4 Section Properties

The purpose of this chapter is to provide guidance regarding routine section property calculation for metallic structural analysis. It provides discussion and methodology for basic hand-calculation, CAE-based determination, and stand-alone computer-aided calculation of section properties.

4.1 References and Nomenclature

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Nomenclature

Symbol	Description	Units
A	Area	in^2
b	Length, generally of a flange	in
b	Width of skin, web or panel	in
B	On Mohr’s circle, projected radius	in
C	On Mohr’s circle, location of center of Mohr’s circle	in
CCW	Counterclockwise	--
C_w	Warping constant about the shear center of the section (sometime referred to as Γ)	in^6
C_{w-c}	Warping constant about the centroid of the section (sometime referred to as Γ_c)	in^6
d, D	Diameter of circle	in
d	distance	in
E	Elastic Modulus of the Material (Young’s Modulus)	psi
E_{tan}	Tangent Modulus of the Material	psi
f, F	Stress	psi
F_{cc}	Crippling Stress	psi
f_{cre}	Elastic initial buckling stress	psi
h, h′, H	Height	in
I	On Mohr’s circle, the horizontal axis	
I	Second or Area Moment of Inertia	in^4
I_{max}	Maximum Principal Moment of Inertia	in^4
I_{min}	Minimum Principal Moment of Inertia	in^4
I_{o-xi}	Second or Area Moment of Inertia of an element of a cross-section about the element’s own centroidal x axis	in^4
I_{o-yi}	Second or Area Moment of Inertia of an element of a cross-section about the element’s own centroidal y axis	in^4
I_p	The Polar Moment of Inertia about the centroid	in^4
I_{p-sc}	The Polar Moment of Inertia about the shear center	in^4
I_{xy}	Product of Inertia in the x-y coordinate system	in^4
I_{uv}	Product of Inertia in the u-v coordinate system	in^4
K	Torsion Constant (in some literature this is also referred to as J)	in^4
k	Buckling Fixity Coefficient	--
L	Length	in
n	Number of elements in a cross-section	--
P	On Mohr’s circle, the vertical axis	--
Q	First or static moment	in^3
q	running shear load	lb/in
r	In warping constant calculations, normal distance from centroid or center of twist to centerline of element	in
R	Radius of Circle	in
R_{xx}	Warping moment about the xx axis	in^5
R_{yy}	Warping moment about the yy axis	in^5
s	length	in

sc	Shear center	--
t	thickness	in
t_{i-1}, t_{i-2}	the thickness of leg 1 or 2 of the i^{th} tee or L intersection	in
u,v	Coordinate system obtained by rotating x,y coordinate system through angle α	--
V	Transverse load	lb
w, w′, W	Width	in
w_s	Warping displacement	in^2
w_{i-a}	Warping displacement of the i^{th} element at end a	in^2
w_{i-b}	Warping displacement of the i^{th} element at end b	in^2
x	Distance along the x axis	in
x_{bar}	x distance to the section centroid from the reference axis	in
x_o	x distance between the shear center and the centroid	in
x_{sc}	x distance to the section shear center from the reference axis	in
y	Distance along the y axis	in
y_{bar}	y distance to the section centroid from the reference axis	in
y_o	y distance between the shear center and the centroid	in
y_{sc}	y distance to the section shear center from the reference axis	in
α	Rotation angle (positive CCW) between x-y axes and the u,v axes	degrees
ε	Material Strain	in/in
ε_{cy}	Material Compression Yield Strain	in/in
ε_{IR}	Material Inter-rivet Buckling Strain	in/in
θ	On Mohr's circle, the half-angle between the arbitrary centroidal x,y axes and the principal axes	radians
ν_e	Elastic Poisson's ratio for the material	--
φ	Rotation angle (positive CCW)	radians
ρ	Radius of gyration	in
ρ_{o-x}	Radius of gyration of a cross-section about the centroidal x axis	in
ρ_x	Radius of gyration of a cross-section about the reference x axis	in
ρ_{o-y}	Radius of gyration of a cross-section about the centroidal y axis	in
ρ_y	Radius of gyration of a cross-section about the reference y axis	in

Subscripts

Symbol	Description	Units
1,2,3,…	Specific elements in a cross-section	--
a,b,c…	Specific elements in a cross-section	--
c	Compression	--
cap	Related to the cap element of the cross-section	--
eff	The portion of the structure which is considered effective for analysis	--
i	The i^{th} element	--
inner	Denoting inner radius or diameter of a hollow circle	--
outer	Denoting outer radius or diameter of a hollow circle	--
sc	In relation to the shear center of the section	--
sk, skin	Related to the skin	--
st	Related to the stiffening element (stiffener, upright, etc.)	--
uu	About the u axis	--
vv	About the v axis	--
web	Related to the web element of the cross-section	--
x_{ref}	About the reference x axis	--
x, xx	About the x axis	--
y_{ref}	About the reference y axis	
y, yy	About the y axis	--

4.2 Basic Theory and Tabulated Shapes

Stress analysis requires the ability to calculate fundamental geometrical values of various shaped cross-sections. This section provides various means for the calculation of these geometrical values, which are essential to calculating the behavior of cross-sections under load.

- Section 4.2.1 provides a practical overview of the theory behind the calculation of area, centroid, first area moment of inertia or "static moment", second area moment or "moment of inertia", the product of inertia, polar moment of inertial, and radius of gyration. This is followed by an example problem to demonstrate building up the geometric properties of a section made up of rectangular components.
- Section 4.2.2 presents the St. Venant torsion constant for thin-walled open sections.
- Section 4.2.3 discusses the use of Mohr's circle in the determination of moments of inertia
- Section 4.2.4 provides equations for the rotation of moment of inertia through an arbitrary angle
- Section 4.2.5 shows a method for the calculation of section properties from standardized shapes
- Section 4.2.6 provides basic equations for commonly used standard shapes.
- Section 4.2.7 discusses the calculation of shear center and the warping constant about the shear center for commonly used standard shapes.
- Section 4.2.8 provides a generalized method for the calculation of the warping constant and warping moments about the section centroid

4.2.1 Basic Equations

Figure 4.2.1-1 shows a generalized section and defines some basic terminology which can be used to calculate the geometrical values for a given cross-section. Note that x and y represent distances along the x and y axes; however, depending on the calculation, they may reference different starting points. Each equation delineates the specific distance in question.

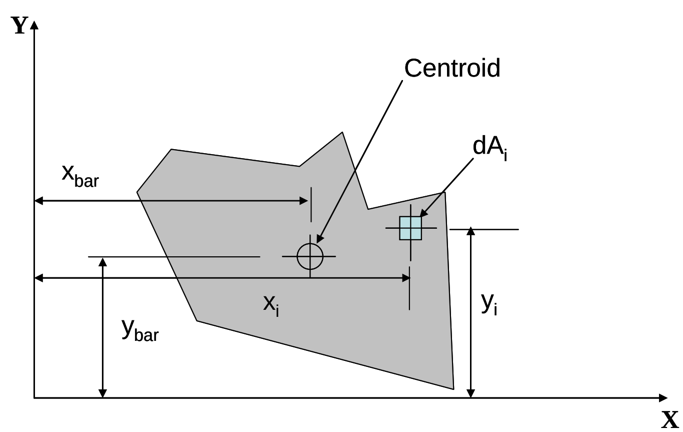


Figure 4.2.1-1 General Section and Terminology

The area, A, can be calculated as

$$A=\int dA = \Sigma dA_i \text{ for } i=1,n$$ **Equation 4.2.1-1**

where
dA_i is an infinitesimal area (in^2)

Although the equation is presented as the summation of infinitesimal areas, it is equally applicable if the area is divided up into logical sub-areas which can easily be calculated.

The locations of the geometric centroid of the part can be found from

$$x_{bar} = \Sigma(x_i dA_i) / A \qquad \text{Equation 4.2.1-2}$$
$$y_{bar} = \Sigma(y_i dA_i) / A \qquad \text{Equation 4.2.1-3}$$
$$\text{for } i=1,n$$

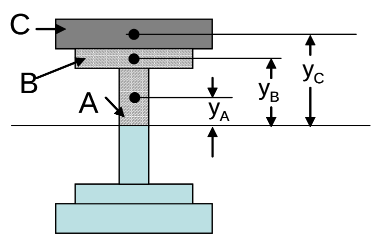
where
x_i and y_i are the locations of the centroid of the infinitesimal area or subarea, dA_i relative to a reference axis (in)
dA_i is the incremental area (in^2)
A is the total cross-sectional area (in)

The first area moment or static moment, Q is the summation of the area moments due to all dA_i elements above or below or to the left or right of the centroidal axes and is given by

$$Q_x=\Sigma y_i dA_i \qquad \text{Equation 4.2.1-4}$$
$$Q_y=\Sigma x_i dA_i \qquad \text{Equation 4.2.1-5}$$

where
x_i is the distance, along the x axis from the centroid of the whole section to the centroid of the infinitesimal section or sub-section (in)
y_i is the distance, along the y axis from the centroid of the whole section to the centroid of the infinitesimal section or sub-section (in)

Figure 4.2.1-2 depicts an I-Beam cross-section which has been subdivided into 3 sub-elements A, B and C. The equation to calculate the static moment Q_x for the top portion of the I-Beam is shown in the figure.



$$Q_{x\text{-}top} = (A_A y_A+A_B y_B+A_C y_C)$$

Figure 4.2.1-2 Sample Calculation of the First Area Moment or Static Moment

The second area moments, or moments of inertia, about the centroidal axes are given by

$$I_{xx} = (\Sigma I_{o\text{-}xi} + \Sigma y_i^2 dA_i) - y_{bar}^2 \Sigma dA_i \qquad \text{Equation 4.2.1-6}$$
$$I_{yy} = (\Sigma I_{o\text{-}yi} + \Sigma x_i^2 dA_i) - x_{bar}^2 \Sigma dA_i \qquad \text{Equation 4.2.1-7}$$
$$\text{for } i=1,n$$

where
x_i, y_i are the x and y distances to the incremental area from a reference axis (in)
x_{bar}, y_{bar} are the x and y distances to the section centroid from the reference axis (in)
dA_i is the incremental area (in^2)
$I_{o\text{-}xi}$ and $I_{o\text{-}yi}$ are the area moments of the incremental area about its own centroidal x and y axes (in^4)

The moments of inertia about any axes parallel to the centroidal axes are found through the use of the parallel axis theorem[2] and are given by

$$I_{x'x'} = I_{xx} + Ay^2 \qquad \text{Equation 4.2.1-8}$$
$$I_{y'y'} = I_{yy} + Ax^2 \qquad \text{Equation 4.2.1-9}$$

[2] The parallel-axis theorem states that the moment of inertia of an area with respect to an arbitrary x′ axis is equal to the moment of inertia of the area with respect to the centroidal x axis, parallel to the arbitrary x′ axis, plus the product of the area times the square of the distance between the x and the x′ axes.

where
x, y are the distances from the centroidal axis to the new axis along the x and y axes (in)

The analyst should note that the moments of inertia relative to the centroidal axes and those relative to other axes parallel to the centroidal axes are often interchangeably referred to as "I_x" or "I_y". It is incumbent on the analyst to know which moment of inertia is required and use the correct value.

The product of inertia, I_{xy}, is given by

$$I_{xy} = \Sigma(x_i)(y_i)dA_i$$ **Equation 4.2.1-10**

where
x_i, y_i are the distances from the centroid to the incremental area along the x and y centroidal axes, respectively (in)

The product of inertia may be positive, negative or zero since x or y may be negative. The product of inertia of an area with respect to two rectangular axes is zero if either one is an axis of symmetry.

The polar moment about the centroidal axis is given by

$$I_p = I_{xx} + I_{yy}$$ **Equation 4.2.1-11**

The polar moment about the shear center, used in torsional buckling calculations is given by

$$I_{p\text{-}sc} = I_{xx} + Ay_o^2 + I_{yy} + Ax_o^2$$ **Equation 4.2.1-12**

where
x_o, y_o are the distances between the shear center and the centroidal axes (in)

The principal axes are the axes which have a zero product of inertia. The principal axes can be found by rotating the centroidal axes through an angle θ as shown in Figure 4.2.1-3. The angle θ (+ CCW) is calculated from

$$\tan(2\theta) = \frac{-2I_{xy}}{(I_{xx} - I_{yy})}$$

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{-2I_{xy}}{I_{xx} - I_{yy}}\right)$$ **Equation 4.2.1-13**

where
I_{xx}, I_{yx} are the moments of inertia about the centroidal x and y axes, respectively. Given by Equations 4.2.1-6 and 4.2.1-7 (in^4)
I_{xy} is the product of inertia given by Equation 4.2.1-10 (in^4)

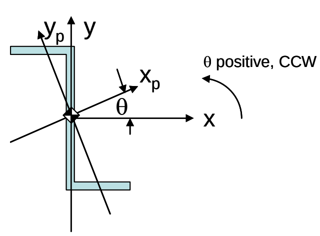


Figure 4.2.1-3 Illustration of Principal Axes

The maximum and minimum moments of inertia about the principal axes are given by

$$I_{\max}, I_{\min} = \frac{I_{xx} + I_{yy}}{2} \pm \sqrt{I_{xy}^2 + \left(\frac{I_{xx} - I_{yy}}{2}\right)^2}$$ **Equation 4.2.1-14**

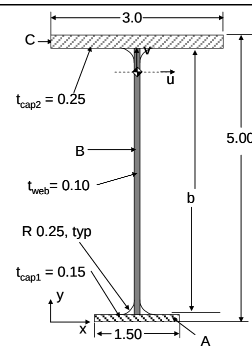
The derivation of the principal axes and maximum and minimum moments of inertia is discussed in Section 4.2.3 Mohr's Circle.

The radius of gyration of a section of a body with respect to any axis is defined as the distance from the axis at which the mass may be concentrated and have the same moment of inertia with respect to the axis as does the distributed mass. The radius of gyration can be calculated about the centroidal axis or an axis parallel to it, depending on the analysis required. For the centroidal axis, it is given by

$$\rho_{xx} = \sqrt{(I_{xx}/A)}$$ **Equation 4.2.1-15**

$$\rho_{yy} = \sqrt{(I_{yy}/A)}$$ **Equation 4.2.1-16**

4.2.1.1 Example Problem – Basic Section Properties

Given the cross-section pictured, use Equations 4.2.1-1 through 4.2.1-16 to calculate the section properties. Subdivide section into subsections A, B and C Assume x,y reference axis origin as shown and the x,y centroidal axes are denoted as u,v.	
1) $A = \Sigma dA_i$	$A = A_A+A_B+A_C$ Height of web, b = (5.00-0.25-0.15) = 4.6 in A_A = 1.5(0.15) = 0.225 A_B =4.6(0.10) = 0.460 A_C =3(0.25) = 0.750 A = 0.75 + 0.46 + 0.225 A = 1.435 in^2
2) $x_{bar} = \Sigma(x_i dA_i) / A$ $y_{bar} = \Sigma(y_i dA_i) / A$	Element A: x_A = 3.0/2 = 1.5; y_A = 0.15/2 = 0.075 in Element B: x_B = 3.0/2 = 1.5; y_B = 0.15 + 4.6/2 = 2.45 in Element C: x_C = 3.0/2 = 1.5; y_C = 0.15 + 4.6 + 0.25(2) = 4.875 in $x_{bar} = \Sigma(x_i dA_i)$ / A = [1.5(0.225) + 1.5(0.460) + 1.5(0.750)]/1.435 = 1.5 $y_{bar} = \Sigma(y_i dA_i)$ / A = [0.075(0.225) + 2.45 (0.460) + 4.875(0.750)]/1.435 = 4.800/1.435 = 3.345 in
3a) $Q_x=\Sigma y_i dA_i$ To calculate the static moments, create a new coordinate system, u,v with the origin at the centroid of the part Divide the section into two parts, above and below the centroid. The upper portion is comprised of Element C	Element A: v_A=3.345-0.15/2 = 3.27 Element B_{lower}: $v_{B\text{-}lower}$=(3.345-0.15)/2 = 1.5975 in Element B_{upper}: $v_{B\text{-}upper}$=(5.00-3.345-0.25)/2 =0.7025in Element C: v_C=(5.00-3.345)-0.25/2 = 1.53 $Q_{x\text{-}upper}$ = 1.53(0.750) + 0.7025[(5.0-3.345-0.25)(0.10)] = 1.246 in^3

and B_{upper} while the lower is comprised of Element A and B_{lower}	$Q_{x\text{-}lower}$ = 3.27(0.225) + 1.5975[(3.345-0.15)(0.10)] = 1.246 in^3
3b) $Q_y = \Sigma x_i dA_i$ Divide the section into two parts, to the left of and to the right of the centroid v axis. The left half is of half of Elements A, B and C while the right half is comprised of the other half of Elements A, B and C. The section is symmetric about B, thus the $Q_{y\text{-}left}$ is identically equal to $Q_{y\text{-}right}$	Element A: u_A=±(1.5/2)/2 = 0.375; Element B: u_B=±(0.10/2)/2 = 0.025; Element C: u_C=±(3.0/2)/2 = 0.75; $Q_{y\text{-}left}$ = 0.375[(1.5/2)(0.15)] + 0.025[(4.6(0.10/2)] + 0.75[(3.0/2)(0.25)] = 0.329 in^3 = $Q_{y\text{-}right}$
4a) $I_{xx} = (\Sigma I_{o\text{-}xi} + \Sigma y_i^2 dA_i) - y_{bar}^2 \Sigma dA_i$ I_{xx} is the moment of inertia of the entire cross-section about the centroidal x axis (defined as "u" in the sketch) $I_{o\text{-}xi}$ is the moment of inertia of the individual subsections about their individual centroidal axes. y_i is the distance, in the y direction, from the subsection centroids to the original x axis. y_{bar} is the distance, in the y direction, from the entire cross-section's centroid to the original x axis.	Element A: $I_{o\text{-}xA} = bh^3/12 = 1.5(0.15)^3/12 = 4.219\text{x}10^{-4}$ in^4; From 1 and 2 above: y_A = 0.075 in; A_A = 0.225 in^2 Element B: $I_{o\text{-}xB} = bh^3/12 = 0.10(4.6)^3/12 = 0.811$ in^4; From 1 and 2 above: y_B = 2.45 in; A_B = 0.460 in^2 Element C: $I_{o\text{-}xC} = bh^3/12 = 3.0(0.25)^3/12 = 3.906\text{ x}10^{-3}$ in^4; From 1 and 2 above: y_c = 4.875 in; A_c = 0.750 in^2 From 1 and 2 above: y_{bar}=3.345 in and A = 1.435 in^2 (calculated above) $I_{xx} = (\Sigma I_{o\text{-}xi} + \Sigma y_i^2 dA_i) - y_{bar}^2 \Sigma dA_i = (4.219\text{x}10^{-4} + [0.075^2(0.225) + 0.811 + 2.45^2(0.460) + 3.906\text{ x}10^{-3} + 4.875^2(0.750)] - 3.345^2(1.435) = 5.342$ in^2
4b) $I_{yy} = (\Sigma I_{o\text{-}yi} + \Sigma x_i^2 dA_i) - x_{bar}^2 \Sigma dA_i$ I_{yy} is the moment of inertia of the entire cross-section about the centroidal y axis (defined as "v" in the sketch) $I_{o\text{-}yi}$ is the moment of inertia of the individual subsections about their individual centroidal axes. x_i is the distance, in the x direction, from the subsection centroids to the reference y axis. x_{bar} is the distance, in the x direction, from the entire cross-section's centroid to the reference y axis.	Element A: $I_{o\text{-}yA} = bh^3/12 = 0.15(1.5)^3/12 = 0.0422$ in^4; From 1 and 2 above: x_A = 1.5 in; A_A = 0.225 in^2 Element B: $I_{o\text{-}yB} = bh^3/12 = 4.6(0.1)^3/12 = 3.833\text{ x}10^{-4}$ in^4; From 1 and 2 above: x_B = 1.5 in; A_B = 0.460 in^2 Element C: $I_{o\text{-}yC} = bh^3/12 = 0.25(3.0)^3/12 = 0.5625$ in^4; From 1 and 2 above: x_c = 1.50 in; A_c = 0.750 in^2 From 1 and 2 above: x_{bar}=1.5 in and A = 1.435 in^2 (calculated above) $I_{yy} = (\Sigma I_{o\text{-}yi} + \Sigma x_i^2 dA_i) - x_{bar}^2 \Sigma dA_i = [0.0422 + 1.5^2(0.225) + 3.833\text{x}10^{-4} + 1.5^2(0.460) + 0.5625 + 1.5^2(0.750)] - 1.5^2(1.435) = 0.6055$ in^2
$I_{x'x'} = I_{xx} + Ay^2$ $I_{y'y'} = I_{yy} + Ax^2$ The transfer could go to any other axis which is parallel to the centroidal axes	Transfer the moments of inertia from the centroid to the reference axis: y=3.345, x=1.5 $I_{x'x'} = I_{xx} + Ay^2 = 5.342 + 1.435(3.345)^2 = 21.398$ in^4 $I_{y'y'} = I_{yy} + Ax^2 = 0.6055 + 1.435(1.5)^2 = 3.834$ in^4
$I_{xy} = \Sigma(x_i)(y_i)dA_i$	Because the section is symmetric about the y-axis, $\Sigma(x_i) = 0$ and $I_{xy} = 0.0$
$I_p = I_{xx} + I_{yy}$	I_p = 5.342 + 0.6055 = 5.948 in^4
$I_{max}, I_{min} = (I_{xx}+I_{yy}) / 2 \pm [I_{xy}^2 + ((I_{xx}-I_{yy})/2)^2]^{0.5}$	Since I_{xy}=0, $I_{max} = I_{xx}$ and $I_{min}=I_{yy}$
$\rho_{xx} = (I_{xx}/A)^{0.5}$ $\rho_{yy} = (I_{yy}/A)^{0.5}$	$\rho_{xx} = \sqrt{(5.342/1.435)}$ = 1.929 in $\rho_{yy} = \sqrt{(0.6055/1.435)}$ = 0.6496 in

4.2.2 St. Venant Torsion Constant

The St. Venant torsion constant is a geometric constant used to describe the relationship between the torsional moment and the angle of twist. The St. Venant torsion constant for thin walled open sections is calculated by dividing the section into elements as shown in Figure 4.2.2.1 and summing the contribution from each element as shown below

$$K=\beta\Sigma(b_i t_i^3) \text{ for } i=1,n$$ **Equation 4.2.2-1**

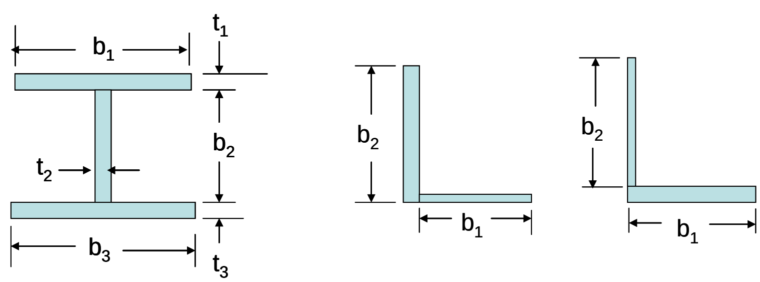
$$\beta=\left[\frac{1}{3}-\frac{0.21}{\frac{b}{t}}\left(1-\frac{1}{12\left(\frac{b}{t}\right)^4}\right)\right]$$

where
b is the flange length (in)
t is the flange thickness (in)

If b>>t, this is often approximated by

$$K=1/3\Sigma(b_i t_i^3) \text{ for } i=1,n$$ **Equation 4.2.2-2**

where
b_i is the length of the i^th^ element (in)
t_i is the thickness of the i^th^ element (in)



For sections comprised of caps and webs, account for the intersection material in the cap element.

Account for the cap as one continuous element

Don't overlap elements or count material twice

In angled sections, account for the material in the intersection in the thicker leg.

Don't overlap elements or count material twice.

Figure 4.2.2-1 Method of Dividing Machined Sections for Calculation of Torsion Constant, K

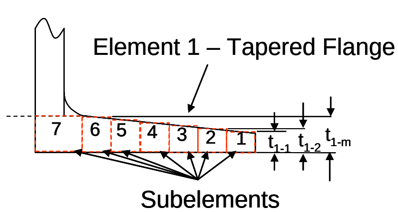
In the case of sheet metal sections b is taken as the developed length of the cross-section. Equation 4.2.2-2 gives accurate results for b/t greater than 60. At a b/t of 30 the difference between this approximation and the actual value is about 2 percent, at b/t of 20 it is about 3 percent and at a b/t of 10 it is above 6 percent for all bent-up sheet metal sections. When b/t is less than 20 for extrusions, omitting the fillet radii and edge effects results in a lower value for the torsion constant resulting in a lower prediction for critical load, a higher calculated applied shear stress and a higher calculated degree of twist and is thus, conservative. Section 4.2.2.1 discusses an approach to reduce this conservatism for extrusions where b/t is less than 20.

4.2.2.1 Calculation of Torsion Constant, K, for Extruded Sections for b/t<20.

As indicated in Section 4.2.2, the use of Equation 4.2.2-1 for extruded sections where the length to thickness ratio of individual flanges is less than 20 is conservative. This section will provide some corrections to improve the accuracy of the calculation. The basic approach described in Section 4.2.2 omits consideration of fillet radii and free edge effects, which for long flanges have a minimal impact on the resulting torsional constant, but for short flanges conservatively underpredicts the torsional constant.

Table 4.2.2-1 describes adjustments that can be made to improve the accuracy of the calculation. Refer to Figure 4.2.2-2 for clarification of dimensions.

Table 4.2.2-1 Calculation of Torsion Constant K for Extruded Sections with b/t < 20

Discussion	Equation
Divide the section into rectangles for webs, flanges or legs and circles for bulbs. This is illustrated in Figure 4.2.2-2 • At the tee joint intersection between a flange and a web, assume the flange to be continuous and the web to start from the side of the flange. • At the L-joint between two legs consider the thicker leg to run to the edge of the section and the thinner leg to start from the side of the thicker leg. • At the joint between a flange and a bulb, consider the flange to start at the bulb.	None
Compute the contributions of the rectangular sections using Equation 4.2.2-1	$K_1=1/3\Sigma(b_i t_i^3)$ *for i=1,n rectangular sections*
Compute the contributions of any circular sections of diameter, D	$K_2 = \Sigma(\pi D_i^4/32)$ *for i=1,n circular sections*
Compute the contributions of any tapered sections where the thickness varies gradually. This approach subdivides a tapered flange into m segments which are approximately constant thickness.	$K_3=1/3\Sigma(b_{i-j} t_{i-j}^3)$ *for j=1,m subsections within* i^{th} *tapered element*
	
Compute the correction for free ends of the rectangular sections. <u>Note this is the one contributor which reduces the torsional constant.</u> Reference 4-5 notes that this correction brings the value of K within the exact solution for b/t>1.5	$K_4 = \Sigma -0.105 t_i^4$ *for i=1,n rectangular sections which have a free end*
Compute the corrections at the junctions between rectangular tee sections. This corrects for fillet radius.	$K_5 = \Sigma d_i^4[0.148+0.102R_i/t_{i-cap}](t_{i-2}/t_{i-1})$ *for i=1,n tee flange intersections* where t_{i-cap} is the tee cap thickness (in) t_{i-1} is the thicker flange(in) t_{i-2} is the thinner flange(in) R_i is the fillet radius (in) d_i is the inscribed circle (in)
Compute the corrections at the junctions between rectangular L-section flanges. This corrects for fillet radius.	$K_6 = \Sigma d^4[0.071+0.077R/t_{i-1}](t_{i-2}/t_{i-1})$ *for i=1,n L flange intersections* where

Discussion	Equation
	t_{i-1} is the thicker flange(in) t_{i-2} is the thinner flange(in) R_i is the fillet radius (in) d_i is the inscribed circle (in)
Compute the corrections at the junction between a rectangle and a circular bulb element. This provides a good approximation for D>2t.	$K_7 = \Sigma\, 0.10 t_i^4$ *for i=1,n rectangular section-circular intersections*
Compute the torsion constant, K	$K = \Sigma\, K_i$ *for i=1,7 torsion constant contributors*

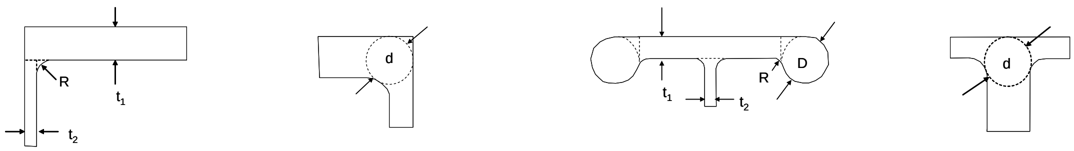
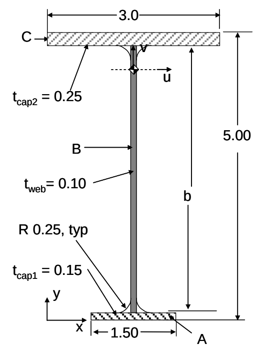
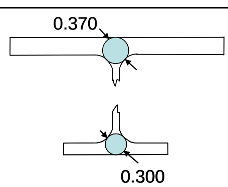


Figure 4.2.2-2 Geometry Definitions for Calculation of Torsion Constant

4.2.2.2 Example Problem – Calculate Torsional Constant

Given the cross-section from Example Section 4.2.1.1, calculate the torsion constant using Equation 4.2.1-15. Then modify the calculation per Table 4.2.8-1.	
Calculate K for the rectangular sections per Equation 4.2.2-1.	$K_1=1/3\Sigma(b_i t_i^3)$ *for i=1,n rectangular sections* $K=(1/3)[3(0.25)^3 + 1.5(0.15)^3 + 4.6(0.10)^3]$ $K_1=0.01885\ in^4$
Using Table 4.2.2-1 for an improved Torsional Constant K_1 – Calculate K for the rectangular sections. (This will be the same numerical value as as is obtained from Equation 4.2.2-1)	$K_1=1/3\Sigma(b_i t_i^3)$ *for i=1,n rectangular sections* $K=(1/3)[3(0.25)^3 + 1.5(0.15)^3 + 4.6(0.10)^3]$ $K_1=0.01885\ in^4$
K_2 – there are no circular sections	
K_3 – there are no tapered sections	
K_4 – Correction for free edges	$K_4 =\Sigma -0.105t_i^4 = -[0.105(0.25)^4+ 0.105(0.25)^4+ 0.105(0.15)^4+0.105(0.15)^4] =$ $-0.0009266\ in^4$

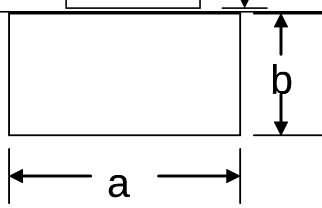
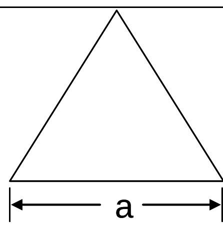
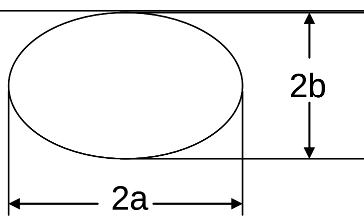
K_5 – Correction for tee intersections d_1 and d_2 were obtained by sketching the cap web intersections to scale and measuring. d_1 = 0.370 in. (measured); $t_{1\text{-}cap}$=0.25 in; R_1 = 0.25 in d_2 = 0.300 in (measured); $t_{2\text{-}cap}$ = 0.15 in; R_2 = 0.25 in 	$K_5 =\Sigma d_i^4[0.148+0.102R_i/t_{i\text{-}cap}](t_{i\text{-}2}/t_{i\text{-}1})$ $K_5 = 0.370^4[0.148+0.102(0.25)/0.25](0.10/0.25)$ $+ 0.300^4[0.148 + 0.102(0.25)/0.15](0.10/0.15)$ $= 0.001874+0.001717 =0.00359$
K_6 – there are no L-sections	
K_7 – there are no bulbs	
Sum up all contributions	$K =\Sigma K_i = 0.01885 – 0.0009266 + 0.00359 = 0.0215\ in^4$

For this section b/t is 10 and 12 for the two caps and 46 for the web. The difference in K from the approximate value given by Equation 4.2.2-1 and the refined value given by Table 4.2.2-1 is a little over 14% higher due to the material in the juncture between the caps and the web.

4.2.2.3 <u>Torsion</u> <u>Constants</u> <u>for</u> <u>Select</u> <u>Sections</u>

The torsion constant, K, is provided for solid non-circular sections in Table 4.2.2-2. They are used to calculate the angular deflection of the section under an applied twisting moment or torque and are used to derive the equations for maximum torsional shear stress. Refer to Section 7.2.2.

Table 4.2.2-2 Exact Solutions for Torsion Constant for Solid Sections

Section	K
	$K = 0.1406a^4$
	$K = ab^3\left[\frac{1}{3} - 0.21\frac{b}{a}\left(1 - \frac{b^4}{12a^4}\right)\right]$
	$K = \frac{\sqrt{3}a^4}{80}$
	$K = \frac{\pi a^3 b^3}{a^2 + b^2}$

For a hollow, non-circular section, depicted in Figure 4.2.2-3, the torsion constant, K, can be calculated from a general equation given as

$$K = \frac{4A_o^2}{\oint \frac{ds}{t}}$$

Equation 4.2.2-3

where
A_o is the area enclosed by the median perimeter line of the section (in^2)
ds is the median incremental length around the perimeter of the section (in)
t is the thickness of the section (in)

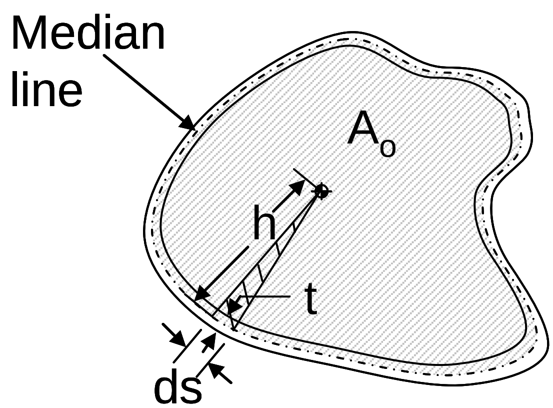


Figure 4.2.2-3 Hollow Non-Circular Section Definition

The enclosed area can be calculated by any convenient means. If the section is an irregularly curved section and the increment ds is sufficiently short to be treated as a straight line and to approximate a right triangle, the area enclosed by the median perimeter line, A_o, can be approximated by

$$A_O = \sum \frac{1}{2} ht$$ **Equation 4.2.2-4**

where
h is the distance from the centroid of the section to the median perimeter line (in)
t is the thickness of the section at the same location (in)

The line integral can be solved by dividing the section into discrete intervals as would be the case for a hollow rectangular section or by using a continuous function as would be the case for a circular ring. See Section 7 for applications and further discussion.

4.2.3 Mohr's Circle and Moments of Inertia

Mohr's Circle can be used to find the principal axes, moments of inertia about those principal axes and the angle between the basic axes and the principal axes. The physical interpretation of Mohr's circle is that as different axis systems rotate about the centroid of the actual section, the values of the moments of inertia and product of inertia are represented by the coordinate points which shift around the circumference of the circle. When the product of inertia is zero, the coordinate points lie entirely on the horizontal axis. These are the moments of inertia about the principal axes.

To use Mohr's circle:

- Calculate the moments of inertia, I_{xx} and I_{yy}, and the product of inertia, I_{xy}, about a convenient x and y centroidal axes.
- On a rectangular coordinate system where I is the horizontal axis and P is the vertical axis, plot the points (I_{xx}, I_{xy}) and (I_{yy}, -I_{xy}).
    - Note that I_{xy} is the calculated value of the product of inertia, which may be positive or negative and is paired with the I_{xx} term. The opposite sign of the product of inertia is paired with I_{yy}.
    - The I and P axes represent the principal axes.
- Connect the two points plotted with a straight line. This forms the diameter of Mohr's circle whose center is on the I coordinate axis. Draw the circle.
- The radius from the center to any point on the circumference represents the axis of inertia corresponding to the I coordinate at that point.

- The angle between any two radii is double the actual angle between the two axes of inertia represented by the two radii.

Figure 4.2.3-1 shows an example of Mohr's circle and the equations below describe how to numerically calculate the values. Notice that when the product of inertia is 0, the values of I are at a minimum and maximum and the line of moment of inertia lies on the horizontal axis indicating the principal axes.

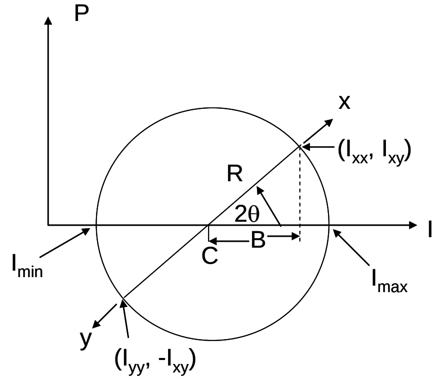


Figure 4.2.3-1 Mohr's Circle Terminology

The location for the center of Mohr's circle, the length of the projected radius, B, and the length of the radius are given by

$$C = (I_{xx} + I_{yy}) / 2 \qquad \textbf{Equation 4.2.3-1}$$

$$B = | I_{xx} - C| \text{ or } |I_{yy} - C|$$

$$R = \sqrt{(I_{xy}^2 + B^2)} \qquad \textbf{Equation 4.2.3-2}$$

where

I_{xx}, I_{yy} are the centroidal moments of inertia about any convenient perpendicular x and y axes, respectively (in^4)

I_{xy} is the product of inertia (in^4)

The value for the maximum and minimum principal moments of inertia are given by

$$I_{max} = C + R$$

$$I_{min} = C - R$$

These equation can be shown to be equivalent to Equation 4.2.1-14.

The angle, in radians, between the arbitrary centroidal x and y axes and the principal axes or any other centroidal axes can be calculated as

$$\theta = \frac{1}{2} \tan^{-1}\left(\frac{I_{xy}}{B}\right)$$

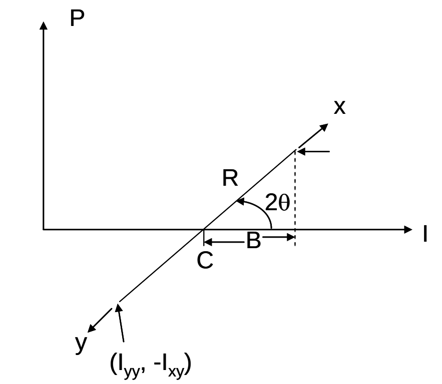
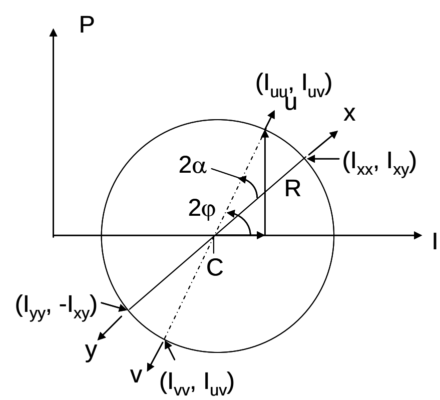
$$\theta = \frac{1}{2} \tan^{-1}\left(\frac{2I_{xy}}{I_{xx} - I_{yy}}\right) \qquad \textbf{Equation 4.2.3-3}$$

The rotational sense of this angle corresponds to the relationship between the initial x and y axis relative to the final axis.

4.2.4 Rotation of Moment of Inertia through an Arbitrary Angle

If it is necessary to rotate the moments of inertia from one coordinate system to another, the problem can be solved using Mohr's circle. Table 4.2.4-1 outlines the steps required for this calculation

Table 4.2.4-1 Mohr's Circle Rotation of Axes Through an Arbitrary Angle

<table>
<tr><td>Step 1: Calculate I_{xx}, I_{yy}, I_{xy} and plot on I, P Coordinate System
Note: The calculated value of I_{xy} is associated with Ixx. If negative, plot as negative.</td><td rowspan="4">
</td></tr>
<tr><td>Step 2: Determine the location of the center of the circle, C, using Equation 4.2.3-1</td></tr>
<tr><td>Step 3: Determine the radius of the circle, R, using Equation 4.2.3-2</td></tr>
<tr><td>Step 4: Determine the angle, θ using Equation 4.2.3-3.</td></tr>
<tr><td>Step 4: Draw Mohr's Circle</td><td rowspan="3">
</td></tr>
<tr><td>Step 5: Rotate x,y axis to new u,v axis through 2α
• α is the angle between the original axis and the desired axis
• α is positive counterclockwise</td></tr>
<tr><td>Step 6: Calculate I_u, I_v, I_{uv} using trigonometry:
For example, for $-\pi/4 < 2\varphi < \pi/4$ radians
$2\varphi = 2\theta + 2\alpha$
$I_{uu} = C + R\cos(2\varphi)$
$I_{uv} = R\sin(2\varphi)$
$I_{uv} = C - R\cos(2\varphi)$</td></tr>
</table>

Another approach is to use the purely mathematical transformation equations given by Equation 4.2.4-1. Figure 4.2.4-1 describes the terminology.

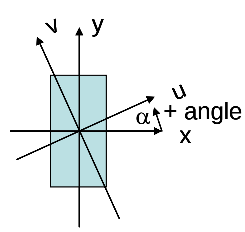


Figure 4.2.4-1 Terminology for Coordinate System Rotation

To rotate the centroidal moment of inertia about the x-y axes through an arbitrary angle α (positive is counterclockwise), the resulting moments of inertia[3] about the u-v axes can be calculated from

$$I_{uu} = I_{xx}\cos^2\alpha + I_{yy}\sin^2\alpha - 2I_{xy}\sin\alpha\cos\alpha \qquad \textbf{Equation 4.2.4-1}$$

[3] Reference 4-8.

$$I_{vv} = I_{xx}sin^2\alpha + I_{yy}cos^2\alpha + 2I_{xy}sin\alpha\, cos\alpha$$

$$I_{uv} = I_{xy}cos2\alpha + [(I_{xx}-I_{yy})/2]sin2\alpha$$

where

α is the angle between the two coordinate systems (+ CCW)

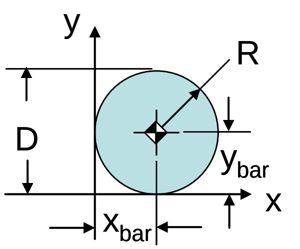
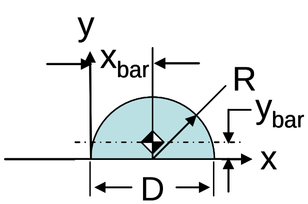
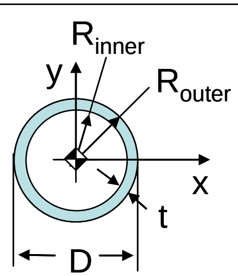
4.2.5 Calculation of Section Properties from Standard Shapes

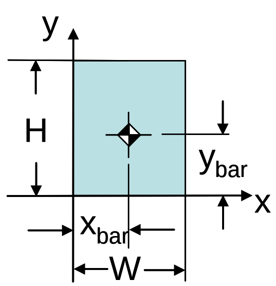
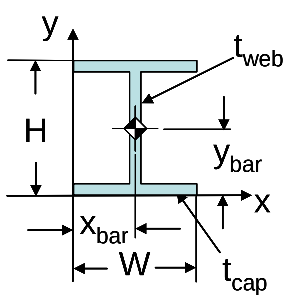
The equations of Section 4.2.1 and the basic shapes of Section 4.2.5 can be used to calculate the moments of inertia of complex sections. The approach subdivides the cross-section into small, logical subsections and uses the parallel axis theorem and Equations 4.2.1-6 and 4.2.1-7 to sum up the contributions to the moment of inertia. The table below is a generic format for this type of calculation. This particular table is set up to calculate the moment of inertia about the x axis only, assuming only rectangular subsections, 1 through n; however a similar calculation could be made for the y axis or including subsections other than rectangles. Note that holes can be removed by modeling them as negative areas in the calculation below.

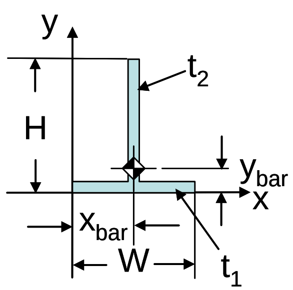
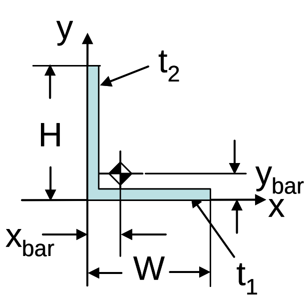
Subsection	Width	Height	y_i	dA_i	$y_i dA_i$	$y_i^2 dA_i$	$I_{o\text{-}x,i}$
1	b_1	h_1	y_1	b_1h_1	y_1dA_1	$y_1^2dA_1$	$I_{o\text{-}x,1}$
2	b_2	h_2	y_2	b_2h_2	y_2dA_2	$y_2^2dA_2$	$I_{o\text{-}x,2}$
…	…	…	…	…	…	…	…
n	b_n	h_n	y_n	b_nh_n	y_ndA_n	$y_n^2dA_n$	$I_{o\text{-}x,n}$
Summations				$A=\Sigma dA$	ΣydA	Σy^2dA	$\Sigma I_{o\text{-}x}$
y_{bar}	$\Sigma(ydA)/A$						
I_{xx}	$\Sigma(I_{o\text{-}x}+y^2dA) - Ay_{bar}^2 = \Sigma I_{o\text{-}x}+\Sigma y^2dA - Ay_{bar}^2$						
where y_i is the distance from the total section axis to the centroid of the individual subsection (in) A_i is the area of the individual subsection (in^2) $I_{o\text{-}x,i}$ is the moment of inertia of the individual subsection about its centroidal axis (in^4)							

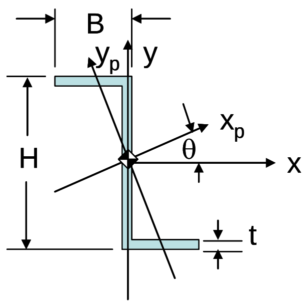
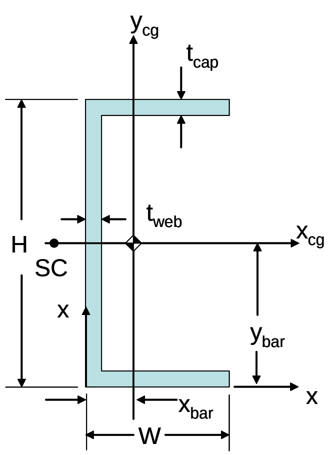
4.2.6 Section Properties for Common Shapes

This section provides equations for the calculation of cross-section properties for some basic shapes. These were taken from References 4-1, 4-2, 4-3, 4-10 and 4-11. Reference 4-2, Volume 1, Section 2 contains an expanded compilation of Section properties for other standard cross-sectional shapes.

Section	Centroid Location	Area	Moment of Inertia, Product of Inertia Radius of Gyration
	$x_{bar} = y_{bar} = D/2 = R$	$A = \pi D^2/4 = \pi R^2$	$I_{xx} = I_{yy} = \pi D^4/64$ $= \pi R^4/4$ $I_{x\text{-}ref} = I_{y\text{-}ref} = (5/4)\pi R^4$ $I_{xy} = \pi R^4$ $\rho_{xx} = D/4 = R/2$ $\rho_{x\text{-}ref} = \rho_{y\text{-}ref} = 1.118D = 2.236R$
	$x_{bar} = D/2 = R$ $y_{bar} = (4/3)R/\pi$ $= 0.4244R$	$A = \pi R^2/2$	$I_{xx} = [\pi/8 - 8/(9\pi)]R^4$ $= 0.1098R^4$ $I_{yy} = \pi R^4/8 = 0.3927R^4$ $I_{x\text{-}ref} = \pi R^4/8 = 0.3927R^4$ $I_{y\text{-}ref} = (5/8)\pi R^4 = 1.964R^4$ $I_{xy} = (2/3)R^4$ $\rho_{xx} = 0.2632R$ $\rho_{yy} = R/2$ $\rho_{x\text{-}ref} = R/2$ $\rho_{y\text{-}ref} = 1.118R$
		$A = \pi(R_{outer}^2 - R_{inner}^2)$	$I_{xx} = I_{yy} = \pi(R_{outer}^4 - R_{inner}^4)/4$ $I_{xy} = \pi R_{outer}^2(R_{outer}^2 - R_{inner}^2)$ *For thin walled tubes where D/t>11 :* $I_{xx} = I_{yy} = \pi t(R_{outer} + R_{inner})^3/8$ $\rho_{xx} = \rho_{yy} = \left(\frac{1}{2}\right)\sqrt{\left(\frac{R_{outer}^4 - R_{inner}^4}{R_{outer}^2 - R_{inner}^2}\right)}$ *For thin walled tubes where D/t>8 :* $\rho_{xx} = \rho_{yy} \sim 0.354(R_{outer} + R_{inner})$ *Error is less than 1%*

Section	Centroid Location	Area	Moment of Inertia, Product of Inertia Radius of Gyration
	$x_{bar} = W/2$ $y_{bar} = H/2$	$A = HW$	$I_{xx} = WH^3/12$ $I_{yy} = HW^3/12$ $I_{x\text{-}ref} = WH^3/3$ $I_{y\text{-}ref} = HW^3/3$ $\rho_{xx} = H/(2\sqrt{3}) = 0.2887H$ $\rho_{yy} = W/(2\sqrt{3}) = 0.2887W$ $\rho_{x\text{-}ref} = H/\sqrt{3} = 0.5773H$ $\rho_{y\text{-}ref} = W/\sqrt{3} = 0.5773W$
Symmetric Section	$h = (H - 2t_{cap})$ $w = (W - t_{web})$ $x_{bar} = W/2$ $y_{bar} = H/2$	$A = HW - hw$	$I_{xx} = (WH^3 - h^3 w)/12$ $I_{yy} = (h t_{web}^3 + 2t_{cap}W^3)/12$ $I_{x\text{-}ref} = [(WH^3 - h^3 w)/12] + A y_{bar}^2$ $I_{y\text{-}ref} = [(h t_{web}^3 + 2t_{cap}W^3)/12] + A x_{bar}^2$ $\rho_{xx} = \sqrt{\dfrac{WH^3 - h^3 w}{12(HW - hw)}}$ $\rho_{yy} = \sqrt{\dfrac{h t_{web}^3 + 2t_{cap}W^3}{12(HW - hw)}}$

Section	Centroid Location	Area	Moment of Inertia, Product of Inertia / Radius of Gyration
 Symmetric Section	$h=H-t_1$ $w=W-t_2$ $x_{bar} = W/2$ $y_{bar}=\frac{t_2H^2+wt_1^2}{2(t_2H+wt_1)}$	$A= HW - hw$	$I_{xx}=\frac{Wt_1^3}{12}+Wt_1\left(y_{bar}-\frac{t_1}{2}\right)^2+\frac{t_2h^3}{12}+t_2h\left(H-\frac{h}{2}-y_{bar}\right)^2$ $I_{yy} = (t_1W^3+ht_2^3)/12$ $I_{x\text{-}ref}= I_{xx}+Ay_{bar}^2$ $I_{y\text{-}ref} = I_{yy} + Ax_{bar}^2$ $I_{xy}=\frac{W^2t_1^2+Wh^2t_2+2Wht_1t_2}{4}$ $\rho_{xx}= \sqrt{(I_{xx}/A)}$ $\rho_{yy}= \sqrt{(I_{yy}/A)}$ $\rho_{x\text{-}ref}= \sqrt{(I_{x\text{-}ref}/A)}$ $\rho_{y\text{-}ref}= \sqrt{(I_{y\text{-}ref}/A)}$
	$h=H-t_1$ $w=W-t_2$ $x_{bar}=\frac{w^2t_1+Ht_2^2+2wt_1t_2}{2(Wt_1+ht_2)}$ $y_{bar}=\frac{Wt_1^2+h^2t_2+2ht_1t_2}{2(Wt_1+ht_2)}$	$A= HW - hw$	$I_{xx} = (1/3)[Wt_1^3+h^3t_2]+t_1^2t_2h+t_1t_2h^2 -Ay_{bar}^2$ $I_{yy} = (1/3)[Ht_2^3+w^3t_1]+t_1t_2^2w+t_1t_2w^2 -Ax_{bar}^2$ $I_{x\text{-}ref}= (1/3)[Wt_1^3+ h^3t_2]+t_1^2t_2h+t_1t_2h^2$ $I_{y\text{-}ref} = (1/3)[Ht_2^3+w^3t_1]+t_1t_2^2w+t_1t_2w^2$ $I_{xy}=Wt_1\left(\frac{W}{2}-x_{bar}\right)\left(\frac{t_1}{2}-y_{bar}\right)+ht_2\left(\frac{t_2}{2}-x_{bar}\right)\left(t_1+\frac{h}{2}-y_{bar}\right)$ $\rho_{xx}= \sqrt{(I_{xx}/A)}$ $\rho_{yy}= \sqrt{(I_{yy}/A)}$ $\rho_{x\text{-}ref}= \sqrt{(I_{x\text{-}ref}/A)}$ $\rho_{y\text{-}ref}= \sqrt{(I_{y\text{-}ref}/A)}$

Section	**Centroid Location**	**Area**	**Moment of Inertia, Product of Inertia** **Radius of Gyration**
 Constant thickness, symmetric section Origin of x, y axis is at centroid of section: x_o is at H/2; y_o is at centerline of web.	$h=H-t$ $b=B-t$ $x_{bar}=0$ $y_{bar}=0$	$A=t(H+2b)$	$I_{xx} = [BH^3-b(H-2t)^3] / 12$ $I_{yy} = [H(B+b)^3 - 2b^3h - 6B^2bh]/12$ $I_{xp}=I_{xx}\cos^2\theta+I_{yy}\sin^2\theta-I_{xy}\sin(2\theta)$ $I_{yp}=I_{xx}\sin^2\theta+I_{yy}\cos^2\theta+I_{xy}\sin(2\theta)$ $\tan\theta = \frac{(Ht-t^2)(B^2-Bt)}{I_{xx}-I_{yy}}$ *(Radians)* $\rho_{xx}= \sqrt{(I_{xx}/A)}$ $\rho_{yy}= \sqrt{(I_{yy}/A)}$
 Section is Symmetric about x axis	$h=H-2t_{cap}$ $w=W-t_{web}$ $x_{bar} = \frac{W^2 t_{cap} + \frac{ht_{web}^2}{2}}{A}$ $y_{bar}=H/2$	$A=2Wt_{cap}+ht_{web}$	$I_{xx} = [WH^3-wh^3]/12$ $I_{yy} = \frac{2t_{cap}W^3 + ht_{web}^3}{3} - \frac{\left(W^2 t_{cap} + \frac{ht_{web}^2}{2}\right)^2}{A}$ $\rho_x= \sqrt{(I_{xx}/A)}$ $\rho_y= \sqrt{(I_{yy}/A)}$

4.2.7 Calculation of Shear Center and Warping Constant about the Shear Center

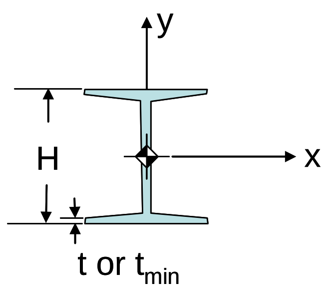
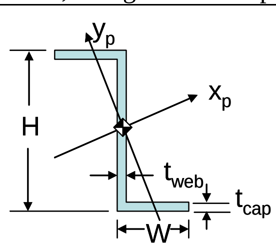
The shear center, (x_s,y_s), is defined as the point on the cross-sectional plane of a beam through which the resultant of the transverse shear load must be applied in order to have no torsion induced. As a result, when the transverse load is applied at the shear center, the stresses in the beam may be determined only from the theories of pure bending and transverse shear. Section 4.2.8 discusses the technique for the determination of the shear center for a general section.

- For any doubly symmetric section or section with point symmetry the shear center coincides with the centroid
- For any singly symmetric section, the shear center is on the axis of symmetry
- For any section made up of two intersecting plates the shear center is at the point of intersection of the plates.

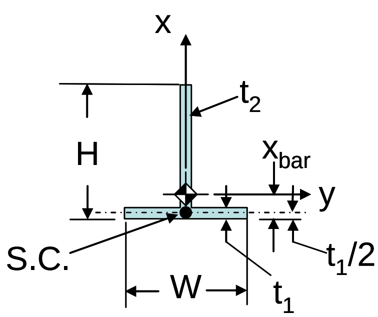
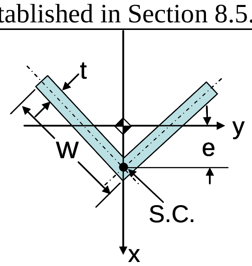
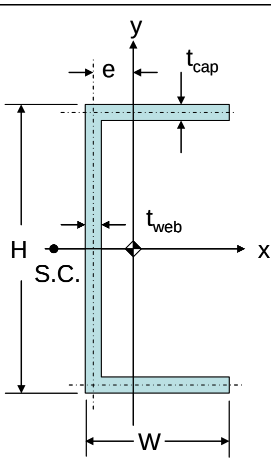
The warping constant, C_w , is an index of the ability of the cross-section to resist twist or torsion due to differential bending or torsion-bending[1]. Section 4.2.9 discusses the derivation of the warping constant for a general section and Section 8.5 discusses the use of C_w in the calculation of torsional instability of open sections.

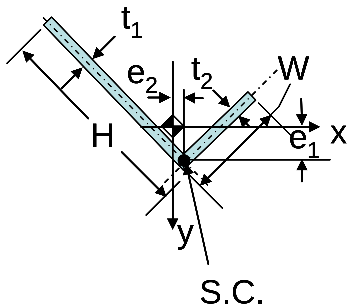
- For tees, angles, and cruciforms which have flanges that meet at a common point the warping constant will be a very small number and a good engineering approximation of the value is 0.0.
- C_w is the warping constant about the shear center.

The shear centers and warping constants about the shear center for various sections are provided in the following table. These formula were taken from References 4-2, 4-3 and 4-11. Reference 4-2, Volume 1, Section 2.6 contains an expanded compilation of shear center locations for other common cross-sectional shapes.

Cross-Section	Shear Center Location	Warping Constant
	$x_{sc} = 0$ $y_{sc} = 0$ *Shear Center is at Centroid*	$h' = H - t_{min}$ (If tapered) $h' = H - t$ (If constant thickness cap) $C_w = h'^2 I_{yy}/4$ where I_{yy} is the section centroidal moment of inertia about the y axis (in^4) H is the height of the I-Beam (in)
Symmetrical I-Beam:Doubly Symmetric; Flanges can be tapered		
	$x_{sc} = 0$ $y_{sc} = 0$ *Shear Center is at Centroid*	$h' = H - t_{cap}$ $w' = W - t_{web}/2$
Zee :Point Symmetry		$C_w = \dfrac{w'^3 h'^2}{12(2w'+h')^2}\left[2t_{cap}\left(w'^2 + w'h' + h'^2\right) + 3t_{web}w'h'\right]$

[1] In some manuals and literature, the warping constant has the symbol Γ.

Cross-Section	Shear Center Location	Warping Constant
	*Shear center is located at intersection of mid-thickness lines of tee legs* $x_{sc} = x_{bar} - t_1/2$ $y_{sc} = 0$	$h' = H - t_1/2$
Symmetrical Tee Single Axis of Symmetry: Symmetric about x per convention established in Section 8.5.2		$C_w = \frac{W^3 t_1^3}{144} + \frac{t_2^3 h'^3}{36}$
	*Shear center is located at intersection of mid-thickness lines of angle legs* $x_{sc} = e$ $y_{sc} = 0$	$C_w = A^3/144$
Symmetrical Equal-Leg Angle Single Axis of Symmetry: Symmetric about x axis per convention established in Section 8.5.2		
	*Shear center is located on the negative x-axis on the opposite side of the web as the flanges* $h' = H - t_{cap}$ $x_{sc} = -e\left(1 + \frac{Ah'^2}{4I_{xx}}\right)$ $y_{sc} = 0$	$C_w = \frac{h'^2}{4}\left[I_{yy} + e^2 A\left(1 - \frac{Ah'^2}{4I_{xx}}\right)\right]$
Symmetrical Channel Single Axis of Symmetry: Symmetric about x		

 Unequal Leg Angle No axis of Symmetry	*Shear center is located at intersection of mid-thickness lines of angle legs* $x_s = e_2$ $y_s = e_1$	$h' = H - t_2/2$ $w' = W - t_1/2$ $C_{w\text{-}sc} = (h'^3\, t_1^3 + w'^3\, t_2^3)/36$

4.2.8 Determination of Shear Center

When the shear center of a thin-walled open section is needed, but it is not tabulated, it can be derived using the following approach. The method is illustrated using a channel section.

The shear center is determined by subdividing the section into elements, assuming an applied transverse load V and a shear center location and then determining expressions for the resultant shearing forces on each element of the section based on the geometry and the assumed shear stress. Using the assumed location for the shear center, the summation of moments about an arbitrary convenient point is performed. Since the shear center is a function of only the geometry of the cross-section, the assumed load is algebraically eliminated. The location of the shear center is the remaining unknown. The method will be illustrated for the channel section shown in Figure 4.2.8-1.

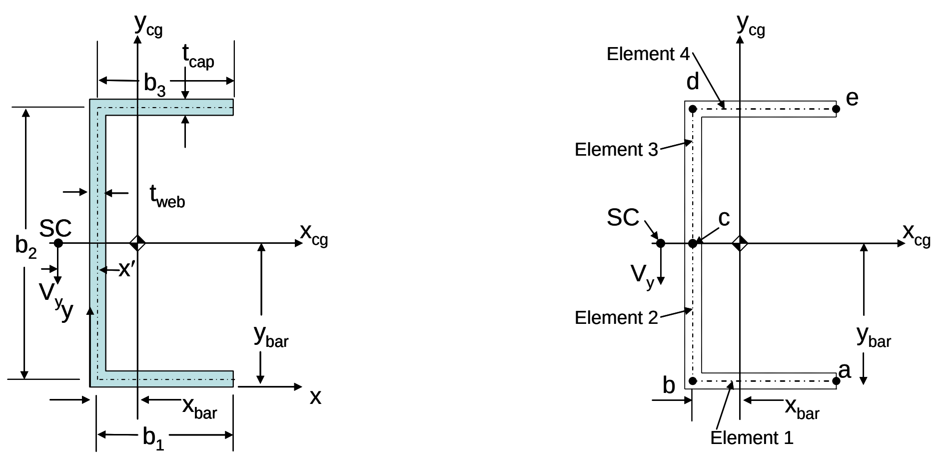
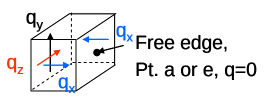
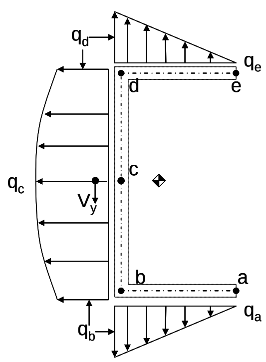


Figure 4.2.8-1 Geometry Used for the Determination of Shear Center

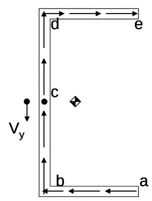
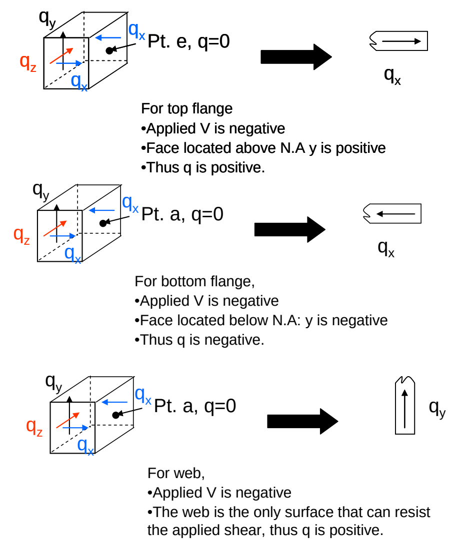
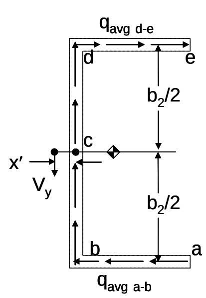
The channel section used for illustration is symmetric about the x-axis, so the shear center is located on the x-axis but at an unknown x location. For asymmetric sections, the steps outlined in Table 4.2.8-1 would need to be repeated to also determine the y location of the shear center.

Table 4.2.8-1Calculation of Shear Center

Step	Process	
1	Assume a shear center location (SC)	x′
2	Apply a transverse shear load V_y at SC	Refer to Figure 4.2.8-1
3	The resulting shear flow in each flange due to the external shear is q[1]  Positive Sign Convention for +V load	$q_z = -\frac{V_y}{I_x}\int_a^b y\,dA = -\frac{V_y}{I_x}\sum_{i=1}^{n} y_i A_i$ where V_y is the applied transverse shear load (positive up) I_x is the moment of inertia of the total section about the x centroidal axis y is the distance from the centroid to the element midline in the y direction A is the area of the element
4	For each element in the cross-section develop the shear flow • Start at free edge where shear flow is 0, work around the section 	*q_{z-a} = 0 shear at a free edge is 0*
		$q_{z-b} = q_{z-a} - \frac{V_y}{I_x} y_1 A_1 = 0 - \frac{V_y}{I_x}\left(-\frac{b_2}{2} b_1 t_1\right) = \frac{V_y}{I_x}\left(\frac{b_2}{2} b_1 t_1\right)$
		$q_{z-c} = q_{z-b} - \frac{V_y}{I_x} y_2 A_2 = \frac{V_y}{I_x}\left(\frac{b_2}{2} b_1 t_1\right) - \frac{V_y}{I_x}\left(-\frac{b_2}{4}\right)\left(\frac{b_2}{2} t_2\right) = \frac{V_y}{I_x}\left(\frac{b_2}{2} b_1 t_1 + \frac{b_2^2}{8} t_2\right)$
		$q_{z-d} = q_{z-c} - \frac{V_y}{I_x} y_3 A_3 = \frac{V_y}{I_x}\left(\frac{b_2}{2} b_1 t_1 + \frac{b_2^2}{8} t_2\right) - \frac{V_y}{I_x}\left(\frac{b_2}{4}\right)\left(\frac{b_2}{2}\right) t_2 = \frac{V_y}{I_x}\left(\frac{b_2}{2}\right) b_1 t_1$
		$q_{z-e} = q_{z-d} - \frac{V_y}{I_x} y_4 A_4 = \frac{V_y}{I_x}\left(\frac{b_2}{2}\right) b_1 t_1 - \frac{V_y}{I_x}\left(\frac{b_2}{2}\right) b_3 t_3$ *But $b_3=b_1$ and $t_3=t_1$, thus $q_{z-e}=0$ (free edge)*
Continued next page		

[1] Reference 4-12

<table>
<tr>
<td>5</td>
<td>Determine direction of shear flow based on external load direction and sign convention given in Step 3. Note, from engineering mechanics, $q_x=q_y=q_z$ at a point.

The figure below shows the assembled resulting shear flow in each flange.

Note the shear flow is continuous around the section and reverses only when it passes through zero.</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Sum moments about an arbitrary point to calculate location of shear center.
• Select a convenient point such as point c which eliminates the shear flow in element 2 and 3 from the moment calculation.
• To calculate the load multiply the average shear flow in each flange by the flange length: $q_{avg\text{-}i} b_i$
</td>
<td>$\Sigma M_c = 0 \quad +CCW$
$$-\frac{q_{avg:d-e} b_3 b_2}{2} - \frac{q_{avg:a-b} b_1 b_2}{2} + V_y x' = 0$$
Calculate average shear flow:
$$q_{avg:d-e} = \frac{\frac{V_y}{I_x}\left(\frac{b_2}{2}\right) b_1 t_1 + 0}{2} = \frac{V_y}{2I_x}\left(\frac{b_2}{2}\right) b_1 t_1$$
$$q_{avg:a-b} = \frac{0 + \frac{V_y}{I_x}\left(\frac{b_2}{2}\right) b_1 t_1}{2} = \frac{V_y}{2I_x}\left(\frac{b_2}{2}\right) b_1 t_1$$
Substitute:
$$\left(-\frac{1}{2}\right)\left(\frac{V_y}{2I_x}\right)\left(\frac{b_2}{2}\right) b_1 t_1 b_3 b_2 + \left(-\frac{1}{2}\right)\left(\frac{V_y}{2I_x}\right)\left(\frac{b_2}{2}\right) b_1 t_1 b_1 b_2 + V_y x' = 0$$
But $b_3=b_1$, thus
$$-\left(\frac{V_y}{2I_x}\right)\left(\frac{b_2}{2}\right) b_1^2 t_1 b_2 + V_y x' = 0$$
Simplifying and solving for x′
$$x' = -\frac{b_1^2 b_2^2 t_1}{4I_x}$$</td>
</tr>
</table>

4.2.9 Calculation of Warping Constant about Section Centroid and Warping Moments

For sections not found in the table of Section 4.2.6, an alternate approach to calculating torsional stability uses the warping constant about the centroid of the section. This requires, additionally, calculation of the warping moments. The following approach is a numerical integration of the equations which can be useful for sections with no axis of symmetry and does not require determination of the shear center location. Section 8.5 explains the use of these equations.

The derivation of the warping constant and warping moments can be found in References 4-5 and 4-6. Figure 4.2.9-1 illustrates the geometry of a section used in the discussion below.

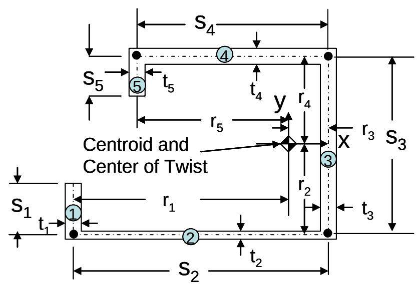


Figure 4.2.9-1 Section Geometry for Warping Constant Calculation

The warping constant is given by

$$C_{w,c} = \int_A w^2 dA - \frac{1}{A}\left[\int_A w dA\right]^2$$

Let A = s*t and assume t is constant for a given flange. Thus,

$$C_{w,c} = \int_s w^2 t ds - \frac{1}{A}\left[\int_s w t ds\right]^2 \qquad \textbf{Equation 4.2.9-1}$$

where
A is the cross-section area (in^2)
w is the warping displacement, discussed below (in^2)
s is the flange midline length (in)
t is the flange thickness (in)

The warping displacement is the axial lengthening of the ith element at an arbitrary point u along its length, for a twist of 1 radian/unit length about the longitudinal z axis located at the center of twist. The warping displacement is given as

$$w_s = \int_s r ds \qquad \textbf{Equation 4.2.9-2}$$

where
r is the normal distance from the centroid or center of twist to the centerline of each element (in)

Equation 4.2.8-1 can be expressed for numerical integration as

$$C_{w,c} = \sum_i \left[\frac{s_i t_i \left(w_{i-a}^2 + w_{i-a} w_{i-b} + w_{i-b}^2 \right)}{3} \right] - \frac{1}{A} \sum_i \left[\frac{s_i t_i \left(w_{i-a} + w_{i-b} \right)}{2} \right]^2 \quad for\ i=1,n \qquad \textbf{Equation 4.2.9-3}$$

where
s_i is the length of element i (in)
$w_{i\text{-}a}$, $w_{i\text{-}b}$ are the warping displacements of i^{th} element at ends a and b, respectively (in^2) given by $w=r_i s_i$
t_i is the thickness of element i (in)

Figure 4.2.9-2 illustrates the element definitions for integration of the cross-section depicted in Figure 4.2.9-1.

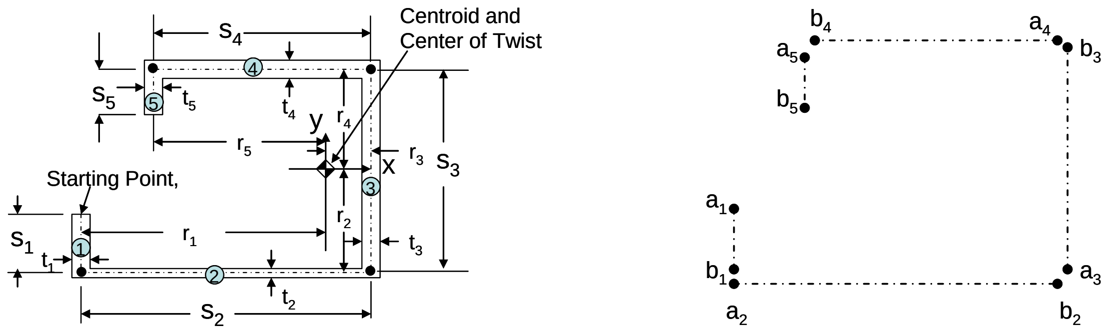


Figure 4.2.9-2 Definition of Terms for Numerical Integration of Warping Constant

The general approach for the calculation of the warping constant and warping moments about the section centroid is as follows:

- Divide the cross-section into appropriate subsections and number each of these elements sequentially around the section shown in Figure 4.2.9-2. This may be done in either a clockwise or counterclockwise direction.
- Using the general approach described in Section 4.2.5, determine the location of the centroid of the total section.
- Determine the normal distance, r from the centroid to the centerline of each element. If an imaginary force vector from a_i to b_i would produce a counterclockwise rotation about the centroid, then the sign of r is positive. If the imaginary vector produces a clockwise rotation, the sign of r is negative.
- Determine the length, s_i, of each element (a_i to b_i) along the centerline of each flange
- Create a table similar to Table 4.2.9-1 to calculate the contribution to the warping constant for each of the subsection.
- Using Equation 4.2.9-3, calculate C_w

Table 4.2.9-1 Calculation of C_w about Centroid for General Element

Col **1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Elem ID	r_i	length s_i: a_i to b_i	$r_i s_i$ **Cols 2**x**3**	w_{i-a} **Col 6(i-1)**	w_{i-b} **Cols 4**+**5**	$w_{i-a}+w_{i-b}$ **Cols 5**+**6**	$(s_i/2)$x $(w_{i-a}+w_{i-b})$ **Cols (3**/2)x**7**
1				0 (always)			
2							
…							
n							

Col **1**	**9**	**10**	**11**	**12**	**13**	$C_{w,c}$ =
Elem ID	t_i	$s_i\,t_i(w_{i-a}+w_{i-b})/2$ **Cols 8**x**9**	$(w_{i-a}^2+(w_{i-a}w_{i-b})+w_{i-b}^2)/3$ Cols (**5**^{2}+(**5**x**6**)+**6**^{2})/3	$s_i t_i$ **Cols 3**x**9**	$s_i t_i(w_{i-a}^2+(w_{i-a}w_{i-b})+w_{i-b}^2)/3$ Cols **12x11**	ΣCol **13** –[ΣCol **10**]^2 / ΣCol **12**
1						
2						
…						
n						
Sums		Σ		A=Σ	Σ	

Notes:

- The warping displacement is always 0 for end “a” of the first element. The warping displacement for end “a” of subsequent elements is equal to the warping constant at end “b” of the previous element.
- Where “Col” is indicated before an arithmetic expression, perform the arithmetic indicated on columns indicate, *e.g.*, Col 2(**2x3)**, multiply 2 time the value from column **2** times the value from column **3** and place it into this column. The underlined, bold number is the column number. Numbers not underlined and bold are numeric values.
- Subscript “i” indicates the current element while “i-1” is the previous element and “i+1” is the next element

Because the warping constant calculated by Equation 4.2.9-3 and described in Table 4.2.9-1 is not at the shear center of the section, there are additional x and y warping moments necessary to calculate the critical load interaction for torsional stability. R_{xx} is the warping moment about the x axis and R_{yy} is the warping moment about the y axis. They are geometric properties of the cross-section and are similar in concept to static and area moments of inertia. If the section is symmetrical about both axes, the warping moments are both zero. If the section is symmetric about x, then R_{yy} is zero and, conversely if the section is symmetric about y, R_{xx} is zero. The mathematical definition[1] of these are

$$R_{xx} = \int_s y w_s t ds = \sum_i \left[\frac{L_i t_i}{6} \left\{ w_{i-a}\left(2y_{i-a} + y_{i-b}\right) + w_{i-b}\left(y_{i-a} + 2y_{i-b}\right) \right\} \right] \qquad \textbf{Equation 4.2.9-4}$$

$$R_{yy} = \int_s x w_s t ds = \sum_i \left[\frac{L_i t_i}{6} \left\{ w_{i-a}\left(2x_{i-a} + x_{i-b}\right) + w_{i-b}\left(x_{i-a} + 2x_{i-b}\right) \right\} \right] \qquad \textbf{Equation 4.2.9-5}$$

Tables 4.2.9-2 and 4.2.9-3 provide a tabular form for the calculation of the warping moments. Section 4.2.9.1 gives an example problem for the calculation of the warping constant and moments for a channel cross-section.

[1] Reference 4-6

Table 4.2.9-2 Calculation of R_{xx} Warping Moment

Col **1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Elem ID	y_{i-a}	y_{i-b}	$2y_{i-a}+y_{i-b}$ **Cols** 2(**2**)+**3**	$y_{i-a}+2y_{i-b}$ **Cols** **2**+2(**3**)	w_{i-a} **Col 5**, Table 4.2.8-1	w_{i-b} **Col 6**, Table 4.2.8-1	$w_{i-a}(2y_{i-a}+y_{i-b})$ **Col 6**x**4**
1							
2							
…							
n							

Col **1**	**9**	**10**	**11**	**12**	**13**	**14**
Elem ID	$w_{i-b}(y_{i-a}+2y_{i-b})$ **Cols 7**x**5**	$w_{i-a}(2y_{i-a}+y_{i-b})$ $+w_{i-b}(y_{i-a}+2y_{i-b})$ **Cols 8**+**9**	length s_i: a_i to b_i	t_i	$s_it_i/6$ **Cols** (**11**x**12**)/6	$\Delta R_{xx} = (s_it_i/6)$x $[w_{i-a}(2y_{i-a}+y_{i-b})+w_{i-b}(y_{i-a}+2y_{i-b})]$ **Col 13**x**10**
1						
2						
…						
n						
					R_{xx} =	Σ

Table 4.2.9-3 Calculation of R_{yy} Warping Moment

Col **1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Elem ID	x_{i-a}	x_{i-b}	$2x_{i-a}+x_{i-b}$ **Cols** 2(**2**)+**3**	$x_{i-a}+2x_{i-b}$ **Col 2**+2(**3**)	w_{i-a} **Col 5**, Table 4.2-1	w_{i-b} **Col 6**, Table 4.2-1	$w_{i-a}(2x_{i-a}+x_{i-b})$ **Col 6**x**4**
1							
2							
…							
n							

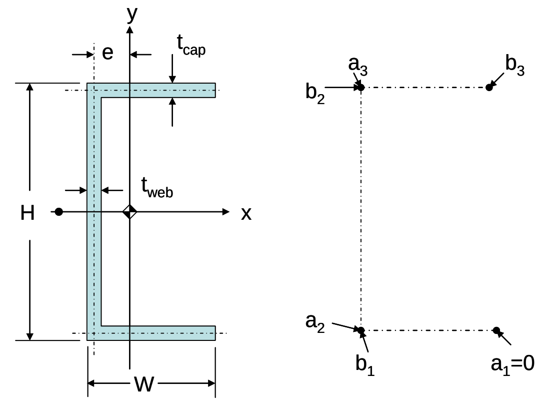
Col **1**	**9**	**10**	**11**	**12**	**13**	**14**
Elem ID	$w_{i-b}(x_{i-a}+2x_{i-b})$ **Cols 7**x**5**	$w_{i-a}(2x_{i-a}+x_{i-b})$ $+w_{i-b}(x_{i-a}+2x_{i-b})$ **Cols 8**+**9**	length s_i: a_i to b_i	t_i	$s_it_i/6$ **Cols** (**11**x**12**)/6	$\Delta R_{yy} = (s_it_i/6)$x $[w_{i-a}(2x_{i-a}+x_{i-b})+w_{i-b}(x_{i-a}+2x_{i-b})]$ **Cols 13**x**10**
1						
2						
…						
n						
Sums					**R_{yy} =**	Σ

4.2.9.1 Example Problem – Calculation of the Warping Constant and Warping Moments about Section Centroid

Calculate the basic Section properties:
From Section 4.2.5:
$h=H-2t_{cap} = 3-2(0.15) = 2.7$ in
$w=W-t_{web} = 1.25-0.15 = 1.10$ in
$A = 2Wt_{cap}+ht_{web} = 2(1.25)0.15+2.7(0.15) = 0.78\ in^2$

$I_{xx} = (WH^3-wh^3)/12 = (1.25(3)^3-1.10(2.7)^3)/12=1.0083\ in^4$
$I_{yy}=[2t_{cap}W^3 + ht^3]/3-[W^2t_{cap}+0.5ht_{web}^2]^2/A = [2(0.15)(1.25)^3 + 2.7(0.15)^3]/3 – [1.25^2(0.15)+0.5(2.7)(0.15)^2]^2 / 0.78 = 0.1984 – 0.0899= 0.1085\ in^4$

$x_{bar} = [W^2t_{cap} + 0.5ht_{web}^2]/A = [1.25^2(0.15)+0.5(2.7)(0.15)^2]/0.78 = 0.3394$ in
$y_{bar} = H/2 = 3.0/2 = 1.5$ in



$H = 3.0$ in; $W = 1.25$ in.; $t_{cap} = t_{web} = 0.15$ in

Determine x_i, y_i, L_i, and r_i

Element	x_{i-a}	x_{i-b}	r_i
1	1.25-0.3394 = 0.9106	-(0.3394-0.15/2) = -0.2644	1.50-0.15/2 = -1.425; +CCW a to b
2	-0.2644	-0.2644	0.3394-0.15/2 = -0.2644; +CCW a to b
3	-0.2644	0.9106	-1.425; +CCW a to b
Element	y_{i-a}	y_{i-b}	L_i
1	-(1.50-0.75/2) = -1.425	-1.425	1.25-0.15/2 = 1.175
2	-1.425	1.5-0.15/2 = 1.425	3.0-0.15/2-0.15/2 = 2.85
3	1.425	1.425	1.25-0.15/2 = 1.175

Calculate $C_{w,c}$

Col **1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Elem ID	r_i	length s_i: a_i to b_i	r_is_i **Cols 2**x**3**	w_{i-a} **Col 6(i-1)**	w_{i-b} **Cols 4**+**5**	$w_{i-a}+w_{i-b}$ **Cols 5**+**6**	$(s_i/2)x(w_{i-a}+w_{i-b})$ **Cols (3**/2)x**7**
1	-1.425	1.175	-1.6744	0	-1.6744	-1.6744	-0.9837
2	-0.2644	2.85	-0.7535	-1.6744	-2.4279	-4.1023	-5.8458
3	-1.425	1.175	-1.6744	-2.4279	-4.1023	-6.5305	-3.8367

Col **1**	**9**	**10**	**11**	**12**	**13**
Elem ID	t_i	$s_it_ix(w_{i-a}+w_{i-b})/2$ **Col 8**x**9**	$(w_{i-a}^2+(w_{i-a}w_{i-b})+w_{i-b}^2)/3$ **Col(5**^{2}+(**5**x**6**)+**6**^{2})/3	s_it_i **Col 3**x**9**	$s_it_i(w_{i-a}^2+(w_{i-a}w_{i-b})+w_{i-b}^2)/3$ **Cols 12x11**
1	0.15	-0.1476	0.9345	0.1763	0.1647
2	0.15	-0.8769	4.2545	0.4275	1.8188
3	0.15	-0.5755	10.8945	0.1763	1.9207
Sums		Σ =-1.60		A=0.78	Σ = 3.9042

$C_{w,c}$ = Σ**Col 13** –[Σ**Col 10**]^2 / Σ**Col 12**
$=3.9042-1.60^2/0.78$
$= 0.622\ in^4$
about section centroid

Calculate the warping moment R_{xx}. The section is symmetric about the x axis, so R_{yy} is 0.

Col **1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Elem ID	y_{i-a}	y_{i-b}	$2y_{i-a}+y_{i-b}$ **Cols** 2(**2**)+**3**	$y_{i-a}+2y_{i-b}$ **Cols 2**+(2)**3**	w_{i-a} Col **5**, Table 4.2-1	w_{i-b} Col **6**, Table 4.2-1	$w_{i-a}(2y_{i-a}+y_{i-b})$ **Cols 6**x**4**
1	-1.425	-1.425	-4.275	-4.275	0	-1.6744	0
2	-1.425	1.425	-1.425	1.425	-1.6744	-2.4279	2.3860
3	1.425	1.425	4.275	4.275	-2.4279	-4.1023	-10.3793

Col **1**	**9**	**10**	**11**	**12**	**13**	**14**
Elem ID	w_{i-b}x $(y_{i-a}+2y_{i-b})$ **Cols 7**x**5**	$w_{i-a}(2y_{i-a}+y_{i-b})$ + $w_{i-b}(y_{i-a}+2y_{i-b})$ **Cols 8**+**9**	length s_i: a_i to b_i	t_i	$s_it_i/6$ **Cols 11**x**12** / 6	$\Delta R_{xx} = (s_it_i/6)[w_{i-a}(2y_{i-a}+y_{i-b}) + w_{i-b}(y_{i-a}+2y_{i-b})]$ **Cols 13**x**10**
1	7.1580	7.1580	1.175	0.15	0.0294	0.2103
2	-3.4598	-1.0738	2.85	0.15	0.0713	-0.0765
3	-17.5373	-27.9166	1.175	0.15	0.0294	-0.8201
					Σ =R_{xx} =	**-0.6863 in^5**

4.3 Skin Effective Width

For various calculations of compression members attached to panels, *e.g.* skins or webs, some portion of the panel is effective in carrying the load in the compression member. This effective panel width can then be used in the calculation of section properties of the member. Care must be taken to add the load in the effective portion of the panel to the member load for analysis. This section discusses how the effective width can be calculated.

It is often the case in aircraft structure that there are laminated composite skins attached to metallic substructure. The method of analysis is very similar to that described below except in the determination of laminate strain at stiffener stress. In that case, principles of composite laminated theory, described in Section 4 of PM4056 should be used. In the case where there are laminated composite skins over laminated composite substructure, the methods described in Section 6 of PM4056 should be used.

4.3.1 Calculation of Skin Effective Width for Compression Loads

Effective panel area is used in column buckling, beam column analysis and torsional instability calculations, but not in crippling calculations. The effective panel area for compression loading is maximum in the case where the stiffener stress is below the initial panel elastic buckling stress, since the panel is unbuckled and is completely effective in carrying the applied compression load. This is illustrated in Figure 4.3.1-1 (A). In this case the effective panel width is half the panel width on each side of the stiffener. While this is the most straightforward calculation it is not the most efficient structural arrangement since it limits the strain in the stiffener to the initial buckling strain of the panel.

The most efficient structural configuration is with the panel effective width based on allowing the stiffener and adjacent panel stress to be at the allowable crippling stress of the stiffener. The resulting distribution is as depicted in Figure 4.3.1-1(B). Implicit in this assumption is that the effective width of the skin and the stiffener together carry the entire load in the panel as is shown in Figure 4.3.1-1(C). The buckling of the skin in compression is given by Equation 10.3.1-3, repeated here for convenience

$$f_{cre} = \frac{k\pi^2 E_c}{12(1-\nu_e^2)}\left(\frac{t}{b}\right)^2$$

Reference Equation 10.3.1-3

where k=k_c for compression from Figure 10.3.2-1

ν_e is the elastic Poisson's ratio for the material

b is the panel width between stiffeners (in)
t is the panel thickness (in)

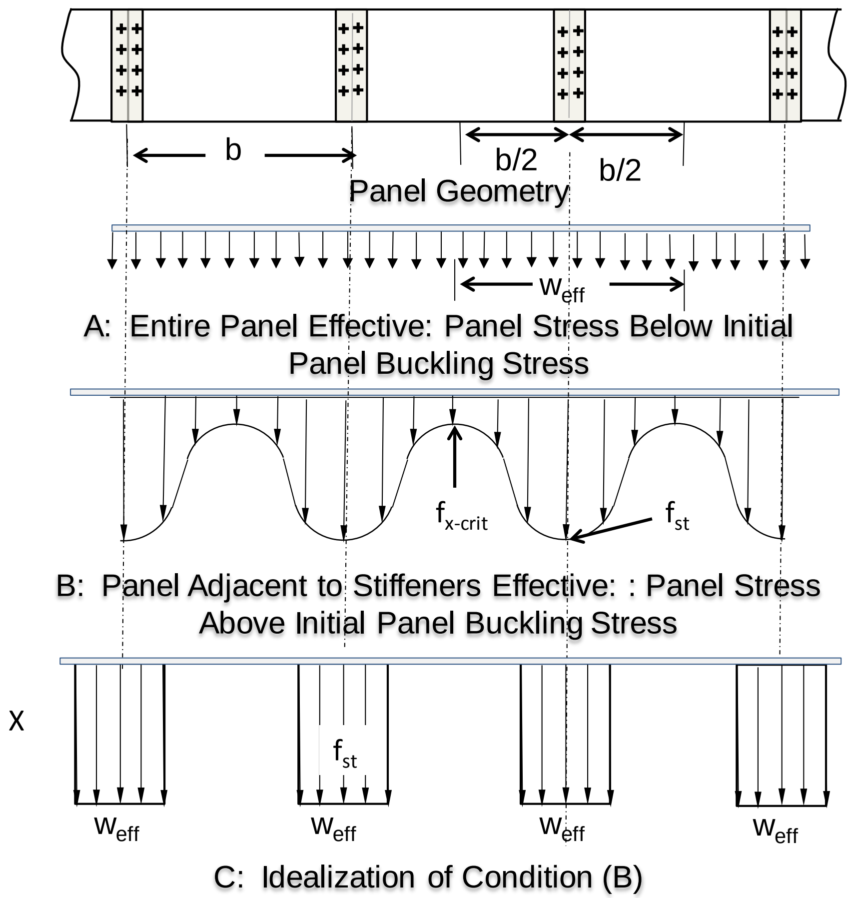


Figure 4.3.1-1 Illustration of Panel Behavior and Effective Width Idealization

For most metallic aerospace materials the elastic Poisson's ratio varies between 0.29 and 0.33. If a value of 0.31 is used in Equation 10.3.1-3, the equation simplifies to[1]

$$f_{cre} = 0.9099 k_c E_c \left(\frac{t}{b}\right)^2 = 0.91 k_c E_c \left(\frac{t}{b}\right)^2 \qquad \textbf{Equation 4.3.1-1}$$

where
$k=k_c$ for compression from Table 4.3.1-1

Above the proportional limit of the stiffener, a plasticity correction is required. References 4-12 and 4-13 recommend the use of the tangent modulus since it is used to correct for plasticity in column buckling analysis and correlates well to panel tests at stress levels above the proportional limit. Rewriting Equation 4.3.1-1, for a more generalized form to include plasticity

$$f_{cr} = 0.91 k_c E_{tan} \left(\frac{t}{b}\right)^2 \qquad \textbf{Equation 4.3.1-2}$$

where
E_{tan} is the tangent modulus of the stiffener (psi)

[1] By using the approximate Poisson's ratio of 0.31 rather than the exact value for typical aerospace metallic materials a maximum difference of about 1.5% in the resulting coefficient is calculated.

This is valid both above and below the proportional limit since below the proportional limit, the tangent modulus is equal to the modulus of elasticity.

If Equation 4.3.1-2 is solved for b when the critical stress, f_{cr}, is set equal to the compression stress of the stiffener, F_{st}, then the portion of the panel that does not buckle at the compression stress level of the compression member can be determined from

$$b_{eff} = t\sqrt{0.91k_c}\sqrt{\frac{E_{tan}}{F_{st}}}$$ **Equation 4.3.1-3**

If the stiffener and the panel are not the same material Equation 4.3.1-3 must be adjusted to the stress level of the panel. The two parts will strain at the same rate so the strain in the stiffener can be calculated and the stress in a metal panel determined by an iterative solution using Equation 3.3.1-6 or 3.3.1-20, depending on the stress level in the parts. If the panel is composite, the stress in the panel must be determined using methods in Section 4 of PM4056. The stress level in the panel can also be determined graphically by plotting the stress strain curve for each material and entering the stiffener stress-strain curve at the stiffener stress level, drawing a vertical line to intersect the panel stress-strain curve and then reading the stress at that strain. The vertical line represents constant strain. This is illustrated in the Section 4.3.3 example problem.

Equation 4.3.1-3 can be generalized to allow for different compression member/panel materials by

$$b_{eff} = t\left(\frac{f_{sk}}{f_{st}}\right)\sqrt{0.91k_c}\sqrt{\frac{E_{tan}}{F_{st}}}$$ **Equation 4.3.1-4**

where
f_{sk} is the stress in the panel at the strain level of the compression member (psi)
f_{st} is the stress in the compression member (psi)
If the skin and stiffener are the same material: $(f_{sk}/f_{st})=1.0$

This effective width represents the panel material adjacent to the stiffener per fastener row which is effective in carrying the stiffener load. It cannot exceed the available material. See Table 4.3.1.2 for further clarification.

The stress in the compression member is generally taken as the allowable stiffener crippling stress, F_{cc}; however, for this to be valid, F_{cc} must be less than the compression yield stress of the skin, $F_{cy\text{-}sk}$, and the minimum inter-rivet buckling stress, F_{IR}. If the skin and stiffener are of different materials, the most straightforward method to determining which of the above stress levels govern is to calculate the strain represented by each.

- If $\varepsilon_{cc}>\varepsilon_{cy\text{-}sk}$ then F_{st} should be taken as the stress in the stiffener at $\varepsilon_{cy\text{-}sk}$.
- If $\varepsilon_{cc}>\varepsilon_{IR}$ then the fastener spacing should be reduced until $\varepsilon_{cc}\le\varepsilon_{IR}$ or minimum fastener spacing of 4D occurs. If the design is limited by minimum fastener spacing, then F_{st} should be taken as the stress in the stiffener at ε_{IR}.

By substitution of the values of k_c from Figure 10.3.2-1, the expressions in Table 4.3.1-1 can be obtained for different panel boundary constraints offered by the stiffeners.

Table 4.3.1-1 Values of k_c as a Function of Panel Constraint Offered by Stiffener

Stiffener Boundary Constraint	k_c	Equation for b_{eff} per Equation 4.3.1-4	Equation Number
Simply Supported	4.0	$b_{eff} = 1.91\left(\frac{f_{sk}}{F_{st}}\right) t \sqrt{\frac{E_{tan}}{F_{st}}}$	**Equation 4.3.1-5**
Fixed	6.98	$b_{eff} = 2.52\left(\frac{f_{sk}}{F_{st}}\right) t \sqrt{\frac{E_{tan}}{F_{st}}}$	**Equation 4.3.1-6**
In-between Fixed and Simply Supported	Figure 10.3.2-14	$b_{eff} = \sqrt{0.91 k_c}\left(\frac{f_{sk}}{F_{st}}\right) t \sqrt{\frac{E_{tan}}{F_{st}}}$	**Equation 4.3.1-7**
One-Edge Free, One Simple	0.429	$b_{eff} = 0.63\left(\frac{f_{sk}}{F_{st}}\right) t \sqrt{\frac{E_{tan}}{F_{st}}}$	**Equation 4.3.1-8**
where f_{sk} is the stress in the panel at the strain level of the compression member (psi) F_{st} is the allowable stress for compression member only (psi). See discussion in box following Equation 4.3.1-4. E_{tan} is the tangent modulus calculated at F_{st} from Equation 3.3.1-8 t is the thickness of the panel (in)			

Equation 4.3.1-7 provides a generalized equation when the edge constraint is between fixed and simply supported. It was reported in Reference 4-12 and 4-13 that in testing it was observed that the panel fixity was related to the b/t ratio of the panel with low b/t ratios, *i.e.*, thick narrow panels, behaving simply supported and high b/t ratios, *i.e.*, thin wide panels behaving fixed. Figure 10.3.2-14 can be used to determine the appropriate k_c for use in Equation 4.3.1-7 based on the panel b/t ratio.

Figure 4.3.1-2 illustrates the potential unconservatism in the choice of effective width if the tangent modulus correction is ignored above the stiffener proportional limit.

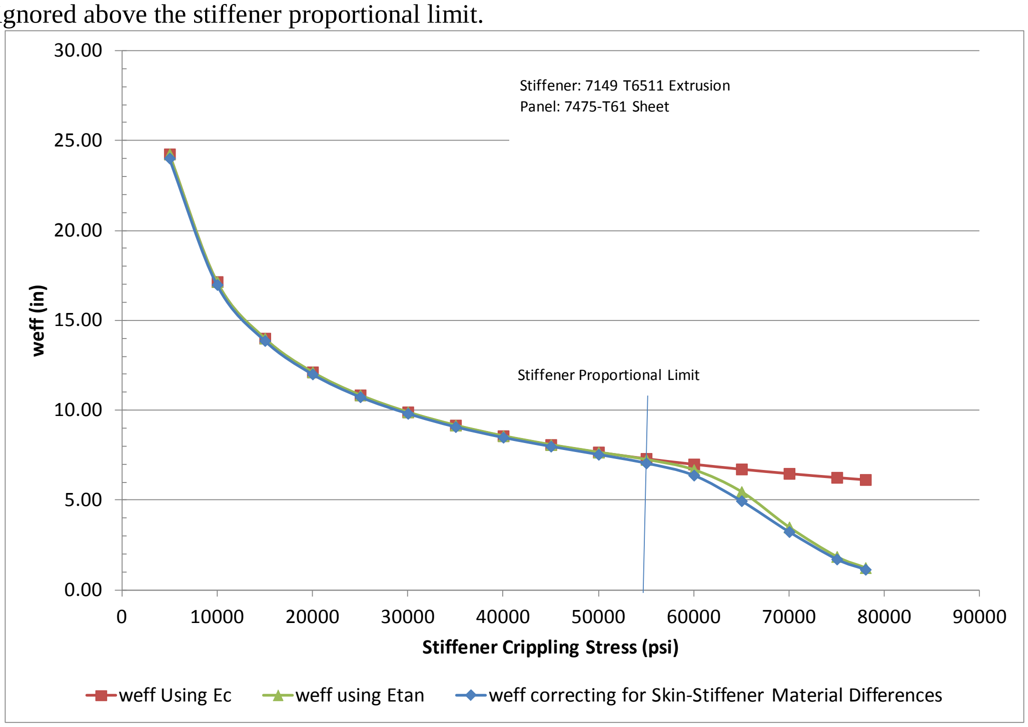
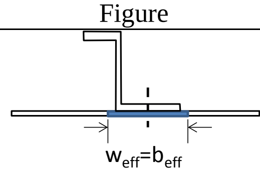
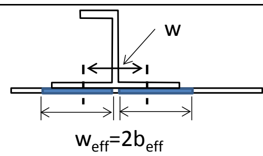
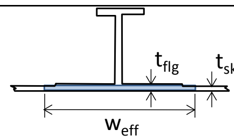
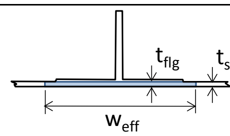
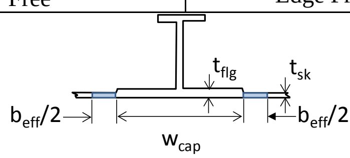


Figure 4.3.1-2 Comparison of Effective Width Calculations

Since b_{eff} is the effective width per fastener row, Table 4.3.1-2 is provided to aid in determining the appropriate calculation of w_{eff} along with the appropriate section for use in the calculation of the crippling stress allowable for the stiffener.

Table 4.3.1-2 Application of Effective Width to Different Web/Stiffener Configurations

Case	Figure		Area for Calculation of Crippling Allowable
1			Stiffener Only
2			Stiffener Only If the distance between fastener rows, w, is less than b_{eff}: $w_{eff}=b_{eff}+w$
3	 $t_{sk} \le t_{flg} < 2t_{sk}$ $w_{eff}=2b_{eff}$ Stiffener Web is "No Edge Free"	 $t_{sk} \le t_{flg} < 2t_{sk}$ $w_{eff}=2b_{eff}$ Stiffener Web is "One Edge Free"	Integral Stiffeners (with or without cap): Crippling is calculated using outstanding stiffener only with no contribution from flange common to skin. Use $t = (t_{flg}+t_{sk})/2$ for calculation of b_{eff} using Table 4.3.1-1
4	 $t_{flg} \ge 2t_{sk}$ $w_{eff}=w_{cap}+b_{eff}$		Integral Stiffeners: Use full I-Section (with cap) or T-Section (without cap) Use $t = t_{sk}$ for calculation of b_{eff} using Table 4.3.1-1

4.3.2 Miscellaneous Calculations of Effective Width

There are a number of cases where an effective width or length must be assumed in order to calculate a reasonable cross-sectional area for analysis. This section addresses some of these situations.

4.3.2.1 Integrally Stiffened Machined Frames or Bulkheads

To perform crippling analysis on integrally stiffened frame sections, a crippling section must be assumed. This is illustrated in Figure 4.3.2-1

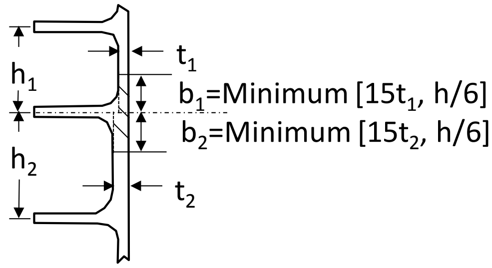


Figure 4.3.2-1 Integrally Machined Frame or Bulkhead Effective Web Crippling Section Assumption

The effective width is then given by

$$w_{eff} = b_1 + b_2 \quad \textbf{Equation 4.3.2-1}$$

where

b_1, b_2 are shown in Figure 4.3.2-1(in)

4.3.2.2 Effective Area of Panels Not Predominantly Loaded in Compression

When skins or webs are not loaded in compression, effective width assumptions for use in the calculation of stiffening element section properties can still be made[1]. While the skin, in tension, is fully effective, the use of the entire skin in the calculation of stiffening element section properties is not typically done. All loads from the skin, flange and web should be obtained by free body diagram either manually or from a finite element model and distributed to the section components based on an EA distribution. This approach works for both metallic and composite skins. If the skin has a pad-up area under the stiffener, engineering judgment is required for the determination of the total effective width. The effective widths are illustrated in Figure 4.3.2-2.

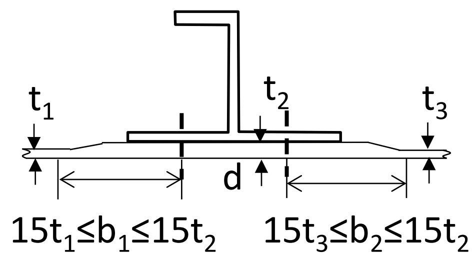


Figure 4.3.2-2 Effective Dimensions for Skins with Pad-ups

The dimensions for b_1 and b_2 should be determined based on the relative lengths of the padded and non-padded portions of the skin. If the pad-up extends further than $15t_2$ to the left of the left fastener and to the right of the right fastener, then a maximum of $b_1=b_2=15t_2$ should be used at each fastener line. The dimension d is taken as the minimum of the fastener spacing or $30t_2$.

If the skin has no pad-up, $t_1=t_2=t_3$, thus $b_1=b_2=15t_1$ and the dimension d is taken as the minimum of the fastener spacing or $30t_1$. The total effective width is then given by

$$w_{eff} = b_1 + b_2 + d \quad \textbf{Equation 4.3.2-2}$$

where

b_1, b_2 and d are shown in Figure 4.3.2-2(in)

Any effective skin area, $A_{skin\text{-}eff}$ would use the appropriate skin thickness for each portion of the effective width: b_1, b_2 or d. Additionally, depending on the type of skin there is a further reduction taken as is given in Table 4.3.2-1 depending on the skin/panel function and installation parameters.

Table 4.3.2-1 Effective Area Reduction for Mechanically Attached Skins and Panels

Panel Type	K_{eff}	A_{eff}
Non-removable panels and skins with nominal Bolt-to-Hole Clearances ≤0.004 in	1.0	$A_{skin\text{-}eff}$
Removable panels and skins with Bolt-to-Hole Clearances ≤ 5% of Fastener Diameter	0.50	$0.50A_{skin\text{-}eff}$
Removable panels and skins with Bolt-to-Hole Clearances > 5% of Fastener Diameter	0.0	0.0
All panels and skins ends beyond panel last two fastener rows. See Figure 4.3.2-3	0.0	0.0

[1] Reference 4-14

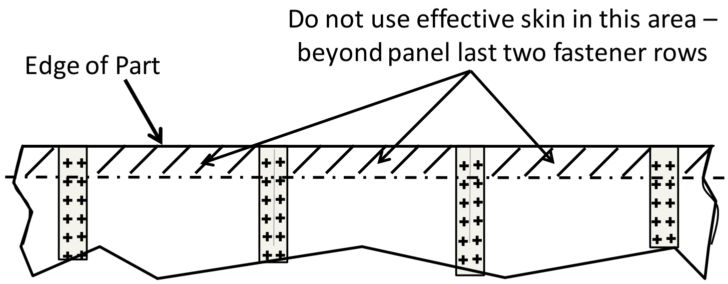


Figure 4.3.2-3 Illustration of Areas Where Skin Cannot be Assumed Effective for Use in Stiffening Element Section Property Calculations

4.3.2.3 Effective Cap Sections for Beams with Buckled Webs

To calculate the section properties of beams with buckled webs, it is necessary to determine the effective cap sections. It is customary for each cap section to include a portion of the web which is considered unbuckled. The effective cap sections for built-up and integrally machined beams are shown in Figure 4.3.2-3.

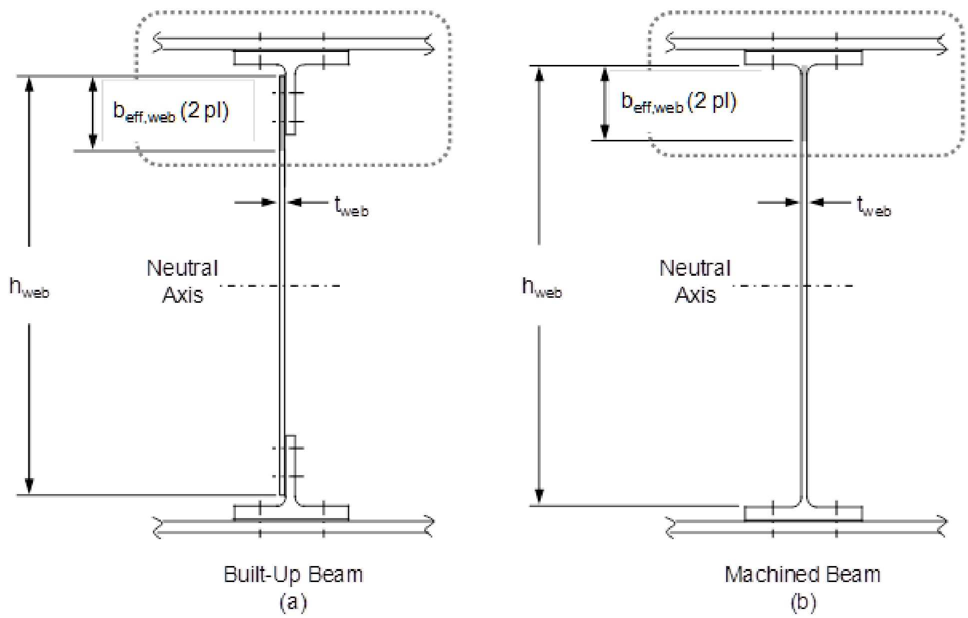


Figure 4.3.2-4 Effective Width of Web for Beam Cap Section

It is recommended that the effective width of web for beam cap section property calculations is limited to the value given by Equation 4.3.2-3.

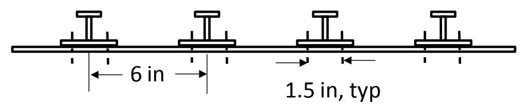
$$b_{eff,web}=Minimum[b_{eff}\ (Table\ 4.3.1\text{-}1),\ h_{web}/6] \qquad \textbf{Equation 4.3.2-3}$$

where
b_{eff} (Table 4.3.1-1) is the web effective width calculated using Table 4.3.1-1 (in)
h_{web} is the web height shown Figure 4.3.2-3 (in)

4.3.3 Effective Width Example Problem

Given: Skin Panel loaded in Compression with Stiffeners at 6 in center-to-center Spacing.

- The skin is 7475-T61 Aluminum Sheet, 0.063 in thick
- The stiffeners are 7149T65111 Extrusions, From IDAT, F_{cc}=65000 psi
- It is assumed that the inter-rivet allowable stress exceeds the stiffener crippling stress



Material Properties:

Material	F_{tu} (ksi)	F_{cy} (ksi)	E_c (psi)	ν	e (in/in)	n
7574-T61 Sheet	75	64	$10.5x10^6$	0.33	0.09	15
7149-T65111 Extrusion	78	71	$11.0x10^6$	0.33	0.07	22

Calculate E_{tan} for the stiffener at F_{cc}

$$E_{tan} = \frac{1}{\frac{1}{E} + 0.002\left(\frac{n}{F_{cy}}\right)\left(\frac{f}{F_{cy}}\right)^{n-1}}$$
$$= \frac{1}{\frac{1}{11x10^6} + 0.002\left(\frac{22}{71000}\right)\left(\frac{65000}{71000}\right)^{(22-1)}}$$
$$= 5320598\ psi$$

Calculate the strain in the stiffener, e_{cc}, at F_{cc}.

$$e = \frac{f}{E_c} + 0.002\left(\frac{f}{F_{cy}}\right)^n$$
$$= \frac{65000}{11x10^6} + 0.002\left(\frac{65000}{71000}\right)^{22} = 0.00620\ in/in$$

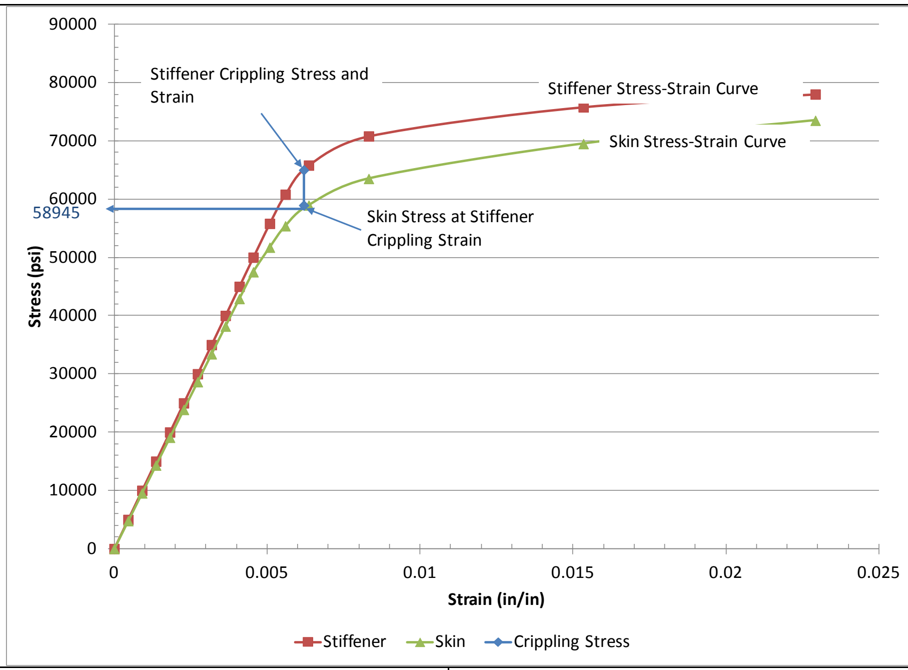
Calculate the Stress in the skin at e_{cc}
This requires an iterative solution – estimate a stress in the skin, solve for the strain and modify the estimate until the strain equals e_{cc}.
Use E, F_{cy} and n for the skin.
First guess – same stress as the stiffener, f=65000 psi

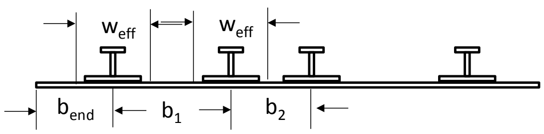
$$e = \frac{f}{E_c} + 0.002\left(\frac{f}{F_{cy}}\right)^n$$
$$= \frac{65000}{10.5x10^6} + 0.002\left(\frac{65000}{64000}\right)^{15}$$
$$= 0.00871\ in/in$$

e_{skin}=0.00871 > $e_{stiffener}$=0.00620. Initial estimate is too high, reduce stress estimate in the skin. Second estimate, f=58945 psi (iterated in Excel)

$$e = \frac{f}{E_c} + 0.002\left(\frac{f}{F_{cy}}\right)^n$$
$$= \frac{58945}{10.5x10^6} + 0.002\left(\frac{58945}{64000}\right)^{15}$$
$$= 0.00620\ in/in$$

The figure below illustrates the stress-strain curve for both stiffener and skin and shows graphically how the skin stress at stiffener strain might be determined.



Calculate b/t for the skin	*b/t = 6.0/0.063 = 95*
From Figure 10.3.2-14, determine k_c	k_c*=6.85 (almost full fixity)*
Calculate b_{eff} using Equation 4.3.1-7	$b_{eff} = \sqrt{0.91k_c}\left(\frac{f_{sk}}{F_{cc-st}}\right)t\sqrt{\frac{E_{tan}}{F_{cc-st}}}$ $= \sqrt{0.91\cdot 6.85}\cdot\left(\frac{58945}{65000}\right)(0.063)\sqrt{\frac{5320598}{65000}}$ $= 1.29\ in$
Determine w_{eff} per Table 4.3.1-2, Case 2 b_{eff}<w: 1.29<1.5 	w_{eff} $= Minimum\left[b_{eff1}, \frac{b_1}{2}\right] + Minimum\left[b_{eff2}, \frac{b_2}{2}\right]$ $= 1.29 + 1.29 = 2.58$
Determine b/t (for Reference)	w_{eff}/t *= 2.58/0.063=40.9*
For comparison only, calculate b_{eff} ignoring the plasticity and material difference corrections using Equation 4.3.0-8 but assuming E_{tan}=E_c and skin and stiffener are made of same material so f_{sk}=F_{cc-st}	$b_{eff} = \sqrt{0.91k_c}\left(\frac{f_{sk}}{F_{cc-st}}\right)t\sqrt{\frac{E_{tan}}{F_{cc-st}}}$ $= \sqrt{0.91\cdot 6.85}\cdot(1.0)(0.063)\sqrt{\frac{11x10^6}{65000}}$ $= 2.05\ in$
For comparison only, Determine w_{eff} per Figure 4.3.1-2, Case 2; But b_{eff} > w: 2.05 > 1.5, thus use b_{eff} and w	$w_{eff} = b_{eff} + w = 2.05 + 1.5 = 3.55$

By not using the plasticity and material corrections, the effective width is overstated at 3.55 inches versus the correct value of 2.58 inches per panel.

4.4 CAD-Based Calculation

Various CAD programs, as well as CATIA, are available for the calculation of some section properties for use in analysis. At various times, these programs have not been clear about the information being provided and whether or not axes have been appropriately rotated and/or translated. The analyst is cautioned to investigate the limitations of the software and to perform checks to ensure an understanding of what information is being calculated.

4.5 Unix/PC-Based Calculation

Another common approach for section property computation is through the use of stand-alone analysis tools or computational aids. The IDAT tool for this purpose is very comprehensive in its ability to accommodate simple-to-complex geometries; it is named SECTION, and a complete description of its features is found in Reference 4-7. IDAT/Section is the preferred tool. However, a variety of uncontrolled MSExcel- and Mathcad-based computational aids are available on individual Programs. Individual analysts are required to verify the accuracy of any such uncontrolled aids prior to initial use.