7 Torsion

The purpose of this chapter is to provide guidance on metallic structural analysis of members loaded in torsion. Members subjected to torsion are categorized by their cross-sections for the purpose of analysis and this section will be organized as shown below.

7.1 References and Nomenclature

Here is a listing of reference used in the preparation of this Section.

7-1. anon., *Lockheed Martin Engineering Stress Memo Manual*, Lockheed Martin Aeronautical Systems, Marietta, GA (October 1998 Release; April 2002 Revision).

7-2. anon., *Structures Analysis Manual, Volume 1 and Volume 2*, General Dynamics Convair and Space Structures Divisions (1988).

7-3. Staff, *LTV Structures Manual*, LTV Aircraft Products Group, Grand Prairie, TX (June 1989 Revision).

7-4. anon., "Metallic Materials and Elements for Aerospace Vehicle Structures," MIL-HDBK-5H, Change Notice 1, Battelle Memorial Institute, Secretariat (1 October, 2001).

7-5. Gerard, G. and Becker, R.,"Handbook of Structural Stability, Part III Buckling of Curved Plates and Shells," NACA TN 3783, National Advisory Committee for Aeronautics, Washington, D.C., 1957

7-6. anon., "Buckling of Thin Walled Circular Cylinders," NASA SP-8007, National Aeronautics and Space Administration, August 1968 Revision.

7-7. Modelinski, J., "Torsion Strength of Round Tubing," LR11474, Lockheed Aircraft Corporation, Burbank, Ca. August 1956.

7-8. Lee, L.H. and Ades, C.S., "Plastic Torsional Buckling Strength of Cylinders Including the Effects of Imperfections," Journal of Aeronautical Sciences, April 1957

7-9. Seely, F and Smith, J, Advanced Mechanics of Materials, McGraw Hill Book Company, New York, 1960

7-10. Timoshenko, S, Strength of Materials, Part II, Krieger Publishing Co. Malabar, FL, 1976

7-11. Nadai, A, Plasticity, A Mechanics of the Plastic State of Matter, John Wiley and Sons, New York, 1931
7-12. Young, W.C., Roark's Formulas for Stress and Strain, McGraw Hill Book Company, New York, 1989

This is a listing of nomenclature used in this Section.

Symbol	Description	Units
a	Length	in
A_O	Enclosed area of median perimeter line	in^2
A_{st}	Area of Differential Element	in^2
b	Length	in
$\left(\frac{b}{t}\right)_{eff}$	Geometric and Boundary Constraint Parameter	--
C	Beam Length Parameter which is a Ratio of the Relative Flange Bending Rigidity to the Torsional Beam Rigidity	in
c	Distance Between Cap Centroids	in
CCW	Counter Clockwise	--
C_W	Section Warping Constant	in^6
d	Distance to the Shear Center of the Cell from the Reference Axis	in
dr	Radial Length of a Differential Element	in
ds	Incremental Length of Perimeter	in
dx	Longitudinal Length of a Differential Element	in
$d\theta$	Angle of Twist of a Differential Element	radians
E	Young's Modulus of the Material	psi
e	Distance to the Shear Center of the Web from the Reference Axis	in
E_c	Young's Modulus of the Material in Compression	psi
E_{sec}	Secant Modulus of the Material	psi
E_{tan}	Tangent Modulus of the Material	psi
f_{cr}	Critical Initial Buckling Stress	psi
F_{cy}	Material Compression Yield Stress	psi
F_{cmax}	Material Maximum Allowable Compression Stress	psi
F_{LA}	Thin Walled Circular Cylinder Torsional Rupture Stress per Energy Formulation	psi
f_s	Applied Shear Stress	psi
$F_{s\text{-}cr}$	Critical Initial Shear Buckling Stress	psi
f_{st}	Applied Torsion Shear Stress	psi
F_{ST}	Thin Walled Circular Cylinder Torsional Rupture Stress Allowable	psi
$F_{st\text{-}cr}$	Critical Initial Torsional Buckling Stress	psi
$f_{st\text{-}max}$	Maximum Applied Torsion Shear Stress	psi
f_{sx}	Applied Longitudinal Stress	psi
F_{su}	Material Ultimate Shear Stress	psi
F_{tu}	Material Ultimate Tension Stress	psi
G	Shear Modulus of the Material	psi
h	Distance Between Cap Midplanes	in
h	Perpendicular Distance from Medial Perimeter Line to Centroid of Section	in
I_p	Polar Moment of Inertia about the Section Centroid	in^4
I_{yy}	Moment of Inertia about the y-axis	in^4
K	St. Venant Torsion Constant	in^4
$k_{st\text{-}med}$	Boundary Constraint Factor for Intermediate Length Cylinder Buckling	--
$k_{st\text{-}short}$	Boundary Constraint Factor for Short Length Cylinder Buckling	--

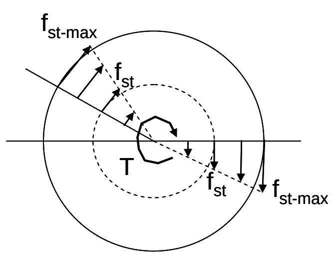
k_T	Geometric Section Factor in Torsion	--
k_{test}	Test Correlation Factor	--
L	Length	in
L_n	Mean Perimeter of Nose	in
M_B	Cap Bending Moment	in-lb
M_o	Moment about Point O	in-lb
M.S.	Margin of Safety	--
n_c	Ramberg Osgood Number in Compression	--
P	Applied Load	lbs
q	Shear Flow	lbs/in
$q_{rupture}$	Allowable Shear Flow at Failure	lbs/in
R	Radius	in
r	radius to differential element	in
R_n	Resultant Load Acting on Nose	lbs
R_{web}	Resultant Load Acting on Web	lbs
t	Thickness	in
T	Torque	in-lb
T_B	Portion of Applied Torque Reacted by In-Plane Cap Bending	in-lb
$T_{rupture}$	Allowable Torque of a Solid Section (including plasticity) at Failure	in-lb
$\left(\frac{Tr}{I_p}\right)_T$	Torsional Modulus of Rupture (Refer to Section 7.3.6.1)	psi
T_s	Portion of Applied Torque Reacted by Torsional Shear	in-lb
V	Sand Heap Volume	in^3
V_T	Lateral Shear Due to Torque in Differential Bending	lb
x	Distance from Restrained End	in
Greek Symbols		
α	Parameter Used in Calculation of Torsional Shear Stress – Rectangular Cross-Section	--
β	Factor for the Correction of K for Thick Sections	--
γ	Shear Strain or Unit Shear Deformation	in/in
γ	Test Correlation Factor for Bucking of Thin-Walled Circular Cylinders	--
η	Plasticity Correction Factor	--
θ	Angle of Twist	radians
ν_e	Elastic Poisson's Ratio	--
ξ	Ratio of Secant Modulus to Tangent Modulus	--
τ_o	Trapezoidal Intercept Shear Stress	psi
τ	Shear Stress	psi
φ	Unit Angle of Twist	radians/in
Subscripts		
1,2,...n	Cell 1, 2,…n	--
avg	average	
i, j	Cell numbers	--
inner	Inside Diameter of Thin-Walled Cylinder	--
outer	Outside Diameter of Thin-Walled Cylinder	--
mean	Dimension at Mid-Thickness Plane in Thin-Walled Closed Shapes	--
n	Nose	--
w	Web	--

7.2 Torsion of Solid Sections

Given the material and section properties for a solid section, the torsional shear stress, f_{st}, and resulting angle of twist, θ, can be calculated for any applied twisting moment or torque. For solid circular cross-sections, the torsional shear stress will vary linearly along radial lines emanating from the geometric centroid and will have the same distribution along all radial lines. For solid non-circular cross-sections, the torsional shear stress is linear only along lines of symmetry where the cross-section contour is normal to the radial line; otherwise, the torsional shear stress distribution is non-linear. Section 7.2.1 discusses torsional shear stresses and deformations in solid circular sections and 7.2.2 discusses torsional shear stresses and deformations in solid non-circular sections.

7.2.1 Behavior of Solid Circular Sections in Torsion

For a solid circular cross-section, subjected to applied twisting moments which lie in the plane of the cross-section or perpendicular to the axis of the shaft, the cross-section remains circular and the diameter remains straight after the application of the torque. As stated above, the torsional shear stress will vary linearly along radial lines emanating from the geometric centroid as long as stresses are less than or equal to the proportional limit and will have the same distribution along all radial lines. The longitudinal shear stress, f_{sx}, which is equal to the torsional shear stress, f_{st}, produces no warping of the cross-section. This is shown in Figure 7.2.1-1 and Figure 7.2.1-3.



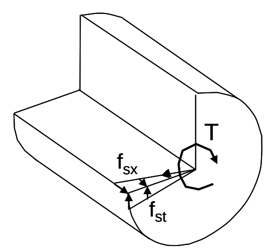


Figure 7.2.1-1 Shear Stress Distribution in a Circular Cross-Section

The analysis of solid circular cross-sections, by the method shown in this and subsequent sections, is subject to the following limitations:

- The material is homogeneous and isotropic
- Deformations are small
- The shear stress does not exceed the shearing proportional limit and is proportional to the shear strain, *i.e.*, Hooke's Law applies
- The applied twisting load cannot be an impact load
- The stresses calculated at points of constraint and at abrupt changes in applied twisting moment are not exact
- If the cross-section abruptly changes, a stress concentration must be considered

Figure 7.2.1-2 shows a straight, solid circular shaft of length "L" with an external torque, or twisting moment, applied at the ends.

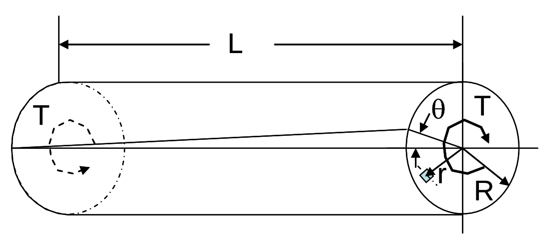


Figure 7.2.1-2 Solid Circular Shaft

The angle of twist, in radians, of the cross-section of a circular shaft (solid or hollow) due to the applied twisting moment, T, is given by

$$\theta = \frac{TL}{GI_p}$$ **Equation 7.2.1-1**

Where T is the applied torque (in-lb)
L is the length of the shaft (in)
G is the shear modulus of the material (psi)
I_p is the polar moment of inertia (in^4)

The shearing deformation of any fiber located a radius r from the center of the circle is given by rθ. The unit deformation or shear strain is given by

$$\gamma = \frac{r\theta}{L}$$ **Equation 7.2.1-2**

where
r is the radius to the fiber (in)
θ is the angle of twist of the shaft (radians)
L is the length of the shaft (in)

The torsional shear stress at any fiber located a radius r in the cross-section can be calculated from Hooke's law and Equations 7.2.2-1 and 7.2.2-2 as

$$f_{st} = G\gamma = \frac{Gr\theta}{L} = \frac{Gr\frac{TL}{GI_p}}{L} = \frac{Tr}{I_p}$$ **Equation 7.2.1-3**

where r is the radius of the cross-section to the point of analysis (in)
γ is shear strain (in/in)

Equation 7.2.1-3 mathematically illustrates that the torsional shear stress is a linear function of the radius of the cross-section. The torsional shear stress is zero at the center (r=0) and increases along any radial line to a maximum at the outer surface where r=R. Thus, the maximum torsional shear stress is given by

$$f_{st-\max} = \frac{TR}{I_p}$$ **Equation 7.2.1-4**

where R is the outer radius of the cross-section (in)

For a section under an applied torsion load, in addition to the torsional shear stress a longitudinal shear stress is also induced. Figure 7.2.1-3 illustrates the shear stress on an incremental element due to an applied torsional moment.

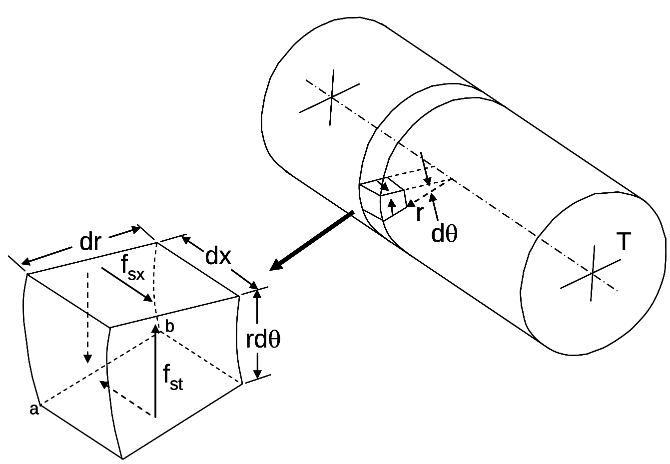


Figure 7.2.1-3 Incremental Element Shear Stresses Due to Torsion

If a summation of moments is done about edge a-b, a longitudinal shear stress, f_{sx}, necessary to meet equilibrium requirements can be determined.

$$\Sigma M_{a\text{-}b} = 0 \ (+CCW)$$

$$f_{st}(A_{st})dx - f_{sx}(A_{sx})rd\theta = f_{st}(rd\theta)(dr)dx - f_{sx}(dx)(dr)rd\theta = 0$$

$$f_{st} = f_{sx}$$

where
f_{st} is the torsional shear stress (psi)
f_{sx} is the longitudinal shear stress (psi)
A_{st} is the area of the differential element used for the calculation of torsional shear stress (in^2)
A_{sx} is the area of the differential element used for the calculation of longitudinal shear stress (in^2)
dr is the length of the differential element in the radial direction (in)
dx is the length of the differential element in the longitudinal direction (in)
dθ is the angle of twist of the differential element (radians)

From this it is seen that the induced longitudinal shear stress is indeed equal to the torsional shear stress as depicted in Figure 7.2.1-1.

The margin of safety for solid sections in shear is calculated as

$$M.S. = \frac{F_{su}}{f_{st}} - 1 \qquad \textbf{Equation 7.2.1-5}$$

where
F_{su} is the ultimate shear strength of the material (psi)
f_{st} is the maximum shear stress due to torsion at ultimate load(psi)

Interaction of shear and other stress components in solid circular sections is treated in Section 2.5.4.2. Rods and other shapes do have capability to carry loads which account for plastic redistribution of stresses. That topic is covered in Section 7.2.3.

7.2.2 Behavior of Solid Non-Circular Sections in Torsion

For a non-circular solid section in torsion, the above equations are not directly applicable because out-of-plane warping of the cross-section occurs; however modified versions of Equations 7.2.1-1 and 7.2.1-3 can be used for these applications. The shear stresses do not vary linearly along radial lines from the centroid of the section. For the case where the ends of the member are free to warp, there are a limited number of shapes for which exact solutions are available. These include rectangles, squares, ellipses and triangles.

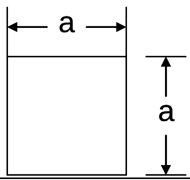
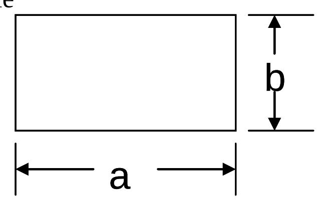
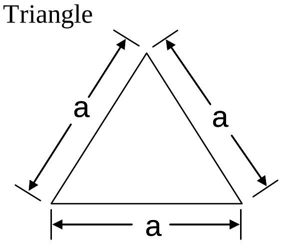
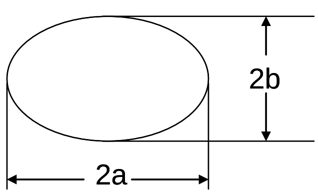
The angle of twist, in radians, of the cross-section of a non-circular shaft due to the applied twisting moment, T, is given by an equation similar to Equation 7.2.1-1 where the polar moment of inertia is replaced by a torsional constant, as shown below. Values of the torsional constant for a limited number of solid cross-sections are tabulated in Section 4.2.2.3.

$$\theta = \frac{TL}{GK} \qquad \textbf{Equation 7.2.2-1}$$

Where
T is the applied torque (in-lb)
L is the length of the shaft (in)
G is the shear modulus of the material (psi)
K is the torsional constant. (Refer to Section 4.2.2.3) (in^4)

The maximum torsional shear stresses can be calculated for some solid non-circular cross-sections from the equations listed in Table 7.2.2-1. Note that because the stresses do not vary linearly along radial lines, the equations are valid only at the points indicated and represent the maximum stresses due to torsion. The margin of safety can be calculated using Equation 7.2.1-5. Equations for other solid cross-sections can be found in Reference 7-12.

Table 7.2.2-1 Formula for Calculation of Torsional Shear Stress for Solid Sections (Reference 7-2)

Section	$f_{st\text{-}max}$
Square 	$f_{st-\max} = \dfrac{T}{0.208a^3}$ at the midpoint of each side
Rectangle 	$f_{st-\max} = \dfrac{T}{\alpha b^2 a}$ maximum for section, midpoint of each long side $f_{st} = \dfrac{T}{\alpha b a^2}$ maximum for short side, at midpoint of each short side $\alpha = \dfrac{1}{3+\dfrac{1.8}{\left(\dfrac{a}{b}\right)}}$ for a>>b, α=1/3
Equilateral Triangle 	$f_{st-\max} = \dfrac{20T}{a^3}$ at midpoint of each side
Elliptic 	$f_{st-\max} = \dfrac{2T}{\pi a b^2}$ maximum for section, at ends of minor axis $f_{st} = \dfrac{2T}{\pi a^2 b}$ at ends of major axis

7.2.3 Plastic Torsion of Solid Sections

When a torsion load is applied to a solid section, it is possible for the section to undergo plastic deformation. The previous method of analysis is based on limit load stress levels in the elastic range. For an efficient design, it can be desirable to allow the section to operate in the plastic region at ultimate load. Compact sections, *i.e.*, those not subject to stability or crippling failures, can be analyzed using the plastic torsion theory at ultimate load. Thin walled circular cylinders require special treatment and are discussed in Section 7.3.6. The method outlined in this Section is called the sand heap method, which is discussed in detail in Reference 7-11 and is outlined in Reference 7-2.

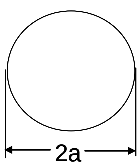
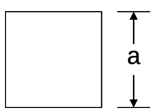
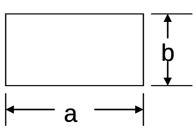
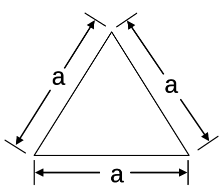
If a surface with the same shape as the cross-section of a beam in torsion is heaped with sand, the slope of the heap represents the shear stress. The shear stress for this condition has the same magnitude over the entire cross-section and the maximum allowable plastic torsional moment is related to the volume of the sand heap, *V*, by

$$T_{rupture} = 2VF_{su} \qquad \textbf{Equation 7.2.3-1}$$

where
F_{su} is the ultimate shear stress of the material (psi)
V is the volume of the sand heap (in^3)

The determination of the sand heap volume is somewhat complex and values for typical solid shapes are provided in Table 7.2.3-1.

Table 7.2.3-1 Sand Heap Volume and Modulus of Rupture for Typical Solid Sections

Cross-Section Shape	Sand Heap Volume	$T_{rupture}$
	$V = \frac{\pi a^3}{3}$	$T_{rupture} = \frac{2\pi a^3}{3} F_{su}$
	$V = \frac{a^3}{6}$	$T_{rupture} = \frac{a^3}{3} F_{su}$
	$V = \frac{b^2(3a-b)}{12}$	$T_{rupture} = \frac{b^2(3a-b)}{6} F_{su}$
	$V = \frac{a^3}{24}$	$T_{rupture} = \frac{a^3}{12} F_{su}$

For other cross-sections, the task of determining the sand heap volume is simplified somewhat by construction of contour lines of constant elevation on the cross-section. The contour line defines the contour of the heap at some constant elevation and is constructed by defining planes passing through the heap parallel to the torsional section. This is illustrated in Figure 7.2.3-1. It is assumed the maximum possible slope of the heap is achieved, *i.e.* the slope is equal to unity. Contour lines will intersect normals through the section boundary at right angles and at a distance from the boundary equal to the elevation. It is then possible to determine the volume of the sand heap by integration.

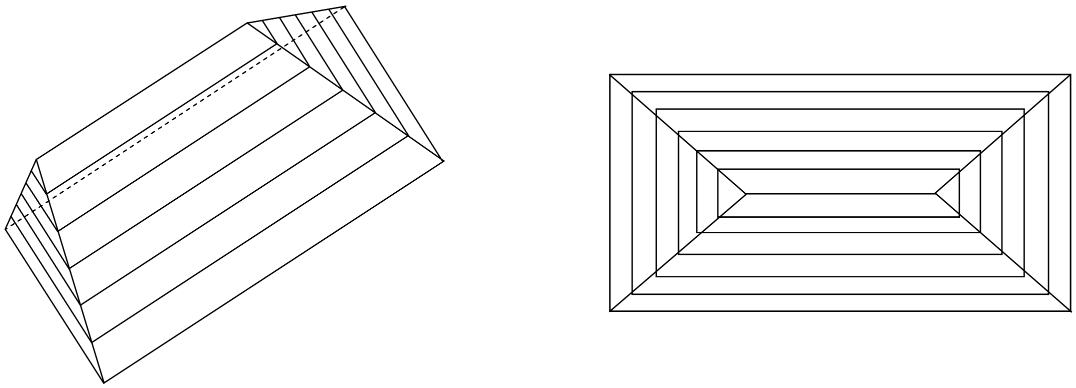


Figure 7.2.3-1 Sand Heap Analogy for Rectangular Cross Section

The Margin of Safety for a solid section can be calculated by

$$M.S. = \frac{T_{rupture}}{T} - 1$$ **Equation 7.2.3-2**

where
$T_{rupture}$ is the maximum allowable torsion of a solid section, including plasticity (in-lb)
T is the applied ultimate torsion (in-lb)

Note that the margin is based on applied torsion load and not calculated stresses since the stresses will be non-linear. If the loading is a combination of torsion stress and other load components, such as axial compression or tension, the torsion stress should not exceed F_{su}. Figure 7.2.3-2, taken from Reference 7-11, illustrates the development of the plastic zone, due to torsion in a square cross section.

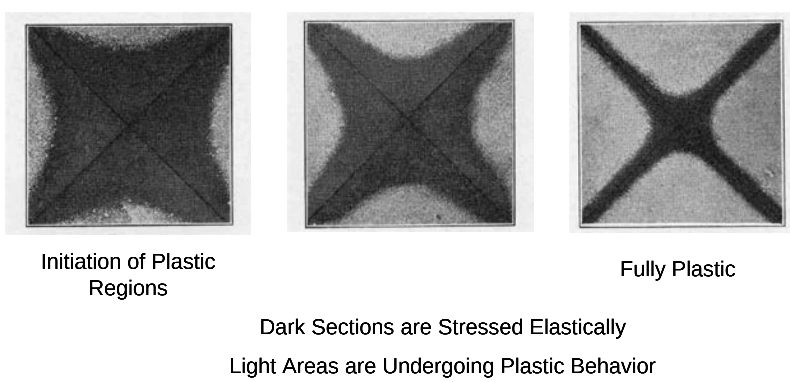


Figure 7.2.3-2 Square Cross-Section Illustrating Increasing Levels of Plastic Torsion

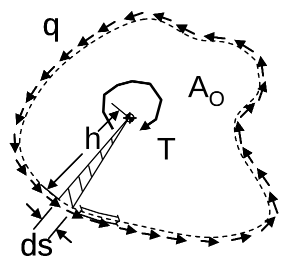
7.3 Torsion of Thin-Walled Sections

A thin-walled section is a cross section which is comprised of elements whose thickness is small relative to the overall dimensions of the section. Thin-walled sections can be open or closed. Closed thin-walled sections are cross-sections where the centerline of the wall forms a closed curve. For example, if the overall section is round, a hollow tube with thin walls would be the thin-walled equivalent and it is a closed section. If the thin walled tube is split along the longitudinal axis, it would be an open section. Other examples of thin-walled open sections are tees, channels and I-beams.

7.3.1 Calculation of Torsion Stress in Single Cell Closed Sections and the Development of Torsional Shear Flow

As discussed in Section 6.5, shear flow (lbs/in) is the product of the shear stress and the thickness. Like shear stress, it is a function of the applied load and the geometry of the part; however, torsional shear flow can be calculated before the thickness has been determined. In this section, shear flow due to torsion is discussed. The torsion may be resultant from an applied torque or it may be a result of an applied shear load not at the shear center of the section. The shear center is defined as the point on the cross-sectional plane of a beam through which the resultant of the transverse shear load must be applied in order to have no torsion induced.
The basic rules and assumptions governing computation of shear flow in flat panels do not change if the panels are curved around into a closed structure as in a section of a fuselage with many longitudinal stringers or a leading edge or an airfoil-shaped control surface.

A general closed section, as shown in Figure 7.3.1-1, with sufficient support to effectively maintain the shape even when loaded, can be used to derive the equations for shear flow under a torsional moment.



Note: The enclosed area, A_O is calculated based on the median-thickness perimeter of the section
Figure 7.3.1-1 Shear Flow in a General Closed Cell Cross-section

In order for the closed cell depicted in Figure 7.3.1-1 to satisfy the equilibrium requirement that the applied torque must not develop any net load unbalance, *i.e.*, ΣM=0, the shear flow, q, due to the torque, T, must be constant around the perimeter and the sum of all the incremental shear flow torques must equal the applied torque. Then, for any incremental distance, ds, along the perimeter, the incremental torque, dT, is given by

$$dT = q(ds)h \qquad \textbf{Equation 7.3.1-1}$$

where
T is the applied torsion or twisting moment (in-lb)
q is the shear flow (lbs/in)
ds is the increment of the perimeter (in)
h is the perpendicular distance from the median perimeter line of the section to the centroid of the section (in)

Assuming ds is sufficiently small to eliminate error due to surface curvature, the area of the cross-hatched section is approximately equal to the area of a triangle, ½(ds)h, so that the incremental torque is the shear flow times twice the area of the triangle. Thus, the sum of all incremental torques and triangles around the section results in

$$T=q(2A_O) \qquad \textbf{Equation 7.3.1-2}$$

$$q=T/(2A_O) \qquad \textbf{Equation 7.3.1-3}$$

where
A_O is the enclosed area of the cross-section to the median thickness of the perimeter. For irregularly curved sections see Equation 4.2.2-4 (in^2)

Thus, to determine the shear flow of a closed section all that is required is the applied twisting moment and the enclosed area of the section. Such a uniform torsional shear flow applied to the end of a closed structure section

develops constant reacting shear flows in the full structure identical to those developed in single flat panels by in-plane loads.
The torsional shear flows can be combined with internal shear flows that might be developed in the closed section due to other externally applied loads. It can also be used in diagonal tension analysis which is discussed in Section 6.

The shear stress due to a shear flow is given as

$$f_{st} = q/t$$ **Equation 7.3.1-4**

where
q is the shear flow (lb/in)
t is the thickness of the panel (in)

Note that if the closed cell is of non-uniform thickness, the shear flow due to torsion is assumed to be uniform, but the highest stress will occur in the thinnest panel.

The angle of twist per unit length of a closed cell section is given by Equation 7.2.2-1, using a unit length of 1 in. as follows

$$\varphi = \frac{T}{GK}$$

but $T=2A_oq$, thus

$$\varphi = \frac{2A_O q}{GK}$$ **Equation 7.3.1-5**

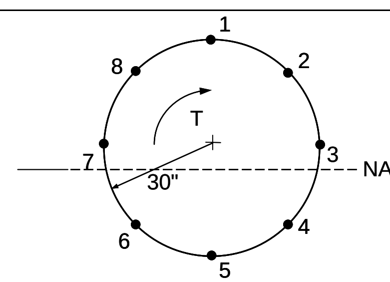
where
T is the applied torque (in-lb)
G is the shear modulus of the material (psi)
K is the torsional constant given by Equation 4.2.2-3

Substituting Equation 4.2.2-3 into Equation 7.3.1-5 yields

$$\varphi = \frac{1}{2A_O}\oint \frac{q}{Gt}ds$$ **Equation 7.3.1-6**

where
φ is the unit angle of twist (radians/in)

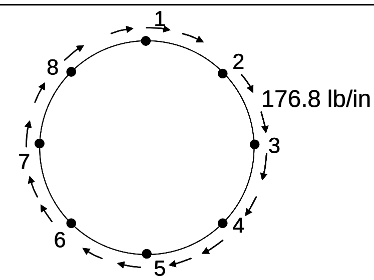
7.3.1.1 Example Problem: Circular Cross-Section

Given the stiffened fuselage section shown in the figure to the right. • $t_{12}=t_{23}=t_{78}=t_{81}=0.040$ in • $t_{34}=t_{45}=t_{56}=t_{67}=0.060$ in The applied torsion is 1,000,000 in-lb. Calculate the shear flow and shear stress in each of the panels due to the applied torsion.	
The enclosed area of the circular cross-section is $A_O = \pi R^2 = \pi(30)^2 = 2827.4in^2$ Where the R>>t, the enclosed area calculated using the outer radius has a very small error over the area calculated using the median radius line (<1%).	

The shear flow, using Equation 7.3.1-3, is

$$q = \frac{T}{2A_O} = \frac{1000000}{2(2827.4)} = 176.8 lb/in$$

f_{st}=q/t = 176.8/0.040 = 4420 psi for panels 1-2,2-3,7-8,8-1
f_{st}=q/t = 176.8/0.060 = 2947 psi for panels 3-4,4-5,5-6,6-7



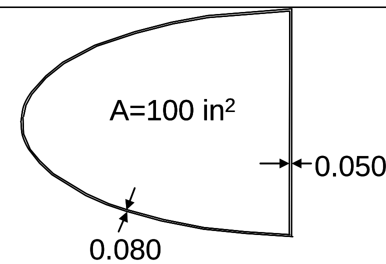
Note that the presence of stiffeners at locations 1-8 does not change the shear flow in the panels. The shear flow due to torsion in all panels is the same, irrespective of thickness. The highest shear stress occurs in the thinnest panels.

7.3.1.2 Example Problem: Non-Circular Cross-Section

Given the leading edge shown in the figure to the right. The enclosed area is 100 in^2. The thickness of the nose is t_n=0.080 in and the thickness of the web is t_w=0.050 in.

The applied torsion is 250000 in-lb

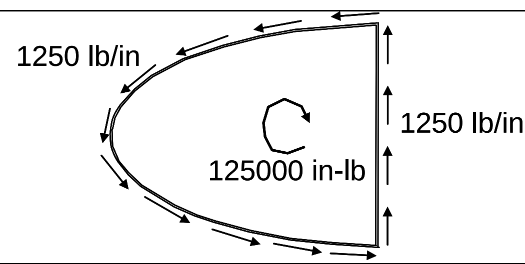
Calculate the shear flow and shear stress in each of the panels due to the applied torsion.



The shear flow, using Equation 7.3.1-3, is

$$q = \frac{T}{2A_O} = \frac{250000}{2(100)} = 1250 lb/in$$

f_{st}=q/t = 1250/0.080 = 15625 psi for the nose
f_{st}=q/t = 1250/0.050 = 25000 psi for web



The technique for determining shear flow due to applied torsion is the same regardless of whether the section is circular or non-circular.

7.3.2 Calculation of Torsion Stress in Multiple Cell Closed Sections and the Development of Torsional Shear Flow

Extending the approach outlined in Section 7.3.1 to shell sections with multiple cells, for an externally applied torque, T, the torque is reacted by an internal shear flow in each cell. This is illustrated in Figure 7.3.2-1.

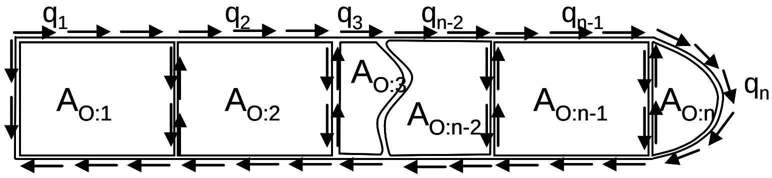


Figure 7.3.2-1 Multiple Cell, Multiple Cap Shell Section

This can be expressed mathematically as

$$T=2q_1A_{O:1}+2q_2A_{O:2}+\ldots+2q_nA_{O:n}$$ **Equation 7.3.2-1**

where
$q_1, q_2, \ldots, q_n$ are the shear flows around each individual cell, 1 to n (lbs/in)
$A_{O:1}, A_{O:2}, \ldots, A_{O:n}$ are the cell areas enclosed by the median lines of cells 1 to n (in^2)

For elastic continuity, the angle of twist of each cell must be equal

$$\varphi_1=\varphi_2=\ldots=\varphi_n$$ **Equation 7.3.2-2**

where
$\varphi_1, \varphi_2, \ldots \varphi_n$ are the unit angular twist of each cell (radians/in)

Equation 7.3.1-6 can be used to determine the unit angle of twist for each cell where the line integral, $\oint \frac{ds}{Gt}$, is replaced with the variable a, representing the segment length divided by the segment thickness.

$$\varphi = \frac{qa_{ij}}{2A_O G_{ij}}$$ **Equation 7.3.2-3**

where
a_{ij} is the length of the side of the cell wall between cells i and j divided by the thickness of the wall. If the wall is between a cell and the outside of the structure, the second subscript is "0".
G_{ij} is the shear modulus for the cell wall between cells i and j
Note: The entire perimeter of a cell must be accounted for.

An equation for the unit angular twist is constructed for each cell of the multi-cell beam shown in Figure 7.3.2-1 using Equation 7.3.2-3

Cell 1:

$$\varphi_1 = \frac{qa_{ij}}{2A_O G_{ij}} = \frac{1}{2A_{O:1}}\left[\frac{q_1 a_{10}}{G_{10}} + \frac{(q_1 - q_2)a_{12}}{G_{12}}\right]$$

Cell 2:

$$\varphi_2 = \frac{qa_{ij}}{2A_O G_{ij}} = \frac{1}{2A_{O:2}}\left[\frac{(q_2 - q_1)a_{12}}{G_{12}} + \frac{q_2 a_{20}}{G_{20}} + \frac{(q_2 - q_3)a_{23}}{G_{23}}\right]$$ **Equation 7.3.2-4**

and so forth until cell n:

$$\varphi_n = \frac{qa_{ij}}{2A_O G_{ij}} = \frac{1}{2A_{O:n}}\left[\frac{(q_n - q_{n-1})a_{n(n-1)}}{G_{n(n-1)}} + \frac{q_n a_{n0}}{G_{n0}}\right]$$

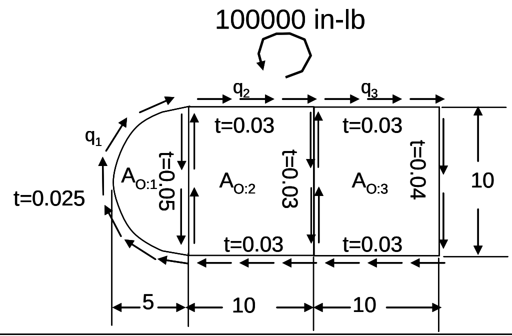
The shear flow in each web can be determined by solving simultaneously Equation 7.3.2-1 and the set of equations described by Equation 7.3.2-4. Once the shear flow is determined for each web, the unit angular twist can be determined using any of the equations developed by Equation 7.3.2-4. The shear stress in each web can be calculated using Equation 7.3.1-4.

7.3.2.1 Example Problem – Multi-Cell Beam

Given the three cell beam shown to the right, determine the shear flow and unit angle of twist for each cell

Assume $G=G_1=G_2=G_3=4x10^6$ psi

Dimensions given are to mean thickness line.



Determine the areas of each cell:

$$A_{O:1} = \frac{1}{2}\left(\pi R^2\right) = \frac{1}{2}\pi(5)^2 = 39.27 in^2$$

$$A_{O:2} = A_{O:3} = LH = 10(10) = 100 in^2$$

Determine the line integrals for each segment

$$a_{10} = \frac{1}{2}\left(\frac{2\pi R}{t_{01}}\right) = \frac{\pi(5)}{0.025} = 628.3$$

$$a_{20} = 2 segments\left(\frac{10}{0.03}\right) = 666.7$$

$$a_{30} = 2 segments\left(\frac{10}{0.03}\right) + \frac{10}{0.04} = 916.7$$

$$a_{12} = \left(\frac{10}{0.05}\right) = 200$$

$$a_{23} = \left(\frac{10}{0.03}\right) = 333.3$$

Equate the torque reacted by each cell to the externally applied torque per Equation 7.3.2-1

$T=2q_1A_{O:1}+2q_2A_{O:2}+2q_3A_{O:3}$

$100000=2\ q_1\ (39.3)\ +2\ q_2\ (100)\ +2\ q_3\ (100)$

$100000=78.6q_1+200q_2+200q_3$

Unit angular twist for Cell 1, recall that from Equation 7.3.2-2, $\varphi = \varphi_1 = \varphi_2 = \varphi_3$

$$\varphi_1 = \frac{qa_{ij}}{2A_O G} = \frac{1}{2A_{O:1}G}\left[q_1 a_{10} + (q_1 - q_2)a_{12}\right]$$

$$G\varphi = G\varphi_1 = \frac{1}{2(39.3)}\left[628.3q_1 + 200(q_1 - q_2)\right]$$

$$G\varphi = \left[8.0q_1 + 2.5(q_1 - q_2)\right] = 10.5q_1 - 2.5q_2$$

Unit angular twist for Cell 2

$$\varphi_2 = \frac{qa_{ij}}{2A_O G} = \frac{1}{2A_{O:2}G}\left[(q_2 - q_1)a_{12} + q_2 a_{20} + (q_2 - q_3)a_{23}\right]$$

$$G\varphi = G\varphi_2 = \frac{1}{2(100)}\left[200(q_2 - q_1) + 666.7q_2 + 333.3(q_2 - q_3)\right]$$

$$G\varphi = \left[(q_2 - q_1) + 3.3q_2 + 1.7(q_2 - q_3)\right] = -q_1 + 6q_2 - 1.7q_3$$

Angular twist for Cell 3

$$\varphi_3 = \frac{qa_{ij}}{2A_O G} = \frac{1}{2A_{O:3}G}\left[(q_3 - q_2)a_{23} + q_3 a_{30}\right]$$

$$G\varphi = G\varphi_3 = \frac{1}{2(100)}\left[333.3(q_3 - q_2) + 916.7q_3\right]$$

$$G\varphi = \left[1.7(q_3 - q_2) + 4.6q_3\right] = -1.7q_2 + 6.3q_3$$

Solve 4 Equations with 4 unknowns:

1) $100000=78.6q_1+200q_2+200q_3$
2) $G\varphi = 10.5q_1 - 2.5q_2$
3) $G\varphi = -q_1 + 6q_2 - 1.7q_3$
4) $G\varphi = -1.7q_2 + 6.3q_3$

Equate Equation 2 with 3 and 2 with 4 to obtain 3 equations in q only and solve for q:
q_1 = 143 lbs/in q_2 = 235 lbs/in q_3 = 209 lbs/in

Then from Equation 2, 3, or 4, with $G=4x10^6$ psi,
φ= 0.000229 rad/in

Note that the shear flow present in the walls between cells is the difference between the shear flows for each cell. For example, the shear flow in wall 1:2 is q_2-q_1 = 235-143 = 92 lbs/in. For wall 2:3, the shear flow is q_3-q_2 = 209-235 = -26 lbs/in.

7.3.3 Open Section Beams in Torsion

Open section beams behave differently depending on whether they are unrestrained or restrained. Section 7.3.3.1 discusses unrestrained behavior, while Section 7.3.3.2 discusses the restrained behavior or differential bending.

7.3.3.1 Calculation of Torsion Stress in Open Section Beams with Unrestrained Ends

An open section is a section in which the centerline of the wall does not form a closed curve. Some examples of this type of section are tees, I-beams, angles, channels and slit tubes. These types of sections are very inefficient in reacting torsion loads and should be avoided if possible where loading is primarily torsion.

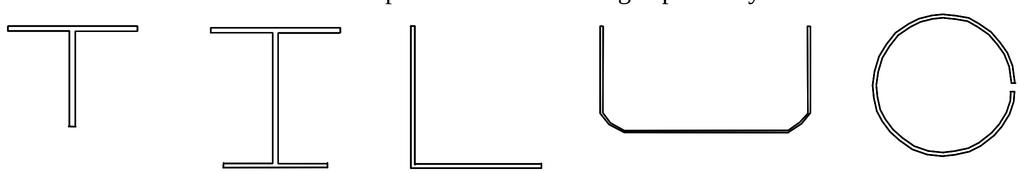


Figure 7.3.3-1 Open Section Configurations

The angle of twist, in radians, of an open section in torsion with unrestrained ends is given by

$$\theta = \frac{TL}{GK} \qquad \textbf{Equation 7.3.3-1}$$

Where
T is the applied torque (in-lb)
L is the length of the beam (in)
G is the shear modulus of the material (psi)
K is the St. Venant torsion constant, discussed in Section 4.2.2 (in^4)

For sections with unrestrained ends made up of multiple segments, the torsional stress for each leg of the section is given by

$$f_{st} = \frac{Tt_i}{K} \text{ for i=1,n legs} \qquad \textbf{Equation 7.3.3-2}$$

where
t_i is the thickness of the leg being analyzed (in)

If the section is bent up sheet metal, the variable, *i*, in Equation 7.3.3-2 is equal to 1 and length used to calculate K is the length of the un-folded section. If the section is a thick extrusion with low b/t and large radii, Section 4.2.2.1 should be used to correct K.

The margin of safety is calculated per 7.2.1-5. For combined stresses, for sections with stable (i.e. non-buckling) flanges the margin of safety can be calculated per Section 2.5.4.2 where the torsion stress is treated as shear stress and may be combined with other shear stress components.

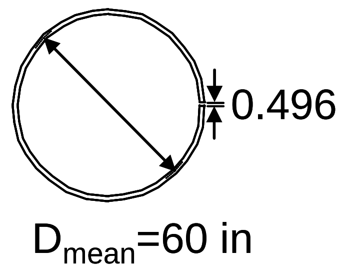
7.3.3.1.1 Example Problem-Open versus Closed Section in Torsion

Given the circular section shown in the figure to the right: t =0.040 in

The applied torsion is 4000 in-lb.

Calculate the shear stress assuming

1) It is a closed circular section
2) It is an open section with a 0.496 in slit as shown
3) Assume the material is aluminum, G~4x10^6 psi, calculate the angular twist per unit length for each section.



Closed Section:
Calculate enclosed area:

$$A_o = \frac{\pi}{4}D_{mean}^2 = \frac{\pi}{4}[60]^2 = 2827.4in^2$$

Equation 7.3.1-3:

$$q = \frac{T}{2A_o} = \frac{4000}{2(2827.4)} = 0.707lb/in$$

Equation 7.3.1-4:

$$f_{st} = \frac{q}{t} = \frac{0.707}{0.040} = 17.67\,psi$$

Open Section:
Calculate circumference of slit tube: b=πD-0.496 = 188 in
t = 0.040
Equation 4.2.2-2:

$$K = \frac{1}{3}\Sigma bt^3 = \frac{1}{3}(188)(0.040)^3 = 0.00401in^4$$

Equation 7.3.3-2:

$$f_{st} = \frac{Tt}{K} = \frac{4000(0.040)}{0.00401} = 39900\,psi$$

Calculate I_p Equation 4.2.1-11 and Section 4.2.6
D_{inner}=60-(0.040/2)=59.98 in; R_{inner}=29.99 in
D_{outer}= 60+(0.040/2)= 60.02 in; R_{outer}=30.01 in

$$I_p = \frac{2\pi}{4}\left(R_{outer}^4 - R_{inner}^4\right) = \frac{\pi}{2}\left(30.01^4 - 29.99^4\right) = 3393in^4$$

L= 1 in
Equation 7.2.1-1

$$\theta = \frac{TL}{GI_p} = \frac{4000(1)}{4x10^6(3393)} = 2.75x10^{-7}\,rad/in$$

Convert to degrees

$$\theta = 2.75x10^{-7}\left(\frac{180}{\pi}\right) = 0.000016°$$

L=1 in

Equation 7.3.3-1

$$\theta = \frac{TL}{GK} = \frac{4000(1)}{4x10^6(0.00401)} = 0.249rad/in$$

Convert to degrees:

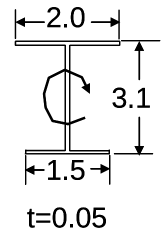
$$\theta = 0.249\left(\frac{180}{\pi}\right) = 14.3°$$

This calculation emphasizes the difference in the ability to carry torsional loads for an open and a closed section. Note that the shear stress in the open section is near F_{su}, while the shear stress in the closed section is quite low at 17.67 psi.

7.3.3.1.2 Example Problem-I Beam in Torsion

For the I-Beam shown, calculate the stress for an applied torsional moment of 100 in-lb.
G=4.0x10^6 psi E=10.4x10^6 psi

Compare the results for stress when K is calculated per Equations 4.2.2-1 and 4.2.2-2
F_{su} = 41,000 psi
If the beam is 10 in long, calculate the rotational deformation. Is this acceptable?



Tabulate the geometry:
b_1=2.0; t=0.05; b/t=40
b_2=[3.0-2(0.05)]=3.0; t=0.05; b/t=60
b_3=1.5; t=0.05; b/t=30

Solve for the stress, using the approximation for K, Equation 4.2.2-2

$$K = \frac{1}{3}\Sigma bt^3 = \frac{1}{3}\left[(2.0)(0.05)^3 + (3.0)(0.05)^3 + (1.5)(0.05)^3\right] = 0.000271 in^4$$

$$f_{st} = \frac{Tt}{K} = \frac{100(0.050)}{0.000271} = 18450\,psi$$

Re-calculate K using the rigorous approach, Equation 4.2.2-1

$$\beta = \left[\frac{1}{3} - \frac{0.21}{\frac{b}{t}}\left(1 - \frac{1}{12\left(\frac{b}{t}\right)^4}\right)\right] = \left[\frac{1}{3} - \frac{0.21}{40}\left(1 - \frac{1}{12(40)^4}\right)\right] = 0.328$$

$$\beta = \left[\frac{1}{3} - \frac{0.21}{\frac{b}{t}}\left(1 - \frac{1}{12\left(\frac{b}{t}\right)^4}\right)\right] = \left[\frac{1}{3} - \frac{0.21}{60}\left(1 - \frac{1}{12(60)^4}\right)\right] = 0.330$$

$$\beta = \left[\frac{1}{3} - \frac{0.21}{\frac{b}{t}}\left(1 - \frac{1}{12\left(\frac{b}{t}\right)^4}\right)\right] = \left[\frac{1}{3} - \frac{0.21}{30}\left(1 - \frac{1}{12(30)^4}\right)\right] = 0.326$$

$$K = \Sigma\beta bt^3 = \left[(0.328)(2.0)(0.05)^3 + (0.330)(3.0)(0.05)^3 + (0.326)(1.5)(0.05)^3\right] = 0.000267 in^4$$

$$f_{st} = \frac{Tt}{K} = \frac{100(0.050)}{0.000267} = 18726\,psi$$

$$\theta = \frac{TL}{GK} = \frac{100(10)}{4.0x10^6(0.000267)} = 0.936 radians = 53.6\deg$$

For these b/t values the difference in the calculated stress between the approximate solution for K and the exact solution for K is 1.5 percent. As noted in Section 4.2.2, for b/t>30 the error is about 2 percent which would be sufficient for engineering applications. For b/t of 10 the error is greater than 5 percent and the rigorous solution might be warranted. IDAT/Section always uses Equation 4.2.2-1.

Note the very large rotational deflection. This violates one of the fundamental assumptions of the method. Thus even though the calculated stresses are low and the “margin of safety” might be numerically positive, it is invalid due to the large deflections.

7.3.3.2 Calculation of Torsion Stress in Open Section Beams with a Restrained End – Differential Bending

When the end of a thin-walled open section beam in torsion is restrained, a portion of the torsion is reacted as a twisting shear in the beam and part is carried as a bending moment in the flanges. A simple approach, which provides a good solution if the web is thin and the beam is long, is to determine the shear on the caps assuming the torque is reacted as a couple as shown in Figure 7.3.3-2 and then calculate the in-plane moment in each flange using simple statics.

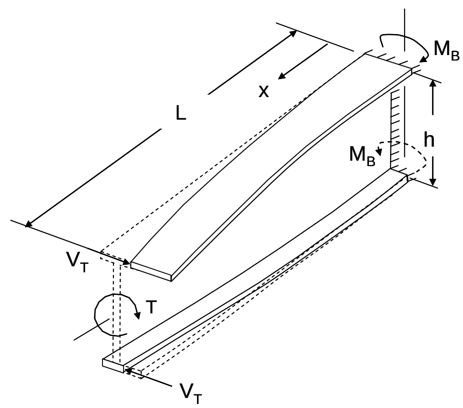


Figure 7.3.3-2 Restrained Beam in Torsion with Caps Carrying Torsion in In-Plane Bending

The lateral shear in each cap due to the applied torsion is given by

$$V_T = T/h \qquad \textbf{Equation 7.3.3-3}$$

where
T is the applied torque (in-lb)
h is the distance between the cap mid-planes (in)

The in-plane bending moment in the cap is given by

$$M_B = V_T(L-x) \qquad \textbf{Equation 7.3.3-4}$$

where
V_T is the lateral shear load due to the applied torque (lbs)
L is the distance between the shear load and the restraint (in)
x is the distance from the restrained end to the point under analysis (in)

The approach used with Equations 7.3.3-3 and 7.3.3-4 assumes all of the applied torque is reacted by the bending resistance of the flanges. A more rigorous approach found in References 7-9 and 7-10 recognizes that with the end restrained, there is a non uniform warpage of the cross-section. This is shown in Figure 7.3.3-3.

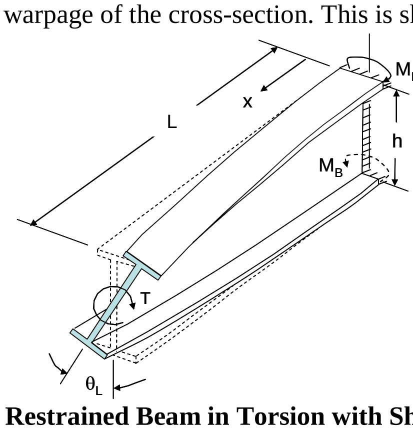


Figure 7.3.3-3 Restrained Beam in Torsion with Shared Torsion Reaction

At the free end, the section can warp freely as in Section 7.3.3.1, however at the constrained end, the cross-section is not free to warp. A state of non-uniform torsion results and the rate of change of the twist angle, θ, will vary along the length of the bar. Additionally, the applied torque is reacted partly by torsional shear stress and partly by lateral shearing forces resulting in in-plane bending of the flanges. This shared reaction can be described mathematically as

$$T = T_S + T_B \qquad \textbf{Equation 7.3.3-5}$$

where
T_S is the portion of the applied torque reacted by torsional shear (in-lbs)
T_B is the portion of the applied torque reacted by cap in-plane bending (in-lbs)

The portion of the torque resulting in lateral shearing forces and reacted as cap in-plane bending is given by

$$T_B = T\,\frac{\cosh\left(\frac{L-x}{C}\right)}{\cosh\left(\frac{L}{C}\right)} \qquad \textbf{Equation 7.3.3-6}$$

where
T is the applied torque (lbs)
x is the distance from the restrained end (in)
L is the distance between the shear load and the restraint (in)
C is the beam length parameter given by Equation 7.3.3-8 (in)

The portion reacted by torsional shear is the difference between the applied torque and the cap in-plane bending portion and is given as

$$T_S = T\left[1 - \frac{\cosh\left(\frac{L-x}{C}\right)}{\cosh\left(\frac{L}{C}\right)}\right] \qquad \textbf{Equation 7.3.3-7}$$

The bending moment in the caps resulting from the portion of the torsion described by Equation 7.3.3-6 is

$$M_B = \frac{CT}{h}\left[\frac{\sinh\left(\frac{L-x}{C}\right)}{\cosh\left(\frac{L}{C}\right)}\right] \qquad \textbf{Equation 7.3.3-8}$$

where
h is the distance between the cap mid-planes (in)

The beam length parameter is a ratio of the relative flange bending rigidity to the beam torsional rigidity and is given in Reference 7-2 by

$$C = \sqrt{\frac{EC_w}{KG}} \qquad \textbf{Equation 7.3.3-9}$$

where
E is the Young's modulus of the material (psi)
C_w is the warping constant from Section 4.2.7 (in^6)
K is the St. Venant torsion constant from Section 4.2.2 (in^4)
G is the shear modulus of the material (psi)

Note that the warping constant is approximately zero for sections where all flanges meet at a common point, such as tees or cruciform, thus C would also be 0 and the amount of torsion resisted by cap bending would be zero.

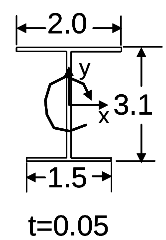
The angle of twist, in radians, of the beam is given by

$$\theta = \frac{T}{GK}\left[x + \frac{C\sinh\left(\frac{L-x}{C}\right)}{\cosh\left(\frac{L}{C}\right)} - C\tanh\left(\frac{L}{C}\right)\right]$$ **Equation 7.3.3-10**

where
x is the distance from the restrained end (in)

At the restrained end the angle of twist is 0 and at x=L the deformation is maximum.

7.3.3.2.1 Example Problem Restrained I-Beam in Torsion

For the I-beam shown in Example 7.3.3.1.2, which is 10 in long, assume it is restrained at one end, calculate the cap moment for an applied torsional moment of 100 in-lb using both the simplified approach of Equation 7.3.3-4 and the rigorous approach of Equation 7.3.3-7. Calculate the maximum angle of twist.	 From IDAT: I_{xx}= 0.515 in^4 I_{yy}=0.047 in^4 K=0.000267

Calculate the stiffness parameter, Equation 7.3.3-8:
From Example 7.3.3.1.2:
K=0.000267 G=4.0x10^6 psi E=10.4x10^6 psi

$$I_{yy-topflange} = \frac{tb^3}{12} = \frac{0.05x2^3}{12} = 0.033in^4 \qquad I_{yy-bottomflange} = \frac{tb^3}{12} = \frac{0.05x1.5^3}{12} = 0.014in^4$$

From Section 4.2.7, for an I-Beam: $C_w = \frac{h'^2 I_{yy}}{4} = \frac{3.05^2 x0.047}{4} = 0.109in^6$

$$C = \sqrt{\frac{EC_w}{KG}} = \sqrt{\frac{10.4x10^6(0.109)}{0.000267(4.0x10^6)}} = 32.58in$$

Calculate the Moment at the restraint, per the simple approach, Equation 7.3.3-4, x=0
V_T=T/h = 100/3.05=32.79 in-lb
M_B=V_T(L-x)=32.79(10)=327.9 in-lb

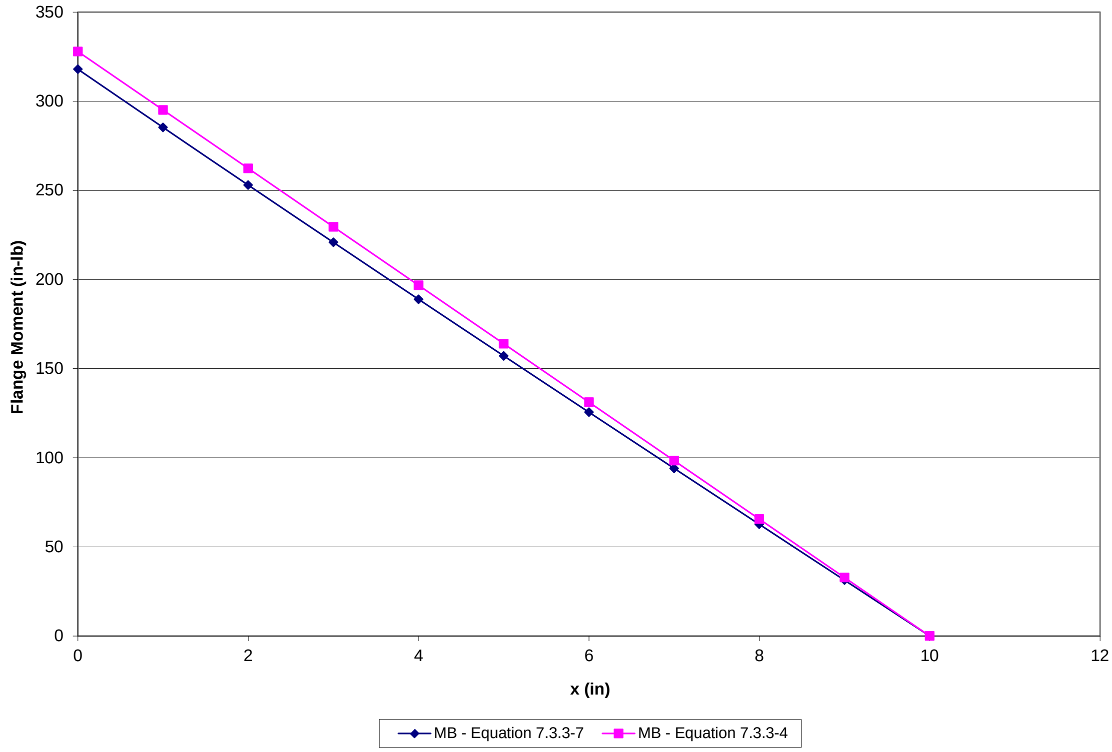
Calculate the Moment at the restraint, per the rigorous approach, Equation 7.3.3-7, x=0

$$M_B = \frac{CT}{h}\left[\frac{\sinh\left(\frac{L-x}{C}\right)}{\cosh\left(\frac{L}{C}\right)}\right] = \frac{32.58(100)}{3.05}\left[\frac{\sinh\left(\frac{10}{32.58}\right)}{\cosh\left(\frac{10}{32.58}\right)}\right] = 317.9in-lb$$

Calculate the maximum angle of twist using Equation 7.3.3-9, x=10

$$\theta=\frac{T}{GK}\left[x+\frac{C\sinh\left(\frac{L-x}{C}\right)}{\cosh\left(\frac{L}{C}\right)}-C\tanh\left(\frac{L}{C}\right)\right]=\frac{100}{4.0x10^{6}(0.000267)}\left[10+\frac{32.58\sinh\left(\frac{0}{24.84}\right)}{\cosh\left(\frac{10}{24.84}\right)}-32.58\tanh\left(\frac{10}{32.58}\right)\right]=0.028rad$$

θ=0.028rad(180degrees/π)=1.6 degrees



The chart shows a comparison between the flange bending moment calculated using the simple approach of Equation 7.3.3-4 where all of the torsion is carried in in-plane bending in the cap and the more rigorous approach where the torsion is split between in plane bending in the cap and torsional shear in the cross-section. This section has a very thin web and the difference in the two approaches is small, less than 5%.
Also note that with a restrained end, the deflections are small and the solution is valid, unlike in Example 7.3.3.1.2 where the deflections were large which invalidated the calculations.

7.3.4 The Development of Shear Flow in Multi-Cell Closed Sections Due to Applied Shear

This section presents a discussion of the shear flow developed in single and multi-cell closed sections due to shear loads. Shear loads, when not applied at the shear center of these cross-sections can result in a torsional moment which may add to or relieve any shear flow due to applied torsion. In addition there will be a shear flow due to applied shear. To present a complete picture, this material is presented as a part of this section on torsion.

7.3.4.1 Shear Flow in an Open Section Deep Beam

Another type of open section that is often encountered in aircraft analysis is determining the shear flow for a beam in bending, which has two caps not supported by a web in the plane of the caps. In this type of structure the caps are assumed to carry all axial loads due to applied bending moment and the web carries the entire shear load. When shear flow acts in a web located between two adjacent caps of a beam in bending, the resultant force system has a twisting moment and shear force unless the web lies in the plane of the two caps. The overall net shear flow acts parallel to a plane connecting the centroids of the caps, but at the shear center of the web which is a distance e from the plane. This is shown in Figure 7.3.4-1.

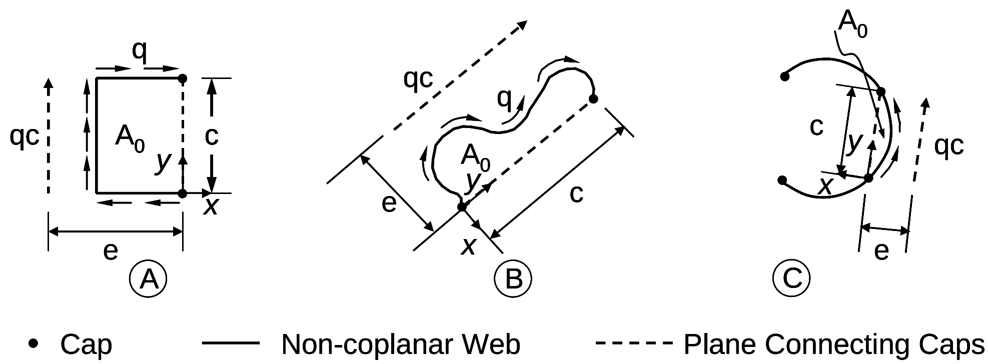


Figure 7.3.4-1 Shear Flow in an Open Section

The resultant of the shear load in the non-planar web acting at the shear center of the web is given by

$$R_{web} = qc \qquad \textbf{Equation 7.3.4-1}$$

where
q is the shear flow in the web (lbs/in)
c is the distance between the cap centroids(in)

The magnitude of the resultant is independent of the shape of the web and is only a function of the height between the caps and the shear flow present in the web. Its direction is parallel to the plane containing both caps; otherwise the system is not in static balance. To balance the twisting moment that results from the resultant shear load, an equal and opposite load must be applied at the same location.

The shear center of the web is located at

$$e = -\frac{2A_o}{c} \qquad \textbf{Equation 7.3.4-2}$$

where
A_O is the enclosed area between the non-planar web and a plane connecting the centroids of the caps (in^2)
c is the distance between the cap centroids. It is the height of the plane connecting the caps (in)

Note that the shear center location is purely a function of geometry and is not a function of the applied load. The negative sign indicates it is located on the negative *x* axis as shown in Figure 7.3.4-1. If the external shear load acts at the shear center, then there is no net twisting moment developed on the section because the external shear load times the shear center distance from the reference axis is balanced by the resultant shear flow developed in the non-planar web times the shear center distance from the reference axis. If, however, the external shear load is applied anywhere other than the shear center, a torsional moment is developed and must be determined by a freebody balance of the beam.

7.3.4.2 Shear Flow in a Two Cap, Single Cell Shell Section

For a two flange, single cell shell, shown in Figure 7.3.4-2, assume P is applied in the plane parallel to the plane joining the caps and torsion is reacted by the closed torque cell. All axial loads due to bending are carried in the cap elements. The dimension *a* is the distance from the reference axis to the applied load, the dimension *e* is the distance from the reference axis to the shear center of the nose web and the dimension *d* is the distance from the reference axis to the shear center of the closed shell section.

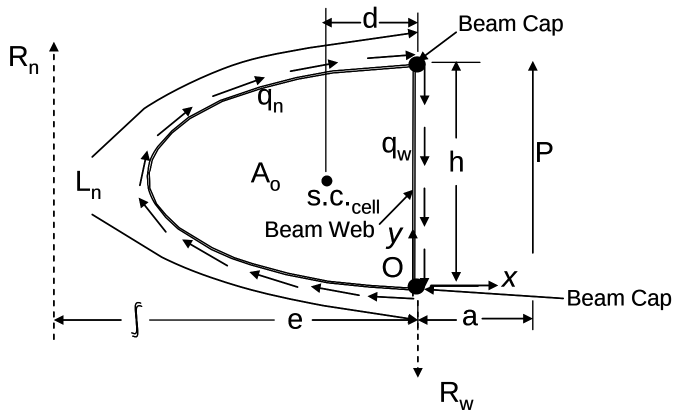


Figure 7.3.4-2 Closed Single Cell beam

The nose is a non-planar web discussed in Section 7.3.4-1. The resultant due to the shear flow q_n in the nose is R_n acting at the shear center of the nose. The resultant due to the shear flow q_w in the web is R_w acting in the plane of the web. They can be calculated from Equation 7.3.4-1 as shown below

$$R_n = q_n h$$ **Equation 7.3.4-3**

$$R_w = q_w h$$ **Equation 7.3.4-4**

where
q_w, q_n are the shear flows in the web and nose, respectively (lbs/in)
h is the height between the cap centroids (in)

Again note that the resultant is calculated in the same manner, whether the web is planar as is the case of the spar web or non-planar, as is the case of the nose of the section.

To determine the shear flows, q_n and q_w, equations for the summation of moments about point O and the summation of forces in the vertical direction are used.

$$\Sigma M_O = 0 \quad \text{+CCW}$$

$$Pa - 2A_O q_n = 0$$

$$q_n = \frac{Pa}{2A_O} \text{ (direction as shown, Figure 7.3.4-2)}$$ **Equation 7.3.4-5**

$$\Sigma F_v = 0$$

$$P + q_n h - q_w h = 0$$

Substituting the value for q_n into the above equation

$$q_w h = P + \frac{Pah}{2A_O}$$

$$q_w = \frac{P}{h}\left(1 + \frac{ah}{2A_O}\right) \text{(direction as shown, Figure 7.3.4-2)}$$ **Equation 7.3.4-6**

where
P is the applied load, parallel to one of the webs joining the caps of the closed section (lbs)
q_n, q_w are the shear flows in the nose and the spar web, respectively (lbs)
h is the distance between the cap centroids. It is the height of the plane connecting the caps. (in)
A_O is the enclosed area of the cell (in^2)
a is the parallel distance from the load application point to the web joining the caps(in)

In examining Equation 7.3.4-5 it is seen that if the load P is applied at the web, then $a=0$ and $q_n=0$ or the entire shear load is reacted by the web. If the load P is applied at the shear center of the nose web ($a=e$), using Equations 7.3.4-2 in Equation 7.3.4-6, $q_w=0$ and the entire shear load is reacted by the nose.

Once the shear flow has been determined, the shear stress can be calculated from Equation 7.3.1-4.

Equation 7.3.4-2 describes the location of the shear center of an open section deep beam. To derive the location of the shear center of the closed cell and the angle of twist of the cell per unit length Equations 7.3.1-2 and 7.3.1-6 are used.

In the case of the single cell closed section, there are two values for q, the shear flow in the web given by Equation 7.3.4-6 and the shear flow in the nose given by Equation 7.3.4-5. In addition, each of the segments of the section may have a different shear modulus, G_w and G_n, respectively. Substituting these into Equation 7.3.1-6 the following general equation for angle of twist per unit length results

$$\varphi = \left(\frac{1}{2A_O}\right)\left[\frac{q_n L_n}{G_n t_n} + \frac{q_w h}{G_w t_w}\right] = \left(\frac{1}{2A_O}\right)\left[\frac{PaL_n}{2A_O G_n t_n} + \frac{P}{G_w t_w}\left(1 + \frac{ah}{2A_O}\right)\right] \qquad \textbf{Equation 7.3.4-7}$$

where
G_w, G_n are the shear moduli of the web and nose, respectively (psi)
t_w, t_n are the thicknesses of the web and nose, respectively (in)
L_n is the mean perimeter of the nose (in)

Note that to calculate the angle of twist, the mean perimeter length of the nose is used in the second term of the equation in association with the shear flow in the nose because this value is part of the calculation of the torsional constant rather than a calculation of the total load in the nose.

If the shear load is applied at the shear center of the cell then the angle of twist of the cell is 0 and the location of the shear center of the cell may be determined. Thus, equating Equation 7.3.4-7 to 0 and solving for *a*

$$a = d = \frac{-2A_O G_n t_n}{L_n G_w t_w + h G_n t_n} \qquad \textbf{Equation 7.3.4-8}$$

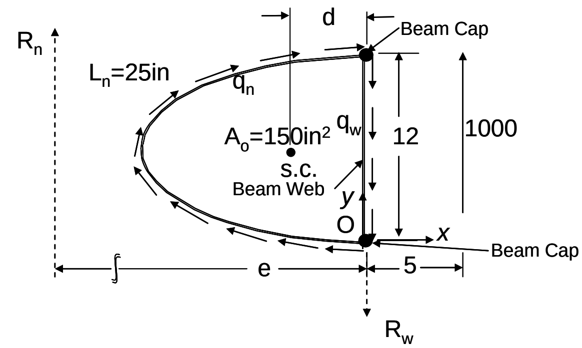
where a is the location of the applied load (in)
d is the location of the shear center of the cell (in)

7.3.4.2.1 Example Problem Two Cap, Single Cell Shell

Given the single cell shell depicted in the figure to the right.

- A_O=150 in^2
- G_n=G_w =4x10^6 psi
- t_n=0.04 in, t_w=0.10 in

Determine the shear center of the nose element and the shear center of the shell.
Determine the shear flow in the nose, q_n and the shear flow in the web, q_w.



from Equation 7.3.4-2, the shear center of the nose is

$$e = -\frac{2A_O}{c} = -\frac{2(150)}{12} = 25in.$$

from Equation 7.3.4-8 for G_n=G_w, the shear center of the cell is

$$d = \frac{-2A_O G_n t_n}{L_n G_w t_w + hG_n t_n} = -\frac{2(150)(0.04)}{[(25)(0.10)+(12)(0.04)]} = -4.02in$$

from Equation 7.3.4-5

$$q_n = \frac{Pa}{2A_O} = \frac{1000(5)}{2(150)} = 16.67 lb/in$$

from Equation 7.3.4-6

$$q_w = \frac{P}{h}\left(1+\frac{ah}{2A_O}\right) = \frac{1000}{12}\left[1+\frac{(5)(12)}{(2)(150)}\right] = 100 lb/in$$

The unit twist can be calculated from Equation 7.3.4-7

$$\varphi = \left(\frac{1}{2A_o}\right)\left[\frac{PaL_n}{2A_O G_n t_n} + \frac{P}{G_w t_w}\left(1+\frac{ah}{2A_o}\right)\right] = \frac{1}{2(150)}\left\{\frac{(1000)(5)(25)}{2(150)(4x10^6)(0.04)} + \frac{1000}{(4x10^6)(0.10)}\left[1+\frac{(5)(12)}{(2)(150)}\right]\right\}$$

$$\varphi = 1.868x10^{-5} radians = 0.00107°$$

7.3.4.3 Shear Flow in a Multiple Cap, Single Cell Shell Section

The same equations used in a single cell, two cap applications can be extended to shell sections with more than two caps. Figure 7.3.4-3 shows a single cell, three cap shell structure. Equation 7.3.4-2 can be used to determine the shear center of each of the nose segments using the area enclosed by the individual cells of the structure, A_{O-1}, A_{O-2}, and A_{O-3}, in the case depicted below. Note that the planes AB and BO do not contain webs. Both planes could contain webs but do not have to. If there are webs present, then additional equations must be developed, similar to the approach used in section 7.3.2.

The applied load, P is resolved into lines of action parallel to each of the three planes. To solve for the shear flows, the summation of forces and moments are developed and n equations result. These are solved simultaneously to determine the shear flows in each web. The number of equations, n, is the same as the number of cells. This will be illustrated in an Example, Section 7.3.4.3.1.

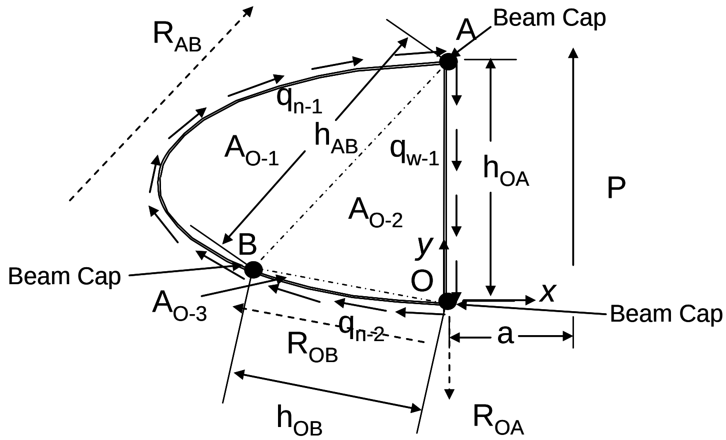


Figure 7.3.4-3 Three Cap Single Cell Shell Section

7.3.4.3.1 Example Problem – Three Cap, Two Cell Shell Structure

Given a leading edge section of a beam with three cap elements.

Directions of the shear flow are assumed.

Solve for the shear flow in the nose and web.

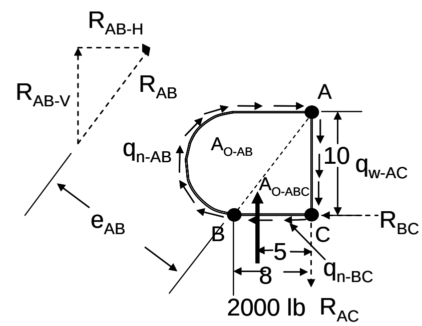


Figure A

Calculate the area of the two nose sections:

$$A_{O-ABC} = \frac{1}{2}bh = \frac{1}{2}(8)(10) = 40in^2$$

$$A_{O-AB} = bh + \frac{1}{2}\left[\frac{\pi}{4}\right]D^2 - A_{O-ABC} = 8(10) + \frac{1}{2}\left[\frac{\pi}{4}\right](10)^2 - 40 = 79.3in^2$$

$e_{BC} = 0$ The resultant for $q_{n\text{-}BC}$ lies in the plane BC because element BC is a planar element.

$e_{AC} = 0$ The resultant for $q_{n\text{-}AC}$ lies in the plane AC because element AC is a planar element.

$$e_{AB} = -\frac{2A_{O-AB}}{c} = -\frac{2(79.3)}{\sqrt{8^2+10^2}} = -12.38in$$

$\sum F_H = 0$

$8q_{n-AB} - 8q_{n-BC} = 0$

$q_{n\text{-}AB} = q_{n\text{-}BC}$

$\sum M_A = 0$ (+ CW). Recall Equation 7.3.1-2, $T=2qA_O$

$2000(5) + T_{AB} + T_{BC} = 0 = 10000 + q_{n-AB}(2)(79.3) + q_{n-BC}(2)(40) = 0$

But $q_{n\text{-}AB} = q_{n\text{-}BC}$,

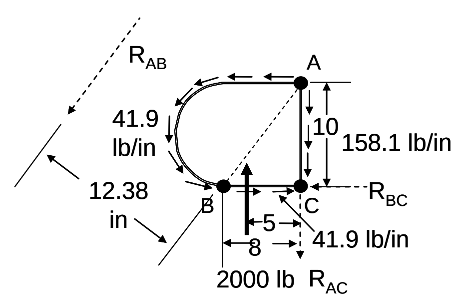
$10000 + (158.6 + 80)q_{n-AB} = 0$

$q_{n\text{-}AB} = q_{n\text{-}BC}$ =-41.9 lbs/in (opposite shown, Figure A)

$\sum F_V = 0$

$2000 - 10q_{w-AC} + 10q_{n-AB} = 0 = 2000 - 10q_{w-AC} + 10(-41.9)$

$q_{w\text{-}AC}$ =158.1 lbs/in (as shown, Figure A)



Corrected sign of loads shown
Figure B

7.3.5 Torsional Buckling of Thin-Walled Circular Sections

For many thin walled sections, torsional buckling is examined using the shear flows developed in the web or panel described in Sections 7.3.1, 7.3.2 and 7.3.4 along with the shear buckling analysis methodology described in Section 10.3. If both external shear and torsion are present, then the internal shear flow used for analysis would be the combined shear flow for the web or panel.

Unstiffened circular thin-wall tubes however have a complex failure mode that must be examined using a different approach. Section 7.3.5 will discuss initial torsional buckling of thin walled circular sections, both elastic and plastic. Section 7.3.6 will discuss the rupture of thin walled circular sections in torsion.

The theory for initial torsional buckling of thin-walled cylinders is documented in detail in Reference 7-5 and will not be presented here. However, the equations governing the behavior of this type of cross-section will be presented. The geometric parameters governing the initial torsional buckling of thin-walled cylinders are the ratio of the length divided by mean radius and the ratio of mean radius divided by thickness. Reference 7-6, provides correlation between the theoretical solution and testing which results in an additional empirical correlation factor, γ, for long and intermediate length cylinders. This correction is not required for short cylinders.

This section will present the initial torsional buckling formulation in a similar format as the approach used in Sections 10.3.1.1 (initial elastic buckling of plates) and 10.3.1.2 (initial plastic buckling of plates) so that the allowable initial buckling curves from Section 10.3.1.6 can be used. Note that Equation 10.3.1-6 for elastic initial buckling is modified by a plasticity correction factor, η, to obtain Equation 10.3.1-10 for inelastic initial buckling. This is a generalized formulation, since η=1 results in the elastic solution. Similarly the equations for initial buckling of cylinders in torsion are presented in the generalized format which includes plasticity correction factor, η. The plasticity correction factor is given by Equation 10.3.1-9.

After determining the critical stress per the approach below, the critical allowable shear flow for the thin-walled, un-stiffened cylinder can be obtained from

$$q_{cr} = F_{s\text{-}cr}(t) \qquad \textbf{Equation 7.3.5-1}$$

where
$F_{s\text{-}cr}$ is the critical initial buckling shear stress (psi)
t is the thickness of the cylinder (in)

The margin of safety can then be determined using the calculated applied shear flow of Equation 7.3.1-3, the critical allowable shear flow of Equation 7.3.5-1 and the equations of Section 2.5. Note that in the combined buckling interaction equations of Section 2.5.2 involving torsion, initial buckling results in failure. This makes torsional combined buckling a strength margin not a ratio-to-requirement calculation. If the cylinder has stiffening members along its length, the method discussed in 7.3.1 should be used for analysis.

The behavior of thin-walled circular cylinders depends on not only their radius to thickness ratio but their length to radius ratio. Based on empirical data, they are divided into 3 categories of response to load: long, intermediate and short. Equations for the critical buckling stress for each category are presented in this section; however, the initial buckling stress solution is iterative since it involves a plasticity correction factor which is a function of the stress. Thus, the most efficient manner to present the solution is by a series of material dependent curves. It has been found that the curves presented in Section 10.3.1.6 for initial buckling can be used if an appropriate effective (b/t) parameter can be specified.

The approach used below is to manipulate the critical stress equations into the general form

$$F_{cr} = \frac{\eta E_c}{\left(\frac{b}{t}\right)_e^2}$$ Reference Equation 10.3.1-10

where
η is the plasticity correction factor given by Equation 10.3.1-8
E_c is the Modulus of Elasticity in compression (psi)
$(b/t)_e$ is a geometric parameter

For long cylinders, defined by Equation 7.3.5-3, the critical shear stress[1] is given by

$$F_{st-cr} = \frac{0.272\gamma E_c \eta}{\left(1-\nu_e^2\right)^{3/4}}\left(\frac{t}{r}\right)^{3/2}$$ **Equation 7.3.5-2**

valid for long cylinders[2] in which $\frac{l}{r} > \sqrt{\left[133.04\left(\frac{r}{t}\right)\left(1-\nu_e^2\right)^{1/2}\right]}$ **Equation 7.3.5-3**

where
γ is a test correlation factor, given by Equation 7.3.5-4
E_c is the elastic modulus of the material in compression(psi)
η is the plasticity correction factor from Equation 10.3.1-8. For elastic buckling, η=1.0
t is the thickness of the cylinder (in)
r is the mean radius of the cylinder (in)
ν_e is the material's elastic Poisson's ratio

The correlation factor shown in Equation 7.3.5-2, given in Reference 7-6, to adjust the theoretical value based on test data is

$$\gamma^{3/4} = 0.67$$

$$\gamma = 0.67^{4/3} = 0.5863$$ **Equation 7.3.5-4**

If Equation 7.3.5-1 is algebraically manipulated to assume the same form as Equation 10.3.1-10, the curves for initial shear buckling described in Section 10.3.1.6 can be used to determine the initial buckling stress of the thin-walled cylinder. Rewriting Equation 7.3.5-2,

$$F_{st-cr} = \frac{\eta E_c}{\left(\frac{1}{0.272}\right)\left(\frac{1}{\gamma}\right)\left(1-\nu_e^2\right)^{3/4}\left(\frac{r}{t}\right)^{3/2}} = \frac{\eta E_c}{\left(\frac{b}{t}\right)_{eff}^2}$$

Thus

$$\left(\frac{b}{t}\right)_{eff} = \sqrt{\left[3.676\left(\frac{1}{\gamma}\right)\left(1-\nu_e^2\right)^{3/4}\left(\frac{r}{t}\right)^{3/2}\right]}$$ **Equation 7.3.5-5**

Note that the parameter $(b/t)_{eff}$ does not have a direct physical significance as a width divided by thickness, but does represent the geometry and boundary restraints of the cylinder.

[1] Reference 7-5 and 7-6
[2] Reference 7-6

For short cylinders, the critical torsional shear stress[1] is given by

$$F_{st-cr} = \frac{k_{st-short}\pi^2 E_c \eta}{12\left(1-\nu_e^2\right)}\left(\frac{t}{L}\right)^2 \qquad \textbf{Equation 7.3.5-6}$$

where
$k_{st-short}$ is 5.35 for simply supported short cylinders
$k_{st-short}$ is 8.98 for clamped cylinders

Manipulating Equation 7.3.5-6 into the same form as Equation 10.3.1-10

$$F_{st-cr} = \frac{\eta E_c}{\frac{12\left(1-\nu_e^2\right)}{\pi^2 k_{st-short}}\left(\frac{L}{r}\right)^2\left(\frac{r}{t}\right)^2} = \frac{\eta E_c}{\left(\frac{b}{t}\right)_{eff}^2}$$

Thus,

$$\left(\frac{b}{t}\right)_{eff} = \sqrt{\left[\frac{12\left(1-\nu_e^2\right)}{\pi^2 k_{st-short}}\left(\frac{L}{r}\right)^2\left(\frac{r}{t}\right)^2\right]} \qquad \textbf{Equation 7.3.5-7}$$

Note that no test correlation factor is recommended for the short length cylinder.

For intermediate length cylinders, the elastic shear buckling stress[3], with correlation factor , is given by

$$F_{st-cr} = \frac{\gamma k_{st-med}\pi^2 E_c \eta}{12\left(1-\nu_e^2\right)^{5/8}}\left(\frac{t}{r}\right)^{5/4}\left(\frac{r}{L}\right)^{1/2} \qquad \textbf{Equation 7.3.5-8}$$

where
k_{st-med} for simply supported intermediate cylinders is 0.93
k_{st-med} for clamped intermediate cylinders is 0.85
γ is a test correlation factor, given by Equation 7.3.5-4

Manipulating Equation 7.3.5-8 into the same form as Equation 10.3.1-10, to obtain an effective (b/t)

$$F_{st-cr} = \frac{\eta E_c}{\frac{12}{\gamma k_{st-med}\pi^2}\left(1-\nu_e^2\right)^{5/8}\left(\frac{r}{t}\right)^{5/4}\left(\frac{L}{r}\right)^{1/2}} = \frac{\eta E_c}{\left(\frac{b}{t}\right)_{eff}^2}$$

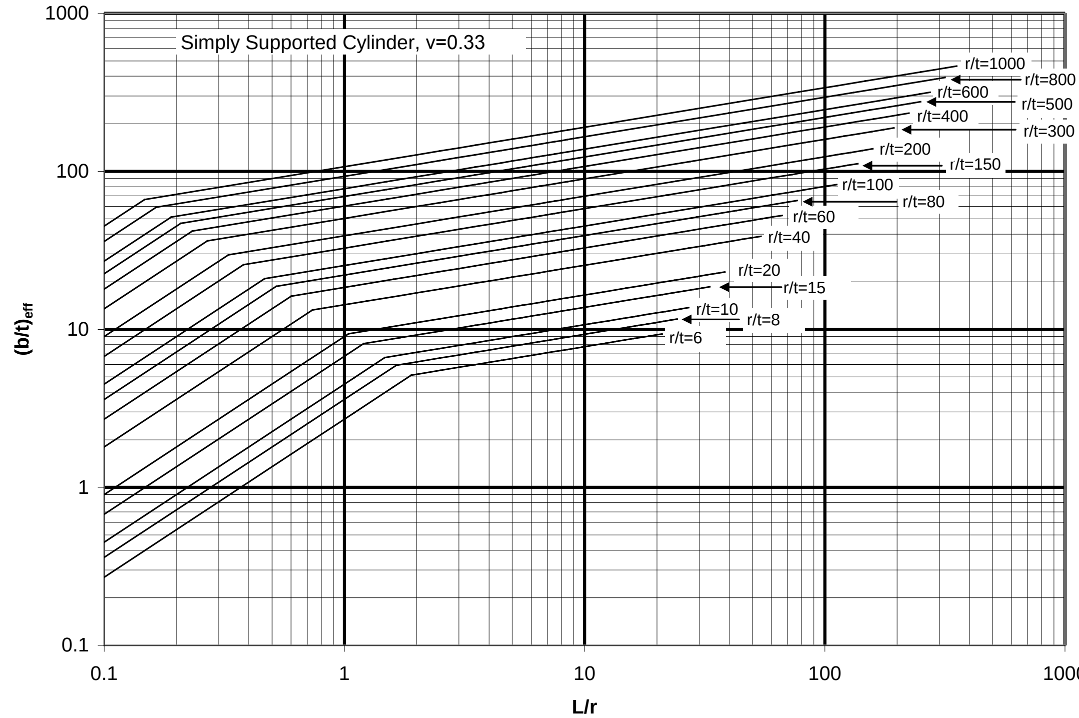
Thus

$$\left(\frac{b}{t}\right)_{eff} = \sqrt{\left[\frac{12}{\gamma k_{st-med}\pi^2}\left(1-\nu_e^2\right)^{5/8}\left(\frac{r}{t}\right)^{5/4}\left(\frac{L}{r}\right)^{1/2}\right]} \qquad \textbf{Equation 7.3.5-9}$$

An examination of the equations reveals that the effect of different Poisson's ratios is small. Figures 7.3.5-1 and 7.3.5-2 provide plots of Equations 7.3.5-7 and 7.3.5-9 for short and intermediate length cylinders with simple and fixed ends, respectively using a Poisson's ratio of 0.33. The location of where the transition from the short to intermediate length formulation is done by iteration to determine the L/r where the two equations provide the same $(b/t)_{eff}$ value. Figure 7.3.5-3 provides a plot of Equation 7.3.5-5 for long cylinders.

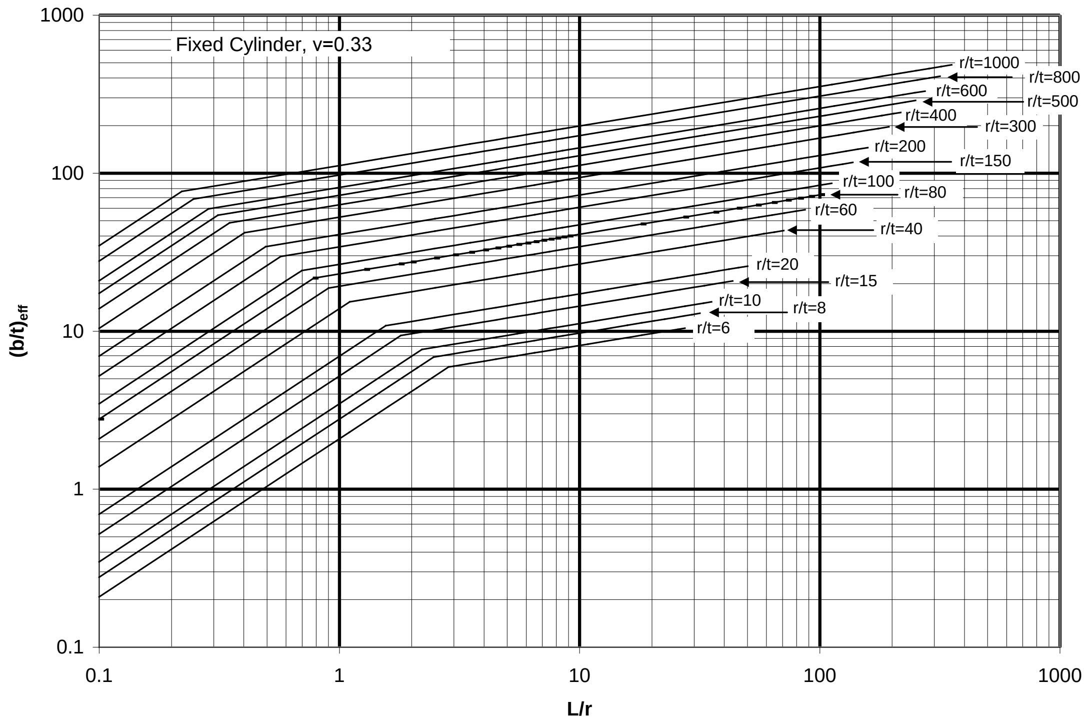
[3] References 7-5 and 7-6.

In this approach, once the determination of effective (b/t) from Figures 7.3.5-1 through 7.3.5-3 is complete, no further consideration of short, intermediate or long categories needs be made, since that is encompassed within the definition of $(b/t)_e$ and the curves of Section 10.3.1.6 for initial shear buckling, $F_{s\text{-}cr}$, are used to determine $F_{st\text{-}cr}$.



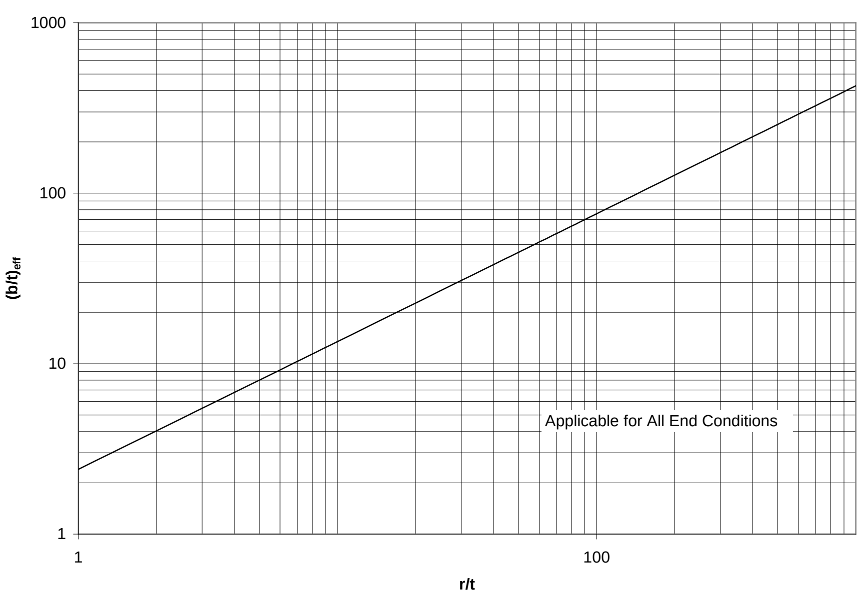
Do not extrapolate. For L/r greater than shown, use Figure 7.3.5-3

Figure 7.3.5-1 Determination of $(b/t)_{eff}$ for Short and Intermediate Length Cylinders with Simple Ends



Do not extrapolate. For L/r greater than shown, use Figure 7.3.5-3

Figure 7.3.5-2 Determination of $(b/t)_{eff}$ for Short and Intermediate Length Cylinders with Fixed Ends



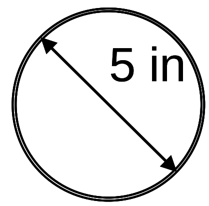
Applicable when $\frac{l}{r} > \sqrt{\left[133.04\left(\frac{r}{t}\right)\left(1-\nu_e^2\right)^{1/2}\right]}$

Figure 7.3.5-3 Determination of $(b/t)_{eff}$ for Long Length Cylinders

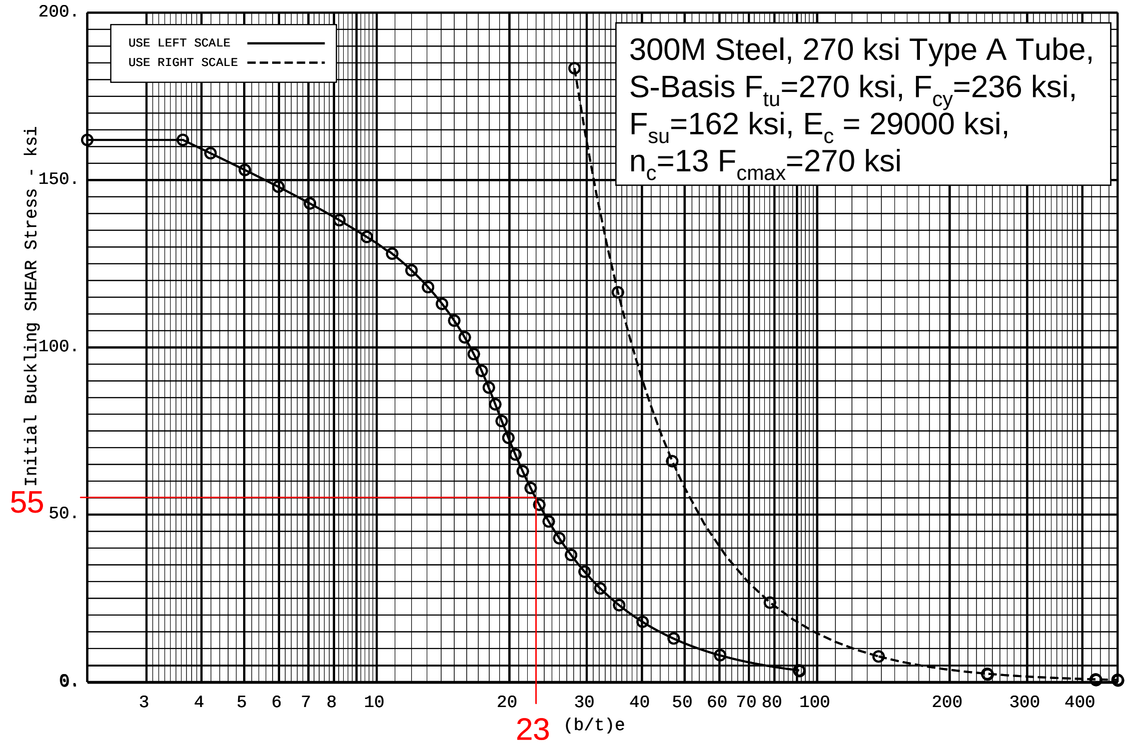
7.3.5.1 Example Problem – Torsional Buckling

<table>
<tr><td>Given the torque tube shown in the figure to the right.
• t=0.0625 in
• D(outside) = 5 in
• L=18 in
• Material: 300M 270 ksi Steel Tube
• Tube is simply-supported and un-stiffened

The applied torsion is 160,000 in-lb

Determine if the tube buckles</td><td></td></tr>
<tr><td>$r_{mean} = \frac{D-t}{2} = \frac{5-0.0625}{2} = 2.469in$
The enclosed area of the circular cross-section is
$A_O = \pi r_{mean}{}^2 = \pi(2.469)^2 = 19.15in^2$</td><td>The shear flow, using Equation 7.3.1-3, is
$q = \frac{T}{2A_O} = \frac{160000}{2(19.15)} = 4178lb/in$</td></tr>
<tr><td>Calculate/Determine effective (b/t)
$\frac{L}{r} = \frac{18}{2.5} = 7.2 \qquad \frac{r}{t} = \frac{2.5}{0.0625} = 40$

From Figure 7.3.5-1, $(b/t)_e = 23$</td><td>Determine Initial Buckling Allowable Stress:
From the figure below, $F_{s\text{-}cr}$= 55000 psi=$F_{st\text{-}cr}$
From Equation 7.3.5-1, $q_{cr}=F_{st\text{-}cr}(t)$ =
55000(0.0625)
q_{cr}=3437.5 lbs/in
q = 4178 lbs/in
$q > q_{cr}$ thus, the Cylinder Buckles</td></tr>
<tr><td colspan="2">Since initial buckling of the cylinder does not constitute failure, a ratio to requirement (RTR, reference Section 2.5.5) rather than a margin of safety might be calculated.
Example 7.3.6.4 will examine if the cylinder fails.</td></tr>
</table>



7.3.6 Torsional Rupture of Thin-Walled Circular Sections

Two independent analytical approaches were initially derived[4] to describe the rupture of thin-walled circular cylinders; however, neither predicts the failure for all L/D, D/t and material combinations but by combining the two approaches with an empirical constant, a reasonably conservative method results. The first approach is to calculate a torsional modulus of rupture which is analogous to the bending modulus of rupture. The second approach obtains a torsional modulus of rupture through an iterative solution of an energy based equation for describing the plastic stress-strain relationship. Both will be described below.

Thus, the allowable failure stress for any D/t is given as

$$F_{ST} = Minimum\{\ (Tr/I_p)_T,\ F_{LA},\ k_{test}[(Tr/I_p)_T+F_{LA}]/2\} \qquad \textbf{Equation 7.3.6-1}$$

where
$(Tr/I_p)_T$ is the torsional modulus of rupture, Equation 7.3.6-3
F_{LA} is the energy method prediction for failure stress, Equation 7.3.6-6
k_{test} is the test correlation factor where k_{test}=0.94

The allowable shear flow at rupture can be calculated as

$$q_{rupture} = F_{ST}(t) \qquad \textbf{Equation 7.3.6-2}$$

where
F_{ST} is the allowable torsional rupture stress from Equation 7.3.6-1 (psi)
t is the thickness of the cylinder (in)

[4] Reference 7-7 and Reference 7-8

7.3.6.1 Torsional Modulus of Rupture of Thin-Walled Circular Sections

The torsional modulus of rupture for a thin walled circular cylinder is derived in Reference 7-7 and is formulated very similarly to the bending modulus of rupture described in Section 6. The torsional modulus of rupture, $(Tr/I_p)_T$, is given by

$$\left(\frac{Tr}{I_p}\right)_T = F_{su} + \tau_o(k_T - 1) \qquad \textbf{Equation 7.3.6-3}$$

where
F_{su} is the ultimate material shear stress (psi)
τ_o is the trapezoidal intercept shear stress given by Equation 7.3.6-4 (psi)
k_T is the torsional geometric section factor given by Equation 7.3.6-5

The trapezoidal intercept stress for torsion is given as

$$\tau_o = 2\tau\left[\frac{3\left(\frac{1}{3} + \frac{n_c+1}{n_c+2}\left\{\frac{3}{7}\left(\frac{\tau\sqrt{3}}{F_{0.7}}\right)^{n_c-1}\right\} + \frac{n_c}{2n_c+1}\left\{\frac{3}{7}\left(\frac{\tau\sqrt{3}}{F_{0.7}}\right)^{n_c-1}\right\}^2\right)}{\left(1+\frac{3}{7}\left(\frac{\tau\sqrt{3}}{F_{0.7}}\right)^{n_c-1}\right)^2} - 1\right] \qquad \textbf{Equation 7.3.6-4}$$

where
n_c is the Ramberg-Osgood number in compression
$F_{0.7}$ is the stress at 0.7E per discussion in Section 3.3.1.1
τ is the shear stress for which the trapezoidal intercept stress is to be calculated. The maximum value is F_{su}. (psi)

The section factor in torsion is given by

$$k_T = \frac{4}{3}\left[\frac{1-\left(1-\frac{2}{D/t}\right)^3}{1-\left(1-\frac{2}{D/t}\right)^4}\right] \qquad \textbf{Equation 7.3.6-5}$$

where D is the outer diameter of the cylinder (in)
t is the thickness of the cylinder wall (in)

7.3.6.2 Energy Formulation for Torsional Rupture of Thin Walled Circular Sections

Reference 7-8 derives an alternative method for determining the rupture strength of thin walled circular sections using strain energy methods. This results in the following equation, which must be solved by iteration

$$3.5910H\left[\frac{L}{t}\right]^2\left[\frac{3F_{s-cr}}{E_{\sec}}\right] = 3.2704H\left[37.3063J^4 + 7.0422\left(1+\frac{1}{\xi_{cr}}\right)J^2 + 2\right] + 0.08213H^8Q \qquad \textbf{Equation 7.3.6-6}$$

where

$$Q = \frac{1}{0.0276H^4 + 0.1660(3\xi_{cr}-1)H^2 + 1} + \frac{1}{0.9680H^4 + 0.9802(3\xi_{cr}-1)H^2 + 1}$$

$$\xi_{cr} = \frac{1 + {}^{3}\!/_{7}\, n_c \left(\dfrac{\sqrt{3} f_{s-avg}}{F_{0.7}} \right)^{(n_c - 1)}}{1 + {}^{3}\!/_{7} \left(\dfrac{\sqrt{3} \tau_{s-avg}}{F_{0.7}} \right)^{(n_c - 1)}} = \frac{E_{sec}}{E_{tan}}$$

$$H = \left(\frac{L^2}{rt} \right)^{1/4}$$

$$J = \frac{1}{H}$$

L is the length of the cylinder (in)
t is the thickness of the cylinder (in)
F_{s-cr} is the critical initial buckling shear stress for the cylinder (psi)
f_{s-avg} is the average applied shear stress in the cylinder (psi)
n_c is the Ramberg-Osgood shape parameter in compression
$F_{0.7}$ is the stress defined by the secant line of slope 0.7E (psi) (Reference Equation 3.3.1-5)

To solve the above equation, the flow chart depicted in Figure 7.3.6-1 may be used.

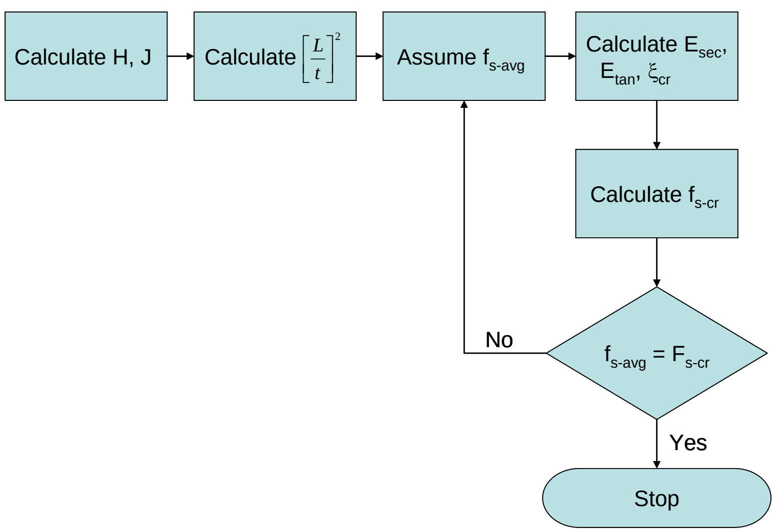


Figure 7.3.6-1 Flow Chart for Solution of Equation 7.3.5-13

In general, the solution provided by Equation 7.3.6-6, is critical for D/t from approximately 20 to 100 for all L/D ratios, while the other prediction methods become more critical outside of this range.

7.3.6.3 Circular Cylinder Torsion Rupture Stress Allowable Curves

IDAT/SM102 provides a means to generate the allowable curves quickly using Equation 7.3.6-1. A sample of curves from SM102 for typical materials used in aircraft applications is provided in Section 7.3.6.4.

No.	Material Alloy/Form	METDB No.	Thickness/ Grain	Basis	F_{tu} (ksi)	F_{cy} (ksi)	F_{su} (ksi)	E_c (ksi)	n_c	F_{cmax} (ksi)	Figure
1	2024-T42 CLAD Sheet	8	0.010-0.062 L	B	59	35	35	9700	13	37.9	7.3.6-2
2	6061-T6 Tube	378	0.025-0.500 L	S	42	34	25.2	10100	20	34.9	7.3.6-3
3	7075-T6 BARE Sheet	40	0.040-0.125 L	B	80	71	48	10500	13	80.0	7.3.6-4
4	7075-T6 CLAD Sheet	44	0.040-0.062 L	B	75	65	45	9700	13	74.2	7.3.6-5
5	300M 280 ksi Steel Tube	379	0.42C L	S	280	247	168	29000	13	280	7.3.6-6
6	260 ksi Low Alloy Steel Tube	383	None	S	260	235	156	29000	13	260	7.3.6-7
7	301 Stainless Sheet 1/2H	320	>0.016 L	B	151	69	82	26000	9	78.5	7.3.6-8
8	Titanium 3AL-2.5V Annealed Sheet	281	>0.016 L	S	90	75	54	15000	20	78.5	7.3.6-9

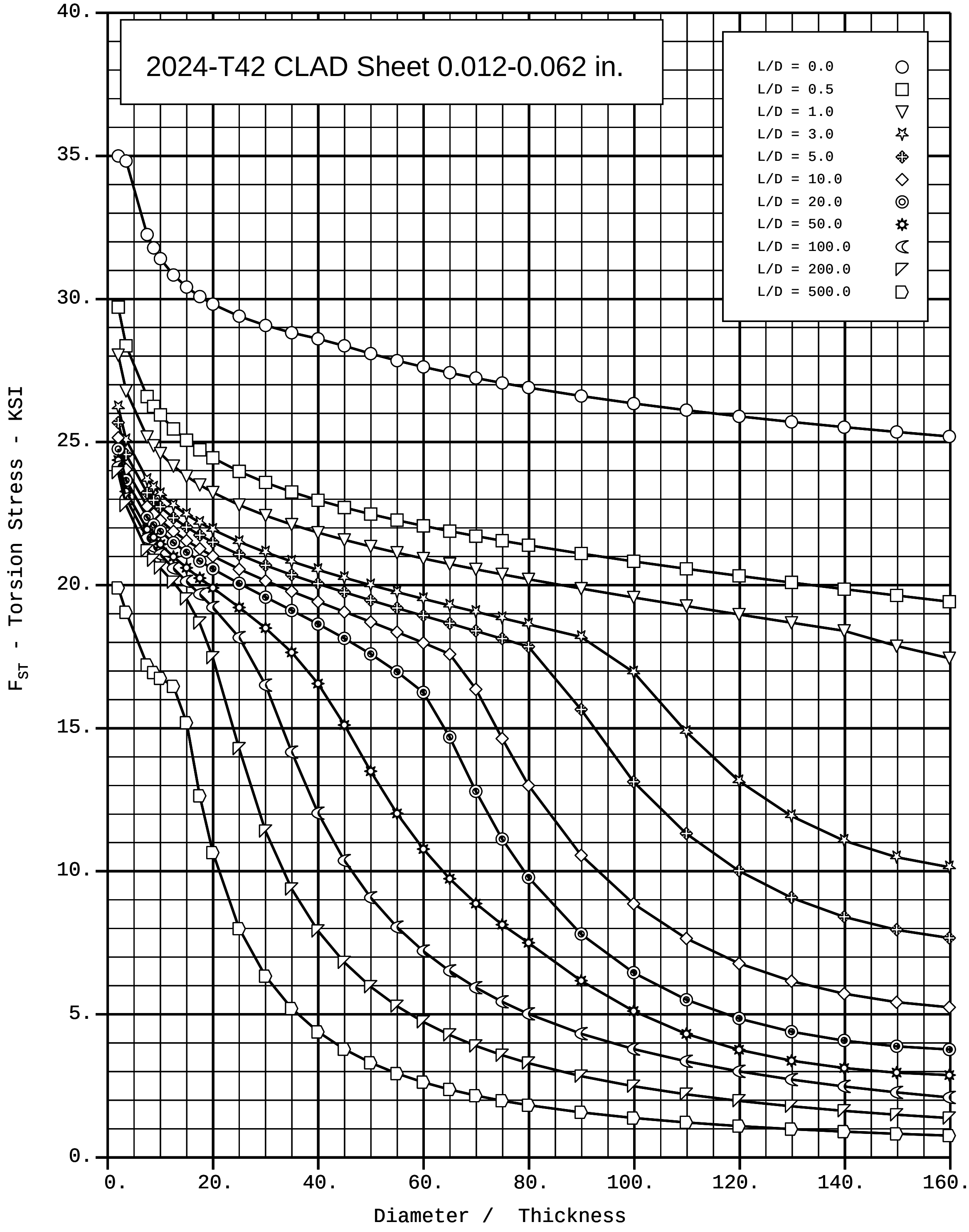


Figure 7.3.6-2 Torsion Rupture Allowable for Circular Cylinder: 2024T-42 CLAD Sheet

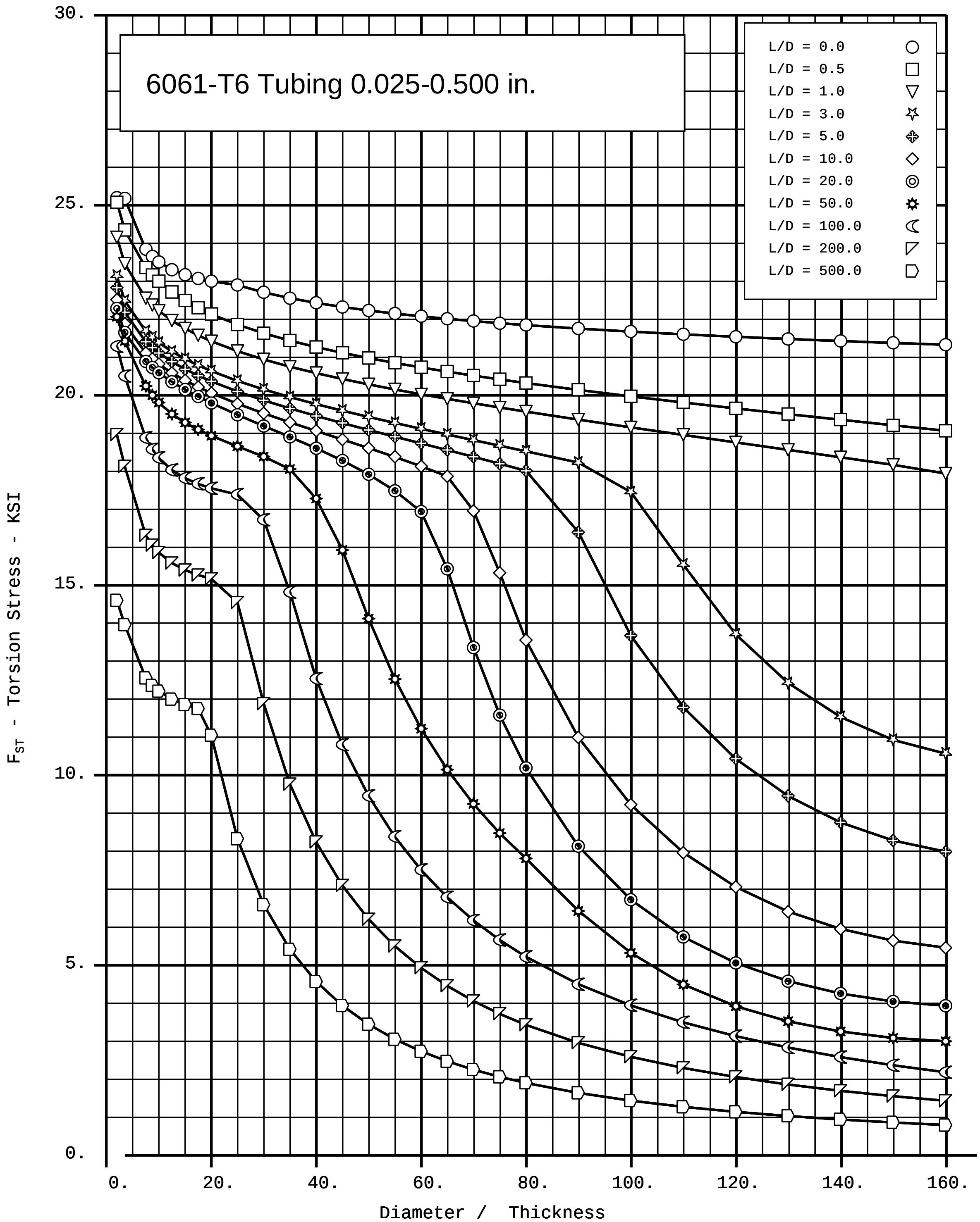


Figure 7.3.6-3 Torsional Rupture Allowable for Circular Cylinder: 6061-T6 Tube

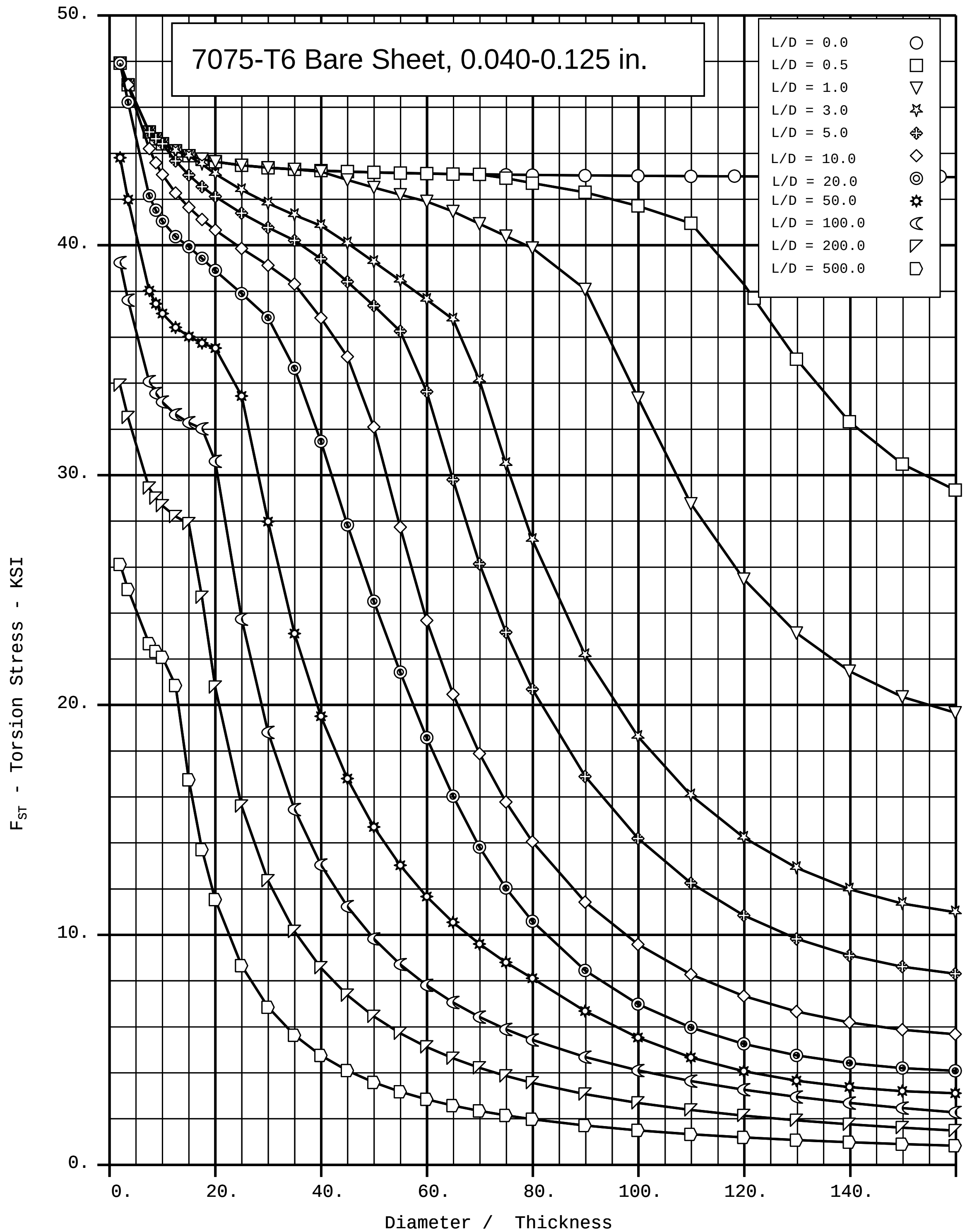


Figure 7.3.6-4 Torsional Rupture Allowable for Circular Cylinder: 7075-T6 Bare Sheet

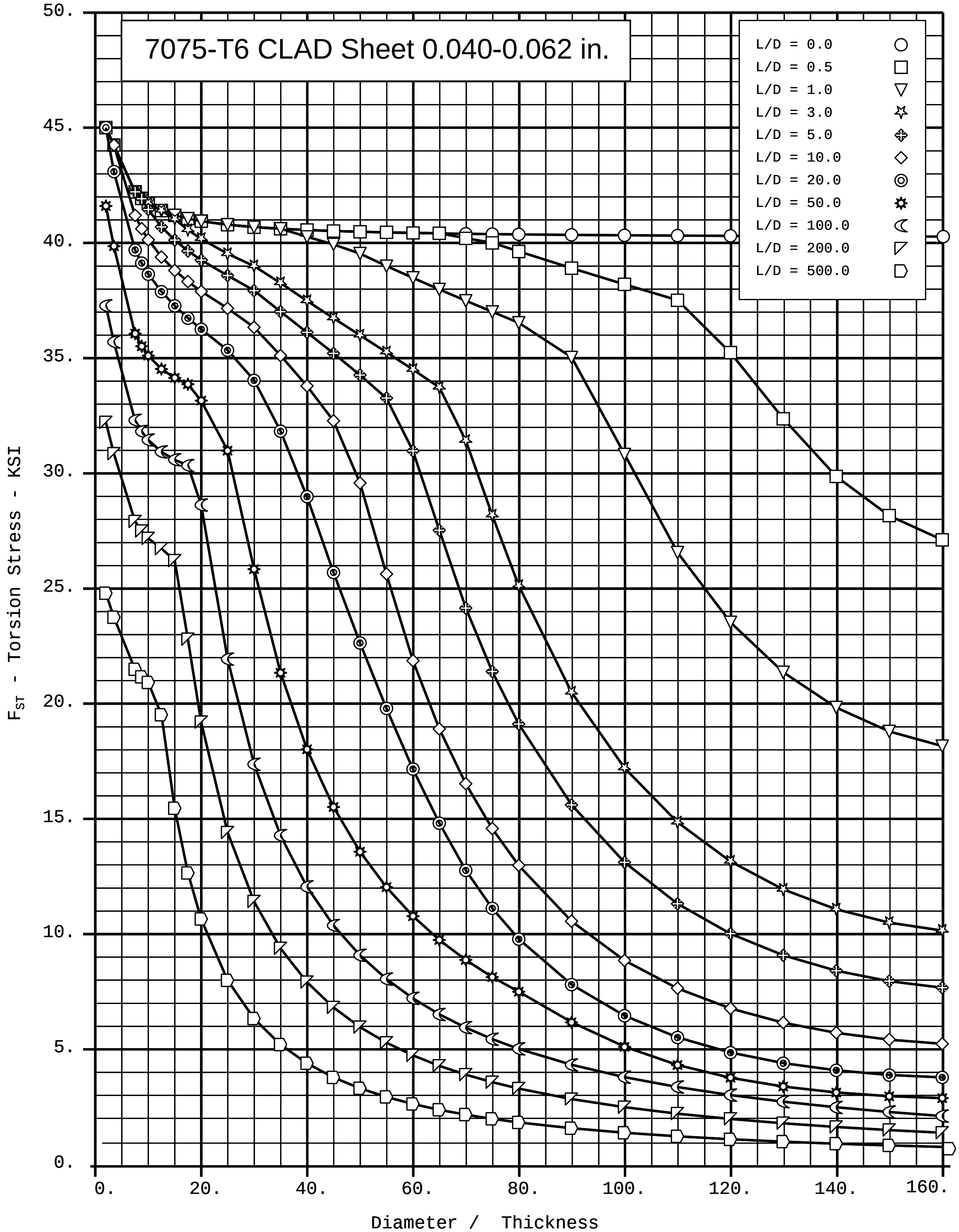


Figure 7.3.6-5 Torsional Rupture Allowable for Circular Cylinder: 7075-T6 CLAD Sheet

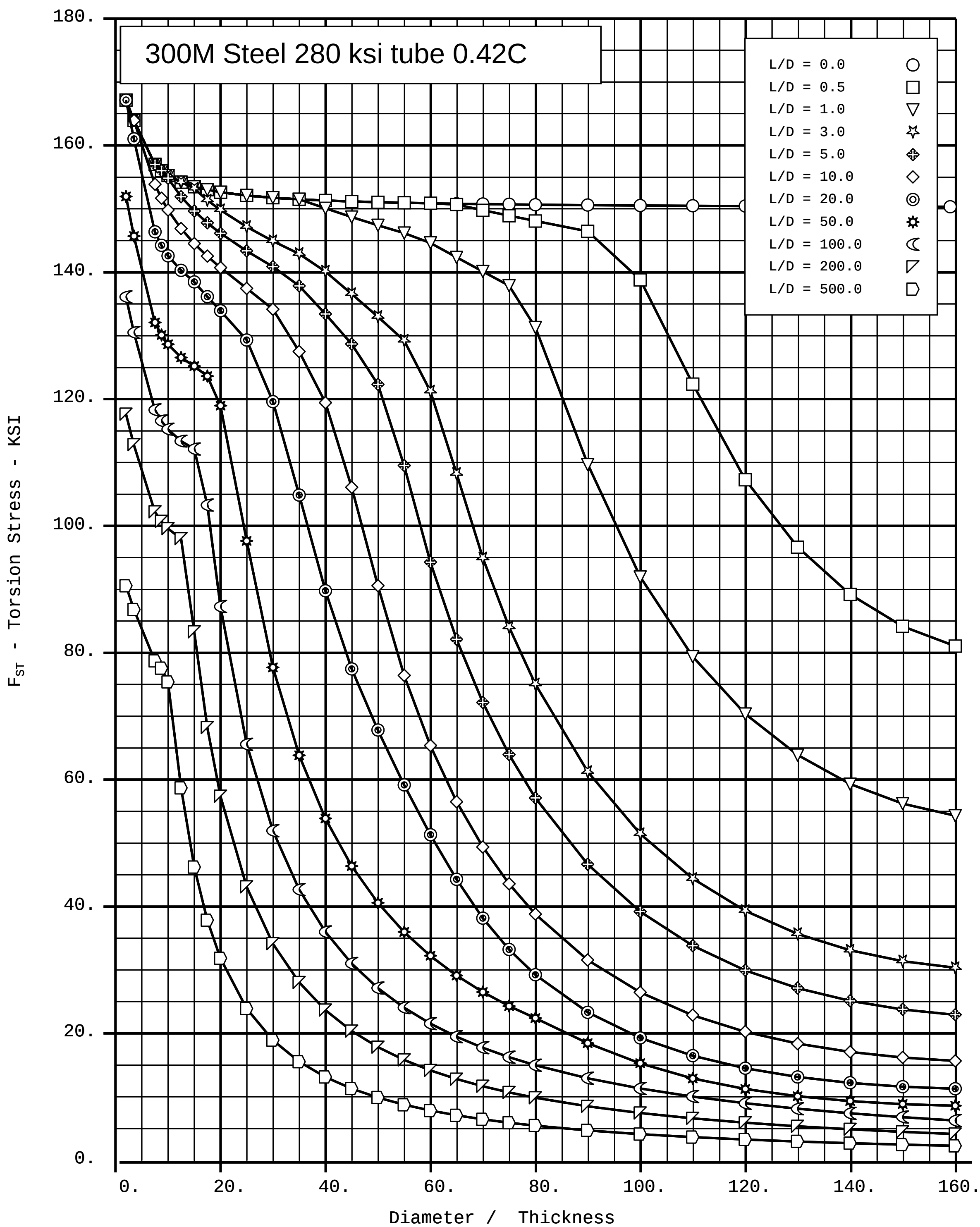


Figure 7.3.6-6 Torsional Rupture Allowable for Circular Cylinder: 300M 280 ksi Steel Tube

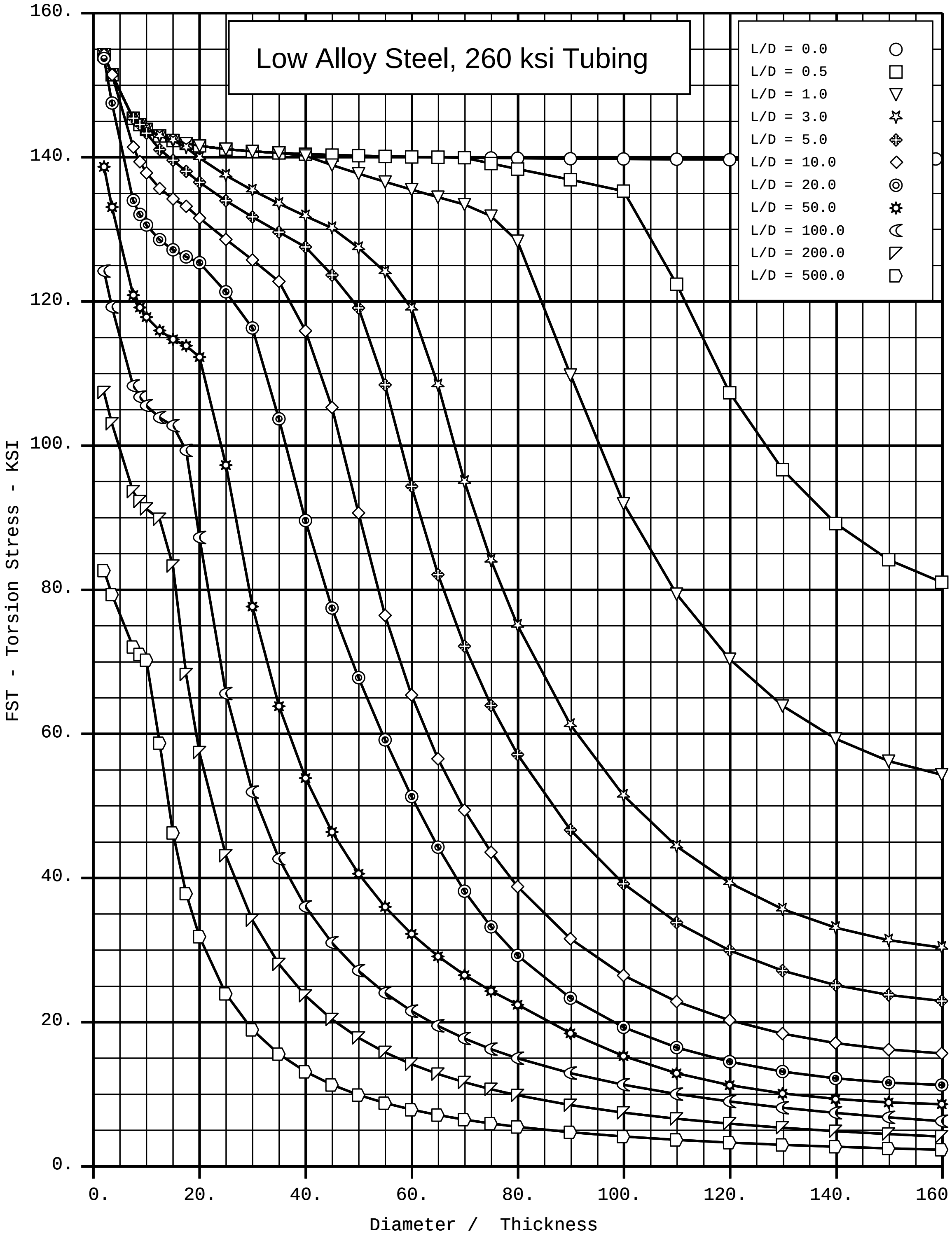


Figure 7.3.6-7 Torsional Rupture Allowable for Circular Cylinder: 260 ksi Low Alloy Steel Tube

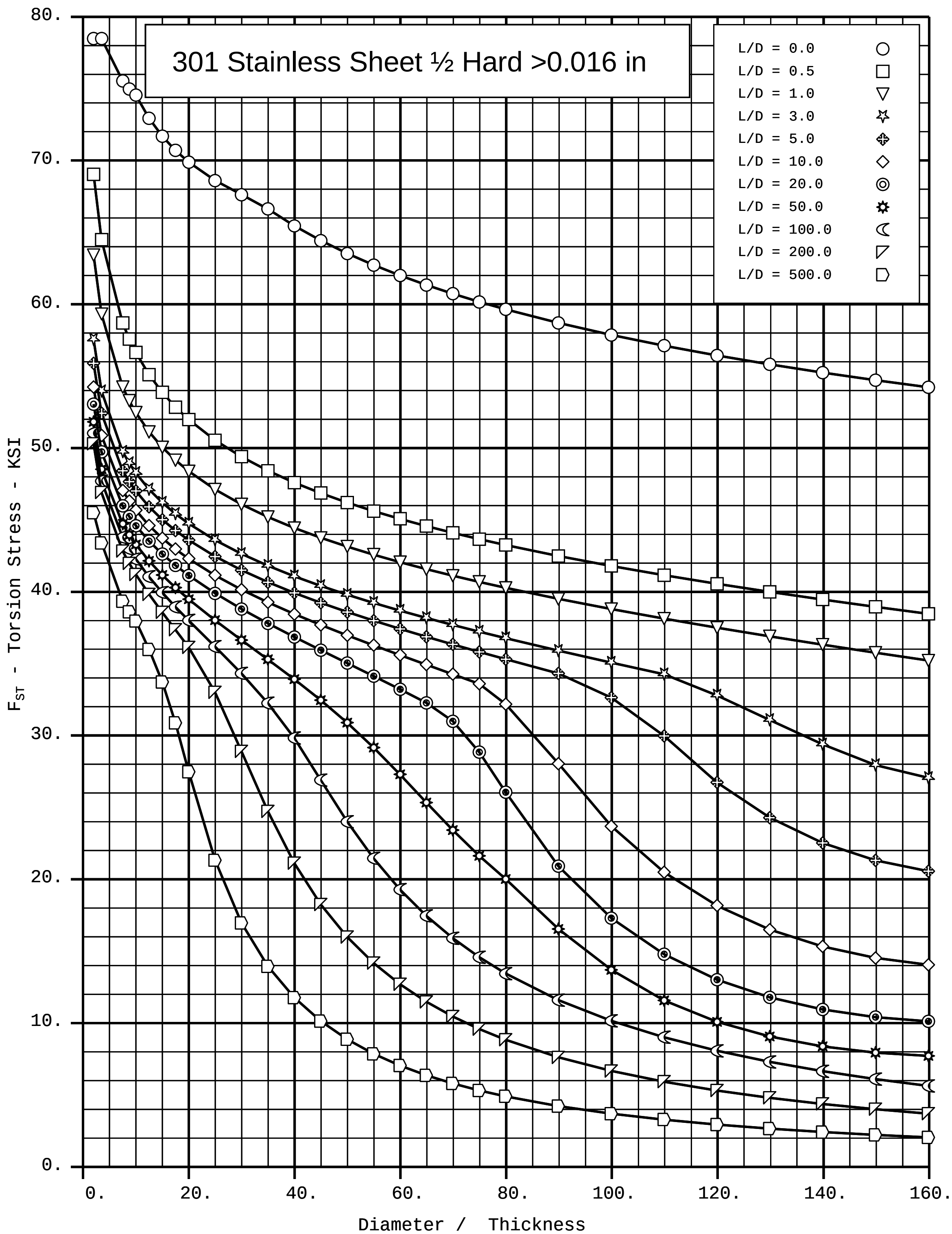


Figure 7.3.6-8 Torsional Rupture Allowable for Circular Cylinder: 301 Stainless Steel Sheet 1/2H

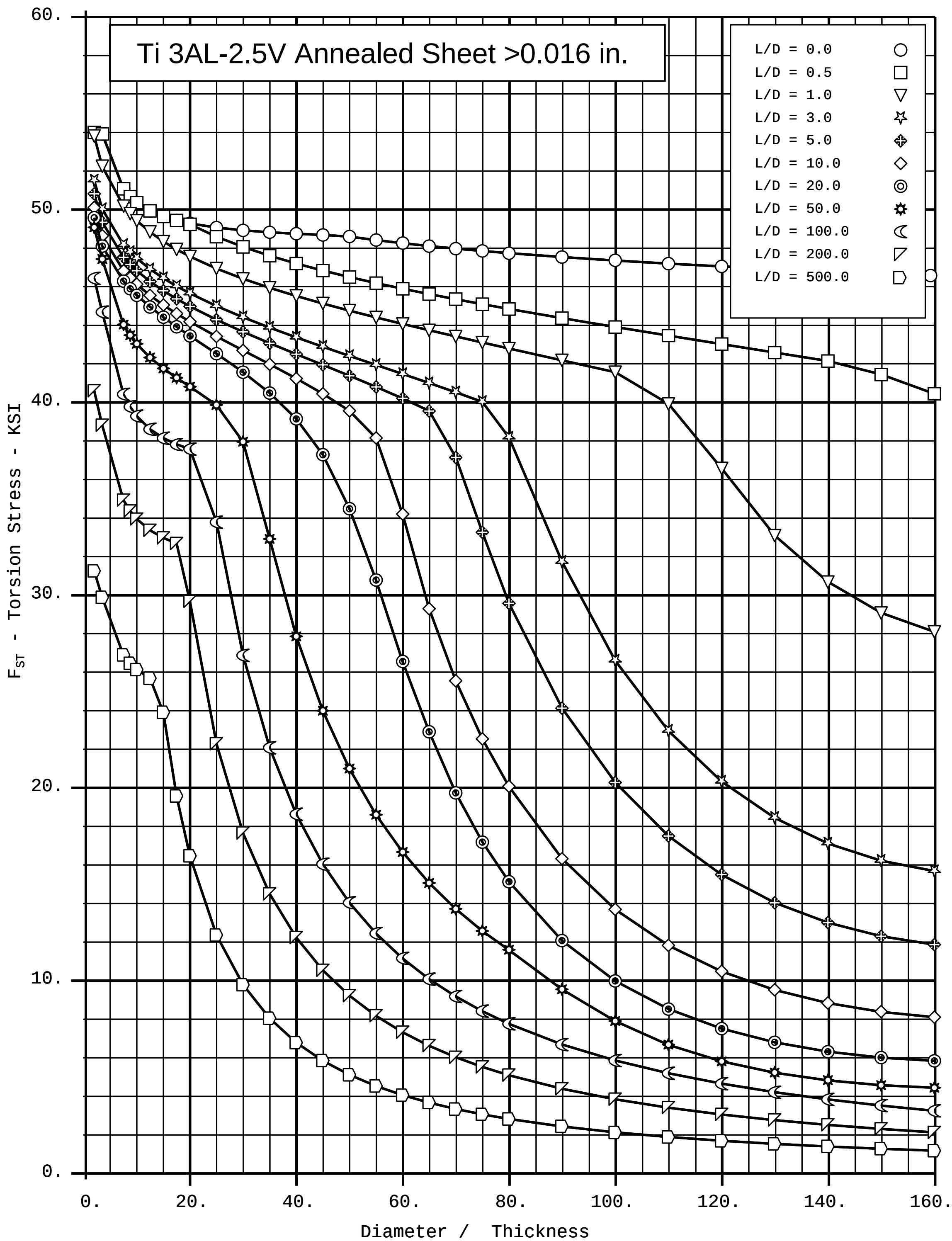


Figure 7.3.6-9 Torsional Rupture Allowable for Circular Cylinder: Titanium 3-AL-2.5V Annealed Sheet

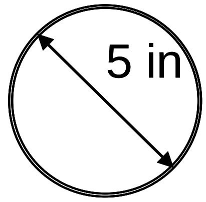
7.3.6.4 Example Problem – Rupture Strength of Thin-Walled Un-stiffened Circular Section

Given the torque tube of Example Problem, Section 7.3.5.1 and shown in the figure to the right.

- t=0.0625 in
- D(outside) = 5 in
- L=18 in
- Material: 300M 270 ksi Steel Tube
- Tube is simply-supported and un-stiffened

The applied torsion is 160,000 in-lb

Determine if the tube fails



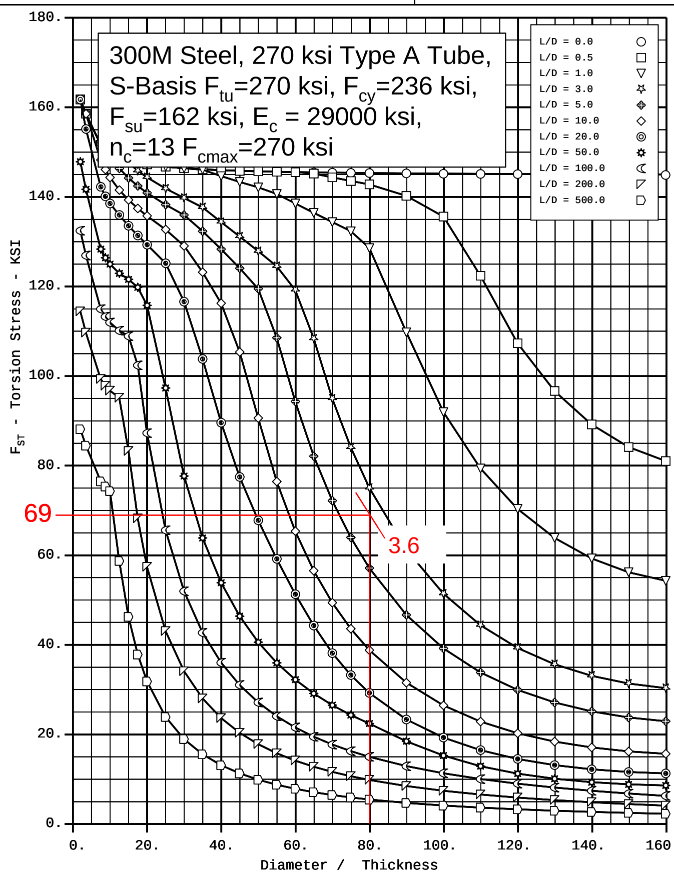
The shear flow, using Equation 7.3.1-3, is

$$q = \frac{T}{2A_O} = \frac{160000}{2(19.15)} = 4178 lb/in$$

Calculate/Determine Geometry Parameters

$$\frac{L}{D} = \frac{18}{5} = 3.6 \qquad \frac{D}{t} = \frac{5}{0.0625} = 80$$

Determine Allowable Torsional Rupture Stress:
From the figure below, f_{st}= 69000 psi
From Equation 7.3.6-2,
$q_{rupture}=F_{st\text{-}allow}(t)$ = 69000(0.0625)
$q_{rupture}$= 4313 lbs/in
M.S. = 4313/4178-1 = <u>0.03 Rupture</u>



7.4 Combined Stresses

The combination of torsion with other stress components is generally done through the use of interaction equations. In some cases torsion stresses are treated as additional shear stress additive to other shear stresses which may be present. In other cases, there are unique equations for torsion stress interaction. Section 2.5 describes the calculation of Margin of Safety and the general use and theory of interaction equations. Section 2.5.2 provides specific interaction equations for use with torsion stress, both for initial buckling and ultimate failure. Table 7.4.0-1 gives a summary of the interactions which may be found in Section 2.5.2 involving torsion.

Table 7.4.0-1 Summary of Torsion Interaction Equations, Section 2.5.2

Case	Description	Structure
12	Compression-torsion	Thin Walled, Unstiffened, Circular Cylinders
13	Tension-Torsion	Thin Walled, Unstiffened, Circular Cylinders
14	Bending-Torsion	Thin Walled, Unstiffened, Circular Cylinders
16	Compression-Bending-Torsion	Thin Walled, Unstiffened, Circular Cylinders
17	Shear-Bending-Torsion	Thin Walled, Unstiffened, Circular Cylinders
19	Bending (Major Axis)-Torsion	Thin Walled, Unstiffened, Elliptical Cylinders
22	Bending-Torsion	Stiffened Circular Cylinders
26	Bending-Torsion	Round Tubes, Failure
27	Compression-Bending-Torsion	Round Tubes, Failure
29	Tension-Torsion	Round Tubes, Failure
30	Tension-Torsion-Internal Pressure	Round Tubes, Failure

7.5 Unix/PC-Based Calculation

Many of the equations which predict torsional buckling and torsional rupture require iterative solutions which lend themselves to computer program. IDAT, the Integrated Detail Analysis Tool suite has programs which can calculate and provide allowable curves for both buckling and rupture. IDAT/SM33 can generate buckling curves for use in initial buckling calculations. Appropriate use of effective b/t per Section 7.3.5 is required. IDAT/SM102 can generate torsional rupture allowable curves per the requirements of Section 7.3.6. Neither program will calculate Margin of Safety for a specific part.