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Prepared by: L. K. Flansburg		22 Dec 2016
2 General Analysis Practice		

2 General Analysis Practice

The purpose of this chapter is to provide generic guidance on metallic stress analysis methods, providing discussion and methodology for topics general to aircraft stress analysis such as accepted methods, sign conventions, combined stresses, drawing tolerance considerations and preparation of stress analysis.

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2.1 References and Nomenclature

Each major section of this handbook has a list of references specific to the material located in that section. References used in this Section and some of the general references used in preparing this document are listed below.

- 2-1. anon., [*Lockheed Martin Engineering Stress Memo Manual*](#), Lockheed Martin Aeronautical Systems, Marietta, GA (October 1998 Release; April 2002 Revision)
- 2-2. anon., [*Structures Analysis Manual, Volume 1 and Volume 2*](#), General Dynamics Convair and Space Systems Divisions (1988).
- 2-3. Staff, *LTV Structures Manual*, LTV Aircraft Products Group, Grand Prairie, TX (June 1989 Revision)
- 2-4. Staff, "PM4056 Composite Structural Analysis Manual," Lockheed Martin Aeronautics, Ft Worth, TX
- 2-5. anon., "Metallic Materials Properties Development and Standardization," *MMPDS-XX¹*, Battelle Memorial Inst., Secretariat (200X).
- 2-6. Peterson, R.E., *Stress Concentration Factors*, John Wiley and Sons, New York, New York, various editions.
- 2-7. anon., "Structures Manual, CVAC #1", Convair Division of General Dynamics Corporation, Ft. Worth TX, (1957)
- 2-8. anon., "Aircraft Structural Integrity Program, Aircraft Requirements", *MIL-STD-1530*, Aeronautical Systems Division, ASD/ENFS, Wright Patterson Air Force Base, Ohio. Various editions.
- 2-9. anon, Lockheed Martin Structural Life Assurance Manual, Lockheed Martin Aeronautical Systems, Marietta, GA., May 1993
- 2-10. Kuhn, P., et. al., "A Summary of Diagonal Tension, Part I – Method of Analysis", NACA-TN-2661, May 1952.
- 2-11. Smith, Clarence R., "Tips on Fatigue," NAVWEPS 00-25-559, Bureau of Naval Weapons, Department of the Navy, Washington D.C., 1963
- 2-12. anon., "Handbook Inspection Program Requirements: Nondestructive For Aircraft and Missile Materials and Parts," Department of Defense, MIL-HDBK-6870A, August 2001
- 2-13. anon., "Sampling Inspection Procedures and Tables for Inspection by Attributes," Mil-Std-105E, Department of Defense, May 1989
- 2-14. Ross, Sheldon, [*Introduction to Probability and Statistics for Engineers*](#), Academic Press, San Diego, CA, Second Edition, 2000

¹ MMPDS revisions are designated by a 2 digit number, for instance MMPDS-03 was published in October 2006. Reference is left generic as MMPDS-XX.

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- 2-15. Timoshenko, S., Theory of Elastic Stability, McGraw Hill Book Company, New York, NY, Second Edition, 1961.
- 2-16. anon. "Strength of Aircraft Elements," ANC-5, Army-Navy-Civil Committee on Aircraft Design Criteria, December 1942
- 2-17. anon. "Strength of Aircraft Elements," ANC-5, Army-Navy-Civil Committee on Aircraft Design Criteria, March 1955
- 2-18. Shanley, F.R. and Ryder, E. I., "Stress Ratios," Aviation Magazine, June 1937
- 2-19. Sadowsky, M.A., "A Principal of Maximum Plastic Resistance," Transcript of ASME 1943, page A-65
- 2-20. Sanderson, P.A., et.al., "Corrugated Panels under Combined Compression and Shear Load," Journal of Aeronautical Sciences, February 1940
- 2-21. Lundquist, E., "Strength Tests of Thin-Walled Duralumin Cylinders in Combined Transverse Shear and Bending," NACA TN523, April 1935
- 2-22. Lundquist, E., "Strength Tests of Thin-Walled Duralumin Cylinders of Elliptic Section," NACA TN527, May 1935
- 2-23. Lundquist, E., "Strength Tests of Thin-Walled Duralumin Cylinders of Elliptic Section in Pure Bending and Torsion," NACA TN851, June 1942
- 2-24. anon., "Some Investigations of the General Instability of Stiffened Metal Cylinders VI Stiffened Metal Cylinders Subjected to Combined Bending and Transverse Shear," NACA TN910, September 1943
- 2-25. anon., "Some Investigations of the General Instability of Stiffened Metal Cylinders VII Stiffened Metal Cylinders Subjected to Combined Bending and Torsion," NACA TN911, November 1943
- 2-26. Bruhn, E., "Tests of Thin Walled Celluloid Cylinders to Determine The Interaction Curves Under Combined Bending, Torsion, and Compression or Tension Loads," NACA TN951, January 1945
- 2-27. Batdorf, S.B., et. al., "Critical Combinations of Shear and Direct Stress for Simply Supported Rectangular Flat Plates," NACA TN1223, March 1947
- 2-28. Batdorf, S.B., et. al., "Critical Combinations of Torsion and Direct Axial Stress for Thin Walled Cylinders," NACA TN1345, June 1947
- 2-29. Batdorf, S.B., et. al., "Critical Combinations of Shear and Longitudinal Direct Stress for Long Plates with Transverse Curvature," NACA TN1347, June 1947
- 2-30. Peters, Roger, "Buckling Tests of Flat Rectangular Plates Under Combined Shear and Longitudinal Compression," NACA TN1750, November 1948
- 2-31. Schildcrout, Murray, et. al., "Critical Combinations of Shear and Direct Axial Stress for Curved Rectangular Panels," NACA TN1928, August 1949
- 2-32. Stowell, E. "Plastic Buckling of a Long Flat Plate Under Combined Shear and Longitudinal Compression," NACA TN1990, 1949
- 2-33. Libove, C., et.al., "Charts for Critical Combinations of Longitudinal And Transverse Direct Stress for Flat Rectangular Plates," NACA ARR L6A05, March 1946
- 2-34. Batdorf, S.B., et. al., "Critical Combinations of Longitudinal And Transverse Direct Stress for An Infinitely Long Flat Plate with Edges Elastically Restrained Against Rotation," NACA ARR L6A05a, March 1946
- 2-35. Batdorf, S.B., et. al., "Critical Combinations of Shear And Transverse Direct Stress for An Infinitely Long Flat Plate with Edges Elastically Restrained Against Rotation," NACA ARR L4L14, January 1945
- 2-36. Stowell, E., et. al., "Critical Stress for an Infinitely Long Flat Plate with Elastically Restrained Edges Under Combined Shear and Direct Stress," NACA ARR 3K13, November 1943
- 2-37. Melcon, M.A., et.al., "Analysis of Stiffened Curved Panels Under Shear and Compression," LR8693, Lockheed Aircraft Corporation, Burbank, CA, August 1952
- 2-38. Timoshenko, S., Strength of Materials, Part I, McGraw Hill Book Company, New York, NY, Second Edition, 1950.
- 2-39. anon., "Airframe Installation and Design Engineering Design Manual, PM4007", Lockheed Martin Aeronautics, Ft. Worth, TX, 2006.

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- 2-40. anon., "Air Force Systems Command (AFSC) Design Handbook, DH2-1, Airframe," ASD/ENES, Wright Patterson AFB, OH, 1992.
- 2-41. anon., "General Specification for Airplane Strength and Rigidity, MIL-A-8860B," Naval Air Engineering Center, Lakehurst, NJ, 1987
- 2-42. anon., "Military Specification: Demonstration Requirements for Aircraft Weapons Systems, MIL-D-8708C," Naval Air Engineering Center, Lakehurst, NJ, 1991
- 2-43. anon., "Department of Defense Joint Services Specification Guide, Aircraft Structures, JSSG-2006," Department of Defense, ASC/ENSI, Wright Patterson Air Force Base, Ohio (2006)
- 2-44. anon., "Airworthiness Standards for Transport Category Airplanes 14CFR25.1 (FAR25)," Federal Aviation Administration, Department of Transportation, Washington, D.C.
- 2-45. anon., "Continued Airworthiness and Safety Improvements for Transport Category Airplanes 14CFR26.1 (FAR26)," Federal Aviation Administration, Department of Transportation, Washington, D.C.
- 2-46. anon., "Standard Test Methods for Tension Testing of Metallic Materials, ASTM E 8," American Society for Testing and Materials, West Conshohocken, PA. (2008)
- 2-47. anon., "Standard Test Methods for Compression Testing of Metallic Materials at Room Temperature, ASTM E 9," American Society for Testing and Materials, West Conshohocken, PA. (2008)
- 2-48. anon., "Standard Test Methods for Shear Testing of Aluminum Alloys, ASTM B 769," American Society for Testing and Materials, West Conshohocken, PA. (2008)

2.1.1 List of Symbols and Nomenclature

Symbol	Description	Units
a	length	in
A	area	in ²
AQL	Acceptable Quality Level	
ASIP	Aircraft Structural Integrity Plan	--
b	width	in
c	distance from neutral axis to outer fiber	in
c	maximum number of defects allowed in the sample before the lot is rejected	
CAT	Computerized axial tomography	
COV	Coefficient of Variation	
D, d	Diameter	in
D	width	in
D _{av}	Average Diameter	in
DOD	Department of Defense	
ESDU	Engineering Sciences Data Unit	
F _{tu}	Material ultimate tensile stress	psi
FAA	Federal Aviation Administration	
FAR	Federal Aviation Regulation	
f _b	Applied bending stress	psi
FC I	Fracture Critical I	
FC II	Fracture Critical II	
FE	Finite Element	
FEA	Finite Element Analysis	
FEM	Finite Element Model	
f	applied stress	psi
F _{B-Basis}	B-Basis stress allowable	
f _c	Applied compression stress	psi

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F_{cu}	Material ultimate compression stress property	psi
f_s	Applied shear stress	psi
F_{stu}	Material torsional ultimate stress property	psi
F_{sty}	Material yield torsional stress property	psi
F_s	Material ultimate shear stress allowable (psi)	psi
F_{su}	Material ultimate shear stress property	psi
F_{sy}	Material yield shear stress property	psi
f_t	Applied tension stress	psi
F_{tu}	Material ultimate tension stress property	psi
F_{ty}	Material tension yield stress property	psi
I	Area moment of inertia	in ⁴
ITAR	International Traffic in Arms Regulations	
J	Polar moment of inertia	in ⁴
JSSG	Joint Service Specification Guide	
K	Shear-tension ratio (F_{su}/F_{tu})	
$K_{one-side}$	One sided tolerance limit factor	
k_t	Stress concentration factor	
K_{Rfe}	Reduction factor based on elongation prolongation testing	
K_{RFu}	Reduction factor based on ultimate strength prolongation testing	
K_{Rfy}	Reduction factor based on yield strength prolongation testing	
LMASAM	Lockheed Martin Aeronautics Structural Analysis Manuals	
LTPD	Lot Percent Defective	
m	Applied moment	in-lb
m	Interaction exponent for shear-bending	
$(mc/I)_b$	Material bending modulus	psi
M.S.	Margin of Safety	
N	Number of observations in sample or population	
NACA	National Advisory Committee for Aeronautics	
NATO	North American Treaty Organization	
NC	Numerical Control	
NDI	Non-destructive Inspection	
OC	Operating Characteristic curve	
p	Pressure	psi
p	Interaction exponent for shear/shear torsion – bending	
P	Axial load	lb
Q	Static moment of inertia	in ³
r	Interaction exponent for tension-bending	
r	Radius	in
R	Stress ratio (applied stress/allowable stress)	
R_{CR}	Critical buckling load ratio	
R_{TR}	Ratio to requirement	
RA	Surface roughness average	μin or 10 ⁻⁶ in
R_{bar}	Combined stress ratio	
R_n	Normal stress ratio due to combined bending and tension stress.	
RSS	Root sum squared	
s_{dev}	standard deviation of a sample	
SPC	Statistical Process control	
t	thickness	in
T	applied torsion load	in-lb
u	probability	
U_{bt}	Bending-tension interaction stress ratio	

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USAF	United States Air Force	
V	Transverse shear	lb
x	coordinate system abscissa	
\bar{x}	mean of sample	
X-RAY	Non-destructive inspection technique using electromagnetic imaging	
y	distance from neutral axis to analysis point	in
y	coordinate system ordinate	
z	Section Modulus (I/c)	in ³
Greek Symbols		
α	level of significance	
β	The least angle between maximum bending stress and maximum shear stress on tube cross-section	Degrees
β	Angle	Degrees
β	Risk of accepting a false hypothesis (or defective part or lot)	
δ	Deflection	in
γ	Material plasticity factor	
θ	Angle	Degrees
σ	mean of population	
σ	stress	psi
σ	standard deviation of population	
$\sigma_{\bar{x}}$	standard deviation of the sample means	
Φ	Cumulative normal distribution function	
Subscripts		
1	first, element 1, or 1-direction	
2	second, element 2 or 2-direction	
3	third, element 3, or 3-direction	
ALLOW, allow	allowable stress, load, bending moment, deflection	
applied	applied stress, load, bending moment, deflection	
b	bending	
c	compression	
ht	hoop tension	
i	ith value or observation	
max	maximum	
min	minimum	
net	net, i.e. remaining area after holes removed	
req	required	
s	shear	
ss	simple shear	
st	torsional shear	
t	tension	
x	x-direction	
y	y-direction	
$\alpha/2$	half interval of level of significance	
β	associated with Risk of accepting a false hypothesis (or defective part or lot)	

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2.2 Classification of Structure

Every component and structural element of an aircraft has structural requirements. The terminology used in describing the criticality of these requirements is generally customer and specification dependent. This section will provide a basic overview of these requirements and terminology. However, the analyst should refer to the pertinent aircraft design criteria document and specification applicable to the aircraft under consideration.

2.2.1 Structural Criticality

This section delineates the general policy for the classification of structure, and meets or exceeds the requirements and policies expressed in the regulatory aircraft specifications such as Rerences 2-40 through 2-45: ASFC Design Handbook, Mil-A-8860, MIL-D-8708C(AS), the Joint Services Specification Guide (JSSG), FAR 25 and FAR 26.

Every component of an air vehicle has structural requirements. Structural components typically carry directly applied aerodynamic loads or to provide a load path to transfer accumulated aircraft loads such as aerodynamic, maneuver, gust, ground, landing, takeoff, etc., loads. Structural components are typically classified based on the inherent significance of the component to continued airworthiness of the vehicle. "Non-structural" components are typically hydraulic, electrical or pneumatic components or components designed only by their own mass as affected by the aircraft inertia environment. However, non-structural should not always be assumed to be inconsequential. The design and analysis of cargo tie-down provisions or the activity involved in electronic equipment qualification is often considered non-structural in terms of primary aircraft design, but both tasks play an important role in aircraft and air crew safety and can represent a significant expenditure of stress engineering effort for certification.

There are, potentially, four types of structural integrity analysis which can be conducted on any part: Static, Durability, Damage Tolerance or Failsafe.

- Static analysis examines the part's ability to resist a single event loading without failure. In this case failure is the inability of the part to function in its intended fashion whether through fracture, excessive deformation or collapse.
- Durability analysis examines the part's resistance to cyclic damage accumulation modes including crack initiation or crack growth.
- Damage tolerance analysis examines the part's resistance to cyclic damage accumulation through the examination of the growth of an assumed crack from initiation or detection to critical crack length. On some programs, failsafe analysis of certain components may be substituted for damage tolerance analysis.
- Failsafe analysis is the examination of the structure's ability to carry load up to a specified load level even though a portion of the structure has failed or been structurally compromised.

The structural criticality, and thus classification of the part, and program criteria govern what analysis must be conducted on each part. For most aircraft, **all** parts must have static and **most** must have some type of durability analysis. For some parts, the "durability analysis" may be in the form of a screening procedure with only those parts failing the screening requiring a full durability check. Aircraft programs generally conduct a preliminary damage tolerance screening analysis on many parts to determine which ones, ultimately, are classified as fracture critical. This screening may be as simple as a comparison of maximum limit stresses or 1g stresses to a cutoff value or it may be more sophisticated with a screening spectrum involving a crack growth analysis with specified flaw sizes. Only the parts deemed fracture critical parts would then require a complete damage tolerance analysis. Only very select parts, designated "failsafe" and agreed to by the customer, would require a failsafe analysis. This manual is concerned with the static analysis aspect of structural integrity analysis.

In addition to the above requirements, many parts or components may have stiffness requirements due to flutter, dynamic, or deflection requirements and the stress analyst may have to perform appropriate stiffness checks or other analyses, including guidance provided by the Flutter and Dynamics groups. This is the case for control surfaces, wing tips, empennage components, nacelle pylons and most equipment installations.

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Section 2.2.1.1 provides information on Static strength classification designations and Section 2.2.1.2 discusses life based classifications.

2.2.1.1 Static Strength Classification Designations

Structures, parts, welds and equipment may be classified as a means for designating the statistical basis for design properties, degree of testing, levels of inspection, and other special requirements. These items may be on the vehicle or on the equipment which interfaces with the vehicle, on the ground, in the air or in space. The appropriate classification shall be determined by the stress engineer and can be useful in creating a common point of discussion with the design organization. The designation should be noted in the stress analysis and can be incorporated in a drawing note by design engineers where appropriate and when specified by program guidance. In any event, use of the terminology defined in this section provides a clearer understanding of the relative criticality of a structural component than the terms “structural”, “non-structural” or “secondary structure”.

Structures, parts, welds, and equipment, are divided into two groups: vital and non-vital as shown in the logic tree provided in Figure 2.2.1-1

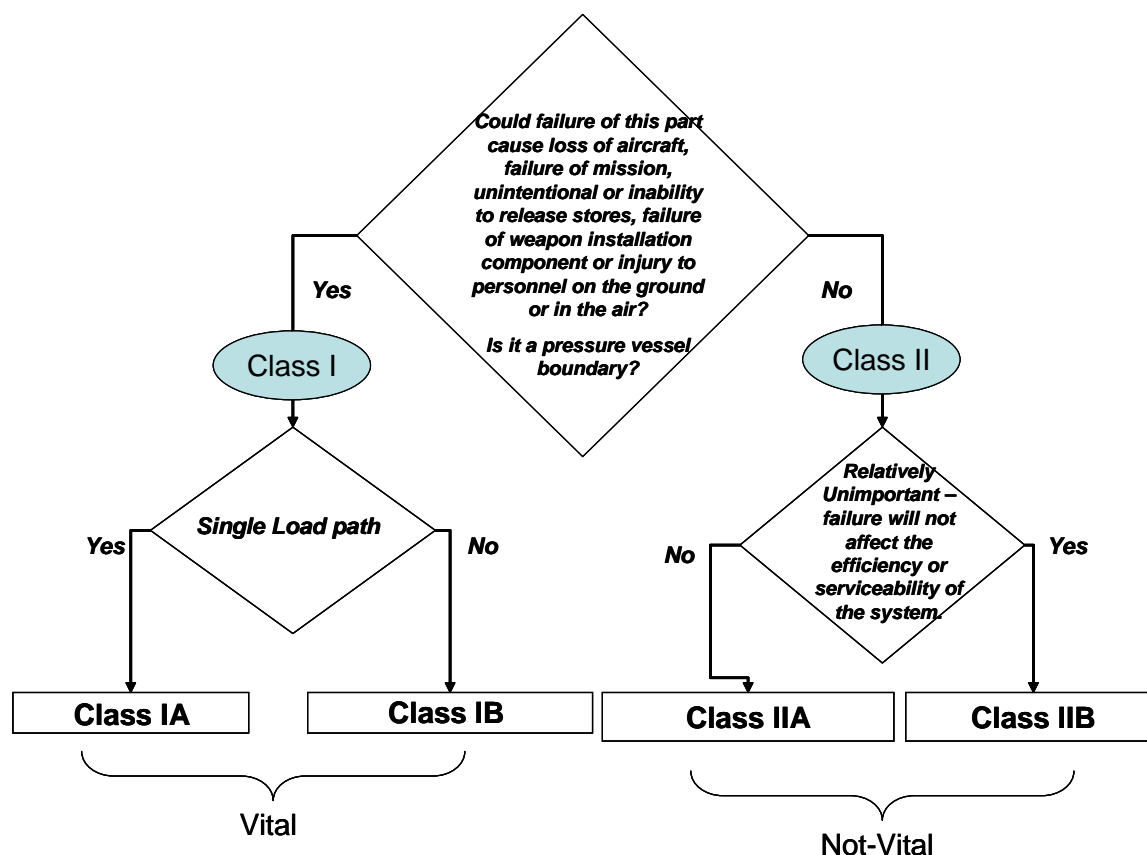


Figure 2.2.1-1 Static Strength Classification Logic Tree

Class I, vital or critical structures, parts, welds, and equipment are those whose failure, by any means, could cause loss of the vehicle or one of its major components, loss of control, failure of the mission, unintentional release of or inability to release an armament or store, failure of parts or components whose function is to retain the weapon in the aircraft, or injury to personnel, on the ground, in the air or in space. This classification is often applied to the same set of parts that receive the fracture critical parts designation described in the next section; however, there can

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be cases where the part is statically a critical part but it is not designated as fracture critical because the service loads spectrum is not the design driver.

This category is further divided into Class IA single load path structure and Class IB multiple load path structure. Vital structures, parts, welds and equipment which are single load path, are classified Class IA. The cognizant structures manager must approve the designation of Class IA. Vital structures, parts, weld and equipment which can be shown by analysis or test not to be Class IA shall be classified Class IB.

All structures, parts, welds and equipment which are not vital or critical are Class II. They are further divided into

- Class IIA: Structural or Important - All important, non-vital structures, parts, welds, and equipment which cannot be classified IIB shall be classified IIA. Class IIA may be changed to IIB if the true margin of safety is greater than 200%.
- Class IIB: Relatively Unimportant - All relatively unimportant non-vital assemblies, parts, welds, and equipment whose structural failure would not affect the efficiency or serviceability of the system, and for which no formal stress analysis is required, shall be classified IIB.

These part classifications do not correspond directly in any way to the durability critical parts or normal control described in the next section.

2.2.1.2 Life-Based Classification Designations

The Air Force's Aircraft Structural Integrity Program (ASIP) requirements ensure the structural integrity of an aircraft throughout its design life. Part of the ASIP approach involves the classification of all structural parts based on their criticality. This section is meant to be a general familiarization overview. For more precise information, refer to References 2-8 and 2-9 and program guidance. The Navy follows a similar approach; however, the details of the Navy's approach are aircraft dependent. The FAA also requires a life-based classification system as defined in FAR 26, Reference 2-45.

As a part of the Air Force and Navy specifications, there are four primary life-based classifications of structure:

- Fracture Critical or Damage Tolerance Critical – these may be traceable or non-traceable
- Mission Critical
- Durability Critical or Maintenance Critical
- Normal Controls

Programs often tailor the specification for their particular aircraft and mission and on a given program there may be fewer life based classification or they may have variations from the names listed above.

Fracture critical or damage tolerance critical parts are parts whose failure could lead to the loss of an aircraft, failure of a mission, unintentional release of stores, inability to release stores, failure of parts or components whose function is to retain the weapon in the aircraft, or injury to personnel in the aircraft or on the ground. It is further broken down into two categories:

- Fracture Critical I, Damage Tolerance Critical I or Fracture Critical – Traceable
- Fracture Critical II, Damage Tolerance Critical II or Fracture Critical – Non-Traceable

Fracture Critical I (FC I) parts will often have very stringent requirements for the manufacturer to provide documentation on every manufacturing step and process that the part has undergone from the creation of the billet, through heat treat, machining, finishing, drilling, assembly, as well as installed aircraft tail number, when it is moved to another aircraft, and when it retires. This reporting requirement is sometimes referred to as "Cradle-to-Grave traceability" and makes FC I parts very expensive and results in the Fracture Critical – Traceable nomenclature. Fracture Critical II parts in general do not have this traceability requirement levied on them.

Durability critical or maintenance critical parts are parts which have a predicted durability life which meets minimum requirements but, perhaps, not much more than that. They may also be parts which have ample durability

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life but are costly to repair or replace (e.g., inaccessible). On very rare occasions, these parts may have traceability requirements levied against them.

Mission critical parts or systems components are parts/components whose continued function is necessary for the completion of the mission. Many times the mission critical designation is given to an aircraft system component such as an avionics, flight control or communications box and is not used for structure; however, in some circumstances the failure of a structural part can compromise mission capability and on some aircraft programs would be designated as mission critical. If the mission critical category is not used for structure, then the part would be listed under one of the other categories.

Normal controls parts are the remaining aircraft structural parts. These parts are important to the structural integrity of the aircraft but may be sized by static loading or have high durability life. These parts are still considered “structural”, although some may be considered Class II parts under Section 2.2.1.1 definitions.

As longer and longer life requirements are levied on aircraft, the mix of fracture critical, durability critical and normal controls parts is evolving. For aircraft designed between 1975 and 2000, it is estimated that fracture critical parts were a small percentage of all aircraft parts, perhaps 5-10%. Durability critical parts were another 20-25% of the parts and normal controls parts were the remaining 65-70%. For aircraft designed since 2000, the mix of parts has been approximately 33% fracture critical, 33% durability critical and 33% normal controls.

Parts follow a classification hierarchy fairly early in the design process to determine what classification is appropriate for each. This decision tree may be fairly complex and will vary by aircraft program. Figure 2.2.1-2 is an example of the approach; however, the analyst should obtain the decision tree appropriate for his application. The most critical parts are usually designated “Safety of Flight Critical.” The determination of a parts status as Fracture Critical is often made by the program’s Fracture Control Board after a part is analytically screened using standard flaw size assumptions and based on its function.

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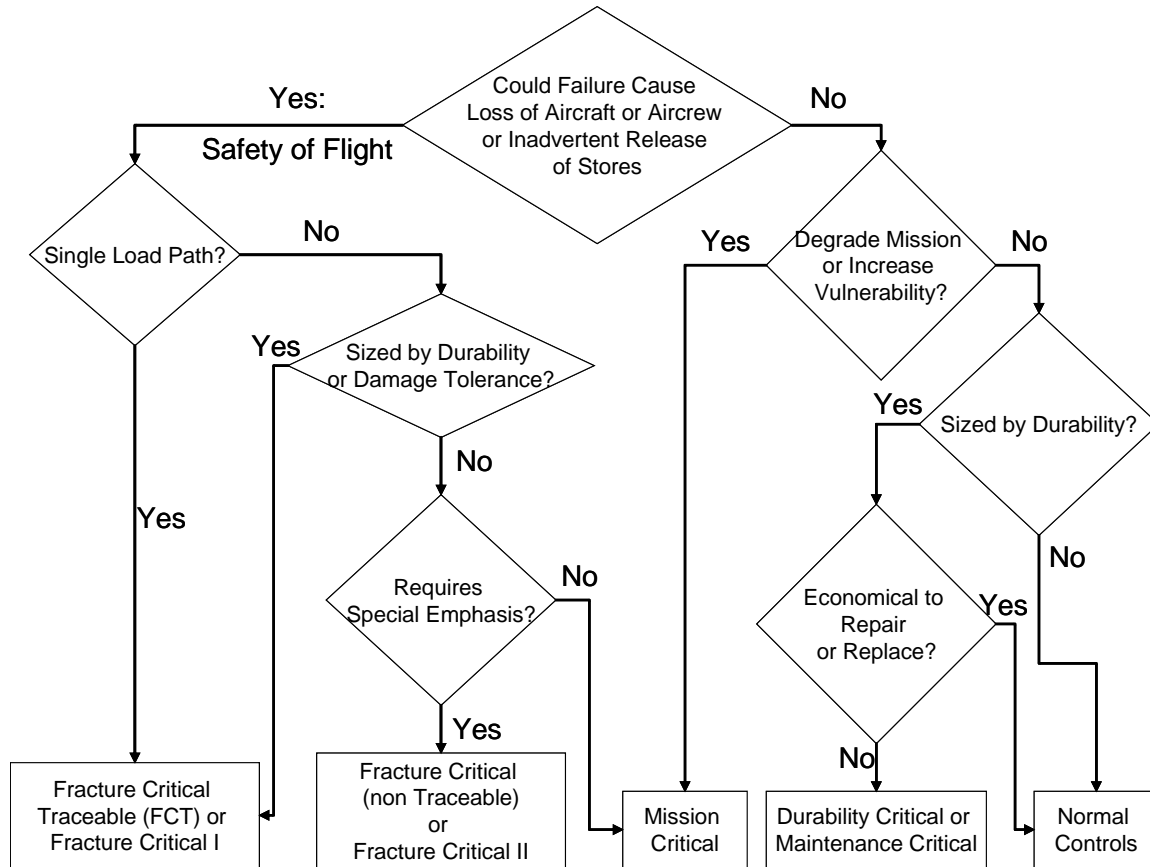


Figure 2.2.1-2 MIL-STD-1530C Parts Classification Decision Tree

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2.3 Accepted Methods of Strength Analysis

For new aircraft programs, the hierarchy of precedence for approved stress analysis methods is as follows. The primary sources are PM-4057 for general guidance and metallic methods and PM4056 (Reference 2-4) for specific guidance and analysis methods for composite materials, as tailored for use on the program through program-specific and/or customer-generated guidance. In the absence of a stress analysis approach in PM-4057, legacy methods such as the Stress Memo Manual, Reference 2-1 and the Structural Analysis Manual (SAM), Volumes 1 and 2, Reference 2-2 are recommended. In addition, there are a number of legacy methods reports from both the Lockheed and General Dynamics heritage companies which can provide guidance. Failing any in-house methodologies, outside sources such as the USAF Stress Analysis Manual, Reference 2-39, NACA reports, ESDU papers, and various textbooks may be used. The legacy manuals, some of the in-house reports as well as some of the NACA reports and the USAF Stress Analysis Manual are posted on the LMASAM website.

A discussion of what is not acceptable might also be appropriate. Most, if not all, aerospace companies hold their stress analysis manuals and tools as company proprietary information; thus, **stress analysis manuals from other Aerospace companies are not acceptable references unless one of the following is true:**

- They come from one of the heritage companies that make up Lockheed Martin, such as the Lockheed Aircraft Company, General Dynamics, Consolidated Vultee, etc. The LTV manual is also a part of this legacy.
- They are approved for use on specific aircraft programs because of teaming arrangements. In the event that a manual (or computer tool) is used in this manner, it should not be removed from the program area and should not be used on other Lockheed Martin aircraft.

If you have any doubt as to the applicability of a manual or tool, please contact 6E5 Core management for clarification.

2.3.1 Sign Conventions and Coordinate Systems

There are several sign conventions of concern when discussing stress analysis. Because the stress analyst must combine inputs from several different organizations, it is important for the analyst to understand the sign conventions that each of these organizations use and to be prepared to adjust these conventions to make them compatible.

2.3.1.1 Global Aircraft Coordinate Systems

Different global coordinate systems may be used to describe the aircraft by the Design organization and other groups within the structural analysis community such as External Loads and Dynamics. The Design community generally uses a “right hand rule” system. with the x axis positive aft along the fuselage, the y axis positive outboard along the right wing and the z axis positive upward. The External Loads on the aircraft are often defined using a “left hand rule” coordinate system where the x axis is positive aft along the fuselage, the positive y axis is outboard along the left wing and the z axis is positive up. Figure 2.3.1-1 illustrates the differences between the two systems, i.e. the direction of the y-axis.

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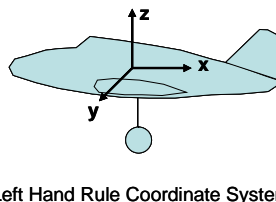
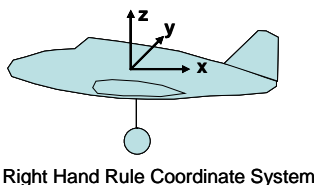


Figure 2.3-1 Aircraft Coordinate Systems

Additionally, the External Loads group defines aircraft accelerations which are used to determine the inertia loading for equipment or fuel and fuel slosh loads. A check should be made to ensure that the sign of the load agrees with the physical behavior of the structure, such as up or down bending. Inertia accelerations are the opposite of applied loads and the sign of the load factors must be reversed to obtain the correct inertia loading. When obtaining this information from the Loads group care must be taken to ensure that precisely what is being provided is understood and what coordinate system applies. Additionally, it should be documented whether the loads are limit or ultimate and how they should be combined, if appropriate.

2.3.1.2 Finite Element Model Coordinate Systems

Finite element analysis (FEA) routines have a global system which may be different from that used by the design organization or the external loads group. Additionally, there are options within many of the FEA routines to define local coordinate systems and material coordinate systems. Internal load results can be output and provided to the analyst in any of the FEA coordinate systems and it is incumbent on the stress analyst to determine in what system the internal loads information is provided. If loads from the FE model must be combined with information obtained directly from the External Loads group, the stress analyst must ensure that the sign conventions are compatible prior to this combination.

Aerodynamic pressure loads as defined by the External Loads group are generally all positive crushing on the airplane, while internal pressures are generally defined as positive bursting the tank or cavity. The definition of the sign convention needs to be provided by the External Loads group with the pressures. The direction vector, called a surface normal, for pressure loads from the NASTRAN FEA model is based on the sequence in which the grid points for the shell elements on the surface are defined. If the model is inconsistent within a particular panel or from panel to panel within the FEM, this can create a situation where the chance for error is significant. Every effort should be made to consistently model surface panels with consistently inward or outward pointing normals, i.e. the sequencing of the grids for the shell elements are all either clockwise or all counterclockwise. This is illustrated in Figure 2.3.1-2

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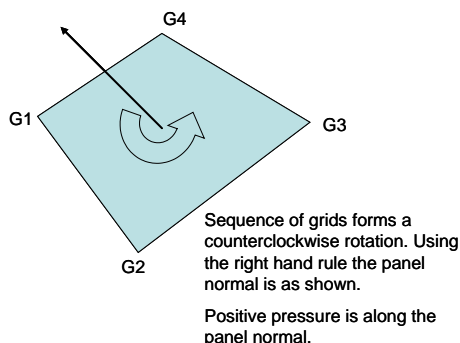
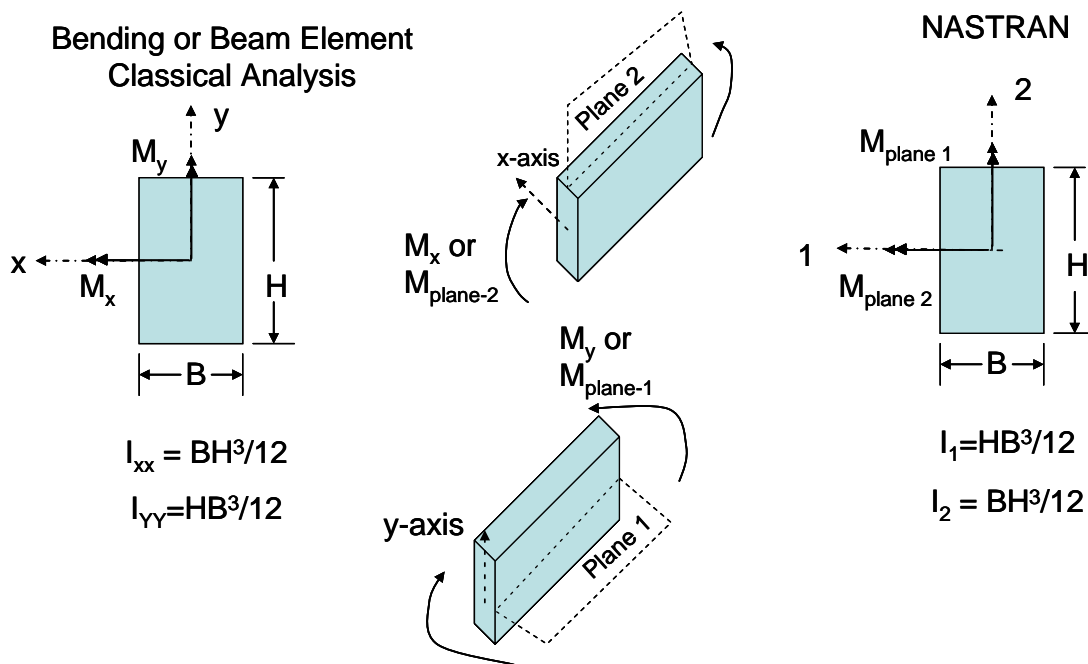


Figure 2.3-2 Determination of Positive Pressure Direction in NASTRAN FEM

One other coordinate system difference in FEA analysis must be addressed. In classical stress analysis bending moment is generally depicted as being about a particular axis, as is shown on the face of a section in Figure 2.3-3. In FEA analysis it is described as being in a particular plane. When the moments of inertia for bending elements in a NASTRAN model are described, it is crucial that they be consistent with the NASTRAN convention and the resulting loads be used appropriately. Figure 2.3-3 illustrates the differences between the classical approach and the NASTRAN FEA approach. Other finite element codes may use other sign conventions.



Note difference in orientation of moment of inertia between classical stress analysis and NASTRAN approaches

In this example, M_2 bending moments are the equivalent of M_x bending moments

Figure 2.3-3 NASTRAN Axial Element Coordinate Systems

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2.3.1.3 General Structural Analysis Sign Conventions

Tension loads or stresses are generally depicted as positive values while compression loads or stresses are negative.

2.3.2 Deflection Effects

There are numerous cases where structural members, when loaded, deflect or rotate such that secondary loads are induced by these deflections or rotations. Materials with low yield stress may increase these deflections and rotations significantly. Such secondary effects can become significant and therefore must be considered.

It has been a long standing practice in certain cases to design structural members for ultimate loads including the effects of deflections at the limit load level only. This criterion, based on small deflection theory, was used and has been successful in the design and substantiation of major components on Lockheed Martin aircraft. If, however, small deflection theory does not apply, large deflection effects should be included. Examples of where this might be applicable are the landing gear and its back-up structure and in control surface analysis.

The purpose of this section is to define specific policies relative to secondary effects caused by deflections. The stress engineer is responsible to ensure that these policies are followed in the design.

2.3.2.1 External Loads Design Analysis Policy

External loads which are a function of structural deflection may be based on limit deflection, not ultimate deflection, providing written approval is given by the Program Chief Structures Engineer or Structures Manager.

2.3.2.2 Internal Loads Design Analysis Policy: Assemblies

The dynamic deflections of some assemblies, such as landing gears, under impact loading conditions at design ultimate loads do not approach static deflections under static loading conditions at design ultimate loads. The stress analysis of such assemblies for impact loads may be based on limit deflection at ultimate load. The decision to use limit instead of ultimate deflections shall be based on the structural specification and static testing requirements. This decision shall be documented as a part of the Program Structural Design Criteria document.

2.3.2.3 Internal Loads Design Analysis Policy: Individual Components

The loads from finite element models should not be used until the reasonableness of the deflections of the nodes, and the loads and stresses in the elements have been verified.

2.3.2.3.1 Compression Members

Individual components such as frames, stringers, and struts designed for compression must react ultimate load at their individual ultimate deflection. Every column has a certain amount of unintentional eccentricity and is, in reality, a beam-column. Neglecting this eccentricity can be justified if its effect on allowable load is small, since the eccentricity arises from manufacturing tolerances, and it is accepted practice to stress analyze to nominal dimensions when adverse tolerances do not exceed 10 percent of a critical section. In the short column range the effect of unintentional eccentricity is negligible; in the long column range it is severe. In deciding whether to consider eccentricity in any particular case, judgment must be used, guided by a calculation in borderline cases. Such a calculation should account for the reduction in allowable load due to the combination of eccentricity, and reduced cross-section area effects. One of the following approaches may be used to account for eccentricity or end effect, whichever is the most severe:

- (a) To account for unintentional eccentricity due to tolerances and manufacturing inaccuracies in Class IA, vital single load path compression members, an assumed magnitude equal to ½ the difference between nominal (usually zero) and the maximum total eccentricity or tolerance buildup permitted by the drawing shall be used.

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For further details relative to drawing tolerances, see Section 2.6. To account for unintentional eccentricity in Class IB, multiple load path structure, an assumed magnitude equal to pin-ended length/1000 shall be used. This is based on experience and the overall correlation to tests. This eccentricity is always taken in the critical direction. For Class II structure unintentional eccentricity is normally not considered.

(b) Joint friction end moments due to rotation of attached structure shall be applied in the stress analysis of all pin-ended compression members when this is a non-relieving effect.

The above items (a) and (b) shall not be applied concurrently; only the greater of the two shall be used.

Some of the legacy major-component analysis programs, such as the transport aircraft integrally stiffened wing skin programs, have a built in check for beam column assuming an eccentricity of pin-ended span length/400.

2.3.2.3.2 Lug/Clevis Parts Subject to Tang Bending

If over ten percent of the total member strength is required to accommodate the effect of ultimate deflections, the criteria in Section 5.4.3.5, lug or clevis tang bending, shall be applied to ultimate loads with limit deflection secondary effects. It must be understood that the stress analysis for ultimate deflection cannot be deviated from except in these areas.

2.3.2.3.3 Control Rods

Control Rods with rod end bearings must carry limit load with an additional end moment due to a frozen bearing, reflecting structural rotation from normal flight load. This criterion is similar to 2.3.2.3.1(b) above.

2.3.2.3.4 Movable Surfaces

Side loads from structural deflection must be applied to flap track rollers. Similar loads must also be applied to spoilers, slats, ailerons, elevators, rudders, speed brakes and other movable surfaces. Deflections of the primary support structure can induce loads at the hinge/attach points which must be taken into account in the analysis. This phenomenon can be especially severe when a wing, for instance, is undergoing maximum bending while a flap is at full deflection, positioning the surface nearly perpendicular to the wing where its in-plane stiffness is substantial.

2.3.2.3.5 FEM Loads

The loads from finite element models must be supplemented with the crushing effect in beams undergoing diagonal tension, Reference 2-10, secondary loads in caps and stiffeners due to buckling in webs and skins, and rib or spar crushing due to wing bending.

Additional loads due to internal (fuel, vapor, dry bay) and external pressures also need to be included. Pressures may or may not be included in the finite element model and even when ostensibly included, the local effects may be incomplete or incorrect if the model is too coarse.

2.3.2.4 Use of Materials whose Allowable Yield Stress is less than Two-Thirds of Ultimate Stress

Aircraft are generally designed to the criteria: Structure shall withstand static ultimate load without failure and static limit load without detrimental permanent deformation.

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The yield properties of the material are good indicators of a potential deformation, yield or creep problems. If F_{cy} or F_{ty} is less than two-thirds of F_{tu} , or F_{bry} is less than two-thirds of F_{bru} , more analysis will be required.¹ Examples of this are 2024-T42 aluminum, depleted uranium and the austenitic stainless steels. A typical problem is faced in using annealed AISI 301 stainless steel where F_{tu} is 73 ksi; however, F_{ty} is only 26 ksi or 35% of F_{tu} . Other materials may exhibit this characteristic and the analyst should be aware when the material of choice has this relationship between yield and ultimate properties.

In general, for 300 series steel, 1/2 hard austenitic heat treat is the minimum hardness recommended for parts which are classified as Class IA, IB, or IIA. For most parts with significant applied load, 6061-T4, 6013-T4 and 2024-T4 should be aged to -T6.

Joint allowables found in Reference 2-1, 2-4 or 2-5 will be appropriately annotated when the fastener material yield strength or the joint material bearing yield strength is less than 2/3 of the ultimate strength, making the joint yield critical.

2.4 Freebody Diagrams

Freebody diagrams are a very important tool for understanding the loading within the structure and communicating with other engineers who are using the analysis. In order to properly size a component of structure, it is crucial that the analyst understand the loads which are applied to and must be reacted by the part and that the loads make sense in the behavior of the airplane. The freebody can be as detailed as a summary of all of the loads on individual fasteners within a joint or as top level as all of the gross loads in a spar cap for a particular condition.

Post-processing software such as PATRAN or VISION can aid in the development of or even produce a freebody; however, in some cases they don't provide adequate information and the freebody diagram must either be drawn by hand or supplemented.

The following is a freebody checklist:

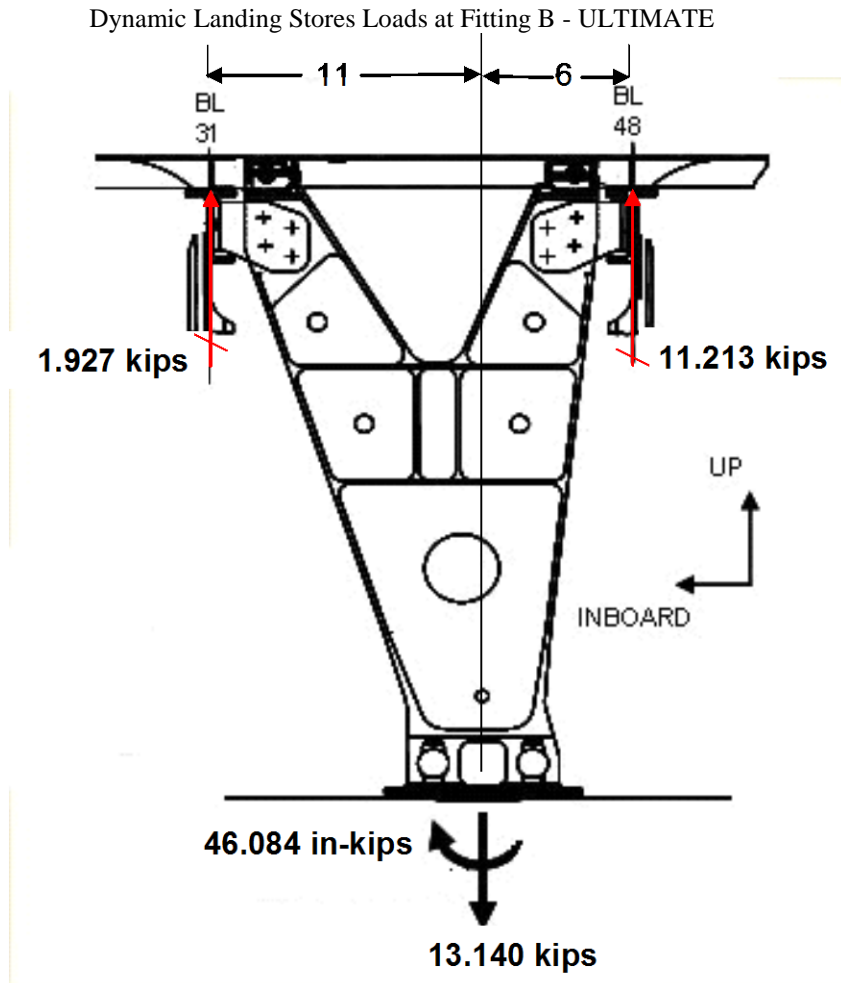
- Single loading condition
- Identify whether the loads shown are limit, ultimate or at some other level
- All loads shown and balanced, with load case identified.
- Source of the loads provided: e.g., finite element model (what version), external loads report, etc.
- Figure should be large enough to be readable and provide information of how the part fits into the larger assembly but not be cluttered with unnecessary detail.
- Provide any pertinent geometry dimensions.
- Assumptions are clearly stated.
- Sign conventions are noted.
- Verify that the summation of applied loads and reactions equal 0.

If a part or component has been modeled in a detailed standalone FEM, the applied loads for the detailed model should come from a freebody of the component taken from some larger, coarse grid model. The freebody should be documented in the FEM report or the stress analysis so that it could be recreated if necessary. Alternately, the detail FEM can be inserted back into the coarse mesh FEM to ensure the proper boundary conditions and loading.

The following figures provide some examples of freebody diagrams. Figure 2.4-1 is a stores fitting which is shown as it is attached to the aircraft. The loads and the calculations are clearly identified for the critical condition and the assumptions used in the calculation are stated.

¹ This assumes that there is a 1.5 ultimate factor of safety and the aircraft criterion is no yielding at limit load. If the aircraft criterion is no yielding at some percent of limit load greater than 100% (i.e. 115% or 125%) then the analysis burden increases.

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Calculate the contribution to the reaction due to the moment:

$$P_{z\text{-moment}} = 46.084/(6+11) = \pm 2.711 \text{ kips}$$

Calculate reactions at BL 31 and BL 48

$$P_{z-48} = 11/(11+6)(13.140) + 2.711 = 11.213 \text{ kips}$$

$$P_{z-31} = 6/(11+6)(13.140) - 2.711 = 1.927 \text{ kips}$$

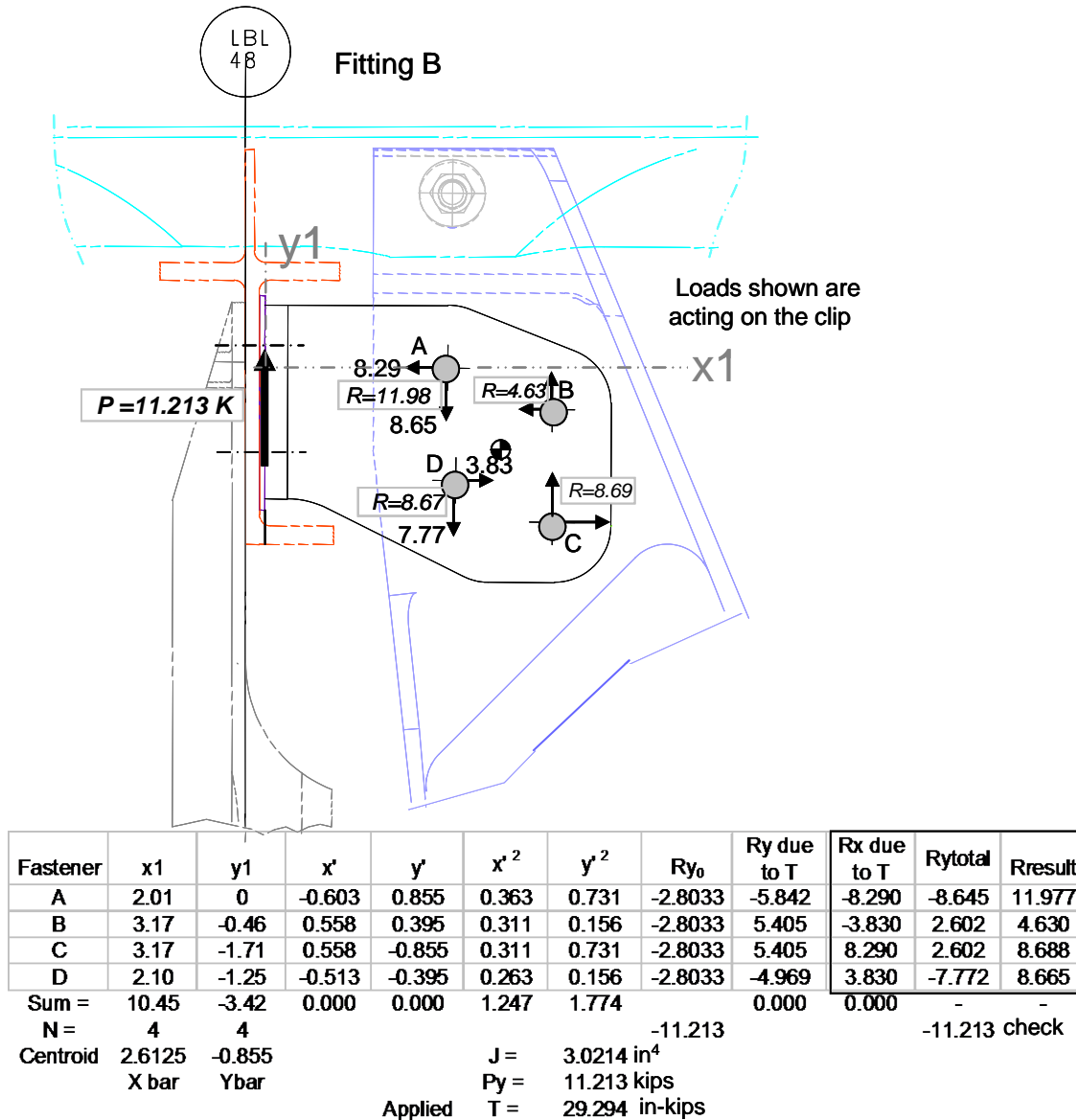
$$\Sigma F_z = 11.213 + 1.927 - 13.140 = 0.0 \text{ (check)}$$

Figure 2.4-1 Freebody Example – Stores Fitting

Figure 2.4-2 is the freebody of the splice clip at BL 48 of Figure 2.4-1. The applied load is 11.213 kips and the fastener pattern is eccentric, requiring the use of the technique for rigid fastener pattern distribution provided in Section 5.2-1. The geometry and results are provided in the table below the sketch and the resultant load on each fastener is shown in the sketch. This provides an illustration that the stress analysis of parts within an assembly may require multiple freebody diagrams to adequately describe the loading in any individual part.

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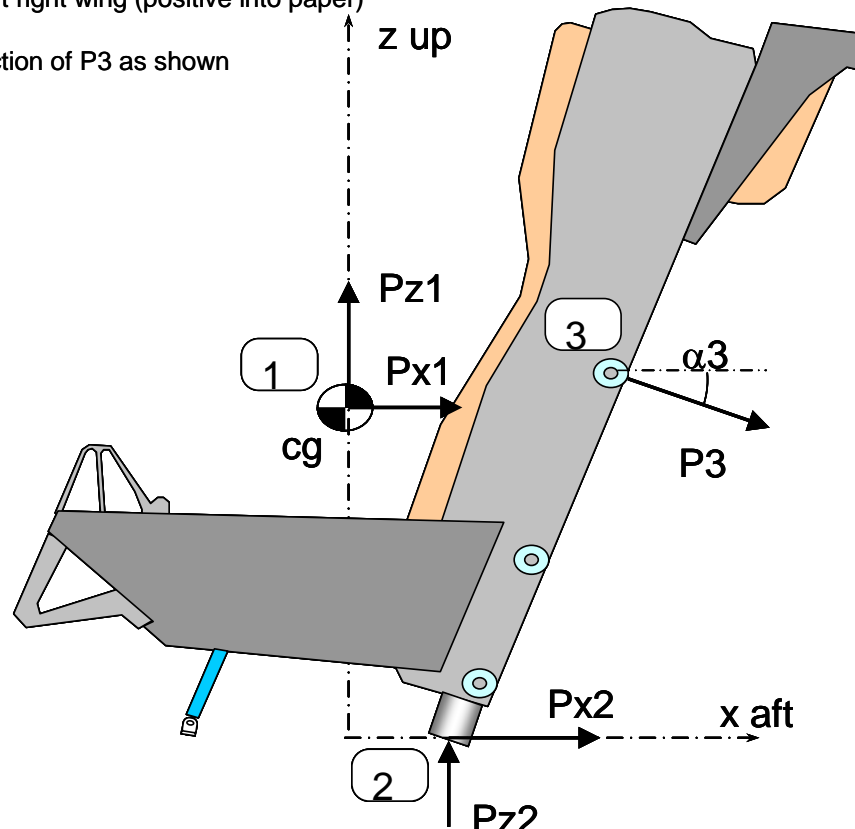
Note x' and y' are distances from the centroidal axes.

Figure 2.4-2 Freebody Diagram – Splice Clip

Figure 2.4-3 is the freebody of an ejection seat. In the analysis, prior to this freebody, would be a discussion of the weight of the seat and pilot and the velocity and acceleration levels during ejections and a derivation of the applied loads based on an external loads report. The geometry is provided, not on the sketch but in tabular form. Each of the pertinent points is clearly labeled on the sketch and the assumptions and sign conventions are clearly stated.

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Positive sign convention for loads applied to seat:
 Px - aft
 Py - out right wing (positive into paper)
 Pz - up
 Positive direction of P3 as shown



Inertia loads are applied to the seat cg.

The seat is attached to the airplane at the base of the rocket catapult housing (Pt. 2) and rollers (Pt. 3).

Roller loads are perpendicular to the seatback rails as shown. P3 represents sum of rollers on 2 seat rails.

Loadpoint geometry and applied loads

	x	z	Px	Pz	angle, $\alpha 3$	Take moments about 2;
1	0	16	-88	-6865		$-(x1 * Px1) - (x2 * Pz1) - (z3 / \cos \alpha 3 * P3) = 0$
2	5	0				$\Sigma Fx \rightarrow; Px1 + Px2 + P3 \cos \alpha 3 = 0$
3	12	22			17.65012	$\Sigma Fz \uparrow; Pz1 + Pz2 - P3 \sin \alpha 3 = 0$

Balancing loads (solved by simultaneous equations of summation of forces shown above)

coefficient matrix [K]				vector	INV [K]			soln	
0	0	-23.087	-35725	0.04128	1	0	0	Px2	-1387
1	0	0.95293	88	-0.0131	0	1	0	Pz2	7334
0	1	-0.3032	6865	-0.0433	0	0	0	P3	1547

check

ΣFx	ΣFz	$\Sigma M3$
0	0	0

Figure 2.4-3 Freebody of Ejection Seat Loads

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2.5 Margin of Safety Calculation

As defined in Section 1.5, the “Margin of Safety” is a quantitative descriptor of the strength of a part or assembly. This descriptor is a fundamental element of the process normally used in aircraft stress analysis to verify that parts or assemblies are adequately strong to perform their desired functions and to withstand potential extreme conditions.

Over decades of experience in aircraft stress analysis, it has been found that structures frequently exhibit the capability to redistribute loads in beneficial ways after parts begin to operate in the plastic range; *i.e.*, the part is beyond the basic elastic behavior. As a result, failure loads would be substantially under-predicted by elastic analysis. Therefore, the usual practice in aircraft stress analysis to deal with total loads rather than stresses; of course, stresses are the analysis mechanism used to predict the load-carrying capability of a total structure, but it is the load capacity that is of the most interest. Because of this, analysis proceeds to characterize the strength of a part or assembly in terms of load.

The margin of safety is a way to express the degree to which the structure is able to sustain the anticipated loads of operational use. Consider first the ratio of the “ultimate” capability of the structure – *i.e.*, “allowable load” – to the expected loads from operation – *i.e.*, “applied load”. If the allowable load is divided by the applied load, the result tells how strong the part is compared to the demands of use. This particular ratio is also sometimes called the factor of safety. The amount by which this factor exceeds unity is a “margin” of capacity. A margin of zero at ultimate load represents the predicted level at which the part will fracture; *i.e.*, the applied load, with required safety factor, equals the allowable load. In order to make this more relevant to the desire to keep weight to a minimum, and realizing that a margin of zero was optimum, the ratio minus 1 was selected as a key measure and has come to be known as the Margin of Safety (M.S.). It can be written as

$$M.S. = \frac{P_{all}}{P} - 1 \quad \text{Equation 2.5.0-1}$$

where

P_{all} is the allowable load (lbs)

P is the applied load (lbs)

The load carrying capacity at $M.S. = 0.0$ is just enough to be safe but not so much as to add unneeded weight. A $M.S. < 0.0$ is clearly unacceptable, as it indicates the applied loads will exceed the capacity of the structure.

The allowable load represents a specific failure mode and load level for the part; thus, a part may have several margins of safety quoted, to address the potential failure modes. This calculation also includes any additional factors required by specification, program guidance or this manual. If the applied load is at limit, it would be multiplied by the required factor of safety (usually 1.5 for manned aircraft) prior to the margin calculation at ultimate load. The use of load for the calculation of margin is preferred and stress may only be used if the stress is linear with load, *i.e.* fully elastic to failure with no plasticity or other non-linear behavior.

For a structure carrying a single load, the Margin of Safety calculations can be as simple as taking an allowable load, dividing by an applied load and subtracting 1. For structures subject to a combination of loads, a margin calculation is complex and requires further guidance. The approach taken in aerospace structures applications to address such combinations is through the use of interaction equations. These equations relate the behavior of structure when loaded in multiple directions through the use of polynomial equations involving a load ratio denoted by “R”. The concept for this approach rests on the idea of utilization. Simply stated, how much of the part’s capacity is being utilized? Combinations of different load ratios are sometimes called utilization factors. These interaction relationships are generally derived through the use of empirical data. This section will provide the equations for some common interactions and provide a discussion of how to use interaction curves when a closed form solution is not available.

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When calculating margin of safety, the preferred approach is to use applied load and allowable load rather than applied stress and allowable stress, or applied stress converted to applied load. This is due to the fact that in many types of analysis, stresses may be nonlinear due to plasticity, but the load remains linear.

Here is a summary of what is contained in this Section

- Section 2.5.1 Provides some general information on how to use interaction curves
Presents closed form solutions for curves of specific form.
- Section 2.5.2 Tabular listing of specific interaction equations for thin sheet and tubular structure which involve the curves presented in Section 2.5.3. Curves are presented for both stability and failure interactions.
- Section 2.5.3 All interaction curves
- Section 2.5.4 Presents an interaction approach for the solution of a more generalized stress state for compact structure where stability is not applicable.
- Section 2.5.5 A discussion of Ratio to Requirements versus Margin of Safety

2.5.1 Interaction Curves

Various types of loading for which interaction curves have been obtained either theoretically or by means of tests are listed in Section 2.5.2. For each loading there is given the interaction equation and/or a reference to a plotted interaction curve. Referring to Figure 2.5.1-1, the margin of safety can be obtained from an interaction curve plot as follows:

- Calculate the load ratios for loadings and failure modes under consideration. Typically
 $R = \text{Applied load} / \text{Allowable load}$
- Plot the load ratios R_1 and R_2 as point A.
- Draw the straight line OA and extend to B, its intersection with the interaction curve.
- Then the margin can be calculated from

$$M.S. = \frac{OB}{OA} - 1 = \frac{R_{1-ALLOW}}{R_1} - 1 = \frac{R_{2-ALLOW}}{R_2} - 1 \quad \text{Equation 2.5.1-1}$$

All three equations provided should result in the same margin of safety and any one of them can be used as is convenient.

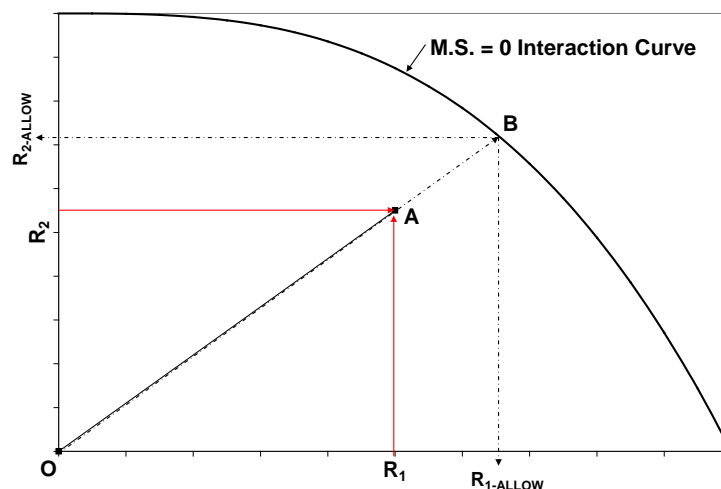


Figure 2.5.1-1 Generic Interaction Curve

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In an interaction plot, the shape of the curve is a function of the order of the terms in the polynomial; however, when plotting point A, the load ratios are plotted directly. Similarly, when the allowable ratio is read from the curve, it is read directly.

Some of the interaction equations in Section 2.5.2 are given in terms of R_{bar} . In this case, no interaction curve is required and the margin can be obtained directly from

$$M.S. = \frac{1}{R_{bar}} - 1 \quad \text{Equation 2.5.1-2}$$

where R_{bar} is the interaction obtained from Section 2.5.2

Additionally, some general closed form solutions for Margins of Safety exist for different forms of the interaction curves. These provide the ability to calculate a margin rather than reliance on interaction curves and the error that can be introduced from reading the curve. These equations are generically provided in terms of R_1 , R_2 , and R_3 and care must be taken to correctly define R_1 , R_2 , and R_3 in terms of the particular stress ratios being analyzed. These general equations are summarized in Table 2.5.1-1.

Table 2.5.1-1 Closed Form Interaction Equation Solutions

Case	Interaction Formula	Margin of Safety
a	$R_1 + R_2 = 1$	$M.S. = \frac{1}{R_1 + R_2} - 1$
b	$R_1^2 + R_2^2 = 1$	$M.S. = \frac{1}{\sqrt{R_1^2 + R_2^2}} - 1$
c	$R_1 + R_2^2 = 1$	$M.S. = \frac{2}{R_1 + \sqrt{R_1^2 + 4R_2^2}} - 1$
d	$R_1 + R_2 + R_3^2 = 1$	$M.S. = \frac{2}{R_1 + R_2 + \sqrt{(R_1 + R_2)^2 + 4R_3^2}} - 1$
e	$R_1 + R_2^2 + R_3^2 = 1$	$M.S. = \frac{2}{R_1 + \sqrt{R_1^2 + 4(R_2^2 + R_3^2)}} - 1$

In addition to the interaction equations depicted in Section 2.5.2, there are other analysis-specific interaction formulas provided in various sections through out the manual.

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2.5.2 Interaction Equations

Case No.	Type of Structure and Loading	Equation or Curve	Figure	Reference	Remarks
Unstiffened Flat Rectangular Panel – Initial Buckling Causes Failure					
Note: For cases where equations are applicable with tension in lieu of compression, see remarks					
1	Shear-compression in longitudinal direction	$R_c + R_s^2 = 1$ $M.S. = \frac{2}{R_c + \sqrt{R_c^2 + 4R_s^2}} - 1$	2.5.3-1 R ₁ =R _c R ₂ =R _s	2-27, 2-30, 2-32, 2-36	Includes tension as negative compression, using compression allowable.
2	Shear-compression in transverse direction	$R_c + R_s^2 = 1$ $M.S. = \frac{2}{R_c + \sqrt{R_c^2 + 4R_s^2}} - 1$	2.5.3-1 R ₁ =R _c R ₂ =R _s	2-27, 2-35	Very conservative when a/b>2; See References 2.1-27 or 2.1-35 for more precise curves, which also include tension
3	Shear – in-plane bending	$R_{bar} = (R_s^2 + R_b^2)^{1/2}$	2.5.3-1	2-18	
4	Compression in-plane bending	$R_b^{1.75} + R_c = 1$	2.5.3-2	2-18, 2-38	
5	Transverse and longitudinal compression, a/b=1	$R_{bar} = R_x + R_y$	2.5.3-1	2-15	Applies only to nearly square panels (a/b ≤ 1.414, where a is the longer side). Correct for tension, only for square panels with small tension (R _x or R _y between 0 and -0.75); treat tension as negative compression using compression allowable.
6	Transverse and longitudinal compression, a/b=∞	$R_x^3 + R_y = 1$	2.5.3-2	2-33, 2-34	Applies when a/b → ∞. R _x is longitudinal compression, R _y is transverse compression. Always conservative; See References 2.1-33 and 2.1-34 for more precise curves, which also include tension.
7	Transverse and longitudinal compression, 1 ≤ a/b ≤ 5	Ref. 2.1-33 and 2.1-34 provide stress equations.		2-33, 2-34	Applies when a/b is between 1 and 5. Includes tension as negative compression, using compression allowable.
8	Shear, compression and bending		2.5.3-3	2-16	Conservative as R _b → ∞ (reduces to R _s ^{1.5} + R _c = 1 instead of R _s ² + R _c = 1).
Unstiffened Curved Rectangular Panel – Initial Buckling					
9	Shear-compression	$R_c + R_s^2 = 1$ $M.S. = \frac{2}{R_c + \sqrt{R_c^2 + 4R_s^2}} - 1$	2.5.3-1 R ₁ =R _c R ₂ =R _s	2-29, 2-31	Applied only to long panels or panels with transverse curvature load in longitudinal compression.

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Case No.	Type of Structure and Loading	Equation or Curve	Figure	Reference	Remarks
<p style="text-align: center;">Thin-walled, Unstiffened, Circular Cylinders – Initial Buckling Causes Failure</p> <p>Note: All of the interaction equations given below for circular cylinders, cases 10-17, are probably not valid for very long ($L/D \geq 100$) or very short cylinders ($L/D < 10$).</p>					
10	Compression-bending	$R_{bar} = R_c + R_b$	2.5.3-1	2-26 Fig. 10	
11	Tension-bending	$R_{bar} = -0.7R_t + R_b$	--	2-26 Fig. 27	For allowable stress in R_t use compression buckling allowable.
12	Compression-torsion	$R_c + R_{st}^2 = 1$ $M.S. = \frac{2}{R_c + \sqrt{R_c^2 + 4R_{st}^2}} - 1$	2.5.3-1 $R_1 = R_c$ $R_2 = R_{st}$	2-26 Fig. 12; 21	
13	Tension-torsion	$R_{bar} = -0.3R_t + R_{st}$	--	2-26 Fig. 28	For allowable stress in R_t use compression buckling allowable.
14	Bending-torsion	$R_b + R_{st}^2 = 1$ $M.S. = \frac{2}{R_b + \sqrt{R_b^2 + 4R_{st}^2}} - 1$	2.5.3-1 $R_1 = R_b$ $R_2 = R_{st}$	2-26 Fig. 14	
15	Shear-bending	$R_{bar} = (R_s^2 + R_b^2)^{1/2}$	2.5.3-1	2-16, 2-21	f_s and f_b are each the maximum value over the section, based upon VQ/I and my/I stress distributions, respectively, even though the locations of the two maxima do not coincide. Use F_{st} from Section 7.2.
16	Compression-bending-torsion	$R_c + R_b + R_{st}^2 = 1$ $M.S. = \frac{2}{R_c + R_b + \sqrt{(R_c + R_b)^2 + 4R_{st}^2}} - 1$	2.5.3-1 $R_1 = (R_c + R_b)$ $R_2 = R_{st}$	2-26 Fig. 31	
17	Shear-bending-torsion	$R_b^p + (R_s + R_{st})^2 = 1$	2.5.3-4		When $R_{st}/R_s \leq 1$ then $p = 2 - 0.5R_{st}/R_s$; when $R_{st}/R_s > 1$ then $p = 1 + 0.5R_s/R_{st}$; f_s and f_b are each the maximum value over the section, based upon VQ/I and my/I stress distributions, respectively, even though the locations of the two maxima do not coincide. Use F_{st} from Section 7.2.

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Case No.	Type of Structure and Loading	Equation or Curve	Figure	Reference	Remarks
Thin-walled, Unstiffened, Elliptic Cylinders – Initial Buckling Causes Failure					
18	Transverse Shear-Bending (Shear applied transverse to longitudinal axis of cylinder)	$R_{bar} = (R_s^2 + R_b^2)^{1/2}$	2.5.3-1	2-16; 2-22	f _s and f _b are each the maximum value over the section, based upon VQ/I and my/I stress distributions, respectively, even though the locations of the two maxima do not coincide. Use F _{st} from Section 7.2.
19	Bending-torsion: Bending in plane of the major axis.	$R_{bar} = (R_{st}^2 + R_b^2)^{1/2}$	2.5.3-1	2-23	
Stiffened Curved Panels - Ultimate					
20	Transverse Shear-compression			2-37	Use analysis method of LR8693, Reference 2.1-37
Stiffened Circular Cylinders - General Instability					
21	Transverse Shear -Bending	$R_{bar} = (R_s^6 + R_b^6)^{1/6}$	2.5.3-5 m=6, R ₁ =R _s ; R ₂ =R _b	2-24	f _s and f _b are each the maximum value over the section, based upon VQ/I and my/I stress distributions, respectively, even though the locations of the two maxima do not coincide. Use F _{st} from Section 7.2.
22	Bending-Torsion	$R_b + R_{st}^2 = 1$ $M.S. = \frac{2}{R_b + \sqrt{R_b^2 + 4R_{st}^2}} - 1$	2.5.3-1 R ₁ =R _b R ₂ =R _{st}	2-25	
Corrugation with Skin - Ultimate					
23	Shear-compression	$R_s^{1.7} + R_c = 1$	2.5.3-2	2-20	

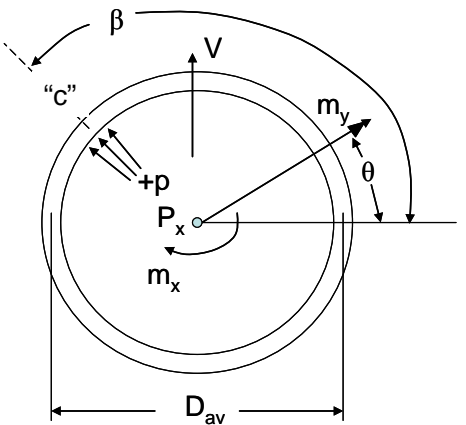
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Case No.	Type of Structure and Loading	Equation or Curve	Figure	Reference	Remarks
Round Tubes - Ultimate Use F_c , F_b , F_{st} , F_s from Section 10.6					
24	Compression-bending	$R_{bar} = R_c + R_b$	2.5.3-1	2-17	<p>General Remarks for round tubes: $R_c = f_c / F_c$ $R_b = f_b / (mc/I)_b$ f_b given as $mc/I = m/z$ $z = I/c = 2I/D_{av}$ and $(mc/I)_b$ is the bending modulus or modulus of rupture of the tube. m = Applied beam-column moment. Include beam column effects in applied moment if compressive axial load is acting. F_b from Section 10.6. $R_t = f_t / F_{tu}$ r, the exponent is given in Figure 2.5.3-8. m, the exponent is given in Figure 2.5.3-7. $(R_1^\infty + R_2^\infty)^{1/\infty}$ is equal to the larger of R_1 and R_2. To determine U_{bt} use $R_1 = R_b$ and $R_2 = R_b$ in Figure 2.5.3-6. Using the correct interaction curve based on r, determine $R_{t-ALLOW}$ or $R_{b-ALLOW}$. $U_{bt} = R_t / R_{t-ALLOW} = R_b / R_{b-ALLOW}$.</p>
25	Tension-bending	$R_t + R_b = 1$	2.5.3-6	2-15	
26	Bending-torsion	$R_{bar} = (R_{st}^2 + R_b^2)^{1/2}$	2.5.3-1	2-17	
27	Compression-bending-torsion	$R_{bar} = R_c + (R_b^2 + R_{st}^2)^{1/2}$	--	2-17	
28	Shear-Bending	$R_{bar} = (R_s^m + R_b^m)^{1/m}$	2.5.3-5 $R_1 = R_s$; $R_2 = R_b$	2-15	
29	Tension-torsion	$R_{bar} = (R_t^2 + R_{st}^2)^{1/2}$	2.5.3-1	2-19	
30	Tension-torsion-internal pressure	$R_{bar} = (R_t^2 + R_{st}^2 + R_p^2)^{1/2}$	--	2-19	
31	Tension-shear-bending	$R_{bar} = (R_s^m + U_{bt}^m)^{1/m}$	2.5.3-5 $R_1 = R_s$; $R_2 = U_{bt}$	2-15	
32	Compression-shear-bending	$R_{bar} = (R_s^m + (R_b + R_c)^m)^{1/m}$	2.5.3-5 $R_1 = R_s$; $R_2 = R_b + R_c$	2-15	
		$R_{bar} = (R_s^m + R_b^m)^{1/m} + R_c$	--	2-15 – use when R_c is large	

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Case No.	Type of Structure and Loading	Equation or Curve	Figure	Ref.	Remarks
Streamline Tubes - Ultimate					
33	Bending-torsion	$R_{bar} = R_b + R_{st}$	2.5.3-1	2-16	
Thick Wall Tubes (Landing Gear) - Ultimate					
34	<p>Axial Stress-Axial Bending-Shear-Torsion-Internal Pressure</p> <p>Note: Relieving loads should be set to zero.</p> <p>If it is not relieving the axial component of the pressure, p, is included in the axial load.</p> 	$R_{bar} = (U_{bt}^2 + R_{sst}^2 + R_{ht}^2 - U_{ht})^{1/2}$	--	Menasco Correspondence	<p>At any point "c"</p> <p>Tension is positive and compression is negative</p> <p>$R_x = P_x / (AF)$</p> <p>If P_x is positive then $F = F_{tu}$</p> <p>If P_x is negative then $F = F_c$</p> <p>Using the "LEFT HAND RULE"</p> <p>$R_b = -m_y \sin(\beta - \theta) / (zF_{bu})$</p> <p>negative R_b is compression at point "c"</p> <p>Combine R_x and R_b</p> <p>If R_x and R_b are not the same sign, then the lesser absolute value of R_x or R_b is set equal to zero.</p> <p>$U_{bt} = R_x + R_b$</p> <p>$R_{sst} = R_s + m_x / (2zF_{st})$</p> <p>$R_s = f_s / F_s$ Obtain F_s from Section 10.6. f_s is the maximum stress over the section</p> <p>$J = 2I$</p> <p>$R_{ht} = pD_{av} / (2tF)$</p> <p>If P_x is positive then $F = F_{tu}$</p> <p>If P_x is negative then $F = F_c$</p> <p>$U_{ht} = \text{Minimum}[(R_{ht}U_{bt}), 0.0]$</p>

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2.5.3 Interaction Curves

Figure 2.5.3-1 presents the curves for linear, quadratic in R_2 , and circular interactions. Since these can be solved directly, the curves are provided for reference only to give insight into the degree of interaction. A linear curve shows the maximum degree of interaction. As the order of the equation increases there is less interaction between the two load components.

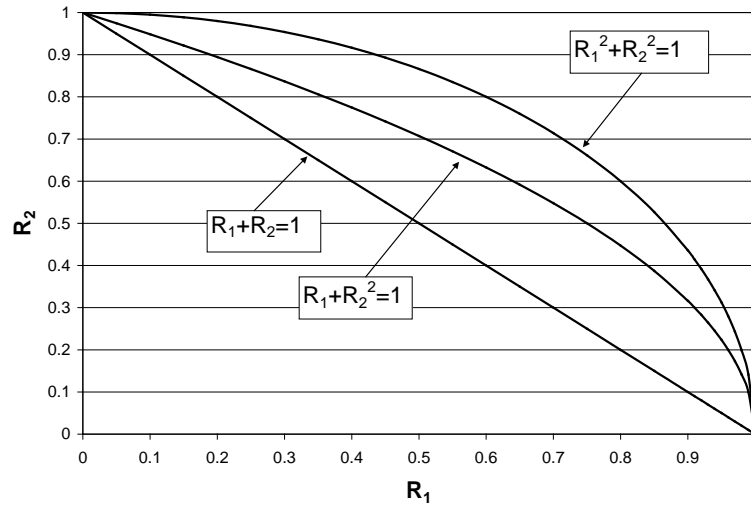


Figure 2.5.3-1 Curves for Various Interaction Equations

Figure 2.5.3-2 provides the curves for Interaction Cases 4, 6 and 23. These are of the general form $R_1^n + R_2 = 1$ where n is 1.70, 1.75 or 3. These do not have closed form solutions and either require graphical or iterative solution.

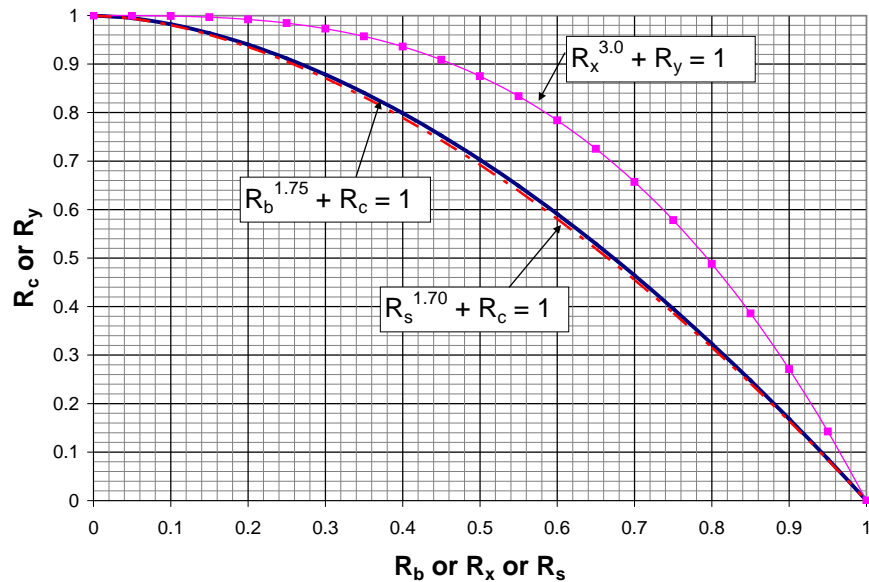


Figure 2.5.3-2 Interaction Curves for Interaction Cases 4, 6 and 23

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Figure 2.5.3-3 provides a graphical solution for interaction Case 8, shear, compression and in-plane bending. To obtain the margin, if $R_s > R_c$, enter the right half of the curve with R_b and R_s , plot that as Point 1 and draw a line from the origin through Point 1 and extend to the intersection with the correct R_c/R_s curve at Point 2. From Point 2, lines can be constructed back to the R_s and R_b axes to obtain the allowable shear ratio $R_{s-ALLOW}$ or bending ratio $R_{b-ALLOW}$. If $R_c > R_s$, enter the left half of the curve with R_b and R_c , plot that as Point 1 and draw a line from the origin through Point 1 and extend to the intersection with the correct R_s/R_c curve at Point 2. From Point 2, lines can be constructed back to the R_c and R_b axes to obtain the allowable compression ratio $R_{c-ALLOW}$ or bending ratio $R_{b-ALLOW}$.

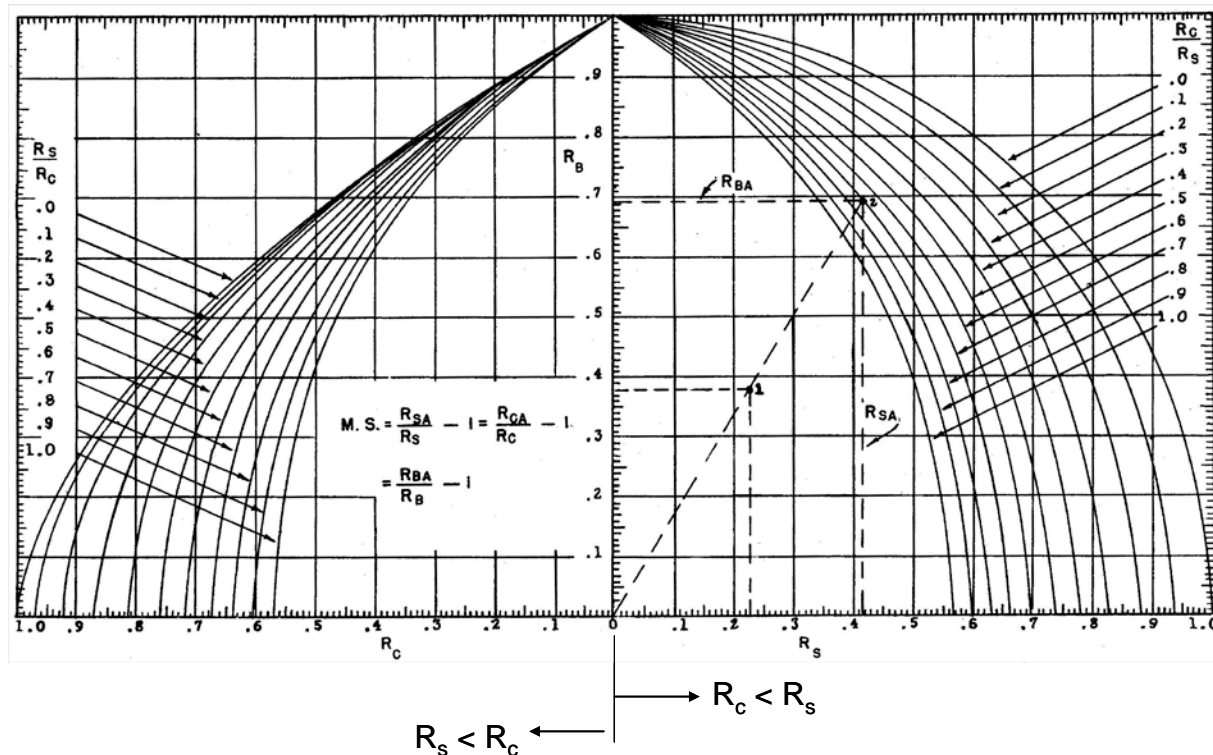


Figure 2.5.3-3 Shear, Compression and In-plane Bending Interaction Curve (Interaction Case 8)

Figure 2.5.3-4 provides the curves necessary to solve for the interactions of Case 17 for various values of p . This equation, except for specific values of the variable p , does not have closed form solutions. The curves can be used in the same manner as described by Figure 2.5.1-1. If the actual value of p falls between values plotted, interpolate between the two adjacent curves.

Figure 2.5.3-5 provides the curves necessary to solve for the symmetrical interactions of Cases 21, 28, 31 and the first interaction for Case 32. These can again be solved by a closed form approach, so they are given for the purpose of visualizing the degree of interaction. As the order of the equations gets higher there is less interaction. As $m \rightarrow \infty$, there is virtually no interaction.

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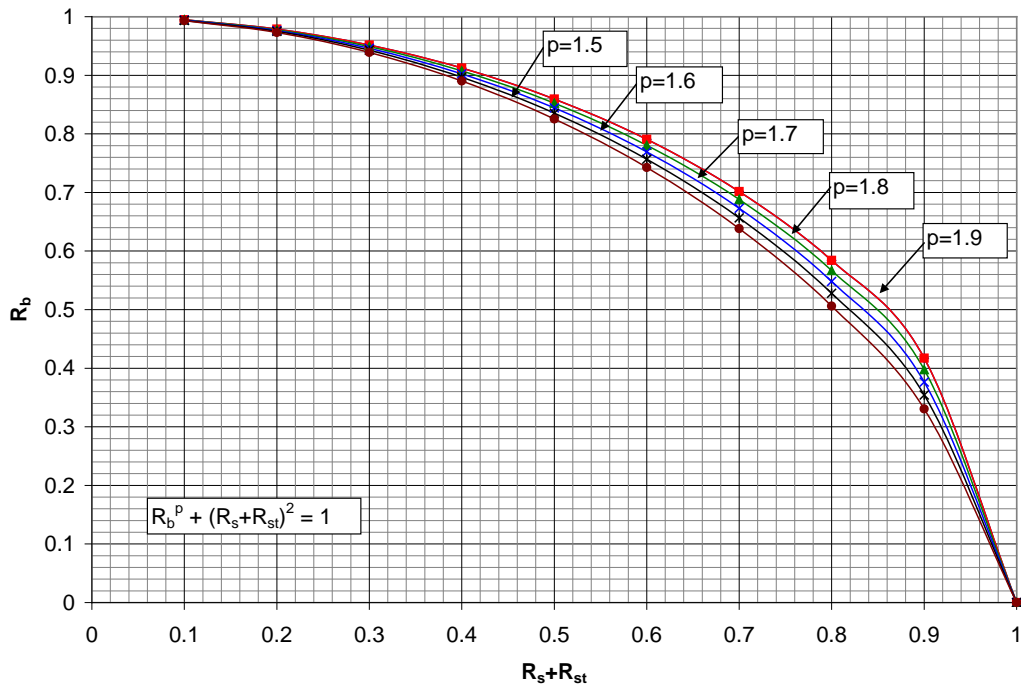


Figure 2.5.3-4 Interaction Curves For Case 17

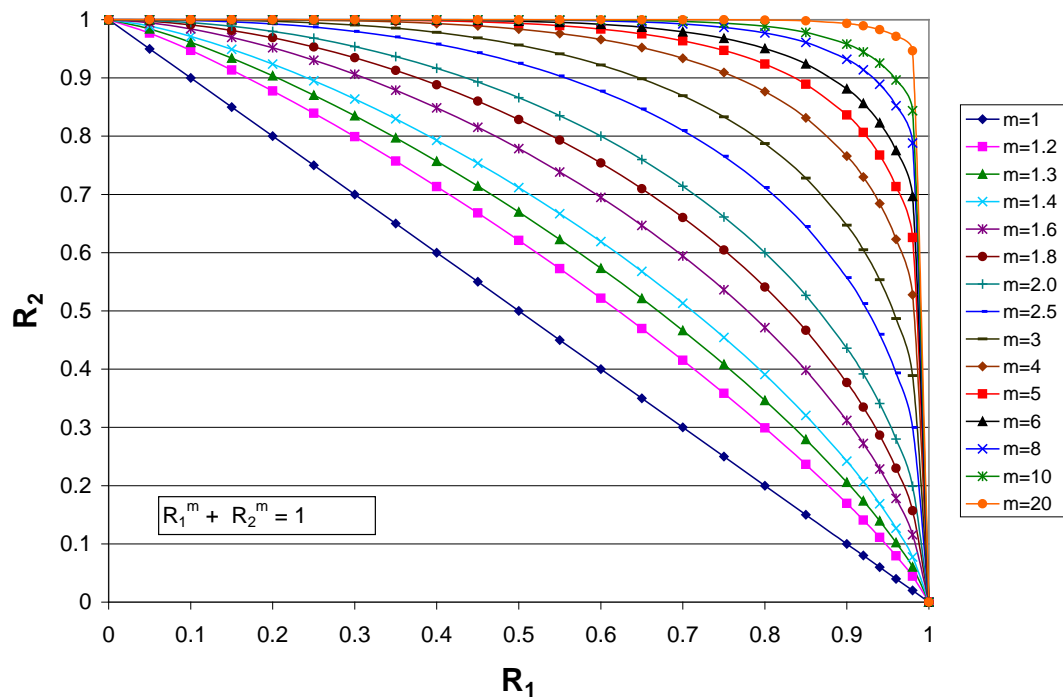


Figure 2.5.3-5 Family of Symmetric Interaction Curves for Interaction Cases 21, 28, 31 and 32

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Figure 2.5.3-6 provides the family of unsymmetric interaction curves for various values of the exponent, r . It is used in Case 25. It is of the same form as the curves presented in Figure 6.3.6-2 but for a larger family of exponents. If more detail is required for exponents between 1 and 2.6, see Figure 6.3.6-2.

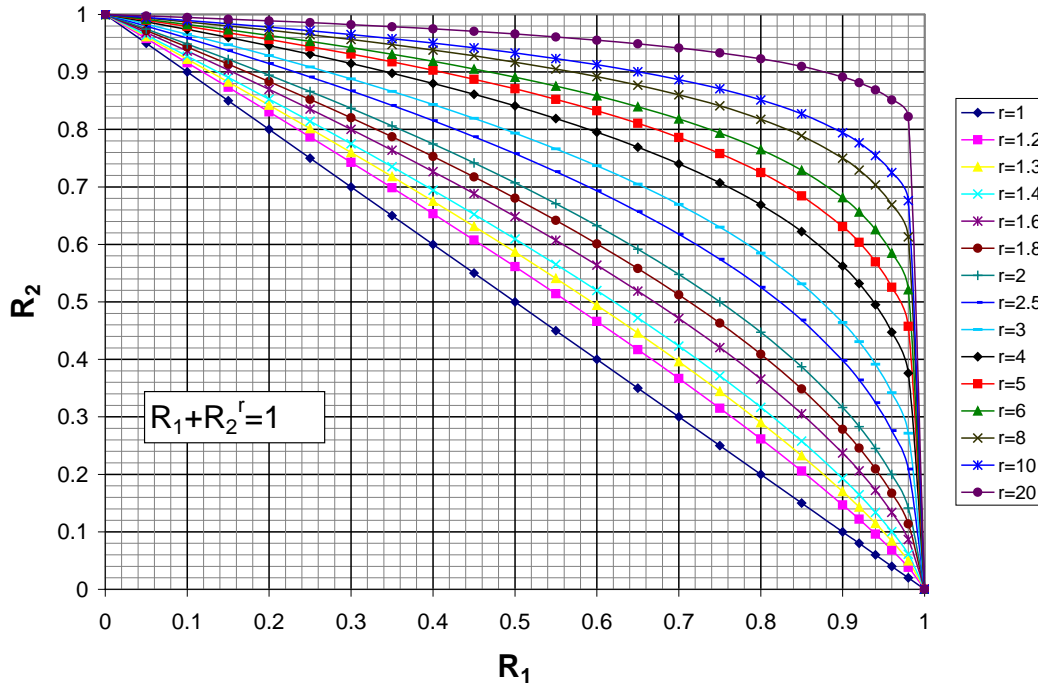


Figure 2.5.3-6 Family of Unsymmetric Interaction Curves (Case 25)

Table 2.5.3-1 gives numerical values and closed form Equations for the calculation of the exponent for the interaction of shear and bending for round tubes, Cases 29, 32 and 33. Figure 2.5.3-7 shows a plot of these values. The angle β is the least angle on the tube cross section between the point of maximum bending stress and the point of maximum shear stress. In the case of simple bending, the angle β is 90 degrees. For tubes, where the shear and bending may be applied at separate planes, the bending and shear interaction angle may be less than 90 degrees. If D/t is greater than 100¹ (thin walled tube), use the value for m obtained at D/t equal to 100. For intermediate angles of β , interpolation may be used.

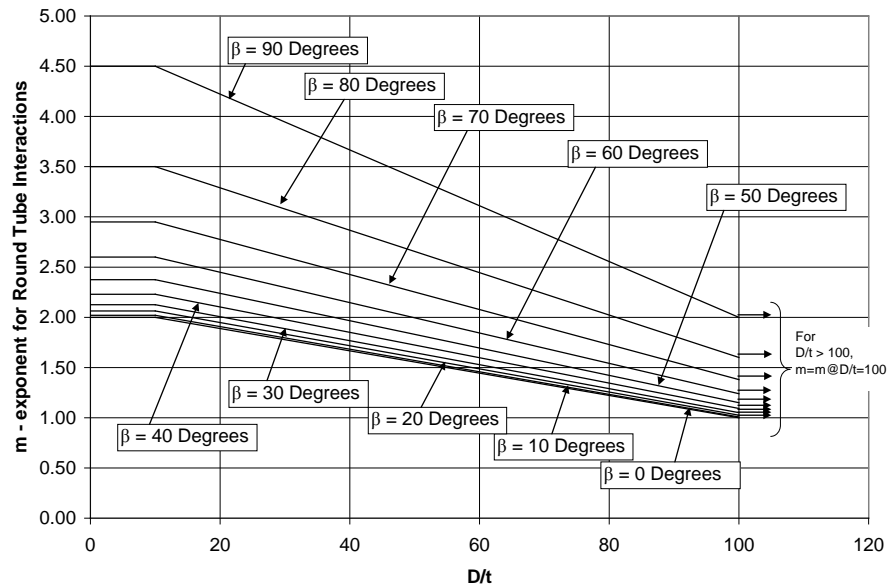
Table 2.5.3-1 Interaction Exponent, m for Cases 29, 32 and 33

Angle β (degrees)	m $D/t \leq 10.0$ Thick Tube	m $D/t \geq 100.0$ Thin Tube	m $10.0 < m < 100.0$
0	2.00	1.000	$m = -0.01111(D/t) + 2.111$
10	2.02	1.008	$m = -0.01124(D/t) + 2.132$
20	2.06	1.025	$m = -0.01153(D/t) + 2.178$
30	2.13	1.050	$m = -0.01194(D/t) + 2.244$
40	2.23	1.092	$m = -0.01264(D/t) + 2.356$
50	2.38	1.150	$m = -0.01361(D/t) + 2.511$
60	2.60	1.240	$m = -0.01511(D/t) + 2.751$
70	2.95	1.380	$m = -0.01744(D/t) + 3.124$
80	3.50	1.600	$m = -0.02111(D/t) + 3.711$
90	4.50	2.000	$m = -0.02778(D/t) + 4.778$

¹ D is the outer tube diameter and t is the tube wall thickness, so that $D/t=2.0$ is a solid round bar.

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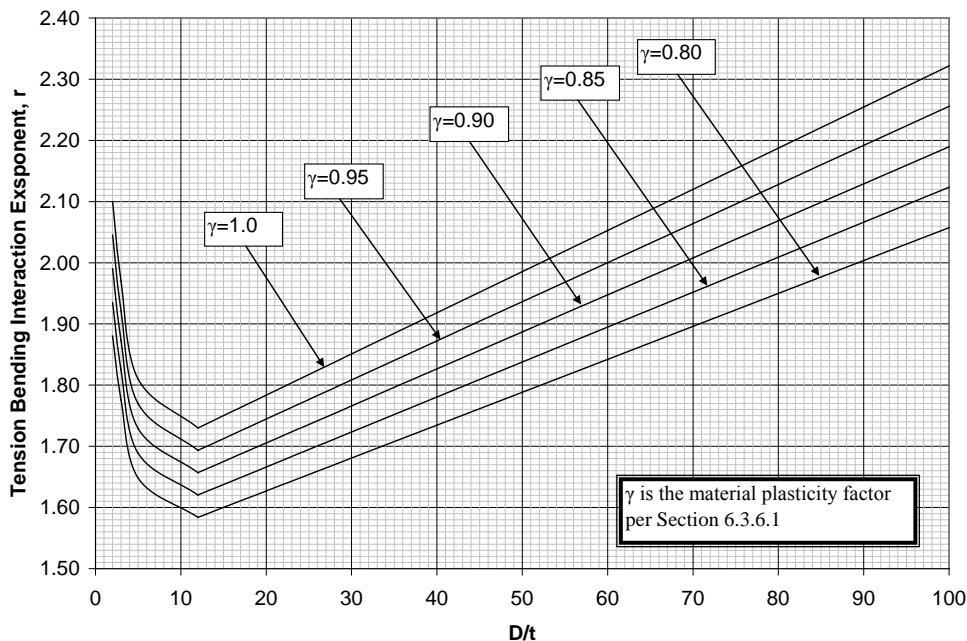
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D is outer diameter of the tube; t is the wall thickness

Figure 2.5.3-7 Shear Bending Interaction Exponent For Round Tubes for Cases 29, 32 and 33

Figure 2.5.3-8 provides the tension-bending interaction exponent for tubes. There is a cusp located at $D/t = 12$. To the left of $D/t = 12$, the thick walled tube fails by plastic rupture. To the right of $D/t = 12$, the thin walled tube fails by crippling collapse. This interaction exponent is similar to that described in Section 6.3.6.1 in the discussion of plastic bending. It has been developed for the specific geometry of round tubes of varying D/t .



D is outer diameter of the tube; t is the wall thickness

Figure 2.5.3-8 Tension Bending Interaction Exponent For Round Tubes for Cases 26

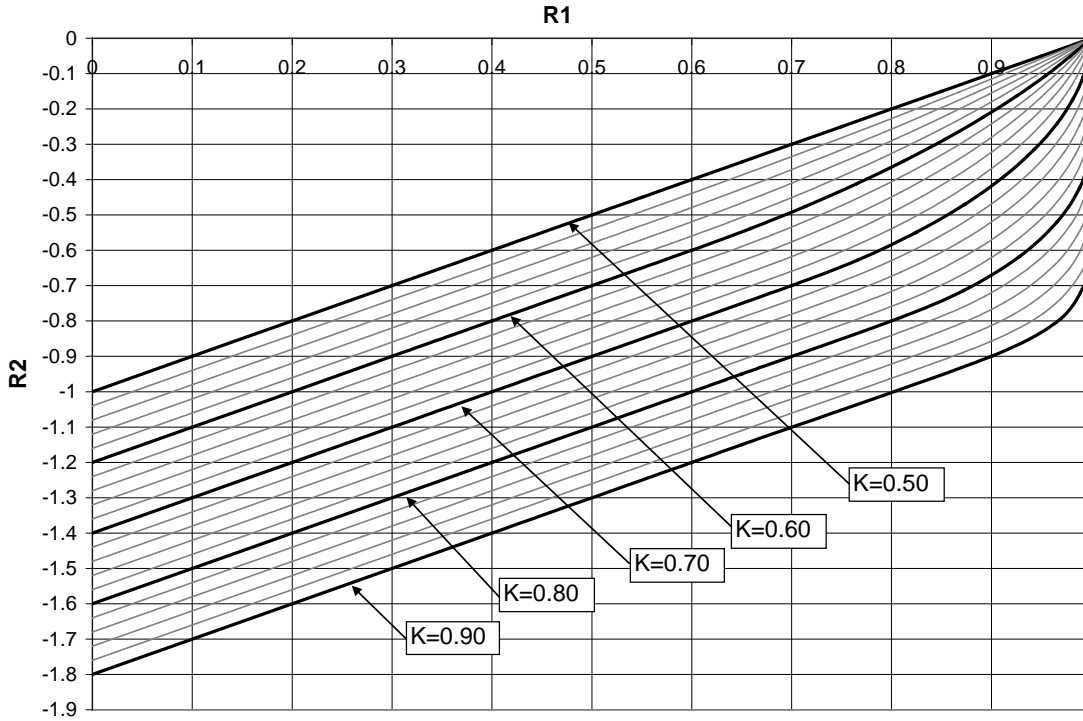
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Figure 2.5.3-9 provides the interaction curves for the ultimate strength margin calculations for compact structures where stability is not pertinent. These curves are derived from a non-dimensional Mohr's circle interaction approach. Section 2.5.4.2.2 describes the appropriate use of these curves.



$K = F_{su}/F_{tu}$ R_1, R_2 from Section 2.5.4.2.2
Figure 2.5.3-9 Interaction Curves for Ultimate Strength of Compact Sections

2.5.4 Combined Stresses

In some cases the use of interaction equations is not appropriate for the calculation of margins of safety. This section will develop the methods to be used in some of these situations.

2.5.4.1 Yielding of Compact Structure

To investigate yielding of a compact structure (ultimate allowable is not pertinent) under the action of 2-dimensional combined stress, interaction curves should not be used. Instead, a maximum "equivalent" load ratio R_{bar} is computed and the margin of safety obtained based on the maximum distortion strain energy criterion for yielding as follows:

$$M.S. = \frac{1}{R_{bar}} - 1$$

$$R_{bar} = (R_{n1}^2 + R_{n2}^2 - R_{n1}R_{n2} + R_s^2)^{1/2}$$

Equation 2.5.4-1

where

R_n is a normal stress ratio due to combining direct stress and bending stress and is given by $R_n = R_t + R_b$ or

$R_n = R_c + R_b$ (Tension is positive and compression is negative)

R_t is the applied stress divided by the allowable tensile yield stress

R_b is the applied bending moment divided by the allowable yield bending moment based on the bending modulus for the material at yield from Section 6.3

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R_c is the applied compression stress divided by the compression yield stress (stability concerns are not applicable)

R_s is the shear ratio due to combining simple shear and torsional shear and is given by $R_s = R_{ss} + R_{st}$

In the above equations, care must be taken that signs are correct. Subscripts 1 and 2 indicate directions which are mutually perpendicular. If it is desired to take account of the difference in yield allowable between with-grain and cross-grain directions, rather than to assume conservatively that the allowable is always the smaller of the two, then the "1" and "2" directions should be chosen to coincide with the grain directions and R_{t1} or R_{c1} and R_{t2} or R_{c2} each evaluated using its appropriate yield allowable. All stress ratios, R , must be evaluated for the same point in the structure so several points may have to be checked to find the most critical one.

R_{ss} and R_{st} may be combined to give R_s only when f_{ss} and f_{st} act in the same or opposite directions and they must be oriented with respect to the normal stresses as shown in Figure 2.5.4-1, for otherwise the stress condition is no longer two dimensional. The same sign convention must be used for both R_{ss} and R_{st} .

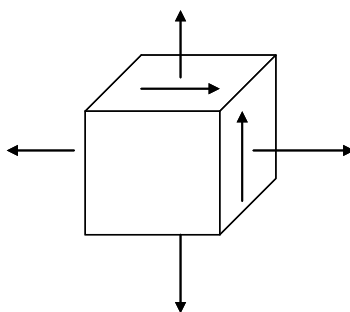


Figure 2.5.4-1 Normal and Shear Stress Orientation

The torsion ultimate material properties are obtained from Section 7.2. The torsion yield material property, F_{sty} , can be calculated as

$$F_{sty} = F_{sy} (F_{stu}/F_{su}) \quad \text{Equation 2.5.4-2}$$

where

F_{sy} the material yield shear property is from Section 3.3.3.4 (psi)

F_{stu} is the material ultimate torsion property (psi)

F_{su} is the material ultimate shear property from Reference 2-4 or 2-5 or other appropriate source (psi)

2.5.4.2 Ultimate Strength of Compact Structure

The ultimate strength of a "compact" structure (cripling and buckling not pertinent) under the action of two-dimensional combined stress may be determined by the following methods. A compact structure is a structure which does not have any long thin sections. Generally the width/thickness ratio of individual flanges in a cross-section is less than approximately 7 for a compact section, although this varies with material. Refer to Section 8 for a discussion of stability.

To use these methods, it is necessary to know the true stresses at ultimate load for all points likely to be critical. These stresses may be different from those obtained from an ordinary elastic analysis, because plastic flow (yielding) redistributes stress away from the points which in the elastic range are most critical and thus permits a greater load to be carried than would be indicated by the elastic analysis. Methods of determining true stresses, or of otherwise evaluating this increase in strength, have been developed only for a few special cases. Situations involving bending and bending combined with tension and shear are treated in Section 6.3. Various load combinations applied to thick walled tubes are treated in Section 2.5.2.

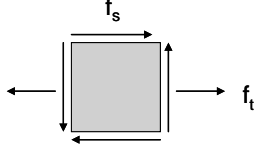
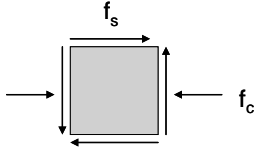
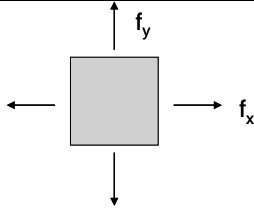
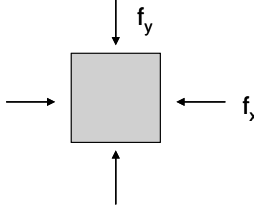
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2.5.4.2.1 Special Cases

Several simple cases of combined stress may be investigated for ultimate strength by means of the interaction equations given in Table 2.5.4-1. In all cases applied stresses are true local stresses. It is to be emphasized that each case represents a particular stress condition in a rectangular volume, not upon a single plane, and that no consideration need be given to the actual plane of fracture.

Stresses are assumed not to be present in directions other than those indicated in the sketches.

Table 2.5.4-1 Ultimate Combined Stress Equations (Applied Stresses are True Local Stresses)

	<p>Tension-Shear for Materials where $F_{su}/F_{tu} < 0.707$ Interaction: $R_t^2 + R_s^2 = 1$ Margin: $M.S. = \frac{1}{\sqrt{R_t^2 + R_s^2}} - 1$ where $R_t = f_t/F_{tu}$; $R_s = f_s/F_s$ and $F_s = \text{MINIMUM}[F_{su}, 0.57735F_{tu}]$</p>
	<p>Compression-Shear Interaction: $R_c^2 + R_s^2 = 1$ Margin: $M.S. = \frac{1}{\sqrt{R_c^2 + R_s^2}} - 1$ where $R_c = f_c/F_{cu}$; $R_s = f_s/F_s$ $F_{cu} = 2F_s$ unless stability critical then $F_{cu} = \text{MINIMUM}[F_c, F_{cc}]$ $F_s = \text{MINIMUM}[F_{su}, 0.57735F_{tu}]$</p>
	<p>Biaxial Tension: No interaction between the axes Margin: $M.S. = \frac{1}{R_{bar}} - 1$ $R_{bar} = \text{MAXIMUM}[R_x, R_y]$ where $R_x = f_x/F_{tu}$; $R_y = f_y/F_{tu}$</p>
	<p>Biaxial Compression: No interaction between the axes. Margin: $M.S. = \frac{1}{R_{bar}} - 1$ $R_{bar} = \text{MAXIMUM}[R_x, R_y]$ where $R_x = f_x/F_{cu}$; $R_y = f_y/F_{cu}$ $F_{cu} = 2F_s$ and $F_s = \text{MINIMUM}[F_{su}, 0.57735F_{tu}]$ Take all signs positive</p>

2.5.4.2.2 General Method

The more general cases shown in Figure 2.5.4-2 cannot be solved by simple interaction equations such as those given in Table 2.5.4-1. Margins of safety for these cases should be obtained by the following general method, from which the interaction equations given in Table 2.5.4-1 were obtained as special cases. Two approaches are provided. The first assumes the value for the ultimate material strength, F_{tu} , is the same in the with-grain and cross-grain directions. In this case the value for F_{tu} is chosen as the minimum of the two. The second approach takes the differences in the material strength in the different grain directions into account. Depending on the degree of anisotropy in the material, this can make a difference in the conservatism of the margins calculated.

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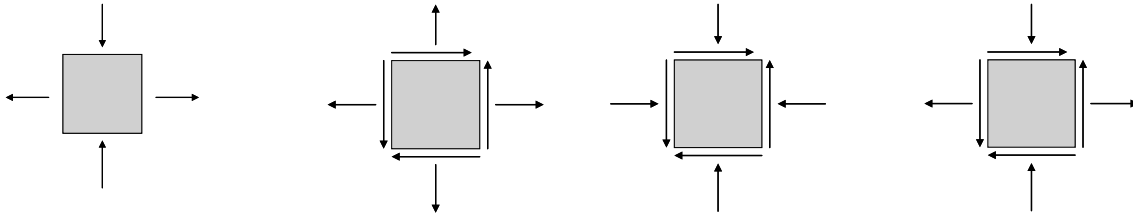


Figure 2.5.4-2 General Stress States

2.5.4.2.2.1 Calculation of R_1 and R_2 if F_{tu} is Same For Both Grain Directions

This section describes the general method for cases where F_{tu} is the same for with-grain and cross-grain directions or where it is desired to assume conservatively that F_{tu} in each direction is the same as in the weaker direction.

Calculate the ratio of the ultimate shear strength divided by the ultimate tensile strength as

$$K = F_{su} / F_{tu} \quad \text{Equation 2.5.4-3}$$

where

F_{su} is the ultimate material shear strength (psi)

F_{tu} is the ultimate material tensile strength (psi)

Calculate the "principal stress ratios" R_1 and R_2 as shown. Figure 2.5.4-3 depicts the stress volume under evaluation.

$$R_1 = \frac{R_{n1} + R_{n2}}{2} + \sqrt{\left(\frac{R_{n1} - R_{n2}}{2}\right)^2 + K^2 R_s^2}$$

$$R_2 = \frac{R_{n1} + R_{n2}}{2} - \sqrt{\left(\frac{R_{n1} - R_{n2}}{2}\right)^2 + K^2 R_s^2} \quad \text{Equation 2.5.4-4}$$

where

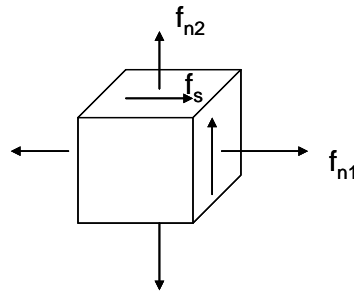
R_{n1} is the normal stress ratio for stress in direction "1". $R_{n1} = f_t / F_{tu}$ or $R_{n1} = -f_c / F_{tu}$

R_{n2} is the normal stress ratio for stress perpendicular to direction "1". $R_{n2} = f_t / F_{tu}$ or $R_{n2} = -f_c / F_{tu}$

K is the ratio calculated by Equation 2.5.4-3

R_s is the shear stress ratio (f_s / F_s) for shear stress acting on the planes indicated in Figure 2.5.4-3

$F_s = \text{MINIMUM}[F_{su}, 0.57735F_{tu}]$



Stresses are assumed not to be present in directions other than those indicated; however, the element may be chosen at any convenient angle, without regard to the actual plane of fracture.

Figure 2.5.4-3 Stress Volume for General Method

Once R_1 and R_2 have been determined, the calculation of Margin of Safety can be performed per Section 2.5.4.2.2.3.

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2.5.4.2.2.2 Calculation of R_1 and R_2 if F_{tu} is Different For Perpendicular Grain Directions

This section describes the general method for cases where F_{tu} is different for with-grain and cross-grain directions and it is desired to take advantage of this difference.

Determine applied stresses on an element with sides parallel and perpendicular to the grain direction as shown in Figure 2.5.4-4

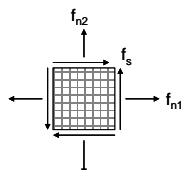


Figure 2.5.4-4 Stress Element Oriented with-Grain

Determine the ratio K from

$$K = \frac{F_{su}}{F_{tu-1}} \sqrt{\frac{1}{2} + \frac{1}{2} \left(\frac{F_{tu-1}}{F_{tu-2}} \right)^2} \quad \text{Equation 2.5.4-5}$$

where

F_{su} is the ultimate material shear strength (psi)

F_{tu-1} is the ultimate material tensile strength parallel with the primary grain direction (psi)

F_{tu-2} is the ultimate material tensile strength perpendicular to the primary grain direction (psi)

Obtain the normal stress ratios R_{n1} and R_{n2} , as follows:

$$\begin{aligned} R_{n1} &= f_{n1} / F_{tu\text{-effective}} \\ R_{n2} &= f_{n2} / F_{tu\text{-effective}} \end{aligned} \quad \text{Equation 2.5.4-6}$$

where

$F_{tu\text{-effective}} = F_s / K$ where K is from Equation 2.5.4-5. Use for tension or compression.

f_{n1} is the applied stress parallel with the primary grain direction. Tension is positive and compression is negative (psi)

f_{n2} is the applied stress perpendicular to the primary grain direction. Tension is positive and compression is negative (psi)

Evaluate the sum of R_{n1} and R_{n2} . If the sum is positive, disregard the values R_{n1} and R_{n2} obtained in Equation 2.5.4-6 and re-compute R_{n1} and R_{n2} using as allowables the appropriate with-grain and cross-grain values of F_{tu} as shown in Equation 2.5.4-7, then use the recomputed R_{n1} and R_{n2} in Equation 2.5.4-8 to determine the "principal stress ratios".

$$\begin{aligned} R_{n1} &= f_{n1} / F_{tu-1} \\ R_{n2} &= f_{n2} / F_{tu-2} \end{aligned} \quad \text{Equation 2.5.4-7}$$

where

F_{tu-1} is the with-grain ultimate allowable tension stress (psi).

F_{tu-2} is the cross-grain ultimate allowable tension stress (psi)

f_{n1} is the applied stress parallel with the primary grain direction. Tension is positive and compression is negative (psi)

f_{n2} is the applied stress perpendicular to the primary grain direction. Tension is positive and compression is negative (psi)

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If the sum of R_{n1} and R_{n2} from Equation 2.5.4-6 is negative or zero, evaluate the principal stress ratios using the values computed by Equation 2.5.4-6.

$$R_1 = \frac{R_{n1} + R_{n2}}{2} + \sqrt{\left(\frac{R_{n1} - R_{n2}}{2}\right)^2 + K^2 R_s^2}$$

$$R_2 = \frac{R_{n1} + R_{n2}}{2} - \sqrt{\left(\frac{R_{n1} - R_{n2}}{2}\right)^2 + K^2 R_s^2}$$

Equation 2.5.4-8

where

R_{n1}, R_{n2} are the normal stress ratios from Equation 2.5.4-6 or Equation 2.5.4-7, as appropriate.

K is the ratio calculated by Equation 2.5.4-5

R_s is the shear stress ratio (f_s/F_s)

$F_s = \text{MINIMUM}[F_{su}, 0.57735F_{tu}]$

Once R_1 and R_2 have been determined, the calculation of Margin of Safety can be performed per Section 2.5.4.2.2.3.

2.5.4.2.2.3 Calculation of Margin of Safety

If R_1 and R_2 calculated per the methods described in Section 2.5.4.2.2.1 or 2.5.4.2.2.2 are both positive the margin of safety can be obtained from

$$M.S. = \frac{1}{R_{bar}} - 1$$

Equation 2.5.4-9

where

$R_{bar} = \text{MAXIMUM}[R_1, R_2]$

If R_1 and R_2 are both negative, the margin of safety can be obtained from

$$M.S. = \frac{2K}{R_{bar}} - 1$$

Equation 2.5.4-10

where

$R_{bar} = \text{MAXIMUM}[|R_1|, |R_2|]$

K is the ratio from Equation 2.5-6

If R_1 is positive and R_2 is negative, choose the appropriate interaction curve in Figure 2.5.3-9, according to the value of K ; enter with R_1 and R_2 and obtain margin of safety as described in Section 2.5.1.

If R_1 is negative and R_2 is positive, interchange R_1 and R_2 . Choose the appropriate interaction curve in Figure 2.5.3-9, according to the value of K ; enter with the interchanged values R_1 and R_2 and obtain margin of safety as described in Section 2.5.1.

It may be noted that application of the foregoing method to the case of simple compression ($R_1 = 0$ or $R_2 = 0$) results in $F_{cu} = 2F_s$, which ordinarily is somewhat greater than F_{tu} . If stability is a factor, limit F_{cu} to the minimum of the column strength or the crippling strength, as applicable.

2.5.5 Ratio to Requirement

In aerospace design, there are times when requirements are levied on structure which do not represent actual failure of the part but are required to ensure the overall structure meets its functional or performance criteria. These requirements can include maximum skin deflections under pressure load, specific buckling criteria such as no buckling at limit load, no buckling at 115% limit load, no buckling at ultimate load, or required minimum gage skin thickness.

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This is documented in the analysis as a ratio to requirement and is the ratio of the allowable load or deflection divided by the applied load or deflection and it must be greater than 1.0. For example, for deflections it can be written as

$$\frac{\delta_{allowable}}{\delta_{applied}} \geq 1.0 \quad \text{Equation 2.5.5-1}$$

where

$\delta_{applied}$ is the deflection resulting from the applied load (in)

$\delta_{allowable}$ is the allowable deflection per the criteria (in)

For calculation of a ratio to requirement under combined loads, a margin of safety should be calculated using the appropriate interaction equation. Then the ratio to requirement, R_{TR} , is given by

$$R_{TR-applied} = M.S. + 1 \quad \text{Equation 2.5.5-2}$$

where

M.S. is the margin of safety calculated from the appropriate margin of safety interaction equation

Plate and shell initial buckling analysis is often more appropriately characterized using a ratio to requirement rather than a margin of safety since no static strength failure is involved. By utilizing R_{TR} to calculate a critical buckling load ratio, the analyst gains insight into whether the structure buckles, at what level and if any additional analysis is required.

The critical buckling load ratio, R_{CR} , is given by

$$R_{CR} = 1/R_{TR} \quad \text{Equation 2.5.5-3}$$

- If $R_{CR} \leq 1.0$, post buckling analysis is not required since a ratio less than 1.0 means the panel is not subject to initial buckling below ultimate load.
- If $R_{CR} > 1.0$, a post buckling analysis shall be performed since the panel buckles below ultimate load.
- If $1.5 \geq R_{CR} > 1.3$, then the panel undergoes initial buckling at 115% of limit load² and, in addition to post buckling strength analysis, the postbuckling effects shall be included on any detrimental deformation calculations.
- If $R_{CR} > 1.5$, then the panel undergoes initial buckling below limit load and, in addition to post buckling strength analysis, the postbuckling effects shall be included in durability and damage tolerance analyses and on any yield analyses and detrimental deformation calculations.

Program and customer requirements will set what critical buckling ratios are appropriate for different types of structure. For further information on critical buckling ratios and panel stability analyses, please refer to Section 10.

The analyst should also be aware that many stability calculations require the calculation of a margin of safety and not merely a ratio to requirement. If the stability mode being analyzed involves column stability, beam column effects, crippling, torsional instability, diagonal tension or any other mode that represents failure of the section, then a margin of safety is required.

² A design criterion on some programs is that there shall be no detrimental deformation at 115% of limit load.

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2.6 Drawing Tolerances and Dimensional Considerations

The strength of a part can be significantly affected by the method of manufacture and the manner in which it is dimensioned and the tolerances specified on the drawing. The stress analyst needs to understand the method of production and the implications of the drawing dimensioning and tolerancing. This section addresses these issues.

2.6.1 Dimensional Considerations Due to Manufacturing

The manner of manufacture for a given part can affect the final thicknesses and the tolerances specified for the part. Typical tolerances for different types of parts are shown in Table 2.6.1-1. Specific tolerances may vary from this due to other considerations.

Table 2.6.1-1 Typical Part Tolerances

Part	Tolerance	Applicability
Machined (NC)	±0.010	Thickness, machined steps
	±0.030	Feature locations, total length
Sheet Metal, Extrusions	±0.005	Stock thickness
	±0.030	Feature Location

When forming a part, the dimensions of that part will change and different types of machining operations have different process capabilities. This results in different forming or machining methods producing different thicknesses for the same part. For example, changing the manufacturing procedure from drawing to spin forming has been known to reduce the critical thickness by 40% and produce unacceptable parts. Varying changes in thickness occur with other forming methods such as stretching, swaging, joggling, etc. and in material products such as extrusion, sheet, tubing, etc.

It is important that the dimensions of the finished part at the critical locations be specified on the drawing.

When performing the stress analysis of a part, the stress engineer must be familiar with this section and consult with the cognizant design and producibility engineers to determine what manufacturing method will most likely be used to produce the part. Be aware that manufacturing has the option of changing production processes within the options allowed by the specification called out on the drawing without notifying Engineering so the analyst should be familiar with the options contained within the specifications called out on drawing and ensure that the drawing and specification are sufficiently restrictive if some methods are not acceptable.

If the part is formed, refer to the Engineering Design Manual PM4007, Reference 2-39, for the applicable forming equation. In the event that the desired forming information is not contained in the Design Manual, consult with the cognizant materials and process engineer. Use this equation to calculate the maximum percent stretch needed to form the part and the corresponding percent thickness reduction. Make an allowance for this thickness reduction in the stress analysis and require that the minimum thickness after forming be specified on the Engineering drawing.

Additionally, the static, fatigue, and fracture properties may change with forming, especially with super plastic forming. See Section 3 for a discussion of static properties. The minimum design properties (F_{tu} , F_{ty} and elongation) may have to be made a requirement and added to the drawing.

2.6.2 Drawing Tolerances

This section establishes the procedure to be followed in making allowances for tolerances in the determination of dimensions to be used in stress analysis.

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There are currently two methods of indicating dimensions: by specifying the maximum and minimum values, or by a nominal dimension with plus and minus tolerances. The tolerances specified may be equal or unequal. The following examples illustrate the alternate methods of dimensioning used

(1) 0.760
0.740

(2) 0.750 ± 0.010

(3) $0.740 + 0.020$
 $0.740 - 0.000$

For stress analysis purposes, the dimensions shall be converted to a mean dimension with equal plus and minus tolerances, as in example (2). Note that in the three examples shown, the mean dimension is the same, i.e., 0.750. The mean dimension may also be the nominal dimension specified on the drawing; however, if a nominal dimension has unequal tolerances, the stress engineer must compute the mean dimension for use in analysis.

The exception to the above discussion occurs when a part is manufactured by numerical control (NC) machining. NC machines are controlled by code generated from CATIA drawings; where the nominal dimensions of the part are targeted, the path of the cutter is the nominal dimension. For all CATIA/NC machined parts, the stress dimension is the lesser of the nominal dimension and the minimum dimension plus ten percent.

The actual dimensions of a part shall be used for salvage or repair.

Standard drawing tolerances are based on the number of significant digits used in dimensioning the part. These are summarized in Table 2.6.2-1. The number of significant digits is often a function of the manufacturing process used to create the part and the size of the part. Each engineering drawing should have the tolerances specified on the face of the drawing; however, when dimensions are obtained directly from a solid model it may be difficult to ascertain the intended tolerances.

Table 2.6.2-1 Typical Drawing Tolerances

Drawing Dimension	Typical Tolerance
X.X in.	± 0.10 in
X.XX in	± 0.03 in
X.XXX in	± 0.010 in

Stress analysis computations shall be based on the mean dimensions subject to the restrictions prescribed below for the particular class of structural components indicated. When classifying components with regard to load paths provided, consideration must be given to the type of loading, failure modes and the method of analysis as discussed in Section 2.2.1.1. A stiffened panel may be considered as a multiple load path component for loads in the stringer direction and single load path for loads in the other direction. The dimensions may be handled as prescribed in Section 2.6.2.3 if this difference is taken into account in the analysis. Spanwise splices are an example of single load path structure which should be handled as prescribed in Section 2.6.2.2.

Drawing dimensions should be configured to minimize the adverse effect of tolerance build-up on critical sections. The analyst should provide information to the designer on where the critical sections are located, so that this can be accomplished.

2.6.2.1 Terminology

The dimensions of interest are the linear dimensions in any view of a section being checked that are used to determine the properties of the section such as area or moment of inertia. These dimensions may be explicitly given or may have to be computed using two or more of dimensions given. Figure 2.6.2-1 provides some examples of how a part might be dimensioned.

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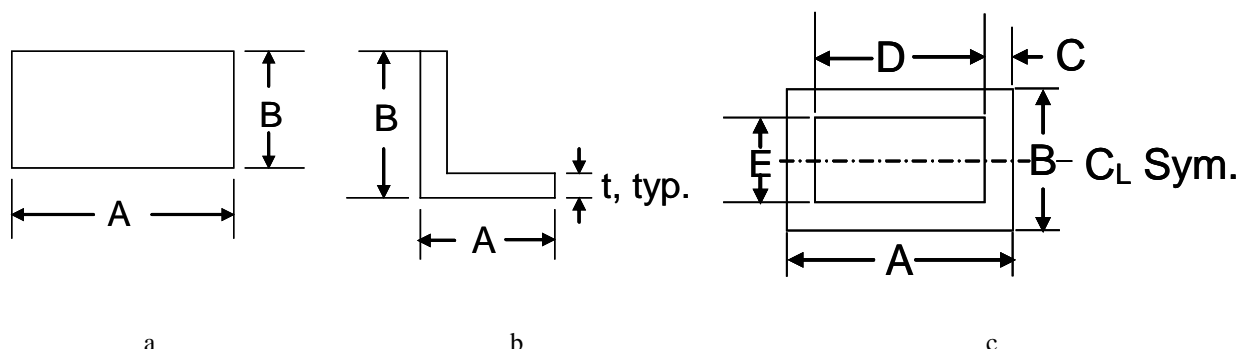


Figure 2.6.2-1 Examples of Part Dimensions

In Figure 2.6.2-1(a), the two dimensions are explicitly given and each one is independently affected by the tolerance specified. In Figure 2.6.2-1(b), the internal height of the vertical leg is determined by subtracting dimension t from dimension B , and the tolerance on this computed dimension is equal to the sum of tolerances on t and B . In Figure 2.6.2-1(c), the thickness of the left side of the rectangle is determined by subtracting dimensions C and D from dimension A , the overall length of the part. The tolerance on this computed dimension is equal to the sum of the 2 worst tolerances on dimensions A , C , and D . For analysis purposes, if a required dimension is determined by using more than two other dimensions, only the two largest tolerances shall be added to obtain the total negative or positive tolerance for the computed dimension. Wherever possible, the analyst should ensure that chained dimensions leading to a critical dimension like the thickness on the left side in Figure 2.6.2-1(c) are avoided. Critical thicknesses should be individually stated and toleranced.

For some applications, it is common practice by the design organization to compute the tolerance stack-up by using either a factored root sum squared procedure (RSS) or a statistical process control procedure (SPC). In the case of the RSS procedure, the square root of the sum of the squares of all of the pertinent tolerances is determined and multiplied by a factor, usually on the order of 1.20 to determine the magnitude for the overall stack-up of individual tolerances. This assumes a Gaussian distribution of tolerances on parts centered within a $\pm 3\sigma$ band. The 1.20 factor equates to approximately 77 percent of the resulting parts falling within this tolerance magnitude. While this approach is sufficient for tolerance studies on assembly stack-ups for fit-up purposes, it is unconservative when calculating the margin of safety of a part and shall not be used for analysis.

The restrictions on the use of the mean dimension, covered in subsequent sections, are based on the relationship of the total negative tolerance to the dimension to which it is applicable. The negative tolerance is the tolerance which reduces the thickness of the part to the minimum value. Note that the tolerances on the specified dimensions may be within acceptable limits but the total negative tolerance on the resulting computed dimension may be outside the acceptable limits.

2.6.2.2 Analysis of Single Load Path Structural Components

This approach shall be used for all CATIA/NC machined parts.

The mean dimension, explicitly given or computed as the case may be, shall be used for analysis, provided the total negative tolerance does not exceed ten percent (10%) of its value. If the tolerance on the mean dimension exceeds 10%, the analyst has three alternatives, the third being the least desirable.

1. To suggest a rearrangement of dimensioning to cut down the total negative tolerance to within ten percent of the pertinent dimension. This works if the dimension and the negative tolerance are the result of calculations consisting of multiple explicit dimensions.
2. To suggest a decrease of tolerances on the given dimensions.
3. To use a dimension equal to 1.11 times its lower limit instead of the mean for analysis.

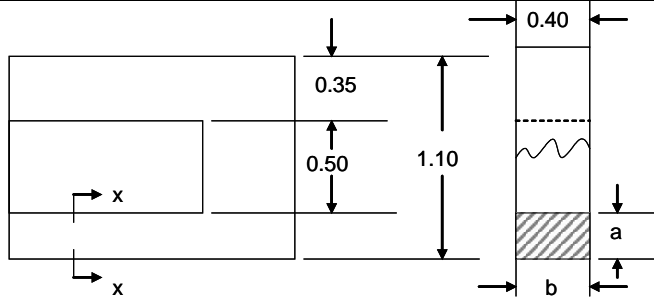
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Standard tolerances appearing in the drawing title block will not be in excess of the ten percent restriction where a two decimal *computed* dimension is 0.60 or greater or a three decimal *computed* dimension is .200 or greater. Standard sheet metal and tubing thickness tolerances are acceptable in all cases and thus need not be considered in the analysis. The tolerances and tolerance build-up on small parts (i.e. machined parts, castings, and forgings) should be given close attention.

Excessive positive tolerances tend to produce overweight production parts. Hence, an effort should be made to hold the maximum total positive tolerances, as well as negative tolerances, to ten percent or less of the given or computed mean dimensions.

2.6.2.2.1 Single Load Path Example

Given the part depicted below, calculate the area at Section x-x.

 <p>Tolerances on all dimensions are ± 0.03.</p>	<p>NOTE: This example is not to be construed as representative of a proper method of dimensioning of proper tolerances, but is presented for the sole purpose of illustrating the handling of tolerances.</p>
<p>Since + and - dimensions are equal, the dimensions shown are the mean dimensions.</p>	
<p>Calculate dimensions a and b: $a = 1.10 - 0.35 - 0.50 = 0.25$ and $b = 0.40$</p>	
<p>Tolerances affecting dimension a are: ± 0.03 on the overall 1.10 in dimension, ± 0.03 on the 0.35 in dimension, and ± 0.03 on the 0.50 in dimension <u>Total</u> negative tolerance for a = $0.03 + 0.03 = 0.06$ (summing the largest 2 of the three tolerances) Percent of dimension: $0.06/0.25(100) = 24\%$ Since the total negative tolerance on dimension a exceeds ten percent, the tolerance must either be reduced or the dimension to be used in the analysis must be calculated as $a = 1.11t_{\min} : 1.11(0.25 - 0.06) = 0.211$ in</p>	
<p>Tolerances affecting dimension b are: ± 0.03 on the 0.40 thickness of the part <u>Total</u> negative tolerance for b = 0.03 Percent of dimension: $0.03/0.40(100) = 7.5\%$ Mean dimension can be used: $b = 0.40$.</p>	
<p>Area of Section x-x: $A = ab = 0.211(0.40) = 0.0844$ in² as opposed to $0.25(0.40) = 0.10$ in² if tolerances are not accounted for.</p>	

2.6.2.3 Analysis of Multiple Load Path Structural Components

This approach shall not be used for CATIA/NC machined parts.

The mean dimension, explicitly given or computed as the case may be, shall be used for analysis provided the total negative tolerance does not exceed twenty percent (20%) of its value. If the tolerance on the mean dimension exceeds twenty percent, the analyst has three alternatives, the third being the least desirable.

1. To suggest a rearrangement of dimensioning to cut down the total negative tolerance to within twenty percent of the pertinent dimension.
2. To suggest a decrease of tolerances on the given dimensions.
3. To use in the analysis, instead of the mean, a dimension equal to 1.25 times its lower limit.

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Standard tolerances appearing in the drawing title block will not be in excess of the twenty percent restriction where a two decimal *computed* dimension is 0.30 or greater or a three decimal *computed* dimension is 0.100 or greater.

2.6.3 High Variability Structure

This section shall not be applicable for single load path, safety of flight structure.

Some structural designs or materials exhibit high variability around the nominal design, which can lead to increased structural risk. It is the responsibility of the structural analyst to seek elimination of such increased structural risk. However, many low cost production initiatives involve opening up the process window, tolerance or specification and it is important to ensure that these structures have the required structural integrity at the extremes of dimensions, tolerances, material properties, processing windows, processing controls, eccentricities, fit ups, fastener flexibilities, fastener fit, end or edge fixities, environments, manufacturing controls, etc. The primary focus of the stress analyst should be to identify those critical dimensions or processes that need extra control or tighter tolerances and provide that information to the designer. The analyst should also ensure, with the aid of materials and processing and design engineering, that any new materials, processes, or design concepts under consideration are sufficiently mature to provide a stable baseline.

The guidance provided in this section should not normally be a design consideration for most conventional designs, manufacturing processes and materials since the normal expected variation would allow the design to meet this requirement. Designs of high variability structure, if qualified using only nominal dimensions and material allowables, could result in under-strength parts while fully complying with the drawing, manufacturing specifications and processes, so additional analyses are required.

Any part exhibiting sufficient variability such that the analyst feels that it falls into the category of highly variable structure should discuss it with his supervision. If the variability cannot be mitigated, the part should be brought to the attention of the program Senior Structures Manager. It may be a candidate part for analysis under the high variability classification. Specific program direction and criteria are required for analysis of such parts.

In any event, the analysis of the part per the guidelines provided in Sections 2.6.1 and 2.6.2 for the usual strength criteria of no failure at ultimate load and no detrimental deformation at design limit load or some program specified increment on design limit load using nominal dimensions is required. Depending on the customer and applicable specification, the analyst must also ensure that for **a critical combination of the acceptable variability extremes** exhibited by these parts the structure shall have no detrimental deformation at the maximum once per lifetime load or at design limit load, whichever is higher, and the part shall not have a structural failure at some program defined level of design limit load, no less than 125% and not to exceed 150%. Particular attention must be paid to structure critical in bending or stability where stresses are not linear with thickness. Any reduction in safety factor will require specific customer concurrence. Additionally the part should be checked that it satisfies stiffness requirements, including aeroelasticity considerations.

In the absence of specific program guidance, Sections 2.6.2.2 and 2.6.2.3 shall apply.

2.7 Stress Concentration Factors

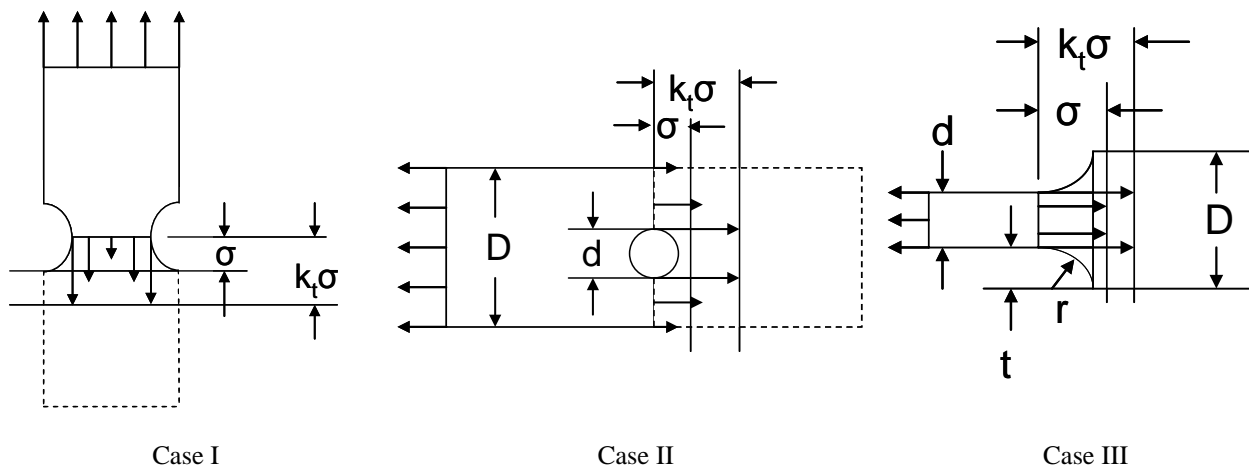
The distribution of stress across the section of a member may be nominally uniform or may vary in some regular manner, as illustrated by the linear distribution of stress in flexure. When the variation is abrupt, so that within a very short distance the intensity of the stress increases greatly, the condition is described as a stress concentration. It is usually due to local irregularities such as holes, screw threads, notches, nicks, keyways, sharp radii at section changes, etc. While stress concentrations are generally thought of in terms of repeated loading and durability or fatigue, on occasion the detail can be so extreme that a single cycle of limit or near limit load can cause failure. This

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section discusses how stress concentrations affect the strength and life of the part and some design improvements to preclude problems.

2.7.1 Static Loading

If an axially loaded member has an abrupt change in section as shown in Figure 2.7.1-1, the maximum elastic stress that occurs in each cross section is greater than the average stress, P/A_{net} , where A_{net} is the area of the net or small portion of the section at the abrupt change of section.



Case I Case II Case III
Figure 2.7.1-1 Examples of Axial Loaded Members with Abrupt Changes in Section

The maximum stress at such changes in section usually is called a stress concentration and the factor by which (P/A_{net}) must be multiplied to obtain the value or the stress for axially loaded members is called an ideal, theoretical or elastic stress concentration factor and is denoted by k_t . Thus, the maximum stress can be calculated from

$$f_{max} = k_t P/A_{net} \quad \text{Equation 2.7.1-1}$$

where

k_t is the theoretical stress concentration

P is the applied load (lbs)

A_{net} is the net section area (in²)

Thus, the stress concentration can be derived from

$$k_t = f_{max} / (P/A_{net})$$

The value of k_t depends on the geometry of the member, that is, on the relative values of the dimensions of the member in the vicinity of the stress concentration. Although the example given in Equation 2.7.1-1 is for axial loading, stress concentrations also exist for any type of loading such as torsion, shear or bending, both in-plane and out of plane. The maximum stresses for torsional and flexural loads are given by

$$f_{max} = k_t (Tr/J)$$

$$f_{max} = k_t (mc/I)$$

where

T is the applied torsion (in-lbs)

r is the radial distance to the extreme fiber (in)

J is the polar moment of inertia (in⁴)

m is the applied bending moment (in-lbs)

c is the distance from the neutral axis to the extreme fiber (in)

I is the moment of inertia (in⁴)

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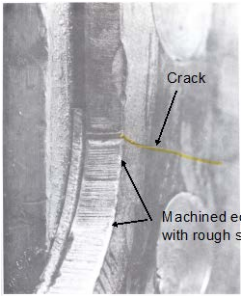
Stress concentration factors can be found in Stress Concentration Factors by Peterson, Reference 2-6, along with legacy manuals, Reference 2-2 and 2-9, and specific program documents and guidance.

2.7.2 Design Guidelines to Minimize Stress Concentrations

For members made of ductile material and sized by static loads and primarily unidirectional stresses, stress concentrations are usually relatively unimportant. However, it is crucial to minimize the damaging effects of localized stress concentrations to improve the durability and damage tolerance life, since on an aircraft the sizing of very few parts is purely a function of static loading.

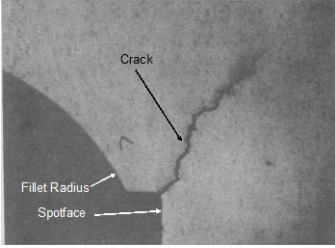
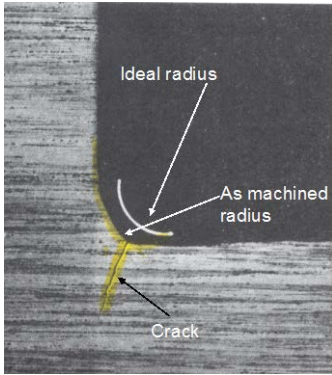
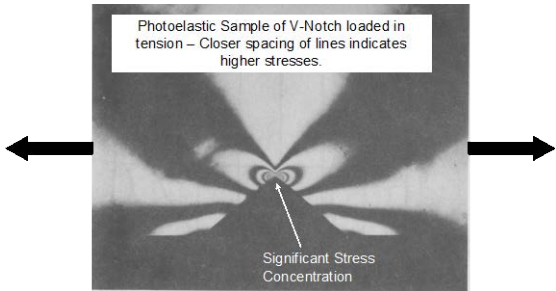
Table 2.7.2-1 addresses some approaches to minimize stress concentrations in parts. All pictures of cracking taken from Reference 2-11.

Table 2.7.2-1 Guidance on Ways to Minimize Stress Concentrations

	Guidance	Rationale/Sketch
1.	Use rubber stamping or silk screening of part numbers and avoid metal stamping and etching.	Metal stamping letters create surface notches in the part.
2.	Highly stressed parts should have surface finishes of RA 125 micro-in or better. See Table 2.7.2-2 for a discussion of surface finishes resulting from manufacturing methods.	 <p>Rough surfaces create crack nucleation sites.</p>
3.	Grain direction should be parallel to the applied load whenever possible.	Loading across grain can cause low failure stresses. Care should be taken in forgings to not apply significant point loads across grain.
4.	Avoid sharp bends and internal corners. Internal corners should not be dimensioned only with a maximum value, since no restriction on the lower bound allows for sharp corners. Radii should always have permissible tolerances specified.	Sharp corners are crack nucleation sites and the abrupt change in section significantly concentrates the stresses.

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5.	<p>Avoid spot facing on highly stressed parts and particularly into a stressed fillet radius.</p> <p>Spot faces generally have very sharp corners and should be avoided where possible. When combined with a stressed fillet radius, k_t's are compounded resulting in a very high local stress.</p>	 <p>Crack</p> <p>Fillet Radius</p> <p>Spotface</p>
6.	<p>Disallow or blend mismatch at fillet or corner radii. Mismatch superimposes stress concentrations and significantly increases local stresses. Do not allow blending to reduce radius.</p>	 <p>Ideal radius</p> <p>As machined radius</p> <p>Crack</p>
7.	<p>Do not allow notches, use generous radii instead.</p> <p>Notches have a sharp root at the point of maximum stress concentration and this serves as a crack nucleation site.</p>	 <p>Photoelastic Sample of V-Notch loaded in tension – Closer spacing of lines indicates higher stresses.</p> <p>Significant Stress Concentration</p>
8.	Blend all sharp edges.	Feather edges can cause injury. Uneven surfaces at a microscopic level can lead to cracks.
9.	Do not knife edge countersunk fasteners.	Knife edge countersunk fasteners have sharp feather edges. See comments on Item 8. A minimum of 0.020 straight shank should be maintained.

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10.	Avoid abrupt changes in sections. Use a generous fillet or faired lines. See Item 6.	<p>MAX STRESS AT A = 1.6 S_n</p> <p>MAX STRESS AT A = 1.1 S_n</p> <p>S_n – NOMINAL STRESS</p> <p>MAX STRESS AT A = 1.8 S_n</p> <p>MAX STRESS AT A = 2.5 S_n</p>
11.	Avoid multiple radii, particularly at changes in cross-sections.	<p>Double Radius and Change in Section</p> <p>Crack</p> <p>Cracking from double radius with sharp radius at thickness transition. Move thickness transition away from corner and increase radius at transition.</p>
12.	Avoid square holes even with corner radii. Round or elliptical holes have significantly lower stresses	<p>Cracks</p> <p>Part is relatively thick with generous corner radii but this is insufficient to overcome the existence of a square cutout.</p>
13.	<p>Drill holes all the way through, i.e. eliminate partial depth holes. Do not allow holes in structural parts to be threaded.</p> <p>The stress concentration at the tip of the drill and the potential for chips can cause cracking. The sharp notches created by the threads cause a significant decrease in life.</p>	<p>"Beach Marks" identifying a fatigue failure</p> <p>Hole drilled partially through (threaded also)</p>

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EXPORT CONTROLLED INFORMATION**

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The surface roughness, RA is the arithmetic deviation of the surface valleys for the mean surface and is a measure of the finer surface irregularities. It is a direct result of the manufacturing process used to create the part or the surface of the part. Surface roughness is measured in microinches of deviation so that a smaller number is a much smoother surface. Table 2.7.2-2 provides a comparison of achievable surface finishes for different manufacturing techniques. These are approximate values. Note that to achieve surface finishes of better than RA125 generally requires special attention during the manufacturing process and can be expensive.

Table 2.7.2-2 Comparison of Manufacturing Operation to Surface Finish Capability

Manufacturing Process	approximate Capability (microinches)
Flame Cutting	RA2000 to RA300
Sawing	RA2000 to RA125
Drilling	RA600 to RA63
EDM	RA250 to RA96
Milling	RA250 to RA32
Broaching	RA125 to RA32
Reaming	RA125 to RA32
Laser Cutting	RA300 to RA32
Burnishing	RA32 to RA8
Grinding	RA125 to RA4
Honing	RA63 to RA8
Polishing	RA32 to RA8
Extruding	RA250 to RA63
Investment Casting	RA250 to RA125
Permanent Mold Casting	RA250 to RA125
Die Casting	RA125 to RA63

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2.8 Statistical Analysis

Statistics and probability offer the engineer powerful tools to evaluate the variations in design parameters. Because statistical methods are techniques used to obtain, analyze and present numerical data, their application to engineering problems is becoming increasingly frequent. Methods for predicting rare occurrences or determining attributes of entire populations from data of small randomly selected samples are of particular importance to structural engineers.

Most manufactured articles are designed to function and to maintain their structural integrity while exposed to adverse environmental conditions. Design levels of load, strength, temperature and other environments must be more severe than the expected average: *e.g.*, one half of the production of manufactured articles designed to average levels would fail. Cost and importance of mission success require that the design of aerospace vehicles be at high levels of structural reliability.

Statistical analyses are used in several ways by the stress analyst. The material properties and allowables used for analysis and which are published in Reference 2-4 or 2-5 are statistically based. Often the acceptance of small manufactured parts is based on sampling plans which are more broadly based on statistics to ensure that lot rejection rates provide an acceptable quality level. The external loads which are applied to the aircraft under design, in some flight regimes such as gust loads or hammer shock events, have magnitudes based on a probability of occurrence. The load or stress spectrum used for durability and damage tolerance analysis is statistically based.

This section provides a brief overview of some statistical fundamentals. Reference 2-14, along with numerous textbooks, provides a more in-depth treatment of the subject.

2.8.1 Normal or Gaussian Distribution

The normal or Gaussian distributions are an important class of statistical distributions because they can be used to model many naturally occurring events and have been found to be useful for modeling of failure data in static strength testing. All normal distributions are symmetric and have bell-shaped density curves with a single peak as shown in Figure 2.8.1-1, described by the equation

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-0.5\left[\frac{x-\mu}{\sigma}\right]^2}$$

where

σ is the population standard deviation

μ is the population mean

x is the abscissa

y is the ordinate

A standard normal distribution is one in which the mean is zero.

Statistics is concerned with modeling the behavior of a population or the total collection of individual elements. Since analysts are often concerned with information that can only be obtained destructively or the population is too large for each part to be tested, statistics can be used to derive information about the population by examining a randomly chosen subgroup called a sample.

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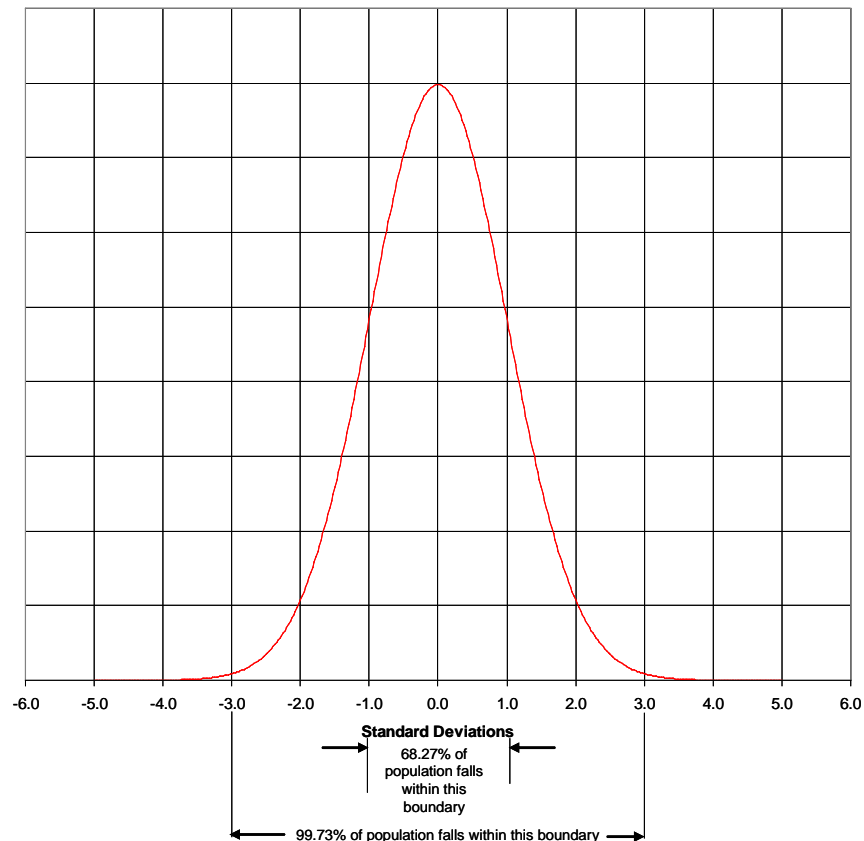


Figure 2.8.1-1 Standard Normal Distribution

Two parameters must be specified for any normal distribution: the mean and the standard deviation. Simply put the mean represents the location on the chart where the maximum number of observations occurs and the standard deviation indicates how wide the curve or scatter in the data is. For the population the mean is denoted as μ , while for a sample of the population it is denoted as \bar{x} . For the population the standard deviation is denoted as σ and for any sample within the population it is denoted as s_{dev} .

The mean or arithmetic average, μ , of the population can be calculated as

$$\mu = (x_1 + x_2 + x_3 + \dots + x_N) / N$$

Equation 2.8.1-1

where

x is the value of the observation

N is the total number of observations in the population

The mean or arithmetic average, \bar{x} , of the sample can be calculated as

$$\bar{x} = (x_1 + x_2 + x_3 + \dots + x_N) / N$$

Equation 2.8.1-2

where

x is the value of the observation

N is the total number of observations in the sample

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The standard deviation, σ , of a population is the root mean square deviation of the values within the population from the population mean and is given by

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (x - \mu)^2}{N}} \quad \text{Equation 2.8.1-3}$$

where

μ is the mean of the observations in the population

N is the total number of observations in the population

Note that Equation 2.8.1-3 should only be used when data is available from the entire population, e.g. a non-destructive test result. Any coupon or specimen testing should use Equation 2.8.1-4.

Often the engineer must calculate the statistics of a sample rather than the entire population. The standard deviation must be estimated from a random sample taken from the population as a whole. In this case the denominator is not the number of observations in the sample, N , but the number of degrees of freedom, $N-1$. The standard deviation, s_{dev} , of a sample set is given by

$$s_{dev} = \sqrt{\frac{\sum_{i=1}^N (x_i - x_{bar})^2}{N-1}} \quad \text{Equation 2.8.1-4}$$

where

N is the number of observations in the sample

The standard deviation is an indicator of the amount of scatter in the data. If the scatter is high or the value of individual data points varies widely from the mean, then the standard deviation will be a large number and the resulting curve will be wide.

The standard error measures the precision of the sample mean to the true population mean and is given by

$$SE = \frac{s_{dev}}{\sqrt{N}}$$

Standard error can be used to judge the uncertainty in the mean and set the confidence interval.

Another useful term is the coefficient of variation, COV. This can be calculated as

$$COV = 100 \frac{s_{dev}}{x_{bar}} \quad \text{Equation 2.8.1-5}$$

where

s_{dev} is the sample standard deviation

x_{bar} is the mean

Sometimes the terms like 3σ or 6σ will be used to describe a statistical value. Since σ represents the standard deviation of the population, 3σ means 3 standard deviations from the mean. In a normal distribution, 1σ encompasses 68.27% of the area under the bell shaped curve or the probability is 68.27% that a randomly selected sample point lies in the area bounded by $\pm 1\sigma$. 6σ would encompass 99.999998% of the population; this measure is often used to indicate that virtually all the population is covered by a finding. Table 2.8.1-1 provides the population density for a normal distribution in terms of standard deviations. Figure 2.8.1-1 depicts this information graphically for several different standard deviations.

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Table 2.8.1-1 Population Density Distribution – Two Tail Problems

Number of Standard Deviations (plus and minus from the mean)	Probability that a randomly selected x_i is in the defined range
$\pm 0.674\sigma$	50
$\pm 1.0\sigma$	68.27
$\pm 2.0\sigma$	95.46
$\pm 3.0\sigma$	99.730
$\pm 4.0\sigma$	99.994
$\pm 5.0\sigma$	99.9999427
$\pm 6.0\sigma$	99.9999998

Another useful statistical value is the standard deviation of the means of a population. This is sometimes referred to as the standard error of the means. The means of a normal distribution will also have a normal distribution, so that their standard deviation is defined as

$$\sigma_{xbar} = \sqrt{\sigma / N} \quad \text{Equation 2.8.1-6}$$

Or, if the population standard deviation is not known, σ_{xbar} can be estimated from

$$\sigma_{xbar} = s_{dev} / (N-1)$$

where N is the number of observations in the sample in both Equations

This value is used in determining whether a sample of a larger population falls within defined accept/reject bounds and is discussed in Section 2.8.2.

2.8.1.1 Calculation of Material Properties or Allowables

The calculation of material properties or design allowables is done using the mathematics discussed in Section 2.8.1.1. A- and B-Basis design properties¹ are provided in References 2-4 and 2-5. Details of the guidelines used for these material property and joint allowables can be found in detail in Chapter 9 of these two references. To determine a material property like F_{tu} , Reference 2-4 requires a minimum of 100 samples from 10 heats² and 10 lots³ of material. Chapter 9 of 2.1-4 and 2.1-5 also allows for a more sophisticated statistical basis when the data warrants.

On occasion a point design allowable is required. Point design testing is conducted on a specific design with specific materials, geometry and loading, usually with a limited number of test specimens and with limited applicability. In this situation, a smaller number of samples are available but a B-Basis property or allowable is still desired. An A-Basis allowable should not be calculated with fewer than 100 samples. This section describes how the normal distribution equations may be used to make this calculation.

For this type of calculation, it is desirable to ensure that some percent of the population does not fall below the calculated allowable. This type of calculation uses a one-sided tolerance interval. Table 2.8.1-2 provides one-sided tolerance limit factors for varying number of sample sizes to calculate a B-Basis allowable⁴ which has a 95% confidence factor. Figure 2.8.1-2 shows an estimated one-sided normal distribution. Note that to the right of the mean value the curve extends continuously, but to the left it is terminated at a lower bound value.

¹ Reference Section 3.3.3 for discussion.

² A heat is all material identifiable to a single molten metal source.

³ A lot is all material from a heat of the same product type having the same thickness or configuration and fabricated as a unit under the same conditions.

⁴ A B-Basis allowable is statistically based such that least 90% of the population of values (90% probability) is expected to exceed the B-basis mechanical design property with a confidence of at least 95%.

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Table 2.8.1-2 One-Sided Tolerance Limit Factors for Normal Distributions, 95% Confidence Factor⁵

Sample Size	K _{one-side, B-Basis}	Sample Size	K _{one-side, B-Basis}
		21	1.905
		22	1.886
3	6.155	23	1.869
4	4.162	24	1.853
5	3.407	25	1.838
6	3.006	26	1.824
7	2.755	27	1.811
8	2.582	28	1.799
9	2.454	29	1.788
10	2.355	30	1.777
11	2.275	31	1.767
12	2.210	32	1.758
13	2.155	33	1.749
14	2.109	34	1.740
15	2.068	35	1.732
16	2.033	36	1.725
17	2.002	37	1.717
18	1.974	38	1.710
19	1.949	39	1.704
20	1.926	40	1.697

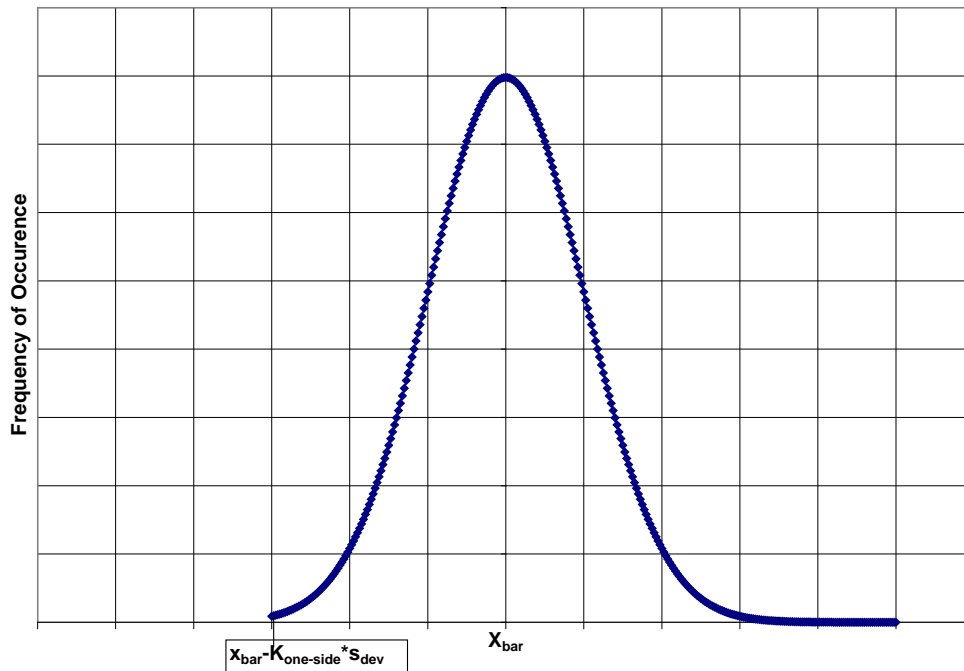


Figure 2.8.1-2 One-Sided or Single Tail Normal Distribution

⁵ Reference 2-4, Chapter 9. Additional values for K_{one-sided} for larger sample sizes can be obtained from this source.

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To calculate the allowable using the one-sided tolerance interval, enter Table 2.8.1-2 with the number of samples and read the value of the one-sided tolerance limit, $K_{one-side}$. Calculate the mean for the sample per Equation 2.8.1-2 and the standard deviation per Equation 2.8.1-4. Then the B-basis allowable can be calculated from

$$F_{B-Basis} = \bar{x} - K_{one-side}(S_{dev}) \quad \text{Equation 2.8.1-7}$$

where

$F_{B-Basis}$ is the B-Basis allowable (psi), generically shown as stress but could be load (lbs) or strain (in/in)

$K_{one-side}$ is the one-sided tolerance limit from Table 2.8.1-2

\bar{x} is the mean of the test sample from Equation 2.8.1-2

S_{dev} is the standard deviation of the test sample from Equation 2.8.1-4

From Table 2.8.1-2 it is obvious that the smaller the number of samples, the larger the multiplying factor on the standard deviation of the sample and thus the larger the reduction to obtain a B-Basis allowable.

2.8.1.1.1 Example Problem – 10 Observations

Testing has provided 10 observations of failure load (lbs) as follows: 100, 92, 90, 102, 105, 98, 100, 99, 105, 94 Calculate a B-Basis allowable assuming a normal distribution.	
Calculate the mean value of the data population, \bar{x}	$(100+92+90+102+105+98+100+99+105+94) / (10) = 98.5$
Calculate the standard deviation of the data population	$S_{dev} = [(\sum_{i=1}^N (x_i - \bar{x})^2) / (N-1)]^{0.5} = \{[(100-98.5)^2 + (92-98.5)^2 + (90-98.5)^2 + (102-98.5)^2 + (105-98.5)^2 + (98-98.5)^2 + (100-98.5)^2 + (99-98.5)^2 + (105-98.5)^2 + (94-98.5)^2] / (10-1)\}^{0.5} = (236.5/9)^{0.5} = 5.126$
From Table 2.8.1-2	$K_{one-side} = 2.355$
$F_{B-Basis} = \bar{x} - K_{one-side}(S_{dev})$	$F_{B-Basis} = 98.5 - 2.355(5.126) = 86.43 \text{ lbs}$
The B-Basis allowable with a 95% confidence level for this set of data is 86.43 lbs.	

2.8.1.1.2 Example Problems – 3 Observations

Testing has provided 3 observations of failure load (lbs) with the same mean and standard deviation as Example 2.8.1.1.2: $\bar{x} = 98.5$ and $S_{dev} = 5.126$ Calculate a B-Basis allowable assuming a normal distribution.	
From Table 2.8.1-2	$K_{one-side} = 6.155$
$F_{B-Basis} = \bar{x} - K_{one-side}(S_{dev})$	$F_{B-Basis} = 98.5 - 6.155(5.126) = 66.95 \text{ lbs}$
The B-Basis allowable with a 95% confidence level for this set of data is 66.95 lbs.	

2.8.1.1.3 Example Problem - 30 Observations

Testing has provided 30 observations of failure load (lbs) with the same mean and standard deviation as Example 2.8.1.1.2: $\bar{x} = 98.5$ and $S_{dev} = 5.126$ Calculate a B-Basis allowable assuming a normal distribution.	
From Table 2.8.1-2	$K_{one-side} = 1.777$
$F_{B-Basis} = \bar{x} - K_{one-side}(S_{dev})$	$F_{B-Basis} = 98.5 - 1.777(5.126) = 89.39 \text{ lbs}$
The B-Basis allowable with a 95% confidence level for this set of data is 89.39 lbs.	

From a comparison of these three examples it can be seen why it is desirable to have a large number of data points. For the same mean and standard deviation, the resulting allowables can vary from 66.95 lbs for 3 observations to 86.43 lbs for 5 observations to 89.39 for 30 observations. More test coupons can result in lighter, more efficient designs. Additionally, the smaller number of tests, the larger the impact of any single reading on the standard deviation is, leading to a much larger effect of a single low reading on the calculated allowable.

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2.8.2 Statistical Inference⁶

The previous sections were concerned with the descriptions of observations and with the estimation of certain characteristics from which a sample is drawn. These same procedures can be used to make decisions concerning the pooling of data and equivalence of samples.

For this type of analysis, a statistical hypothesis is proposed. This is a theory concerning the behavior or property of a specific physical population. The only way to truly prove or disprove the hypothesis is to test all members of the population. Since many tests in this industry are destructive, testing of every member of the population is unreasonable. As result, samples of the population are taken and tested and based on those results, the hypothesis is accepted or rejected. This is called a statistical proof.

The level of significance, α , is a numerical evaluation of the risk that is taken in rejecting a true hypothesis: *e.g.* rejecting a part or lot which meets all of the requirements. At $\alpha=0.05$ a true hypothesis will be rejected 5 times out of 100. The smaller the value of the level of significance, the less likely the rejection of a true hypothesis but the chance of accepting a false hypothesis increases. Figure 2.8.2-1 depicts the level of significance on a normal distribution curve. Since the total probability of occurrence of any event is 1.0, for a level of significance of $\alpha = 0.05$, the negative boundary is at $\alpha/2 = 0.025$ or the lower 2.5 percent of the area under the curve and the positive boundary would be at $1-0.025 = 0.975$ or the upper 2.5% of the curve

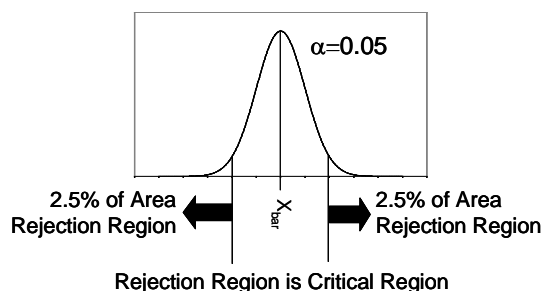


Figure 2.8.2-1 Graphical Interpretation of Level of Significance

The β risk is the risk taken of accepting a hypothesis when it is false; *e.g.*, accepting a discrepant part or lot as good. With a fixed number of observations, N , upon establishing α , β becomes fixed. If α is increased, β is decreased. The converse is also true. The only way to decrease both is to increase the number of observations.

To determine what values on the normal distribution curve define the boundaries of the rejection region, the cumulative normal distribution function, $\Phi(u)$ is used. The values for $\Phi(u)$ are tabulated Table 2.8.2-2 and Table 2.8.2-3. The use of these tables will be illustrated in the examples provided in this section.

To determine the sample means which form the boundaries of the rejection region, the following is used

$$\frac{|x_{bar} - \mu|}{\sigma_{xbar}} \leq u_{\alpha/2} \quad \text{Equation 2.8.2-1}$$

where

x_{bar} is the mean of the sample set

μ is the mean of the population

σ_{xbar} is the standard deviation of the means of the sample sets given by Equation 2.8.1-6

$u_{\alpha/2}$ is the cumulative normal distribution variable.

⁶ Reference 2-14

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Once the boundaries of the rejection range have been set using the level of significance, the β risk, or the probability of accepting a defective part or lot, can be calculated. Another name for the β risk function is the operating characteristic curve. The operating characteristic curve can also be used to determine how many samples are needed to obtain an acceptable defect acceptance rate. The β risk function is given by

$$\beta(\mu) = \Phi\left(\frac{\mu - x_{bar}}{\sigma_{xbar}} + u_{\alpha/2}\right) - \Phi\left(\frac{\mu - x_{bar}}{\sigma_{xbar}} - u_{\alpha/2}\right) \quad \text{Equation 2.8.2-2}$$

where

Φ is the cumulative normal distribution function

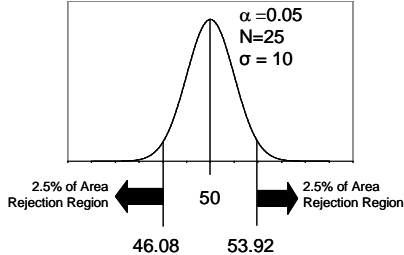
The number of observations required to achieve a specific rejection rate for defective parts can be approximated from

$$N_{req} \approx \frac{\left(u_{\alpha/2} + u_{\beta}\right)^2 \sigma^2}{(x_{bar} - \mu)^2} \quad \text{Equation 2.8.2-3}$$

where

u_{β} is the probability of acceptance desired for a defective part or lot

2.8.2.1 Example Problem – Statistical Inference: Level of Significance

<p>Given: A population which is normally distributed. Population mean, $\mu = 50$ Population standard deviation, $\sigma = 10$ Number of observations in sample, $N=25$ Level of Significance, $\alpha = 0.05$ Determine: Data values defining the boundaries of the rejection region</p>	
Calculate the standard error of the means: $\sigma_{xbar} = \sigma / N^{0.5} = 10 / 25^{0.5} = 10 / 5 = 2$	
At $\alpha = 0.05$, the lower 2.5% boundary is at 0.025 and the upper 2.5% boundary is at $1.0 - 0.025 = 0.975$	
Enter Table 2.8.2-2 for $\Phi[u] = 0.975$ and read the upper (positive) cumulative normal distribution variable: $u_{upper} = 1.96$	
Enter Table 2.8.2-3 for $\Phi[u] = 0.025$ and read the lower (negative) cumulative normal distribution variable: $u_{lower} = -1.96$	
<p>Calculate the boundaries: $x_{bar} - \mu / \sigma_{xbar} \leq u_{\alpha/2}$ $x_{bar} - 50 / 2 \leq 1.96$ $46.08 < x_{bar} < 53.92$</p>	
Thus for $\alpha = 0.05$, there is a 5% probability that the sample mean will be outside of the interval defined by 46.08 to 53.92 and, thus would be rejected.	

2.8.2.2 Example Problem – Statistical Inference: β Risk

<p>Given: A population which is randomly distributed from the previous example: Population mean, $\mu = 50$ Population standard deviation, $\sigma = 10$ Number of observations in sample, $N=25$ Level of Significance, $\alpha = 0.05$ Determine: Determine the β risk that a sample population will be accepted if the mean of the sample is $x_{bar}=52$</p>
--

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The limits for the population are the same as in the previous example, $46.08 < \bar{x}_{\text{bar}} < 53.92$, so the mean of the sample falls within the accept range.

From the previous example, $\sigma_{\bar{x}_{\text{bar}}} = 2$ and $u_{\alpha/2} = \pm 1.96$

Calculate $\beta(\mu)$

$$\begin{aligned}\beta(\mu) &= \Phi[(\mu - \bar{x}_{\text{bar}})/\sigma_{\bar{x}_{\text{bar}}} + u_{\alpha/2}] - \Phi[(\mu - \bar{x}_{\text{bar}})/\sigma_{\bar{x}_{\text{bar}}} - u_{\alpha/2}] \\ &= \Phi[(50 - 52)/2 + 1.96] - \Phi[(50 - 52)/2 - 1.96] = \Phi[0.96] - \Phi[-2.96]\end{aligned}$$

From Table 2.8.2-2 for $\Phi[0.96] = 0.8315$

From Table 2.8.2-3 for $\Phi[-2.96] = 0.001538$

$\beta = 0.8315 - 0.001538 = 0.8299$ or 82.99%. So the probability of accepting set which is defective is 83%.

If the number of observations within the sample set were increased, the risk of accepting a defective set would reduce. Table 2.8.2-1 illustrates this for the above problem for 2 different levels of significance and several different sample sizes.

Table 2.8.2-1 Variation of β with Sample Size and Level of Significance

Number of Sample Observations, N	β Risk for $\alpha = 0.05$	β Risk for $\alpha = 0.01$
1	0.95	0.99
5	0.93	0.99
10	0.90	0.97
25	0.83	0.94
50	0.71	0.88
100	0.48	0.72
1000	0.00001	0.0001

From Table 2.8.2-1 it can be seen that for the same sample size, the probability of accepting a defective set increases as the level of significance is reduced. For example, for a sample size of 10, there is a 90% probability of accepting a defective set for a level of significance of 0.05 but a 97% probability for a level of significance of 0.01. Similarly, as the number of observations is increased from 5 to 50, the probability of accepting a defective set drops from 93% to 71% for $\alpha = 0.05$.

2.8.2.3 Example Problem – Estimate Number of Observations Required

Given: A population which is randomly distributed from the Example 2.8.2.3:

Population mean, $\mu = 50$ Population standard deviation, $\sigma = 10$

Number of observations in the sample, $N = 25$

Level of Significance, $\alpha = 0.05$

Determine: Determine the number of observations required to have at least a 75% probability of rejection when the mean of the sample is $\bar{x}_{\text{bar}} = 52$

The limits for the population are the same as in the Example 2.8.2.3, $46.08 < \bar{x}_{\text{bar}} < 53.92$, so the mean of the sample falls within the accept range. From the Example 2.8.2.3, $\sigma_{\bar{x}_{\text{bar}}} = 2$ and $u_{\alpha/2} = \pm 1.96$

The probability of acceptance desired for a defective part or lot is $1 - 0.75 = 0.25$

Determine u_{β} for $\Phi[0.25]$

From Table 2.8.2-3 For $\Phi[u] = 0.2514$, $u = -0.67$ and $\Phi[u] = 0.2483$, $u = -0.68$

Interpolating: $0.0014/0.0031 = x / 0.01$ so $x = 0.0045161$ and $u_{\beta} = -0.67 - 0.0045161 = -0.674516$ for $\Phi[u] = 0.25$

Calculate the estimated number of specimens required

$$N_{\text{req}} \sim [(u_{\alpha/2} + u_{\beta})^2 \sigma^2] / [(\bar{x}_{\text{bar}} - \mu)^2] \sim [(1.96 - 0.674516)^2 (10)^2] / [52 - 50]^2 = 173.6 \Rightarrow 174$$

A sample size of approximately 174 is required.

Check the result : $\sigma_{\bar{x}_{\text{bar}}} = 10/174^{0.5} = 0.758$

Calculate $\beta(\mu)$

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$$\begin{aligned}
 \beta(\mu) &= \Phi[(\mu - \bar{x}_{\text{bar}}) / \sigma_{\bar{x}_{\text{bar}}} + u_{\alpha/2}] - \Phi[(\mu - \bar{x}_{\text{bar}}) / \sigma_{\bar{x}_{\text{bar}}} - u_{\alpha/2}] \\
 &= \Phi[(50 - 52) / 0.758 + 1.96] - \Phi[(50 - 52) / 0.758 - 1.96] \\
 &= \Phi[-0.68] - \Phi[-4.60]
 \end{aligned}$$

From Table 2.8.2-2 for $\Phi[-.68] = 0.2483$; From Table 2.8.2-3 for $\Phi[-4.60] = 0.000002112$
 $\beta = 0.2483 - 0.000002112 = 0.2482$ or 24.8%. Thus using 174 specimen results in a 24.8% probability of acceptance for a sample whose mean is 52. The probability of rejection is $1 - 0.2482 = 0.7518$ or 75.2%.

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Figure 2.8.2-2 The Cumulative Normal Distribution Function: Positive u

u	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8960	0.8997	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96905	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.926319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999

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u	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3.1	0.9990324	0.9990646	0.9990957	0.9991260	0.9991553	0.9991836	0.9992112	0.9992378	0.9992636	0.9992886
3.2	0.9993129	0.9993363	0.9993590	0.9993810	0.9994024	0.9994230	0.9994429	0.9994623	0.9994810	0.9994991
3.3	0.9995166	0.9995335	0.9995499	0.9995658	0.9995811	0.9995959	0.9996103	0.9996242	0.9996376	0.9996505
3.4	0.9996631	0.9996752	0.9996869	0.9996982	0.9997091	0.9997197	0.9997299	0.9997398	0.9997493	0.9997585
3.5	0.9997674	0.9997759	0.9997842	0.9997922	0.9997999	0.9998074	0.9998146	0.9998215	0.9998282	0.9998347
3.6	0.9998109	0.9998469	0.9998527	0.9998583	0.9998637	0.9998689	0.9998739	0.9998787	0.9998834	0.9998879
3.7	0.9998922	0.9998964	0.9999004	0.9999043	0.99990799	0.99991158	0.99991504	0.99991838	0.99992159	0.99992468
3.8	0.99992765	0.99993052	0.99993327	0.99993593	0.99993848	0.99994094	0.99994331	0.99994558	0.99994777	0.99994988
3.9	0.99995190	0.99995385	0.99995573	0.99995753	0.99995926	0.99996092	0.99996253	0.99996406	0.99996554	0.99996696
4.0	0.99996833	0.99996964	0.99997090	0.99997211	0.99997327	0.99997439	0.99997546	0.99997649	0.99997748	0.99997843
4.1	0.99997934	0.99998022	0.99998106	0.99998186	0.99998263	0.99998338	0.99998409	0.99998477	0.99998542	0.99998605
4.2	0.99998665	0.99998723	0.99998778	0.99998832	0.99998882	0.99998931	0.99998978	0.999990226	0.999990655	0.999991066
4.3	0.99999146	0.999991837	0.999992199	0.999992545	0.999992876	0.999993193	0.999993497	0.999993788	0.999994066	0.999994332
4.4	0.999994587	0.999994831	0.999995065	0.999995288	0.999995502	0.999995706	0.999995902	0.999996089	0.999996268	0.999996439
4.5	0.999996602	0.999996759	0.999996908	0.999997051	0.999997187	0.999997318	0.999997442	0.999997561	0.999997675	0.999997784
4.6	0.999997888	0.999997987	0.999998081	0.999998172	0.999998258	0.999998340	0.999998419	0.999998494	0.999998566	0.999998634
4.7	0.999998699	0.999998761	0.999998821	0.999998877	0.999998931	0.999998983	0.999999032	0.999999079	0.9999991235	0.999999166
4.8	0.9999992067	0.9999992453	0.9999992822	0.9999993173	0.9999993508	0.9999993827	0.9999994131	0.9999994420	0.9999994696	0.9999994958
4.9	0.9999995208	0.9999995446	0.9999995673	0.9999995889	0.9999996094	0.9999996289	0.9999996475	0.9999996652	0.9999996821	0.9999996981

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Figure 2.8.2-3 The Cumulative Normal Distribution Function: Negative u

u	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1040	0.1003	0.0985
-1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
-1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
-1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
-1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
-1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.02938
-1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03095	0.02330
-1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.01831
-2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01426
-2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01101
-2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.00842
-2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00639
-2.4	0.008198	0.007976	0.007760	0.007549	0.007344	0.007143	0.006947	0.006756	0.006569	0.004799
-2.5	0.006210	0.006037	0.005868	0.005703	0.005543	0.005386	0.005234	0.005085	0.004940	0.003573
-2.6	0.004661	0.004527	0.004396	0.004269	0.004145	0.004025	0.003907	0.003793	0.073681	0.002635
-2.7	0.003467	0.003364	0.003264	0.003167	0.003072	0.002980	0.002890	0.002803	0.002718	0.001926
-2.8	0.002555	0.002477	0.002401	0.002327	0.002256	0.002186	0.002118	0.002052	0.001988	0.001395
-2.9	0.001866	0.001807	0.001750	0.001695	0.001641	0.001589	0.001538	0.001489	0.001441	0.001001
-3.0	0.001350	0.001306	0.001264	0.001223	0.001183	0.001144	0.001107	0.001070	0.001035	0.000711

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u	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.1	0.0009676	0.0009354	0.0009043	0.0008740	0.0008447	0.0008164	0.0007888	0.0007622	0.0007364	0.0005009
-3.2	0.0006871	0.0006637	0.0006410	0.0006190	0.0005976	0.0005770	0.0005571	0.0005377	0.0005190	0.0003495
-3.3	0.0004834	0.0004665	0.0004501	0.0004342	0.0004189	0.0004041	0.0003897	0.0003758	0.0003624	0.0002415
-3.4	0.0003369	0.0003248	0.0003131	0.0003018	0.0002909	0.0002803	0.0002701	0.0002602	0.0002507	0.0001653
-3.5	0.0002326	0.0002241	0.0002158	0.0002078	0.0002001	0.0001926	0.0001854	0.0001785	0.0001718	0.0001121
-3.6	0.0001891	0.0001531	0.0001473	0.0001417	0.0001363	0.0001311	0.0001261	0.0001213	0.0001166	0.0000501
-3.7	0.0001078	0.0001036	0.0000996	0.0000957	0.0000920	0.0000884	0.0000850	0.0000816	0.0000784	0.0000330
-3.8	0.00007235	0.00006948	0.00006673	0.00006407	0.00006152	0.00005906	0.00005669	0.00005442	0.00005223	0.00002157
-3.9	0.00004810	0.00004615	0.00004427	0.00004247	0.00004074	0.00003908	0.00003747	0.00003594	0.00003446	0.00001395
-4.0	0.00003167	0.00003036	0.00002910	0.00002789	0.00002673	0.00002561	0.00002454	0.00002351	0.00002252	0.000008934
-4.1	0.00002066	0.00001978	0.00001894	0.00001814	0.00001737	0.00001662	0.00001591	0.00001523	0.00001458	0.000005668
-4.2	0.00001335	0.00001277	0.00001222	0.00001168	0.00001118	0.00001069	0.000010220	0.000009774	0.000009345	0.000003561
-4.3	0.000008540	0.000008163	0.000007801	0.000007455	0.000007124	0.000006807	0.000006503	0.000006212	0.000005934	0.000002216
-4.4	0.000005413	0.000005169	0.000004935	0.000004712	0.000004498	0.000004294	0.000004098	0.000003911	0.000003732	0.000001366
-4.5	0.000003398	0.000003241	0.000003092	0.000002949	0.000002813	0.000002682	0.000002558	0.000002439	0.000002325	0.000000834
-4.6	0.000002112	0.000002013	0.000001919	0.000001828	0.000001742	0.000001660	0.000001581	0.000001506	0.000001434	0.000000504
-4.7	0.000001301	0.000001239	0.000001179	0.000001123	0.000001069	0.000001017	0.000000968	0.000000921	0.000000877	0.000000302
-4.8	0.0000007933	0.0000007547	0.0000007178	0.0000006827	0.0000006492	0.0000006173	0.0000005869	0.0000005580	0.0000005304	0.0000005042
-4.9	0.0000004792	0.0000004554	0.0000004327	0.0000004111	0.0000003906	0.0000003711	0.0000003525	0.0000003348	0.0000003179	0.0000003019

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2.8.3 Production Quality Control

Statistical inference, described in Section 2.8.2, is often used for quality control applications either through post-manufacture part sampling for lot acceptance or through control of the manufacturing process through surveillance and continuous sampling as parts are manufactured. This section provides an introduction to both.

2.8.3.1 Part Sampling and Acceptable Quality Levels

A part sampling plan is often a requirement for quality control of manufactured parts. This is a way of ensuring the lot of parts has an acceptable quality level through the inspection of some random subset of the parts within the lot. The specification for the required sampling is called a sampling plan. Sampling can be based on attributes or variables. An attributes-based sampling plan is specified by

- N – the total lot size from which the sample is drawn
- n – the sample size to be taken, randomly, from the lot
- c – maximum number of defects allowed in the sample before the lot is rejected.

From this information, using probability calculations similar to those detailed in Section 2.8.2, the probability that a lot with a given defect rate will be accepted is calculated. This calculation is repeated for all possible percent defect rates within the lot and an operating characteristic (OC) curve is generated. A sample OC curve is shown in Figure 2.8.3-1.

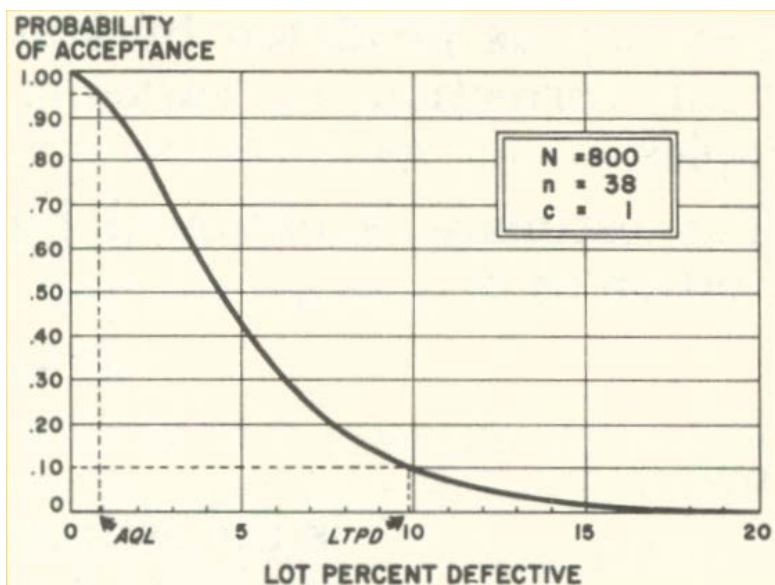


Figure 2.8.3-1 Sample Operating Characteristic Curve

On the basis of the curve, the lot percent defective, LTPD, can be determined. This number is the percent defective for a given inspected lot for which the inspection plan will give a small probability of being accepted. This is usually 10% or less. Figure 2.8.3-1 shows a LTPD of 9.9 or approximately 79 defects in the entire lot of 800 have a 0.10 or 10% probability of being accepted. Also the Acceptable Quality Level (AQL) can be determined. This number is the percent defective for an inspected lot for which there is a high probability of acceptance. This is usually around 95%. In Figure 2.8.3-1, the AQL of 0.95 or 95% shows a lot percent defective of 1% or 8 defects in the 800 item lot. So, for this sample Operating Characteristic Curve and the lot of 800 parts, there is a 95% probability that there will be 8 defective parts and a 10% probability that the number of defective parts will be as high as 79.

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There is an added complexity to sampling to ensure quality which involves how many times samples are drawn from the lot to determine the accept/reject status of the lot. Single sample plans allows a single sample set to be drawn randomly from the lot and it is used to determine whether or not the lot is accepted or rejected. Another approach is to allow for double sampling. That approach draws a single sample set and based on the results of the inspection of those items the lot may be rejected (if the defect level is above a certain criterion), the lot may be accepted (if the defect level is below a certain value), or a second smaller sample is randomly drawn and inspected (if the defect level is in between the upper and lower levels) to determine the accept/reject status of the lot. Another approach allows for multiple or sequential sampling which allows for multiple re-sampling if certain criteria are met.

Reference 2-13 provides numerous operating characteristic curves and sampling plan options which include accept/reject rates and varying levels of AQL. Stress analysts generally are not involved in the determination of what sampling plan may be appropriate for lot acceptance. Their involvement occurs during the material review of defective parts to accept or reject lots which have failed. A basic understanding of the terminology and application of the statistics will aid in making an informed decision.

2.8.3.2 Process Control

Process control is based on continuous inspection and sampling from a production process such as the machining, drilling, or assembly of parts and can be either one of two types depending on the method of inspection: attributes or variables.

Attribute testing accepts or rejects a part based on a defined attribute like defective or non-defective while variable testing involves measurement and recording of specific part characteristics. For instance, under attribute testing, the 0.150 inch thickness of a bracket could be either defective or non-defective depending on its measured thickness. If it were defective, that would be the only information recorded relative to that part, so the engineer evaluating the part would know only that it was defective, but would not know if the 0.150 inch flange measured 0.125, 0.05 or 0.250 inch thick. The same part under variable testing would also be rejected; however, since the actual thickness measurement is recorded, the engineer would know what the thickness of the flange measured. In general, attribute testing is used on small parts such as hardware while variable testing is used on large, expensive parts which may have hundreds of measurements, any one of which could cause rejection. This allows the opportunity to perform a salvage analysis.

The basic premise of process control uses a statistically constructed control chart based on sampling data obtained during the monitoring and inspection process. The control chart is a tool to determine whether or not the process is statistically controlled. If the chart shows the process is under control, *i.e.*, highly repeatable within specified limits, then it can be used to predict how well the process will perform in the future. If it shows the opposite, then the information can be used to adjust the process until the desired level of quality is achieved. The construction of the control chart varies depending on whether the testing is attribute- or variable-based. In order to put a statistical control process in place, the engineering drawing requires the identification of key characteristics or factors. The stress analyst should play an active role in defining the key characteristics based on areas which have critical margins because the key characteristics are feature dimensions which are subject to the monitoring and control process.

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2.9 Special Inspection and Test Requirements

The acceptance of certain airplane parts requires special inspection and testing techniques to ensure that the as-manufactured part meets all of the design requirements. It is the responsibility of the stress and design engineers to specify these special techniques and to ensure that they appear as notes on production drawings when necessary. Structure should be classified according to Section 2.2.1.1 and then the inspections based on this classification.

Reference 2-4, Section 19 has an extensive discussion on the inspection techniques used for composite materials.

2.9.1 Non-Destructive Testing

There are five primary non-destructive testing methods in use:

- Dye penetrant inspection which is useful for surface defects in non-ferromagnetic (aluminum, magnesium, stainless steels, etc.) parts
- Magnetic particle inspection to find surface defects or near surface defects for ferromagnetic parts only.
- Ultrasonic inspection is useful for internal defects.
- Radiographic (X-RAY) inspection, which is useful for internal defects for all types of parts but can be relatively expensive.
- Computerized Axial Tomography (CAT) scan inspection, useful for three dimensional mapping of internal defects but can be very expensive.

The following sections discuss each of these in more detail.

2.9.1.1 Dye Penetrant Inspection

When penetrant inspection is required for a part, a note shall be placed on the drawing indicating the requirement and the applicable process specifications. Penetrant inspection is used to reveal defects open to the surface of a material. Penetrant inspection involves painting, dipping or spraying the surface of the part with a fluorescent dye. The dye wicks into any surface irregularities and cracks through capillary action. After a period of time, the excess dye is removed and a developer is applied. The function of the developer is to draw the dye out of the surface defects and make it visible under ultraviolet light. This inspection may be performed on any metallic material and is very useful in determining surface cracking or damage. It is variously referred to as “dye penetrant inspection”, “fluorescent penetrant inspection” or “FPI”.

All Vital, Class I per Section 2.2.1-1, non-ferromagnetic (including 3xx series stainless steel) parts shall be 100% fluorescent penetrant inspected. Non-vital, Class II, zero-draft forgings shall be penetrant inspected when deemed necessary on the basis of size, complexity, and function. Section 3.5.4 has additional discussion on the inspections required for forged parts.

Heat treat strengthened, non-ferromagnetic parts fabricated from nickel base, copper base, or cobalt base alloys and precipitation hardening stainless steel alloys 17-7PH, 17-4PH, 15-7PH and A-286 shall be 100% penetrant inspected.

2.9.1.2 Magnetic Particle Inspection

If ferromagnetic parts require magnetic particle inspection, then a note should be placed on the drawing specifying this requirement and appropriate processes. Magnetic particle inspection uses a magnetic field or DC current running through the part to reveal defects on or near the surface of ferromagnetic parts. This method relies on the principle that there is a difference in the signal in and around the area of the defect. A suspension of iron oxide particles, sometimes containing a fluorescent dye to enhance visibility, is sprayed on the part which has been magnetized. This localizes area where the magnetic field is different on the surface of the part. It is easier and more cost effective than either dye penetrant or ultrasonic inspection but only works with magnetic materials.

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Austenitic stainless steels (e.g. 3xx series, and 21–6–9) are not ferromagnetic. Precipitation hardening steel alloys of 17–7PH, 17–4PH, 15–7PH, and A–286 which are somewhat magnetic, but exhibit non–relevant indications should be inspected using dye penetrant inspection. Martensitic 4xx series stainless steels (e.g. 421, 422, 440C), precipitation hardening steels such as PH13–8Mo and carbon steels (e.g. 1010 and 1020) and low alloy steels (e.g. 4130, 4340 and 300M) are ferromagnetic. 17–7PH, 17–4PH, 15–7PH and A–286 may have local ferromagnetic indications.

All ferromagnetic stainless steel alloys, regardless of classification, shall be 100% magnetic particle inspected. This magnetic inspection is to be performed after all machining and heat treating operations. An exception is precipitation hardening steel alloys of 17–7PH, 17–4PH, 15–7PH, and A–286 which are somewhat magnetic but exhibit non–relevant indications. These alloys should be inspected using dye penetrant inspection.

All Class I, vital parts per Section 2.2.1-1 ferromagnetic parts shall be 100% magnetic inspected.

A 100% magnetic inspection is recommended for ferromagnetic bolts, nuts, and other standard utility parts, 0.25 inch and larger in nominal diameter, for which the specified minimum hardness is above Rockwell C26 (approximately 125 ksi Ultimate Tensile Stress) unless otherwise specified in the Procurement Specification for the specific item. In general, the Procurement Specifications for tensile bolts and standard utility parts heat treated to 125 ksi Ultimate Tensile Stress and shear bolts and screws heat treated to 160 ksi Ultimate Tensile Stress specify magnetic inspection on a statistical sampling basis.

Procurement Specifications for high tensile, high fatigue strength bolts, such as MS20004 through MS20020 internal wrenching bolts (160 ksi Ultimate Tensile Stress) and MS21250 12 point external wrenching bolts (180 ksi Ultimate Tensile Stress) require 100% magnetic inspection.

All ferromagnetic castings, regardless of classification, shall be 100% inspected after all machining and heat treating operations.

2.9.1.3 Ultrasonic Inspection

When ultrasonic inspection is required for a part, a note shall be placed on the drawing indicating the requirement and the applicable process specifications. Ultrasonic inspection uses sound waves to reveal internal defects in materials. To perform ultrasonic testing sound pulses, ranging from 0.1-15 MHz, are sent into the part. This can be done using hand-held devices or with the part suspended in a tank filled with liquid. One technique uses a single transducer which acts as both source and receiver. This approach is often called “pulse echo” and looks for the reflection of the signal either off the backside of the part or off an internal defect face. Another approach, called “through-transmission,” uses two separate transducers with one providing the sound source and the other acting as the receiver. The receiver is located some distance away separated by a liquid medium.

One drawback of ultrasonic inspection is that standards must be developed before the process provides meaningful results. Standards are the results of inspection of good parts and parts with known defect and defect sizes used for comparison with parts whose defects have not been characterized.

All aluminum and titanium hand forgings and all stock for aluminum and titanium alloy die forgings shall be ultrasonic inspected in accordance with their respective material specifications. This inspection shall be done before machining and may be done before heat treatment. Section 3.5.4 has additional discussion on the inspections required for forged parts and Section 3.6.7 has additional discussion on inspections of castings.

2.9.1.4 Radiographic (X–RAY) Inspection

When X-RAY inspection is required on a part, a note shall be placed on the drawing indicating the requirement for the inspection and the applicable process specification. X–RAY inspection is used to reveal internal defects of materials. An X-ray image is generated by sending electromagnetic rays through a part and recording the amount of

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transmission on a layer of film. Different sections in the part absorb different amounts of the radiation due to the presence of defects or differences in thickness. This creates a map of internal defects. For some products, such as plate and tube, ultrasonic inspection is a more efficient method. Section 3.6.7 provides additional discussion on the requirements for the X-RAY inspection of castings. Section 5.7 discusses the requirements for X-RAY inspection of fusion welds.

2.9.1.5 Computerized Axial Tomography (CAT) Scan Inspection

When a CAT scan is required on a part, a note shall be placed on the drawing indicating the requirement for the inspection and the applicable process specification. Computerized axial tomography (CAT) is used to find and map three dimensional images of internal defects in parts. This particular procedure is a radiographic procedure which takes a series of images at various depth through the part and from various angles around the part and then combines them into a three dimensional image of the body which may be interrogated to determine if defects are present. This procedure has limits on the size of the part which can be examined, depending on the size and shape of the machine, and can be extremely expensive; however, it has proven useful in detecting defects in large castings.

2.9.2 Hydraulic Proof and Burst Testing

Fluid systems are subjected to proof pressure tests to disclose any porosity, leaks, or malfunction of the system. It is also required that some units be subjected to burst tests. Since the requirements for such tests are based upon functional and structural considerations it shall be the responsibility of the Design and Stress Groups to specify the proper notes on the engineering drawing, when such tests are required. Because these tests are required to substantiate the stress analysis, assembly and the method of fabrication of the parts, the stress engineer must ascertain that the proper notes are on the drawings.

2.9.3 Physical Testing

Specific requirements for physical testing vary with part and manufacturing process. For parts which use processing intensive manufacturing, such as castings, forgings, super plastic-formed parts, etc., there will be physical testing required to determine final part integrity. Individual sections within Section 3 which address these processes also address special testing requirements.

Physical testing can either be as simple as a one-time proof or destructive test, repeated lot proof or destructive tests to as complex as development of part-specific property generation or some combination of these. The simple burst or proof test approach might be used for a welded tube assembly while development of part-specific allowables might be done for a large Class I forged part.

For large parts designated as Class I per Section 2.2.1.1, each part would have excess material which is made into process control coupons that provide test data to validate the guaranteed minimum design properties. These areas are called prolongations. Prolongations are used where the shape and material are made simultaneously, such as castings, forgings, super-plastic, and electro-deposited formed parts, to obtain certification properties of parts. Without the prolongation material, cutting test coupons from the parts would destroy the part. The prolongations need to be integral to the part and need to undergo all of the processing steps that the part undergoes including but not limited to forging, casting or forming, heat treating, and quenching. The prolongation area should not be thinner than the minimum thickness of the part. The test coupons are shown on the final part drawing in position on the part and are tested for each part manufactured. All tensile test coupons should be two inch gage length but not less than one inch gage length. Standard test specimen configurations can be found in References 2-46 through 2-48. Typical properties tested by prolongations include yield and ultimate tensile strength and elongation. If the critical failure mode for the part is compression crippling, compression yield coupons should also be included in the prolongations.

In order to provide a qualitative assessment of the prolongations, a first article part as described in Section 2.9.4 is required. A correlation factor, K_{RF} , for the prolongation parts can then be established by dividing the prolongation

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test properties of the part by the test properties of the prolongations of the destroyed first article part. If the result of this division is greater than one, RF is set equal to one; thus, this ratio is always less than or equal to one.

$$K_{RFu} = F_{tu-prolongation} / F_{tu-prolongation-firstarticle}$$

$$K_{RFy} = F_{ty-prolongation} / F_{ty-prolongation-firstarticle}$$

$$K_{RFe} = e_{prolongation} / e_{prolongation-firstarticle}$$

To guarantee the design properties of the parts, the test values obtained from prolongations ($F_{tu-prolongation}$, $F_{ty-prolongation}$, $e_{prolongation}$) on all production parts, multiplied by the appropriate K_{RF} must be higher than the specification minimum properties. All the test results and reduction factors must be on the certification document (cert) delivered with the part from the vendor.

2.9.4 First Article Inspections

For parts which are Class I per Section 2.2.1.1 or fracture or durability critical per Section 2.2.1.2 there should be a first article part drawing which is either a separate dash number of the part drawing or a separate drawing. The first article is the first part which fully conforms to the final manufacturing steps and processes and represents the production part. It is not the “first part off of the tool” if the process has not been finalized. The first article has test coupons located in the stress designated, non-designated, and prolongation areas of the part. If it is a forging then specimens should encompass all grain directions.

The first article part is cut up and tested, as a minimum, from the first manufacturing lot of the part or any time any significant processing or chemistry changes have occurred or if a new die, mold or tool is used or if the manufacturing is moved from one facility to another. In addition, for some critical parts, there may be one part per production lot cut up and tested.

In some cases, such as with parts machined from plate or bar, the first article can be a purely dimensional and configurational inspection and has no destructive testing associated with it. Such a first article can be used for a production aircraft.

The stress analyst should ensure, on all first article parts, that the expected configuration has been produced, without any bad intersections of load lines or machining details producing unexpected reentrant corners, overlapping radii or other such bad fatigue details.

2.10 Preparation of Stress Analysis Reports

Preliminary stress analysis reports contain the calculations performed in analyzing structural components of the contract item prior to the stress group approval of the drawing, drawing changes, engineering orders, etc. and may include load determination calculations as well as stress analysis. While preliminary reports are generally not submitted to the procuring agency, much of the information should be used, to the degree possible to generate the final stress report.

Final Stress analysis reports are compiled for submittal to the licensing or procuring agency as substantiation of the structural adequacy of the design and to provide proof that all of the static strength requirements have been satisfied. They are also used within Lockheed for establishing placards or modifications due to changed weight, speed, mission or criteria. A final use is for the purpose of production salvage and airplane fleet maintenance or repairs.

At this point a discussion of what is not a stress analysis might be appropriate. Finite element models are wonderful tools which stress analysts use to aid in the internal distribution of loads in a complex, highly redundant structure. **However, a finite element model does not do the analysis for the analyst and color contour stress plots are not stress analysis.** Model plots are extremely useful, at times, in pointing the analyst to critical areas that should be analyzed and for visualizing how the structure is responding to the applied loads. Plots of loads can be made quickly

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and efficiently and can be a tremendous benefit in documenting critical load cases. Some of the post-processing software can create freebody diagrams. But none of these items is a substitute for a stress analysis which summarizes pertinent loads, documents allowables and calculates margins of safety for all appropriate failure modes.

2.10.1 Preliminary Stress Report

Preliminary reports often have more detail than final reports and can serve as a reference for use in salvage and repair activities; however, in order to be useful they must be clear and well defined although not necessarily as neat as the final report.

There are two types of preliminary stress reports which will be discussed in this section. The first type may be initial analysis of a part for the purposes of preliminary sizing, trade studies, or configuration decisions. The analysis is generally less rigorous and documentation less formal. At this point in the design cycle the parts may not have individual part numbers but are part of a layout drawing. It is incumbent on the analyst to make the analysis as complete as possible, providing sketches and stating assumptions, particularly any configuration assumptions which are not yet part of a defined drawing, such as number and size or type of fasteners. The loads and loading sources should be defined to the extent possible as well as criteria used in the sizing, such as whether or not the structure is allowed to buckle, edge fixity assumptions, deflection or stiffness requirements.

Equally important is to note are things not considered but which may be a factor in the analysis of this part at a later date. If it is known that the structure is pressurized or in a high temperature environment, but no temperatures or pressures are yet available, this should be noted along with any assumed temperatures or pressures.

Each page of this type of preliminary stress analysis shall contain the following information:

- o Name of Analyst
- o Date of Preparation
- o Revision Dates
- o Temporary Page Number

This information can be crucial at a later date when other analysts might be trying to determine the pedigree and applicability of this set of preliminary analyses. All computations should be shown; if a non-standard program or spreadsheet is used, sample calculations with equations need to be provided and reproducible. Appropriate sketches, pictures, and freebody diagrams should be included. Margins of safety should be shown with critical condition and failure mode clearly stated. If, in the judgment of the analyst, the margin is conservative and it is noted as such, the rationale for the statement must be explained.

The second type of preliminary stress report is the precursor to the final stress report which is submitted to the certifying or procurement agency. This form of a preliminary report has all of the information described in Section 2.10.2 for a Formal Report; however it may contain significantly more analysis because it is used to as the basis for substantiating the structure before the stress approval of an engineering drawing for release. It may be hand written rather than electronically generated and the organization requirements are less rigid. Nevertheless, it should be clearly written and organized but does not have the same the requirements for full referencing and checking. All computations should be shown: the standard is that a formula is shown and sourced, values are substituted, and the result stated. If a non-standard program or spreadsheet is used, sample calculations with equations need to be provided and reproducible. Margins of safety should be shown with critical condition and failure mode clearly stated. If, in the judgment of the analyst, the margin is conservative and it is noted as such, the rationale for the statement must be explained. A preliminary stress report may also contain durability analysis.

2.10.2 Final Stress Report

Here is a list of the guiding principles of final stress reports:

- o Final stress reports must provide sufficient information to substantiate the structure.
- o All numbers must be re-computable

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- Analysis must be easily readable
- The report should be done in a cost effective manner which can be interpreted as the minimum required content.

Final stress analysis shall be given a thorough independent check prior to their submission to the regulatory authorities. This may be done by the stress group lead, manager or some other independent analysis structure matter expert (SME) and the check should be documented by the checker's signature on the report. In the event that the final stress report is not submitted to a regulatory agency, the requirements of this section still apply.

Stress analysis reports are to have a uniform degree of coverage for all structures. All primary, Class I and Class IIA structure are to be fully substantiated. The term fully substantiated means that all elements (caps, stiffeners, webs, panels, joints, etc.) of the part are analyzed with critical loads summarized and margins of safety at the critical points calculated and provided. In the event that a part is substantiated only by ultimate strength static testing, all significant internal loadings shall be summarized for that part.

Repetitive structures (typical ribs, bulkhead cutouts, stiffeners, brackets, intermediate spars, etc) should be covered by analysis of the critical or typical one of each group. Additional information sufficient to check the strength of the remaining typical structure should be supplied, i.e. internal loads, pressures, etc.

Class II Structure whose minimum Margin of Safety exceeds 1.0 can be omitted from the Final Stress Report.

It is essential that a Final Stress Report contain all of the critical required static and design load conditions applicable to the structure covered in the report. For instance, if the report is for a wing, balanced load conditions would be expected for maximum wing up- and down-bending, maximum torsion, maximum shear, maximum aero pressures over the surface, all applicable stores/separation conditions and, if fueled, fuel pressure maximum cases, negative fuel pressure cases and any failed valve pressure cases.

The extent to which the "near critical loading conditions" are shown and the number of locations analyzed for any part will be guided by the classification of the part per Section 2.2.1.1, the number and life of the airplanes and the ease of showing a complete analysis. A more complete analysis would be expected of a Class I component in an aircraft which is part of a build of 300 that has an expected 30 year life than would be expected for a Class IIA part or a part which is part of a run of 3 unique aircraft or an aircraft which is a demonstrator or flying test bed.

Detailed analysis can be shown without increasing the bulk of the report by tabulating results or using the results from computer tabulations as well as eliminating routine calculations.

If spreadsheets or programs are used to analyze these repetitive structures and only tables summarizing loads, margins, etc are provided, at least one sample set of calculations, with equations, shall be provided so that the analysis can be easily checked or recreated.

2.10.2.1 Final Stress Report Arrangement

If the report is a compiled stand-alone document of the aircraft or a major component of the aircraft, the report should contain the elements shown in Table 2.10-1. The format and sequence of these elements can be customized per program direction.

Some programs have elected to provide stress analysis as a part of the drawing release package on a part by part basis. Some of the elements of information listed in Table 2.10-1 will need to be provided for each part. Some will need to be contained in a higher level document. Specific program direction is necessary to define the details of this type of approach.

Table 2.10-1 Elements of Formal Stress Report

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Title
Security Cover Sheet
Cover
Title Page – this page repeats the title but additionally has the signature which authorizes release
Abstract or Executive Summary – this is a short summary (one paragraph) which describes why the report has been generated, who the customer is, and a high level summary of what it contains.
Revision List – this page is primarily for updates but by inserting it upon initial release, it serves as a reminder that changes need to be documented
List of Pages – total number of pages by major section.
Minimum Margin of Safety Summary – the lowest margin on each structural element in the report in tabular form with part number, description of part, margin of safety, failure mode, and failure load case id and description. The margin is preferably calculated based on load and allowable load. See Section 2.5.
Table of Contents
Drawing List – list of all drawings analyzed and their titles
List of References – list of other reports, specifications, text books, etc. which are used for reference. The list should be numbered so that it can be cited in the report text.
Symbols, Acronyms, Abbreviations – information provided should be the Symbols, Acronyms, or Abbreviations, with a definition of each and its units.
Sign Conventions – provide sketches if necessary
Introduction
General Arrangement – Overall view of the airplane or airplane section describing where the component fits
Loads – provide all critical load conditions and design conditions applicable to the covered structure/drawings
Detailed Analysis By Component or Part: <ul style="list-style-type: none"> ○ Physical Description ○ Functional Description ○ Geometry ○ List of Load Conditions/Descriptions, including environmental requirements ○ Summary of Specific Design Criteria ○ Summary of Method of Analysis and Assumptions ○ Load Analysis and Summary ○ Detailed Margin of Safety List/Figure ○ Stress Analysis

2.10.2.2 Details of Coverage

A formal stress report contains static analysis and margins of safety or ratios to requirements. It generally does not contain durability or damage tolerance analysis or estimates of life, unless by direction of the program. Life information is generally contained in another report.

All analysis included in the final report must be clear and complete. It should always start with an overall picture and work down to the details:

- What is being analyzed (specific section or detail), drawing or part number, material and material size, heat treat, other processing, etc.
- Allowables and source of allowables – MMPDSXX, program document, stress manual, etc
- Why it is being analyzed – if it has any critical function or type of loading which isn't obvious
- How is it being analyzed – always state the assumptions and method, referencing stress manuals, text books, program guidance, etc.

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- Loads being used and load application. Provide load case id and description, free-body diagram(s), references to loads document or FEM. If internal loads are from the FEM clearly identify element or grid point id.
- Calculations and tabulations of stresses
- Final results – margin of safety or ratio to requirement (stability, deformation), double underlined, stating type of failure (bearing, net section, buckling, crippling, etc.) and load condition id and descriptor.
- Picture of part with critical location(s) indicated.

For calculations not shown, there should be sufficient information to reproduce the results. This includes pertinent dimensions, loads, eccentricities, secondary loads, pressures and so forth. If section property calculations are made by computer or spreadsheet, provide these in the summary.

A complete file of unsubmitted analysis shall be maintained by each IPT stress organization for use in material review, aircraft repair, and aircraft modifications until the entire fleet is retired from use.

2.10.2.3 Computer Generated Data

Final computer generated data from FEA models, spreadsheets or computer programs from which critical loadings, stresses and margins are generated, if not included in the stress report, shall be maintained by the stress group. This can be done electronically or by hard copy. It is the responsibility of the aircraft program to establish a data repository in which this data can be placed and maintained. Data organization should allow for easy access and retrieval and, as a minimum, information should be identified and described in summary report available in the same location. It is the responsibility of the cognizant stress person to ensure that the files/reports are placed in the data repository.

Any personal computational aids (spreadsheets, computer programs, etc) which are used to perform calculations to generate data for the final report need to have sufficient testing for verification that the results generated are as expected. This verification needs to be documented and retrievable.

2.10.2.4 Report Format

Program guidance will determine the electronic format of the final report. Options currently include both Microsoft Word and Excel. The organizations and final content is governed by program guidelines; however this section provides some recommendations for use by the program in its final determination.

Each page of the stress analysis shall contain the following information:

- Company Name
- Report Number
- Date of Preparation
- Name of Analyst
- Name of Checker
- Revision Dates
- Page Number
- Name and Part number of part being analyzed.

Even when being prepared, pages should be numbered. These temporary numbers can be updated or replaced when the analysis become part of a larger report.

All stress reports shall be written in the third person present tense and written in such a manner so that it can be easily understood and checked. The data should be developed in the sequence needed. The analyst preparing the report should place himself in the position of someone unfamiliar with the part looking at the analysis for the first time. If he needs some information or a picture to understand the analysis, then it should be included.

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Sketches, drawings or illustrations should be readable without the drawing and do not have to contain all of the information that the actual drawing would contain. It may require multiple figures to provide sufficient information.

The figures should contain the following:

- What is the part and part number
- Its location in the aircraft
- Location, direction and magnitude of and external/applied loads
- Location, direction and magnitude of reactions. A freebody diagram, in balance, should be provided.
- Identify all parts by part number and where appropriate provide information on next assembly, installation, forging, casting or other pertinent drawings.
- Provide material and heat treat information
- Show the dimensions used in the analysis. See Section 2.6.
- Locate the sections analyzed and the reference axis
- Title the sketch or drawing if different from the page heading. If more than one figure appears on a page, numbering may be used for clarification. Do not use a running numbering of figures throughout the report.
- Use break lines to show boundaries of parts.
- Provide sketches which are approximately to scale with only the information necessary. If not approximately to scale, indicate this.

Every figure, formula and method should be clearly referenced to its source. The only exceptions are the following:

- Items (numbers, formulas, sketches) easily found on the same page
- Common algebraic equations and theorems
- Simple trigonometric relationships
- On tabular forms or outputs, columns should have explanatory headings and a second row provided for column references. If necessary, column results should be specified, e.g. (Col 1)(Col 2)/(Col 5).

When using numbers from elsewhere in the report or from another report, do not round off the number. The reader should be able to easily find the exact number when a reference is checked. This gives assurance that the correct number is being used.

Symbols, acronyms and abbreviations should be introduced and defined within the text or in a subsection of the report.

2.10.2.5 Check of Final Stress Report

It is paramount that **adequate and independent checking** of the final stress report occurs. This is the responsibility of the group lead stress engineer who may provide the check himself or designate the check to another more senior analyst familiar with stress analysis methodology.

The checking required includes

- Numerical verification
- Concurrence with the method(s) of analysis
- Completeness of coverage of the part
- Completeness of coverage relative to the types of analysis required/performed
- Compliance with contract or regulatory requirements
- Readability and completeness of the analysis for organization
- Complete references.

Once checked, the group lead stress engineer shall sign the report as having checked it. It is preferable that each individual page have checker identity noted.

2.10.2.6 Release and Distribution of Final Stress Report

The report should be marked with appropriate security or proprietary restrictions. This includes, but is not limited to United States Department of Defense (US DOD) classification markings, North American Treaty Organization

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(NATO) security markings, International Traffic in Arms Regulations (ITAR) markings/release information, or any company proprietary markings.

Technical approval and distribution of all stress reports is required before release. Such approval shall be given by the responsible Chief Structures Engineer or equivalent within the IPT plus any other approvals required by program direction.

2.11 Errata in Standard Texts

Refer to the errata page on the LMASAM Structural Analysis Website.

2.12 Analysis Checklist

There are many items a stress analyst must consider when performing analysis and reviewing design drawings for signature. This section provides checklists which may be used to aid in the process.

2.12.1 Loads, Criteria, and Environment	
	Balanced Freebody Diagram
	Current Load Sheet identifying Case ID and condition descriptions
	Balanced, Sensible FEM Loads
	Critical Load Conditions Identified, including any design load conditions.
	Applicable specifications checked for criteria. Document any special criteria in analysis
	Temperature and environment known and documented
	Buffet / Acoustic fatigue checked if applicable.
	Dynamic environment identified and documented. Dynamic stiffness criteria or other analysis performed.
	Flutter and control loop stiffness considered
	Applicable pressures and sign conventions documented. Fuel (positive and negative), ullage (fuel overpressure), cockpit, dry bay/compartment, hammershock, vent, ditching, water pressure, etc.
	Applicable local conditions (which MAY NOT be FEM based) and criteria documented: crash, tire burst, gun blast, canopy jettison, actuator jam conditions, bird strike, landing gear loads, ground conditions, hoisting, tie-down, etc.
	Any local inertia/acceleration loads and sign conventions understood, identified and documented.
	Handling / assembly loads considered.
2.12.2 Detail Analysis	
	Function and Loads of all attached parts are known and documented.
	Local and point loads are summarized and shown on freebody diagrams.
	Appropriate material factors, e.g. casting, considered.
	The part has been given a static strength classification designation per Section 2.2.1.1 and this is documented in the analysis.
	All critical sections analyzed with margins indicated.
	No failure at ultimate load.
	No detrimental permanent (plastic) deformation at limit load (or as specific aircraft criteria requires).
	Elastic deformation must not interfere with safe operation.
	Structure is stable, satisfying initial buckling and post buckling requirements, including crippling and beam column effects, as applicable.

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	Has general instability of the structure been checked? This can be an issue even if all other stability criteria have been met.
	Design has direct efficient load lines; structure can transmit loads from source to reaction.
	Any changes in load line (kicks or eccentricities) have been minimized and are properly supported
	Minimum tolerance / accumulation have been considered. Analysis thicknesses comply with Section 2.6
	Stress concentrations have been minimized or eliminated to the extent possible. Refer to Section 2.7 for guidance.
	The curved flange / curved beam effects have been analyzed.
	Structural stiffness for flutter, load sharing, dynamic analysis is sufficient.
	Effects of thermal mismatch and thermal loading have been included in the analysis.
	Effects of thermal environment used to appropriately reduce allowable loads.
	In hot regions of the aircraft near engines and other heat sources, creep must be considered.
	No square cutouts – even with generous radii.
	No hard points in pressurized structure.
	Design has been analyzed to minimum weight
	Design is minimum cost

2.12.3 Secondary Effects

	Secondary induced loads have been defined and analyzed: membrane effects, Brazier effect, friction, beam column, lateral longeron bending due to bulkhead loads, flanges adjacent to lugs, forced crippling.
	Diagonal Tension: For post buckled structure, have loads been appropriately redistributed to caps and analyzed. Has forced crippling of stiffeners been taken into account?
	Deflections checked for changes in load path.

2.12.4 Special Considerations

	Have external skins, panels and doors been analyzed for gap, waviness or deflection per program aerodynamic, radar cross-section or propulsion requirements?
	Have external skins, panels and doors been analyzed for sonic fatigue and flutter?
	Have minimum program requirements for structural thicknesses of metallic parts for lightning strike or impact damage been satisfied?
	Have external doors been analyzed for design pressures (in-flight and ground-based), stiffness requirements due to ram pressure divergence, dynamic response, and flutter requirements as well as sonic environment?
	Has the structure in cavities that are open to the air stream been analyzed for open bay pressure differentials (positive and negative) and sonic fatigue?

2.12.5 Material, Processing, and Inspections

	Has the potential for Stress Corrosion been considered (preload, interference fits, cold work, clamp-up, material selection, grain orientation) and eliminated to the extent possible. Refer to Section 3.4 for a discussion of corrosion and prevention.
	Has the optimum material for the part been selected.
	Thermal expansion mismatch between materials should be minimized. The combination of aluminum and graphite composite is least compatible.
	Have the appropriate manufacturing processes been specified.
	Is the proper finishing specified
	Has the impact of the manufacturing process been accounted for in the stress analysis? See Section 2.6 for a discussion of potential impacts to tolerances and material thinning.

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	Does the selected material have a yield allowable less than 2/3 of the ultimate allowable and has this been accounted for in the analysis?
	Is there adequate non-destructive inspection (NDI) for structural integrity? Refer to Section 2.9 for a discussion of NDI methods.
	Have the need for inspections been balanced with the criticality of the part and the inspections minimized to the extent possible?
	Have appropriate First Article Inspections, per Section 2.9, been specified.

2.12.6 Fastener Joints

	Are there sufficient size and type of fasteners specified to carry the load?
	Joints have been analyzed for applicable fastener failure modes (fastener shear, fastener bending, fastener tension) and material failure modes (bearing, net section tension, shear tear-out).
	Have fitting/ joint/ lug factors been appropriately considered per program guidance?
	Is the proper torque specified, either in the fastener installation specification or on the drawing? Fasteners in tension applications require higher torque than those in shear applications. Most Lockheed Martin torque specifications are intended for shear applications. Refer to Section 5.2 for discussion.
	Edge distance is nominal 2D for metals and 2.5D for composites.
	Fastener spacing is minimum of 4D and a maximum of 8D.
	Check inter-rivet buckling
	No threads in bearing allowed.
	No beneficial effects of friction accounted for except in friction joints per Section 5.9.
	No mixed fastener types in joints.
	No mixed hole fits in joints.
	Match the capability of nuts/collars with fasteners in tension applications, i.e. no aluminum collars/nuts in tension..
	Proper fastener callouts on drawing.
	Single fastener joints avoided. Joint can spin, additional bending moments induced, etc.
	Eccentricities considered in single shear and one sided lap joints.
	Joint fastener patterns are designed such that load lines end at centroid of fastener patterns to minimize secondary loads.
	Fastener fatigue considered, as applicable. This can be very important in composite/metallic joints.
	Fastener bending considered, as applicable. See Section 5.2.3.4 for discussion.
	End fastener load peaking considered.
	Joints are not fastener shear critical, but preferably bearing critical.
	No knife edge countersunk fasteners. Refer to Section 5.2.3.1.7 for discussion.
	Safety-of-flight-critical fasteners require separate locking mechanism.
	All bolts subject to rotation require separate locking mechanism.
	All bolts subject to rotation and high vibration environments should be oriented such that the bolt does not fall out if the nut becomes loosened.
	Any requirements for sub-flush heads have been taken into account in the analysis.
	Any reduction in allowables for wet installed fasteners been taken into account in the analysis.
	If a few holes in a joint require cold working, consider cold working all to ensure the critical ones are cold worked.
	Destack and debur all joints (after drilling, joint is separated, i.e. destacked, and all burrs and chips removed, i.e. deburred).
	Stresses from press-fit bushings must be analyzed both for the effects on the bushing as well as the surrounding structure.

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2.12.7 Durability and Damage Tolerance Considerations

	Has durability analysis or screening occurred per program requirements?
	Has the part been designated Fracture Critical per the Fracture Control board?
	Are the appropriate durability/fracture critical classifications designated on the drawing along with the appropriate process specifications and inspections?

2.12.8 Fabrication Considerations

	Surface finish RA 125 or better specified. If not, include effects in durability and damage tolerance analysis, as required.
	Multiple corner radii / fillet radii have blend and smooth requirements specified
	Combined stress concentrations eliminated to extent possible.
	Have any required cold worked or peened areas been designated on the drawing?
	No excessive stresses in short transverse grain direction or parting plane area of forgings. If possible, locate parting plane off edge of part.
	Residual tension stresses checked from compression yielding and bend forming
	Part thin out due to super-plastic forming, deep drawing or bends analyzed
	Checked stress concentrations from plate nut rivets, tooling holes and drain holes. Ensure plate nut rivets are oriented in least harmful direction. Relocate tooling and drain holes to low stress areas, if possible. Verify that any required pad ups have been added to the drawing.
	Water trap areas avoided.
	Potential for galvanic corrosion through the use of compatible materials and appropriate surface protection minimized.
	Ability for repair must be considered. Working stresses should not be so high that fasteners cannot be installed.
	Has the drawing oriented the part correctly with primary grain direction specified?
	Is the proper heat treatment specified?
	Are the proper lubricants specified?
	Have joints been checked for compatibility to preclude galling?
	Use proper weld efficiency factors in analysis.
	Has the appropriate weld classification and inspection requirements, commensurate with part criticality, been specified on the drawing
	If welding is required, are the selected materials weldable?

2.12.9 Systems Installations

	Correct operating, proof and burst pressures specified for all fluid systems
	Drawing should include clear, achievable rigging procedures to prevent excessive mis-rigging loads
	Mechanism loads should be determined in a manner that accounts for friction, e.g. friction circle analysis
	Removable items should have captive fasteners
	Attachments for systems brackets in primary structure such as bulkheads should be made in stiffeners, not in the webs. Fasteners should have sufficient edge distance in the stiffener. Since the systems design phase generally occurs after the structural design phase, stiffeners should be assumed to have fastener holes present during the structural design phase.
	Check mounting of heavy objects, such as pumps, to ensure no hard points are created.
	No fully threaded fasteners in aircraft structure.

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	Avoid tall tube and equipment brackets which can cause significant secondary loads in back-up structure.
	No butterfly clamping of tubes.
	For high pressure fuel or hydraulic lines, make sure that loads are properly accounted for.

2.12.10 Testing and Certification

	A and/or B-Basis Material allowables available as necessary
	Is special testing per Section 2.9.2 or 2.9.3 required for part?
	Can this part be fully tested as part of full-scale static and fatigue test articles? Considerations include redundancy of structure, part being sized primarily by other considerations than flight loads (such as pressure)
	If a part is being tested, what is the test criteria and what constitutes a successful test?
	Is component or element level testing required for certification of part?
	Does vulnerability testing need to be conducted on this part or component?
	Does birdstrike testing need to be conducted on this part or component?
	Is the test article representative of the final design using production methods?
	Are all necessary parts installed on the test article?
	Are all the finishes representative of production
	Does the test article contain all of the holes and stress concentrations present in production aircraft?
	If the article is instrumented, have strain predictions been made?
	Prior to test have no-go stress/strain or load levels been identified for all gages?