered, if appropriate or per program direction.

6.3.2 Repair and Durability

Because the repair of composite parts often involves the use of fasteners, allowing for the eventual use of fasteners is a good design practice. This results in a reduction of the part allowables from ultimate pristine levels to filled-hole or open-hole compression stress levels. Composite column structure should be sized for future repair as described in Section 2.4.2.4. Section 6.6 provides details on how the laminated composite columns are sized for future repair.

6.4 Stability of Columns

Some aspects of the stability analysis of columns made of composite material are similar to isotropic materials. These methods follow those for isotropic materials discussed in PM4057 Section 8 (Reference 6-1) with some minor modifications to the material properties that are explained below.

6.4.1 Slenderness Ratio and Fixity Coefficient

A column is assumed to have failed if it cannot fulfill its designed function. Failure of a column does not necessarily imply permanent damage or catastrophic failure of the overall structure. The geometry of the column is the primary predictor of column failure. Column definition is provided in Figure 6.4-1.

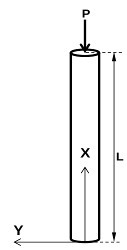


Figure 6.4-1 Column Orientation

The geometry can be characterized by the slenderness ratio L'/ρ, which is the effective length divided by the beam's radius of gyration. The radius of gyration (ρ in) is given by

$$\rho = \sqrt{\frac{\sum_{i=1}^{n}[(E_{xc}I)_{min}]_i}{\sum_{i=1}^{n}[E_{xc}A]_i}}$$ **Equation 6.4.1-1**

where,

$[(E_{xc}I)_{min}]_i$ = modulus weighted moment of inertia of each branch of the section about the weak-axis centroid of the cross section,lb-in^2

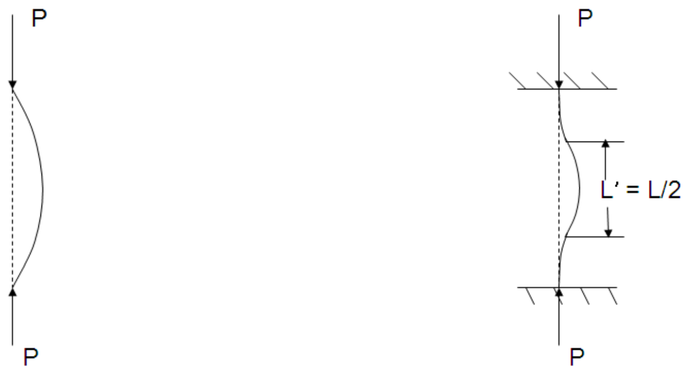
$(E_{xc}A)_i$ = modulus weighted area of the cross section of each branch of the section, lb

n = number of branches in the section

E_{xc} = compressive modulus of the laminate in the direction of the load, psi

Note that since it is possible that each branch of the section is made up of different materials, ply orientations and cross sections the product of (EI) and (EA) should always be evaluated together for each branch of the section and should not be separated as it is done for metals. E_{xc} is the compressive modulus of the laminate in the direction of the applied uniaxial load.

The effective length L' is a function of how the column is restrained at its ends and it relates a column with pinned ends to a column with other support conditions. For example, for a column that is pinned at both ends, the buckled deflected shape is as shown in Figure 6.4-2 (a) which is a half-sine wave. For a column that is fixed at both ends the deflected shape is as shown in Figure 6.4-2 (b). From this it can be noted that there is a reversal of curvature, called the point of contraflexure or inflection, at some point along the length of the column. In the case of a fully fixed column the point of contraflexure occurs at the quarter points of the column length. Since at the inflection points there is no curvature and no bending moment, the column behaves as pinned end column in-between the two points of inflection. Therefore the buckling of the column with fixed support is similar to the column that is pinned at both ends except that the effective column length is half the length of the column that is pinned at both ends. The column with fixed end is more stable than the pinned end.



(a) Pin Ended Column (b) Fixed Ended Column

Figure 6.4-2 Column Deflected Shapes

The compressive strength of the column can be plotted as a function of the slenderness ratio as depicted in Figure 6.4-3.

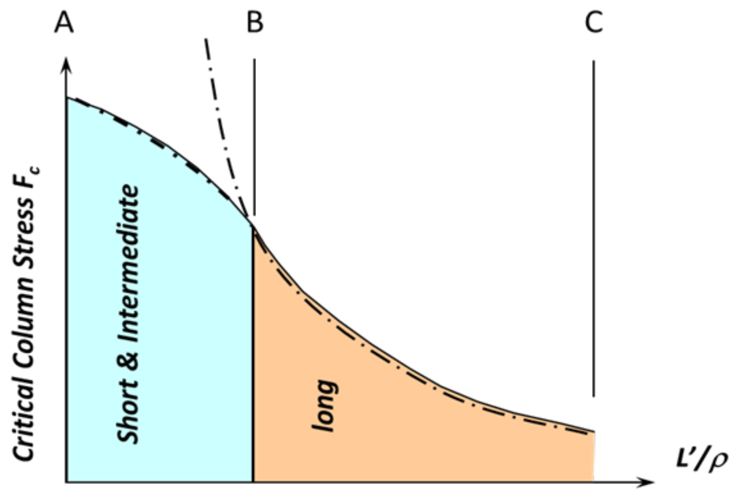


Figure 6.4-3 Column Instability Curve

Three modes of compressive failures can be related to the response shown in Figure 6.4-3. In region AB, depending on the length of the column, the column can fail by (i) compression, (ii) crippling and/or axial nonlinearity and in region BC the column can fail by (iii) Euler buckling. In region AB, for very short columns, the failure is primarily due to compression or a combination of buckling and compression. In the very short column range, the buckling strength of the web and flanges are higher than the compressive strength, therefore the compressive strength of the section is assumed to be the critical stress. For short and intermediate columns with thin walled sections, the flanges initially buckle, but the column as a whole can sustain higher loads if the corners are stable since the postbuckled load is now transferred to the corners. The failure of the entire column occurs when the corners are unable to sustain the additional load and fails. The crippling failure is characterized by the local deformation of the section while the column as a whole has very little out-of-plane distortion.

Crippling in short columns for metals is characterized by a region where the crippling strength is independent of L'/ρ [Reference 6–9]. Experimental efforts [Reference 6-5] to identify such a region for composites for all materials types and cross sections were inconclusive. Therefore, it is assumed that the length has an effect in the entire region AB until further test data is available. In other words the interaction between global Euler buckling and local crippling exists in the entire transition region AB. The failure in region AB is predicted by employing the semiempirically derived Johnson-Euler curve using the crippling strength and geometry of the column. The failure mode in region AB is further discussed in detail in Section 6.6.

The instability failure in the region BC typically occurs in long slender beams on a global scale (Euler Buckling). The column undergoes large out of plane deformation about the axis of weakest principal moment of inertia. Columns should be carefully designed to avoid this kind of failure. Note that the columns can also fail by twisting instead of bending about the weakest bending axis. This type of failure associated with torsional instability will be addressed in Section 6.8.

6.4.2 Column Analysis Method

The stability allowable for columns is the minimum of all possible failure modes which include column buckling, crippling, and for open sections possibly torsional buckling. When an open-section column is loaded axially in compression the flanges usually buckle first since one of their sides is not restrained. After the flange buckles the web portion of the stiffener can carry additional load, since the bending and axial rigidity of the web can restrain the lateral and rotational movement of the flanges. The stiffener ultimately fails in crippling after the corner fails. In such cases the crippling load can be considered as the allowable load. Whenever necessary, the assumed load redistribution should be validated through testing of the component.

All of the column stability failure modes except torsional buckling are discussed in Section 6.4. If eccentricity or lateral loads or both are present beam-column analysis must be performed. It is prudent to perform a beam column analysis assuming an eccentricity even if the structure does not have an eccentricity per design. Note that the beam column analysis is a strength check and does not replace the stability analysis of this Section.

The general flow of column stability analysis is shown in Figure 6.4-4. A margin of safety can be written for each failure mode and the minimum chosen as the column allowable subject to the discussion in the previous paragraph.

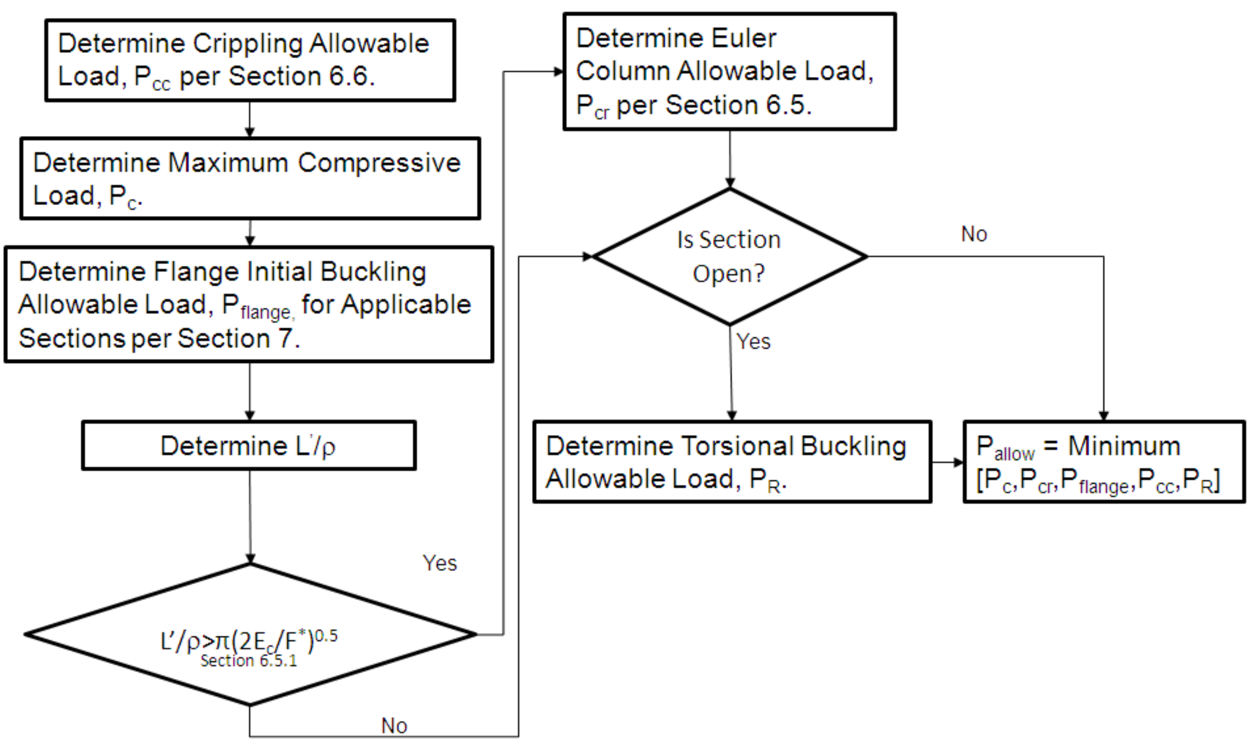


Figure 6.4-4 Column Analysis Flow Chart

In addition to the above analysis use Sections 6.7.2 to determine if Beam Column analysis is required. F* = minimum (P_{cc}, P_{cmax}). Ignore P_{flange} if other elements of the section such as the web can carry the additional load subsequent to flange buckling. This can be ascertained, for example, by comparing the flange buckling stress to the web buckling stress.

For certain geometries the flange buckling can be critical compared to crippling. For example when an angle section with similar flange geometries and material properties is subjected to axial compressive load it is possible for both the flanges to buckle at the same load. This will result in additional load transfer to the corner region which may not be able to sustain the excess load. If the capability of the corner to sustain the excess load cannot be verified through testing, the flange buckling stress should be used as the critical stress. Flange buckling will be discussed in Section 7 (Laminated Plates and Shells).

If the column is very short ($L'/\rho \approx 0$) the column will fail in compression without crippling since the web and the flange lengths are short enough to not cause buckling.

6.5 Euler Buckling

This type of instability failure occurs in long slender columns where the flange elements of the cross section are stable but the column itself becomes unstable due to large lateral deflection caused by bending. The failure is due to lack of stiffness rather than lack of strength and occurs on a global scale. Buckling can result in delamination which may not be easily detected. The Euler buckling of columns with constant cross section is discussed first followed by stability of variable cross sections. Fine grid Finite Element Analysis employing linear buckling solution can be used to determine the Euler buckling load. However, the analyst should verify the results of the analysis with a suitable closed form solution.

6.5.1 Constant Cross Section

The constant cross section columns can be analyzed using the Euler's equation given by

$$F_{col} = \frac{\pi^2 E_{xc}}{\left(L'/\rho\right)^2}$$

Equation 6.5.1-1

F_{col} is the column critical buckling stress, psi
E_{xc} is the equivalent compressive modulus of the section in the direction of the load, psi (Figure 6.4-1)
ρ is the radius of gyration given by Equation 6.4.1-1, in
L' is the effective length of the column in, $L' = L/\sqrt{c}$
L is the total length of the column, in
c is the end fixity coefficient in Table 6.5-1for constant cross sections and in Reference 6-1for variable cross sections.

Since it is possible that the column is made up of sections of different materials the area weighted average modulus must be used as E_{xc}. For uniaxial loading, E_{xc} is determined using Equation 4.4-23 and is shown below for convenience.

$$E_{xc} = \frac{1}{t a_{11}}$$

$$a_{11} = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2}$$

where,
t = total thickness of the laminate, in
A_{ij} = terms in the laminate in-plane extensional stiffness matrix [A] defined in Section 4, lb/in

For preliminary design purposes E_{xc} can be determined with less accuracy using Equation 5.3-3 which is repeated here for convenience.

$$E_{xc} = \frac{E_{11}\ \%0^o + E_{22}\ \%90^o + E_{45}\ \%45^o}{100}$$

where,
E_{45} = E_{xx} for a $[\pm 45^o]_s$ laminate {≈$(A_{11}-(2A_{12})/A_{22}$ or approx 2.8 msi for Carbon/Epoxy}

The loads acting on a cross section can be accurately determined using finite element analysis.

The slenderness ratio beyond which an Euler buckling check is required can be determined by equating the Euler buckling equation Equation 6.5.1-1 to the Johnson-Euler formula, Equation 6.6.1-5. From this, the following criterion is obtained. If L'/ρ is greater than this value an Euler buckling check is required.

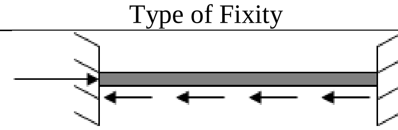
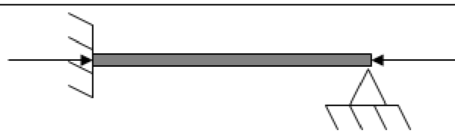
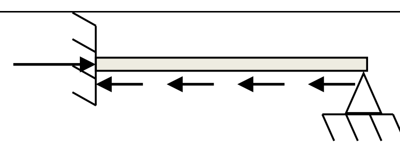
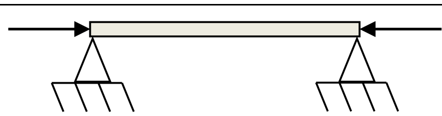
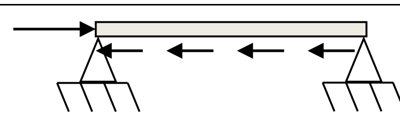
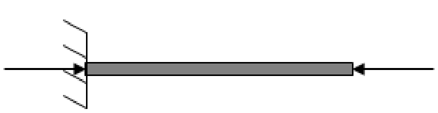
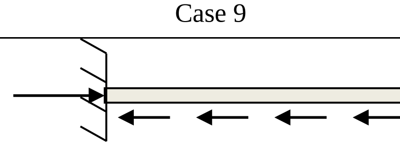
$$\frac{L'}{\rho} > \pi\sqrt{\frac{2E_{xc}}{F_{cc}}}$$ **Equation 6.5.1-2**

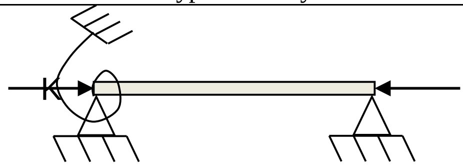
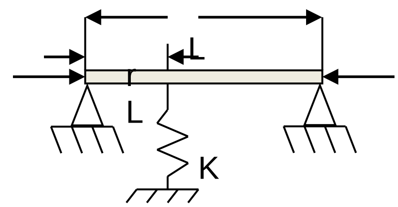
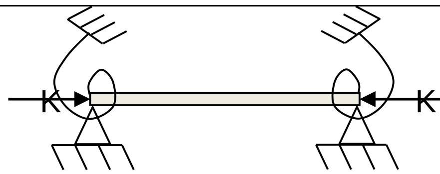
where,
F_{cc} = crippling strength of the cross section, psi, as defined in Equation 6.5.6-1.

Note that Equation 6.5.1-1 can be applied to columns that satisfy the following requirements
1. The column must be straight without initial curvature
2. No eccentrically applied loads in compression or tension
3. Compression loads must be applied only at the ends.

The end fixity coefficient (c) introduced in Equation 6.5.1-1depends on the boundary conditions and its values are tabulated in Table 6.5-1.

Table 6.5-1 End Fixity Coefficients for Constant Cross Section Columns

Type of Fixity	c	Type of Fixity	c
 Case 1	4	 Case 7	7.5
 Case 2	2.05	 Case 8	6.08 (appx.)
 Case 3	1.0	 Case 9	1.87
 Case 4	0.25	 Case 10	0.794

Type of Fixity	c	Type of Fixity	c
 Case 5	Figure 8.2.1-2 (Reference 6-1)	 Case 11	Figures 8.2.1-4, 8.2.1-5, 8.2.1-6, (Reference 6-1)
 Case 6	Figure 8.2.1-2 (Reference 6-1)		

Note that the end fixity constraint c for Cases 5, 6 and 11 depends on the stiffness of the support relative to the column and the location of the support (Case 11). The spring constant K is a measure of the restraint enforced on the column by the supporting structure and can be determined theoretically by finite element analysis or through testing. It is desirable that the K should be verified through testing.

The spring stiffness is the applied force divided by the deflection of the structure resulting from the force and is given as

$$K = P/\delta \text{ or } K = M/\phi$$

where,

K = axial stiffness, lb/in or rotational stiffness, in-lb/rad
P = applied force, lb
δ = deflection, in
M = applied moment, in-lb
ϕ = slope, radians

Figure 8.2.1-2 (Reference 6-1) provides curves to determine end fixity coefficient, c, for Case 5 and 6 of Table 6.5-1. The end fixities are plotted in the figure as a function of the variable β, which represents the relative stiffness of the elastic end support, K, to the rotational stiffness of the column and is given by

$$\beta = \frac{KL}{\sum(E_{xc}I)_{min}}$$

where,

K = rotational stiffness of the elastic end support, in-lb/rad
L = length of the column, in
$\sum(E_{xc}I)_{min}$ = modulus weighted moment of inertia of the section about the weak-axis centroid of the cross section, lb-in^2. ($E_{xc}I$) should be calculated for each branch of the cross section about the neutral axis of the cross section and summed.

Reference 6-1, Figures 8.2.1-4, 8.2.1-5 and 8.2.1-6 provides curves for the determination of c for columns that are pinned-pinned but with an intermediate axial elastic support as shown in Table 6.5-1, Case 11. The variable B which is the ratio of the support stiffness to the column stiffness is given by

$$B = \frac{KL^3}{\sum(E_{xc}I)_{min}}$$

where,

K = stiffness of the intermediate support, lb/in
L = total length of the column, in

$\sum(E_{xc}I)_{min}$ = modulus weighted moment of inertia of the section about the weak-axis centroid of the cross section, lb-in^2. $(E_{xc}I)$ should be calculated for each branch of the cross section about the neutral axis of the cross section and summed.

The spring stiffness of the intermediate support should be carefully determined and verified using finite element analysis or by testing. If verification is not possible a conservative value should be chosen for K.

6.5.2 Effect of Transverse Shear Stiffness on Column Buckling

For columns made with thick cross sections or with honeycomb core the critical loads are reduced because of lower transverse shear stiffness of the composite materials. Figure 6.5-1 shows an example of athick rectangular cross-sectional area geometry.

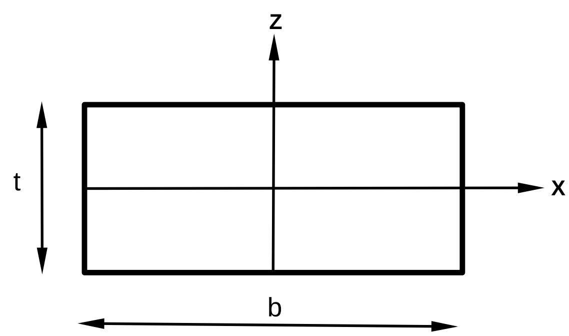


Figure 6.5-1 Cross sectional Area that Resists Shear

The reduced Euler buckling load is given by [Reference 6-2]

$$P_{CR} = \frac{P_c}{[1 + \frac{n_s P_C}{\sum(AG_{xz})}]}$$

where,

P_{CR} = the reduced column critical load, lb
P_c = the column critical load, lb
A = the cross sectional area of each branch, in^2, that resists shear. For a rectangular section as shown in Figure 6.5-1, A = (b)(t).
G_{xz} = the transverse shear modulus of each branch, psi
For a section made of several branches the product AG_{xz} is summed over all branches.
n_s = shape factor.

Table 6.5-2 provides the shape factors from Reference 6-10 for various cross sections. For cross sections not shown in Table 6.5-2 finite element analysis should be performed.

Table 6.5-2 Shape Factors for Various Cross Sections

Shape	**Shape factor (n)**
Rectangle	1.2
Circle	1.11
I-section – bending in the plane of the flange	$1.2\sum(G_{xz}A)/\sum(G_{xz}A)_f$
I-section – bending in the plane of the web	$\sum(G_{xz}A)/\sum(G_{xz}A)_w$

The product of the transverse shear modulus and area for the whole section, $(G_{xz}A)$, is obtained by summing the product for each element of the section. $(G_{xz}A)_f$ and $(G_{xz}A)_w$ are the product of the transverse shear modulus and the area of the flange and web respectively.

6.5.3 Example Problem - Constant Cross Section Column

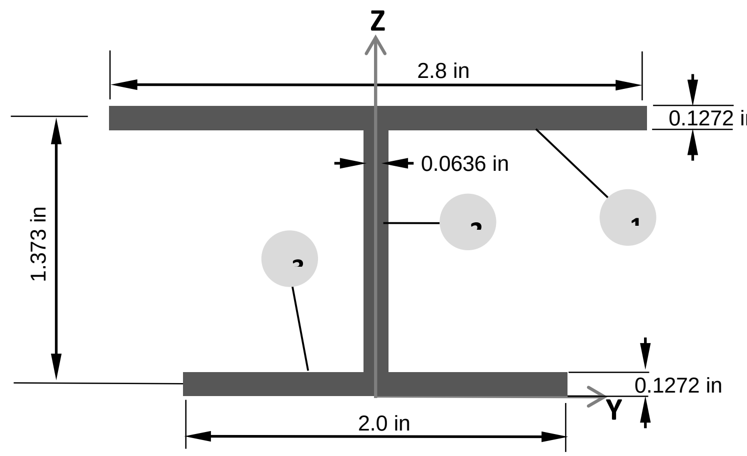
Calculate the Margin of Safety if the column is loaded axially with a 730 lb load.

Assume:

- End fixity: 1) Pinned-Pinned.
- Ambient temperature conditions
- Ignore initial flange buckling for this problem.

Total depth (H) = 1.5 in.

Compare results by calculating (EI) about the maximum and minimum principal axis.



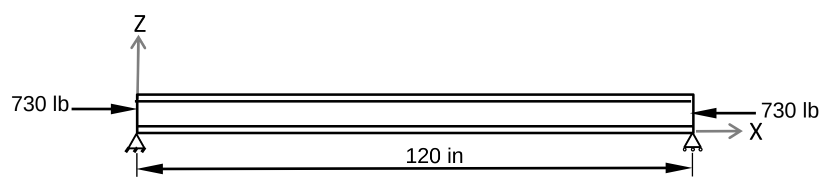
The crippling strength of the section is 51110 psi.

Material: IM7-5250-4 tape

Flange Elements (1,3): laminate ply percentage: [58.3/33.4/8.3], laminate thickness = 0.1272 in.

Web Element (2): laminate ply percentage: [16.7/66.6/16.7], laminate thickness = 0.0636 in.

Both flange and web laminates are symmetric and balanced.



The section properties are calculated using the Table shown below

Calculate the Modulus Weighted Centroid of the section:

Element	E_{xi} (lb/in^2)	Gxi (lb/in^2)	t_i (in)	w_i (in)	A_i (in^2)	$\overline{z_{ci}}$ (in)	$E_{xi}A_i$ (lb)	GxiAi (lb)	$E_{xi}A_iz_{ci}$ (in-lb)
1	$1.46\text{x}10^7$	$2.5\text{x}10^6$	0.1272	2.8	0.3562	1.4364	$5.2\text{x}10^6$	$8.8\text{x}10^5$	$7.5\text{x}10^6$
2	$7.05\text{x}10^6$	$4.1\text{x}10^6$	0.0636	1.25	0.0792	0.75	$5.6\text{x}10^5$	$3.2\text{x}10^5$	$4.2\text{x}10^5$
3	$1.46\text{x}10^7$	$2.5\text{x}10^6$	0.1272	2.0	0.2544	0.0636	$3.7\text{x}10^6$	$0.6\text{x}10^5$	$2.4\text{x}10^5$
Sum					0.6898		$9.5\text{x}10^6$	$1.8\text{x}10^5$	$8.1\text{x}10^6$

Calculate the Modulus Weighted Centroid ($\overline{z}_c$)

$\overline{z}_c = \sum E_{xi}A_iz_{ci} / \sum E_{xi}A_i = 0.858$ in

Calculate the Modulus Weighted Moment of Inertia:

Element	E_{xi} (lb/in^2)	A_i(in^2)	I_i(in^4)	$d_i = \lvert\overline{z}_c - \overline{z}_{ci}\rvert$	$EI_{yy} = E_{xi}I_i+E_{xi}A_id_i^2$	I_{zz}	EI_{zz}
1	$1.46\text{x}10^7$	0.3562	0.00048	0.5787	$1.75\text{x}10^6$	$2.3\text{x}10^{-1}$	$3.4\text{x}10^6$
2	$7.05\text{x}10^6$	0.0792	0.01024	0.1077	$7.87\text{x}10^4$	$2.7\text{x}10^{-5}$	$1.9\text{ x}10^2$
3	$1.46\text{x}10^7$	0.2544	0.00034	0.7941	$2.35\text{x}10^6$	$8.5\text{x}10^{-2}$	$1.2\text{x}10^6$
Sum					$4.17\text{x}10^6$		$4.64\text{x}10^6$

	Using $(E_{xc}I)_{min}$	Using $(E_{xc}I)_{max}$ for comparison only. Margin should be calculated only using $(E_{xc}I)_{min}$
Calculate Area A	$\sum A_i = 0.3562+0.0792+0.2544$ $= 0.6898$ in^2	0.6898 in^2
Calculate E_{xc}	$\frac{\sum E_{xc}A_i}{\sum A_i} = \frac{9.5x10^6}{0.6898}$ $= 13.7x10^6$ psi	$13.7x10^6$ psi
Calculate $E_{xc}I$	From Table $E_{xc}I_{yy} = 4.17x10^6$ lb-in^2	From Table $E_{xc}I_{zz} = 4.64x10^6$ lb-in^2
Calculate $(E_{xc}I)_{min}$	$(E_{xc}I)_{min}$	$(E_{xc}I)_{max}$
Calculate $\rho = (E_{xc}I/E_{xc}A)^{0.5}$	$\rho = \sqrt{\frac{4.17x10^6}{9.5x10^6}}$ =0.6638 in	$\rho = \sqrt{\frac{4.64x10^6}{9.5x10^6}}$ =0.6995 in
Calculate $L' = \frac{L}{\sqrt{c}}$	$\frac{L}{\sqrt{c}} = \frac{120}{\sqrt{1}}$ = 120 inches	120 inches
Calculate L'/ρ	$L'/\rho = 120/0.6638$ = 181	$L'/\rho = 120/0.6995$ = 171.5
Calculate $\sqrt{\frac{2E_{xc}}{F_{cc}}}$	$\pi\sqrt{\frac{2E_{xc}}{F_{cc}}} = \pi\sqrt{\frac{2x13.7x10^6}{51110}} = 73$	73
Since $\frac{L'}{\rho} > \pi\sqrt{\frac{2E_{xc}}{F_{cc}}}$ buckling check has to be performed		
Determine F_{col}	$\frac{\pi^2 E_{xc}}{\left(L'/\rho\right)^2} = \frac{\pi^2 13.7x10^6}{181^2}$ = 4147 psi	$\frac{\pi^2 E_{xc}}{\left(L'/\rho\right)^2} = \frac{\pi^2 13.7x10^6}{171.5^2}$ = 4606 psi
Calculate the allowable column load, $P_{cr}=F_{col}A$	$P_{cr} = (4147)(0.6898)$ = 2861 lb	$P_{cr} = (4606)(0.6898)$ = 3177 lb
Calculate the Margin of Safety M.S. = P_{cr}/P-1	M.S = 2861/730 -1 = +2.9	M.S = 3177/730 -1 = +3.35 For comparison only

Note that if $(E_{cx}I)_{max}$ is used instead of $(E_{cx}I)_{min}$ the margins are unconservative, therefore the minimum (EI) should be always used.

6.5.4 Shear Loaded Columns

In some applications, in addition to axial loading columns are also loaded in shear as shown in Figure 6.5-2. This type of loading combination is analyzed using the same methods used for metals. For more complex boundary conditions Finite Element Analysis should be used. In reference 6-1, Figure 8.2.1-8 shows a plot of effective end fixity coefficients for columns loaded both axially and in shear as a function of the ratio of shear to the applied load P'. In addition Table 8.2.1-3 (Reference 6-1) provides curve fit equations for these curves.

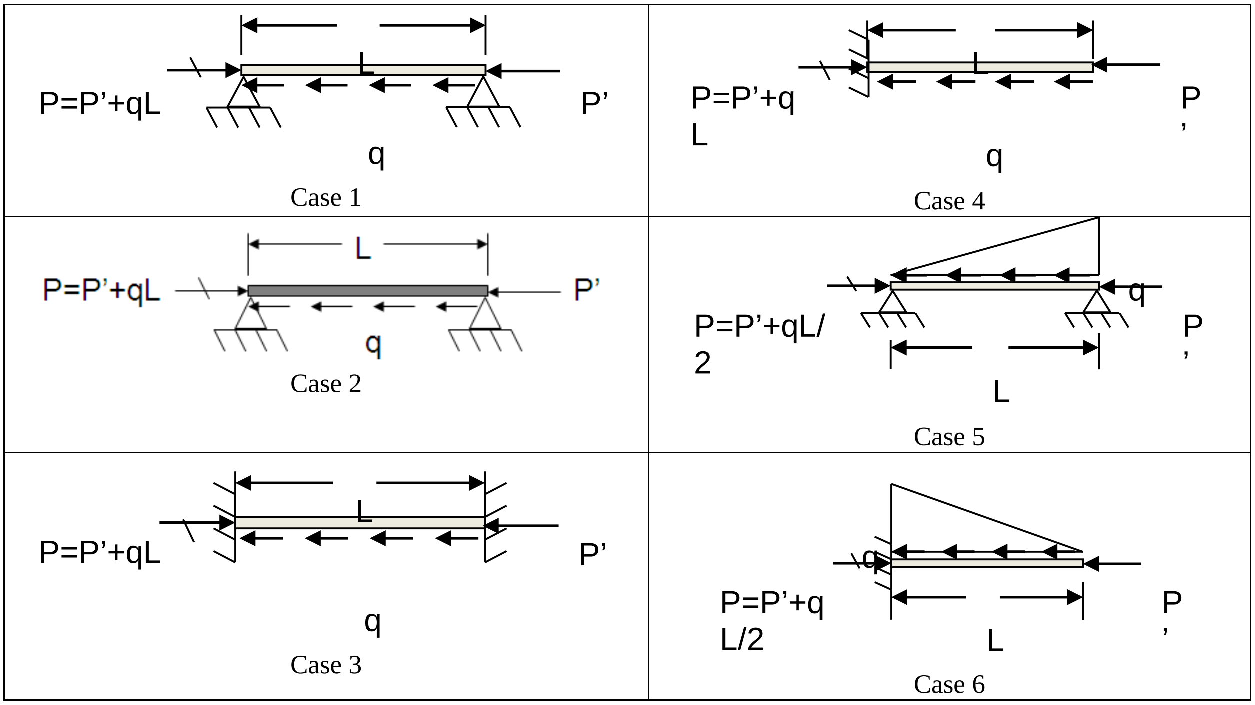


Figure 6.5-2 Combined Axial and Shear Loaded Column Geometry and Parameters

The column buckling stress can be determined using Equation 6.5.1-1. Once the critical column buckling load is determined the margin of safety can be determined using the total axial load reaction, P.

$$P = P' + qL \qquad \textbf{Equation 6.5.4-1}$$

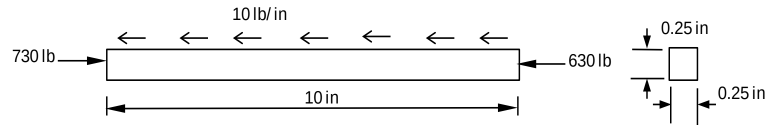
$$M.S. = \frac{P_{cr}}{P} - 1 \qquad \textbf{Equation 6.5.4-2}$$

where,
P' is the applied axial load (lb)
q is the applied running shear load (lb/in)
L is the total column length (in)
P_{cr} is the allowable column buckling load determined from Equation 6.5.1-1.

Details for analyzing additional boundary conditions for metals are provided in Reference 6-1, Section 8. These can be extended for composite materials by using appropriate modulus weighted averages for moment of inertia I and area A.

6.5.5 Example Problem - End Fixity – Constant Cross Section with Axial and Shear Load

Given a square beam of cross section 0.2544 in. X 0.2544 in, 10 inches long. It is made from IM7_5250-4 ply layup: $[45/90/-45/0_3/-45/0_3/45/0]_{2s.}$ E_c = 14.6 msi.
Calculate the Margin of Safety if the column is loaded axially with a 10 lb/in shear load and a 630 lb axial load that are reacted as an axial load at the end of the beam.
Assume the following end fixities: 1) Pinned-Pinned 2)Fixed-Pinned 3)Fixed-Fixed 4)Fixed-Free
Compare the results



	Pinned-Pinned	Fixed-Pinned	Fixed-Fixed	Fixed-Free
Calculate Area A=wh	A=(0.2544)(0.2544) = 0.0647 in^2	A=(0.2544)(0.2544) = 0.0647 in^2	A=(0.2544)(0.2544) = 0.0647 in^2	A=(0.2544)(0.2544) = 0.0647 in^2
Calculate I $I=bh^3/12$	$I=(0.254)(0.254)^3/12$ $=3.491 \times 10^{-4}$ in^4	$I=(0.254)(0.254)^3/12$ $=3.491 \times 10^{-4}$ in^4	$I=(0.254)(0.254)^3/12$ $=3.491 \times 10^{-4}$ in^4	$I=(0.254)(0.254)^3/12$ $=3.491 \times 10^{-4}$ in^4
Calculate $\rho = (E_cI/E_cA)^{0.5}$	$\rho = \sqrt{\frac{(3.491x10^{-4})(14.6x10^6)}{(0.0647)(14.6x10^6)}}$ =0.07344 in	$\rho = 0.07344$ in	$\rho = 0.07344$ in	$\rho = 0.07344$ in
Calculate $q_{avg}L/P'$	$q_{avg}L/P'$ = 10(10/630)=0.1587	$q_{avg}L/P'$ = 10(10/660)=0.1587	$q_{avg}L/P'$ = 10(10/660)=0.1587	$q_{avg}L/P'$ = 10(10/660)=0.1587
Calculate c	$0.134982(0.1587)^3$+ $0.393818(0.1587)^2$ +0.288494(0.1587) +1.055461 = 1.112	$3.690161(0.1587)^3$- $1.668933(0.1587)^2$ +1.958186(0.1587) +2.099683 = 2.383	$1.074757(0.1587)^3$+ $0.282579(0.1587)^2$ +2.137104(0.1587) +4.006007 = 4.357	$0.393771(0.1587)^3$- $0.108402(0.1587)^2$ +0.255179(0.1587) +0.259000 = 0.298
Calculate $L' = \frac{L}{\sqrt{c}}$	L'= 10 /√(1.112) =9.4842	L'= 10 /√(2.383) = 6.4777	L'= 10 /√(4.357) = 4.791	L'=10/√(0.298) = 18.308
Calculate L'/ρ	L'/ρ =10/0.07344 =129.14	L'/ρ =6.478/0.07344 =88.20	L'/ρ =4.791/0.07344 =65.24	L'/ρ =18.308/0.0734 =249.29
Determine F_{col}	8640 psi	18521 psi	33858 psi	2319 psi
Calculate the allowable column load, $P_{cr}=F_{col}A$	P_{cr} = (8640)(0.0647) = 559 lb	P_{cr} =(18521)(0.0647) = 1199 lb	P_{cr} =(33858)(0.0647) = 2191 lb	P_{cr} = (2319)(0.0647) = 150 lb
Calculate $P=P'+q_{avg}L$	P=630+10(10) = 730 lb	P=630+10(10) = 730 lb	P=630+10(10) = 730 lb	P=630+10(10) = 730 lb
Calculate the Margin of Safety M.S. = P_{cr}/P-1	MS = 559/730-1 = -0.23	MS = 1199/730-1 = +0.64	MS = 2191/730-1 = +2.00	MS = 150/730-1 = -0.79

6.5.6 Variable Cross Section

The buckling analysis of variable cross section laminated composite columns follows that of metals. Therefore the analysis procedures employed for metals, discussed in Reference 6-1, Section 8, can be extended to composite columns with minor modifications such as replacing the moment of inertia (I) and area (A) by the modulus weighted averages. Also, the modulus in the direction of load application should be used. For more complex boundary conditions Finite Element Analysis should be used.

6.6 Crippling

Crippling failure occurs by deformation of the cross section of the column in its own plane without any global out-of -plane deflection of the column. This type of failure generally occurs in stable columns that are either short or long that are stabilized by the attached skin. The crippling process is similar for both metals and composites. For example, when a channel section is loaded in compression, the outer flanges buckle initially. On further loading the additional load is now distributed to the web which also eventually buckles. As loading further progresses the corners take the excess load and ultimately fail in compression, resulting in the collapse of the entire column. Crippling should be considered a strength rather than stability failure and typically positive ultimate margin of safety is required.

The crippling allowable is determined using crippling curves that are based on experimental data documented in References 6-3 and 6-4. These test data along with related supporting documents are organized coherently in Reference 6-5. Employing the available test data, statistically based crippling curves are derived in Reference 6-5. The methodology for analyzing a section for crippling involves the following steps:

1. Subdivide the cross section into elements which have either one edge free or no-edges free as shown in Figure 6.6-1. Note that the modified element length ‘b’ defined in Figure 6.6-1 is different than the length defined in PM4057 for metals.

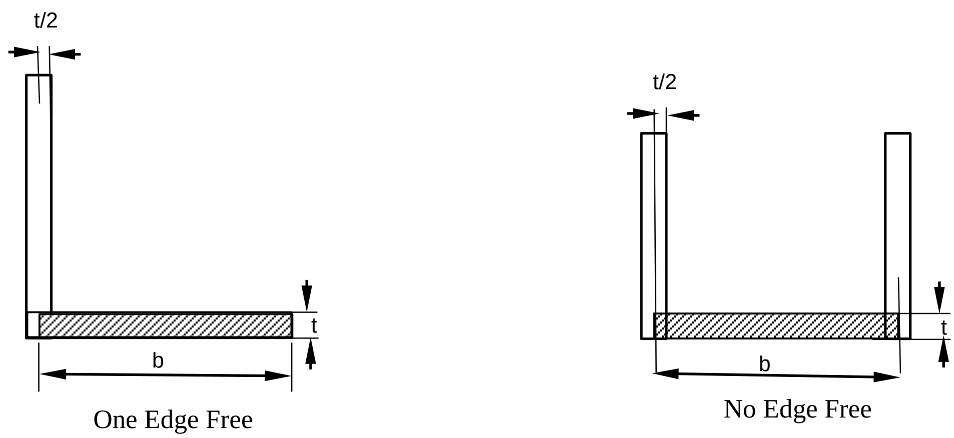


Figure 6.6-1 One Edge Free (OEF) and No Edge Free (NEF) Geometries

2. Determine the crippling allowable for each segment using the method described in Section 6.6.1. A Table such as Table 6.6-1can be constructed to facilitate steps 1 and 2.

Table 6.6-1 Sample Format for Calculating Crippling Stress Allowable for a Composite Section

Element	Edge	b	t	A_i	ν_{xy}	ν_{yx}	D_{11}	E_x	E_{xci}	E_{yc}	F_c	X	Y	Min(F_{cci},F_c)	A_iE_{xci}	$A_iE_{xci}F_{cci}$
		(in)	(in)	(in²)			(in-lb)	(psi)	(psi)	(psi)	(psi)			(psi)	(lb)	(lb²-in²)
1	OEF															
2	NEF															
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
SUM															ΣA_iE_{xci}	$\Sigma A_iE_{xc}F_{cci}$

3. Calculate the crippling allowable of the entire section by summing the modulus and area-weighted crippling strengths of each element (Reference 6-6):

$$F_{cc} = \frac{\sum_{i=1}^{n}(F_{cci}E_{xc}A_i)}{\sum_{i=1}^{n} E_{xc}A_i} \qquad \textbf{Equation 6.5.6-1}$$

where,

F_{cc} = Stiffener crippling strength, psi
F_{cci} = Element crippling strength, psi
E_{xc} = Compression modulus in the direction of the axis of the column, psi
A_i = Element area, in^2
n = number of elements in the stiffener

Note that 'b' in Table 6.6-1is the modified element length shown in Figure 6.6-1 that should be used only for calculating the crippling strength and should not be used for calculating the element area (A). In the next section the method to determine the crippling strength of an element is discussed.

6.6.1 Crippling Allowable

The element crippling strength (F_{cci}) introduced in Equation 6.5.6-1 is determined using the non-dimensional curve given in Figure 6.6-2. The curves were obtained using test data and methods described in Reference 6-5.

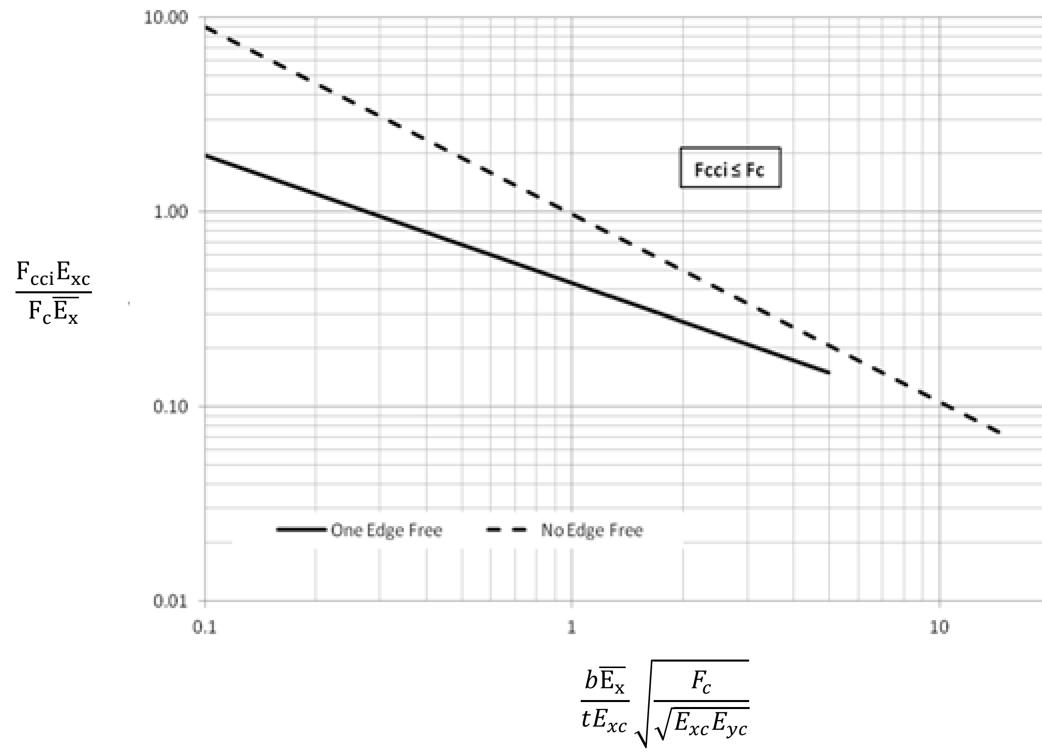


Figure 6.6-2 Non-Dimensional Composite Crippling Curves

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The geometries that correspond to the one edge free and no edge free conditions are shown in Figure 6.6-1.

The curves depicted in Figure 6.6-2 are defined as

$Y = 0.432X^{-0.653}$ (OEF) **Equation 6.6.1-1**

$Y = 0.971X^{-0.962}$ (NEF) **Equation 6.6.1-2**

where the X and Y terms are defined as follows.

$$X = \frac{b\overline{E_x}}{tE_{xc}}\sqrt{\frac{F_c}{\sqrt{E_{xc}E_{yc}}}}$$ **Equation 6.6.1-3**

$$Y = \frac{F_{cci}E_{xc}}{F_c\bar{E}_x}$$ **Equation 6.6.1-4**

where,

$\bar{E}_x$ = Effective laminate flexural modulus, accounting for the stacking sequence
$= \frac{12(1-\gamma_{xy}\gamma_{yx})D_{11}}{t^3}$, psi.
b = Element effective crippling width (Figure 6.6-1) in.
D_{11} = Bending stiffness of the laminate in load direction, in-lb.
E_{xc} = Laminate compressive modulus in load direction, psi
E_{yc} = Laminate compressive modulus perpendicular to the load direction, psi
F_{cci} = Crippling strength of the element, psi
F_c = Laminate allowable axial compressive strength, psi: F_{cu} or FHC or OHC
F_{cu} = Laminate ultimate compressive strength, psi
FHC = Laminate Filled hole Compression strength (FHC), psi
OHC = Open Hole Compression (OHC) strength, psi
t = Element thickness, in
ν_{xy}, ν_{yx} = Poisson's ratios for the laminate

It is recommended that filled hole compression, FHC be used for F_c to accommodate repairability and durability requirements. If fastener hole clearance is in excess of 0.004 inches use open hole compression. F_{cu} should not be used unless the part is labeled with a "no fastener zone".

All the material properties used in the analysis should be at the operating temperature of the structure. If the calculated value of F_{cci} is greater than F_c then set $F_{cci} = F_c$ since the crippling strength cannot exceed the allowable axial compressive stress.

After the F_{cci} value is determined for each element the crippling strength of the entire section is determined using Equation 6.5.6-1. Furthermore, a correction for slenderness ratio (L'/ρ) is made using the Euler-Johnson equation:

$$F_{cl} = F_{cc} - \frac{F_{cc}^2}{4\pi^2 E_{xc}}\left(\frac{L'}{\rho}\right)^2$$ **Equation 6.6.1-5**

where,
F_{cl} = Length-corrected allowable crippling stress, psi.
L' = Effective stiffener length, in, $L' = \frac{L}{\sqrt{c}}$
c = End fixity coefficient per Table 6.5-1
ρ = radius of gyration, in, $\rho = \sqrt{\frac{\Sigma(E_{xc}I)_{min}}{\Sigma(E_{xc}A)}}$
$\sum(E_{xc}I)_{min}$ = the modulus weighted moment of inertia about the weak-axis centroid of the cross section ,lb-in^2. ($E_{xc}I$) should be calculated for each branch of the cross section about the neutral axis of the cross section and summed.
$\sum E_{xc}A$ = the area weighted modulus of the section, lb

There is limited empirical evidence that beyond a slenderness ratio of approximately 20, the crippling strength of the section is influenced by the column length (Reference 6-5). In order to account for this length effect, after the F_{cc} value is determined using Equation 6.6.1-4, a correction for slenderness ratio (L'/ρ) is made using the Euler-Johnson equation. This correction is made for all slenderness ratios from 0 to the point where the Euler-Johnson curve is tangent to the Euler curve. This approach may be conservative but in the absence of test data the degree of conservatism, if any, is unknown.

The length corrected allowable crippling load P_{cl} is defined as:

$$P_{cl} = AF_{cl}$$ **Equation 6.6.1-6**

where,
A = area of the cross section, in^2

The margin of safety is calculated as

$$M.S = \frac{P_{cl}}{P} - 1$$ **Equation 6.6.1-7**

where,
P_{cl} = allowable crippling load calculated using Equation 6.6.1-6, lb
P = applied load, lb

The length correction factor need not be applied for the crippling calculations where portions of the cross section are in tension. However, the column checks as recommended in Figure 6.4-4 should be performed.

In the absence of program direction it is recommended that the filled hole compression strength, FHC be used for F_c to satisfy the durability and repairability requirements. If manufacturing variability exists then its effect on crippling should be introduced through an Effects-of-Defect (EOD) factor which has to be experimentally determined. Factors that need to be considered for EOD can include the following:

- manufacturing induced delamination
- variability in fiber orientation
- porosity
- damage during handling of the component.

If through testing a reduction factor of k_{EOD} is determined for EOD, this factor is applied to the crippling allowable determined using Equation 6.6.1-5 as shown below.

$$F_{cl-EOD} = (k_{EOD})(F_{cl})$$

where,
$F_{cl\text{-}EOD}$ = Crippling allowable corrected for length and Effects-of-Defects (EOD), psi
k_{EOD} = Effects-of-Defects empirical reduction factor

Note that the material properties at the prescribed operating temperature should be used in these equations when calculating X and Y.

The crippling equations are valid for cross sections where the element thicknesses are not the same. For example the cap, flange and web thicknesses could be different. The cross section could consist of straight elements or curved elements. The analysis of a curved cross section in bending is discussed in Section 6.6.6.

In I, J or Hat stiffeners, corner radii exist resulting in a void as shown in Figure 6.2-1 I-section illustration. It is recommended that these areas be filled with a stiff fiber material, running along the axis of the part, to improve the crippling strength (References 6-8 and 6-11) and to reduce the potential for local distortion or wrinkling of the plies. However, this increase in crippling strength is not accounted for in the analysis.

The opening of the corner angle between the flange and the web will set up tensile peel stresses in the composite sections that can result in delamination. These modes of failure will be addressed in a different section.

An effective width of the skin adjacent to the flanges, if it exists, should be included in crippling calculations. The method for calculating effective width is discussed in Section 6.6.3.

6.6.2 Example Problem - Crippling

Determine the crippling Margin of Safety for the section given in Example Problem 6.5.3.

Assumptions:

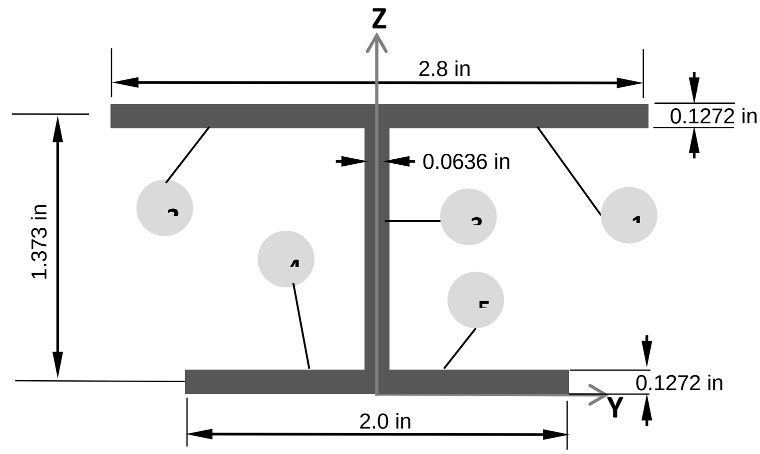
- End fixity of pinned-pinned
- Ambient temperature conditions.
- Ignore initial flange buckling for this problem.

The material and geometrical properties are repeated here for convenience.

Material: IM7-5250-4 tape

Flange Elements (1,2,4,5): laminate ply percentage: [58.3/33.4/8.3], laminate thickness = 0.1272 in.

Web Elements (3): laminate ply percentage: [16.7/66.6/16.7], laminate thickness = 0.0636 in.



$(EI)_{min}$ (modulus weighted minimum moment of Inertia) is given as 4.1743×10^6 lb-in^2.

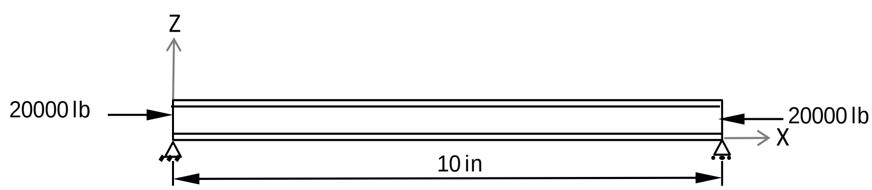
Both flange and web laminates are symmetric and balanced.

Total depth (H) = 1.5 in.

Anticipating future repair using a 0.25 in dia. Titanium protruding head fastener use F_c = FHC = 68255 psi for elements 1 & 2.

For all other elements use their respective unnotched compression allowable for F_c as shown in the Table below.

D_{11} is calculated using the lamina properties as described in Section 4 or this can be calculated using IDAT LAMINATE



tool. The calculated D_{11} = 1974 lb/in for flange and 131 lb/in for web.	
Element #1:	
Calculate element effective crippling width b	b = 2.8/2 in b = 1.4 in
Calculate Area	A = (2.8/2)(0.1272) in^2 = 0.17808 in^2
D_{11} (given)	D_{11} = 1974 lb/in
Calculate $\overline{E_x}$ as shown in Equation 6.6.1-4	$\bar{E}_x = \frac{12\left[1-(\nu_{yx})(\nu_{xy})\right](D_{11})}{t^3}$ $\bar{E}_x = \frac{12[1-(0.411)(0.133)](1974)}{0.1272^3}$ = 10.8798x10^6 psi
Calculate X using Equation 6.6.1-3	$X = \frac{b\overline{E_x}}{tE_{xc}}\sqrt{\frac{F_c}{\sqrt{E_{xc}E_{yc}}}}$ $X = \frac{(1.4)(10.8798x10^6)}{(0.1272)(14.6x10^6)}\sqrt{\frac{68255}{\sqrt{(14.6x10^6)(4.73x10^6)}}}$ =0.743321
Calculate Y using Equation 6.6.1-1	Y = (0.432)($X^{-0.653}$) OEF Y = $(0.432)(0.743321)^{-0.653}$ Y=0.52433
Calculate F_{cci} using Equation 6.6.1-4	$F_{cci} = \frac{YF_c\bar{E}_x}{E_{xc}}$ $F_{cci} = \frac{(0.52433)(68255)(10.8798x10^6)}{(14.6x10^6)}$ =26669 psi

Repeating for each element results in the following table:

Table 6.6-2 Table for Calculating Crippling Stress Allowable

Element	Edge	b	t	A	ν_{xy}	ν_{yx}	D_{11}	$\bar{E}_x$
		in	in	in^2			lb/in	psi
1	OEF	1.4	0.1272	0.17808	0.411	0.133	1974	10.9x10^6
2	OEF	1.4	0.1272	0.17808	0.411	0.133	1974	10.9x10^6
3	NEF	1.373	0.0636	0.07922	0.411	0.411	131	5.1x10^6
4	OEF	1	0.1272	0.1272	0.411	0.133	1974	10.9x10^6
5	OEF	1	0.1272	0.1272	0.411	0.133	1974	10.9x10^6
SUM				0.6898				

Element	Edge	E_{xc}	E_{yc}	F_c	X	Y	F_{cci}	EA	EAF_{cci}
		psi	psi	psi			psi	lb	lb^2/in^2
1	OEF	14.6x10^6	4.73x10^6	68255	0.74332	0.52433	26669	2.5999x10^6	6.934x10^{10}
2	OEF	14.6x10^6	4.73x10^6	68255	0.74332	0.52433	26669	2.5999x10^6	6.934x10^{10}
3	NEF	7.05 x10^6	7.05 x10^6	78000	1.63569	0.60483	33983	0.5585x10^6	1.898x10^{10}
4	OEF	14.6x10^6	4.73x10^6	160000	0.81291	0.52433	58968	1.8571x10^6	10.95x10^{10}
5	OEF	14.6x10^6	4.73x10^6	160000	0.81291	0.52433	58968	1.8571x10^6	10.95x10^{10}
SUM								9.4727x10^6	3.7668x10^{11}

Calculate EA	$\Sigma(EA) = 9.4727\text{x}10^6$
Determine the crippling stress using Equation 6.6.3-3 Equation 6.5.6-1	$F_{cc} = \frac{\sum EAF_{cci}}{\sum EA}$ $= 3.7668\text{x}10^{11}/9.4727\text{x}10^6$ $= 39765$ psi
Determine effective stiffener length. Assume simple-simple boundary condition	$L' = \frac{L}{\sqrt{c}}$; $L'=10/(1)^{0.5} = 10$ in, c=1 for simple-simple boundary condition
Determine the radius of gyration (ρ), in	$\rho = [\Sigma\,(EI)_{min}/\,\Sigma\,(EA)]^{0.5}$ $\rho = [(4.1743\text{x}10^6)/\,9.4727\text{x}10^6]^{0.5}$ $= 0.6638$ in
Calculate E_{xc}	$E_{xc} = \frac{\sum(EA)_i}{\sum A_i}$ $=(9.4727\text{x}10^6)/0.6898$ $=13.732\text{x}10^6$
Determine the crippling allowable stress using Equation 6.6.1-5.	$F_{cl} = F_{cc} - \frac{F_{cc}^2}{4\pi^2 E_{xc}}\left(\frac{L'}{\rho}\right)^2$ $F_{cl} = 39765 - \frac{39765^2}{4\pi^2(13.732x10^6)}\left(\frac{10}{0.6638}\right)^2$ $F_{cl} = 39103$ psi
Determine the allowable crippling load	$P_{cl} = (F_{cl})(A) = (39103)(0.6898) = 26973$ lb
Determine the Margin of Safety (M.S.)	M.S. = $P_{cl}/P - 1 = 26973/20000 - 1 = +0.34$

6.6.3 Effective Width of Composite Skin

Most structural beam/beam-column members have a skin attached to them and therefore rarely do they carry the loads completely alone. These thin skin members by themselves are unable to provide adequate buckling strength but in combination with load carrying structural members such as spars, stringers etc. they do provide some additional ability to carry load in compression, which should be accounted for in deriving margins. Note that in tension the skins become fully effective in carrying the load. Also, if the skin is stable up to DUL in compression then it is fully effective in compression and the effective width is the stiffener spacing.

To calculate the effective skin width in compression it is assumed that the stiffener bending stiffness is sufficiently high to force buckling in the skin between the stiffeners first. Subsequent to the skin buckling any additional increase in load is taken up by the skin attached to the stiffeners, provided the stiffeners themselves do not buckle. The effective width of the skin that can take the additional compressive load is determined following the steps provided in References 6-7 and 6-8 and discussed below. Figure 6.6-3 illustrates the skin-stiffener geometry used in effective width calculations.

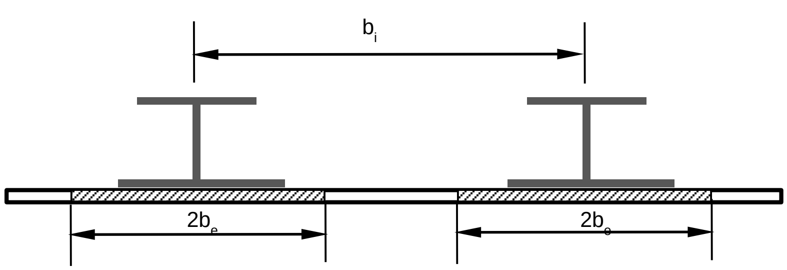


Figure 6.6-3 Definition of Effective Width

The effective width is determined by modifying the expression for effective width given in Reference 6-7 as

$$b_e = \frac{b_i}{2}\sqrt{\frac{F_{cr}}{F_{max}}}$$ **Equation 6.6.3-1**

where,
b_e = effective width, in (Figure 6.6-3)
b_i = space between the stiffeners, in (Figure 6.6-3)
F_{cr} = initial buckling stress of the skin between the stiffeners, psi
F_{max} = maximum stress acting on the effective area of the skin, psi

The maximum stress (F_{max}) acting on the effective area of the skin is the sum of the initial buckling stress and a portion of the stress in excess of the initial buckling stress.

$$F_{max} = F_{cr}\frac{b_i}{2b_e} + \frac{(P_x - P_{cr})}{A_{skin}}\frac{(AE)_{skin}}{(AE)_{skin} + (AE)_{stiff}}$$ **Equation 6.6.3-2**

where,
P_x = Load acting on the whole cross section, lb
P_{cr} = Initial bucking load of the skin, lb
F_{cr} = Initial buckling stress of the skin, psi
A_{skin} = Area of cross section of the skin, in^2
E_{skin} = Young's modulus of the skin, psi
A_{stiff} = Area of the cross section of the stiffener, in^2
E_{stiff} = Young's modulus of the stiffener, psi

Using Equation 6.6.3-1 in Equation 6.6.3-2 and simplifying b_e can be expressed as follows:

$$b_e = \frac{-C_2 \pm \sqrt{C_2^2 + 4C_1C_3}}{2C_1}$$ **Equation 6.6.3-3**

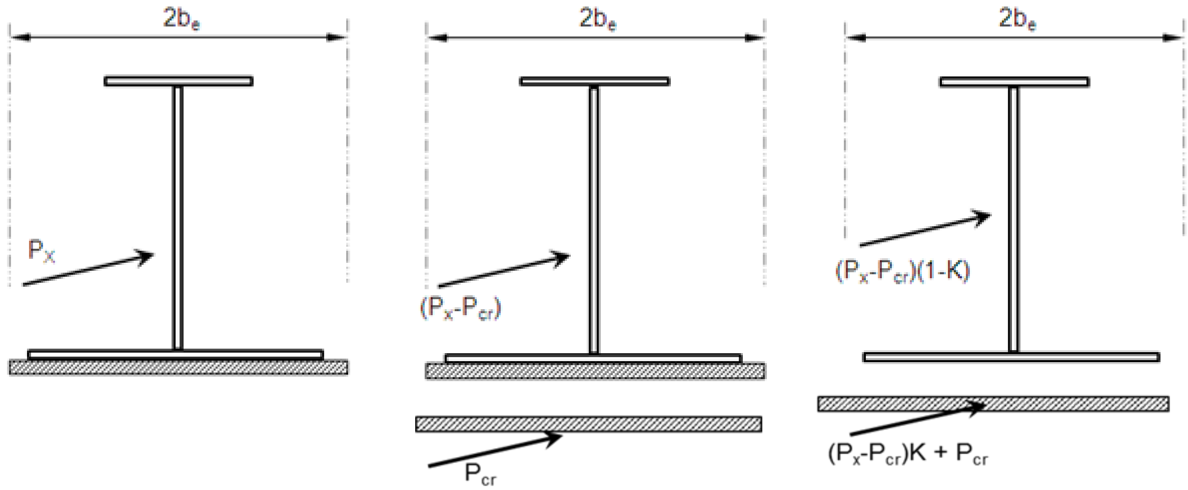
where,

$$C_1 = F_{cr}b_it_{skin}E_{skin} + (P_x - P_{cr})E_{skin}$$
$$C_2 = F_{cr}\frac{b_i}{2}(AE)_{stif} - \frac{b_i^2}{2}F_{cr}t_{skin}E_{skin}$$
$$C_3 = \frac{b_i^2}{4}F_{cr}(AE)_{stif}$$

In Equation 6.6.3-3 t_{skin} is the thickness of the skin. Since b_e should be always positive, choose the positive value for b_e.

The load P_x is distributed between the skin and stiffener as shown in Figure 6.6-4, where K is defined below. It is conservatively assumed that the skin contains the entire initial buckling load and the additional load in excess of the buckling load is shared by the skin in proportion to its (AE).

$$K = \frac{(AE)_{skin}}{(AE)_{skin} + (AE)_{stif}}$$ **Equation 6.6.3-4**



(a) Load P_x Applied to the Skin-Stiffener Combination.

(b) Load Distribution in Skin-Stiffener Combination and the Skin Shown Separately Just After the Skin Buckles. (P_x-P_{cr}) is the Load Acting on the Skin-Stiffener Combination.

(c) Load Distribution in Skin and Stiffener Shown Separately Just After the Skin Buckles. K is defined in Equation 6.6.3-4

Figure 6.6-4 Load Distribution in the Skin-Stiffener Combination After the Skin Buckles

Employing Equation 6.6.3-3 b_e can be determined iteratively as a function of P_x subject to the following constraints:

- the total load in the stiffener $(P_x - P_{cr})(1 - K)$, Figure 6.6-4(c), does not exceed the crippling strength of the stiffener
- the total load applied (P_x) does not exceed the crippling strength of the stiffener and the effective skin combination.

This method for determining b_e is explained in Example 6.6.4. Note that the effective width be can be smaller, greater than or equal to the flange width.

The skin width is conservatively assumed to extend from the center of one stiffener to the centers of the adjacent stiffeners. Simply supported boundary condition is assumed on all sides, however, if the skin is not stiffened at one of its ends, free boundary condition should be assumed along that edge and the buckling strength of the skin (F_{cr}) determined.

The stiffeners need not be equally spaced. If the stiffeners are not evenly spaced the effective width will be assumed as shown in Figure 6.6-5. If P_1 is the initial buckling strength (lb) of the skin of width b_1 and P_2 is the initial buckling strength (lb) of the skin of width b_2 then the initial buckling load acting on the effective skin width is $(P_1+P_2)/2$.

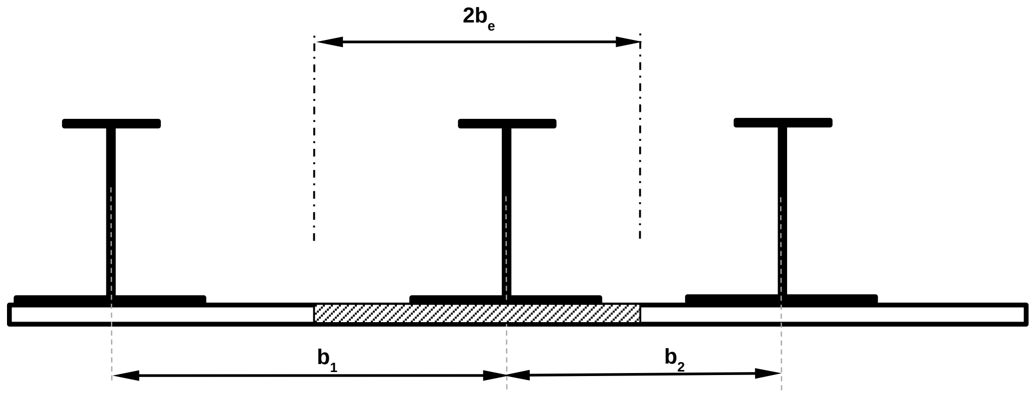


Figure 6.6-5 Effective width for an unequally spaced stiffener.

The methodology for calculating the effective width of skin is shown in Figure 6.6-6.

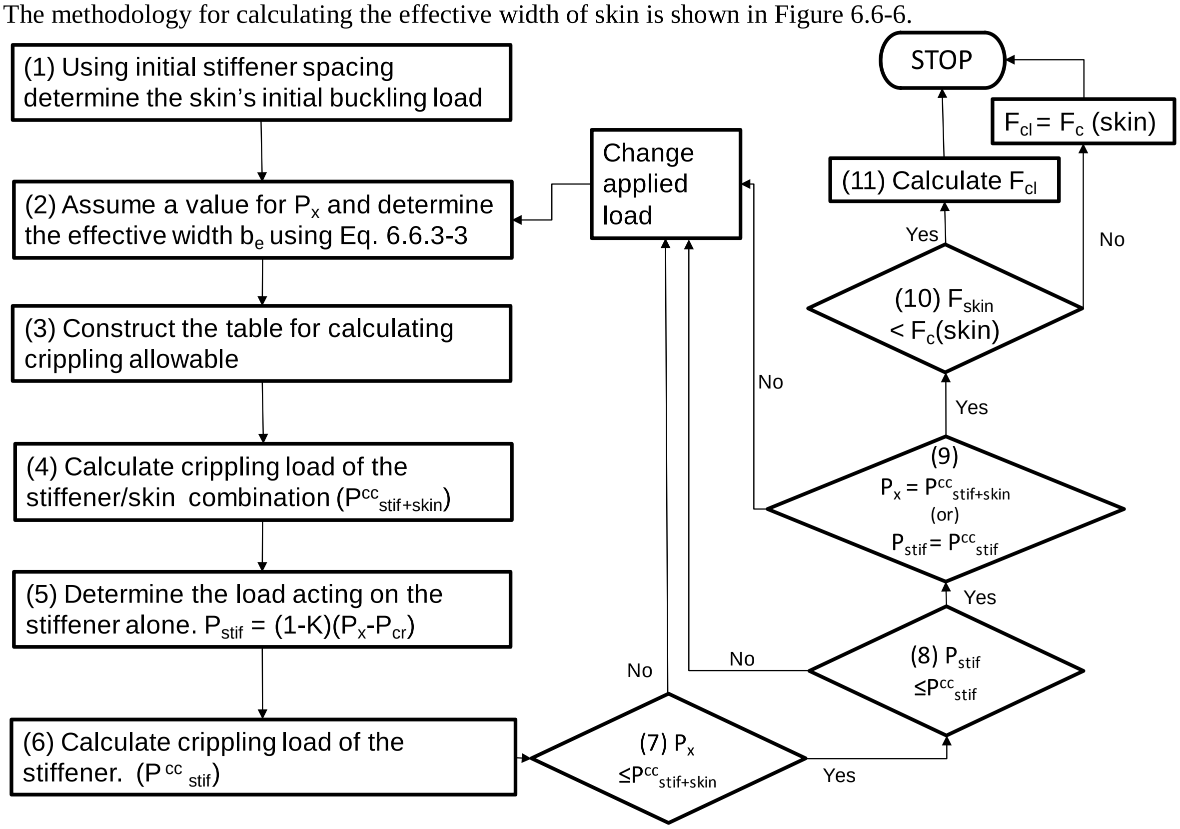


Figure 6.6-6 Method for Calculating Effective Width

These steps are demonstrated in the Example Problem 6.6.4.

6.6.4 Example Problem – Effective Width Calculation

Determine the effective width for the skin/stiffener combination shown on the right hand side. The section geometry is given in Example Problem 6.6.2. The stiffeners are spaced equally as shown.

Assumptions:

- End fixity of pinned-pinned
- Ambient temperature conditions.
- Ignore initial flange buckling for this problem.

The material and geometrical properties are repeated here for convenience.

Material: IM7-5250-4 tape

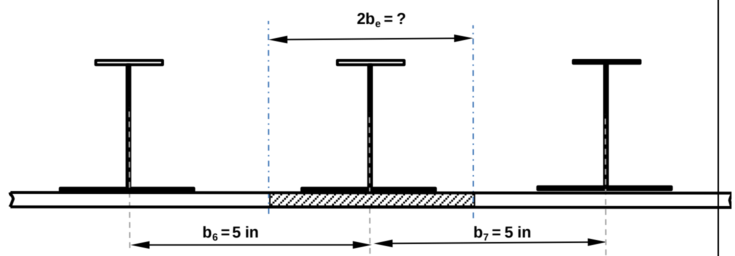
Flange Elements (1,2,4,5): laminate ply percentage: [58.3/33.4/8.3], laminate thickness = 0.1272 in.

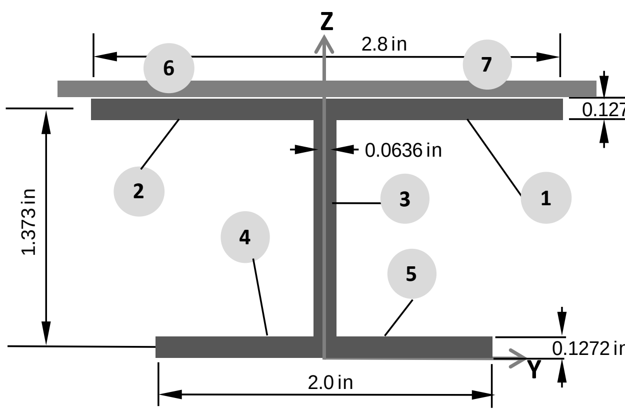
Web Elements (3): laminate ply percentage: [16.7/66.6/16.7], laminate thickness = 0.0636 in.

Skin Elements (6,7) Outer plies – IM7-5250-4-FAB-CSW, inner plies –IM7-5250-4 tape$[45/-45/0/45/90/0/45/90/-45]_s$, Laminate thickness = 0.1014 in. Skin modulus = $E_{skin} = 7.8 \times 10^6$ psi.

Anticipating future repair using a 0.25 in dia. Titanium protruding head fastener use F_c = FHC = 68255 psi for elements 1, 2 and the skin.

For all other elements use their respective unnotched compression allowable for F_c as shown in the Table below.





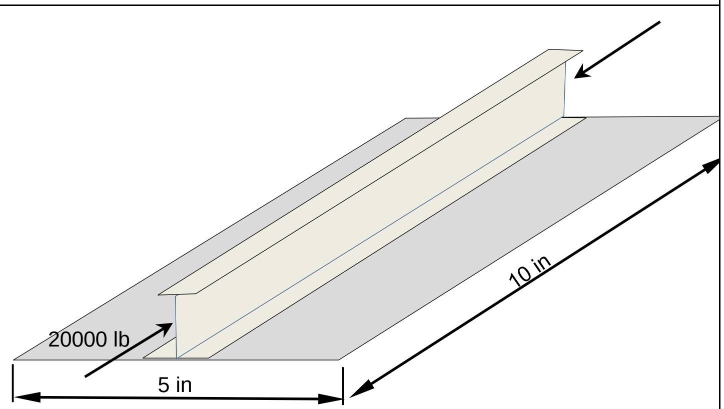
Skin (6,7) is shown detached from flange (1,2) for clarity

Length of the section is 10 inches. Both flange and web laminates are symmetric and balanced.

Total depth (H) = 1.5 in.

The initial buckling strength for the skin between the stiffeners is 1365 lb/in assuming simply supported boundary conditions on all sides. The laminate D_{11} properties, which are as given in the table below, are calculated using the lamina properties as described in Section 4 or using IDAT LAMINATE tool.

Applied axial compressive load to the skin/stiffener combination = 20000 lb.



Step 1 Skin Initial Buckling Load:

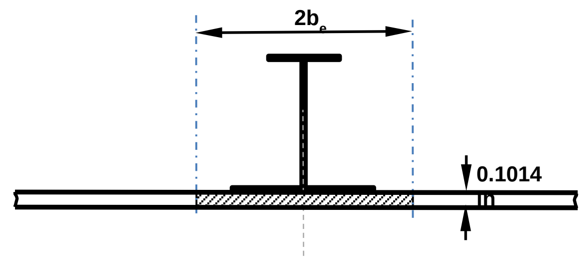
N_{x1cr}, N_{x2cr} = 1365 lb/in. where N_{x1cr} and N_{x2cr} are the critical buckling load of the skin on either side of the stiffener.

The critical buckling stress of the skin attached to the stiffener and stretched equally on either side of the stiffener is determined as

F_{cr} = $[(N_{x1cr})(b_6)/2+(N_{x2cr})(b_7)/2]/[(b_6/2+b_7/2)t_{skin}]$= $[(1365)(5)/2+(1365)(5)/2]/[(5/2+5/2)(0.1014)]$
= 13462 psi

b_6 and b_7 are shown above. Since the stiffeners are evenly spaced $b_6 = b_7 = 5$ inches.

The critical buckling load
P_{cr} = (13462)(5)(0.1014) = 6825 lb



Step 2 Assume P_x and determine the effective width b_e using Equation 6.6.3-3.

Assume the initial load (P_x) = 26973 lb. This value is chosen to be equal or greater than the stiffener crippling stress.

b_i=5 in, $E_{skin} = 7.8x10^6$, $\Sigma(EA)_{stif} = 9.5x10^6$ lb (From Example Problem 6.6.2).

$C_1 = F_{cr} b_i t_{skin} E_{skin} + (P_x - P_{cr}) E_{skin}$
$= (13462)(5)(0.1014)(7.8x10^6) + (26973 – 6825)(7.8x10^6)$
$= 2.1x10^{11}$

$C_2 = F_{cr} \frac{b_i}{2} (AE)_{stif} - \frac{b_i^2}{2} F_{cr} t_{skin} E_{skin}$
$= (13462)(5/2)(9.5x10^6)-(5^2/2)(13462)(0.1014)(7.8x10^6)$
$= 1.86x10^{11}$

$C_3 = \frac{b_i^2}{4} F_{cr} (AE)_{stif}$
$= (5^2/4)(13462)(9.5x10^6)$
$=7.97x10^{11}$

$2b_e$ = 3.11 in

Step 3
Determine the critical crippling load for the skin/stiffener combination ($P^{cc}_{skin+stif}$) by adding the skin elements 6 and 7 to the crippling stress allowable calculation from Example 6.6.2.

Table 6.6-3 Table for Calculating Crippling Stress Allowable for Skin/Stiffener at Initial Load

Element	Edge	b	t	A	ν_{xy}	ν_{yx}	D_{11}	$\bar{E}_x$
		in	in	in^2			lb/in	psi
1	OEF	1.4	0.1272	0.1781	0.411	0.133	1974	$10.9x10^6$
2	OEF	1.4	0.1272	0.1781	0.411	0.133	1974	$10.9x10^6$
3	NEF	1.373	0.0636	0.0792	0.411	0.411	131	$5.1x10^6$
4	OEF	1	0.1272	0.1272	0.411	0.133	1974	$10.9x10^6$
5	OEF	1	0.1272	0.1272	0.411	0.133	1974	$10.9x10^6$
6	NEF	1.554	0.1014	0.1576	0.411	0.411	788	$7.54x10^6$
7	NEF	1.554	0.1014	0.1576	0.411	0.411	788	$7.54x10^6$
SUM				1.0049				

Element	E_{xc}	E_{yc}	F_c	X	Y	F_{cci}	EA	EAF_{cci}	**$(EA)/(EA)_{tot}$**	**P_x-P_{cr}**	**P_i**
	psi	psi	psi			psi	lb	lb^2/in^2		lb	lb
1	$14.6x10^6$	$4.73x10^6$	68255	0.74332	0.52433	26669	$2.6x10^6$	$6.934x10^{10}$	0.2179	20148	4391
2	$14.6x10^6$	$4.73x10^6$	68255	0.74332	0.52433	26669	$2.6x10^6$	$6.934x10^{10}$	0.2179	20148	4391
3	$7.1 x10^6$	$7.1 x10^6$	78000	1.63569	0.60484	33984	$5.6x10^5$	$1.898x10^{10}$	0.0468	20148	943
4	$14.6x10^6$	$4.73x10^6$	160000	0.81291	0.49457	58968	$1.9x10^6$	$10.95x10^{10}$	0.1557	20148	3136
5	$14.6x10^6$	$4.73x10^6$	160000	0.81291	0.49457	58968	$1.9x10^6$	$10.95x10^{10}$	0.1565	20148	3136
6	$7.8 x10^6$	$7.8 x10^6$	68255	1.3854	0.70962	46806	$1.2 x10^6$	$5.747x10^{10}$	0.103	20148	5488
7	$7.8 x10^6$	$7.8 x10^6$	68255	1.3854	0.70962	46806	$1.2 x10^6$	$5.747x10^{10}$	0.103	20148	5488
SUM							$11.931 x10^6$	$5.783x10^{11}$	1.0		26973

In the above table elements 6 and 7 are the effective skin attached to the stiffener. Since the effective skin is bounded by the skins on either side a no-edge-free (NEF) condition is assumed for crippling calculations.	
Step 4 The crippling load for the skin/stiffener combination ($P^{cc}_{skin+stif}$) is calculated as follows:	
$P^{cc}_{skin+stif} = \frac{\sum(EAF_{cci})}{\sum(EA)} \sum A$ $P^{cc}_{skin+stif} = \frac{4.9174x10^{11}}{11.931x10^{6}}(1.0049)$ = (41216)(1.0049) = 41418lb	
Step 5 Determine the load acting on the stiffener. P_i is the load in each element of the stiffener.	For element 1: $P_1 = (EA)_1/(EA)_{tot}\ (P_x-P_{cr}) = (2.6x10^6/11.9x10^6)(26973-6825)$ = (0.2179)(20148) = 4391 lb $P_{stif} = \sum_{i=1}^{5} P_i = (4391 + 4391 + 943 + 3136 + 3136)$ = 15997 lb
Step 6 Determine the critical crippling load for the stiffener (see Example Problem 6.6.2)	$P^{cc}_{stif} = 26973\ lb$
Determine the total load acting on the skin/stiffener combination and check that it sums up to the total applied load.	$P_x = P_{skin+stif} = \sum_{i=1}^{7} P_i$ $= (4391 + 4391 + 943 + 3136 + 3136 + 5488 + 5488)$ = 26973 lb The total load in Elements 6 and 7 are determined as: $P_{6,7} = (EA)_1/(EA)_{tot}\ (P_x-P_{cr}) + P_{cr}$, where P_{cr} is the initial skin buckling load (lb). $P_{6,7}$ = (0.103)(20148) + (1365)(5)/2 = 5488 lb
Step 7 Determine $(P^{cc}_{skin+stif} - P_x)$	$(P^{cc}_{skin+stif} - P_x) = 41418 - 26973$ = 14445 lb
Step 8 Check if $P_{stif} \leq P^{cc}_{stif}$	$P_{stif}(= 15997\ lb) < P^{cc}_{stif}\ (= 26973\ lb)$
Step 9 Check if $P_{stif} = P^{cc}_{stif}$ or $P_x = P^{cc}_{stif+skin}$	From Steps 7 and 8 $P_x \neq P^{cc}_{stif+skin}$ and $P_{stif} \neq P^{cc}_{stif}$
Since $P_x \neq P^{cc}_{stif+skin}$ and $P_{stif} \neq P^{cc}_{stif}$ increase P_x and repeat analysis from Step 1 until $P_{stif} = P^{cc}_{stif}$ or $P_x = P^{cc}_{stif+skin}$ Increase P_x to 39782 lb	
Step 2 Determine b_e again using the load P_x = 39782 lb.	$2b_e$ = 1.33 + 1.33 = 2.66 inches

Step 3
The loads in the element at this applied load are shown in the following Table.

Table 6.6-4 Table for Calculating Crippling Stress Allowable for Skin/Stiffener at Final Load

Element	Edge	b	t	A	ν_{xy}	ν_{yx}	D_{11}	$\bar{E}_x$
		in	in	in^2			lb/in	psi
1	OEF	1.4	0.1272	0.1781	0.411	0.133	1974	10.9x106
2	OEF	1.4	0.1272	0.1781	0.411	0.133	1974	10.9x106
3	NEF	1.373	0.0636	0.0792	0.411	0.411	131	$5.1 x 10^6$
4	OEF	1.0	0.1272	0.1272	0.411	0.133	1974	$10.9 x 10^6$
5	OEF	1.0	0.1272	0.1272	0.411	0.133	1974	$10.9 x 10^6$
6	NEF	1.331	0.1014	0.1350	0.411	0.411	788	$7.54 x 10^6$
7	NEF	1.331	0.1014	0.1350	0.411	0.411	788	$7.54 x 10^6$
SUM				0.9597				

Element	E_{xc}	E_{yc}	F_c	X	Y	F_{cci}	EA	EAF_{cci}	**$(EA)/(EA)_{tot}$**	**P_{app}-P_{cr}**	**P_i**
	psi	psi	psi			psi	lb	lb^2/in^2		lb	lb
1	$14.6 x 10^6$	$4.73 x 10^6$	68255	0.74332	0.52433	26669	$2.60 x 10^6$	$6.934 x 10^{10}$	0.2246	32967	7403
2	$14.6 x 10^6$	$4.73 x 10^6$	68255	0.74332	0.52433	26669	$2.60 x 10^6$	$6.934 x 10^{10}$	0.2246	32967	7403
3	$7.1 x 10^6$	$7.1 x 10^6$	78000	1.63569	0.60484	33984	$5.59 x 10^5$	$1.898 x 10^{10}$	0.0482	32967	1591
4	$14.6 x 10^6$	$4.73 x 10^6$	160000	0.81291	0.49457	58968	$1.86 x 10^6$	$1.095 x 10^{11}$	0.1604	32967	5288
5	$14.6 x 10^6$	$4.73 x 10^6$	160000	0.81291	0.49457	58968	$1.86 x 10^6$	$1.095 x 10^{11}$	0.1604	32967	5288
6	$7.8 x 10^6$	$7.8 x 10^6$	68255	1.18659	0.82365	54327	$1.05 x 10^6$	$5.719 x 10^{10}$	0.0909	32967	6410
7	$7.8 x 10^6$	$7.8 x 10^6$	68255	1.18659	0.82365	54327	$1.05 x 10^6$	$5.719 x 10^{10}$	0.0909	32967	6410
SUM							$1.1578 x 10^7$	$4.9107 x 10^{11}$	1.0		39792

Determine the crippling stress (psi) for the skin/stiffener combination.

$$F^{cc}_{skin+stif} = \frac{\sum(EAF_{cci})}{\sum(EA)}$$

$$F^{cc}_{skin+stif} = \frac{4.9107x10^{11}}{11.57810^6}$$

=42413 psi

Step 4
Determine the crippling load for the skin/stiffener combination ($P^{cr}_{skin/stif}$)

$$P^{cc}_{skin+stif} = F^{cc}_{skin/stif} \sum A$$

= (42413)(0.9597)
=40704 lb

Step 5
Determine load acting in the stiffener alone

$$P_{stif} = 7403 + 7403 + 1591 + 5288 + 5288$$

= 26973 lb

Step 7
Check if $P_x \leq P^{cc}_{stif+skin}$

$$P_x\,(= 39792\ lb) < P^{cc}_{stif+skin}(= 40704\ lb)$$

Step 8
Check if $P_{stif} \leq P^{cc}_{stif}$

$$P_{stif}\,(= 26973\ lb) = P^{cc}_{stif}(= 26973).$$

Step 9
Since $P_{stif} = P^{cc}_{stif}$ stop the iteration.

Calculate load in the skin

$P_{skin} = (P_6 + P_7)$
= (6410+6410)
= 12820 psi

Step 10
Calculate Margin of Safety for the skin

Skin FHC allowable = 68255 psi
P_{cu} = (68255)(0.1350)(2)
= 18429 lb
$M.S._{skin} = P_{cu}/P_{skin} - 1$
= 18429/12820 -1
= +0.44

Determine the crippling allowable after including the length correction using Equation 6.5.1-1. A table is constructed as shown in Example Problem 6.5.3 to determine the section properties with the effective width of the skin included. The minimum modulus weighted moment of inertia of the section with the skin included still occurs about the axis parallel to the Y axis of the section.

Element	E_{xi} (lb/in^2)	t_i (in)	w_i (in)	A_i (in^2)	$\overline{z_{ci}}$ (in)	$E_{xi}A_i$ (lb)	$E_{xi}A_iz_{ci}$ (in-lb)
1,2	1.46x10^7	0.1272	2.8	0.3562	1.4364	5.20x10^6	7.47x10^6
3	7.05x10^6	0.0636	1.25	0.0792	0.75	5.58x10^5	4.19x10^5
4,5	1.46x10^7	0.1272	2.0	0.2544	0.0636	3.71x10^6	2.36x10^5
6	7.80x10^6	0.1014	1.33	0.1350	1.5507	1.053x10^6	1.63x10^6
7	7.80x10^6	0.1014	1.33	0.1350	1.5507	1.053x10^6	1.63x10^6
Sum				0.9597		1.1578x10^7	1.14x10^7

Calculate the Modulus Weighted Centroid ($\overline{z}_c$) $\overline{z}_c = \sum E_{xi}A_iz_{ci} / \sum E_{xi}A_i = 0.9837$ in

Calculate Modulus Weighted Moment of Inertia

Element	I_i(in^4)	$d_i = \lvert \overline{z}_c - \overline{z}_{ci} \rvert$	$EI_{yy} = E_{xi}I_i + E_{xi}A_id_i^2$
1	0.0004802	0.4527	1.07x10^6
2	0.0102426	0.2337	1.03x10^5
3	0.000343	0.9201	3.15x10^6
4	0.0001156	0.5670	3.39x10^5
5	0.0001156	0.5670	3.39x10^5
Sum			5.0035x10^6

Modulus weighted moment of inertia = 5.0035x10^6 lb-in^2

Calculate the radius of gyration	$\rho = \sqrt{\frac{(EI)_{min}}{(EA)}} = \sqrt{\frac{5.0035\text{x}10^6}{1.1578\text{x}10^7}}$ = 0.6574 in
Calculate L'	$L' = L/\sqrt{c} = 10/\sqrt{1} = 10$ inches
Calculate E_{xc}	$E_{xc} = \frac{\sum(EA)_i}{\sum A_i}$ =(1.1578x10^7)/(0.9597) =12.064x10^6
Step 10 Determine the crippling allowable stress using Equation 6.6.1-5.	$F_{cl} = F^{cc}_{skin/stif} - \frac{(F^{cc}_{skin/stif})^2}{4\pi^2 E_{xc}}\left(\frac{L'}{\rho}\right)^2$ $F_{cl} = 42413 - \frac{42413^2}{4\pi^2(12.064x10^6)}\left(\frac{10}{0.6574}\right)^2$ F_{cl} = 41539 psi
Determine the allowable crippling load	$P_{cl} = (F_{cl})(A) = (41539)(0.9597) = 39865$ lb
Determine the Margin of Safety (M.S.)	M.S. = $P_{cl}/P_{app} - 1 = 39865/20000 - 1 = 0.99$
Calculate the post-buckling (PB) ratio of the skin	$PB\ ratio = \frac{P^{cc}_{skin+stif}}{P_{cr}}$ = 40704/ 6825 = 6 This implies that if the skin is allowed to buckle locally between the stiffeners and the stiffener is allowed to take additional load then the skin/stiffener combination can resist six times the initial buckling load of the skin.

The above method for calculating effective width is valid when the stiffener is bonded or bonded and fastened to the skin. If the skin is fastened to the stiffener flange then inter-rivet buckling check, Section 6.6.7, must also be performed.

6.6.5 Crippling Under Combined Axial and Bending

Bending loads can cause certain portions of the beam/column section to be in compression. In such situations the region that is in compression should be checked for crippling. Crippling check is performed by first determining an equivalent compressive stress acting on the compressive portion of the cross section as shown below.

$$\sigma_{ceq} = \frac{2(\sigma_{cmax} - \sigma_{zc})}{3} + \sigma_{zc}$$ **Equation 6.6.5-1**

where,

σ_{zc} is the stress at the modulus weighted centroid of the cross section, psi
σ_{cmax} is the maximum compressive stress, psi

For a rectangular cross section σ_{zc} is defined as

$$\sigma_{zc} = \frac{(\sigma_{cmax} + \sigma_{cmin})}{2}$$ **Equation 6.6.5-2**

where,

σ_{cmin} is the minimum compressive stress, psi

The maximum, minimum and equivalent compressive stresses are shown in Figure 6.6-7.

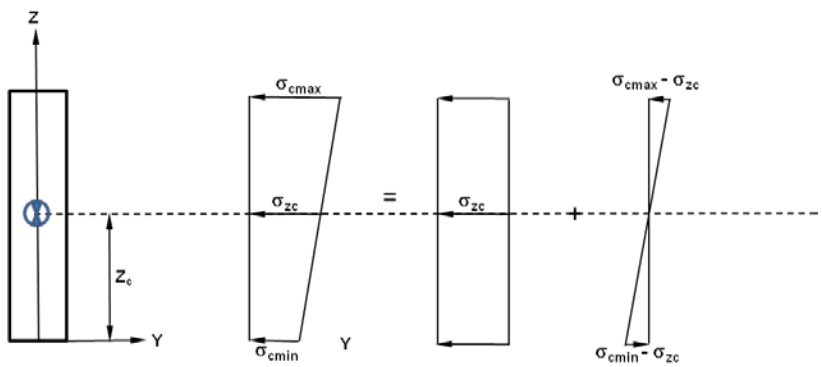
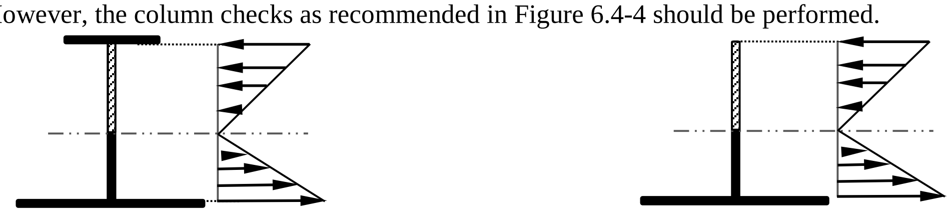


Figure 6.6-7 Section Subjected to Combined Axial and Bending Loads

The equivalent compressive stress is assumed to act uniformly over the compressive region of the cross section and the crippling margin of safety is determined.

The segment that is included for the crippling check can have either the no edge free (NEF) or the one edge free (OEF) boundary condition as shown shaded in Figure 6.6-8. Only that portion of the web length that is in compression should be used for crippling calculations and for determining the applied load.

The length correction factor need not be applied for the crippling calculations where portions of the cross section are in tension. However, the column checks as recommended in Figure 6.4-4 should be performed.



(a) NEF (web) (b) OEF

Figure 6.6-8 NEF and OEF Condition for Crippling Under Bending

The loads acting on the cross-sections can be extracted from a finite element model subsequent to performing a nonlinear finite element analysis.

6.6.5.1 Example Problem – Crippling Under Bending

Determine the crippling Margin of Safety for the section given in Example Problem 6.5.3.

Assume:

- End fixity: 1) Pinned-Pinned.
- Ambient temperature conditions
- Ignore initial flange buckling for this problem.

The material and geometrical properties are repeated here for convenience.

Material: IM7-5250-4 tape

Flange Elements (1,2,4,5): laminate ply percentage: [58.3/33.4/8.3], laminate thickness = 0.1272 in.

Web Elements (3) laminate ply percentage: [16.7/66.6/16.7], laminate thickness = 0.0636 in.

Applied axial compressive load (P) = 20000 lb. Applied moment (M) = -1000 in-lb, which results in compression in elements 3, 4 and 5. The stress distribution due to the applied moment and axial load in each of the element is provided in the table. Since the crippling occurs in bending where portions of the element are in tension no length correction factor is applied. The beam-column is stable and does not undergo Euler buckling.

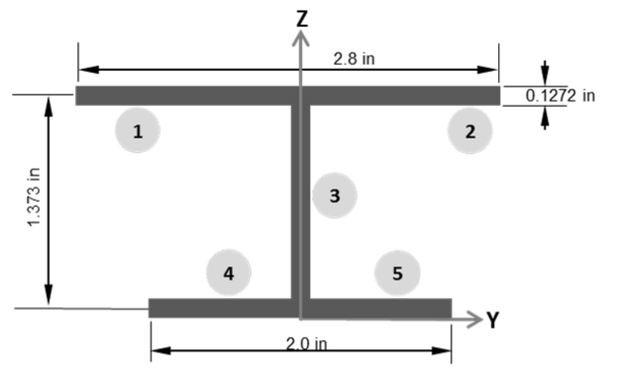
$(EI)_{min}$ (modulus weighted minimum moment of Inertia) is given as 4.1743×10^6 in^4.

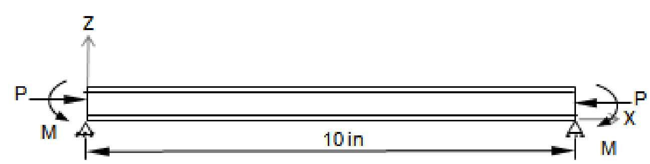
Both flange and web laminates are symmetric and balanced.

Total depth (H) = 1.5 in.

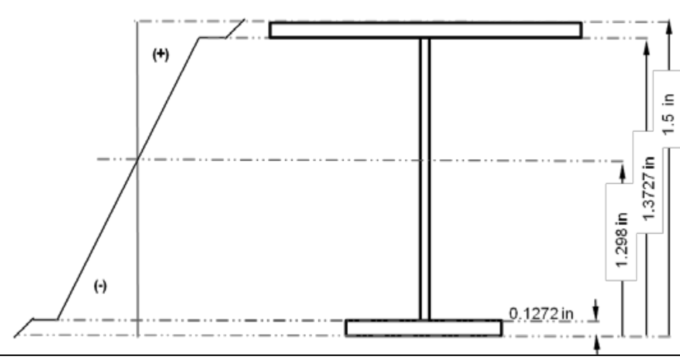
Anticipating future repair using a 0.1875 in dia Titanium protruding head fastener use F_c = FHC = 73830 psi for elements 4 & 5.

D_{11} can be calculated using the lamina properties as described in Section 4 or using IDAT LAMINATE tool.





Elm #	z(in)	σ_x(psi)
4,5	0	-45411
4,5	0.1272	-40963
3	0.1272	-19780
3	1.2984	0
3	1.3727	1257
1,2	1.3727	2603
1,2	1.5	7052



Calculate the average applied compressive stresses in element 3, 4 and 5. This is required for determining the margins of safety.	Using Equation 6.6.5-2 $$\sigma_{zc}^{elm\ 4,5} = \frac{-45411 - 40963}{2}$$ = -43187 psi Using Equation 6.6.5-1 $$\sigma_{ceq}^{elm\ 4,5} = \frac{2(-45411 - (-43187))}{3} + (-43187)$$ = -44670 psi Similarly $$\sigma_{zc}^{elm\ 3} = \frac{-19780 - 0}{2}$$ = -9890 psi $$\sigma_{ceq}^{elm\ 3} = \frac{2(-19780 - (-9890))}{3} + (-9890)$$ = -16483 psi
Determine the length of the web (element 3) that is in compression.	The compressive stress in element 3 acts over a segment of length = 1.2984 – 0.1272 = 1.1712 inches

Calculate the crippling allowable

Element	Edge	b	t	A	ν_{xy}	ν_{yx}	D_{11}	$\bar{E}_x$
		in	in	in^2			lb/in	psi
3	NEF	1.1712	0.0636	0.0745	0.411	0.411	131	$5.1x10^6$
4	OEF	1	0.1272	0.1272	0.411	0.133	1974	$10.9x10^6$
5	OEF	1	0.1272	0.1272	0.411	0.133	1974	$10.9x10^6$
SUM				0.3289				

Element	Edge	E_{xc}	E_{yc}	F_c	X	Y	F_{cci}	EA	EAF_{cci}
		psi	psi	psi			psi	lb	lb^2/in^2
3	NEF	$7.1\ x10^6$	$7.1\ x10^6$	78000	1.39524	0.7048	39600	$5.3x10^5$	$2.08x10^{10}$
4	OEF	$14.6x10^6$	$4.73x10^6$	73830	0.55220	0.63664	35027	$1.9x10^6$	$6.505x10^{10}$
5	OEF	$14.6x10^6$	$4.73x10^6$	73830	0.55220	0.63664	35027	$1.9x10^6$	$6.505x10^{10}$
SUM								$4.2394x10^6$	$1.5089x10^{11}$

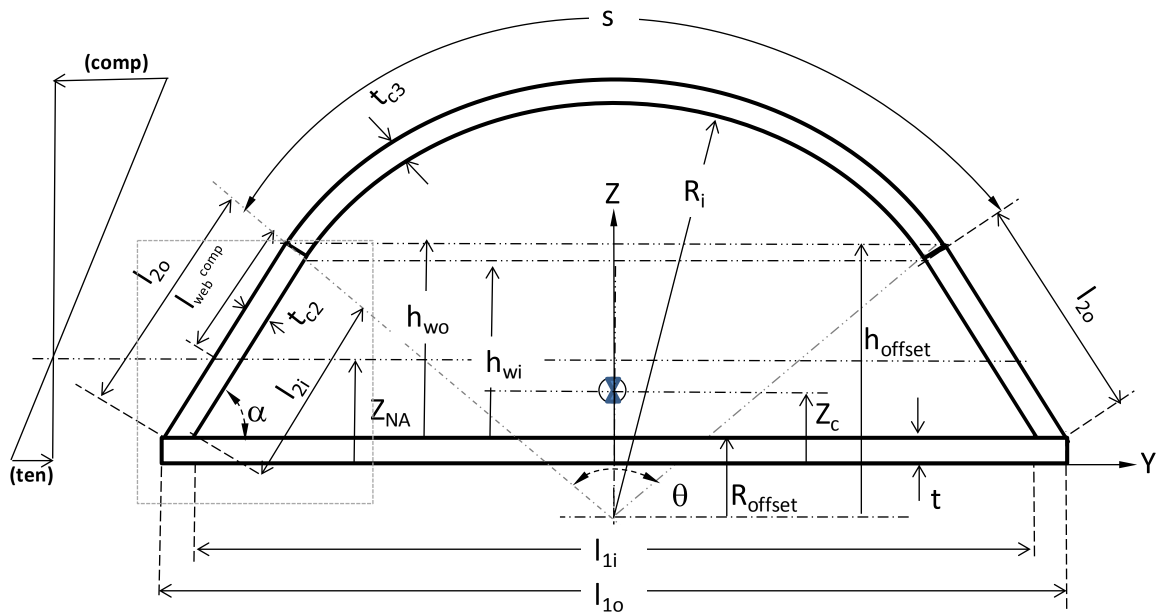
Determine the crippling stress (psi) using Equation 6.5.6-1.	$$F_{cc} = \frac{\sum(EAF_{cci})}{\sum(EA)}$$ $$F_{cc} = \frac{1.5089x10^{11}}{4.2394x10^6}$$ = 35592 psi
Determine the crippling load allowable for the section in bending. A is that portion of the area of the stiffener cross section that is in compression.	$$P_{cc} = F_{cc}A$$ P_{cc} = (35592)(0.3289) = 11706 lb
Calculate the average compressive load applied to the section in bending (P_{app}^{bend}) A_3 = Area of element 3 that is in compression, in^2	$$P_{app}^{bend} = \sigma_{ceq}^{elm\ 3}A_3 + \sigma_{ceq}^{elm\ 4,5}A_{4,5}$$ = (-16483)(0.0745)+(-44670)(0.2544) = -12592 lb

$A_{4,5}$ = Area of element 4 and 5 in^2.	
Calculate the Margin of Safety for crippling under bending loads	M.S. = $\frac{P_{cl}}{P_{app}^{bend}} - 1$ =11706/12592 – 1 =-0.07

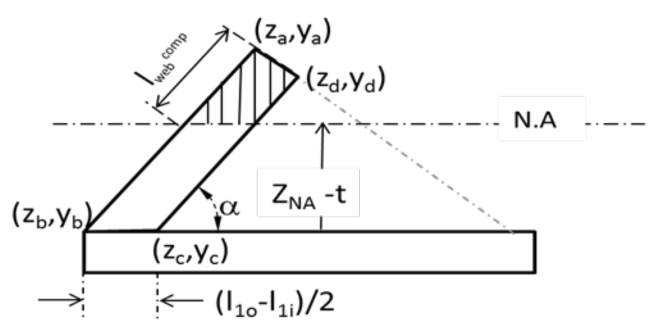
6.6.6 Crippling Analysis of a Bead in Bending

Crippling analysis of the semi-circular cross section in bending is obtained by modifying the method for crippling analysis of metals with a circular cross section under uniform compression (Reference 6-12). The analysis discussed below should be used only for preliminary design until crippling test data is available for curved sections.

The cross section of a bead in bending is shown in Figure 6.6-9 .



(a)



(b)

Figure 6.6-9 Geometry of the Bead

In Figure 6.6-9 (a), Z_c defines the location of the geometric centroid of the whole section and $Z_{NA}(\geq 0)$ defines the neutral axis of the section under combined axial and bending load. The portion of the web enclosed in a box is shown magnified inFigure 6.6-9b. Due to the bending loads a portion of the cross section is under compression which is shown shaded in Figure 6.6-9 b. If the quadrilateral that defines the web is defined in terms of the coordinates as shown in Figure 6.6-9 b, the lengths l_{2o} and l_{2i} are determined as

$$l_{2o} = \sqrt{(z_a - z_b)^2 + (y_a - y_b)^2}$$ **Equation 6.6.6-1**

$$l_{2i} = \sqrt{(z_d - z_c)^2 + (y_d - y_c)^2}$$ **Equation 6.6.6-2**

If the coordinates are not given l_{2o} and l_{2i} are determined as follows:

$$l_{2o} = \frac{h_{wo}}{\sin(\propto)}$$ **Equation 6.6.6-3**

$$l_{2i} = \frac{h_{wi}}{\sin(\propto)}$$ **Equation 6.6.6-4**

where,

$$h_{wo} = R_o \cos(\frac{\theta}{2})$$ **Equation 6.6.6-5**

$$h_{wi} = R_i \cos(\frac{\theta}{2})$$ **Equation 6.6.6-6**

s = outer arc length of the cap, in
R_o = outer radius of the bead cap, in
R_i = inner radius of the bead cap, in
θ = s/R_o, radians
α = $\theta/2$

If R_{offset} and h_{offset}, shown in Figure 6.6-9a, are known then h_{wo} can also be expressed as

$$h_{wo} = h_{offset} - R_{offset}$$ **Equation 6.6.6-7**

where,
h_{offset} =distance from origin to the top of the quadrilateral that encloses the web, in
R_{offset} =distance from the origin to the inner skin, in

The length l_{1o} and l_{1i} are defined as

$$l_{1o} = 2R_o \sin\left(\frac{\theta}{2}\right) + 2l_{2o}\cos(\propto)$$ **Equation 6.6.6-8**

$$l_{1i} = l_{1o} - 2\left(\frac{t_{c2}}{\sin(\propto)}\right)$$ **Equation 6.6.6-9**

The area of the web is defined as

$$A_{web} = t_{c2}\frac{(l_{2i} + l_{2o})}{2}$$ **Equation 6.6.6-10**

The area of the web in compression (shaded part in Figure 6.6-9 b) is determined as

$$A_{web-comp} = A_{web} - \frac{(l_{1o} - l_{1i})}{2}(Z_{NA} - t)$$ **Equation 6.6.6-11**

For crippling analysis in bending applications only the portion of the section that is in compression, which is determined by the location of Z_{NA}, has to be analyzed.

The curved portion of the bead is approximated as its chord for the purpose of crippling analysis as shown Figure 6.6-10.

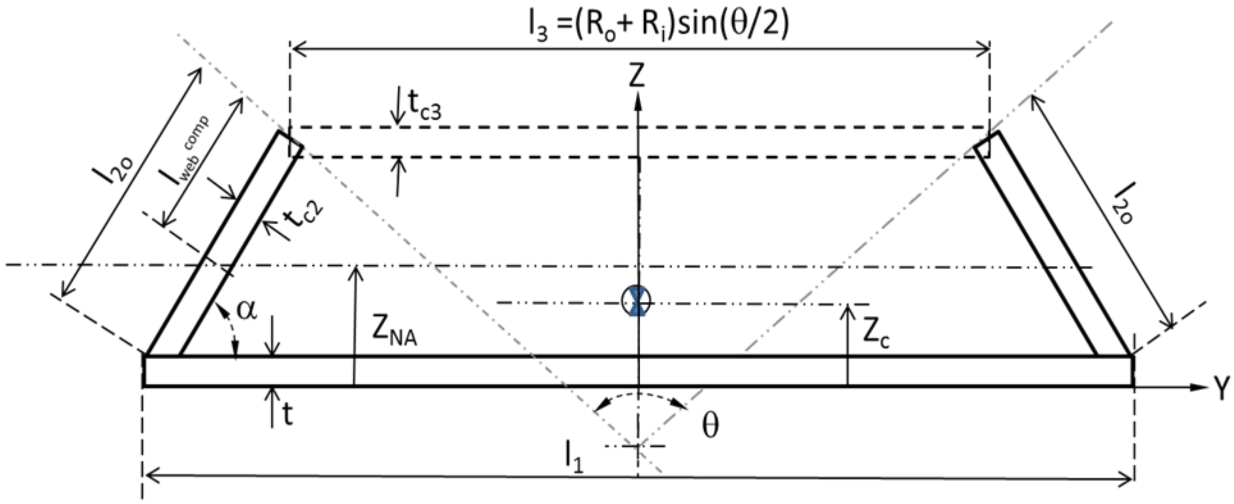


Figure 6.6-10 Geometry of the Bead Showing the Segmentation of the Curved Portion

The length of chord that is used in the crippling analysis is defined using trigonometric relations as follows:

$$\mathrm{l}_3 = (\mathrm{R_o} + \mathrm{R_i})\sin\left(\frac{\theta}{2}\right) \qquad \textbf{Equation 6.6.6-12}$$

The area of each segment can be expressed as

$$\mathrm{A_{segment}} = \frac{\theta \mathrm{t_{c3}}}{2}(2\mathrm{R_o} - \mathrm{t_{c3}}) \qquad \textbf{Equation 6.6.6-13}$$

The length of the web portion of the bead that is in compression is determined as:

$$\mathrm{l_{web}^{comp}} = 0.5\left[\frac{\mathrm{l_{2i}}\sin(\propto) - \mathrm{Z_{NA}} + \mathrm{t}}{\sin(\propto)} + \frac{\mathrm{l_{2o}}\sin(\propto) - \mathrm{Z_{NA}} + \mathrm{t}}{\sin(\alpha)}\right] \qquad \textbf{Equation 6.6.6-14}$$

If the flange portion has to be included in the crippling analysis the length of the flange that has to be used in the crippling analysis is defined as:

$$l_1^{crip} = \frac{l_{1o} + l_{1i}}{2} \qquad \textbf{Equation 6.6.6-15}$$

All the parameters used in the above equation are defined in Figure 6.6-9 . The crippling allowable for the whole section can then be determined as discussed before. Note that ‘X’ defined by Equation 6.6.1-3 can be re-written for curved cross section as shown below.

$$X = \frac{(\mathrm{R_o} + \mathrm{R_i})\sin\left(\frac{\theta}{2}\right)\overline{E_x}}{tE_{xc}}\sqrt{\frac{F_c}{\sqrt{E_{xc}E_{yc}}}} \qquad \textbf{Equation 6.6.6-16}$$

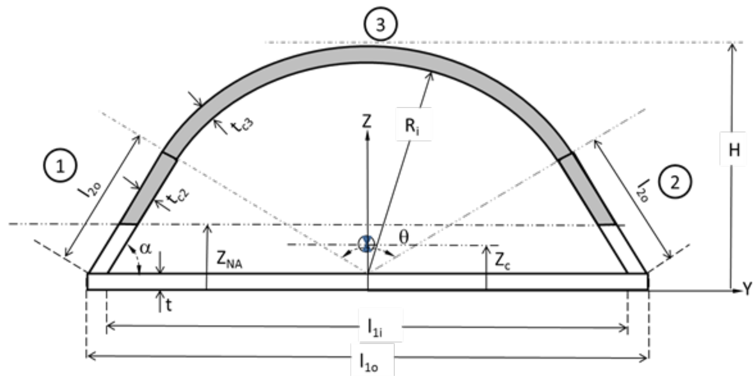
where ‘b’ in Equation 6.6.1-3 is replaced with l_3 given by Equation 6.6.6-12.

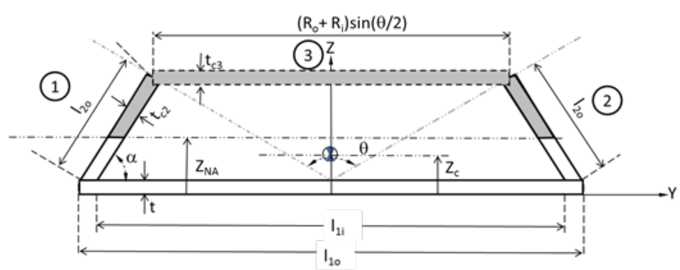
6.6.6.1 Example Problem: Crippling Analysis of Bead Section

Calculate the crippling allowable and the safety margin for the bead cross section shown on the right. The cross section is subjected to a bending and axial load that results in a stress distribution that is given in the table on the right. The elements that are relevant for crippling analysis are shown numbered from 1 to 3. The top figure shows shaded the section that is in compression. The bottom figure shows the curved cap region approximated as the chord of the curve. The geometry and material properties are defined below.

$t = 0.0584$ in
$t_{c2} = t_{c3} = 0.0438$ in
$Z_{NA} = 0.1$ in
$s = 1.3876$ in (see Figure 6.6-9 a)
$\theta = 118^o$
$R_i = 0.63$ in
$H = 0.73$ in
$\alpha = 59^o$

The material properties for the bead cap and web at 160°F are provided below.
$D_{11} = 58$ lb/in
$\nu_{xy}=0.231$
$E_{xc}=E_{yc}=6.8x10^6$ psi (compressive Young's modulus in the direction of load application and perpendicular to it in the plane of the element)
The side walls and the cap region are designated as "no fastener zone". Therefore use $F_c = F_{cu} =$ 45ksi. (unnotched compression allowable)





The geometric definition of the web is given in the table below. Refer to Figure 6.6-9 (b) for the location of each of the co-ordinates.

Location	Y (in)	Z (in)
a	-0.5775	0.4055
d	-0.5399	0.3829
c	-0.735	0.0584
b	-0.7861	0.0584

<table>
<tr><td></td><td>Stress distribution across the cross section.
<table>
<tr><th>Z (in)</th><th>σ (psi)</th></tr>
<tr><td>0</td><td>10000</td></tr>
<tr><td>0.1</td><td>0</td></tr>
<tr><td>0.73 (at H)</td><td>-20000</td></tr>
</table></td></tr>
<tr><td>Calculate l_{2o} using Equation 6.6.6-1</td><td>$l_{2o} = \sqrt{(z_a - z_b)^2 + (y_a - y_b)^2}$
$l_{2o} = \sqrt{(0.4055 - 0.0584)^2 + (-0.5775 + 0.7861)^2}$
= 0.4050 in</td></tr>
<tr><td>Similarly calculate l_{2i} using Equation 6.6.6-2</td><td>$l_{2i} = 0.3786\ in$</td></tr>
<tr><td>Calculate l_{1o} using Equation 6.6.6-8</td><td>$\mathrm{R_o} = \mathrm{R_i} + \mathrm{t_{c3}}$
= 0.63 + 0.0438
= 0.6738 in
$l_{1o} = 2\mathrm{R_o}\sin\left(\frac{\theta}{2}\right) + 2\mathrm{l_{2o}}\cos(\propto)$
$l_{1o} = 2(0.6738)\sin\left(\frac{118}{2}\right) + 2(0.4050)\cos(59)$
= 1.5723 in</td></tr>
<tr><td>Calculate l_{1i} using Equation 6.6.6-9</td><td>$l_{1i} = \mathrm{l_{1o}} - \frac{\mathrm{t_{c2}}}{\sin(\propto)}$
$l_{1i} = 1.5723 - \frac{0.0438}{\sin(59)}$
= 1.5212 in</td></tr>
<tr><td>Calculate the length of element 1 in compression using Equation 6.6.6-14</td><td>$\mathrm{b_1} = \mathrm{l_{web}^{comp}} = 0.5\left[\frac{\mathrm{l_{2i}}\sin(\propto) - \mathrm{Z_{NA}} + \mathrm{t}}{\sin(\propto)} + \frac{\mathrm{l_{2o}}\sin(\propto) - \mathrm{Z_{NA}} + \mathrm{t}}{\sin(\alpha)}\right]$
$= 0.5\left[\frac{(0.3786)(\sin 59^o) - 0.1 + 0.0584}{\sin(59^o)} + \frac{(0.405)(\sin(59^o)) - 0.1 + 0.0584}{\sin(59^o)}\right]$
$\mathrm{b_1} = 0.3433\ \mathrm{in}$</td></tr>
<tr><td>Calculate length b for element 3 using Equation 6.6.6-12.</td><td>$\mathrm{b_3} = \mathrm{l_3} = (\mathrm{R_o} + \mathrm{R_i})\sin\left(\frac{\theta}{2}\right)$
$\mathrm{b_3} = (0.63 + 0.6738)\sin\left(\frac{118^o}{2}\right)$
= 1.1175 in</td></tr>
<tr><td>Calculate the area of the web using Equation 6.6.6-10</td><td>$\mathrm{A_{web}} = \mathrm{t_{c2}}\frac{(\mathrm{l_{2o}} + \mathrm{l_{2i}})}{2}$
= (0.0438)($\frac{0.405+0.3786}{2}$)
=0.0172 in^2</td></tr>
<tr><td>Calculate the area of the web in compression (Elements 1 and 2) using Equation 6.6.6-11.</td><td>$\mathrm{A_{web}^{comp}} = \mathrm{A_{web}} - \frac{(\mathrm{l_{1o}} - \mathrm{l_{1i}})(\mathrm{Z_{NA}} - \mathrm{t})}{2}$
$\mathrm{A_{web}^{comp}} = 0.0172 - \frac{(1.5723 - 1.5212)(0.1 - 0.0584)}{2}$
= 0.0161 in^2</td></tr>
<tr><td>Calculate the area of Element 3 using Equation 6.6.6-13</td><td>$\mathrm{A_{segment}} = \frac{\theta \mathrm{t_{c3}}}{2}(2\mathrm{R_o} - \mathrm{t_{c3}})$</td></tr>
</table>

	$A_{segment} = \frac{\left(\frac{(118)(\pi)}{180}\right)0.0438}{2}(2(0.6738) - 0.0438)$ = 0.0588 in^2
Calculate $\overline{E_x}$ for all elements.	$\bar{E}_x = \frac{12[1-(\nu_{yx})(\nu_{xy})](D_{11})}{t_{c2}^3}$ $\bar{E}_x = \frac{12[1-(0.231)(0.231)](58)}{0.0438^3}$ = 7.84x10^6 psi
Calculate X for the portion of element 1in compression	$X = \frac{b\overline{E_x}}{tE_{xc}}\sqrt{\frac{F_c}{\sqrt{E_{xc}E_{yc}}}}$ $X = \frac{(0.3433)(7.84x10^6)}{(0.0438)(6.86x10^6)}\sqrt{\frac{45000}{\sqrt{(6.86x10^6)(6.86x10^6))}}}$ =0.7255
Calculate Y for element 1	Y = (0.971)X$^{-0.962}$ (No Edge Free Condition) Y = (0.971)(0.7255)$^{-0.962}$ Y=1.3221
Calculate F_{cci} for element 1	$F_{cci} = Min(\frac{YF_c\bar{E}_x}{E_{xc}}, F_c)$ $F_{cci} = Min\left(\frac{(1.3221)(45000)(7.84x10^6)}{(6.86x10^6)}, 45000\right)$ =Min(67994 ,45000) = 45000 psi
Calculate EA for the portion of element 1in compression	=(6.86x10^6)(0.0161) = 110446 lb
Calculate EAF_{cci} for element 1	=(110446)(45000) = 4.969x107 lb^2/in^2

Repeat this for all elements as shown in the table below

Element	Edge	b	t	A	ν_{xy}	ν_{yx}	D_{11}	$\bar{E}_x$
		in	in	in^2			lb/in	psi
1	NEF	0.3433	0.0438	0.0161	0.231	0.231	58	7.84x10^6
2	NEF	0.3433	0.0438	0.0161	0.231	0.231	58	7.84x10^6
3	NEF	1.1175	0.0438	0.0588	0.231	0.231	58	7.84x10^6
SUM				0.0910				

Element	Edge	E_{xc}	E_{yc}	F_c	X	Y	F_{cci}	EA	EAF_{cci}
		psi	psi	psi			psi	lb	lb^2/in^2
1	NEF	6.86x10^6	6.86x10^6	45000	0.7255	1.3211	45000	110446	4.969x10^9
2	NEF	6.86x10^6	6.86x10^6	45000	0.7255	1.3211	45000	110446	4.969x10^9
3	NEF	6.86x10^6	6.86x10^6	45000	2.363	0.42474	21847	403368	8.812x10^9
SUM								624260	1.875x10^{10}

Note that if repair and durability conditions have to be satisfied, in the absence of program direction, the Filled Hole Compressive (FHC) strength should be used instead of the unnotched compressive strength in the F_c column above. If the cap has drain hole(s) then OHC should be used for F_c for element 3.

Determine the allowable crippling stress using Equation 6.5.1-1	$F_{cc} = \frac{\sum EAF_{cci}}{\sum EA}$ $= 1.875\text{x}10^{10}/624260$ $= 30035$ psi (compression)
The crippling allowable for the bead section is 30035 psi.	
Calculate the average applied compressive load (P_{app})	$\sigma_{zc} = \frac{(\sigma_{cmax} + \sigma_{cmin})}{2}$ = (-20000+0)/2 = -10000 psi $\sigma_{ceq} = \frac{2(\sigma_{cmax} - \sigma_{zc})}{3} + \sigma_{zc}$ $\sigma_{ceq} = \frac{2(-20000 - (-10000))}{3} + (-10000)$ $\sigma_{ceq} = -16667\ psi$ psi $P_{app} = \sigma_{ceq} \sum A$ where, $\sum A$ = sum of area of elements in compression From, Table above $\sum A = 0.091\ \text{in}^2$ $P_{app} = (16667)(0.0910)$ = 1517 lb (compression)
Calculate allowable crippling load (P_{crip_allow})	$P_{crip-allow} = F_{cc} \sum A$ P_{crip_allow} = (30035)(0.0910) = 2733 lb (compression)
Calculate the crippling margin	$M.S = \frac{P_{crip-allow}}{P_{app}} - 1$ $M.S = \frac{2733}{1517} - 1$ = +0.8

Since only a portion of the stiffener is in compression the stiffener will not experience global Euler buckling and therefore the length correction factor need not be applied.

As mentioned earlier only the elements of the section in compression should be included in the crippling analysis. If a portion of the bead cap is in compression then only that portion should be analyzed by using the appropriate value for its arc length (s), determining θ and using this θ in Equation 6.6.6-12. If the bottom flange is in compression then the flange should be included in the crippling analysis. The length of the flange that should be used in the crippling analysis is determined using Equation 6.6.6-15.

6.6.7 Inter-rivet Buckling

Inter-rivet buckling occurs when the skin is attached to the flange by fasteners and the flange or the skin buckles between the fasteners before crippling can occur. The fastener may also be used in addition to bonding to satisfy fail-safe requirements or to satisfy repair requirements. In such situations, a complete disbond is assumed between fasteners and inter-rivet buckling calculations are performed to ensure that the skin or the flange will not buckle between the fasteners.

The fastener pitch(s) at which the failure mode switches from crippling to inter-rivet buckling is given in Reference 6-8 as

$$s = \pi\sqrt{\frac{c_i D_{11}}{t F_{allw}}} \qquad \text{Equation 6.6.7-1}$$

where,
c_i = 1, for countersunk fasteners
= 3, for protruding head fasteners
t = thickness of the flange, in
D_{11} = laminate bending stiffness, in-lb
F_{allw} = allowable crippling stress for the skin or the stiffener section, psi.

If the distance between the fasteners is more than *s* inter-rivet buckling will occur. Also, the fastener spacing, as suggested in PM4056 Section 2, should not be less than 4D or more than 8D where D is the diameter of the fastener.

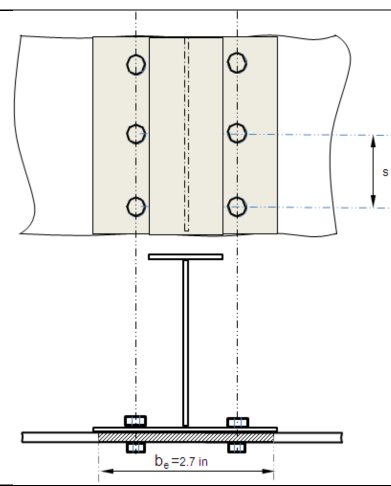
If the inter-rivet spacing s is given then Equation 6.6.7-1can be rewritten to determine the inter-rivet buckling allowable stress.

$$F_{ir} = \left(\frac{\pi}{s}\right)^2 \frac{D_{11} c_i}{t} \qquad \textbf{Equation 6.6.7-2}$$

where,
F_{ir} = inter-rivet buckling allowable stress, psi.

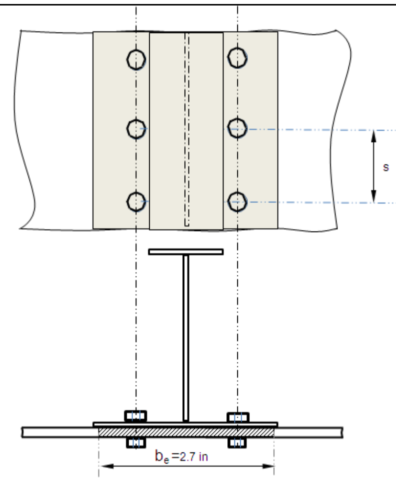
Compare F_{ir} with the crippling allowable (F_{cc} or F_{cl}) for that specific element and if F_{ir} is the minimum determine the inter-rivet buckling margin.

6.6.7.1 Example Problem – Fastener Pitch

Calculate the fastener pitch below which inter-rivet buckling cannot occur for the Example Problem 6.6.4. The fastener diameter is 0.25 inches and the fastener pattern is shown on the right. Assume protruding head fasteners are used. Relevant parameters are repeated here for convenience. Flange: D_{11}^{flange}= 1974 in-lb t_{flange} = 0.1272 in Skin: D_{11}^{skin}= 788 in-lb t_{skin} = 0.1014 in	
Calculate the fastener pitch below which inter-rivet buckling will not occur for the flange using Equation 6.6.7-1.	$s_{flange} = \pi\sqrt{\frac{c_i D_{11}^{flange}}{t_{flange} F_{allw}^{section}}}$ From Table 6.6-4, $F_{allw}^{section} = \frac{2(7403) + 1591 + 2(5288)}{2(0.1781) + 0.0792 + 2(0.1272)}$ = 39101 psi

	$s_{flange} = \pi\sqrt{\dfrac{(3)(1974)}{(0.1272)(39101)}}$ = 3.4 inches
Calculate the fastener pitch below which inter-rivet buckling will not occur for the skin using Equation 6.6.7-1.	$s_{skin} = \pi\sqrt{\dfrac{c_i D_{11}^{skin}}{t_{skin}F_{allw}^{skin}}}$ From Table 6.6-4, $F_{allw}^{skin} = \dfrac{2(6410)}{2(0.135)}$ = 47481 psi $s_{skin} = \pi\sqrt{\dfrac{(3)(788)}{(0.1014)(47481)}}$ = 2.2 inches
Since $s_{skin} < s_{flange}$, the fastener pitch s = 2.2 inches is chosen.	
Calculate s/D	s/D = 2.2/0.25 = 9
In the absence of any program requirement, since s/D should be between 4 and 8, the fastener pitch is chosen such that s/D=8	
Calculate the new fastener pitch, s	s =(8)(0.25) = 2 inches

6.6.7.2 Example Problem – Inter-rivet Buckling

Determine the inter-rivet buckling allowable and the safety margin for the Example Problem 6.6.7.1 given the inter-rivet spacing is 4 inches. The fastener pattern is shown on the right. Flange: D_{11}^{flange}= 1974 in-lb t_{flange} = 0.1272 in $F_{applied}^{Flange} = 24470\ psi$ Skin: D_{11}^{skin}= 788 in-lb t_{skin} = 0.1014 in $F_{applied}^{skin} = 13074\ psi$	
Calculate the inter-rivet buckling allowable for the flange.	$F_{ir} = \left(\dfrac{\pi}{s}\right)^2 \dfrac{D_{11}c_i}{t}$ $F_{ir\_flange} = \left(\dfrac{\pi}{4}\right)^2 \dfrac{(1974)(3)}{(0.1272)}$ = 28718 psi
Determine crippling allowable stress for the flange from Table 6.6-4	$F_{cc}^{flange} = \dfrac{2(7403)}{2(0.1781)}$ = 41567psi
Since $F_{ir\_flange} < F_{cc}^{flange}$ the inter-rivet buckling margin has to be determined.	

Calculate the Margin of Safety for flange inter-rivet buckling.	$MS = \frac{F_{ir\_flange}}{F_{applied}^{flange}} - 1$ $MS = \frac{28718}{24470} - 1$ =+0.17
Calculate the inter-rivet buckling allowable for the skin.	$F_{ir\_skin} = \left(\frac{\pi}{4}\right)^2 \frac{(788)(3)}{(0.1014)}$ = 14381psi
Determine crippling allowable stress for the skin from Table 6.6-4	$F_{cc}^{skin} = \frac{2(6410)}{2(0.1350)}$ = 47481 psi
Since $F_{ir\_skin} < \mathrm{F}_{cc}^{skin}$ determine the inter-rivet buckling margin for the skin.	
Calculate the Margin of Safety for skin inter-rivet buckling.	$MS = \frac{F_{ir\_skin}}{F_{applied}^{skin}} - 1$ $MS = \frac{14381}{13074} - 1$ = +0.10

6.7 Beam Column Analysis

A beam-column is a structural member that react both axial and bending loads as opposed to column, which reacts only axial loads. The axial load can be either tension or compression. Beam-column analysis for composites follows closely the analysis method for metals with minor modifications. Details pertaining to beam-column analysis are provided in Reference 6-1, Section 8. The transverse loads that causes bending do not usually affect the buckling therefore the stability analysis has to be performed in addition to the beam-column analysis.

The beam-column analysis of the aircraft structures is typically performed by determining the maximum bending moment in a section and comparing it with the allowable bending moment. Two methods, an approximate method and a standard solution method are discussed below.

6.7.1 Beam Column Analysis – Approximate Solution

An approximate solution for determining the beam column amplification factor is discussed here followed by the more accurate standard solution. If the eccentricity (e) of a beam-column is known then the approximate analysis can be used to determine the effects of the eccentricity on the moments and the axial load. The same procedure outlined in Reference 6-1 for metals can be used for composites also by suitably modifying the material parameters to suit composites. This is done by calculating the expression j as follows:

Equation 6.7.1-1

$$j = \sqrt{\frac{\sum(E_x I)_i}{P}}$$

where,

$(E_xI)_i$ = the modulus weighted moment of inertia (lb-in^2) of each element of the section
P = the applied load (lb).

Once the expression for j is calculated then the method discussed in Reference 6-1, Section 8 can be employed to perform beam-column analysis of the composite.

6.7.2 Beam Column Analysis - Standard Solution

The details of the standard solution are provided in Reference 6-1, which is for metals but can be extended to composites by using appropriate orthotropic material properties. It is summarized here for convenience.

The moment, shear force, displacement and rotation are given as

$$M = C_1 sin\left(\frac{x}{j}\right) + C_2 \cos\left(\frac{x}{j}\right) + f(w)$$ **Equation 6.7.2-1**

$$V = \frac{C_1}{j} cos\left(\frac{x}{j}\right) - \frac{C_2}{j}\cos\left(\frac{x}{j}\right) + f'(w)$$ **Equation 6.7.2-2**

$$\delta = \frac{(M_o - M)}{P}$$ **Equation 6.7.2-3**

$$\theta = \frac{(V_o - V)}{P}$$ **Equation 6.7.2-4**

where,
f'(w) is the first derivative of f(w)
M_o is the bending moment of the beam with no beam-column magnification, in-lb
M is the bending moment calculated from Equation 6.7.2-1, in-lb
V_o is the shear of the beam with no beam-column magnification, lb
V is the shear calculated from Equation 6.7.2-2, lb
P is the beam axial load, lb
The constants C_1 and C_2 are defined in Reference 6-1 Table 8.3.2-1

The bending moment in a section due to different transverse loads and moments can be obtained by superposition as long as the total axial load is the same in each of the cases that are superimposed. This is discussed in detail in Section 8.3.1-2 Reference 6-1.

The above equations are valid for uniform cross sections. Non-uniform cross sections should be modeled using non-linear Finite Element Analysis.

6.7.3 Beam-Column with Axial Tension

Axial tension in a beam-column acts to reduce the eccentricity which in turn reduces the primary bending moment. However, this beneficial effect of axial tension is not considered in sizing the component. Further details are provided in Section 8, Reference 6-1.

6.8 Torsional Instability Analysis

This section is reserved for future use. See Section 8.5 PM4057 for a basic discussion of Torsional Instability.

6.9 Unix/PC-Based Calculations

The basic allowables discussed above can be obtained from IDAT/MATUTL for ultimate compression, IDAT/IBOLT and IDAT/MATUTL for FHC and OHC, and IDAT/CDADT for durability and damage tolerance cutoffs. The methods for crippling discussed above lend themselves to spreadsheet calculations.

6.10 FEA Modeling Conventions and Considerations

This section is reserved for future use.