4 Composite Laminates

The purpose of this chapter is to provide information and guidance on analysis and design of structural composite laminates. The information in this chapter is intended for general guidance only. Refer to your program for specific guidance on analysis of composite laminates.

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Table 4.1-1 Symbols and Nomenclature

Symbol	Description	Units
[A]	Laminate in-plane extensional stiffness matrix	lb/in
[B]	Laminate in-plane coupling stiffness matrix	lb

[1] In 2002, administration of MIL-HDBK-17 was transferred to FAA. Future releases will be released as Composite Materials Handbook 17 (CMH-17), Materials Sciences Corporation, Secretariat. The CMH-17 organization is an all voluntary organization comprised of engineers and scientists from government, academia, and industry.

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[C]	Ply stiffness matrix	
C_{ij}	Ply stiffness matrix components	psi
CLPT	Classical Laminated Plate Theory	
[D]	Laminate in-plane bending stiffness matrix	in-lb
E_{11}, E_1	Axial stiffness in the 1 direction. (Young's Modulus)	psi
E_{22}, E_2	Axial stiffness in the 2 direction. (Young's Modulus)	psi
E_{22}^{sec}	Secant axial stiffness in the 2 direction.	psi
E_{33}, E_3	Axial stiffness in the 3 direction (Young's Modulus)	psi
F_{11}^{tu}	Ultimate tensile stress allowable in the fiber direction	psi
F_{11}^{cu}	Ultimate compressive stress allowable in the fiber direction	psi
F_{22}^{tu}	Ultimate tensile stress allowable in the transverse direction	psi
F_{22}^{cu}	Ultimate compressive stress allowable in the transverse direction	psi
F_{33}^{tu}	Ultimate tensile stress allowable in the out of plane direction	psi
F_{33}^{cu}	Ultimate compressive stress allowable in the out of plane direction	psi
F_{12}^{su}	Ultimate engineering shear stress allowable in the 1-2 plane	psi
F_{13}^{su}	Ultimate engineering shear stress allowable in the 1-3 plane	psi
F_{23}^{su}	Ultimate engineering shear stress allowable in the 2-3 plane	psi
FEA	Finite Element Analysis	
G_{12}	Shear stiffness in the 1-2 material plane.	psi
G_{12}^{sec}	Secant shear stiffness in the 1-2 material plane.	psi
G_{13}	Shear stiffness in the 1-3 material plane.	psi
G_{23}	Shear stiffness in the 2-3 material plane.	psi
IDAT	Integrated Detailed Analysis Toolset	
LASSE	IDAT Laminate Stacking Sequence Evaluator	
MATUTL	IDAT composite material editing and viewing utility	
M_x	Laminate moment resultant force in the x direction	in-lb/in
M_y	Laminate moment resultant force in the y direction	in-lb/in
M_{xy}	Laminate moment resultant force in the x-y plane	in-lb/in

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N_x	Laminate resultant force in the x direction	lb/in
N_y	Laminate resultant force in the y direction	lb/in
N_{xy}	Laminate resultant shear force in the x-y plane	lb/in
$[Q]$	Reduced stiffness matrix	
Q_{ij}	Reduced stiffness components	psi
$[\overline{Q}]$	Reduced transformed stiffness matrix	
$[R]$	Reuter Matrix	
S	Load Spectrum	
[S]	Compliance matrix	
S_{ij}	Compliance matrix components	1/psi
T	Operating temperature	˚F
T_g	Glass Transition Temperature	˚F
T_0	Stress Free Temperature	˚F
$[T]$	Transformation Matrix	
u^o	Laminate mid-plane displacement in x direction	in
v^o	Laminate mid-plane displacement in y direction	in
w^o	Laminate mid-plane displacement in z direction	in
W/D	Coupon specimen width to hole diameter ratio	
α_1	Coefficient of thermal expansion in material direction 1.	in/in/˚F
α_2	Coefficient of thermal expansion in material direction 2.	in/in/˚F
α_{12} or α_6	Coefficient of thermal expansion in material direction 1-2.	in/in/˚F
α_3	Coefficient of thermal expansion in material direction 3.	in/in/˚F
β_1	Coefficient of moisture expansion in material direction 1.	in/in/ ΔC
β_2	Coefficient of moisture expansion in material direction 2.	in/in/ ΔC
β_{12} or β_6	Coefficient of moisture expansion in material direction 1-2.	in/in/ ΔC
β_3	Coefficient of moisture expansion in material direction 3.	in/in/ ΔC
ΔC	% Moisture content by weight	
ΔT	Change in temperature	˚F

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ε_{ij}	Axial strain components (Tensor notation)	in/in
ε_{11}^{tu}	Ultimate tensile strain allowable in the fiber direction	in/in
ε_{11}^{cu}	Ultimate compressive strain allowable in the fiber direction	in/in
ε_{22}^{tu}	Ultimate tensile strain allowable in the transverse direction	in/in
ε_{22}^{cu}	Ultimate compressive strain allowable in the transverse direction	in/in
ε_{22}^{tp}	Pseudo tension transverse strain allowable	in/in
ε_{33}^{tu}	Ultimate tensile strain allowable in the out of plane direction	in/in
ε_{33}^{cu}	Ultimate compressive strain allowable in the out of plane direction	in/in
ε_{x}^{o}	Mid-plane strain in the x direction	in/in
ε_{y}^{o}	Mid-plane strain in the y direction	in/in
γ_{xy}^{o}	Mid-plane engineering shear strain in the x-y plane	in/in
γ_{12}^{su}	Ultimate engineering shear strain allowable in the 1-2 plane	in/in
γ_{13}^{su}	Ultimate engineering shear strain allowable in the 1-3 plane	in/in
γ_{ij}	Engineering shear strain components (Tensor notation)	in/in
κ_x	Laminate curvature in x direction	1/in
κ_y	Laminate curvature in y direction	1/in
κ_{xy}	Laminate twist curvature	1/in
θ_k	Angle of ply k	degrees
t_k	Thickness of ply k	in
z_k	Out of plane coordinate for the top surface of ply k	in
ν_{12}	Poisson's ratio for the 1-2 plane	
ν_{13}	Poisson's ratio for the 1-3 plane	
ν_{23}	Poisson's ratio for the 2-3 plane	
σ_i	Axial stress components	psi
τ_{ij}	Shear stress components (Tensor notation)	psi

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4.2 Introduction

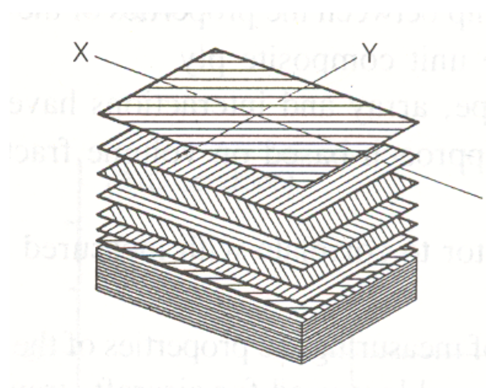

Figure 4.2-1 Structural Composite Laminate

A structural composite laminate is a consolidated stack of two or more oriented laminae. The order and orientation of laminae in the stack is referred to as the “stacking sequence”. Laminates are typically constructed by stacking and orienting a design specified set of lamina material forms on a tool surface. The laminate is then cured using process specific temperature and pressure profiles. The temperature and pressure profiles are designed to prevent defects and optimize properties of the laminate. For thermoset material forms, the cure process causes an irreversible chemical change that hardens the resin material and bonds the individual laminae together. Cure transforms the individual stacked oriented laminae material forms into a solid block of resin containing tightly packed two dimensional layers of oriented fiber material.

Material selection, stacking sequence, and laminate thickness are the primary determinants of effective in-plane strength and stiffness properties for a laminate. Stacking and orienting laminae material forms to achieve a desired set of engineering properties in one or more directions is referred to as “tailoring”. Properties that are often “tailored” include in-plane stiffness, strength, bending stiffness, and coefficients of thermal expansion. Different lamina materials and material forms can also be combined in the stack to create hybrid laminates. Hybrid laminates can show improved durability, damage tolerance, and manufacturability.

Tailoring stacking sequence and materials for a specific set of design conditions can result in a more efficient use of material and reduce structural weight of the laminate. Weight reductions achieved by tailoring are in addition to those seen from the higher specific strength and stiffness of composite material forms. The ability to tailor laminate strength and engineering properties to minimize weight is one of the primary structural advantages of composite laminates.

Analysis of laminates for strength and stiffness is complex when compared to the equivalent analysis for isotropic materials. Laminates in general exhibit step discontinuities in strength and stiffness from ply to ply throughout the laminate thickness. These discontinuities in stiffness can lead to stiffness couplings between shear, extension, and bending that are not possible in isotropic material forms. Strength and stiffness of a laminate is in general dependent on orientation of the laminated material with respect to the load. In-plane analysis of laminates for strength and stiffness is typically performed using Classical Laminated Plate Theory – which is often designated by the initials CLPT.

This section provides information on specification of laminates, analysis for strength and stiffness, and development of material properties, failure criteria and allowables.

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4.3 Laminate Specification and Notation

A stacking sequence description specifies the order, orientation, and material for each lamina in the laminate stack. Laminate stacking sequences are specified using two basic notational schemes. Each notational scheme orients the laminae with respect to a laminate coordinate system. This section describes the laminate coordinate system, stacking sequence notation, and laminate terminology. Stacking sequences with special properties are identified.

4.3.1 Laminate Coordinate System

Each lamina in the laminate stack is oriented with respect to a reference laminate material coordinate system (see Figure 4.3-1). Laminate material coordinate systems follow the right hand rule, with in-plane axes designated "x-y", and the out of plane axis designated "z". The angle from the laminate material coordinate system x-axis to the lamina 1-2 coordinate system is designated θ. (See section 3.5.3.3 for a description of the lamina coordinate system)

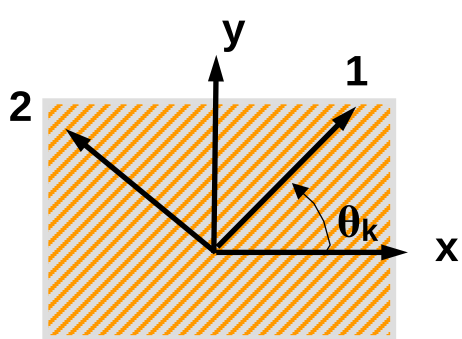

Figure 4.3-1 Laminate Material Coordinate System

The origin of the laminate coordinate system is located on the laminate mid-plane with the positive *z* direction indicating the "analytical" top surface of the laminate. Laminae in the laminate stack are numbered from *1* to *N*, starting at the bottom surface of the stack. The *z* coordinate from the mid-plane to the top surface of the k^{th} layer is designated z_k, with the *z* coordinate to the bottom surface of the k^{th} layer designated z_{k-1}. The thickness, t_k, of any layer is then calculated as $t_k = z_k - z_{k-1}$. The bottom surface of the laminate is denoted z_0, and the total laminate thickness is designated *t*. The fiber orientation of lamina *k* is denoted θ_k. The laminate coordinate system and coordinate notation are illustrated in Figure 4.3-2. Note that positive z is pointing down in the reference Figure 4.3-2.

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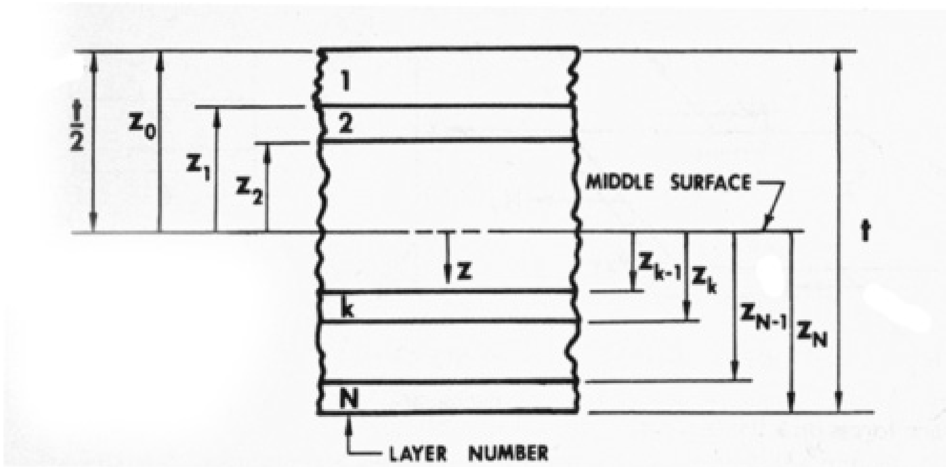

Figure 4.3-2 Laminate Coordinate System and Notation

4.3.2 Stacking Sequence Terminology

The stacking sequence terms "Balance" and "Symmetry" are introduced in this section. When applied to a laminate description, these terms indicate that a laminate meets specific stacking sequence criterion which eliminate coupling terms in the laminate stiffness matrix. The basic definitions are given here. The mathematics will be discussed in section 4.4.1.

4.3.2.1 Balance

Balanced laminates have a +θ ply for every –θ ply. The +θ and –θ ply pairs must have the same material properties and be the same thickness. The +θ plies and the –θ plies can be located randomly through the thickness. A balanced stacking sequence eliminates in-plane coupling between shear and extension.

4.3.2.2 Symmetry

Symmetric laminates have stacking sequences that are mirrored about the laminate mid-surface ($z = 0$). Unsymmetric laminates will bend when in-plane axial forces are applied, and will deform axially when pure bending loads are applied. A symmetric stacking sequence eliminates in-plane coupling between bending and extension.

Stacking sequence guidelines discussed in section 2.3.2.1 recommend that all laminates be balanced and symmetric. Note that there is no link between balance and symmetry. A laminate can be balanced and not symmetric, a laminate can be symmetric and not balanced, and a laminate can be both balanced and symmetric and conversely a laminate can be both unbalanced and unsymmetric.

4.3.3 Stacking Sequence Notation

Composite laminates are currently specified using two basic notational forms: 1) ply percentages, and 2) stacking sequence.

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4.3.3.1 Ply Percentage Notation

There are two variations of the ply percentages notational form. Both variations follow laminate stacking sequence guidelines which limit ply orientation angles to 0°, ±45°, and 90° (see section 2.3.2.1). The ply percentage notation methods specify laminates by listing the percentage of plies in each allowed orientation. The ply percentages are surrounded by parenthesis and separated by forward slashes. The first variation of this form specifies the total percentage of ±45°plies, and the second variation lists the +45° and the -45° ply percentages separately. Examples of the two ply percentage notational forms follow in Table 4.3-1:

Table 4.3-1 Ply Percentage Laminate Notation Example	
(50/40/10)	50% 0°, 40% ±45°, 10% 90°
(50/20/20/10)	50% 0°, 20% +45°, 20% -45°, 10% 90°

The first number in parentheses indicates the percentage of 0° laminae in the stack. The last number indicates the percentage of 90° laminae in the stack. The middle number(s) indicate the total percentage of ± 45° laminae. The first ply percentage form specifies a "balanced" $\pm 45^\circ$ ply laminate, and for the example shown is equivalent to the laminate coded in second form. The second form lists the percentage of +45° plies using the second number and the percentage of -45° plies using the third number. For both methods, the total of the numbers in parenthesis must sum to 100%.

Ply percentage notational forms are used primarily for preliminary design work. These notational forms provide a simple but incomplete description of a laminate. Total laminate thickness and the order of plies in the stack are not specified. Ply percentage notational forms are useful because in-plane extensional stiffness and in-plane shear stiffness are adequately described by ply percentages. If laminate thickness is also specified then failure strength can be approximated and preliminary design can proceed.

A NASTRAN PCOMP laminate option "SMEAR" is available to perform preliminary finite element analysis using ply percentages and thickness specification. The "SMEAR" method ignores stacking sequence and assumes bending stiffness equal to an equivalent homogeneous shell. Because stacking sequence guidelines specify balanced symmetric laminates with ply orientations distributed evenly through the thickness, the "SMEAR" approximation is sufficient for preliminary design. Exact bending stiffness is not specified until the stacking order is specified.

4.3.3.2 Stacking Sequence Notation

Stacking sequence notation is the primary method used to specify the order and orientation of each ply in a composite laminate. The stacking sequence laminate description is surrounded by braces. Inside the braces, each ply is represented by an orientation angle in degrees and is separated from the next ply using a forward slash. Plies are specified proceeding from the laminate bottom to the laminate top. For the default form, the laminate stacking sequence begins with ply 1 at the left brace and proceeds to ply N at the right brace. Refer to section 4.3.1 for a description of the laminate coordinate system and notation. An example of the default stacking sequence notational form with no shorthand notation follows in Table 4.3-2:

Table 4.3-2 Stacking Sequence Laminate Notation Example	
[+45/0/0/-45/0/0/+45/0/-45/90/90/-45/0/+45/0/0/-45/0/0/+45]	50% 0°, 40% ±45°, 10% 90°

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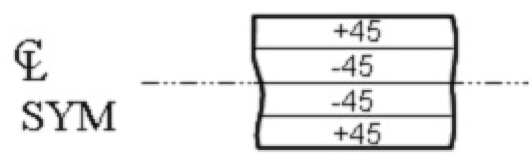
The stacking sequence example in Table 4.3-2 is one of many possible solutions to the (50/40/10) ply percentage laminate specified in the Table 4.3-1 ply percentage notation examples. The twenty plies indicated in the example are the minimum needed to satisfy the ply percentage requirements and stacking sequence guidelines for balance, symmetry, and ply distribution. Ply percentage laminate specifications can be converted to "rule compliant" stacking sequence form with the aid of the IDAT utility LASSE (Laminate Stacking Sequence Evaluator).

The stacking sequence notation form includes various subscripts and shorthand conventions used to simplify and compact the laminate description. Stacking sequence shorthand notation is listed in Table 4.3-3.

Table 4.3-3 Laminate Stacking Sequence Notation Shorthand	
$[\ ]_T$	Total stacking sequence (default)
$[\ ]_s$	symmetric[1]
$[\pm X\ ]$	[+X/-X]
$[\mp X]$	[-X/ +X]
$[X_n]$	n plies at angle X
$(\ )_n$	Designates that the sequence in parentheses is repeated n times.

Examples showing the use of stacking sequence notation and shorthand notation are given in the tables that follow: (Table 4.3-4, Table 4.3-5, Table 4.3-6, Table 4.3-7, Table 4.3-8, and Table 4.3-9)

Table 4.3-4 Laminate Stacking Sequence Notation Shorthand Example 1



Equivalent Stacking Sequence Notation for the Laminate Shown	
$[\ +45/-45/-45/+45]_T$	Total stacking sequence (default)
$[+45/-45\ ]_s$	Symmetric
$[\pm 45\ ]_s$	[+X/-X] Symmetric
$[\pm 45/\mp 45]$	[+X/-X /-X/ +X]

[1] The laminate ply angles, ply thicknesses, and materials specified in the brackets represent one half of the laminate, and are "mirrored" about the mid-plane to create the whole laminate.

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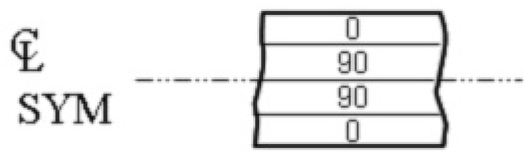
Table 4.3-5 Laminate Stacking Sequence Notation Shorthand Example 2

Equivalent Stacking Sequence Notation for the Laminate Shown	
$[+45/-45/0/-45/+45]_T$	Total stacking sequence (default)
$[+45/-45/\bar{0}]_s$	Symmetric, Overbar indicates a single centerline ply
$[\pm 45/0/\mp 45]$	[-X/ +X/ 0/ -X/ +X]

Table 4.3-6 Laminate Stacking Sequence Notation Shorthand Example 3



Equivalent Stacking Sequence Notation for the Laminate Shown	
$[0/90/90/0]_T$	Total stacking sequence (default)
$[0/90]_s$	Symmetric
$[0/90_2/0]$	N plies at angle X

Table 4.3-7 Laminate Stacking Sequence Notation Shorthand Example 4

Equivalent Stacking Sequence Notation for the Laminate Shown	
$[-45/-45/+45/+45]_T$	Total stacking sequence (default)
$[-45_2/+45_2]$	N plies at angle X

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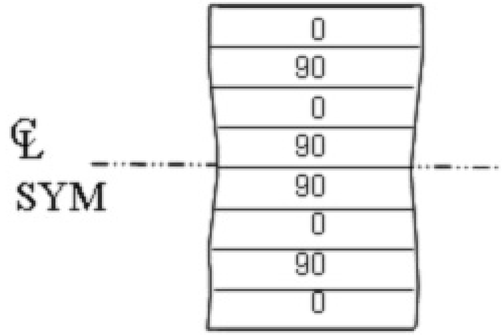
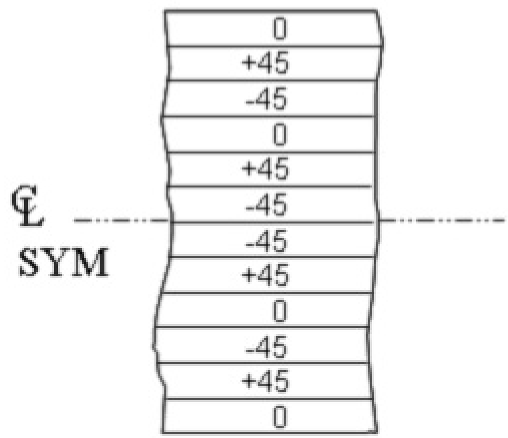
Table 4.3-8 Laminate Stacking Sequence Notation Shorthand Example 5



Equivalent Stacking Sequence Notation for the Laminate Shown	
$[\ 0/90/0/90/90/0/90/0]_T$	Total stacking sequence (default)
$[0/90/0/90\ ]_s$	Symmetric
$[(0/90)_2]_s$	$()_n$ Sequence in parentheses is repeated n times, Symmetric

Table 4.3-9 Laminate Stacking Sequence Notation Shorthand Example 6



Equivalent Stacking Sequence Notation for the Laminate Shown	
$[\ 0/+45/-45/0/+45/-45/-45/+45/0/-45/+45/0]_T$	Total stacking sequence (default)
$[0/\pm45/0/\pm45/\mp45/0/\mp45/0\ ]_T$	Total., [+X/-X], [-X/ +X]
$[0/+45/-45/0/+45/-45\ ]_s$	Symmetric

A tape–fabric hybrid laminate would be designated as follows:
$[45_f/0_f/45_t/0_t/90_t/-45_t]_s$, where ‘t’ stands for tape, and ‘f’ stands for fabric.

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4.4 Analytical Methods for Laminate Strength and Stiffness

This section addresses basic theory and techniques of laminate analysis for strength and stiffness. Other important failure modes and criteria such as stability, durability, and damage tolerance are addressed in other sections of the manual. Basic analysis of composite laminates for strength and stiffness can be divided into four primary phases:

- Classical Laminated Plate Theory (CLPT) is used to calculate the ideal in-plane strength and stiffness of an undamaged laminate. CLPT calculations are done using empirically derived average engineering properties for a specified set of environmental conditions. CLPT calculations rarely size a part in isolation, but are the first steps leading to more detailed composite analysis. CLPT calculations also serve as a basis for the weight impact of durability and damage tolerance requirements.

- Internal residual strains caused by changes in laminate temperature and moisture content from “stress free temperature” must also be accounted for. Internal residual lamina strains are calculated as the difference between laminate hygrothermal expansion and free hygrothermal expansion for each ply.

- Interlaminar shear and interlaminar tension stresses are calculated for specific laminate geometry and load conditions. CLPT is a two dimensional plane stress plate theory and does not predict interlaminar stress. Critical locations are identified and additional analysis techniques are used to predict failure caused by interlaminar shear and interlaminar tension.

- CLPT analysis assumes no defects, stress concentrations, or imperfections in the manufactured laminate. The effects of imperfections, stress concentrations at holes, cutouts, and notches are considered and accounted for with the use of “notched” analysis and allowables.

This section presents and explains each of these laminate structural analysis components.

4.4.1 Classical Laminated Plate Theory

Classical Laminated Plate Theory (CLPT) represents a collection of structural and kinematic assumptions which govern the analysis of laminate in-plane strength and stiffness. CLPT provides a methodology for calculating an in-plane linear elastic stiffness matrix that governs extension and bending of a laminated plate. The CLPT stiffness matrix relates a set of laminate in-plane force and moment resultants to laminate mid-plane strains and curvatures. CLPT provides the laminate stiffness matrix, which is required to solve the governing equilibrium equations for thin flat plates.

CLPT is derived from Classical Thin Plate Theory. The key difference between theories is the constitutive or stress-strain relations. Classical Thin Plate Theory assumes a single homogeneous layer of material while Classical Laminated Plate Theory allows for an arbitrary number of bonded orthotropic laminae. Each orthotropic layer of material must be linear elastic and of constant thickness. The stiffness for each layer is calculated using the transformed reduced constitutive relations for orthotropic laminae as developed in section 3. The CLPT stiffness matrix is assembled from the reduced constitutive relations for each lamina.

The generally accepted and well-known linear differential equations for static equilibrium of flat plates subjected to plain stress are given by Equation 4.4-1.

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$$N_{x,x} + N_{xy,y} = 0$$
$$N_{xy,x} + N_{y,y} = 0 \qquad \textbf{Equation 4.4-1}$$
$$M_{x,xx} + 2M_{xy,xy} + M_{y,yy} = -p$$

Where:

N_i $i = x, y, xy$, are the in-plane force resultants.
M_i $i = x, y, xy$, are the in-plane moment resultants.
p is a pressure force applied normal to the plate surface.
The commas denote partial differentiation of the principal symbol with respect to the subscript variable that follows.

The equilibrium differential Equation 4.4-1 has been expressed in terms of the force and moment resultants for thin flat plates. Equation 4.4-1 has been reduced from the more general form of the equilibrium equations for thin plates by making the following assumptions and simplifications that are common to both classical thin plate theory and CLPT:

- Nonlinear terms are neglected. The secondary effects of membrane forces on bending are lost as a result of this assumption.
- Static equilibrium – All terms containing derivatives with respect to time are eliminated.
- Rotatory inertia terms are negligible.
- There are no body forces.

The CLPT approach is to formulate the laminate force resultants "*N*", and laminate moment resultants "*M*", as expressions of the laminate mid-surface displacements (u^o, v^o, and w^o) and rotations (k). After the stress and moment resultants have been expressed in terms of plate mid-surface displacements and rotations, a solution for the equilibrium differential equations is typically obtained using the finite element method. CLPT kinematic assumptions are used to calculate the principle stress in each lamina after a solution to the equilibrium differential equation has been obtained.

4.4.1.1 Kinematic Equations and Assumptions

Classical Laminated Plate Theory utilizes Kirchhoff's thin plate theory assumptions as a basis for development of kinematic expressions relating displacements of points in the plate cross section to displacements and rotations of the plate mid-surface. Kirchhoff's assumptions for thin plates are summarized as follows:

- Straight lines originally normal to the mid-surface remain straight and normal after the plate is extended or bent (see Figure 4.4-1).
- These normal lines are inextensible.

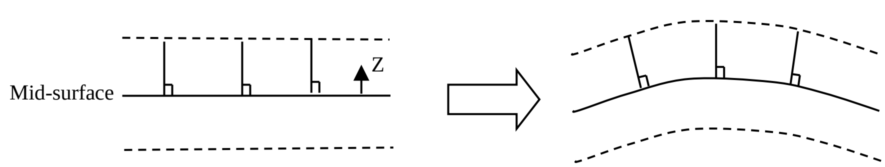

Figure 4.4-1 Kirchhoff's Thin Plate Theory Assumptions

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The Kirchhoff assumptions impose a state of plane strain on the laminate. Requiring the normal to the middle surface to remain straight and perpendicular to the middle surface after deformation is equivalent to ignoring shear strains in planes perpendicular to the middle surface. That is, $\gamma_{xz} = \gamma_{yz} = 0$. If the normal lines are also inextensible then strain perpendicular to the middle surface is ignored as well, meaning that $\varepsilon_z = 0$.

The Kirchhoff assumption of a plane strain state is an inconsistency in the formulation of Classical Laminated Plate Theory that is commonly ignored. The constitutive relations derived in section 3 assume that out of plane stresses σ_z, τ_{xz}, and τ_{yz}, are zero. As a result, the plane strain state implied by the Kirchhoff assumptions is ignored and CLPT is considered to be a plane stress plate theory. Stresses acting in the plane of the plate are assumed to dominate behavior.

The Kirchhoff assumptions make it possible to write simple kinematic equations relating displacements of points in the plate cross section to extension and bending of the plate middle surface. These kinematic equations are given by Equation 4.4-2. The subscript "o" in Equation 4.4-2 indicates mid-surface values.

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\,\partial w_0(x,y)/\partial x \\ v(x,y,z) &= v_0(x,y) - z\,\partial w_0(x,y)/\partial y \\ w(x,y,z) &= w_0(x,y) \quad \text{(constant thru the thickness)} \end{aligned}$$

Equation 4.4-2

The kinematic equations calculate in-plane displacements of material points in the plate cross section as linear functions of the mid-surface displacements and z times the mid-surface slope at the specified surface x-y coordinates. *This kinematic approximation is valid for small rotations of the plate mid-surface ($\sin(\theta) \approx \theta$, and $\cos(\theta) \approx 1$).* The kinematic relationship for a "u" displacement of point "*P*" in the plate cross section is illustrated in Figure 4.4-2.

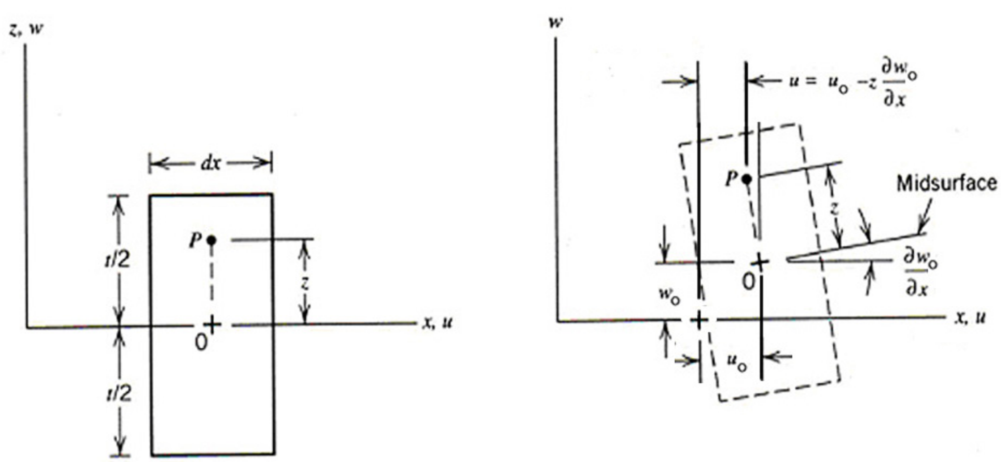

Figure 4.4-2 Kinematic relationship for the "u" displacement of point "P"

An implied assumption of the kinematic Equation 4.4-2 is that individual laminae are bonded together to act as an integral mass. The bonds between laminae are assumed to be infinitesimally small and do

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not deform in shear. There is no relative motion between adjacent lamina at the bond lines. These assumptions are implied by the assumption of a continuous linear distribution of in-plane displacement through the lamina thickness.

4.4.1.2 Strain Displacement Relations

The strain displacement relations for thin plates are given by Equation 4.4-3. The form of Equation 4.4-3 is reduced from the more general form of the strain displacement relations by the assumption of flat plates and small displacement theory. Displacements u, v, and w are assumed small compared to the plate thickness. Strains, ε_x, ε_y, and γ_{xy} are assumed small compared to unity.

$$\begin{aligned}
\varepsilon_x &= \partial u / \partial x \\
\varepsilon_y &= \partial v / \partial y \\
\varepsilon_z &= \partial w / \partial z \\
\gamma_{xy} &= \partial u / \partial y + \partial v / \partial x \\
\gamma_{xz} &= \partial u / \partial z + \partial w / \partial x \\
\gamma_{yz} &= \partial v / \partial z + \partial w / \partial y
\end{aligned}$$

Equation 4.4-3

Substitution of the kinematic Equation 4.4-2 into the strain-displacement Equation 4.4-3 yields strain at any point in the cross section as a function of displacement and rotation of the plate mid-surface. The result of this substitution is given by Equation 4.4-4.

$$\begin{aligned}
\varepsilon_x &= \partial u / \partial x = \partial u_0 / \partial x - z\, \partial^2 w_0 / \partial x^2 = \varepsilon_x^0 + z\kappa_x \\
\varepsilon_y &= \partial v / \partial y = \partial v_0 / \partial y - z\, \partial^2 w_0 / \partial y^2 = \varepsilon_y^0 + z\kappa_y \\
\varepsilon_z &= \partial w / \partial z = \partial w_0 / \partial z = 0 \\
\gamma_{xy} &= \partial u / \partial y + \partial v / \partial x \\
&= (\partial u_0 / \partial y + \partial v_0 / \partial x) - 2z(\, \partial^2 w_0 / \partial x \partial y) = \gamma_{xy}^0 + z\kappa_{xy} \\
\gamma_{xz} &= \partial u / \partial z + \partial w / \partial x = -\partial w_0 / \partial x + \partial w_0 / \partial x = 0 \\
\gamma_{yz} &= \partial v / \partial z + \partial w / \partial y = -\partial w_0 / \partial y + \partial w_0 / \partial y = 0
\end{aligned}$$

Equation 4.4-4

Where:

κ_i *i* = *x*, *y*, *xy*, are the mid-surface curvatures. Note that there are negative signs associated with the curvatures {κ} due to the sign convention for moments.
The superscript ”0” indicates a mid-surface value.

The state of plane-strain implied by use of Kirchhoff’s assumptions in development of the kinematic equations is evident in Equation 4.4-4. Shear strains γ_{xz}, and γ_{yz} are zero, which is *a reasonable assumption for thin plates*, but not for thick plates. The assumption of zero out of plane shear strain requires that *the thickness of the plate be much smaller than any other physical dimension*.

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4.4.1.3 Stress Strain Relation for Lamina k

The non-zero components of the strain field given by Equation 4.4-4 are expressed in vector form using mid-surface strains and curvatures as follows in Equation 4.4-5.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Equation 4.4-5

Using the transformed reduced stiffness matrix for lamina k developed in section 3.5.3.5, and the strain field relation given by Equation 4.4-5, the stress-strain relation for lamina k can be expressed in terms of mid-surface strains and curvatures as follows in Equation 4.4-6.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left\{ \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right\}$$

Equation 4.4-6

Where:

$[\overline{Q}]_k$ is the transformed reduced stiffness matrix for lamina k.

$\{\varepsilon^o\}$ is the mid-surface strain

$\{\kappa\}$ is the curvature

The variation of in-plane stiffness $[\overline{Q}]_k$ through the laminate thickness is a discontinuous function of ply material, ply orientation, and stacking sequence. Step discontinuities in in-plane stiffness and stress occur at the boundaries of dissimilar plies. These discontinuities are illustrated by the representative profiles of laminate cross section values shown in Figure 4.4-3.

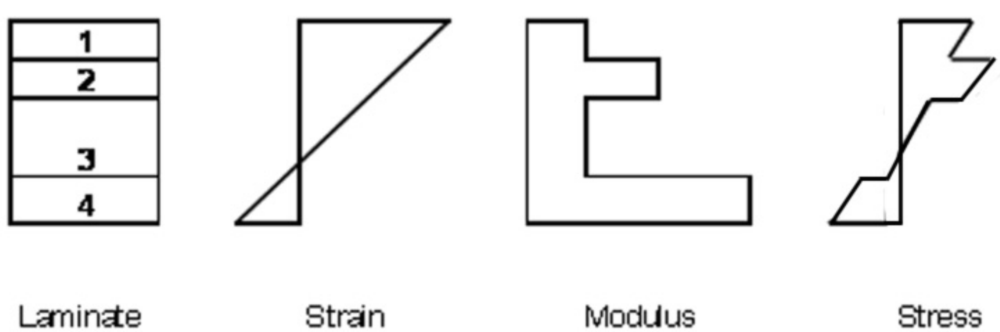

Figure 4.4-3 Discontinuous Variation of Modulus and Stress

The discontinuous variation of in-plane properties through the thickness is the primary reason that laminates cannot in general be modeled and analyzed using methods developed for homogeneous anisotropic materials. It will be shown that the discontinuous variation in stiffness can create in-plane couplings between bending and extension, and between bending and shear. These couplings cannot be modeled accurately with homogeneous anisotropic material behavior and analysis methods.

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4.4.1.4 Laminate Resultant Forces and Moments

{N} is the resultant force vector. {N} is defined as the force per unit length on the laminate cross section and is calculated as follows in Equation 4.4-7:

$$N = \int_{-t/2}^{t/2} \sigma \, dz \qquad \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k dz$$

Equation 4.4-7

{M} is the resultant moment vector. {M} is defined as the moment per unit length on the laminate cross section and is calculated as follows in Equation 4.4-8:

$$M = \int_{-t/2}^{t/2} \sigma z \, dz \qquad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z \, dz = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k z \, dz$$

Equation 4.4-8

The force and moment resultants represent a system which is statically equivalent to the forces and moments acting on the laminate. Resultant forces and resultant moments are defined as the integration of stress through the entire laminate thickness. Since stress is a discontinuous function through the laminate thickness, the integration of stress through the laminate thickness is replaced by a summation of the integrals through each lamina thickness. The force resultants defined by Equation 4.4-7 follow the sign conventions illustrated in Figure 4.4-4. The moment resultants defined by Equation 4.4-8 follow the sign conventions illustrated in Figure 4.4-5.

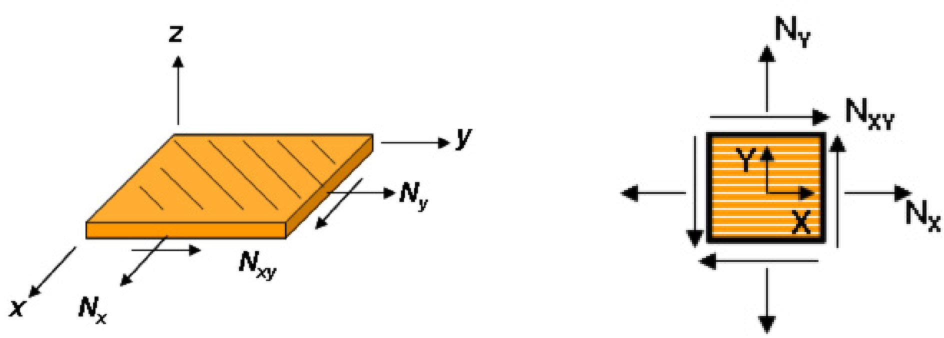

Figure 4.4-4 Sign Conventions for Force Resultants

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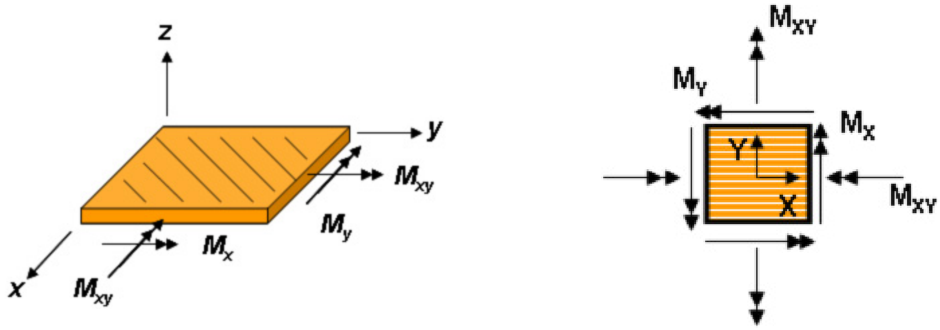

Figure 4.4-5 Sign Conventions for Moment Resultants

M_{xy} is the moment due to "twist curvature". To maintain equilibrium at the element corners, the orthogonal values of M_{xy} must be equal and opposite.

The sign conventions for CLPT shown in Figure 4.4-4 and Figure 4.4-5 are consistent with popular reference texts, ABAQUS, and the Lockheed Martin Aeronautics composite analysis programs (IBOLT, SPAM, CDADT, SQ5, SO0, SS8 and others). These Lockheed Martin Aeronautics analysis tools are part of the IDAT toolset. Be aware that the NASTRAN sign convention for M_x, M_y and M_{xy} is reversed from CLPT, ABAQUS, and IBOLT. The NASTRAN moment resultant sign convention is shown in Figure 4.4-6.

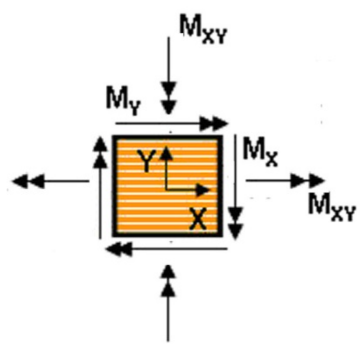

Figure 4.4-6 Sign Conventions for NASTRAN Moment Resultants

4.4.1.5 Laminate Stiffness Matrix

The laminate stiffness matrix relates the force resultants $\{N\}$ and moment resultants $\{M\}$ to the mid-plane strains $\{\varepsilon^o\}$ and the curvatures $\{\kappa\}$. The top half of the laminate stiffness matrix is obtained by substituting the stress-strain relation for lamina k given by Equation 4.4-6 into the force resultant Equation 4.4-7, to yield Equation 4.4-9.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left\{ \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} dz + \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} z\, dz \right\}$$

Equation 4.4-9

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The bottom half of the laminate stiffness matrix is obtained by substituting the stress-strain relation for lamina *k* given by Equation 4.4-6 into the moment resultant Equation 4.4-8 , to yield Equation 4.4-10.

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left\{ \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} z\,dz + \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} z^2\,dz \right\}$$ **Equation 4.4-10**

Equation 4.4-9 expresses the laminate force resultants $\{N\}$ as a summation of the force resultants for each lamina. Equation 4.4-10 expresses the laminate moment resultants $\{M\}$ as a summation of the moment resultants for each lamina. For both equations, the z-coordinate is referenced to the laminate mid-surface, and the transformed reduced stiffness matrix $\left[\overline{Q}\right]_k$ has been brought outside the integration over layer k, because lamina stiffness is not a function of z within layer k.

The mid-surface strains and curvatures are not functions of the thickness coordinate z. Like stiffness, these terms can be removed from the integration. The mid-surface strains and curvatures are also not a function of the lamina parameter k. As a result, these terms can also be factored out of the summation operation. After this re-arrangement, the resultant forces are given by Equation 4.4-11, and resultant moments are given by Equation 4.4-12.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \left(\left[\overline{Q}\right]_k \int_{z_{k-1}}^{z_k} dz \right) \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + \sum_{k=1}^{n} \left(\left[\overline{Q}\right]_k \int_{z_{k-1}}^{z_k} z\,dz \right) \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$ **Equation 4.4-11**

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \left(\left[\overline{Q}\right]_k \int_{z_{k-1}}^{z_k} zdz \right) \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + \sum_{k=1}^{n} \left(\left[\overline{Q}\right]_k \int_{z_{k-1}}^{z_k} z^2\,dz \right) \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$ **Equation 4.4-12**

The summation terms in the force and moment resultant equations now represent three unique 3x3 matrices of constant terms that are determined by laminae material properties, orientation, and stacking sequence. If these three unique matrices are designated [A], [B], and [D], the resultant equations can be written as follows in Equation 4.4-13.

$$\begin{matrix} \{N\} \\ \{M\} \end{matrix} = \begin{matrix} [\mathrm{A}]\{\varepsilon^o\} + [\mathrm{B}]\{\kappa\} \\ [\mathrm{B}]\{\varepsilon^o\} + [\mathrm{D}]\{\kappa\} \end{matrix}$$ **Equation 4.4-13**

The definite integrals in the summations of Equation 4.4-11, and Equation 4.4-12 are evaluated to give expressions for each term of the [A], [B], and [D] stiffness matrices in terms of transformed lamina stiffness [$\overline{Q}$]$_k$, and stacking sequence(z_k,z_{k-1}); where z_k is defined as the z coordinate of the top surface of ply k. Note that the transformed reduced stiffness matrix is symmetric ($\overline{Q}_{ij} = \overline{Q}_{ji}$) therefore the [A], [B], and [D] matrices are also symmetric.

The [A] matrix represents extensional stiffness. Terms of the [A] matrix are given by Equation 4.4-14.

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$$[A] = \sum_{k=1}^{n} \left([\overline{Q}]_k \int_{z_{k-1}}^{z_k} dz \right) \Rightarrow A_{ij} = \sum_{k=1}^{n} \left(\left(\overline{Q}_{ij} \right)_k \left(z_k - z_{k-1} \right) \right)$$ **Equation 4.4-14**

The [B] matrix represents coupling between bending and extension. Terms of the [B] matrix are given by Equation 4.4-15.

$$[B] = \sum_{k=1}^{n} \left([\overline{Q}]_k \int_{z_{k-1}}^{z_k} z dz \right) \Rightarrow B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(\left(\overline{Q}_{ij} \right)_k \left(z_k^2 - z_{k-1}^2 \right) \right)$$ **Equation 4.4-15**

The [D] matrix represents bending stiffness. Terms of the [D] matrix are given by Equation 4.4-16.

$$[D] = \sum_{k=1}^{n} \left([\overline{Q}]_k \int_{z_{k-1}}^{z_k} z^2 dz \right) \Rightarrow D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \left(\left(\overline{Q}_{ij} \right)_k \left(z_k^3 - z_{k-1}^3 \right) \right)$$ **Equation 4.4-16**

After collecting terms, the laminate stiffness matrix is expressed as follows in Equation 4.4-17:

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [\mathrm{A}] & [\mathrm{B}] \\ [\mathrm{B}] & [\mathrm{D}] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^o\} \\ \{\kappa\} \end{Bmatrix}$$ **Equation 4.4-17**

Stiffness of the laminate is defined by the [A], [B], and [D] sub-matrices. [A], [B], and [D] are the stiffness matrices of Classical Laminated Plate Theory (CLPT). The CLPT laminate stiffness matrix is shown in expanded form in Equation 4.4-18.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ \cdots \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & \vdots & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & \vdots & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & \vdots & B_{16} & B_{26} & B_{66} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ B_{11} & B_{12} & B_{16} & \vdots & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & \vdots & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & \vdots & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \cdots \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$ **Equation 4.4-18**

The [A][B][D] matrices of CLPT relate resultant forces and resultant moments in the laminate coordinate system, to the mid-plane strains and curvatures of a laminate. Classical Laminated Plate Theory accounts for shear extension coupling, bending extension coupling, and bend twist coupling that can be present in a laminate.

4.4.1.6 Balance, Symmetry, and Coupling Revisited

Laminate stacking sequences classified as "Balanced" and/or "Symmetric" were discussed in section 4.3.2. This section relates specific CLPT stiffness matrix terms to coupling behavior and shows how balance and symmetry eliminates CLPT stiffness terms to de-couple laminate stiffness.

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4.4.1.6.1 Balanced Laminates and Shear-Extension Coupling

The extensional stiffness matrix [A] determines in-plane laminate behavior. The in-plane laminate stress strain relation with the extensional stiffness matrix [A] is shown extracted in Equation 4.4-19.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Equation 4.4-19

An examination of Equation 4.4-19 shows that the A_{16}, and A_{26} terms in the extensional stiffness matrix represent coupling between shear and extension. Laminates with non-zero A_{16}, and A_{26} terms will shear when an axial load is applied.

In section 4.3.2.1, the concept of a “balanced” laminate was introduced. To review, balance means that the thickness and stiffness of the $+\theta$ plies is the same as for $-\theta$ plies. Normally this means a $+\theta$ ply for every $-\theta$ ply. Balance eliminates the A_{16} and A_{26} terms and thus eliminates coupling between shear and extension. To understand that the shear extension coupling terms become zero for a balanced laminate, first examine the expressions for A_{16} and A_{26} given by Equation 4.4-20.

$$A_{16} = \sum_{k=1}^{n} \left(\left(\overline{Q}_{16} \right)_k \left(t_k \right) \right)$$
$$A_{26} = \sum_{k=1}^{n} \left(\left(\overline{Q}_{26} \right)_k \left(t_k \right) \right)$$

Equation 4.4-20

Using the expressions for transformed reduced stiffness given in section 3.5.3, it can be proven that $\overline{Q}_{16}(+\theta) = -\overline{Q}_{16}(-\theta)$, and $\overline{Q}_{26}(+\theta) = -\overline{Q}_{26}(-\theta)$. Therefore, if there is a same thickness $+\theta$ ply for every $-\theta$ ply, the $\overline{Q}_{16}$ and $\overline{Q}_{26}$ terms for the $\pm\theta$ ply pairs will sum to zero, and the shear extension coupling terms (A_{16} and A_{26}) in the extensional stiffness matrix will be zero. The stiffness matrix and coupling behavior for a balanced unsymmetric laminate is shown in Figure 4.4-7.

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Shear-Extension Coupling Terms are Zero

Bend-Extension Coupling Terms

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ \cdots \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \left[\begin{array}{ccc|ccc} A_{11} & A_{12} & 0 & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & B_{26} \\ 0 & 0 & A_{66} & B_{16} & B_{26} & B_{66} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{array}\right] \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \cdots \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Bend-Extension Coupling Terms

Bend-Twist Coupling Terms

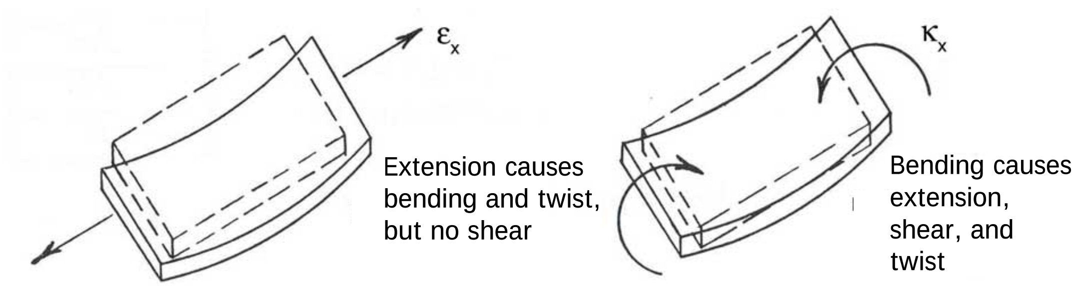

Figure 4.4-7 Balanced Unsymmetric Laminate Behavior

4.4.1.6.2 Symmetric Laminates and Bend-Extension Coupling

An examination of Equation 4.4-17 shows that non-zero terms in the [B] stiffness matrix will create coupling between bending and extension. Laminates with a non-zero [B] matrix will bend if an axial load is applied. If [B] = 0, no bending-extension coupling will be present.

In section 4.3.2.2, the concept of a “symmetric” laminate stacking sequence was introduced. To review, symmetry indicates that ply orientation, ply thickness, and ply material are mirrored about the laminate mid-surface. A mid-plane symmetric stacking sequence will set all terms in the [B] matrix to zero and thus eliminate coupling between bending and extension.

To understand how mid-plane symmetry sets the [B] matrix to zero, consider the four ply symmetric laminate shown in Figure 4.4-8. Elements of the [B] matrix are given by Equation 4.4-15. The calculations that follow show the contribution of $\left(z_k^2 - z_{k-1}^2\right)$ to B_{ij} for the outermost mid-plane symmetric laminae pair in Figure 4.4-8. Because each z term in Equation 4.4-15 is squared, the contributions to [B] from +z, and –z laminae pairs will cancel each other out if the laminae also have the same $\overline{Q}_{ij}$ and the same thickness.

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For the top lamina in Figure 4.4-8, $z_k = 2t$, and $z_{k-1} = 1t$

$$\left(z_k^2 - z_{k-1}^2\right) = ((2t)^2\text{-}(1t)^2) = ((2^2\text{-}1^2)t^2) = (4\text{-}1)t^2 = 3t^2$$

For the bottom lamina in Figure 4.4-8, $z_k = -1t$, and $z_{k-1} = -2t$

$$\left(z_k^2 - z_{k-1}^2\right) = ((-1t)^2\text{-}(-2t)^2) = (((-1)^2\text{-}(-2)^2)t^2) = (1\text{-}4)t^2 = -3t^2$$

Therefore, the contribution to B_{ij} for the symmetric lamina pair presented in this example calculation is: $B_{ij} = \frac{1}{2}\overline{Q}_{ij}3t^2 - \frac{1}{2}\overline{Q}_{ij}3t^2 = 0$.

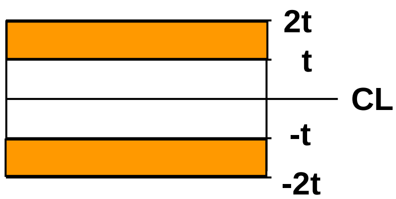

Figure 4.4-8 Mid-Plane Symmetric Laminate

The stiffness matrix and coupling behavior for a unbalanced symmetric laminate is shown in Figure 4.4-9. As shown in the figure, symmetry eliminates the bend extension coupling terms of the [B] matrix.

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Shear-Extension Coupling Terms

Bend-Extension Coupling Terms are Zero

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ \cdots \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \left[\begin{array}{ccc:ccc} A_{11} & A_{12} & \boxed{A_{16}} & 0 & 0 & 0 \\ A_{12} & A_{22} & \boxed{A_{26}} & 0 & 0 & 0 \\ \boxed{A_{16}} & \boxed{A_{26}} & A_{66} & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & D_{11} & D_{12} & \boxed{D_{16}} \\ 0 & 0 & 0 & D_{12} & D_{22} & \boxed{D_{26}} \\ 0 & 0 & 0 & \boxed{D_{16}} & \boxed{D_{26}} & D_{66} \end{array}\right] \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \cdots \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Bend-Extension Coupling Terms are Zero

Bend-Twist Coupling Terms

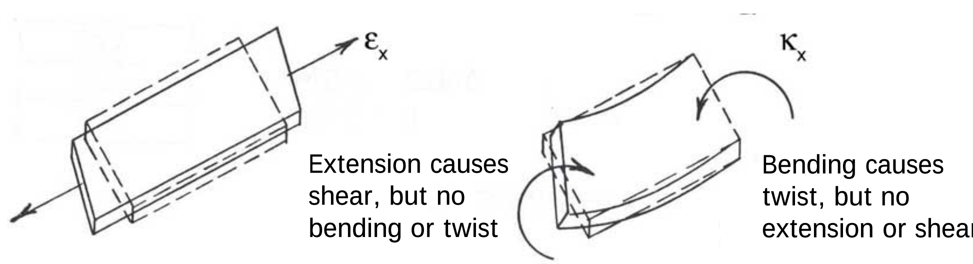

Figure 4.4-9 Unbalanced Symmetric Laminate Behavior

Stacking sequence guidelines in section 2.3.2.1 recommend that laminates use only 0º, ±45 º and 90 º ply angles with maximum and minimum ply percentages specified for each allowed orientation. Because a minimum percentage of ±45 º plies must be present, bend-twist coupling terms in the [D] matrix can not be eliminated. Guidelines also recommend that laminates be both balanced and symmetric. The stiffness matrix and coupling behavior for a typical balanced symmetric laminate that meets stacking sequence guidelines is shown in Figure 4.4-10.

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Shear-Extension Coupling Terms are Zero

Bend-Extension coupling terms are Zero

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ \cdots \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & \vdots & 0 & 0 & 0 \\ A_{12} & A_{22} & 0 & \vdots & 0 & 0 & 0 \\ 0 & 0 & A_{66} & \vdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \vdots & D_{11} & D_{12} & D_{16} \\ 0 & 0 & 0 & \vdots & D_{12} & D_{22} & D_{26} \\ 0 & 0 & 0 & \vdots & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \cdots \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Bend-Extension coupling terms are Zero

Bend-Twist Coupling Terms

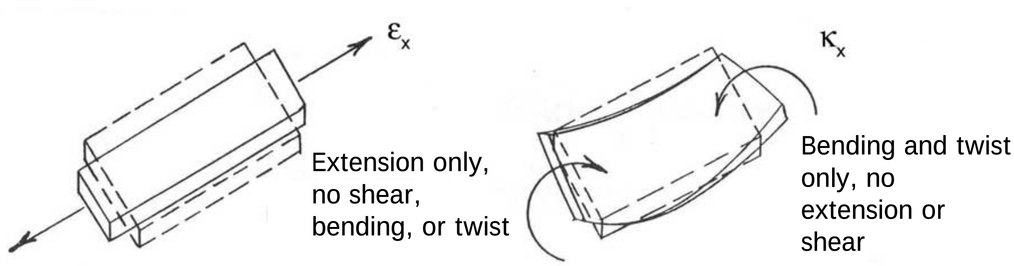

Figure 4.4-10 Balanced Symmetric Laminate Behavior

4.4.1.6.3 Bending Stiffness

The bending stiffness matrix [D] determines laminate bending behavior. The in-plane laminate stress strain relation for pure bending is shown extracted in Equation 4.4-21.

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$ **Equation 4.4-21**

The order of laminae orientation in the stacking sequence is most significant when calculating terms in the bending stiffness matrix [D]. Equation 4.4-16 indicates that the contribution of a ply to bending stiffness is proportional to that ply's distance from the mid-surface cubed. Lamina located farther away from the mid-surface will contribute much more to bending stiffness than a similar stiffness lamina located near the mid-surface. The most efficient way to increase the bending stiffness of a laminate is to add zero degree plies near the outer surfaces.

Example calculations of the $\left(z_k^3 - z_{k-1}^3\right)$ bending stiffness terms for a 10 ply symmetric laminate are shown in Table 4.4-1. The example calculations show that the 5th lamina from the mid-surface carries

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a 61 times factor on the contribution to bending stiffness compared to the 1 times factor on the 1st lamina.

Table 4.4-1 Position Contribution to Bending Stiffness				
k (ply)	**Z_k**	$\left(z_k^3 - z_{k-1}^3\right)$		
1	-4t	$(-4t)^3-(-5t)^3$	$(-64+125)t^3$	$61t^3$
2	-3t	$(-3t)^3-(-4t)^3$	$(-27+64)t^3$	$37t^3$
3	-2t	$(-2t)^3-(-3t)^3$	$(-8+27)t^3$	$19t^3$
4	-1t	$(-1t)^3-(-2t)^3$	$(-1+8)t^3$	$7t^3$
5	0	$(-0t)^3-(-1t)^3$	$(-0+1)t^3$	$1t^3$
--mid-plane--				

Unlike isotropic plates, the bending stiffness of a laminate is dependent on the bending axis. Figure 4.4-11 shows a series of "beam cross section analogs" that depict the difference in bending stiffness for four orientations of a $[0/+45/-45/90]_s$ laminated plate. For the 0˚ direction, the 0˚ plies are on the outside of the laminate and bending stiffness is maximized. For the 90˚ direction, the 0˚ plies are on the mid-surface of the laminate and bending stiffness is minimized.

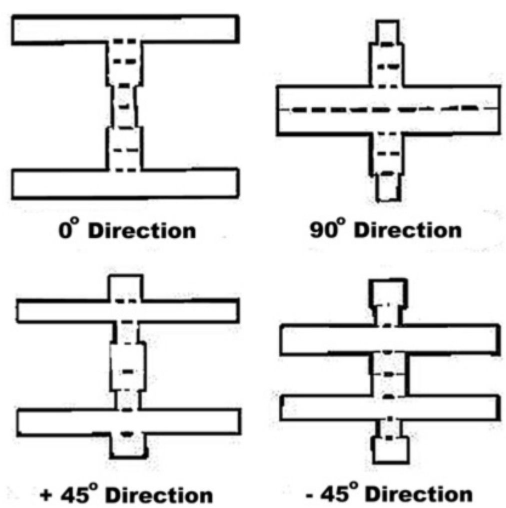

Figure 4.4-11 "Beam Section Analogs" for Bending Stiffness

The D_{16}, and D_{26} terms in Equation 4.4-21 represent coupling between bending and twist. D_{16}, and D_{26} will only vanish for cross ply laminates. Cross ply laminates have 0˚, and 90˚ plies only, with no off-axis angle plies. Laminates with no bend-twist coupling violate the stacking sequence guidelines and are not usually practical. The bend-twist coupling terms for laminates that follow stacking sequence guidelines are usually small compared to the main diagonal terms in the bending stiffness matrix. The effect of bend-twist coupling is negligible.

4.4.1.7 Laminate Effective Elastic Properties

The effective in-plane elastic engineering properties for a laminate are E_x, E_y, G_{xy}, and ν_{xy}. Effective properties approximate the laminate as a homogenous orthotropic plate and are useful for preliminary design calculations.

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Effective elastic properties can be calculated using sub-matrix terms of the inverted CLPT stiffness matrix. As shown in Equation 4.4-22, sub-matrices of the inverted ABD matrix are represented by the lower case letters "*a b h d*". Note that the 3 x 3 sub-matrices *b* and *h* may not necessarily be symmetric themselves, but that the inverted 6 x 6 stiffness matrix is symmetric.

$$[ABD]^{-1} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} = \begin{bmatrix} a & b \\ h & d \end{bmatrix}$$ **Equation 4.4-22**

Sub-matrices *a*, *b*, *d*, and *h*, from the right hand side of Equation 4.4-22 can be calculated using sub-matrices of the ABD matrix as follows:

Let $A^* = A^{-1}$ $\quad H^* = BA^{-1}$
$B^* = -A^{-1}B$ $\quad D^* = D - BA^{-1}B$

$\boldsymbol{a} = A^* - B^*D^{*-1}H^*$
$\boldsymbol{b} = B^*D^{*-1}$
$\boldsymbol{h} = -D^{*-1}H^*$
$\boldsymbol{d} = D^{*-1}$

The effective elastic properties of a laminate are calculated using terms of the inverted sub-matrix "a" as follows in Equation 4.4-23:

$$E_x = 1 / (t * a_{11})$$
$$E_y = 1 / (t * a_{22})$$
$$G_{xy} = 1 / (t * a_{66})$$
$$\nu_{xy} = a_{12} / a_{11}$$
Equation 4.4-23

Where t is the total laminate thickness.

4.4.2 Environmental Effects on Laminate Strength

4.4.2.1 Hygrothermal

The response of lamina materials exposed to variation in temperature and humidity was discussed in section 3.6. The two classes of structurally significant environmental effects on lamina are summarized as follows:

- Changes in material properties including: stiffness, strength, glass transition temperature (T_g), strain energy release rates, and density
- Material expansion and contraction primarily in the matrix-dominated material directions

Environmental changes in engineering properties at the lamina material level affect the strength and stiffness of laminates. Lamina property values used to calculate the strength, stiffness, and failure of a laminate must be consistent with the environment of the analysis load condition.

Material expansion and contraction of laminae due to changes in environmental conditions results in two distinct effects on laminates: First, laminates will expand and contract due to hygrothermal forces

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caused by the difference between the analysis environment and the ambient environment. Second, laminae within the laminate may develop additional strains due to the difference between the analysis environment and the stress-free environment[1] of the laminate. These additional internal lamina strains are referred to as "residual strains". Residual strains are caused by the differential expansion rates of individual plies and ply orientations. Residual strains are real and distinct from hygrothermal and mechanical strains. Residual strains must be combined with the hygrothermal and mechanical strains to accurately predict the failure of a laminate.

Practical analysis of laminates for strains caused by changes in environmental conditions is accomplished in two distinct steps: first the mechanical and hygrothermal laminate strains are calculated and second the internal residual strains are added. This section presents the theory and technique used to calculate the hygrothermal and residual strains on laminae within a laminate.

4.4.2.1.1 Environmental Resultant Forces and Moments

Hygrothermal strains are converted to equivalent resultant forces and resultant moments, and are then applied to the left hand side of the stiffness equation as loads. This rearrangement is possible because the strain components caused by variations in hygrothermal conditions are independent from the mechanical strains. The transformed reduced constitutive relation for a lamina was derived in section 3 and is repeated here as Equation 4.4-24.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left\{ \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k + \begin{Bmatrix} -\alpha_x \Delta T \\ -\alpha_y \Delta T \\ -\alpha_{xy} \Delta T \end{Bmatrix}_k + \begin{Bmatrix} -\beta_x \Delta C \\ -\beta_y \Delta C \\ -\beta_{xy} \Delta C \end{Bmatrix}_k \right\}$$ **Equation 4.4-24**

Examination of Equation 4.4-24 reveals that the hygrothermal components of strain are a function of the change in hygrothermal conditions and are independent of the mechanical strain.

To obtain expressions for the resultant forces caused by changes in temperature and moisture, first substitute the transformed reduced constitutive relation given by Equation 4.4-24 into the resultant force Equation 4.4-7. The result of this substitution is given by Equation 4.4-25.

$$\{N\} = \sum_{k=1}^{n} \left[\overline{Q}\right]_k \left\{ \int_{z_{k-1}}^{z_k} \{\varepsilon^0\} dz + \int_{z_{k-1}}^{z_k} \{\kappa\} z dz + \int_{z_{k-1}}^{z_k} \{-\{\alpha\}_k \Delta T\} dz + \int_{z_{k-1}}^{z_k} \{-\{\beta\}_k \Delta C\} dz \right\}$$ **Equation 4.4-25**

Using CLPT theory, Equation 4.4-25 can be simplified and expressed as follows in Equation 4.4-26.

$$\{N\} = [A]\{\varepsilon^0\} + [B]\{\kappa\} - \{N^T\} - \{N^C\}$$ **Equation 4.4-26**

Where the thermal resultant forces $\{N^T\}$ are given by Equation 4.4-27, and the moisture resultant forces $\{N^C\}$ are given by Equation 4.4-28.

$$\{N^T\} = \int \left[\overline{Q}\right]_k \{\alpha\}_k \Delta T \, dz$$ **Equation 4.4-27**

[1] See section 4.4.2.7 Cure and Stress Free Temperature

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$$\left\{N^C\right\} = \int \left[\overline{Q}\right]_k \{\beta\}_k \Delta C \; dz$$ **Equation 4.4-28**

To obtain expressions for the resultant moments caused by changes in temperature and moisture, first substitute the transformed reduced constitutive relation given by Equation 4.4-24 into the resultant moment Equation 4.4-8. The result of this substitution is given by Equation 4.4-29.

$$\{M\} = \sum_{k=1}^{n} \left[\overline{Q}\right]_k \left\{ \int_{z_{k-1}}^{z_k} \left\{\varepsilon^0\right\} z dz + \int_{z_{k-1}}^{z_k} \{\kappa\} z^2 dz + \int_{z_{k-1}}^{z_k} \left\{-\{\alpha\}_k \Delta T\right\} z dz + \int_{z_{k-1}}^{z_k} \left\{-\{\beta\}_k \Delta C\right\} z dz \right\}$$ **Equation 4.4-29**

Using CLPT theory, Equation 4.4-29 can be simplified as follows in Equation 4.4-30.

$$\{M\} = [B]\left\{\varepsilon^0\right\} + [D]\{\kappa\} - \left\{M^T\right\} - \left\{M^C\right\}$$ **Equation 4.4-30**

Where the thermal resultant moments $\{M^T\}$ are given by Equation 4.4-31, and the moisture resultant moments $\{M^C\}$ are given by Equation 4.4-32.

$$\left\{M^T\right\} = \int \left[\overline{Q}\right]_k \{\alpha\}_k \Delta T \, z dz$$ **Equation 4.4-31**

$$\left\{M^C\right\} = \int \left[\overline{Q}\right]_k \{\beta\}_k \Delta C \, z dz$$ **Equation 4.4-32**

The hygrothermal contributions to combined stress are now expressed as resultant forces and moments which must be added to the mechanical forces and moments applied to a laminate. The thermal resultant forces and the moisture resultant forces are moved to the left side and combined with the applied mechanical loads as shown in Equation 4.4-33 and Equation 4.4-34.

$$\left\{\overline{N}\right\} = \{N\} + \left\{N^T\right\} + \left\{N^C\right\} = [A]\left\{\varepsilon^0\right\} + [B]\{\kappa\}$$ **Equation 4.4-33**

$$\begin{Bmatrix} \overline{N}_x \\ \overline{N}_y \\ \overline{N}_{xy} \end{Bmatrix} = \begin{bmatrix} N_x + N_x^T + N_x^C \\ N_y + N_y^T + N_y^C \\ N_{xy} + N_{xy}^T + N_{xy}^C \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{16} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{16} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$ **Equation 4.4-34**

The thermal resultant moments and the moisture resultant moments are moved to the left side and combined with the applied mechanical loads as shown in Equation 4.4-35 and Equation 4.4-36.

$$\left\{\overline{M}\right\} = \{M\} + \left\{M^T\right\} + \left\{M^C\right\} = [B]\left\{\varepsilon^0\right\} + [D]\{\kappa\}$$ **Equation 4.4-35**

$$\begin{Bmatrix} \overline{M}_x \\ \overline{M}_y \\ \overline{M}_{xy} \end{Bmatrix} = \begin{bmatrix} M_x + M_x^T + M_x^C \\ M_y + M_y^T + M_y^C \\ M_{xy} + M_{xy}^T + M_{xy}^C \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{16} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{16} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$ **Equation 4.4-36**

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Equation 4.4-34 and Equation 4.4-36 are in the original form developed using CLPT theory, but now include the effects of temperature and moisture changes in the expressions for laminate resultant forces and moments.

Let:

$$\left\{N^h\right\} = \left\{N^T + N^C\right\} \qquad \left\{M^h\right\} = \left\{M^T + M^C\right\}$$ **Equation 4.4-37**

The total laminate force is now related to the total laminate strain using CLPT

$$\begin{Bmatrix} \overline{N} \\ \overline{M} \end{Bmatrix} = \begin{Bmatrix} N \\ M \end{Bmatrix} + \begin{Bmatrix} N^h \\ M^h \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix}$$ **Equation 4.4-38**

Where Total Laminate Strain = Mechanical Strain + Hygrothermal Strain.

$$\begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} = \begin{bmatrix} a & b \\ h & d \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} + \begin{bmatrix} a & b \\ h & d \end{bmatrix} \begin{Bmatrix} N^h \\ M^h \end{Bmatrix}$$ **Equation 4.4-39**

When analyzing a laminated aircraft structure, each load condition will typically have a global temperature and a distribution of local temperatures that are applied at each node of the FEA model.[1] Temperature dependent material properties are mapped to each element based on the temperatures applied at adjacent nodes. The net thermal loads applied to the FEA model represent localized changes in temperature from ambient conditions. The applied temperature changes cause the structure to expand or contract resulting in thermal strains and/or thermal resultant forces. Output from the FEA analysis will be the total laminate resultants $\overline{N}$ and $\overline{M}$ as given by Equation 4.4-38, and the total laminate strain as given by Equation 4.4-39.

Expansion or contraction of the laminate due to changes in moisture content is typically ignored when performing an FEA analysis. Moisture absorption primarily affects material properties (including density) and allowables. At Lockheed Martin Aeronautics, the current practice is to include the worst case effects of moisture absorption in the material properties and allowables for a given analysis temperature. Materials are tested “hot wet” and “cold-dry”. Temperature dependent properties and allowables with worst case moisture effects included are obtained from IDAT MATUTL. Residual strains caused by assumed worst case changes in moisture content are included in calculations done by IDAT SQ5.

Thermal expansion or contraction of a laminate can be estimated using an effective Thermal Expansion Coefficient. Using the methods presented in this section, calculate the thermal resultant forces and associated strains caused by a temperature change ΔT. Divide the thermal strains by the change in temperature to obtain the effective Thermal Expansion Coefficient. Equation 4.4-40 shows the two step process.

Thermal Strains $\left\{\varepsilon^0\right\}^T = [a]\left\{N(\Delta T)^T\right\}$ **Equation 4.4-40**

[1] Refer to your program for guidance on applying thermal loads to finite element models.

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Effective Thermal Expansion Coefficient $\{\alpha\}_{effective} = \{\varepsilon^0\}^T / \Delta T$

Carpet plots of laminate Thermal Expansion Coefficients can be produced from within the IDAT tool MATUTL by selecting the "*Carpet Plot/ Thermal Expansion Coef.*" menu item after a material has been selected. An example carpet plot for Thermal Expansion Coefficient is shown in Figure 4.4-12. To use the carpet plot, enter the X axis at the percentage of ±45º plies in the laminate. Proceed in the Y direction to intersect the appropriate percentage 0º plies line. The value of the thermal coefficient is then read at the level of the intersect point on the Y axis scale. The referenced figure shows an example for the X direction of a (50/40/10) laminate.

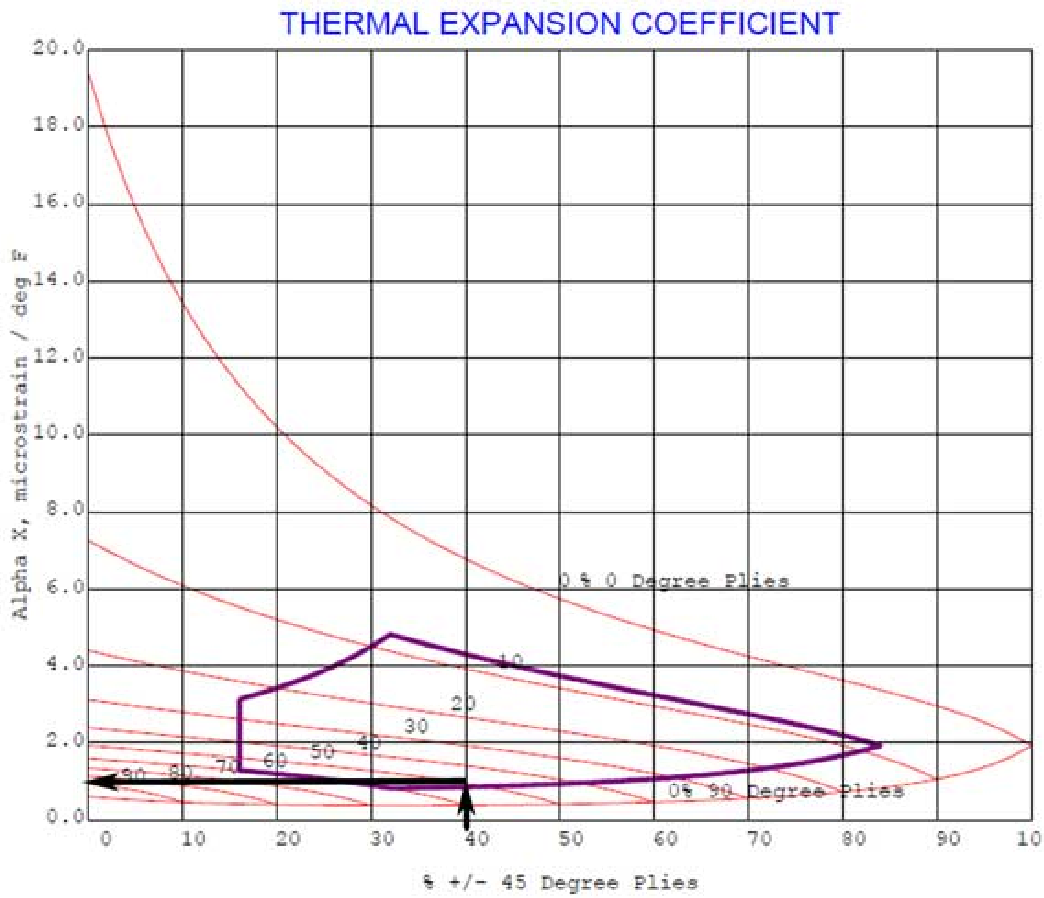

Figure 4.4-12 Carpet plot of Laminate Thermal Expansion Coefficient

4.4.2.1.2 Residual Strains

When considering hygrothermal forces and moments applied to a composite material, it is important to distinguish between forces caused by expansion or contraction of a laminate, and the internal forces caused by differential expansion or contraction of individual plies within the laminate.

In laminates with multiple ply orientations or materials, temperature and moisture changes create differential strains between plies inside the laminate. Stiff fibers in one orientation will constrain contraction or swelling in adjacent plies of different materials and orientations. These internal strains

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are referred to as residual strains. Residual strains are calculated as the difference between free ply hygrothermal strain of the lamina and actual hygrothermal strain of the laminate.

At cure temperature with no external forces applied, we assume the laminate is stress free. (See section 4.4.2.1.4 for a discussion of stress free temperature) As the temperature or moisture content changes, internal stress and strain will develop inside the laminate independent of any external loading. In-plane residual strains are partially[1] responsible for the warping and distortion of composite parts after cool down from cure. Residual stresses and strains are superimposed on external/combined stresses and strains when performing a stress analysis.

If the temperature is low, and/or the conditions are dry (CTD = Cold Temperature Dry), the lamina transverse direction wants to contract, but often can't because of strong stiff fibers in adjacent plies. This creates transverse tension in the lamina.

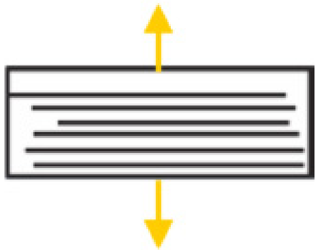

Figure 4.4-13 Low Temperatures Result in High Matrix Tensile Stresses

If the temperature is high, and/or the conditions are wet (ETW = Elevated Temperature Wet), the lamina transverse direction wants to swell, but often can't because of strong stiff fibers in adjacent plies. This creates transverse compression in the lamina.

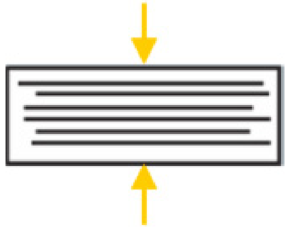

Figure 4.4-14 High Moisture Content Results in High Matrix Compressive Stresses

To solve for the lamina residual strains, first solve for the residual hygrothermal resultants N^h*, and* M^h using Equation 4.4-37 with ΔT and ΔC from "stress free" conditions. Then use CLPT to calculate the laminate residual hygrothermal strain as follows in Equation 4.4-41.

$$\begin{Bmatrix} \varepsilon^{h0} \\ \kappa^{h} \end{Bmatrix} = \begin{bmatrix} a & b \\ h & d \end{bmatrix} \begin{Bmatrix} N^{h} \\ M^{h} \end{Bmatrix}$$ **Equation 4.4-41**

Each ply within the laminate is forced to expand or contract with the laminate. Lamina residual strain is equal to the laminate residual hygrothermal strain from Equation 4.4-41, minus the lamina strain due to free ply expansion. Lamina residual strain is given by Equation 4.4-42 and Equation 4.4-43.

[1] Chemical contraction of the resin primarily in the out of plane direction during cure causes curved sections to increase in curvature

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$$\left\{\varepsilon^{R}\right\}_{k}=\left\{\varepsilon^{h}\right\}_{k}-\left\{\alpha\right\}_{k}\Delta T-\left\{\beta\right\}_{k}\Delta c$$ **Equation 4.4-42**

$$\begin{Bmatrix}\varepsilon_1^R\\ \varepsilon_2^R\\ \gamma_{12}^R\end{Bmatrix}_k=\left[T\right]_k^{-T}\left\{\begin{Bmatrix}\varepsilon_x^{h0}\\ \varepsilon_y^{h0}\\ \gamma_{xy}^{h0}\end{Bmatrix}+\begin{Bmatrix}\kappa_x^h\\ \kappa_y^h\\ \kappa_{xy}^h\end{Bmatrix}z_k\right\}-\begin{Bmatrix}\alpha_1\\ \alpha_2\\ \alpha_{12}\end{Bmatrix}_k\Delta T-\begin{Bmatrix}\beta_1\\ \beta_2\\ \beta_{12}\end{Bmatrix}_k\Delta c$$ **Equation 4.4-43**

The net residual strain in each ply is equal to the strain of the laminate in the ply direction, minus the free expansion of the ply due to ΔT and ΔC.

4.4.2.1.3 Example Problem for Residual Hygrothermal Strain Calculation

Consider a quasi-isotropic [+45/0/-45/90]$_s$ laminate.
Calculate the thermal residual strains at 75ºF (room temperature)

Material: B_IM7/977 tape

T ply = .0053"

Material Properties At 75º F

E_{11} =	22180000.0	psi
E_{22} =	1290000.0	psi
G =	7100000.0	psi
ν_{12} =	0.329	

Thermal Expansion Coefficients

α_{11} =	0.00000060	in/in/ ºF
α_{22} =	0.00001940	in/in/ ºF
γ_{12} =	0.00000000	in/in/ ºF

Stress Free Temperature =	300	ºF
Temperature Change(ΔT) =	-225	ºF
% Moisture Content (ΔC)=	0.0	

Strain allowables

	Compression	**Tension**	
ε_{11} =	-0.011199	0.013261	in/in
ε_{22} =	-0.019438	0.012962	in/in
γ_{12} =	-0.032872	0.032872	in/in

Step 1: Calculate the Thermal Resultant Forces

Equation 4.4-27 $$\left\{N^{T}\right\}=\int\left[\overline{Q}\right]_{k}\left\{\alpha\right\}_{k}\Delta T\,dz$$

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Simplify equation 4.4-27 for a constant ply thickness laminate:

$$\{N^T\} = \left[\sum_{k=1}^{n} [\overline{Q}]_k \{\alpha\}_k\right] t_{ply}\ \Delta T$$

Calculate the $\{\alpha_x\}_k$ vectors by transforming the lamina thermal expansion coefficients to the laminate system using Equation 3.5-31

$$\{\alpha_x\} = [T]^T \{\alpha_1\}$$

	45º	-45º	0º	90º
α_x =	0.0000100	0.0000100	0.00000060	0.00001940
α_y =	0.0000100	0.0000100	0.00001940	0.00000060
α_{xy} =	-0.0000188	0.0000188	0.00000000	0.00000000

Multiply the transformed reduced stiffness matrix by the transformed thermal expansion coefficient for each ply orientation.

$$= [\overline{Q}]_k \{\alpha\}_k$$

45º	-45º	0º	90º
23.56	23.56	21.68	25.44
23.56	23.56	25.44	21.68
-1.88	1.88	0.00	0.00

Perform the summation over all plies and multiply by the ply thickness to obtain the Coefficients of Thermal Force

$$CFT = \sum_{k=1}^{n} [\overline{Q}]_k \{\alpha\}_k t_k$$

CTF_x =	(23.56+23.56+21.68+25.44) x 2 x .0053 =		0.9989
CTF_y =	(23.56+23.56+25.44+21.68) x 2 x .0053 =		0.9989
CTF_{xy} =	(-1.88+1.88+0.0+0.0) x 2 x .0053 =		0.0000

Multiply the coefficients of thermal force by the temperature change to obtain the thermal resultant forces
N = CTF x (-225ºF)

N_x =	-224.76	lb/in
N_y =	-224.76	lb/in
N_{xy} =	0.00	lb/in

Note: In this example the change in moisture content is 0.0%. If the change in moisture content specified for the analysis temperature is not zero, the procedure specified in step one is repeated using alpha moisture x ΔC, and the thermal and moisture forces are added to yield the total hygrothermal force.

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Step 2: Calculate the Laminate Hygrothermal Strain

Calculate the laminate thermal strain using a simplified form of equation 4.4-39 and the hygrothermal resultant force vector calculated in step 1.

$$\left\{\varepsilon^{0}\right\}^{T} = [a]\left\{N(\Delta T)^{T}\right\}$$

ε_x =	-0.0004332	in/in
ε_y =	-0.0004332	in/in
γ_{xy}=	0.0000000	in/in

Step 3: Calculate the Lamina residual strain in each ply using equation 4.4-43

$$\begin{Bmatrix}\varepsilon_1^R\\ \varepsilon_2^R\\ \gamma_{12}^R\end{Bmatrix}_k = [T]_k^{-T}\left\{\begin{Bmatrix}\varepsilon_x^{h0}\\ \varepsilon_y^{h0}\\ \gamma_{xy}^{h0}\end{Bmatrix} + \begin{Bmatrix}\kappa_x^{h}\\ \kappa_y^{h}\\ \kappa_{xy}^{h}\end{Bmatrix} z_k\right\} - \begin{Bmatrix}\alpha_1\\ \alpha_2\\ \alpha_{12}\end{Bmatrix}_k \Delta T - \begin{Bmatrix}\beta_1\\ \beta_2\\ \beta_{12}\end{Bmatrix}_k \Delta c$$

a.) Transform the laminate hygrothermal strain to each ply orientation using the inverse transpose of the transformation matrix.

	45º	**-45º**	**0º**	**90º**
$\boldsymbol{\varepsilon_{11}}$	-0.0004332	-0.0004332	-0.0004332	-0.0004332
$\boldsymbol{\varepsilon_{22}}$	-0.0004332	-0.0004332	-0.0004332	-0.0004332
$\boldsymbol{\gamma_{12}}$	0.0000000	0.0000000	0.0000000	0.0000000

b.) Subtract the free-ply-expansion from each of the transformed ply strains to obtain the hygrothermal residual strains in lamina coordinates.

	45º	**-45º**	**0º**	**90º**
$\boldsymbol{\varepsilon_{11}}$	-0.0002982	-0.0002982	-0.0002982	-0.0002982
$\boldsymbol{\varepsilon_{22}}$	0.0039318	0.0039318	0.0039318	0.0039318
$\boldsymbol{\gamma_{12}}$	0.0000000	0.0000000	0.0000000	0.0000000

Compare the hygrothermal strains to the allowables to see the impact of hygrothermal strains on lamina strength.

Percent of original allowable strain remaining

	45º	**-45º**	**0º**	**90º**
$\boldsymbol{\varepsilon_{11}}$	97.34%	97.34%	97.34%	97.34%
$\boldsymbol{\varepsilon_{22}}$	69.67%	69.67%	69.67%	69.67%
$\boldsymbol{\gamma_{12}}$	100.00%	100.00%	100.00%	100.00%

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4.4.2.1.4 Cure and the Stress Free Temperature

Aerospace laminates made from thermoset resin materials are often cured in an autoclave. An autoclave is an industrial machine that uses heat and controlled pressure to process materials. Heat reduces the viscosity of resin which allows it to flow and evenly coat the fibers. The elevated pressure compacts the material, forcing out excess resin to achieve a higher fiber volume fraction.

In a typical autoclave cure, the temperature is ramped up at a rate of 1º to 5º F per minute while a vacuum is applied. The heat and vacuum assist with removal of moisture and volatile substances present in the resin. Failure to remove these substances prior to cure may result in porosity and other defects in the finished part. Temperature and pressure profiles must be carefully controlled to achieve high quality parts. At a pre-determined temperature the vacuum is vented and the part “soaks” while autoclave pressure is slowly ramped up at approximately 3 psi/minute. The maximum autoclave pressure will vary depending on the material and part being cured. Any volatiles remaining in the matrix material will remain dissolved provided the pressure is maintained while the resin cures. After the desired pressure is reached, the temperature is increased again at a rate of 1º to 5º F per minute until the final cure temperature is reached.

Resins are generally formulated to cure at 250ºF to 350ºF and higher. The part will “soak” at the cure temperature for approximately 3 hours. During this phase, a chemical reaction solidifies the resin and the part is considered to be in a stress free condition. At any temperature other than the cure temperature, variable expansion rates of lamina at different orientations may cause distortion and internal strain. Thermally induced internal strains are referred to as residual strains and are discussed fully in section 4.4.2.1.2. After the “soak” at cure temperature, the temperature is ramped down at approximately 5º F per minute. The pressure is released when the temperature is below 140 ºF.

The stress free temperature for a laminate (designated T_o) is defined as the material and process dependent temperature at which the resin solidifies during cure. Stress free temperature is used primarily in the calculation of residual strains as described in section 4.4.2.1.2. Current practice for structural analysis at Lockheed Martin Aeronautics is to use an effective stress free temperature that is a function of analysis temperature. Figure 4.4-15[1] shows an example of effective $K\Delta T$ vs. use or analysis temperature for a material with T_o equal to 350º F. The factor $K\Delta T$ represents the difference in stress free temperature from the use temperature. For example, at a use temperature of -65°F, the dry $K\Delta T$ obtained from the chart is approximately 225°F. Adding these numbers together indicates that the effective stress free temperature for this material is 160°F at a use temperature of -65°F. At a use temperature of 75°F, the dry $K\Delta T$ obtained from the chart is also approximately 225°F. Adding these numbers together indicates that the effective stress free temperature for this material is 300°F at a use temperature of 75°F. Figure 4.4-15 is provided as an example only and is not intended to be used for calculation of effective stress free temperature. IDAT MATUTL provides the effective stress free temperature for each use or operating temperature of IDAT materials on the “Environmental and Fracture Properties” tab of the “Material Properties Shell”.

[1] Figure 4.4-12 is a reproduction of Figure A2 from Reference 4-13 *CSE-101, Composite Design Methods Handbook*

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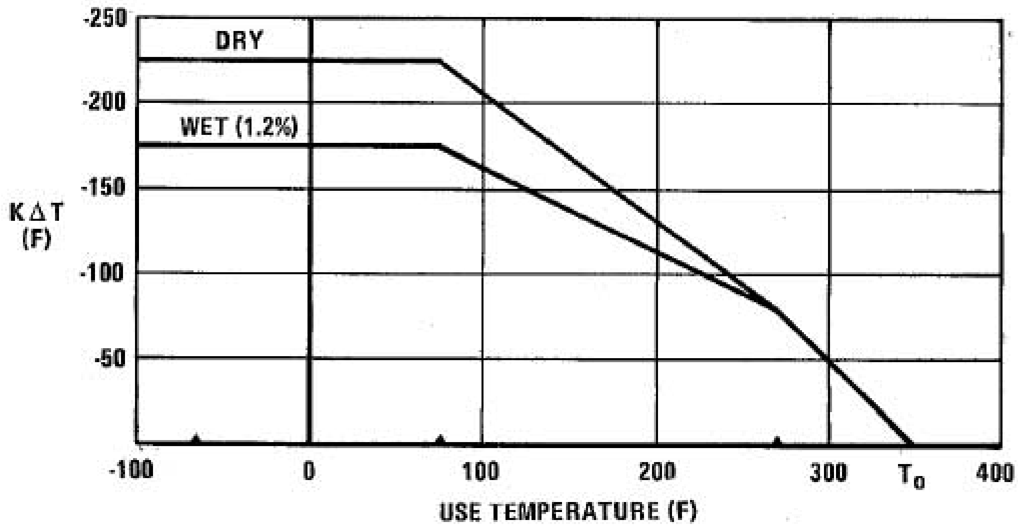

Figure 4.4-15 Effective Delta Temperature

4.4.2.2 Electromagnetic

4.4.2.2.1 Lightning

Lightning damage to aircraft systems can be classified as direct or indirect. Direct damage is caused by direct attachment of the lightning channel to the aircraft structure. Attachment of the lightning channel can cause tearing, bending, burning, vaporization or blasting of the aircraft structure and/or electrical systems. Lightning strikes can cause holes to be burned in the structure, puncturing and splintering, and have been known to weld bearings and hinges. Indirect lightning damage is caused by voltage or current transients induced by lightning in aircraft wiring and or electrical components and systems.

Composite airframe structures must be designed to withstand the direct effects of lightning strikes that are expected to be encountered in service. The extent of damage caused by a lightning strike is a function of the inherent electrical conductivity of the airframe materials. High electrical conductivity dissipates the concentrated energy from a lightning strike and reduces the severity of structural damage. Conductive materials such as aluminum foil, conductive paint, fabric metalized with aluminum or nickel, and embedded aluminum or copper grids have been used to increase conductivity and thus reduce the direct damage caused by a lightning strike.

Requirements for lightning strike protection are program specific. Among the preferred solutions for lightning strike protection of laminated composite materials at Lockheed Martin Aeronautics are sufficient skin thickness and/or an embedded Expanded Copper Foil (ECF) ply. Minimum skin thickness requirements for lightning strike are given in Section 2.3.1.

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4.4.2.2.2 Galvanic corrosion

Galvanic corrosion occurs when two different materials electrically contact each other and are immersed in an electrolyte[1]. In order for galvanic corrosion to occur, an electrically conductive path and an ionically conductive path are necessary. The electrical path occurs when dissimilar materials are in contact. The ionic path is usually moisture or humidity present in the air. These conductive paths create a galvanic couple where the more active metal corrodes at an accelerated rate and the more noble metal corrodes at a retarded rate. The galvanic series for common aerospace materials is shown in Table 4.4-2.

Table 4.4-2 Galvanic Series

MOST ACTIVE (ANODIC)

Magnesium alloys

Aluminum 5000 series

Aluminum 7000 series

Pure aluminum and clad 2000 series

Cadmium

Aluminum 2000 series

Steel and iron

Brass and bronze alloys

Titanium

Nickel

Graphite and carbon

LEAST ACTIVE (CATHODIC)

When dissimilar materials are in contact, large potential differences represented by separation in the galvanic series will cause a galvanic corrosion problem. Note the separation between graphite and aluminum in the galvanic series given by Table 4.4-2. When graphite and /or carbon are in contact with aluminum, the aluminum is susceptible to damage by galvanic corrosion. Imides and metals are also problem combinations.

To prevent galvanic corrosion, dissimilar materials must be separated by a durable non-conductive material. A fiberglass barrier or scrim ply separating dissimilar materials is the preferred method of galvanic corrosion prevention at Lockheed Martin Aeronautics. Insulation, paint, and other coatings have also been used to prevent galvanic corrosion. Design requirements for prevention of galvanic corrosion are typically specified in a program specific “Corrosion Prevention and Control Plan”.

4.4.2.3 Fluids and Chemicals

Epoxy matrix composite laminates are resistant to fluids such as fuel, fuel vapor, hydraulic fluids, and de-icing fluids. Surface sealing to overcome porosity or roughness is recommended in those areas subject to exposure to these fluids and vapors.

The epoxy matrix may be affected by exposure to strong chlorinated solvents or highly acidic or alkaline paint strippers. Strong acidic or alkaline strippers may break down the epoxy molecules

[1] For most aerospace applications, the electrolyte is moisture or humidity present in the air.

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structure and cause complete loss in structural integrity of the surface plies where the solvent is applied. Avoid exposure to these chemicals and apply protective paints.

Applicable standards and test methods for exposure to fluids are specified in Reference 4-15, LMA-PT001B *Standard Test Procedures for Advanced Fiber Reinforced Composite Materials.*

4.4.2.4 Environmental Impact and Erosion

Composite structures must be resistant to rain erosion and impact damage on the leading edges. Some upper and lower composite surfaces are also susceptible to hail and foreign object damage. Materials such as polyurethane or neoprene have proven satisfactory for rain erosion, while materials such as Kevlar or metal caps will afford a higher level of protection from hail or foreign object damage. Most structure is designed with minimum thickness requirements to mitigate damage from impact and erosion. The form of protection used must be repairable or replaceable, and must consider the effects of galvanic corrosion.

Failure analysis for environmental impact and erosion is included within the criteria for composite durability and damage tolerance. Composite durability and damage tolerance criteria are discussed in section 4.5.5.

4.4.3 Interlaminar Stresses

The three stress components associated with the thickness or out-of-plane direction of a laminate (σ_z, τ_{xz}, and τ_{yz}) are commonly referred to as interlaminar stresses. Out-of-plane stresses are typically critical at the interlaminar boundaries where a thin layer of relatively weak matrix material bonds laminae together. Sufficient out-of-plane stress at an interlaminar boundary will initiate an area of delamination[1]. Once initiated, a delamination can grow leading to failure of the laminate. The term interlaminar stress follows from the critical role of out-of-plane stress in delamination failure of the interlaminar bonds. Delamination is a primary failure mode in laminated composite plates.

In addition to interlaminar stress caused by out-of-plane loading, specific laminate geometry and in-plane loading conditions can result in significant levels of interlaminar stress that are not predicted by Classical Laminated Plate Theory. CLPT assumes a plane stress state and does not predict interlaminar stress. Typical areas of concern are corner radii, near structural discontinuities such as ply drop-offs, and near laminate free edges (cutouts, stiffener flanges, or edges of laminates). Accurately predicting interlaminar stress in these areas calls for a solution approach that considers three-dimensional elastic equilibrium. Stress distributions in the laminate away from corner radii, free edges, and discontinuities are adequately described by CLPT.

For a given in-plane load condition and critical geometric configuration, the magnitude and distribution of interlaminar stress through the thickness is highly influenced by stacking sequence. Balanced symmetric laminates with the same ply orientations but different stacking sequences have been shown to fail at different axial loads. The differences in failure load are caused by differences in interlaminar stresses.

Prevention of laminate failure caused by high interlaminar stress is addressed through both laminate design guidelines and specialized analysis methods. The laminate stacking sequence and design guidelines outlined in section 2.3.2 of this manual include rules intended to minimize the magnitude of interlaminar stress. Additional analysis in critical areas is accomplished using tools such as IDAT ESDU 9528 for interlaminar free edge stress and IDAT BEND for interlaminar stress in corner radii.

[1] Delamination is defined as a failure of the bond between two laminae

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The geometry and loading conditions which lead to high interlaminar stress are discussed in this section. Theory and recommended methodologies to predict interlaminar stresses are discussed in the remainder of section 4.4.3

4.4.3.1 Interlaminar Shear

CLPT does not account for shear that can occur in thick laminates or shear from locally applied resultants. Interlaminar shear can become significant if the plate thickness is greater than 10 percent of the length or width of the plate. This section provides equations which can be used to approximate the interlaminar shear stress on ply *k* due to shear loads in laminates. Derivation of the equations for interlaminar shear stress that follow can be found in Reference 4-14 *FZM 5494*. These equations assume a symmetric laminate, and neglect cross derivative terms.

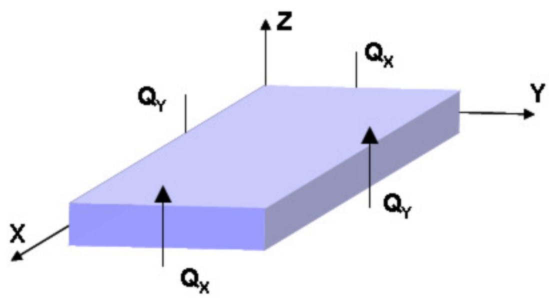

Figure 4.4-16 Shear Loads Applied to a Laminate Edge

Given an infinite mid-plane symmetric (but not necessarily balanced) laminated plate loaded with shear resultants Q_x and Q_y as shown in Figure 4.4-16, and neglecting certain plate theory cross terms, interlaminar shear stresses in the kth ply (τ_{xz}^k, τ_{yz}^k) can be calculated using Equation 4.4-44 and Equation 4.4-45.

$$\tau_{xz}^k = (1/8)(4z_k^2 - h^2)(\overline{Q}_{11}^k \times W_x + \overline{Q}_{26}^k \times W_y)$$ **Equation 4.4-44**

$$\tau_{yz}^k = (1/8)(4z_k^2 - h^2)(\overline{Q}_{16}^k \times W_x + \overline{Q}_{22}^k \times W_y)$$ **Equation 4.4-45**

Where:

$$W_x = (-D_{22} \times Q_x + D_{26} \times Q_y)/(D_{11} \times D_{22} - D_{16} \times D_{26})$$ **Equation 4.4-46**

$$W_y = (D_{16} \times Q_x - D_{11} \times Q_y)/(D_{11} \times D_{22} - D_{16} \times D_{26})$$ **Equation 4.4-47**

Physically, $W_x = d^3w/dx^3$, and $W_y = d^3w/dy^3$

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4.4.3.2 Interlaminar Stress Due to Bending in Curved Laminates

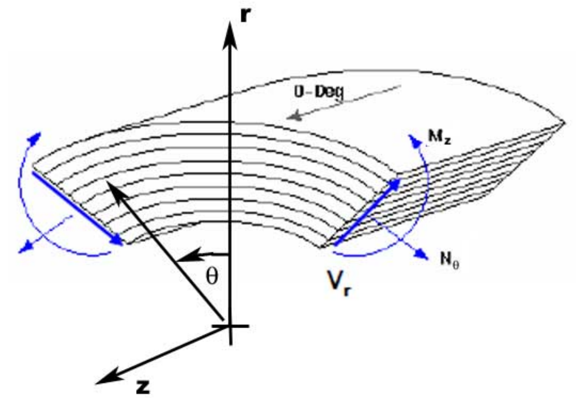

Figure 4.4-17 Curved Composite Section

Interlaminar stresses occur in corner radii as deformations from applied loads increase or decrease the radius of curvature. Figure 4.4-17 shows a curved laminate section with applied loads and a local cylindrical coordinate system. The strain displacement relations in cylindrical coordinates are given by Equation 4.4-48.

$$\varepsilon_r = \frac{\partial u}{\partial r}, \quad \varepsilon_\theta = \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta}, \quad \varepsilon_z = \frac{\partial w}{\partial z}$$ **Equation 4.4-48**

The radial displacement coupling term (u/r) in the circumferential strain (ε_θ) of Equation 4.4-48 allows potential energy due to bending about the curved axis to be minimized through a combination of circumferential strain and radial displacements. When a curved section is bent, laminae move towards the neutral axis or away from the neutral axis according to the sign of the moment applied. The variation of radial displacements through the laminate thickness creates out of plane normal stress σ_r within the curved cross section. Following the moment sign convention and curvature of Figure 4.4-17, positive moments create tensile normal stresses which act to pull the laminae apart (delamination) and negative moments create compressive normal stresses which act to push the laminae together.

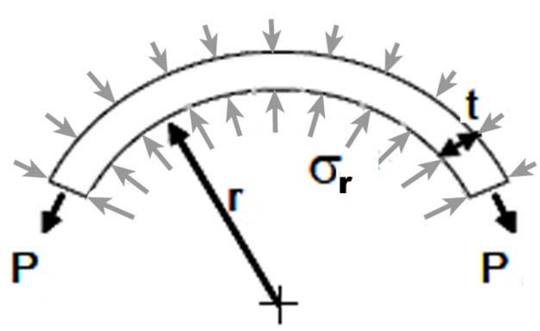

Figure 4.4-18 Forces Acting on a Single Curved Lamina

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The coupled forces acting on each ply can be compared to the hoop tension forces in pressure vessels. In a pressure vessel, internal radial pressure force is balanced by circumferential tension in the pressure vessel wall. In a curved laminate, the circumferential ply loads caused by bending are in part balanced by net radial forces that develop between laminae. Figure 4.4-18 shows the net radial force on a ply caused by the variation in radial stress between the inner and outer surfaces of the ply. A key difference for curved laminates is that the sum of radial forces through the thickness will be zero provided that no net external pressure forces are applied.

In the general case for orthotropic laminates, circumferential stiffness varies discontinuously from ply to ply through the thickness. The discontinuous variation in stiffness causes a discontinuous variation in circumferential stress which in turn affects the magnitude and distribution of interlaminar normal stress through the laminate thickness. Methods used to predict interlaminar normal stress for curved laminates must properly account for the discontinuous variation in stiffness.

The detailed methodology for calculation of interlaminar stress in curved sections is presented in FZM 9254 *Composite Curved Beam Interlaminar Tension Analysis*. The methodology presented in the reference FZM is implemented in the IDAT software BEND. Use of the IDAT software BEND is recommended for prediction of interlaminar stress due to bending in curved laminated sections.

4.4.3.3 Interlaminar Free Edge Stresses

Interlaminar free edge stresses are a boundary layer or edge effect. A free edge boundary for interlaminar stress may exist along any unsupported-unloaded laminate edge and along discontinuities such as ply drop-offs and splices. The width of the affected boundary region is approximately equal to the thickness of the laminate.

A free edge boundary orientation parallel to the laminate x-axis is shown in Figure 4.4-19. For the orientation shown, free edge boundary conditions specify that stress components σ_y, τ_{xy}, and τ_{yz} are zero along the free edge indicated in the figure.

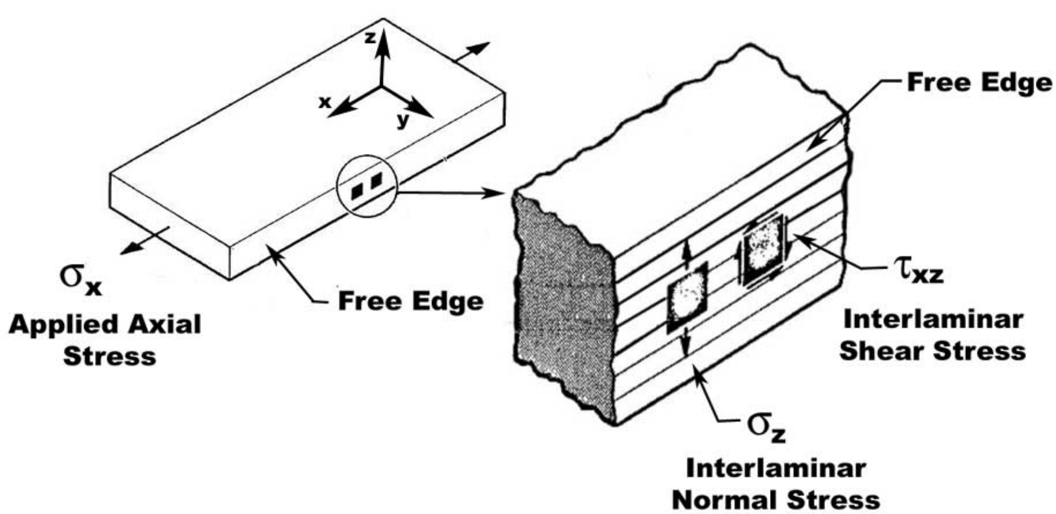

Figure 4.4-19 Interlaminar Free Edge Stress Due to Applied Axial Load

Boundary conditions for the free edge orientation shown in Figure 4.4-19, are given by Equation 4.4-49.

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$$\sigma_y = 0, \quad \tau_{xy} = 0, \quad \tau_{yz} = 0$$ **Equation 4.4-49**

The variation of in-plane shear stress and the development of interlaminar shear and interlaminar normal stresses within the free edge boundary region are not predicted by CLPT. Within the free edge boundary shown in Figure 4.4-19, interlaminar normal stress (σ_z) and interlaminar shear (τ_{xz}) become non-zero as the free edge is approached, while in-plane shear (τ_{xy}) goes to zero. Axial stress (σ_x) changes very little within the boundary and is adequately predicted by CLPT. A representative profile of the changes in interlaminar normal stress at a free edge boundary is shown in Figure 4.4-20.

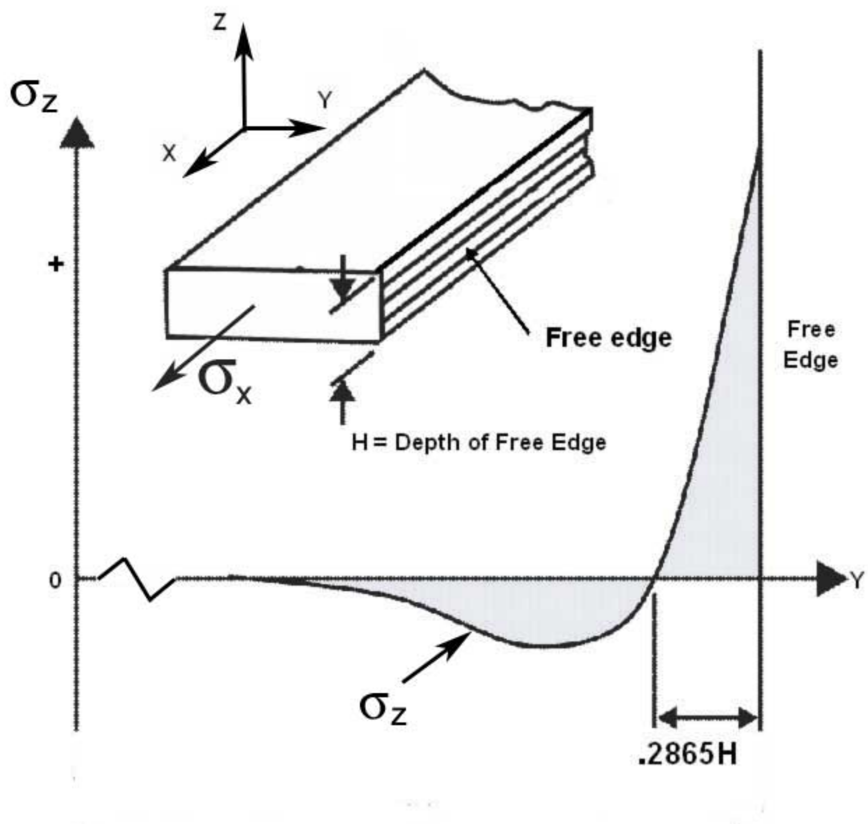

Figure 4.4-20: Interlaminar Normal Stress (σ_z) Profile at a Free Edge Boundary

Interlaminar free edge stresses are a byproduct of orthotropic ply material stiffness, stacking sequence, and axial loading. Orthotropic off-axis laminae develop in-plane shear stresses (τ_{xy}) due to axial loads on the laminate. The in-plane shear stress that develops in off-axis laminae must go to zero at the free edge if no externally applied in-plane shear is available to balance the internal force.

Three-dimensional elastic equilibrium conditions given by Equations 4.4-50 are satisfied in the boundary region through a complex redistribution of load into σ_z and τ_{xz}, as τ_{xy} goes to zero.

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$$\begin{aligned}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0 \\
\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0 \\
\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0
\end{aligned}$$

Equation 4.4-50

When the axial direction is X, we assume that stress does not vary in the x direction ($\frac{\partial}{\partial x} \approx 0$). This assumption leads to the simplification of the equilibrium Equations 4.4-50 given by Equations 4.4-51.

$$\begin{aligned}
\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0 \\
\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0 \\
\frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0
\end{aligned}$$

Equation 4.4-51

The simplified equilibrium Equations 4.4-51 are solved to give the expressions for the interlaminar stresses found in Equations 4.4-52.

$$\begin{aligned}
\tau_{xz} = -\int_{z_{k-1}}^{z_k} \frac{\partial \tau_{xy}}{\partial y}\, dz + \text{Constant} \\
\sigma_z = -\int_{z_{k-1}}^{z_k} \frac{\partial \tau_{yz}}{\partial y}\, dz + \text{Constant}
\end{aligned}$$

Equation 4.4-52

Where:

$$\tau_{yz} = -\int_{z_{k-1}}^{z_k} \frac{\partial \sigma_y}{\partial y}\, dz + \text{Constant}$$

In Equation 4.4-52, the constant terms are evaluated for continuity at the ply interfaces and the interlaminar stresses are set to zero at the laminate top and bottom surfaces.

The distribution of interlaminar stress through the thickness at a free edge boundary is governed by <u>ply values</u> for τ_{xy}, and σ_y. Ply values of τ_{xy}, and σ_y for off-axis laminae are in general non-zero even in the free edge case where no external shear or transverse loads are applied. Cross ply laminae will have σ_y, and no τ_{xy}. Because the constant terms in Equation 4.4-52 are evaluated for continuity at the ply interfaces, the interlaminar stresses are a strong function of stacking sequence order. Figure 4.4-21 shows an example of the variation in interlaminar normal stress σ_z due to changes in stacking sequence. Note that the laminates in Figure 4.4-21 are subject to the same loading and contain the same ply orientations arranged in different stacking sequence order.

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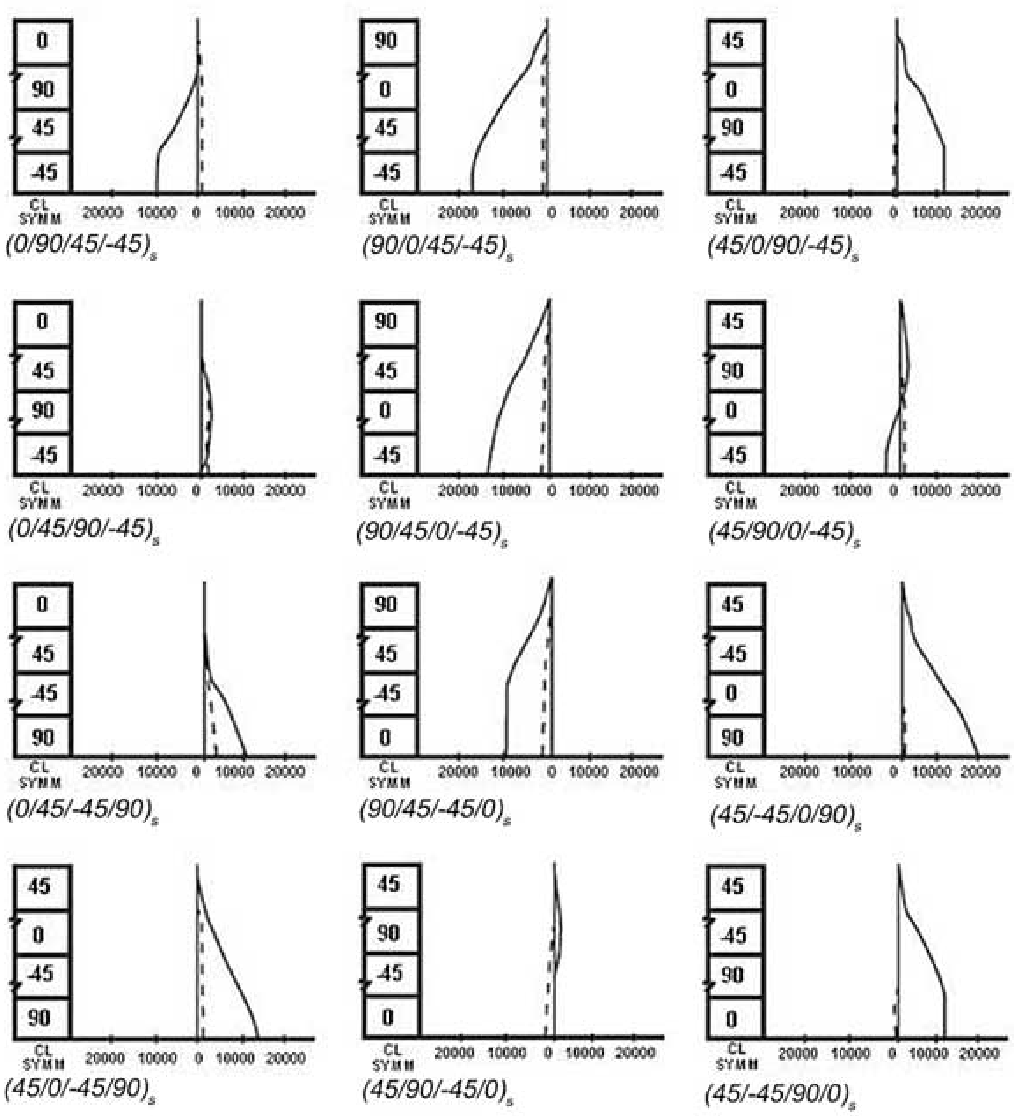

Figure 4.4-21 Free Edge Stress σ_z (psi) for 50,000 psi Applied Axial Load

Laminate design and stacking sequence guidelines given in section 2.3.2 are in part intended to minimize interlaminar free edge stresses. Further optimization of stacking sequences to minimize free edge stresses must be approached carefully. A stacking sequence that minimizes free edge stresses for one load condition will have large free edge stresses under reverse loading. Follow the stacking sequence guidelines provided in section 2 of this manual to minimize the magnitude of interlaminar stresses.

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4.5 Laminate Failure Criteria

The ideal failure load of a laminate is calculated using CLPT strain predictions and un-notched failure criteria. Un-notched failure criteria assume a perfect laminate with no defects or imperfections. The effects of holes, notches, manufacturing defects, interlaminar tension, interlaminar shear, and anticipated in-service damage are accounted for using a variety of other failure criteria. Notched failure criteria are generated using a combination of empirical data and theoretical calculations. The use of a specific criterion is driven by the presence of specific design features such as holes, notches, and corners, as well as general requirements for composite durability and damage tolerance.

4.5.1 Un-notched Laminate Failure

Un-notched failure criteria assume that composite laminates contain no damage or defects. Detrimental effects of common design features such as fastener holes, ply drop-offs, and part edges are also not considered. Un-notched laminate strength predictions represent “ideal” conditions and are rarely used in isolation to size composite parts. Un-notched failure prediction forms the basis for notched failure criteria which will be discussed later.

4.5.1.1 Maximum Strain Failure Criteria

Un-notched laminate failure analysis at Lockheed Martin Aeronautics uses Maximum Strain Failure Criteria with CLPT. Maximum strain failure criteria require that lamina strain components measured in the lamina “1-2” coordinate system are within the bounds of maximum and minimum failure strain values. No interaction formulas are applied to the strain components. Equation 4.5-1 provides a mathematical representation of the Maximum Strain Failure Criteria.

$$\begin{aligned} &\varepsilon_{11}^{c} < \varepsilon_{11} < \varepsilon_{11}^{t} \\ &\varepsilon_{22}^{c} < \varepsilon_{22} < \varepsilon_{22}^{t} \\ &\left|\gamma_{12}\right| < \gamma_{12}^{s} \end{aligned} \qquad \textbf{Equation 4.5-1}$$

Where:

ε_{11}^{c} is the maximum allowable compressive strain in the 11 direction.

ε_{11}^{t} is the maximum allowable tensile strain in the 11 direction.

ε_{22}^{c} is the maximum allowable compressive strain in the 22 direction.

ε_{22}^{t} is the maximum allowable tensile strain in the 22 direction.

γ_{12}^{s} is the maximum allowable shear strain in the 12 direction.

The Maximum Strain Failure Criteria envelope is shown in Figure 4.5-1.

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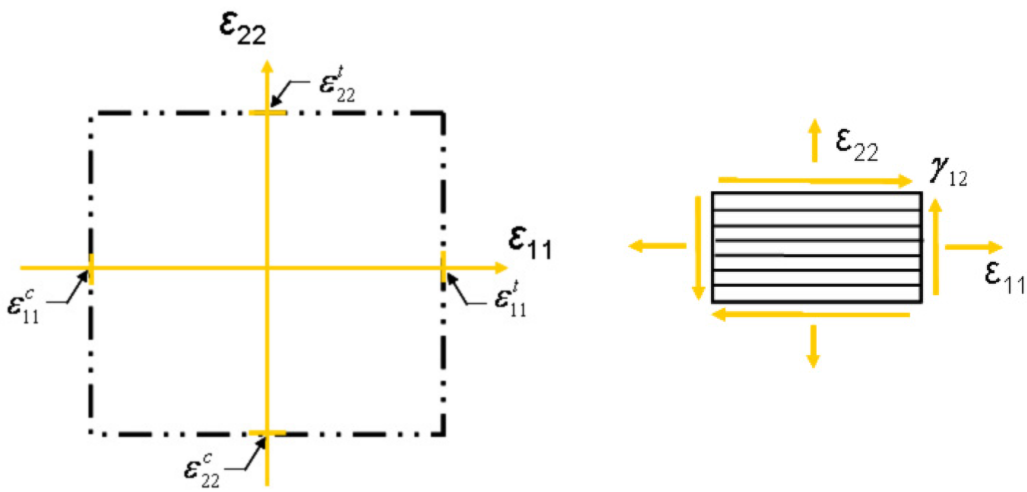

Figure 4.5-1: Maximum Strain Failure Criteria envelope

Maximum Strain Failure Criteria are applied to each ply in a laminate to predict failure. First ply failure determines the ultimate allowable load for a laminate and limit load is calculated as 2/3 of the ultimate value.

4.5.1.2 Un-notched Laminate Margin of Safety

The critical Margin of Safety (M.S.) reflects how much mechanical strength remains given an applied load and a residual condition. Applied mechanical strains in each ply are compared to an allowable mechanical strain minus the residual strain using the Margin of Safety Equation 4.5-2. Each orthotropic lamina will have five allowable strains (ε_{11}^{c}, ε_{11}^{t}, ε_{22}^{c}, ε_{22}^{t} , and γ_{12}^{s}) and thus five margins will be calculated for each lamina.

$$M.S. = \frac{\varepsilon_{allow} - \varepsilon_{residual}}{\varepsilon_{applied}} - 1 \qquad \textbf{Equation 4.5-2}$$

The residual strain ($\varepsilon_{residual}$) in Equation 4.5-2 is primarily the result of hygrothermal effects and assembly forces. Residual strains must be accounted for either explicitly or implicitly in the margin calculation. In some cases, hygrothermal residual strains are implicitly included in the material allowable strains. Methodology for calculating hygrothermal residual strains was discussed in section 4.4.2.1.2. IDAT SQ5[1] currently includes calculations for residual strains from temperature and moisture. Refer to your program for guidance on the current policy for inclusion of residual strain in Margin of Safety calculations.

4.5.1.3 Un-notched Material Properties and Allowables

Un-notched allowable maximum and minimum strain values are established through a process of laminate coupon testing and analysis. Applicable standards and test methods for laminated composite coupons are specified in Reference 4-15, LMA-PT001B *Standard Test Procedures for Advanced Fiber*

[1] SQ5 is a Lockheed Martin Aeronautics analysis program that predicts un-notched failure of composite laminates using CLPT, secant stiffness properties, hygrothermal effects, and the maximum strain failure criteria.

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Reinforced Composite Materials. IDAT SQ5 predictions are correlated to laminate coupon test data by adjusting lamina stiffness data and material failure allowables to obtain a best conservative fit. The un-notched allowables development methodology creates a link between material properties, failure allowables, and the failure prediction methodology and criteria in SQ5.

The un-notched allowables development process is shown in Figure 4.5-2. Lamina stiffness properties are established using coupon tests of $[0]_t$, $[90]_t$, and $[\pm 45]_t$ laminates. The lamina maximum/minimum strain allowables for tension and compression (ε_{11}, ε_{22}) are established by testing a family of (0/±45/90) laminates at specified temperature and moisture combinations. A minimum of three different (0/±45/90) stacking sequences are considered necessary to establish valid allowables, with seven different laminates being ideal. The values for shear (γ_{12}) are developed from tests of ±45˚ laminate tensile coupons. Material properties and failure allowables are adjusted to achieve a best conservative fit of the test data with predictions from SQ5. The process establishes allowable strain values for a specific material that are temperature and moisture content dependent.

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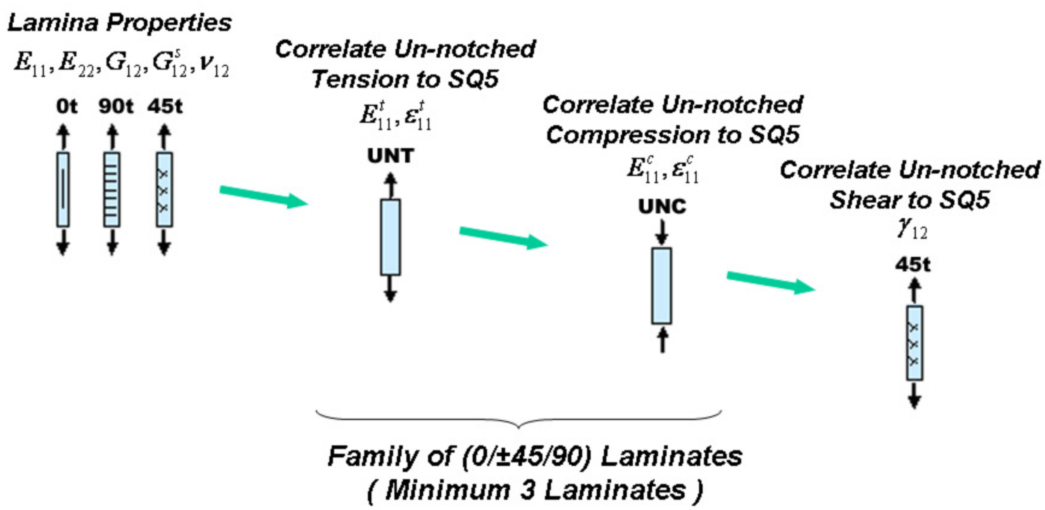

Figure 4.5-2: Un-notched allowables development process

Approved un-notched lamina stiffness properties and allowables are available in the IDAT utility MATUTL. Elastic properties and allowables are obtained by selecting the "Basic Properties" tab on the MATUTL application. Maximum and minimum allowable strain values are located in the areas identified by the red outlines in the MATUTL example shown in Figure 4.5-3.

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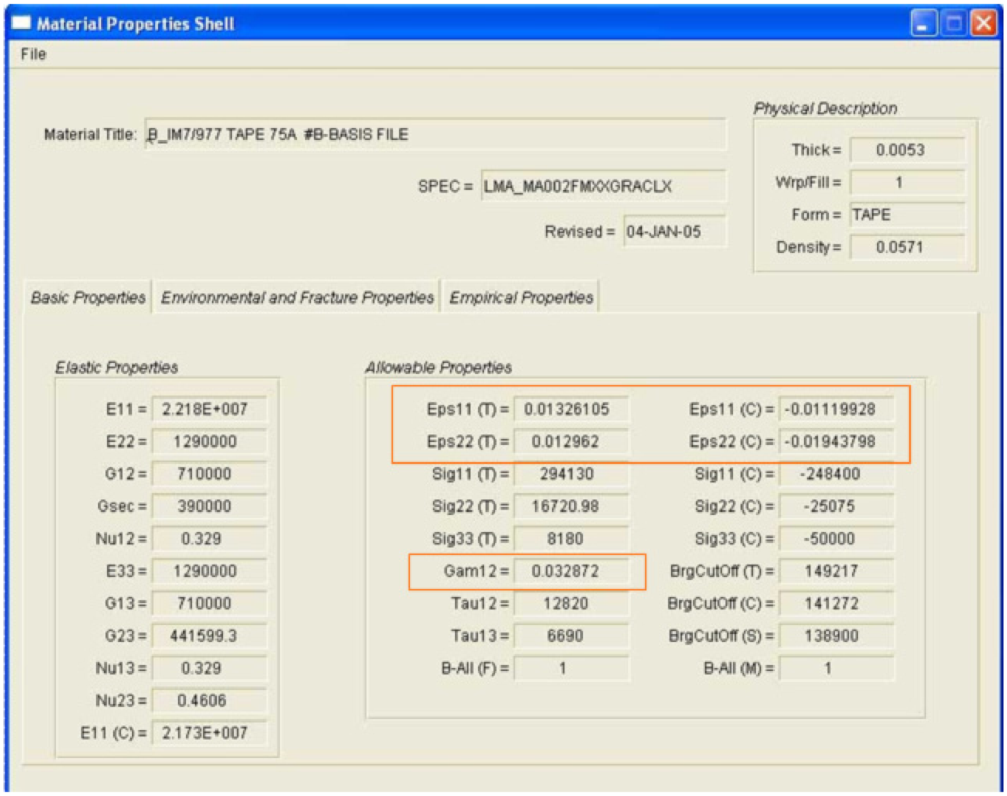

Figure 4.5-3: IDAT MATUTL Material Properties

4.5.1.4 Laminate Transverse Tension Failure

The value that MATUTL provides for ε_{22}^{t} (=Eps22 (T)) appears higher than might be expected for the transverse direction of a uni-axial tape material. The artificially high transverse tension allowable is a mechanism that prevents laminate failure predictions caused by transverse matrix tension. By increasing the allowable, transverse matrix tension failures are automatically ignored with no special action required from the analyst. Use of this value is only valid for (0/±45/90) laminates which comply with the minimum ply percentage guidelines. Plies oriented in the transverse direction are assumed to carry the load while the adjacent transverse plies develop matrix tension cracks. The value of ε_{22}^{t} provided by MATUTL is retained for legacy analysis methods, but is not currently used by SQ5. SQ5 uses a pseudo tension transverse strain allowable (ε_{22}^{tp}), that is calculated using Equation 4.5-3.

$$\varepsilon_{22}^{tp} = \varepsilon_{11}^{t} - \alpha\Delta T - \beta\Delta M \qquad \textbf{Equation 4.5-3}$$

Where:
α = coefficient of thermal expansion, β = coefficient of moisture expansion,
ΔT = temperature change, and ΔM = moisture change

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The value of ε_{22}^{tp} calculated by Equation 4.5-3 is similar in magnitude to the legacy value presented in MATUTL.

A closer look at the MATUTL allowable data reveals that while the transverse allowable tensile strain has been artificially increased, the allowable stress and transverse stiffness values remain unchanged. The lamina transverse failure response is assumed to be elastic plastic, with the maximum tension load governed by the transverse stress allowable and the maximum strain governed by the pseudo tension strain allowable. The transverse secant modulus is introduced in the un-notched failure analysis to limit the stress (load) carried by plies in the transverse direction and approximate the elastic plastic transverse response.

The transverse secant modulus (E_{22}^{s}) approximates the elastic plastic non-linearity in the assumed transverse response. The transverse secant modulus is calculated from the pseudo tension allowable as follows in Equation 4.5-4.

$$E_{22}^{s} = \frac{\sigma_{22}^{t}}{\varepsilon_{22}^{tp}}$$ **Equation 4.5-4**

The assumed transverse elastic plastic response and the approximation given by the secant shear modulus is shown in Figure 4.5-4.

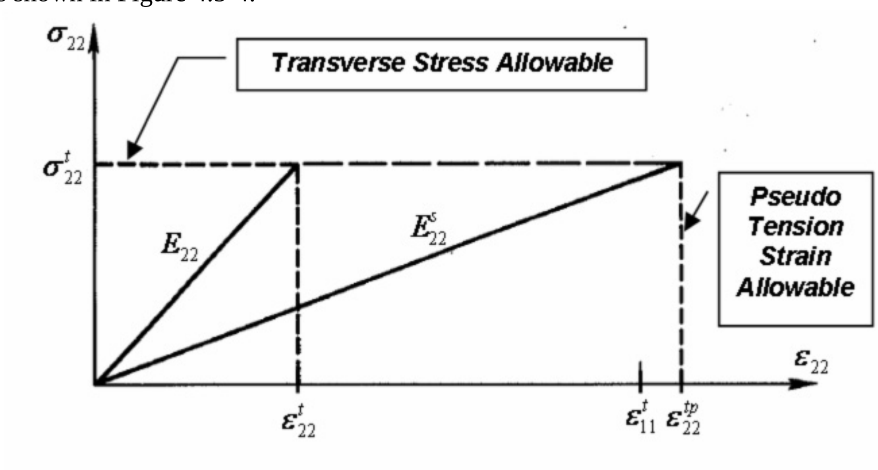

Figure 4.5-4: Transverse Elastic Plastic Response and the Secant Shear Modulus

4.5.1.5 Un-notched Failure Strain Calculation

Element resultants (usually obtained from a finite element analysis) are input to CLPT as shown by Equation 4.5-5, to obtain predicted laminate mid-plane strains and curvatures that will be used for un-notched failure analysis.

$$\begin{Bmatrix} \{\varepsilon^{\circ}\} \\ \{\kappa\} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} a & b \\ h & d \end{bmatrix} \begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix}$$ **Equation 4.5-5**

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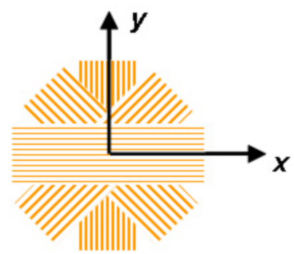
The inverse [ABD] matrix in Equation 4.5-5 is calculated using secant properties for shear ($G_{12} = G_{12}^{\sec}$) and secant properties for transverse stiffness ($E_{22} = E_{22}^{\sec}$). Secant properties approximate non-linear shear stiffness G_{12}^{s} as discussed in section 3.5.4.3, and non-linear transverse stiffness E_{22}^{s} as discussed in section 4.5.1.4. Note that secant properties are not used when performing a Finite Element Analysis to obtain stress resultants. Secant properties are only used in strain calculation for failure prediction.

Equation 4.5-5 predicts mid-plane strains and curvatures in the laminate coordinate system that must be transformed to ply-level strains using the kinematic Equation 4.4-5 and the appropriate transformation matrix for each ply angle. Ply-level strains in the lamina coordinate system can be compared to the un-notched maximum strain allowables to predict failure. The process for calculating strains in the lamina coordinate system is illustrated in Figure 4.5-5.

Each lamina has a unique set of strains which can be determined from the laminate mid-plane strains and curvatures using the kinematic equation and the ply z coordinates.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + z_k \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

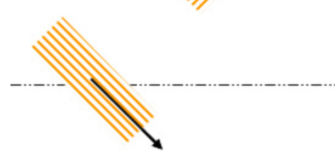


Ply strains in the laminate coordinate system are then transformed to the appropriate coordinate system for each lamina.

$$At\ 0^\circ, \{\varepsilon_{(1,2)}\} = [T(0^\circ)]^{-T} \{\varepsilon_{(x,y)}\}$$

$$At\ +45^\circ, \{\varepsilon_{(1,2)}\} = [T(+45^\circ)]^{-T} \{\varepsilon_{(x,y)}\}$$

$$At\ -45^\circ, \{\varepsilon_{(1,2)}\} = [T(-45^\circ)]^{-T} \{\varepsilon_{(x,y)}\}$$



$$At\ 90^\circ, \{\varepsilon_{(1,2)}\} = [T(90^\circ)]^{-T} \{\varepsilon_{(x,y)}\}$$

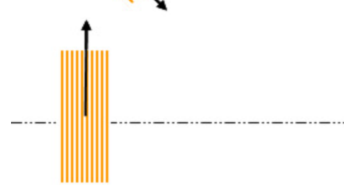


Figure 4.5-5: Calculation of Un-notched Lamina Strains

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4.5.1.6 Un-notched Laminate Failure Interaction Diagrams

An un-notched interaction diagram shows the allowable in-plane stress envelope for laminates with a specified combination of (0/±45/90) ply percentages. The failure envelope is calculated using maximum strain failure criteria. Laminate failure interaction diagrams are actually three dimensional in σ_x, σ_y, and τ_{xy}. Interaction diagrams are depicted in two dimensions, σ_x and σ_y, with the third dimension variable τ_{xy}, appearing as cutoff lines. Un-notched interaction diagrams can be produced using IDAT SQ5 by choosing “view interaction diagram” from the “graphics” menu. An example interaction diagram for a (25/50/25) laminate is shown in Figure 4.5-6.

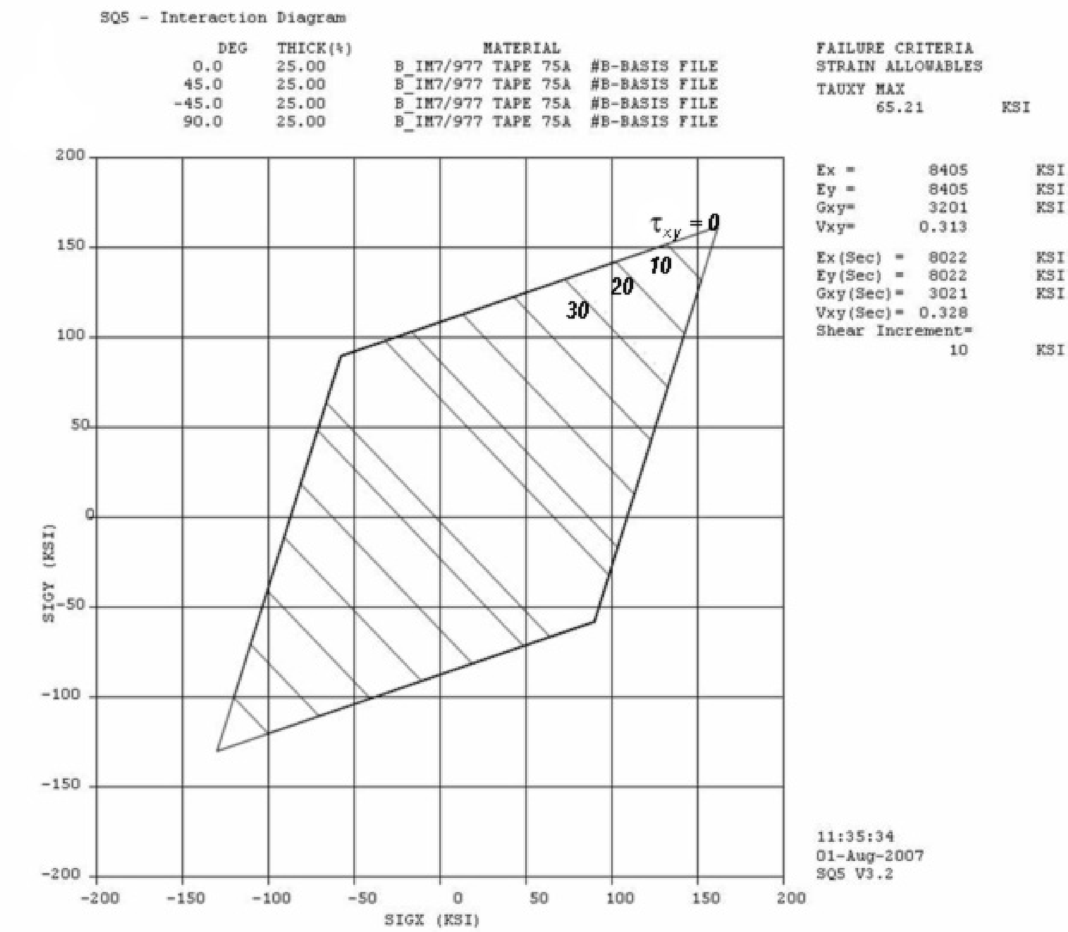

Figure 4.5-6: SQ5 Interaction Diagram

To use the interaction diagram, determine the stress state σ_x (SIGX), σ_y (SIGY), and τ_{xy} for the laminate being analyzed. On the interaction diagram for the laminate, plot a point where the axial and transverse stresses intersect. If the point lies within the boundary of the diagram (including the cut-off lines for shear) the un-notched margin of safety will be positive. An example use of the interaction diagram is shown in Figure 4.5-7.

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Example:

$$\sigma_x = 100\ Ksi$$
$$\sigma_y = 100\ Ksi$$
$$\tau_{xy} = 20\ Ksi$$

The green boundary on the figure at right shows the allowable stress region for a laminate with 20 Ksi shear load. Since the intersection point for the applied stress σ_x, and σ_y is inside the boundary, the margin for this load condition will be positive when calculated using SQ5.

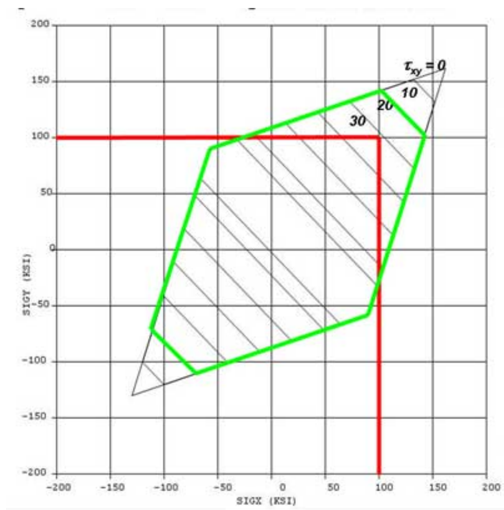

Figure 4.5-7: Example Use of Interaction Diagram

4.5.2 Notched Laminate Failure Criteria

Practical design requirements have led to the development of “notched” analysis methods and allowables. Practical composite structures must retain positive structural margins when undetected flaws, small holes, defects, free edges and minor damage are present. Un-notched failure as predicted by CLPT assumes an ideal laminate and does not consider the detrimental effects of holes and defects on laminate strength. Notched failure criteria implicitly accounts for the presence of small holes, notches, matrix cracks, free edges, certain ply drop-offs, undetected flaws, and damage by equating their effect on laminate strength to a filled or open circular hole. Notched analysis is generally considered sufficient to satisfy the analysis requirements for durability and many small stress concentrations.

4.5.2.1 IBOLT Notched Failure Analysis and Criteria

Notched failure analysis at Lockheed Martin Aeronautics is performed using the Lockheed Martin proprietary theory and methodology contained in IBOLT. IBOLT is an IDAT software tool that implements the IBOLT theory and methodology contained in Reference 4-11, *FF-93-92* and Reference 4-12, *FZM-9065*. IBOLT calculates a fracture mechanics based static strength prediction for a rectangular composite element with a single open or filled hole. The failure loads predicted by IBOLT software and methodology have been shown to correlate well with experimental data.

IBOLT supports a combination of biaxial membrane loads, shear loads, out of plane bending moments, and an off axis bolt load. The rectangular IDAT IBOLT analysis element and load sign conventions are shown in Figure 4.5-8.

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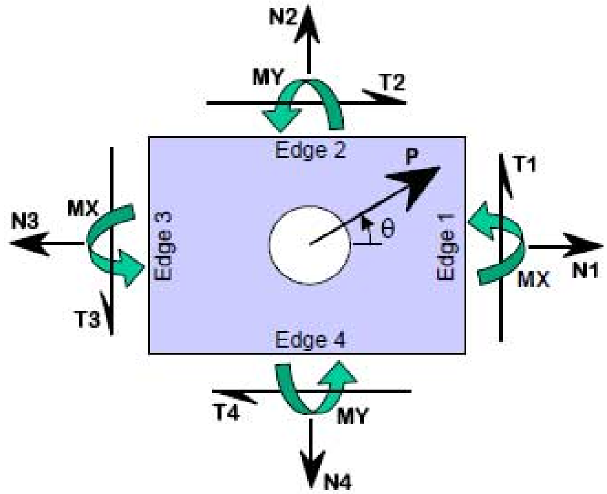

Figure 4.5-8: IBOLT Analysis Element

IBOLT calculates stress and performs a failure analysis at each of the eight locations around the perimeter of the hole where ply fiber direction is tangent to the hole edge. Analysis is performed using the tangent stacking sequence at each location. Figure 4.5-9 shows an example (50/40/10) laminate with the eight IBOLT analysis points and the tangent ply percentage stacking sequences identified. Because the IBOLT solution is dependent on tangent stacking sequence, it is important that the analysis element X-axis is aligned with the 0º ply orientation of the analysis laminate.

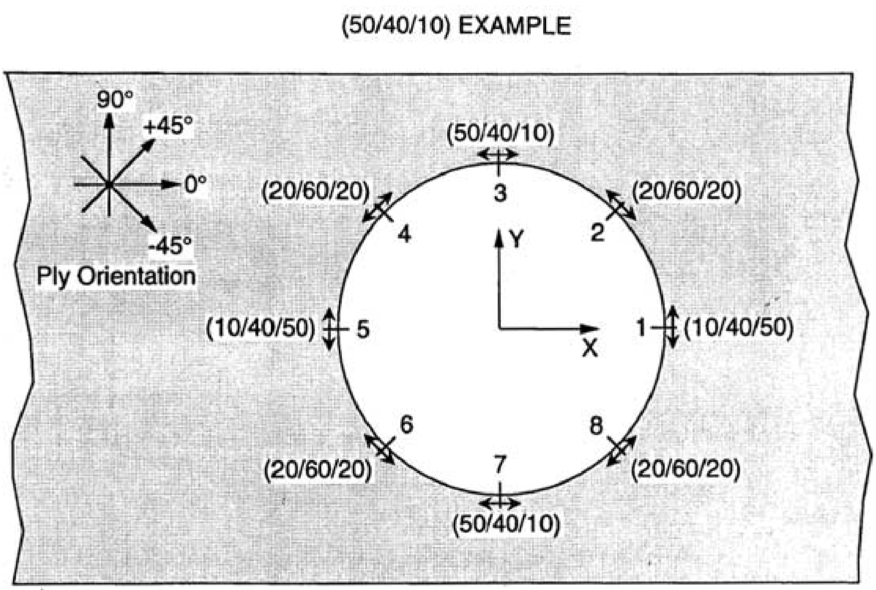

Figure 4.5-9 IBOLT Analysis Locations and Example Tangent Ply Percentages

IBOLT analysis is strongly dependent on a set of theoretical stress concentration factors. IBOLT defines stress concentration factors as the ratio of the local tangential stress at a given position on the hole boundary to the far-field applied stress. IBOLT stress concentration factors are a function of

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laminate ply percentage, the type of applied loading (tension, compression, shear, and bearing), and position around the hole boundary. Figure 4.5-10 shows a plot of the stress concentration factor KTX = σ_θ/σ_x vs. θ for three stacking sequences, where SIG THETA = σ_θ, and SIGTX = σ_x. The theoretical stress concentration factors for holes in composite laminates are calculated using simple formulas but assume an infinite plate. Additional correction factors are needed to correct the results for a finite geometry analysis element. The finite geometry correction factors are computed using stored numerical data from boundary element analysis at each of the eight IBOLT analysis points. Additional empirically derived factors are applied to account for variations in joint configuration such as filled vs. open holes, countersink, torque level, and fastener/hole clearance.

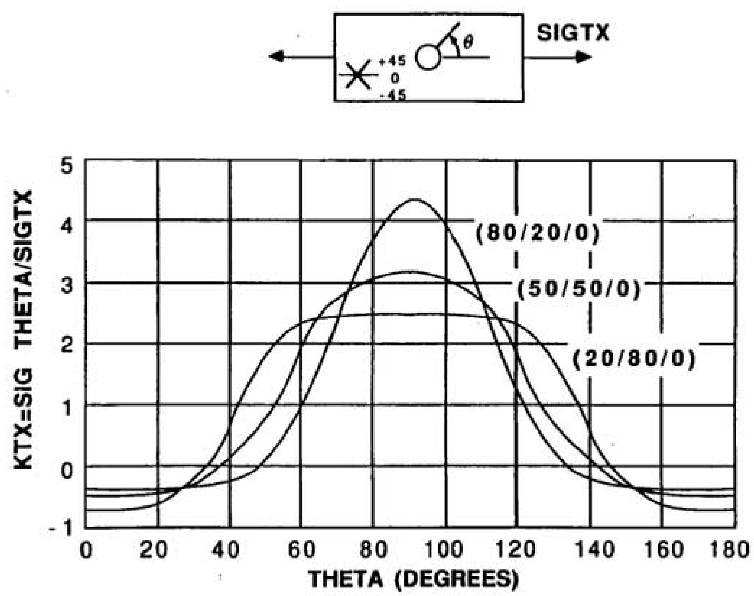

Figure 4.5-10 IBOLT Stress Concentration Factors

The prior discussion of IBOLT analysis dealt primarily with bypass loading. In this context, "bypass loading" indicates that there is no force reacted at the hole. When the hole is loaded, IBOLT calculates peak bearing stress using a beam on elastic foundation analysis combined with a set of empirically derived factors.

At each of the eight analysis locations on the holes perimeter, solutions and factors from each applied load condition are superimposed as shown in Figure 4.5-11 to yield the total stress at each analysis point "I".

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At any point i on the hole boundary...

F_{tu} (i) = + SIGTX • LTX(i) • FTXi [a(i)/r] • FTF	Tension, X-direction
+ SIGTXX • LTXX(i) • FTXXi [a(i)/r] • FTF	Membrane/shear, X-direction
+ SIGTY • LTY(i) • FTYi [a(i)/r] • FTF	Tension, Y-direction
+ SIGTYY • LTYY(i) • FTYYi [a(i)/r] • FTF	Membrane/shear, Y-direction
+ SIGTXY• LTXY(i) • FTXYi [a(i)/r] • FTF	Shear, X-Y
+ SIGBX • LBX(i) • FBXi [a(i)/r] • FBF	Bearing/tension, X-direction
+ SIGBXX • LBXX(i) • FBXXi [a(i)/r] • FBF	Bearing/shear, X-direction
+ SIGBY • LBY(i) • FBYi [a(i)/r] • FBF	Bearing/tension, Y-direction
+ SIGBYY • LBYY(i) • FBYYi [a(i)/r] • FBF	Bearing/shear, Y-direction

Joint configuration factors
Effective stress concentration factors
Finite geometry correction factors
Applied stresses

Where;

SIGTX = Tension stress in x direction, bypass loading
SIGTXX = Tension stress in x direction, shear reaction
SIGTY = Tension stress in y direction, bypass loading
SIGTYY = Tension stress in y direction, shear reaction
SIGTXY = Shear stress in xy direction, bypass loading
SIGBX = Bearing stress in x direction reacted in tension
SIGBXX = Bearing stress in x direction reacted in shear
SIGBY = Bearing stress in y direction reacted in tension
SIGBYY = Bearing stress in y direction reacted in shear

Figure 4.5-11: IBOLT Superposition of Stress at a Point[1]

Conventional failure criteria do not apply to composite laminates containing holes. For a constant width-to-diameter ratio, the failure strength of a composite laminate containing a hole is sensitive to the hole diameter. Figure 4.5-12, and Figure 4.5-13 illustrate the variation of notched failure strength with hole size for constant W/D[2] specimens. As shown by the figures, strength is significantly reduced for large holes and then rapidly approaches the un-notched laminate strength as hole size becomes smaller. Failure criteria applied to composite laminates containing holes must properly account for failure sensitivity to hole diameter.

IBOLT implements a fracture mechanics solution to account for the sensitivity of composite laminate failure strength to hole size. The fracture mechanics solution implemented by IBOLT uses an assumed flaw size “a” that is dependent on the laminate critical strain energy release rate G_{1C}, the hole radius r, the un-notched failure strength F_{tu}, and the tangential laminate compliance matrix C. The assumed flaw size varies with location around the hole. As shown by Figure 4.5-13, the IBOLT fracture mechanics

[1] The top portion of Figure 4.5-11 is a reproduction of page 62 from reference 4-11, *FF-93-92, IBOLT Theory Manual for Analysis of Composite Bolted Joints.*
[2] W/D = width to diameter ratio

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solution agrees with test data and accurately predicts the variation in laminate strength as hole diameter changes.

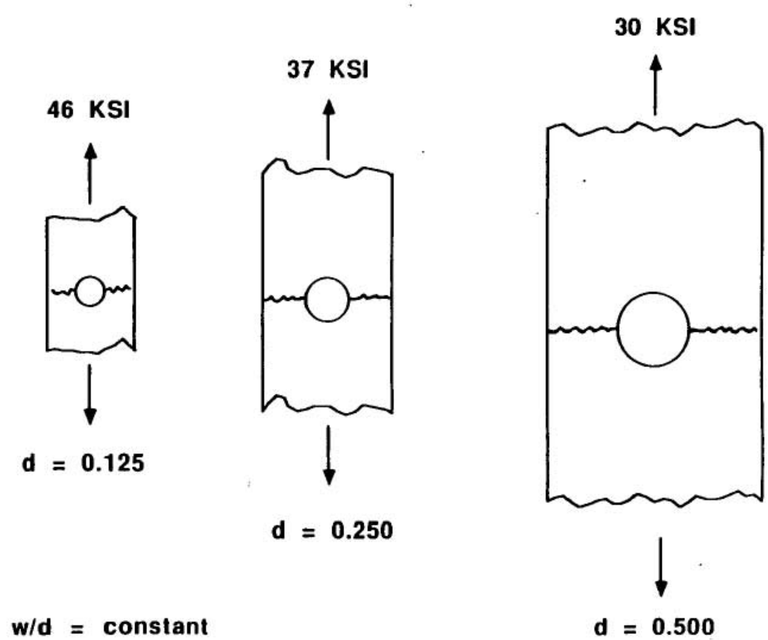

Figure 4.5-12 Laminate Strength versus Hole Size

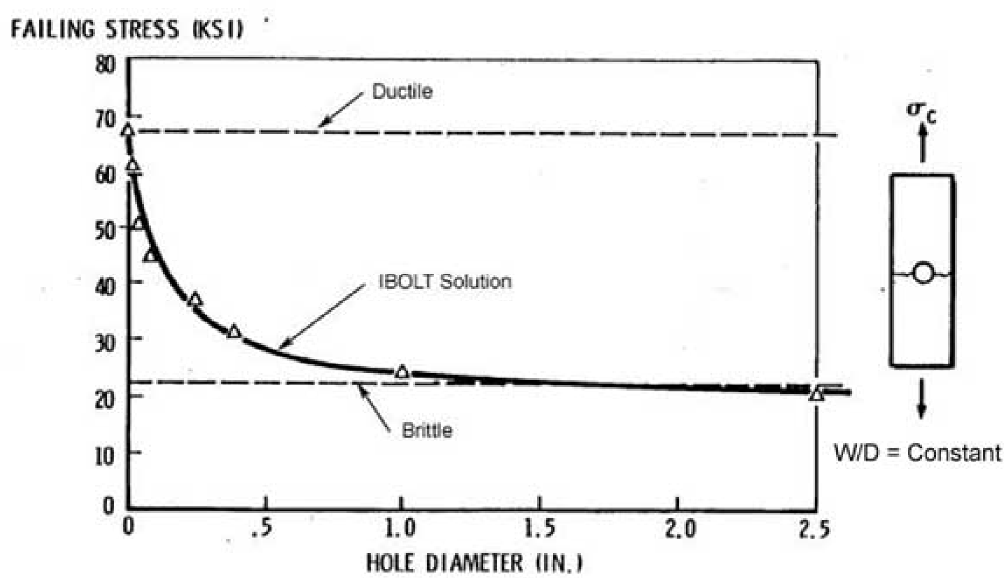

Figure 4.5-13 IBOLT Fracture Mechanics Solution

The IDAT IBOLT application input screen is shown in Figure 4.5-14.

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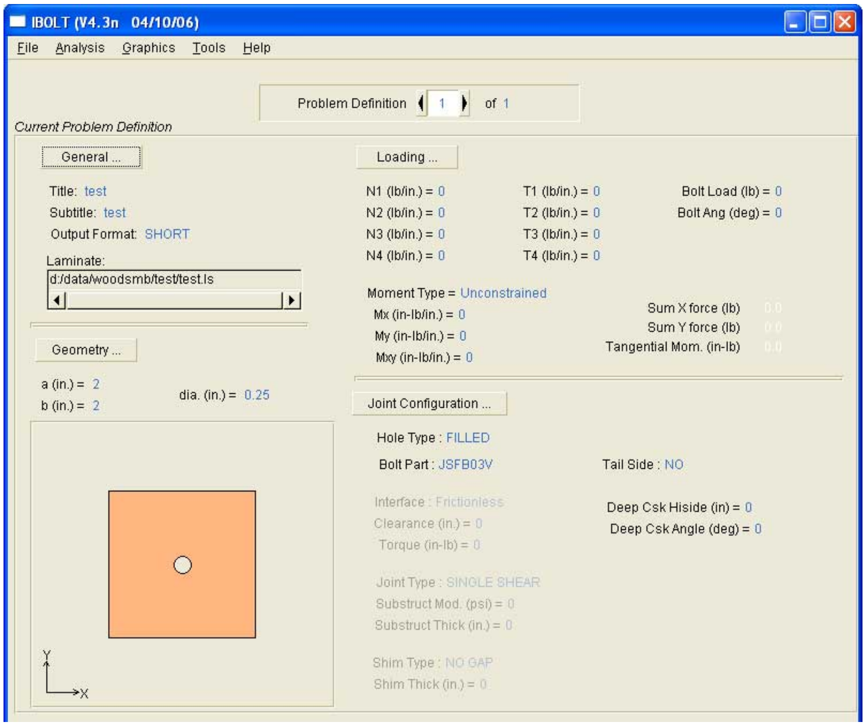

Figure 4.5-14 IDAT IBOLT Input Screen

The failure loads predicted by IBOLT have been shown to correlate well with experimental data. For additional information on IBOLT theory and charts showing comparison to experimental data refer to reference 11, *FF-93-92, IBOLT Theory Manual for Analysis of Composite Bolted Joints*, and reference 12, *FZM-9065, IBOLT Theory Manual*.

4.5.2.2 Notched Material Properties and Allowables

In addition to lamina strength and stiffness data, the IBOLT notched analysis methodology is dependent on a large set of empirical data derived in part from the material coupon tests described in Table 4.5-1, and Figure 4.5-15. Variations in the configuration of the notched analysis element are accounted for by a set of empirically derived analysis factors. Examples of the variables that require analysis factors include: filled versus open hole, hole clearance, countersink, fastener torque, single or double shear joint configuration, and many more. Appendix A of the IBOLT theory manual (ref 11) contains a complete listing and description of the IBOLT analysis factors. A portion of the IBOLT factors are material specific. The material specific factors and the strain energy release rates must be empirically derived as part of the material characterization for notched analysis.

Characterization of a material for notched analysis using IBOLT requires a significant test program. The complete notched allowables test program requires twelve distinct coupon tests. Each coupon test is typically done on a family of three to seven (0/±45/90) laminates (stacking sequences), with the "notched" condition satisfied by a ¼" diameter hole. Ideally, the same coupon test conditions are repeated for the entire family of laminates at four temperatures (-65˚, 75˚, 220˚, 275˚). Moisture content is set at a critical value for each test temperature. Higher temperature tests are conducted wet and lower temperature laminates are tested dry. The twelve notched coupon test configuration names and abbreviations are listed in Table 4.5-1, and the test configurations are shown in Figure 4.5-15.

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Table 4.5-1 Notched Allowables Coupon Tests
UNT – Un-notched Tension
OHT – Open Hole Tension
FHT – Filled Hole Tension
UNC – Un-notched Compression
OHC – Open Hole Compression
FHC – Filled Hole Compression
FHB – Filled Hole Bending
OHB – Open Hole Bending
FHBC – Filled Hole Bending with Countersink
BCT – Bearing with Tension Reaction
BCC – Bearing with Compression Reaction
BCS – Bearing with Shear Reaction

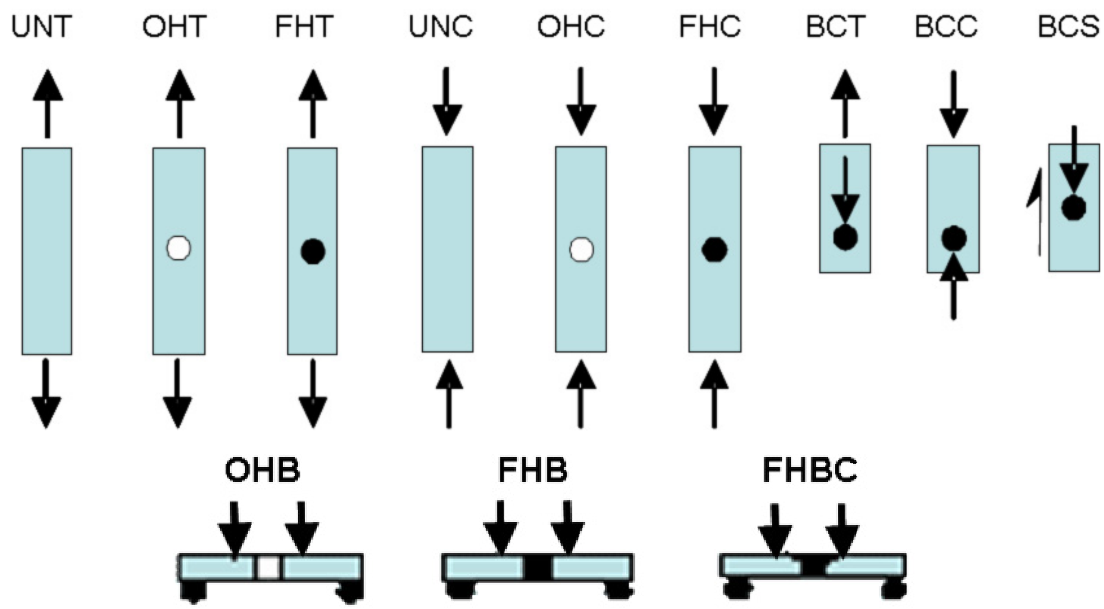

Figure 4.5-15 Coupon Test Configurations for Notched Failure Data

The notched coupon tests are conducted in the order shown by the arrows in Figure 4.5-16 to establish correlated values of un-notched laminate strength, strain energy release rates (G0, G45, G90), bearing strengths, and the various “empirical properties” used by IBOLT to predict notched laminate strength.

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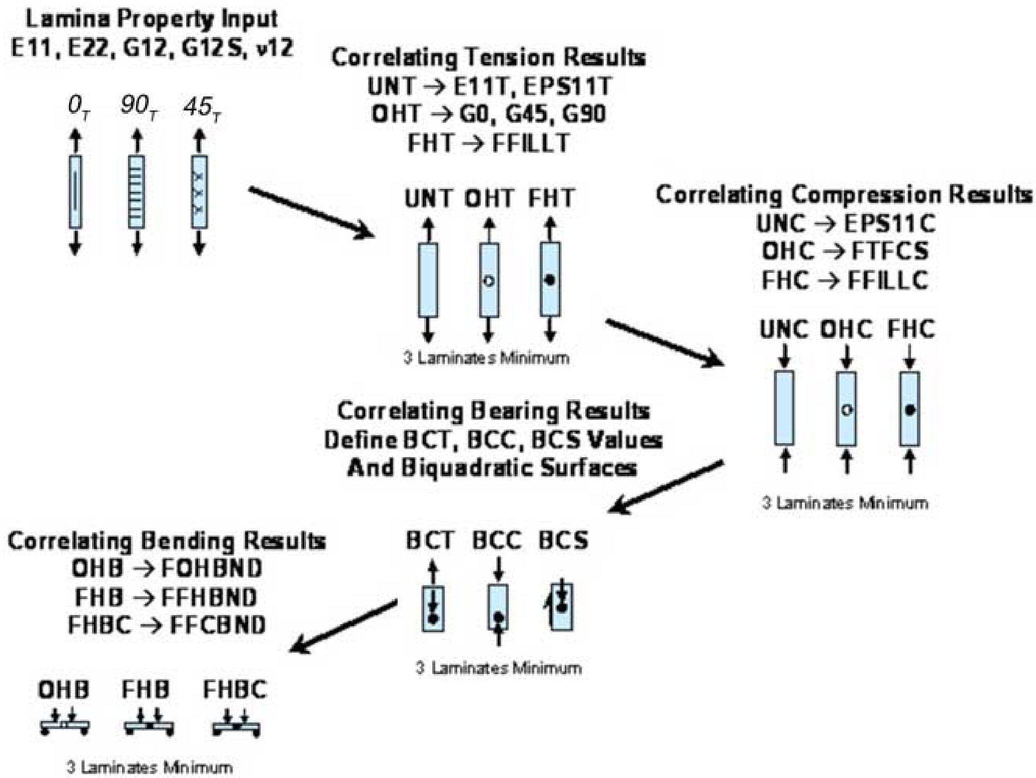

Figure 4.5-16 Order of Notched data Testing and Data Reduction

Names on Figure 4.5-16 such as FFILLT, FFILLC, FTFCS, etc, refer to the factors used when calculating notched strength. For example "FFILLT" is the Factor for a Filled hole in Tension, and is defined as the laminate strength with an open hole divided by the strength with a filled hole. A complete list and description of the empirical factors used by IBOLT for notched strength prediction can be found in Appendix A of the IBOLT Theory Manual (See References). These factors are typically variable with respect to some laminate or geometric parameter such as %0 degree plies, and are expressed as a best fit equation with constant coefficients "C". The coefficients are grouped in a vector designated "C", which can be accessed by selecting the "empirical properties" tab in IDAT MATUTL. Associations between the notched strength factors and components of the coefficient vector "C" are also described in appendix A of the IBOLT Theory Manual.

Note that "Notched Allowables" are not found in the IDAT MATUTL material data files. The IDAT MATUTL material data files contain un-notched allowables, environmental properties, fracture properties, and the IBOLT factors (the C vector). For each lamina material, the notched allowable/margin of safety is dependent upon stacking sequence, notch condition, load condition, and environment. The notched allowable for a specific analysis element must be calculated using IBOLT theory combined with the un-notched allowables, fracture properties, and appropriate elements of the C vector.

4.5.2.3 IBOLT Notched Strength Carpet Plots

IBOLT notched strength carpet plots are available for materials in the IDAT MATUTL material database. Notched strength carpet plots show the variation in allowable x-direction stress due to stacking sequence for a specified notched analysis element geometry and material. Carpet plots are

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accessible from the IDAT MATUTL menu (not from the IBOLT application). IBOLT carpet plots are useful for preliminary design of laminates using notched strength criteria. Access of the IBOLT notched strength carpet plot from IDAT MATUTL is shown in Figure 4.5-17. An IBOLT carpet plot for notched filled hole compression of a B_IM7/977 laminate is shown in Figure 4.5-18.

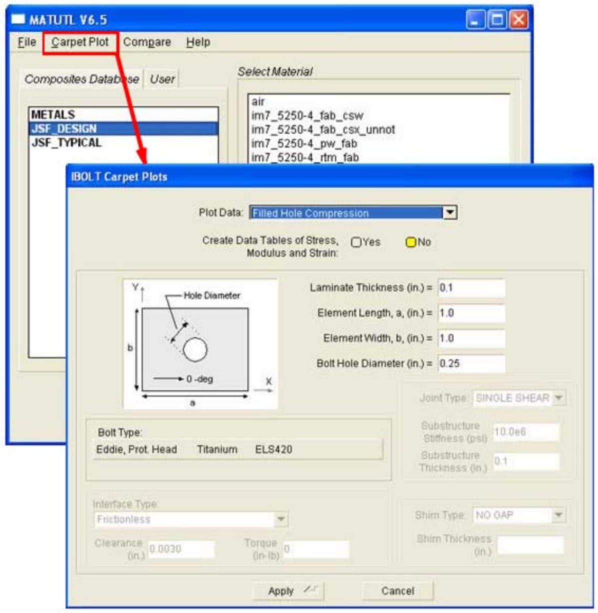

Figure 4.5-17 Access IBOLT Carpet Plots From IDAT MATUTL

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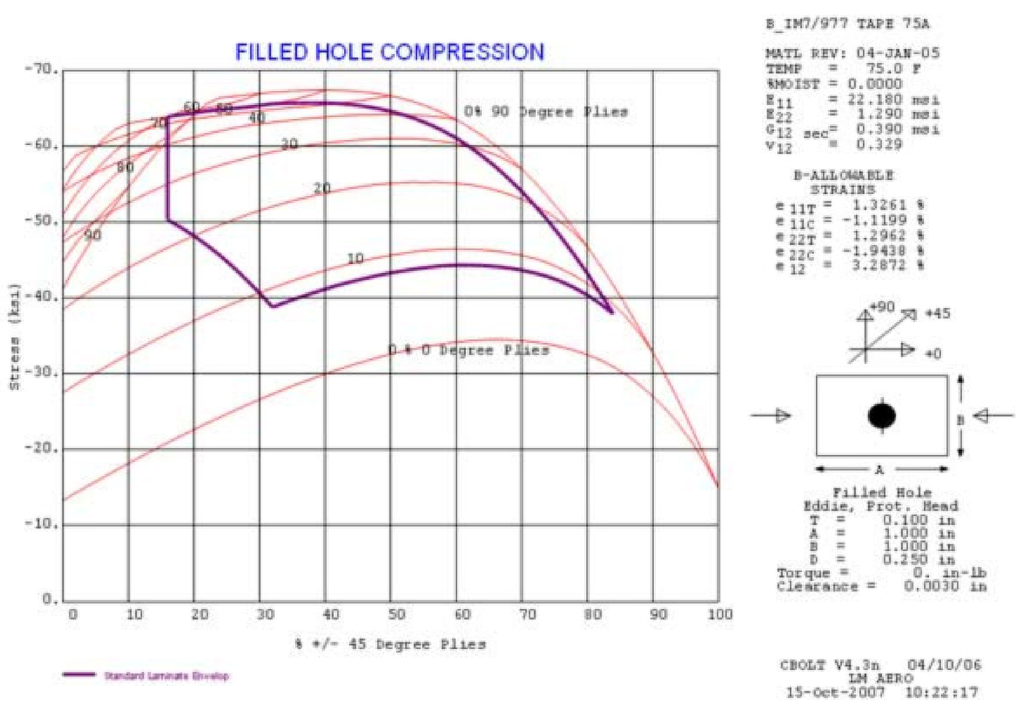

Figure 4.5-18 IBOLT Carpet Plot for Filled Hole Compression

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To use the IBOLT notched strength carpet plot:

1. Locate the total percentage of ±45˚ plies on the x axis
2. Move in the positive y direction until you intersect the curve representing the percentage of 0˚ plies in the laminate.
3. Move left horizontally until you intersect the y axis and read the allowable x-direction stress at that point.

Table 4.5-2 shows example notched 100% bypass filled and open hole allowable strength predictions for three stacking sequences of B_IM7/977 tape. 100% bypass means that the fastener (if present) does not react any of the applied load. Figure 4.5-19 shows an example carpet plot for open hole tension.

Table 4.5-2 Laminate Notched Allowable Stress Predictions

Stacking Sequence	Allowable Stress (Psi)			
	OHT	OHC	FHT	FHC
(25/50/25)	51300	38300	48560	58500
(50/40/10)	77600	50600	66300	65500
(10/40/50)	36300	30800	35500	43700

Figure 4.5-19 Stress Carpet Plot for Open Hole Tension

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4.5.3 Interlaminar Failure Criteria

Obtain the interlaminar stresses using an approved methodology such as IDAT ESDU 9528 or IDAT BEND. The allowables for interlaminar stress (F_{xz}^{su} , F_{yz}^{su} , F_{zz}^{tu}) are found in the IDAT MATUTL composite material database. Failure can be calculated either using the Maximum Stress Failure Criteria given by Equation 4.5-6, or using the Modified Hill failure criteria for interlaminar stress given by Equation 4.5-7.

$$M.S._{xz} = \frac{F_{xz}^{su}}{\tau_{xz}} - 1$$
$$M.S._{yz} = \frac{F_{yz}^{su}}{\tau_{yz}} - 1 \qquad \textbf{Equation 4.5-6}$$
$$M.S._{zz} = \frac{F_{zz}^{tu}}{\sigma_{zz}} - 1$$

$$M.S. = \left[\left(\frac{\tau_{xz}}{F_{xz}^{su}} \right)^2 + \left(\frac{\tau_{yz}}{F_{yz}^{su}} \right)^2 + \left(\frac{\sigma_{zz}}{F_{zz}^{tu}} \right)^2 \right]^{-\frac{1}{2}} - 1 \qquad \textbf{Equation 4.5-7}$$

IDAT BEND calculates a margin based on the maximum stress theory given by Equation 4.5-6.

4.5.4 Three-Dimensional Failure Criteria

Solid body modeling of composite laminates is sometimes required in conditions where the assumptions of Classical Laminated Plate Theory (CLPT) are not valid (when the normal stress σ_3, or the transverse shears τ_{13} and τ_{23}, are significant). Example cases are out of plane loads, corner radii, and part edges. Specialized IDAT analysis programs such as “CORNER” and “ESDU 9528” have been developed to handle the most common load and structural configurations with potential for significant out of plane stress, meaning that solid body models are not usually required. In situations where a 3-dimensional state of stress exists, the 3-dimensional failure criteria given by Equation 4.5-8 may be used.

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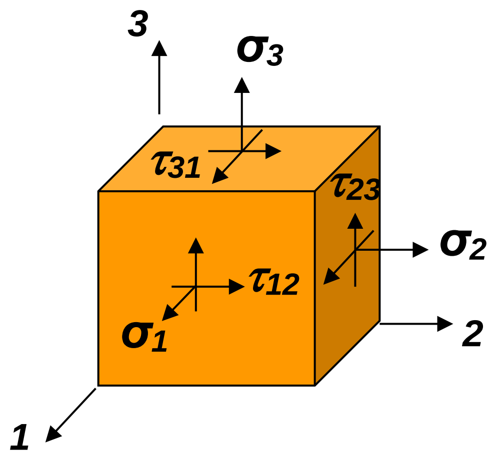

Figure 4.5-20:

$$\left(\frac{\sigma_1^2-\sigma_1\sigma_3}{X_tX_c}\right)+\left(\frac{\sigma_2^2-\sigma_2\sigma_3}{Y_tY_c}\right)+\left(\frac{\sigma_3}{Z}\right)^2+\left(\frac{\tau_{13}}{S_{13}}\right)^2+\left(\frac{\tau_{23}}{S_{23}}\right)^2+\left(\frac{\tau_{12}}{S_{12}}\right)^2\geq 1 \qquad \textbf{Equation 4.5-8}$$

Where:

$X_t = F_{11}^{tu}$

$X_c = F_{11}^{cu}$

$Y_t = F_{22}^{tu}$

$Y_c = F_{22}^{cu}$

$Z = F_{33}^{tu}\ if\ \sigma_3 \geq 0, else\ Z = F_{33}^{cu}$

$S_{13} = F_{13}^{su}$

$S_{23} = F_{23}^{su}$

$S_{12} = F_{12}^{su}$

Use caution when attempting to model composite behavior using solid finite elements. A single traditional anisotropic solid finite element can not in general be used to model the behavior of a laminated composite. Stacked solid models or specialized solid elements are normally required. Depending on the results you are seeking (e.g.: interlaminar stress), it might be necessary to model the adhesive layers between the plies as separate solid bodies. Be aware that mesh dependent behavior can be seen in three-dimensional models at singular or near singular stress fields. Singular stress fields often occur at locations such as free edges and reentrant corners. Application of three-dimensional failure criteria to FEA results that are not converged is very dangerous and misleading. Empirically derived distance rules have to be utilized. Seek guidance from your program FEA subject matter expert when applying three-dimensional FEA modeling techniques to laminated composite materials.

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4.5.5 Criteria for Composite Durability and Damage Tolerance

Composite laminates in service are subject to potential damage from sources such as maintenance personnel, tools, runway debris, service equipment, hail, lightning, etc. Damage may also occur during initial manufacturing and assembly.

Durability:	Resistance to damage for a specified period of time.
Damage tolerance:	Resistance to failure initiated by damage for a specified period of time.
Fatigue:	Process of damage accumulation due to repeated and/or sustained exposure to load and environment.

To alleviate the effects of expected damage, most composite components are designed to specific damage tolerance and durability criteria. The intent is to limit working strain levels for critical primary structure such that in-service damage and undetected manufacturing defects will not degrade structural performance or flight safety. Durability and damage tolerance criteria also reduce maintenance and support costs during the design service life of the vehicle.

The composite damage tolerance and durability criteria discussed in this section are commonly used and originate from guidance provided in Reference 4-16 *JSSG-2006.*[1] Program specific guidance is typically found in formal documents such as the *Durability and Damage Tolerance Control Plan,* and the *Durability and Damage Tolerance Analysis Operating Plan*.

The information provided in this section is intended to provide a brief introduction to composite durability, damage tolerance, and related failure criteria. Durability criteria drive the use of notched allowables as discussed in section 4.5.2. Additional information related to durability and damage tolerance is located in section 12.

4.5.5.1 Durability

Durability refers to the ability of the material/structure to resist cracking, environmental degradation, delamination, and wear for a specified period of time. Durability criteria are intended to account for material behavior, inherent manufacturing flaws, and general damage from handling, low velocity impact, etc. In general, a durable structure is the one that does not incur excessive maintenance cost during its service life. Durability criteria ensure that adequate strength and stiffness are maintained throughout the service life of the composite structure.

Damage resistance may be improved by increasing laminate thicknesses and for sandwich applications by using denser core. Reinforcement fibers with high strain capability and toughened matrix materials are other options to enhance damage resistance. Stiffened panel skins are usually more durable when compared to equivalent sandwich construction. Increased bending action typically absorbs more energy and increases damage resistance.

Other items to improve damage resistance include the use of a layer of fabric as the exterior ply over tape. An exterior fabric layer will resist scratches and abrasion, provide softening of impact, and will also reduce fiber breakout during drilling of fastener holes. Laminate edges are subject to delamination when exposed directly into the air stream. Avoid delamination by using non-erosive edge protection, replaceable sacrificial materials, or locating the forward edge below the level of the aft edge of the next

[1] Reference 4-16 JSSG-2006, Appendix A, Tables VII, VIII, IX, and X.

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forward panel. Utilize replaceable conductive materials with special attention to tips and trailing edge surfaces for protection in areas prone to high energy lightning strike.

Composites have excellent fatigue characteristics when compared to metals. In composites, fatigue damage due to repeated mechanical loads usually initiates as cracks in the matrix material at laminate edges, notches, and stress discontinuities and then may progress as interlaminar delamination. Damage tolerance and repair requirements usually limit the loads of current composite structures to levels below the threshold that would cause extensive matrix cracking and fatigue. One exception is in the vicinity of fastener holes. High bearing stresses and hole elongation may cause bolt fatigue failures and other anomalies due to internal load redistribution. Thus, good supportability design should feature low bearing stresses.

Refer to Section 15 for more information on Durability and Damage Tolerance.

4.5.5.2 Durability Criteria

Composite Durability Criteria are typically impact based. Table 4.5-3 specifies the typical composite durability criteria for a 4 ft-lb and a 6 ft-lb impact.

Table 4.5-3 Composite Durability Criteria

- For a 4 ft-lb impact
    - No visible damage. (Visible damage = .1" deep dent, surface scratches, paint cracking, fuel leakage)
    - Must meet full durability for 2 lifetimes
    - No failure for design ultimate load including environmental effects
- For a 6 ft-lb impact
    - Withstand two lifetimes without failure
    - No failure for P_{xx} load
        - P_{xx} load will depend on the certifying authority and might be: Once in 20 lifetimes load (P_{lt}), Maximum limit load, Maximum design load, or some percent of limit load.
- 0.5" diameter hemispherical steel impact device.

Usually both the 4 ft-lb and 6 ft-lb impact criteria require no functional impairment of the structure (strength, leakage, maintenance actions). The 6 ft-lb criteria are only applied in "probable impact zones". The 4 ft-lb criteria are used for less likely impact zones and should result in no visible damage. Durability criteria usually drive the minimum gage requirements for composite laminates.

Programs must demonstrate that composite parts meet the established durability criteria by analysis and/or test. Durability analysis is performed across the broad acreage of the part, away from edges and stress concentrations. A positive margin from an IBOLT 100% by-pass analysis of a .25" filled hole at 150% DLL is currently used and accepted by some programs as satisfying requirements for durability analysis. Allowables and analysis for durability are always established and verified by test. Each program will coordinate with the customer and/or certifying agency to establish durability criteria and analysis requirements for composite parts.

4.5.5.3 Damage Tolerance

Damage Tolerance is a measure of the ability of the material/structure to resist failure due to the presence of damage for a specified period of time. The damaged part must maintain functionality with

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sufficient residual strength and stiffness for the specified loads and time period. Damage tolerance is achieved by reducing allowable strain levels in critical areas and/or providing multiple load paths.

Refer to Section 15 for more information on Durability and Damage Tolerance.

4.5.5.4 Damage Tolerance Criteria

Damage tolerance criteria are applied to structure designated as "fracture critical", where the "fracture critical" designation indicates that in-service failure of the part would be a hazard for loss of the entire aircraft, or in some cases inability to complete the mission - including inadvertent loss or delivery of a weapon. Durability and damage tolerance specifications require that fracture critical parts are tracked and records are maintained from "cradle-to-grave". Typical Composite Damage Tolerance Criteria are given in Table 4.5-4.

Table 4.5-4 Composite Damage Tolerance Criteria

- Must withstand two lifetimes without failure
- No failure for P_{xx} load
  - P_{xx} load will depend on the certifying authority and might be: Once in 20 lifetimes load (P_{lt}), Maximum limit load, Maximum design load, or some percent of limit load.
- Impact threat is minimum of:
  - Zoned impact energy (maximum of 100 ft lbs)
  - Clearly visible Damage
- Testing uses a 1.0" diameter hemispherical steel impact device.
- Damage is assumed at the critical location in structure.

Each program will coordinate with the customer and/or certifying agency to establish damage tolerance criteria for composite parts. Allowables for damage tolerance are always established by test. Programs must demonstrate that composite parts meet the established criteria by analysis and/or test.

4.5.5.5 IDAT CDADT Analysis

IDAT CDADT software is used for damage tolerance analysis of fracture critical structures. The CDADT analysis methodology is based in-part upon a semi-empirical curve fit developed from a large body of durability and damage tolerance data. Impact tests on various laminates, thicknesses, and stacking sequences were used to establish a relationship between impact energy and visibility of damage. CDADT analysis results are only applicable to laminates constructed from the tested materials - IM7/977-3 and IM7/5250-4 tape and fabric. For a given impact energy level, the damage area is calculated as a function of ply thickness, laminate thickness, laminate lay-up, environmental effects, and material type. The delamination is assumed to occur at the critical location in the laminate thickness.

The CDADT failure allowable is calculated based on the critical strain energy release rate required to grow the delamination. Strain energy release rate is the energy dissipated by fracture per unit of the new fracture surface. Testing has shown that the axes of the delamination ellipse are aligned with the

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principal strain axes of the load condition being analyzed. The strain energy release rate is calculated along each principal strain axis of the delamination ellipse to give strain energy release rates (G_a and G_b) for delamination growth in the direction of the ellipsis axes a and b. The load level at which G_a or G_b is equal to the critical strain energy release rate G_{1C}, is the critical load level for the part. Empirical factors are applied for correlation with test data.

IDAT CDADT Software predicts Compression After Impact (CAI) and Tension After Impact (TAI) allowables for specified laminate and fracture criteria. CAI allowables for damage tolerance are often critical for composite laminates. The assumed flaw is a 4.0 in^2 circular delamination at the critical ply interface for the load condition and stacking sequence being analyzed. Averaged loads appropriate to the assumed flaw size are used. The CDADT margin is calculated by comparing the predicted strength after impact to the predicted stress for the maximum occurring load.

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