Answer: 9.1

- Body in Pure translatory Motion (e.g. sliding object)
- Motion of car can be also described by assuming as a particle if our interest is to find displacement, velocity & Accel^n.
- Motion of lift (Kinetics).

9.2 Centre of mass.

9.3 $\frac{dv}{dt} = a(t)$

$\int dv = \int a\,dt \Rightarrow v = \int a\,dt + C$

No, we need initial velocity to find the Const C.

9.4 $\frac{m\,dv}{dt} = F = ma$

rate of change of linear momentum is constant.

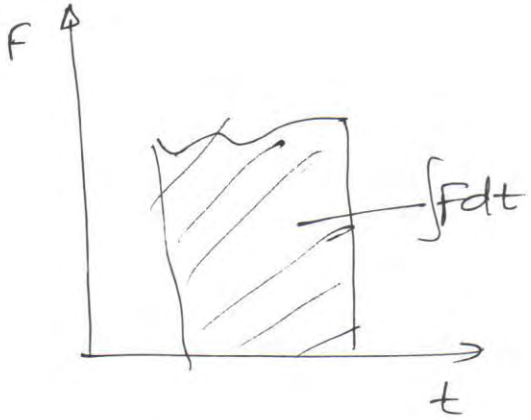
Hence, linear momentum changes linearly with time (b)

9.5

$v = 0.$

$a = \frac{dv}{dt} \Rightarrow v_f = v_i + \int a\,dt$

(9.6)



$$\int F dt = \int d(mv)$$

(c)

(9.7) (b) No net force acting on it.

(9.8) Given,

$F = 5KN$

$m = 1Kg$

$t = 5sec.$

$$F = m\frac{dv}{dt} = 5$$

$$\Rightarrow m\,dv = 5dt$$

$$\Rightarrow m\int_0^v dv = 5\int_0^t dt$$

$$\Rightarrow m[v] = 5t$$

(a) $\therefore v\Big|_{t=5sec} = \frac{5\times 5\times 10^3}{1} = 25ms^{-1}.$

(b) $v\Big|_{t=10sec} = \frac{5\times 10\times 10^3}{1} m/s = 50 Km/sec.$

9.10 Given

$\dot{x} = K_1 + K_2 t$

$K_1 = 1\,ft\,s^{-1}, K_2 = 1\,ft\,s^{-2}$

$x(0) = 1\,ft$

$$\frac{dx}{dt} = K_1 + K_2 t \Rightarrow \int_1^x dx = \int_0^t (K_1 + K_2 t)\,dt$$

$$\Rightarrow (x-1) = K_1 t + K_2 t^2/2$$

$$\therefore x\Big|_{t=10sec} = 1 + 1\times 10 + 1\times\frac{10^2}{2}$$

$$= 1 + 10 + 50$$

$$= 61\,ft.$$ Ans

9.11 Given,

$$\dot{x} = \frac{dx}{dt} = cx$$

$$\Rightarrow \frac{dx}{x} = c\,dt$$

Integrating both sides, we get

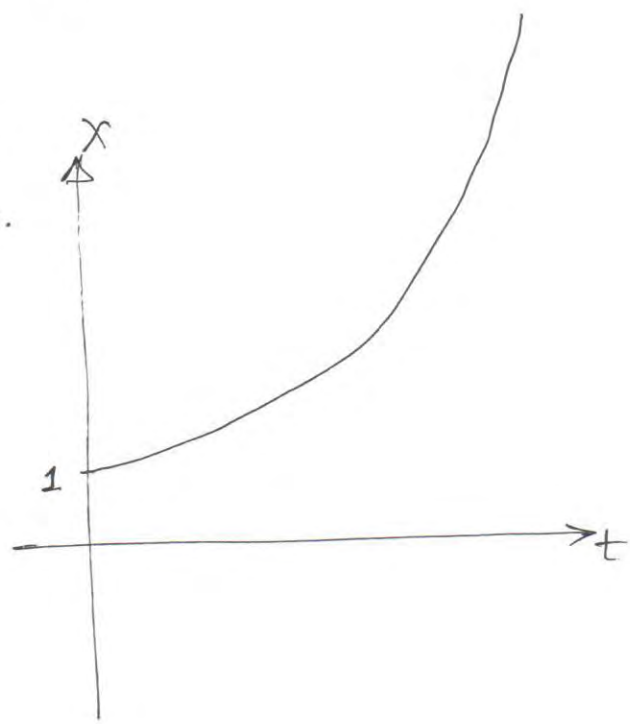
$$\ln x = ct + C_1$$

at $t=0$, $x(0)=1$, $C=1$

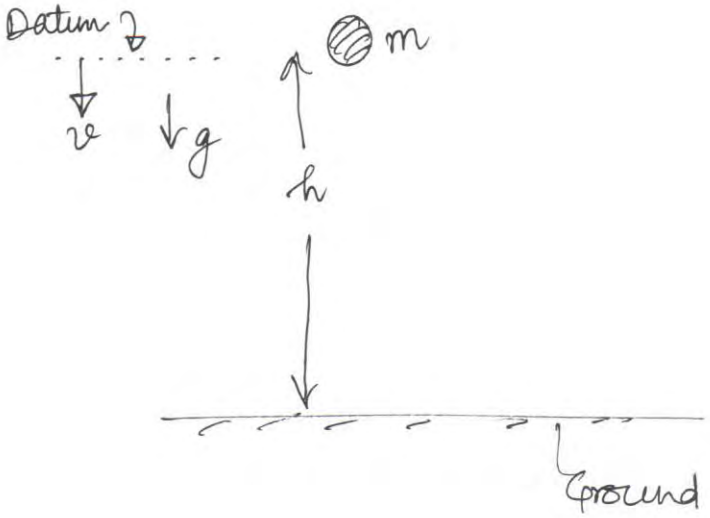
$$\Rightarrow 0 = 0 + C_1 \Rightarrow C_1 = 0.$$

$$\therefore \ln x = t$$

$$\Rightarrow \boxed{x = e^t}$$



9.12



$$\frac{dv}{dt} = g$$

$$\Rightarrow v = gt + c$$

at $t=0$,
$v=0$

$\therefore c=0$

$\therefore \boxed{v = gt}$ Ans

$$\frac{dx}{dt} = gt$$

$$\Rightarrow x = gt^2/2 + c$$

at $t=0$, $x=0$

$\therefore x = 0 + gt^2/2 = gt^2/2$

$\therefore h = gt^2/2$

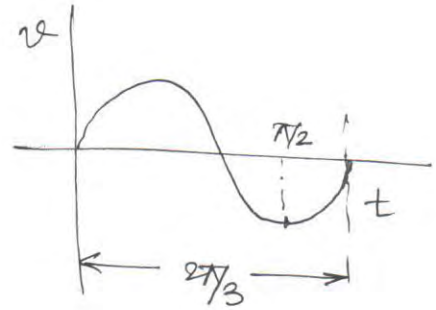
$\Rightarrow t = \sqrt{\frac{2h}{g}}$.

$\therefore v = \sqrt{2h/g} \cdot g$

$\Rightarrow v = \sqrt{2gh}$. Ans

9.13

$$v = A\sin[ct]$$
$$A = 0.5\,ms^{-1}.$$
$$c = 3\,rad\,s^{-1}.$$



$$\frac{dx}{dt} = A\sin(ct)$$

$$\Rightarrow \int_0^x dx = \int_0^{\pi/2} A\sin(ct)\,dt$$

$$\Rightarrow x = \left[\frac{-A\cos[ct]}{c}\right]_0^{\pi/2}$$

$$= A/c = \frac{0.5}{3}$$

$$\Rightarrow x = 0.1667\,m.$$ Ans.

$$x = -A/c\left[\cos(ct) - 1\right]$$

$$\boxed{x = A/c\left[1 - \cos(ct)\right]}$$

9.14

Given,

$$\dot{x} = x/s$$

$$x(0s) = 1m.$$

$$x(t = 5sec) = ?$$

$$\frac{dx}{dt} = x \Rightarrow \int dx/x = \int dt$$

$$\Rightarrow \ln x = t + C$$

$$\Rightarrow \boxed{C = 0} \quad \because t = 0, x = 1.$$

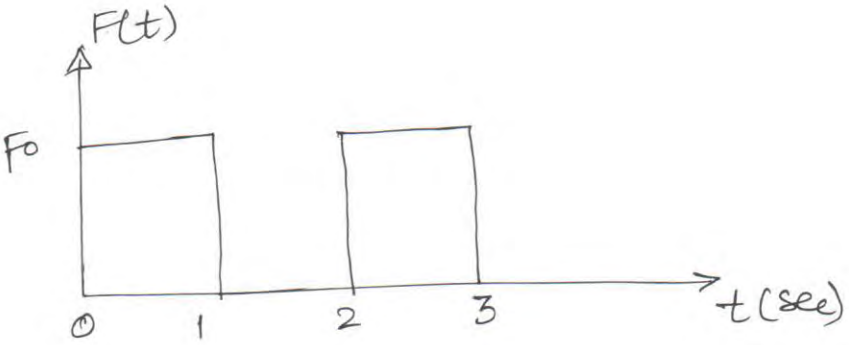
$$\therefore x = e^t$$

$$\therefore x\Big|_{t=5sec} = e^5\,m$$

$$\Rightarrow x(5s) = 148.413\,m.$$ Ans

9.15

(a)



$$m\,dv = F_o\,dt \quad \text{—— (1)}$$

$$\Rightarrow m(v_1 - v_0) = F_o(t_1 - t_0) \quad [\because F_o \text{ is const}].$$

$$\Rightarrow v_1 = \frac{10 \times (1-0)}{10} = 1\,ms^{-1}.$$

Since external force acting b/w point (1) & (2) is 0,

$$v_1 = v_2.$$

<u>Again</u>,

$$v_3 = v_2 + \frac{F_o}{m}[t_3 - t_2] \quad \text{(from (1))}$$

$$\Rightarrow v_3 = 1 + \frac{10}{10}(1)$$

$$= 2\,ms^{-1}.$$

(b) $$v_2 = v(0) + \frac{F_o}{m}[t_2 - t_0]$$

$$= 2\,ms^{-1}.$$

$\therefore v_3 = v_2 = 2\,ms^{-1}$. [since no force acting b/w point (2) & (3)].

9.16

$$F = m\frac{dv}{dt}$$

$$\Rightarrow m\,dv = F\,dt$$

$$m(v_f - v_i) = \int F\,dt$$

To get same final velocity, v_f. $\int_0^T F\,dt$ must be same for both cases.

$$\therefore \frac{1}{2} \times F_T \times T = \int_0^T F_S \sin\left(\frac{\pi t}{T}\right) dt$$

$$\Rightarrow \frac{F_T \cdot T}{2} = \left[F_S \cdot -\cos\left(\frac{\pi t}{T}\right) \cdot \frac{T}{\pi}\right]_0^T$$

$$= \frac{T F_S}{\pi}\left[1 - \cos\frac{\pi \cancel{T}}{\cancel{T}}\right]$$

$$\Rightarrow \frac{F_T \cdot \cancel{T}}{2} = \frac{2\cancel{T} F_S}{\pi}$$

$$\Rightarrow \boxed{F_S = \frac{\pi}{4} F_T}$$

Ans

9.17

Given,

$m = 1\ Kg.$

$$F(t) = \begin{cases} F_0 t & 0 \le t \le 1s \\ F_0(2-t) & 1s \le t \le 2s \end{cases}$$

$F_0 = 5\ N.$

$v(t=0) = 0.$

For $0 \le t \le 1s$

$$m\frac{dv}{dt} = F_0 t$$

$$\Rightarrow dv = \frac{F_0}{m} t\,dt$$

Integrating both sides, we get

$$\int_0^v dv = \frac{F_0}{m}\int_0^t t\,dt$$

$$\Rightarrow \boxed{v = \frac{F_0}{2m} t^2} \Rightarrow \boxed{v = 2.5t^2}$$

$$\therefore v\Big|_{t=1sec} = 2.5\ m/s.$$

$1s \le t \le 2s.$

$F(t) = F_0(2-t)$

$$\Rightarrow m\int_{v_1}^{v_2} dv = F_0\int_1^2 (2-t)\,dt$$

$$\Rightarrow v_2 = v_1 + \frac{F_0}{m}\cdot\left[2t - t^2/2\right]_1^2$$

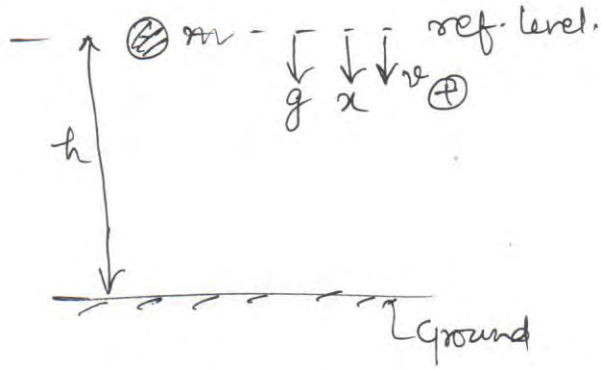
$$\Rightarrow v_2 = 2.5 + \frac{5}{1}\left[(4-2)-(2-1/2)\right]$$

$$\Rightarrow v_2 = 5\ m/s.$$ Ans

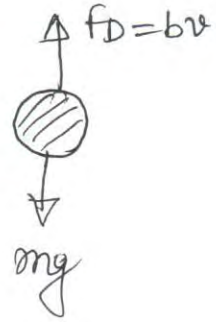
9.18

$v(t) = ?$

$x(t) = ?$



Free body diagram:-



Net force on body of mass 'm' in x-dirn.

$$mg - bv = m\ddot{x}$$

$$\Rightarrow \quad m\frac{dv}{dt} = mg - b\dot{x} \qquad [v = \dot{x}]$$

$$\Rightarrow \quad \frac{m\,dv}{\left(mg - \frac{bv}{m}\right)} = dt$$

$$\Rightarrow \quad \ln\left(mg - \frac{bv}{m}\right) = t + C$$

at $t = 0$, $v = 0$

$$\Rightarrow \quad C = \ln(mg).$$

$$\therefore \quad \ln\left(1 - \frac{bv}{mg}\right) = t$$

$$\Rightarrow \quad 1 - \frac{bv}{mg} = e^{t}$$

$$\Rightarrow \quad \frac{bv}{mg} = 1 - e^{t}$$

$$\Rightarrow \quad \boxed{v = \frac{mg}{b}\left[1 - e^{t}\right]}$$

Ans

$$v = \frac{mg}{b}\left[1 - e^{t}\right]$$

$$\Rightarrow \frac{dx}{dt} = \frac{mg}{b}\left[1 - e^{t}\right]$$

$$\Rightarrow \int_0^x dx = \frac{mg}{b}\int_0^t (1 - e^{t})\,dt$$

$$\Rightarrow x = \frac{mg}{b}\left[t - e^{t}\right]_0^t$$

$$= \frac{mg}{b}\left[t - e^{t} + 1\right]$$

$$\Rightarrow \boxed{x = \frac{mg}{b}\left[1 + t - e^{t}\right]}$$ Ans

9.19

$x(0) = v(0) = 0 \quad , m = 1\,Kg.$

$F(t) = 5\sin t$

$$m\frac{dv}{dt} = 5\sin t$$

$$\Rightarrow \int_0^v dv = \frac{5}{m}\int_0^t \sin t\,dt$$

$$\Rightarrow v = 5\left[-\cos t\right]_0^t$$

$$\Rightarrow \boxed{v = 5[1-\cos t]}$$ Ans

(a) $v\big|_{t=2\pi} = 0.$

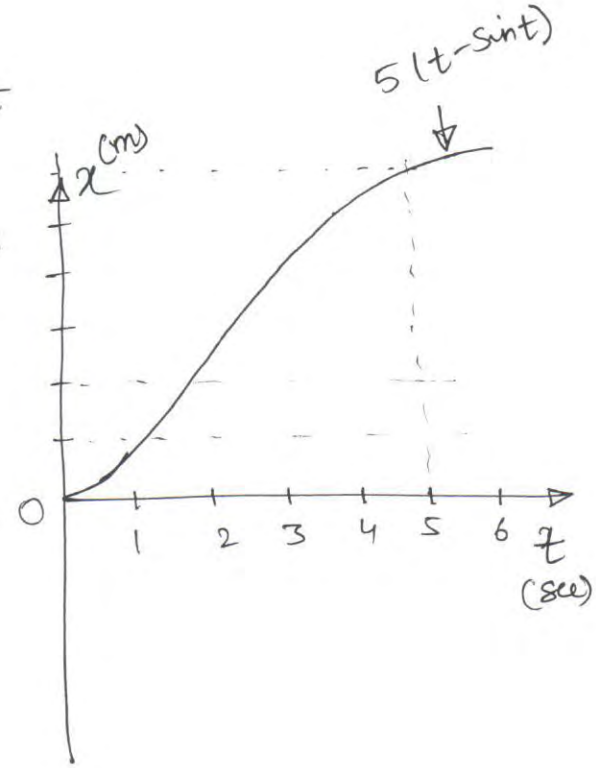
(b) $v = 5(1-\cos t)$

$$\frac{dx}{dt} = 5(1-\cos t)$$

$$\Rightarrow \int_0^x dx = \int_0^t 5(1-\cos t)\,dt$$

$$\Rightarrow x = 5\left[t - \sin t\right]_0^t$$

$$\Rightarrow \boxed{x = 5[t - \sin t].}$$



(9.20) $v_i = 0\,mph$ 1 mile $\simeq 1.6\,Km$.

$v_f = v(t=5sec) = 60\,mph \simeq \frac{60 \times 1.6 \times 10^3}{60 \times 60}$

$= 26.67\,ms^{-1}$.

Average acceleration: $a_{avg} = \frac{v_f - v_i}{t_f - t_i}$

$= \frac{26.67}{5}\,ms^{-2}$

$\Rightarrow \boxed{a_{avg} = 5.334\,ms^{-2}}$ Ans

$\Rightarrow \boxed{a_{avg} = 0.5437\,g.}$

(9.21) $0 \leq t \leq 5sec$

$v_0 = 30\,ms^{-1}$.

$\therefore x(5sec) = v_0 t$
$= 150m$.

for $t \geq 5sec$, $a_x = -10\,ms^{-2}$.

$\Rightarrow \frac{dv}{dt} = -10$

$\Rightarrow \int_{30}^{v} dv = \int_{5}^{t} -10\,dt$

$\Rightarrow v - 30 = -10(t-5)$

$\Rightarrow \boxed{v = 80 - 10t}$

(9.21) Continue ...

$$v = 80 - 10t \quad \text{—— (1)}$$

$$\frac{dx}{dt} = 80 - 10t$$

$$\Rightarrow \int_{150}^{x} dx = \int_{5}^{t} (80 - 10t)\, dt$$

$$\Rightarrow x = 150 + \left(80t - 10t^2/2\right)_5^t$$

$$= 150 + 80t - 5t^2 - 400 + 125$$

$$\Rightarrow \boxed{x = -5t^2 + 80t + 125}$$

$$\therefore \; v\Big|_{t=8\,sec} = 0 \qquad \text{Ans}$$

$$v\Big|_{t=12\,sec} = 80 - 10 \times 12$$

$$= -40\,m/s. \qquad \text{Ans}$$

body started moving in negative-x-dir^n with 40 m/s at t = 12 sec.

$$x\Big|_{t=8\,sec} = -5 \times 8^2 + 80 \times 8 - 125$$

$$= 195\,m. \qquad \text{Ans}$$

$$x\Big|_{t=12\,sec} = 115\,m. \qquad \text{Ans}$$

9.22

$$a \propto -(v - v_t)$$

$$a = -b(v - v_t) \quad ; \; b = \text{proportionality Constant.}$$

$$\Rightarrow m\frac{dv}{dt} = -b(v - v_t)$$

$$\Rightarrow \frac{dv}{dt} = -\frac{b}{m}(v - v_t)$$

$$\Rightarrow \frac{dv}{(v - v_t)} = -\frac{b}{m}dt$$

Integrating, we get

$$\int_{v_0}^{v} \frac{dv}{(v - v_t)} = -\frac{b}{m}\int_{t=0}^{t} dt$$

$$\Rightarrow \left[\ln(v - v_t)\right]_{v_0}^{v} = -\frac{b}{m}t$$

$$\Rightarrow \left(\frac{v - v_t}{v_0 - v_t}\right) = e^{-b/m\, t}$$

$$\Rightarrow v = (v_0 - v_t)e^{-b/m\, t} + v_t$$

$$\Rightarrow \boxed{v = v_t + (v_0 - v_t)e^{-b/m\, t}}$$

9.22 Continue —

$$\frac{dx}{dt} = v_t + (v_0 - v_t)\, e^{-b/m\, t}$$

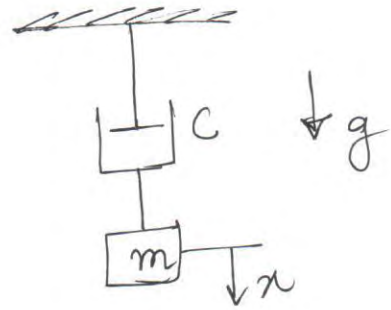
$$\int_{x_0}^{x} dx = \int_0^t \left[v_t + (v_0 - v_t)\, e^{-b/m\, t} \right] dt$$

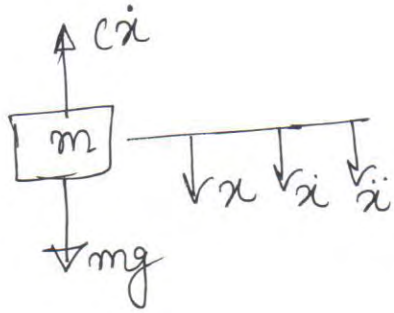
$$\Rightarrow \quad x = x_0 + v_t \cdot t + (v_0 - v_t)\frac{m}{b}\left[1 - e^{-b/m\, t}\right]$$

Ans

9.23

Free body diagram of mass "m"





~~$m\ddot{x} + mg = c\dot{x}$~~ $F_{net} = ma$

$$\Rightarrow mg - c\dot{x} = m\ddot{x}$$

$$\Rightarrow \boxed{m\ddot{x} + c\dot{x} - mg = 0.}$$

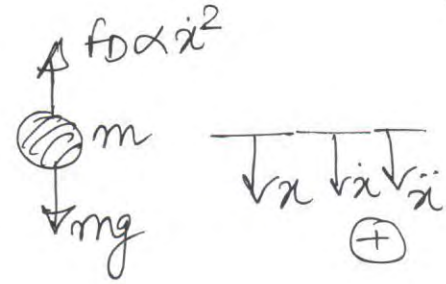
$\dot{x} = y$

$$\boxed{m\dot{y} + cy = mg}$$

9.24 1. $F_D \propto \dot{x}^2$

$\Rightarrow F_D = +b\dot{x}^2$

$m\ddot{x} = mg - b\dot{x}^2$



$$\Rightarrow \boxed{m\ddot{x} + b\dot{x}^2 - mg = 0}$$

2. Constant speed motion, $\ddot{x} = 0$

$$\Rightarrow b\dot{x}^2 = mg.$$

$$\dot{x}^2 = \frac{mg}{b}.$$

$$\Rightarrow \dot{x} = \sqrt{\frac{mg}{b}}.$$

3. $\ddot{x} = -\frac{b}{m}\dot{x}^2 + mg.$

Matlab : For Numerical solution

~~function (xdot~~

```
function [xdot] = myeqn (t,x)

xdot = [x(2) ;(-b*x(2)^2)/m + m*g.]

(t,x) = ode45(@myeqn, [tspan], [Z0]);
```

9.25

$$a = -Kv^2 = \frac{dv}{dt}$$

$$\Rightarrow -\int_{v_0}^{v} \frac{dv}{v^2} = \int_0^t K\,dt$$

$$\Rightarrow \left[\frac{1}{v} - \frac{1}{v_0}\right] = Kt$$

$$\Rightarrow \frac{1}{v} = \frac{1}{v_0} + Kt$$

$$\Rightarrow v = \frac{v_0}{(1+Kv_0t)}$$

$$v = v_0/2 \Rightarrow 1 + Kv_0t = 2$$

$$\Rightarrow Kv_0t = 1$$

$$\Rightarrow \boxed{t = \frac{1}{Kv_0}}$$

$$\frac{dx}{dt} = \frac{v_0}{1+Kv_0t}$$

$$\Rightarrow \int dx = v_0 \int_0^t \frac{dt}{(1+Kv_0t)}$$

$$\Rightarrow x - x_0 = v_0 \left[\ln(1+Kv_0t)\right] \frac{1}{Kv_0}$$

$$= \frac{1}{K} \ln(1+Kv_0t)$$

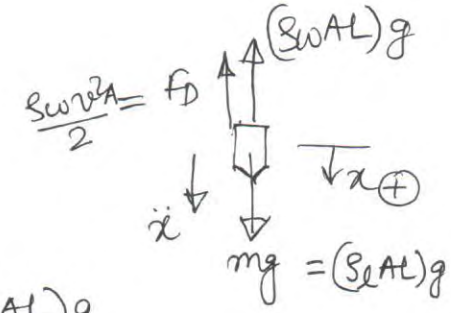
$$= \frac{1}{K}\left[\ln\left(1 + Kv_0 \cdot \frac{1}{Kv_0}\right)\right]$$

9.26

Free body diagram of bullet –

$\rho_L = 11.38$

$\Rightarrow \rho_w = 0.08858$



$$\Rightarrow \ddot{x}(\rho_L AL) = (\rho_L AL)g - (\rho_w AL)g - \frac{\rho_w v^2 A}{2}$$

$$\Rightarrow \cancel{\rho_L AL}\frac{dv}{dt} = 0.9115(\cancel{\rho_L AL})g - \frac{0.0885(\cancel{\rho_L A})v^2}{2}$$

$$\Rightarrow \frac{L\,dv}{dt} = 0.9115Lg - \frac{0.0885v^2}{2}$$

$$\Rightarrow \frac{L\,dv}{(8.942L - 0.04425v^2)} = dt$$

$$\Rightarrow \frac{dv}{\left(8.942 - \frac{0.04425v^2}{L}\right)} = dt$$

Integrate to find velocity & position as a function of time.

9.33.

a) Initial potential energy $= \frac{1}{2}kx^2$

$= \frac{1}{2}kx_0^2.$

$= \frac{1}{2} \times 5 \times 0.5^2$

$\Rightarrow (E_P)_i = 0.625 J.$

Initial K.E $= (KE)_i = 0$ (since it was released from rest position).

(b) Since there are no losses, system is considered to be conservative.

$\therefore$ KE + P.E = Const.

$\therefore 0.625 = (E_P)_{eq.} + (K_E)_{eq.}$

$\Rightarrow (K_E)_{eqm} = 0.625 J.$ (as $E_P = 0$).

(c) $\frac{1}{2} m \times v^2 = 0.625$

$\Rightarrow v = \left[\frac{2 \times 0.625}{2}\right]^{1/2}$

$\therefore v = 0.7905 \, ms^{-1}.$

9.31.

$$F = C_0 + C_1 x$$

workdone, $W = \int_{x_1}^{x_2} F\,dx$

$$= \int_{x_1}^{x_2} (C_0 + C_1 x)\,dx = \left[C_0 x + \frac{C_1 x^2}{2} \right]_{x_1}^{x_2}$$

$$\Rightarrow W = C_0 (x_2 - x_1) + \frac{C_1}{2}\left[x_2^2 - x_1^2\right]$$

$$\Rightarrow W = (x_2 - x_1)\left[C_0 + \frac{1}{2} C_1 (x_2 + x_1)\right]$$

9.32.

$$F = -\frac{dE_p}{dx}.$$

a) $F = 0$, $\frac{dE_p}{dx} = 0 \Rightarrow \boxed{E_p = C = \text{constt.}}$

b) $F = F_0$, $\int dE_p = \int F_0\,dx$

$$\Rightarrow \boxed{E_p = F_0 x + C}$$

c) $F = Kx$.

$$\Rightarrow \int dE_p = \int Kx\,dx$$

$$\Rightarrow \boxed{E_p = \frac{1}{2} K x^2 + C.}$$

d) $F = A \sin(x/x_0)$

$$\Rightarrow \boxed{E_p = -A x_0 \cos(x/x_0) + C.}$$

(9.35)

$F_p = \mu m g$

$= 1 \times 1000 \times 10$ N.

$\therefore$ acceleration, $a = 10 \, ms^{-1}$.

Since motion is constant acceleration.

$v = u + at$ — (1)

$v^2 = u^2 + 2as$ — (2)

& $s = ut + \frac{1}{2}at^2$ — (3) can be used.

here $u = 0$.

(a) $\therefore$ from (2)

$$s = \frac{v^2}{2a}.$$

$$v = 60 \text{ mph} = \frac{60 \times 1.609 \times 10^3}{60 \times 60} \, ms^{-1} = 26.8166 \, ms^{-1}.$$

$\therefore S_1 = 35.9566$ m. Ans

(b) from (1) $\Rightarrow$ $v_2 = 70$ mph $= 31.286 \, ms^{-1}$.

$\therefore$ $31.286 = 26.8166 + 10t$

$\Rightarrow t = 0.44695$ sec. Ans

(c) ~~Power used~~ $P = \int_0^t F \cdot v \, dt$

peak Power used by engine in getting up to $v_1 = 60$ mph

$P = F v_1$

9.36

$m = 1000\,Kg.$

$v_0 = 30\,ms^{-1}.$

$d = 1\,m.$

a). Total energy dissipated in crash

~~Initial KE~~ $= (K.E)_{before\,crash} - (KE)_{after\,crash}.$

(neglecting change of potential energy).

$= \frac{1}{2} \times 1000 \times 30^2 - 0$

$= 450\,KJ$ Ans

(b) Since constant accel^n is assumed during crash.

$F = Const.$

$F \times 1 = 450\,KJ$

$\Rightarrow F = 450\,KN.$ Ans.

(c) Newton Second law → force of the wall on car $= 450\,KN.$

(d) $F = 450\,KN$

$\therefore a = \frac{450 \times 10^3}{10^3} = 450\,m/s^2.$

$\boxed{a = 45.871g.}$ $g = 9.81\,ms^{-2}.$

Ans

(e) Force of the seat belt on the person

$= 50 \times 450\,N$

$= 2293.577$ times body weight.

(f) $F = 450 \times 15\,N.$ If parent is held stationary by belt.

$= 6.750\,KN. = 6750\,N.$ Ans

$= 450$ times child body weight.

9.39.

$m = 100\,kg$

$d_o = 20\,cm = 0.2\,m$ (under stationary weight mg).

(a) Stiffness of the spring,

$$K = \frac{mg}{d_o} = \frac{100 \times 9.81}{0.2} = 4905\,N/m$$

(b) Initial (K.E + PE) of system, when person at height 1m.

$$= 0 + mg \times 1$$

$$= 981\,J$$

Final (KE + PE) of system, when spring is having maximum compression

$$= 0 + \frac{1}{2} K d_{max}^2 - mg d_{max}$$

Since Assuming there is no dissipation.

$$-mg d_{max} + \frac{1}{2} K d_{max}^2 = 981\,J$$

$$\Rightarrow \frac{1}{2} \times 4905\, d_{max}^2 - 981\, d_{max} - 981 = 0$$

$$\Rightarrow 2452.5\, d_{max}^2 - 981\, d_{max} - 981 = 0.$$

$$\Rightarrow d_{max} = 0.8633\,m,\ -0.46332\,m.$$

$$\boxed{d_{max} = +0.8633\,m}$$ Ans

(c)

$$F_{peak} = K d_{max}$$

$$= 4905 \times 0.8633$$

9.45

$$x = c_1 e^{\lambda t} + c_2 e^{-\lambda t}$$

$$\dot{x} = \lambda c_1 e^{\lambda t} - \lambda c_2 e^{-\lambda t}$$

$$\ddot{x} = \lambda^2 c_1 e^{\lambda t} + \lambda^2 c_2 e^{-\lambda t}$$

$$= \lambda^2 (x)$$

$$\underline{\ddot{x} - \lambda^2 x = 0.}$$

It doesn't satisfy the harmonic oscillator equation

$$\ddot{x} + \lambda^2 x = 0.$$

9.46

$$\ddot{x} = -cx.$$

$$x = A\sin\sqrt{c}\,t + B\cos\sqrt{c}\,t$$

$$\dot{x} = A\sqrt{c}\cos\sqrt{c}\,t - B\sqrt{c}\sin\sqrt{c}\,t.$$

$$x(0) = 1, \quad \dot{x}(0) = 0$$

$$\Rightarrow \quad 1 = \left(A\sin\sqrt{c}\,t + B\cos\sqrt{c}\,t\right)\Big|_{t=0}$$

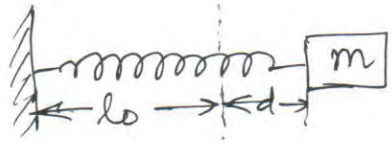
$$\Rightarrow \quad 0 = A\sqrt{c} \Rightarrow A = 0$$

$$\Rightarrow \quad x = \cos\sqrt{c}\,t$$

$$\therefore \quad x(\pi s) = \cos\left(\frac{1}{s}\right)t = \pm 1. = \pm 1. \checkmark$$

$$\dot{x}(\pi s) = -\left|\frac{1}{s}\right| \sin(\sqrt{c}\,t) = 0. \checkmark$$

9.50.



The equation of motion can be written as

$$m\ddot{x} + Kx = 0.$$

$$\& \quad x = d\cos\sqrt{\frac{K}{m}}\,t \qquad ; \omega = \sqrt{K/m}.$$

$$\dot{x} = -d\sqrt{K/m}\,\sin\sqrt{\frac{K}{m}}\,t.$$

Potential energy of system at any time 't'

$$= \frac{1}{2}Kx^2$$

$$= \frac{1}{2}Kd^2\cos^2\left(\sqrt{K/m}\right)t$$

Kinetic energy of system at any time 't'

$$= \frac{1}{2}m\dot{x}^2$$

$$= \frac{1}{2}Kd^2\sin^2\left(\sqrt{K/m}\right)t.$$

The potential & K.E will be equal at

$$\sqrt{K/m}\,t = (2n+1)\pi/4 \qquad ; n = 0, 1, 2, 3 \ldots$$

$$\Rightarrow t = \frac{(2n+1)\pi}{4\omega} = \frac{(2n+1)}{8}\left(\frac{2\pi}{\omega}\right).$$

$$\Rightarrow t = (2n+1)\frac{T}{8}. \qquad ; n = 0, 1, 2, 3, \ldots \quad \text{(b) Ans}$$

Potential energy is $\pi/2$ phase ahead of KE ~~and~~.

9.47

$$\ddot{x} + \lambda^2 x = C_0 \,, \quad x(0) = x_0 \,, \quad \dot{x}(0) = 0.$$

The general solution of above equation can be written as

$$x = C_1 \cos \lambda x + C_2 \sin \lambda x \quad ; \text{ where } (C_1 + C_2)\lambda^2 = C_0$$

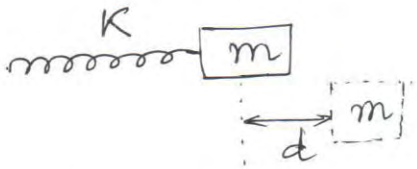
$$x(0) = x_0 \Rightarrow x_0 = C_1.$$

$$\dot{x}(0) = 0 \Rightarrow C_2 \lambda = 0 \Rightarrow C_2 = 0.$$

$$\therefore \quad x = x_0 \cos \lambda x \qquad ; x_0 = \frac{C_0}{\lambda^2}.$$

$$\text{or } x = \frac{C_0}{\lambda^2} \cos \lambda x.$$

9.48.



The differential equation governing the motion can be written as $-m\ddot{x} + Kx = 0$.

$$x(0) = d \,, \quad \dot{x}(0) = 0.$$

$$\therefore \quad x = d \cos \sqrt{\frac{K}{m}}\, t \qquad ; \omega_n = \sqrt{\frac{K}{m}}.$$

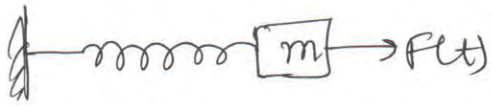
Time passed from release until the mass moves through the equilibrium position for the 2nd time $= \frac{3}{4} T$.

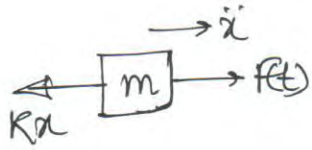
$$T = \frac{2\pi}{\omega}.$$

$$\therefore t = \frac{3}{4} \cdot 2\pi \sqrt{\frac{m}{K}} \quad \Rightarrow \boxed{t = \frac{3}{2}\pi \sqrt{\frac{m}{K}}} \text{ Ans}$$

9.51.

a)





$$\Rightarrow m\ddot{x} + K(x) = F(t) \quad \text{--- (1)}$$

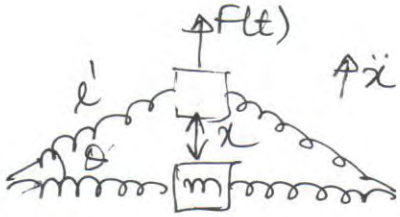
(b)

The above system is equivalent to two spring in series attached to mass m.

$$\therefore K_{eq} = \frac{K_1 K_2}{K_1 + K_2}.$$

Eqn of motion can be governed by $\boxed{\ddot{x} + \frac{K_{eq}}{m} x = 0}$

(c)



$\tan\theta = x/l_0$, $\sin\theta = x/l' \simeq \theta$ (for small θ)

$$m\ddot{x} = -2\left(\sqrt{l_0^2+x^2} - l_0\right) K \sin\theta + F(t)$$

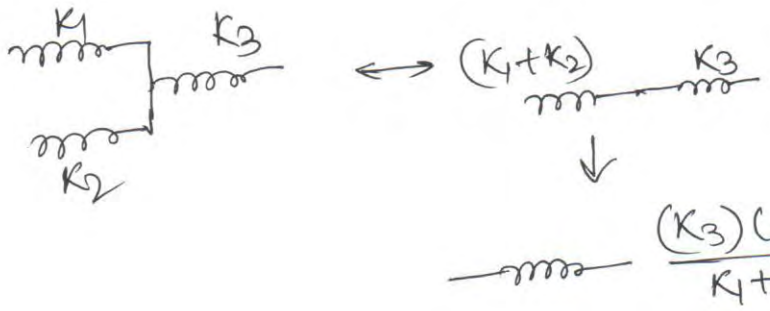
$$\Rightarrow m\ddot{x} + 2K\left(x - \frac{l_0 x}{(l_0^2+x^2)^{1/2}}\right) = F(t).$$

Ans

9.52

(a) $\omega_n = \sqrt{\dfrac{K_4}{M}}$.

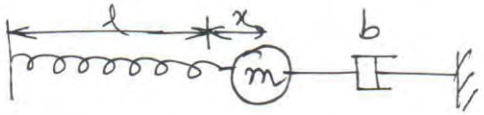
(b)

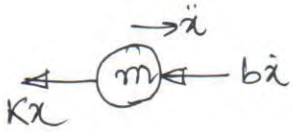


$$K_{eq} = \frac{K_1K_3 + K_3K_2 + K_4K_1 + K_4K_2 + K_4K_3}{K_1 + K_2 + K_3}$$

$$\therefore \omega_n = \sqrt{\frac{K_{eq}}{M}}$$ Ans

9.56.





$$m\ddot{x} = -Kx - b\dot{x} \Rightarrow m\ddot{x} + b\dot{x} + Kx = 0.$$

Let the solution is of form $x = e^{st}$

$$\Rightarrow \quad s^2 + \frac{c}{m}s + \frac{K}{m} = 0$$

$$s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - K/m} \Rightarrow x = e^{-(c/2m)t}\left[A e^{\left(\sqrt{(c/2m)^2 - K/m}\, t\right)} + B e^{-\left(\sqrt{(c/2m)^2 - K/m}\, t\right)}\right]$$

$$\xi = \frac{c}{2\sqrt{Km}}, \quad \omega_n = \sqrt{K/m}.$$

$$\frac{s_{1,2}}{\omega_n} = -\xi \pm i\sqrt{1-\xi^2} \quad ; \xi < 1.$$

$$\Rightarrow x = e^{-\xi\omega_n t}\left(A e^{i\sqrt{1-\xi^2}\,\omega_n t} + B e^{-i\sqrt{1-\xi^2}\,\omega_n t}\right).$$

for $\xi > 1$, $x = A e^{\left(-\xi + \sqrt{\xi^2 - 1}\right)\omega_n t} + B e^{\left(-\xi - \sqrt{\xi^2 - 1}\right)\omega_n t}$

where A & B can be found from initial conditions.

9.57.

$m = 0.4\,kg$, $c = 10\,kg/s$, $K = 5\,N/m$, $\omega_n = \sqrt{\frac{K}{m}} = 3.535$

a) $$\xi = \frac{c}{2\sqrt{Km}}$$

$$= \frac{10}{2\sqrt{0.4 \times 5}} = 3.535$$

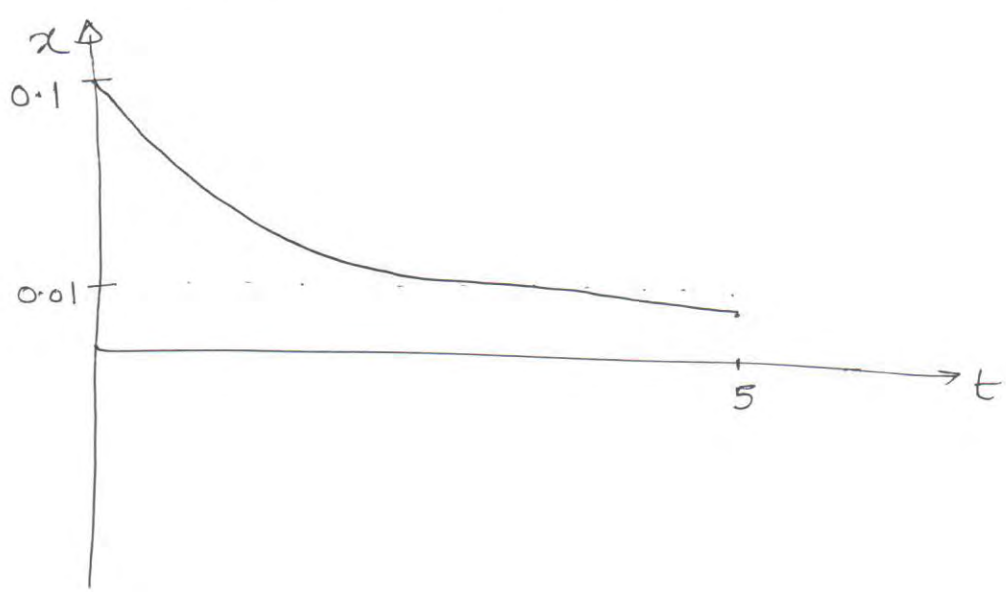
Since $\xi > 1$, system is over damped.

$$x = Ae^{(-\xi + \sqrt{\xi^2 - 1})\omega_n t} + Be^{(-\xi - \sqrt{\xi^2 - 1})\omega_n t}$$

where $$A = \frac{\dot{x}(0) + (\xi + \sqrt{\xi^2 - 1})\,\omega_n\, x(0)}{2\omega_n\sqrt{\xi^2 - 1}} = \cancel{2.129 \times 10^{-3}}\ 0.1021$$

$$\& \quad B = \frac{-\dot{x}(0) - (\xi - \sqrt{\xi^2 - 1})\,\omega_n\, x(0)}{2\omega_n(\sqrt{\xi^2 - 1})} = -0.00213.$$

$$\therefore x = 0.1021\, e^{-0.5104t} - 0.00213\, e^{-24.482t}$$



9.58

$$x = X e^{-\xi \omega_n t} \sin\left(\sqrt{1-\xi^2}\,\omega_n t + \phi\right).$$

$$\frac{x_4}{x_1} = 0.25 = \frac{e^{-\xi\omega_n(t_1+3\tau_d)} \cancel{\sin\left[\sqrt{1-\xi^2}\,\omega_n(t+3\tau_d)+\phi\right]}}{e^{-\xi\omega_n t_1} \cancel{\sin\left(\sqrt{1-\xi^2}\,\omega_n t_1\right)}}$$

$$= e^{-3\xi\omega_n\tau_d}$$

$$= e^{-3\xi \cancel{\omega_n} \cdot \frac{2\pi}{\cancel{\omega_n}\sqrt{1-\xi^2}}}$$

$$\Rightarrow 0.25 = e^{-3\xi \cdot \frac{2\pi}{\sqrt{1-\xi^2}}}$$

$$\Rightarrow \ln\left(\frac{1}{0.25}\right) = \frac{6\pi\xi}{\sqrt{1-\xi^2}}$$

$$\Rightarrow \frac{\xi^2}{1-\xi^2} = 3.9017 \times 10^{-3}$$

$$\boxed{\xi = 0.06234}$$ Ans.

$$\omega_n = \sqrt{\frac{K}{m}}.$$

$$\tau_d = 0.6 = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$\Rightarrow \omega_n = 10.4924$$

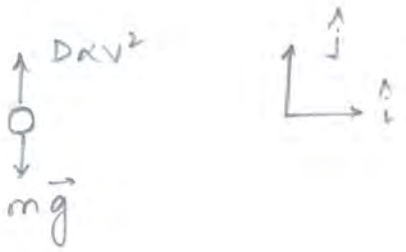
$$\Rightarrow K = 132.108\ N/m.$$ Ans

9.1.8

$d = 10\,Km$; $V = 15\,mph$

$$t = \frac{d}{V} = \frac{10 \times 10^3}{15 \times 1609.34} \times 60 = 24.85 \text{ mins}$$

9.1.24



i) $\sum F = m\vec{a}$

$\Rightarrow m\vec{a} = -mg\hat{j} + D\hat{j}$

Where $D \propto V^2 \Rightarrow D = KV^2$

$m\vec{a} = (KV^2 - mg)\hat{j} \quad \Rightarrow \quad (m\ddot{y} = K\dot{y}^2 - mg)$

where $|\vec{a}| = \dfrac{dv}{dt}$

and K = constant

ii) For constant speed motion

$|\vec{a}| = 0$

$Kv^2 - mg = 0$

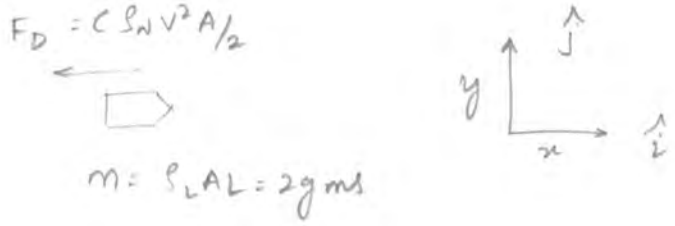
$V = \sqrt{\dfrac{mg}{K}} \quad \Rightarrow \quad \dot{y} = \sqrt{\dfrac{mg}{K}}$

iii) Equation for numerical solution

$\dot{y} = V$

$\dot{V} = \dfrac{K\dot{y}^2 - mg}{m}$

9.1.26



$V_0 = 400$ m/s

$\rho_e/\rho_w = 11.3$

$d = 5.7$ mm

$m = \rho_L A L = 2g$ ms

$F_D = C \rho_w V^2 A/2$

$C \approx = 1$

a) Equation of motion

$$m\ddot{x} = -\frac{C \rho_w \dot{x}^2 \pi d^2}{8} \approx -\frac{\rho_w \dot{x}^2 \pi d^2}{8}$$

$$\ddot{x} = \frac{-\rho_w \dot{x}^2}{2 \rho_L L}$$

9.2.10

$m = 90$ lbm

$h = 10$ ft

$g = 32$ ft/sec^2

$d = 1$ ft

a) Total distance:

$$h + d = 10\text{ft} + 1\text{ ft}$$
$$= 11\text{ ft}$$

b) Potential energy lost

$$mg(h+d) = 90 \times 32 \times 11 \quad \text{lbm ft}^2/\text{sec}^2$$

$$\text{P.E lost} = 31680 \text{ lbm ft}^2/\text{sec}^2$$

c) Work absorbed by legs

All P.E corresponding to 11 ft is absorbed by her legs

$\Rightarrow$ Work absorbed by legs = P.E lost = 31680 lbm.ft^2/sec^2

d) $$\underbrace{F \cdot d}_{\text{Work absorbed}} = \underbrace{mg(h+d)}_{\text{P.E lost}}$$

$$F = \frac{mg(h+d)}{d}$$

$$= \frac{90 \times 32 \times 11}{1}$$

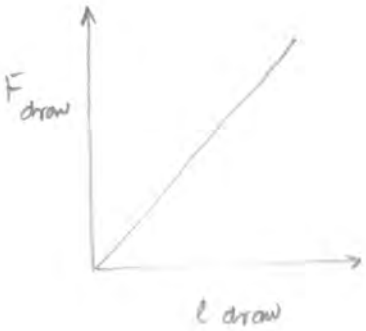
$$F = 31680 \text{lbm ft/sec}^2 \quad \Rightarrow \quad \frac{F}{mg} = \frac{h+d}{d} = 11$$

9.2.11

$F_{draw} = 25\ lbf - 75\ lbf$

$l_{draw} = 2ft - 2.5\ ft$

$M = 300 \times 64.8\ mg \simeq \frac{3}{4}$ ounce



a) Range of speed:

Energy stored in pulled bow-arrow

$= \frac{1}{2} F_d \times l_d =$ K.E of arrow $= \frac{1}{2} m v^2$

$$V = \sqrt{\frac{F_d l_d}{m}}$$

$$\sqrt{\frac{F_{d_{min}} \cdot l_{d_{min}}}{m}} \leq V \leq \sqrt{\frac{F_{d_{max}} \cdot l_{d_{max}}}{m}}$$

$$6.03 \times 10^3\ ft/sec \leq V \leq 11.67 \times 10^3\ ft/sec$$

b) when shot up; at maximum height

$$mgh = \frac{1}{2} F_d l_d \Rightarrow h = \frac{2 F_d l_d}{mg} = \frac{2V^2}{g}$$

$$2.272 \times 10^3\ ft \leq h \leq 8.511 \times 10^3\ ft$$

9.2.13

$$F_p = \mu mg \simeq mg$$

$m = 1000\ kg$

$\mu = 1$

$g = 10\ m/s^2$

$V_1 = 60\ mph$

$$\Rightarrow \quad a = \frac{mg}{m} = g.$$

for $V_1 = 60\ mph = \dfrac{60 \times 1.6093 \times 10^3}{60 \times 60}\ \dfrac{m}{sec}$

$= 26.8\ m/sec$

at any point $V = gt$; $x = \frac{1}{2}gt^2$; $V = \sqrt{2gx}$

$F = mg$;

Instantaneous power

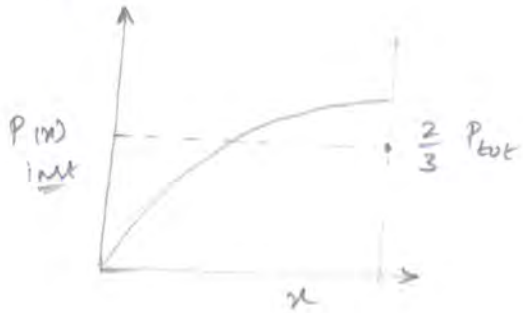
$P = F \cdot V = mgv$

$P(x) = mg\sqrt{2gx} = mg^2 t$

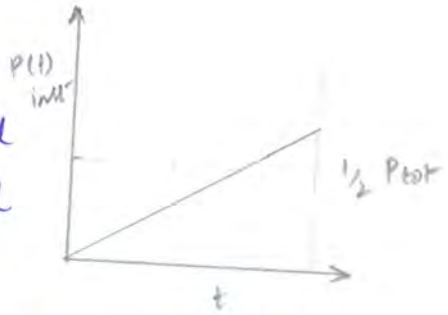
$$\bar{P}_1 = \frac{1}{x}\int_0^x mg\sqrt{2gx'}\,dx' = \frac{mg\sqrt{2g}\,x^{3/2}}{x\,(3/2)} = \frac{2}{3}mg\sqrt{2gx} = \frac{2}{3}mg^2 t \longrightarrow (i)$$

$$\bar{P}_2 = \frac{1}{t}\int_0^t mg^2 t'\,dt' = \frac{mg^2 t}{2} \longrightarrow (ii)$$

The two methods give diffrent powers
Power averged over distance travelled and since $P \propto \sqrt{x}$, this avg power is more than the time averaged power $\bar{P}(t)\ \Delta t$



If acceleration is Zero and input force is balanced by some dissipative force, say friction, then the input power averaged over distance or time will be same and equal to instantaneous power during the ride.



when the differ the time averged power is right because $\bar{P}(t) \cdot t$ then represents total input energy;

ie $\bar{P}(t) = \frac{\Delta E}{\Delta t}$

9.2.14

If deacceleration is not constant, all answer will change except for part (a) ie the total energy dissipated.

answer to (b), (c), (d), (e) and (f) will depend on instantaneous deacceleration.

9.2.15

a) $$F = -\frac{mgR^2}{r^2}$$

$$m\ddot{y} = -\frac{mgR^2}{(y+R)^2}$$

where y is the altitude of rocket

$$m\left(\frac{d}{dt}v\right) = -\frac{mgR^2}{(g+R)^2}$$

$$mv\frac{dv}{dy} = -\frac{mgR^2}{(y+R)^2}$$

$$\Rightarrow \int_{v_0}^{v} mv\,dv = -\int_0^h \frac{mgR^2}{(y+R)^2}\,dy$$

$$\frac{1}{2}m(v^2 - v_0) = \frac{mgR^2}{(y+R)}\Big|_0^h = \left(\frac{mgR^2}{h+R}\right) - mgR$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 - \frac{mgR^2h}{R(R+h)}$$

$$v^2 = v_0^2 - \frac{2gRh}{R+h}$$

$$v = \sqrt{v_0^2 - \frac{2gRh}{R+h}}$$

b) Maximum height

$$0 = v_0^2 - \frac{2gRh_{max}}{R+h_{max}} \Rightarrow \frac{2gR}{v_0^2} = 1 + \frac{R}{h_{max}}$$

$$h_{max} = \frac{R}{\frac{2gR}{v_0^2} - 1} = \frac{Rv_0^2}{2gR - v_0^2}$$

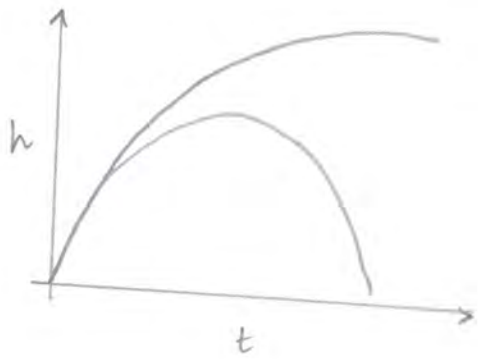
c) For V_0 = Velocity of Escape

$h_{max} \longrightarrow \infty$

$\Rightarrow 2gR - V_0^2 \longrightarrow 0$

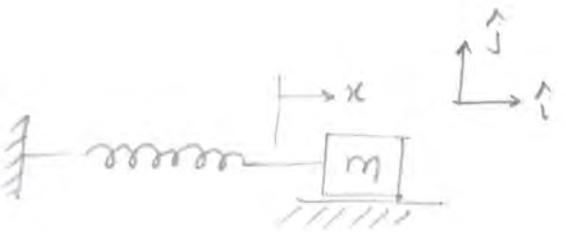
$V_{escape} = \sqrt{2gR}$

d) $\frac{dh}{dt} = \sqrt{V_0^2 - \frac{2gRh}{R+h}}$

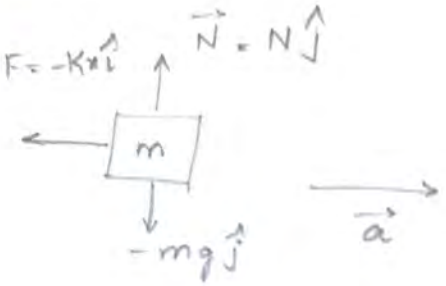


9.3.1

(a, b)



c) F.B.D of Mass



d) Equation of linear momentum balance

$$ma\hat{i} = -Kx\hat{i} - mg\hat{j} + N\hat{j} \quad \text{——} \quad (i)$$

e) $(i)\cdot\hat{i} \Rightarrow ma = -Kx$

$$m\ddot{x} = -Kx$$

$$\ddot{x} + \frac{K}{m}x = 0$$

$$\Rightarrow \ddot{x} + \omega^2 x = 0 \quad , \quad \omega = \sqrt{\frac{K}{m}}$$

Equation of harmonic oscillator

f) General solution

i) $x = A\sin\omega t + B\cos\omega t$, where $\omega = \sqrt{\frac{K}{m}}$

ii) $x = C\sin(\omega t + \phi)$, $\omega = \sqrt{\frac{K}{m}}$

g) Natural frequency $\omega_1 = \sqrt{\frac{K}{m}}$

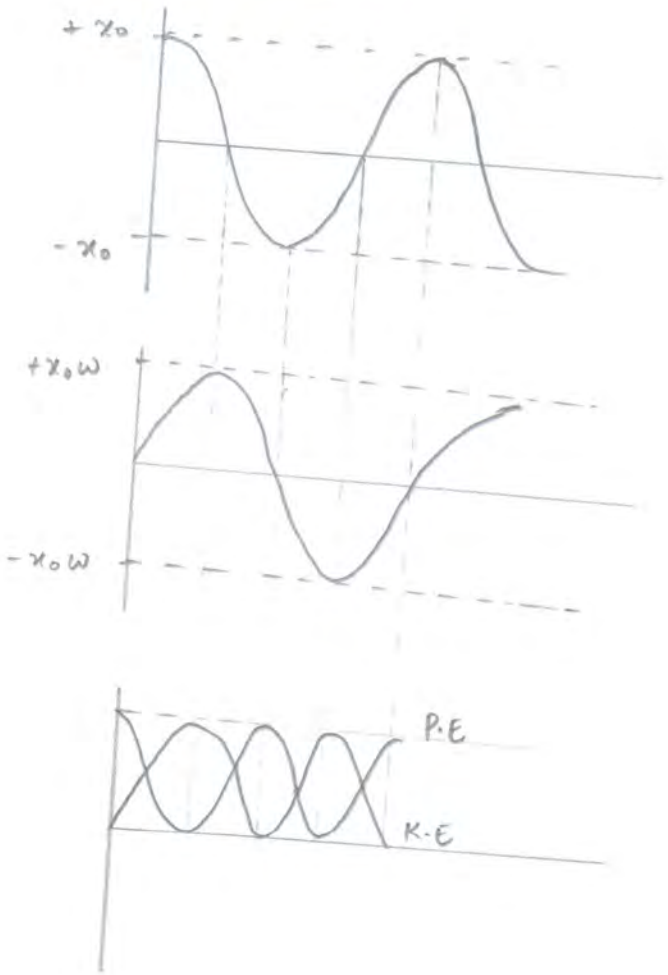
h) Period $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$

i) Frequency, $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$

j) Solution for $x(0) = x_0$

$\dot{x}(0) = 0$

$x = x_0 \cos \omega t$



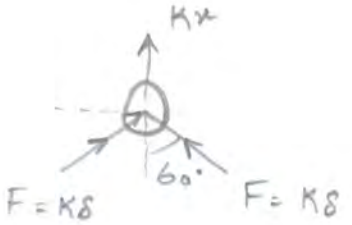
$P.E = \frac{1}{2} K x^2$

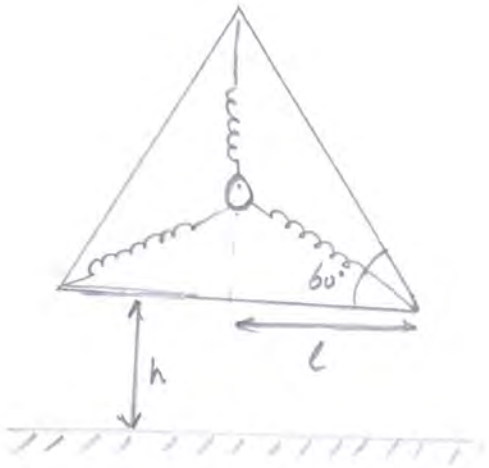
$K.E = \frac{1}{2} m v^2$

K) $\left.\begin{array}{l} \dot{x}(0) = v_0 \\ x_0 = x_0 \end{array}\right\} \Rightarrow \begin{array}{l} x = A \sin \omega t + B \cos \omega t \\ B = x_0 \ , \ A = \frac{v_0}{\omega} \end{array}$

$x = \frac{v_0}{\omega} \sin \omega t + x_0 \cos \omega t$

9.3.7





Equation of Motion

$$m\ddot{x} = -Kx - 2\left[K\,\frac{x}{2}\cos 60^\circ\right]$$

$$m\ddot{x} = -Kx - \frac{Kx}{2}$$

$$m\ddot{x} = -\frac{3}{2}Kx$$

Vibration Frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{3K}{2m}}$$

b) Maximum vertical deflection of the Egg

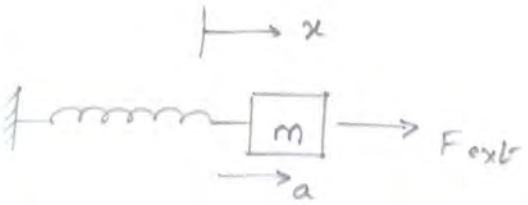
$$\frac{1}{2}m\dot{x}_0^2 = \frac{1}{2}K\cdot\frac{3}{2}\cdot x^2$$

$$\frac{1}{2}\cdot K\cdot\frac{3}{2}\,x^2 = mgh$$

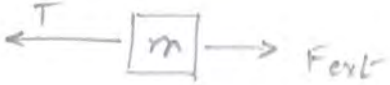
$$x = \sqrt{\frac{2mgh}{\frac{3K}{2}}} = \sqrt{\frac{4mgh}{3K}}$$

9-2-7

Equation of motion



$m\ddot{x} = F_{ext} - T$



$m\ddot{x} = F_{ext} - Kx$

$m\ddot{x} + Kx = F_{ext}$

9.2.3

$$F = F_0 \sin(ct)$$

$$m = 3kg$$

$$\dot{x}\Big|_{t=2} = 5\ m/s \quad , \quad x\Big|_{t=2} = 3m, \quad c = 2/s$$

a) $m\ddot{x} = F = F_0 \sin ct$

$$x = -\frac{F_0}{mc} \sin ct + At + B$$

$$a = \ddot{x} = \frac{F_0 \sin ct}{m} = \frac{4\sin(4)}{3} = \frac{4(-0.756)}{3}$$

$$a = -1.008\ m/s$$

a) $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \cdot m \cdot (5)^2 = \frac{75}{2}$ Joules

$= 37.5$ Joules

b) $P = F \cdot V = 4(-0.756)5 = -15.12$ watts

c) $\dot{E}_K = \frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2\right) = \frac{1}{2}m\dot{x} \times 2 \times \ddot{x} = m\dot{x}\ddot{x} = -3 \times 5 \times 1.008$

$= -15.1$ watts

e) Rate at which Force is doing work

$W_F = F \cdot V = -15.12$ watts

9.4.5

$$\dot{x}_1 = x_3 \quad , \quad \dot{x}_3 = 4\Omega^2 x_2 - 5\Omega^2 x_1 + 2\Omega^2 V_1^*$$

$$\dot{x}_2 = x_4 \quad , \quad \dot{x}_4 = 4\Omega^2 x_1 - 5\Omega^2 x_2 - \Omega^2 V_1^*$$

$$\begin{Bmatrix} x_1 = x \\ x_2 = y \\ x_3 = \dot{x} \\ x_4 = \dot{y} \end{Bmatrix} = \underline{x}$$

$$[A] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5\Omega^2 & 4\Omega^2 & 0 & 0 \\ 4\Omega^2 & -5\Omega^2 & 0 & 0 \end{bmatrix} \quad \& \quad \underline{C} = \begin{bmatrix} 0 \\ 0 \\ 2\Omega^2 \\ -\Omega^2 \end{bmatrix} V_1^*$$

$$\therefore \quad \dot{\underline{x}} = \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix}$$

$$\dot{\underline{x}} = [A]\,\underline{x} + \underline{C}$$

9.4.6

a) $\ddot{\theta} + \lambda^2\theta = \cos t \Rightarrow \ddot{\theta} = -\lambda^2\theta + \cos t$

Let

$$\underline{z} = \begin{Bmatrix} z_1 = \theta \\ z_2 = \dot{\theta} \end{Bmatrix}$$

$$\dot{\underline{z}} = \begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\lambda^2 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cos t$$

b) $\ddot{x} = -2p\dot{x} - Kx$,

$$\underline{z} = \begin{bmatrix} z_1 = x \\ z_2 = \dot{x} \end{bmatrix}$$

$$\dot{\underline{z}} = \begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -K & -2P \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

c)

$$\ddot{x} = -2c\dot{x} - K\sin x$$

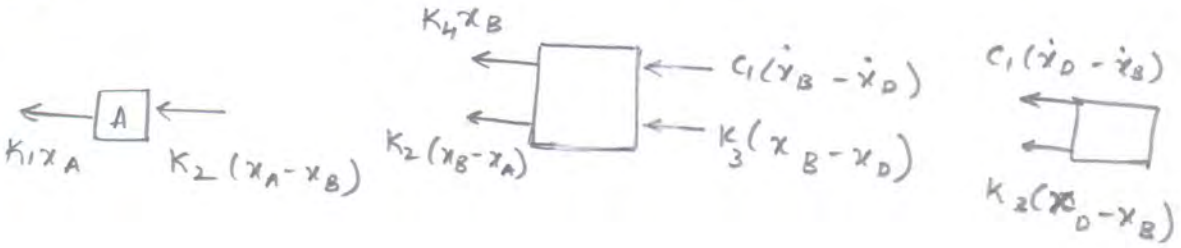
$$\Rightarrow \ddot{x} = -2c\dot{x} - Kx$$

Assuming x is $< 10^\circ$ or small , $\sin x = x$

$$\Rightarrow \ddot{x} = -2c\dot{x} - Kx$$

$$\Rightarrow \dot{\underline{z}} = \begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -K & -2c \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

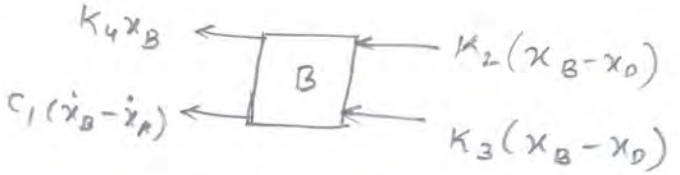
9.4.10



Acceleration of 'B'.

$$a_B = -\frac{1}{m_B}\left[\left[-K_2 x_A + (K_2 + K_3 + K_4)x_B - K_3 x_D\right] + c_1\dot{x}_B - c_1\dot{x}_D\right]$$

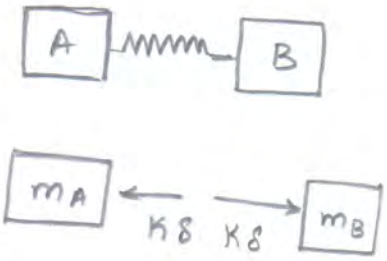
9.4.11



$$m_B \ddot{x}_B \hat{i} = -K_4 x_B - K_2(x_B - x_D) - K_3(x_B - x_D) - c_1(\dot{x}_B - \dot{x}_A)$$

$$\ddot{x}_B = \frac{-1}{m_B}\left[(K_2 + K_3 + K_4)x_B - (K_2 + K_3)x_D + (c_1 \dot{x}_B + c_1 \dot{x}_A)\right]$$

9.4.12

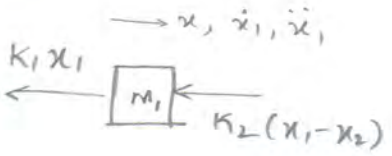


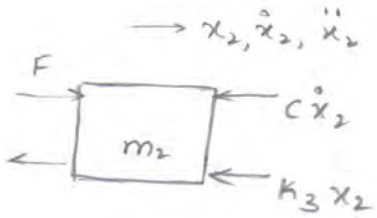
$$m_A a_A (-\hat{i}) = K\delta(-\hat{i})$$

$$a_A = \frac{K\delta}{m_A}$$

$$V_A = \int a_A \, dt = \int_0^t \frac{K\delta}{m_A} \, dt = \frac{K\delta t}{m_A} = V_A$$

9.4.14





$$m_1 \ddot{x}_1 = -K_1 x_1 - K_2 x_1 + K_2 x_2$$

$$m_2 \ddot{x}_2 = -K_3 x_2 - K_2 x_2 + K_2 x_1 - c\dot{x}_2 + F$$

Let

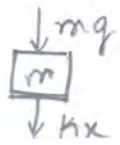
$$Z_1 = x_1 \qquad \dot{Z}_1 = Z_2$$

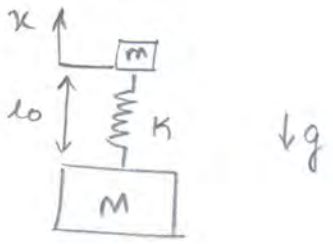
$$Z_2 = \dot{x}_1 \qquad \dot{Z}_2 = \left[-(K_1 + K_2) Z_1 + K_2 Z_3\right] \times \left(\frac{1}{m_1}\right)$$

$$Z_3 = x_2 \qquad \dot{Z}_3 = Z_4$$

$$Z_4 = \dot{x}_2 \qquad \dot{Z}_4 = \left(\frac{1}{m_2}\right)\left[K_2 x_1 - (K_2 + K_3) x_2 - c\dot{x}_2\right] + \frac{F}{m_2}$$

9.4.16





$m\ddot{x} = -kx - mg$

a) Static equilibrium

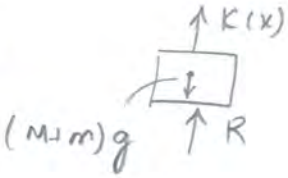
$\ddot{x} = 0$

$\Rightarrow \quad -kx - mg = 0$

$x = -\frac{mg}{k}$

b) $m\ddot{x} + kx = mg$

c)



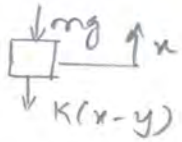
d) M lifts off if $R = 0$

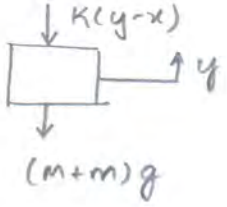
$K(x) + R = (M+m)g$

$K(x) = (M+m)g$

$x = \frac{(M+m)g}{K}$

e)

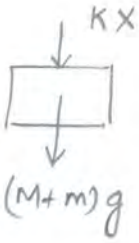




$m\ddot{x} = -k(x-y) - mg$

$M\ddot{y} = -k(y-x) - (m+M)g$

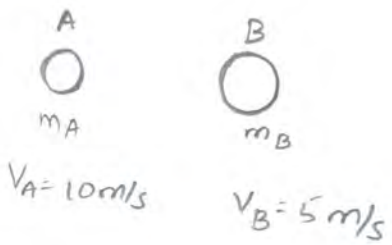
f)



$R = 0$

$X = -\frac{(M+m)g}{K}$

9.5.1



a) $L_A = m_A V_A = 10 \text{ kg m/s}$

b) $L_B = m_B V_B = 10 \text{ kg m/s}$

c) $L_A + L_B = 20 \text{ kg m/s}$

d) $L_A^+ = m_A V_A^+ = 1 \times 6 = 6 \text{ kg m/s}$

e) $L^+ = L^- = L^+ = L_A^- + L_B^- = 20 \text{ kg m/s}$

f) $L^+ = m_A V_A^+ + m_B V_B^+ = m_A V_A^- + m_B V_B^- = 20 \text{ kg m/s}$

$m_B V_B^+ = L_B^+ = 20 - 6 = 14 \text{ kg m/s}$

g) $P^B = m_B (V_B^+ - V_B^-) = 2(7-5) = 4 \text{ kg m/s}$

h) $P^A = m_A (V_A^+ - V_A^-) = 4 \text{ kg m/s}$

i) $KE^- = \frac{1}{2} m_A V_A^{-2} + \frac{1}{2} m_B V_B^{-2}$

$= \frac{1}{2} \times 100 + \frac{1}{2} \times 2 \times 25 = 75 \text{ Joules}$

j) $KE^+ = \frac{1}{2} m_A V_A^{+2} + \frac{1}{2} m_B V_B^{+2} = \frac{1}{2} \times 36 + \frac{1}{2} \times 2 \times 49 = 67 \text{ Joules}$

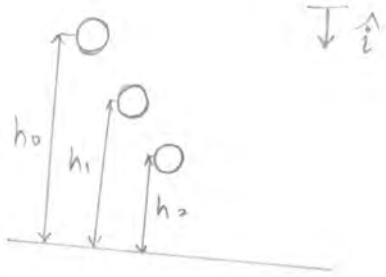
k) $e = -\frac{(V_B^+ - V_A^+)}{(V_B^- - V_A^-)} = -\frac{(7-6)}{(5-10)} = \frac{1}{5} = 0.2$

9.5.2

$h_0 = 10\,m$

$h_1 = 6.4\,m$

$e = ?$



$$\frac{1}{2}mv^2 = mgh_0$$

$$v^- = \sqrt{2gh_0}$$

$$\frac{1}{2}mv^{+2} = mgh_1$$

$$v^+ = \pm\sqrt{2gh_1}$$

$$e = -\frac{(v^+ - 0)}{(v^- - 0)} = \frac{-(-\sqrt{2gh_1})}{\sqrt{2gh_0}} = \sqrt{\frac{h_1}{h_0}} = \sqrt{\frac{6.4}{10}} = 0.8$$

$$\frac{1}{2}mv_1^2 = mgh_1$$

$$v_1^- = \sqrt{2gh_1}$$

$$e = -\frac{(v_1^+ - 0)}{(v_1^- - 0)} \Rightarrow v_1^+ = -0.8 \times \sqrt{2gh_1} \longrightarrow (1)$$

$$\frac{1}{2}mv_1^{+2} = mgh_2 \quad (\text{Apply } (1))$$

$$\frac{1}{2}(0.8^2 \times 2gh_1) = gh_2$$

$$h_2 = 0.8^2 h_1$$

$$h_2 = \frac{8^2}{1000} \times 64 = 4.096\,m$$

9.5.3

$m = 0.02\ kg$, $V = 500\ m/s$

$$m v_m^- + M v_m^- = m v_m^+ + M v_M^+$$

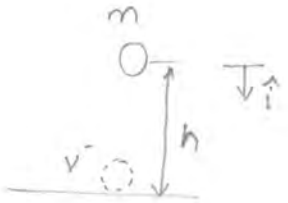
$$0.02 \times 500 + 50 \times 0 = (0.02 + 50)\, v^+$$

$$V^+ = \frac{10}{50.02} = 0.2\ m/s$$

$e = 0$

9.5.4

$$e = \frac{v^+}{v^-}$$



$$\frac{1}{2} m v^{-2} = mgh$$

$$v^{-2} = 2gh$$

$$\frac{1}{2} m v^{+2} = mgh_1$$

$$\frac{1}{2} e^2 v^{-2} = gh_1$$

$$\frac{1}{2} e^2 2gh = gh_1$$

$$\Rightarrow \quad h_1 = e^2 h$$

9.5.5

$$m_A V_0 = m_A V_A + m_B V_B$$

$$-eV_0 = V_A - V_B$$

$$\begin{bmatrix} m_A \\ -e \end{bmatrix} V_0 = \begin{bmatrix} m_A & m_B \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix}$$

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = -\frac{1}{m_A + m_B} \begin{bmatrix} -1 & -m_B \\ -1 & m_A \end{bmatrix} \begin{bmatrix} m_A \\ -e \end{bmatrix} V_0$$

$$\Rightarrow \quad V_A = -\frac{(em_B - m_A) V_0}{m_A + m_B}$$

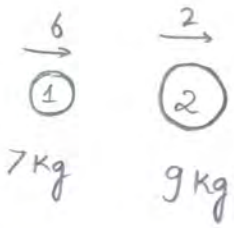
$$V_B = \frac{(m_A + em_A) V_0}{m_A + m_B}$$

$m_A = 2\,kg$, $m_B = 500\,kg$, $V_0 = 2.6\,m/s$, $e = 0.8$

$$V_A = -\frac{(0.8 \times 500 - 2)\ 2.6}{502} = -2.06\ m/s$$

$$V_B = \frac{(2 \times 1.8) \times 2.6}{502} = 0.018\ m/s$$

9.5.6



$$P^{on\,2} = m_2(v_2^+ - v_2^-)$$

$$= 9(v_2^+ - 2)$$

$$-P^{on\,1} = m_1(v_1^+ - v_1^-) = 7(v_1^+ - 6) = -9(v_2^+ - 2)$$

$$9v_2^+ + 7v_1^+ = 42 + 18 = 60$$

$$9v_2^+ + 7v_1^+ = 60 \longrightarrow (1)$$

$$e = -\frac{(v_2^+ - v_1^+)}{(v_2^- - v_1^-)} \Rightarrow 0.5(2-6) = -(v_2^+ - v_1^+)$$

$$v_2^+ - v_1^+ = 2 \Rightarrow v_2^+ = 2 + v_1^+ \longrightarrow (2).$$

Apply (2) in (1).

$$18 + 9v_2^+ + 7v_1^+ = 60$$

$$16v_1^+ = 42$$

$$v_1^+ = \frac{21}{8} \text{ m/s}$$

$$v_2^+ = \frac{37}{8} \text{ m/s}$$

$$p^{on2} = 9\left(\frac{37}{8} - 2\right) = 23.625 \text{ kg m/s}$$

$$p^{on1} = 7\left(\frac{21}{8} - 6\right) = -23.625 \text{ kg m/s}$$

$$\therefore \; p^{on2} = -p^{on1}$$

9.5.9

a) $mV_0 = m_A V_A + (m + m_B) V_B$

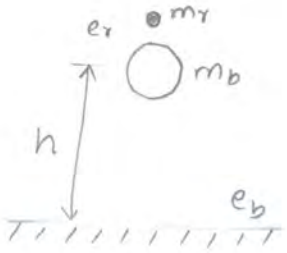
b) $mV_0 = (m_B + m) V_B$

$$V = \frac{(m+m_B)}{m} V_B$$

c) 1) $E_{loss} = -\frac{1}{2} m \left[\left(\frac{m+m_B}{m}\right) V_B\right]^2 + \frac{1}{2} m V_0^2 - \frac{1}{2} m_A V_A^2$

2) $E_{loss} = \frac{1}{2} m \left[\left(\frac{m+m_B}{m}\right) V_B\right]^2 - \frac{1}{2} (m+m_B) V_B^2$

9.5.10

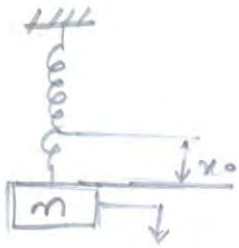


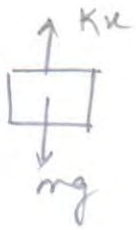
$$h_f = \frac{h}{\left(1 + \frac{m_b}{m_r}\right)^2}\left[\frac{m_b}{m_r}(e_r + e_b + e_r e_b) - 1\right]^2$$

if $m_b >> m_r$

$$h_f = h\,(e_r + e_b + e_r e_b)^2$$

10.1.1





From equation of motion

$$-k(x+x_0)+mg = m\ddot{x} \longrightarrow (1)$$

at equilibrium, the upward force is equal to the downward force

$$-Kx_0+mg=0$$

$$mg = Kx_0$$

substitute in (1)

$$m\ddot{x} = -Kx$$

$$\ddot{x}+\left(\sqrt{\frac{K}{m}}\right)^2 x = 0$$

on solving the differential equation

$$x = C.\cos\left(\sqrt{\frac{K}{m}}\cdot t+\phi\right)$$

At, $t=0$, $\dot{x}_0 = 0$, $x_0 = d$

$$x = C \cdot \cos\left(\sqrt{\frac{K}{m}}\, t + \phi\right)$$

$$\dot{x} = -C \cdot \sqrt{\frac{K}{m}} \sin\left(\sqrt{\frac{K}{m}}\, t + \phi\right)$$

$$x_0 = C \cdot \cos\phi$$

$$\dot{x}_0 = 0 = -C \cdot \sqrt{\frac{K}{m}} \cdot \sin\phi$$

this means that

$$\sin\phi = 0$$

hence $\phi = 0$

$$C = d$$

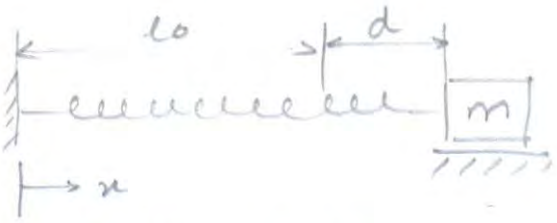
$$\therefore \quad x = d \cos\left(\sqrt{\frac{K}{m}}\, t\right)$$

The time required by the mass to cross the equilibrium position for the second time is 3/4 the period of oscillation

$$t = \frac{3}{4} T = \frac{\frac{3}{4} \times 2\pi}{\sqrt{\frac{K}{m}}}$$

$$t = \frac{3\pi}{2} \sqrt{\frac{m}{K}}$$

10.1.2



a) Acceleration just after release

$$= -\frac{kd}{m} \quad \text{towards wall}$$

b) $$-k(x - l_0) = m\ddot{x}$$

Initial conditions at $t=0$

$$\dot{x} = 0 \quad \text{at } t=0$$

$$x = l + d \quad \text{at } t=0$$

c) position at arbitrary time

$$-k(x - l_0) = m\ddot{x}$$

Put $y = x - l_0$

$$\ddot{y} = \ddot{x}$$

$$m\ddot{y} = -ky$$

then the solution of the differential equation

$$y = C\cos\left(\sqrt{\frac{k}{m}} + \phi\right)$$

from the initial conditions

$$y = d\cos\left(\sqrt{\frac{k}{m}}\, t\right)$$

$$x - l_0 = d\cos\left(\sqrt{\frac{k}{m}}\,t\right)$$

$$x = l_0 + d\cos\left(\sqrt{\frac{k}{m}}\,t\right)$$

d)

$$\dot{x} = -d\sqrt{\frac{k}{m}}\sin\left(\sqrt{\frac{k}{m}}\,t\right) \longrightarrow (1)$$

When the mass is at the point where $x = l_0$ it has performed $\frac{1}{4}$ of the oscillation hence the time t is given by

$$t = \frac{\frac{2\pi}{4}}{\sqrt{\frac{k}{m}}} = \frac{\frac{\pi}{2}}{\sqrt{\frac{k}{m}}}$$

on substuting the value in (1)

we get :

$$\dot{x}_{l_0} = -d\sqrt{\frac{k}{m}}$$

10.1.3

a) Potential energy $= \frac{1}{2} k (x - l_0)^2$

Kinetic energy $= \frac{1}{2} m \dot{x}^2$

Where $(x - l_0) = d \cos\left(\sqrt{\frac{k}{m}}\, t\right)$

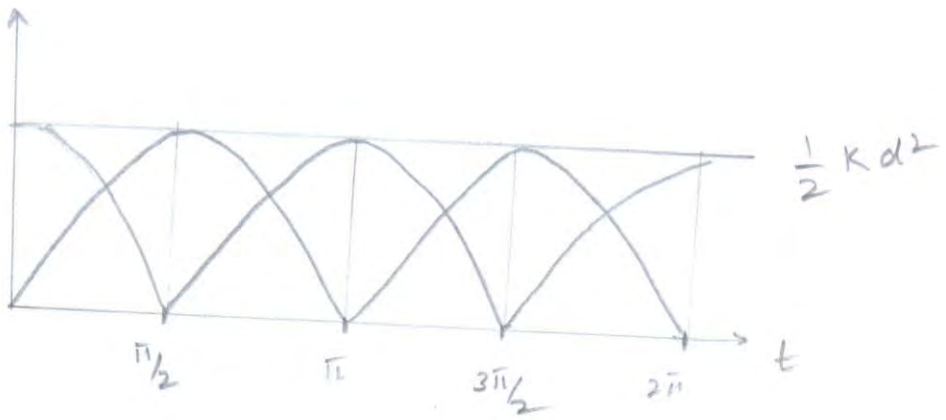
$$\dot{x} = -d \sqrt{\frac{k}{m}} \sin\left(\sqrt{\frac{k}{m}}\, t\right)$$

$$V = \frac{1}{2} k d^2 \cos^2\left(\sqrt{\frac{k}{m}}\, t\right)$$

$$K_i = \frac{1}{2} d^2 k \sin^2\left(\sqrt{\frac{k}{m}}\, t\right)$$

$$\boxed{V + K = \frac{1}{2} k d^2}$$

c)



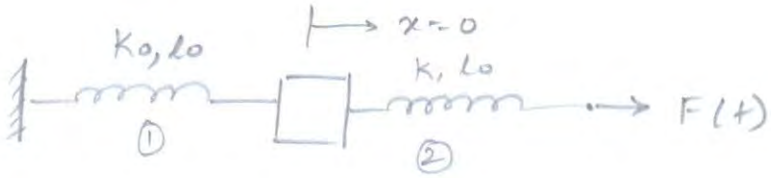
10.1.4

a)

$$m\ddot{x} = -Kx + F(t)$$

$$Kx + m\ddot{x} = F(t)$$

b)

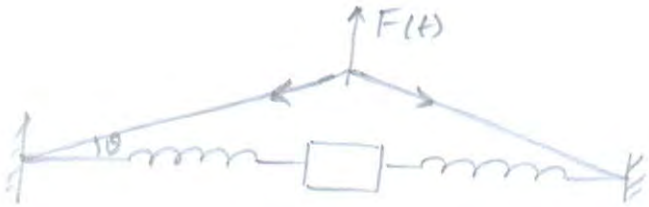


$$-Kx_1 + K(x_2 - x_1) = m\ddot{x}_1$$

$$K(x_2 - x_1) - F(t) = \cancel{m_s \ddot{x}_2}^{\,0}$$

$$\ddot{x} + \frac{K}{m}x = \frac{F(t)}{m}$$

c)



$$-2K\left(l_0/\cos\theta - l_0\right)\sin\theta + F(t) = m l_0 \frac{d^2}{dt^2}(\sin\theta)$$

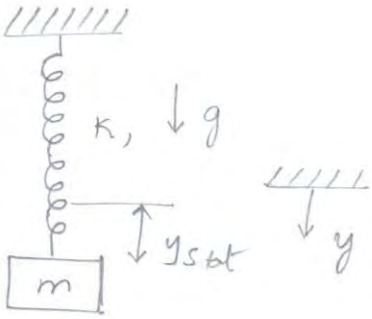
$$-2K\left(\frac{l_0}{\cos\theta} - l_0\right)\sin\theta + F(t) = m l_0\left[-\sin\theta\,\dot{\theta}^2 + \cos\theta\cdot\ddot{\theta}\right]$$

if $\theta \to 0$ then $\sin\theta = \theta$

$\cos\theta = 1$

$$F(t) = m l_0 (\ddot{\theta} - \theta \dot{\theta}^2)$$

10.1.5



$$m\ddot{y} = mg - Ky$$

let $x = y - y_{stat}$

$$m\ddot{x} = mg - K(x + y_{stat})$$

$$m\ddot{x} + Kx = 0$$

$$\omega = \sqrt{\frac{K}{m}}$$

since from equilibrium condition

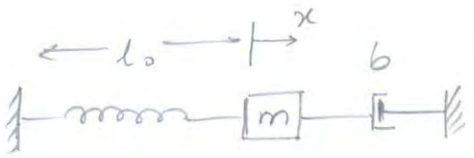
$$mg = K y_{stat}$$

Period of oscillation

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{m}{K}}$$

10.1.6



a)

b) Motion in x direction

$$m\ddot{x} + b\dot{x} + Kx = 0$$

10.1.7

$$m\ddot{x} + b\dot{x} + kx = 0 \longrightarrow (1)$$

The solution is of the form

$$x_c = Be^{st}$$

Substitute in (1)

$$(ms^2 + bs + k)\, Be^{st} = 0$$

$$ms^2 + bs + k = 0$$

the roots are

$$s_{1,2} = \frac{1}{2m}\left(-b \pm \sqrt{b^2 - 4mk}\right)$$

Since there are two roots the complementary solution is

$$x_c = B_1 e^{s_1 t} + B_2 e^{s_2 t}$$

Now we rewrite the equation as

$$\frac{k}{m} = \omega_n^2 \quad , \quad \frac{b}{m} = 2\xi\omega_n$$

$$S_{1,2} = -\xi\omega_n \pm \sqrt{\xi^2 - 1}\ \omega_n$$

Now the Natural frequency

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10^4}{1}} = 100 \text{ rad/sec}$$

Damping ratio

$$\xi = \frac{b}{2\sqrt{km}}$$

$$\xi = \frac{5}{\sqrt{4\times10^4}} = 2.5\times10^{-2}$$

$$\xi < 1$$

The system is underdamped

Damped frequency,

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_d = 99.96 \text{ rad/sec}$$

10.1.8

$$\omega = 150 \text{ rad/s}$$

$$\omega d = \omega \sqrt{1-\xi^2}$$

$$100 \geq 150 \sqrt{1-\xi^2}$$

$$0.6666 \geq \sqrt{1-\xi^2}$$

$$0.4444 \geq 1-\xi^2$$

$$-0.5556 \geq -\xi^2$$

$$0.5556 \leq \xi^2$$

$$\xi \geq 0.7453$$

10.1.9

$$m\ddot{x} + c\dot{x} + kx = 0$$

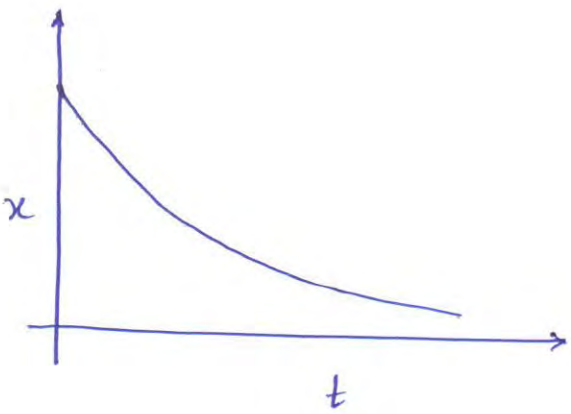
$$S_{1,2} = -\xi\omega_n \pm \sqrt{\xi^2 - 1} \cdot \omega_n$$

a) $$\xi = \frac{c}{2\sqrt{km}} = \frac{10}{2\sqrt{0.4 \times 5}} = 3.535$$

$$\xi > 1$$

The system is overdamped.

b)



c) The equation of motion is given by.

$$x_c = A e^{-\xi\omega_n t} \cdot \sin(\omega_d t + \psi)$$

$$\dot{x}_c = A\left[-\xi\omega_n e^{-\xi\omega_n t} \sin(\omega_d t + \psi) + e^{-\xi\omega_n t} \cdot \omega_d \cdot \cos(\omega_d t + \psi)\right]$$

c) The equation of motion is given by.

$$x = B_1 e^{s_1 t} + B_2 e^{s_2 t} \longrightarrow (1)$$

$$\dot{x} = B_1 s_1 e^{s_1 t} + B_2 s_2 e^{s_2 t} \longrightarrow (2)$$

$$s_1 = -\xi \omega_n + \sqrt{\xi^2 - 1} \cdot \omega_n$$

$$s_1 = -\xi \omega_n - \sqrt{\xi^2 - 1} \quad \omega_n$$

Now, put $\xi = 3.535$

then,

$$s_1 = -3.535\ \omega_n + 3.39\ \omega_n = -0.144\ \omega_n$$

$$s_2 = -3.535\ \omega_n - 3.39\ \omega_n = -6.925\ \omega_n$$

Now, $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{5}{0.4}} = 3.535$ rad/s

then,

$$s_1 = -0.5$$

$$s_2 = -24.5$$

From equation (1) and (2) at $t = 0$

$$x_0 = B_1 + B_2$$

$$\dot{x}_0 = B_1 s_1 + B_2 s_2$$

From the initial condition we know that

$$x_0 = 0.1\ m$$

$$\dot{x}_0 = 0$$

then

$$B_1 S_1 = -B_2 S_2$$

$$B_1 = -B_2 \frac{S_2}{S_1}$$

$$B_1 = -49\ B_2$$

$$B_1 + B_2 = 0.1$$

$$-49 B_2 + B_2 = 0.1.$$

$$-48 B_2 = 0.1$$

$$B_2 = -2.08 \times 10^{-3}$$

$$B_1 = 0.102$$

we get,

$$x = 0.102\ e^{-0.5t} - 2.08 \times 10^{-3}\ e^{-24.5t}$$

10.1.10

$K = 500\ N/m$

$C = 10\ kg/s$

Range of mass = 1 kg to 10 kg in the increment of half kg.

Suppose that the system is critically damped then

$\xi = 1$.

we have.

$$\xi = \frac{c}{\sqrt{4mk}}$$

$$\sqrt{m} = \frac{c}{\xi\sqrt{4k}}$$

$$m = \frac{c^2}{\xi^2 \cdot 4 \cdot k}$$

$$m = \frac{c^2}{4 \cdot k} = \frac{10 \times 10}{4 \times 500}$$

$$m = 0.05\ kg$$

Now, let us consider the mass given in the problem

$m = 1\ kg.$

$$\xi = \frac{c}{\sqrt{4mk}}$$

$$\xi = \frac{10}{\sqrt{4 \times 1 \times 500}}$$

$$\xi = 0.22$$

Now take, $m = 10\ kg.$

$$\xi = \frac{c}{\sqrt{4mk}}$$

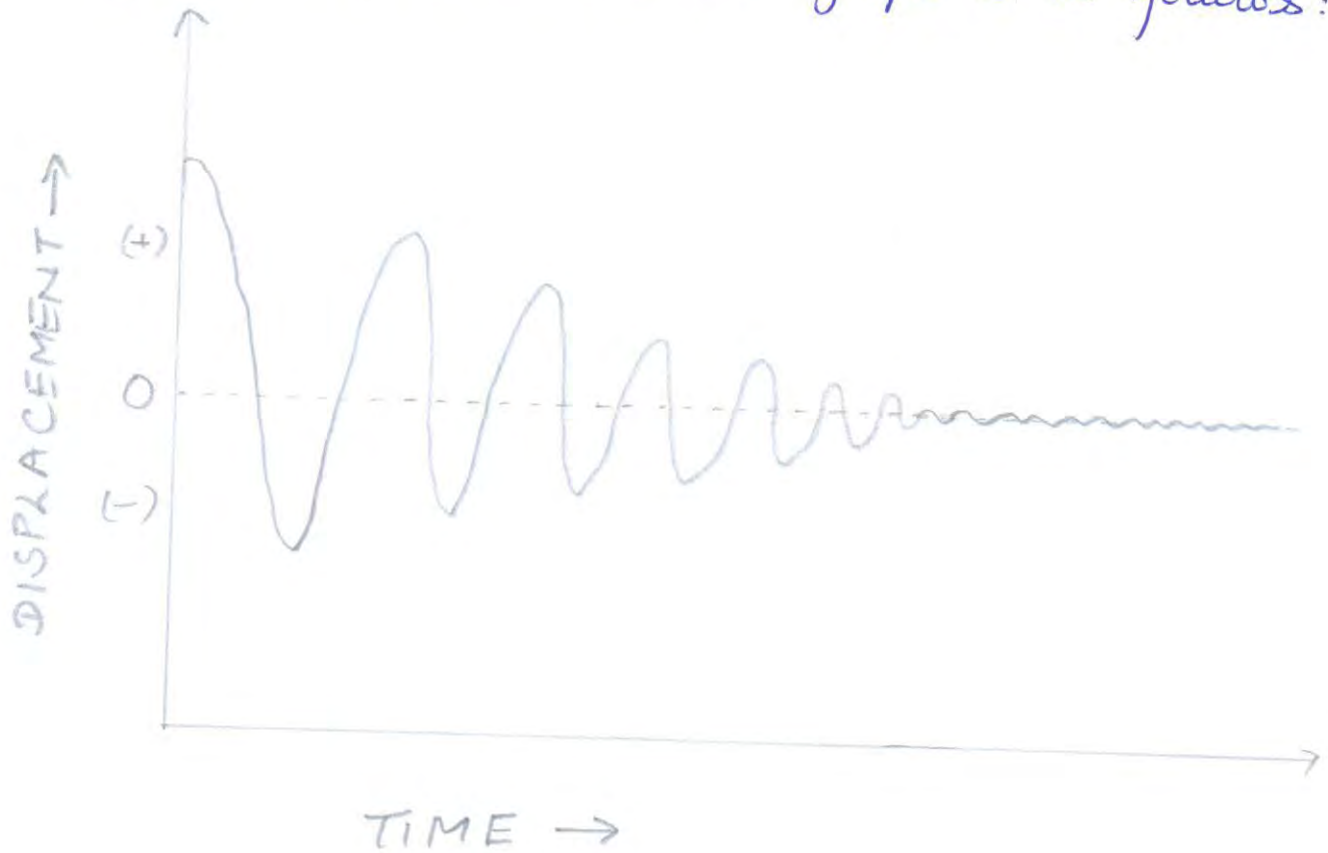
$$\xi = \frac{10}{\sqrt{4 \times 10 \times 500}}$$

$$\xi = 0.0707$$

within the mass range of 1 kg to 10 kg the system is underdamped and it oscillates.

10.1.11 Irrespective of k and initial displacement, the SDOF oscillator with lower 'c' will undergo more number of oscillations, before reaching the steady state.

For damped oscillations without continuous external forcing, the displacement v/s time graph is as follows:



So, as can be seen, displacement decays with progress of time. The steady state displacement for both SDOF systems, therefore, would be zero.

10.1.12

$\tau_d = 0.6\ s$

$m = 1.2\ kg$

Aplitude drops to 25% of the peak value in 3 periods of oscillation.

Logarithmic Decrement

$$x_i = X e^{-\xi \omega_n t_i}$$

next cycle occurs one period later at $t_i + \tau_d$

$$x_{i+1} = X e^{-\xi \omega_n (t_i + \tau_d)}$$

ratio of successive amplitudes

$$\frac{x_i}{x_{i+1}} = e^{\xi \omega_n \tau_d} = \text{constant}.$$

now,

$$\frac{x_0}{x_3} = \frac{x_0}{x_1} \cdot \frac{x_1}{x_2} \cdot \frac{x_2}{x_3} = e^{3\xi \omega_n \tau_d}$$

$$\ln\left(\frac{x_0}{x_3}\right) = 3\xi \omega_n \tau_d$$

$$\ln\left(\frac{x_0}{x_3}\right) = \frac{3 \times 2\pi \xi}{\sqrt{1-\xi^2}}$$

$$\tau_d = \frac{2\pi}{\omega_d}$$

$$= \frac{2\pi}{\omega_n \sqrt{1-\xi^2}}$$

$$\ln\left(\frac{x_0}{0.26\,x_0}\right) = \frac{3 \times 2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\left(\frac{1.38}{3 \times 2\pi}\right)^2 = \frac{\xi^2}{\left(\sqrt{1-\xi^2}\right)^2}$$

$$5.4 \times 10^{-3} = \frac{\xi^2}{1-\xi^2}$$

$$5.4 \times 10^{-3} = 1.0054\,\xi^2$$

$$5.37 \times 10^{-3} = \xi^2$$

$$\xi = 0.07$$

Now,

$$\tau_d = \frac{2\pi}{\omega_d}$$

$$\omega_d = \frac{2\pi}{\tau_d} = \frac{2\pi}{0.6} = 10.47 \text{ rad/s}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$10.47 = \omega_n \times 0.997$$

$$\omega_n = 10.49 = \sqrt{\frac{k}{m}}$$

$$110.04 = \frac{k}{m}$$

$$110.04 \times m = k$$

$K = 132.04 \, N/m$

$$\xi = \frac{c}{\sqrt{4Km}}$$

$$\xi \sqrt{4Km} = c$$

$$c = 0.07 \left(\sqrt{4 \times 1.2 \times 132.04} \right)$$

$$c = 1.76 \, kg/s$$

10.1.13 $m = 1\,kg$ $c = 10\,kg/sec.$ $k = ?$

For the system to return to equilibrium position in the quickest amount of time possible, it has to be critically damped.

$$\left(\frac{c_c}{2m}\right)^2 - \frac{k}{m} = 0$$

$$\Rightarrow \frac{c_c^2}{4m^2} = \frac{k}{m} \qquad \Rightarrow k = \frac{c_c^2}{4m} = \frac{10 \times 10}{4 \times 1} = 25\,N/m$$

10.2.1

$$\ddot{\theta} + K^2\theta = \beta \sin\omega t$$

$$\theta(0) = 0 \quad \text{and} \quad \dot{\theta}(0) = \dot{\theta}_0$$

the homogenous solution

$$\theta_h = A_1 \sin Kt + A_2 \cos Kt$$

The perticular solution

$$\theta_p = \phi \sin\omega t$$

$$\ddot{\theta}_p = -\omega^2 \phi \sin\omega t$$

then,

$$-\omega^2\phi + K^2\phi = \beta$$

$$\phi(K^2 - \omega^2) = \beta$$

$$\phi = \frac{\beta}{K^2 - \omega^2}$$

soln : $\theta = A_1 \sin Kt + A_2 \cos Kt + \dfrac{\beta}{K^2-\omega^2} \sin\omega t$

when $t = 0$, $\theta = \theta_0$

$$\theta_0 = A_2 + \frac{\beta}{K^2-\omega^2} \sin 0$$

$$A_2 = \theta_0 = 0$$

$$\dot{\theta}_0 = KA_1 \cos Kt + A_2 K(-\sin Kt) + \frac{\beta\omega}{K^2-\omega^2}\cos \omega t$$

$$\dot{\theta}_0 = KA_1 + \frac{\beta\omega}{K^2-\omega^2}$$

$$A_1 = \frac{1}{K}\left[\dot{\theta}_0 - \frac{\beta\omega}{K^2-\omega^2}\right]$$

$$\theta = \frac{1}{K}\left[\dot{\theta}_0 - \frac{\beta\omega}{K^2-\omega^2}\right]\sin Kt + \beta\frac{\sin\omega t}{K^2-\omega^2}$$

10.2.2 $k_1 = k$ $c_1 = c$ $F = F_0 \sin pt$

$k_2 = 2k$ $c_2 = 2c$

$k_3 = 4k$ $c_3 = c/2$

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin pt \quad (1)$$

$$x_p(t) = X\cos(pt - \phi) \quad (2)$$

Substituting (2) in (1),

$$X\left[(k - mp^2)\cos(pt-\phi) - cp\sin(pt-\phi)\right] = F_0 \sin pt \quad (3)$$

$\cos(pt-\phi) = \cos pt \cos\phi + \sin pt \sin\phi$

$\sin(pt-\phi) = \sin pt \cos\phi - \cos pt \sin\phi$ } Using these and comparing co-efficients in (3)

$$X\left[(k - mp^2)\cos\phi + cp\sin\phi\right] = 0 \quad (4)$$

$$X\left[(k - mp^2)\sin\phi - cp\cos\phi\right] = F_0 \quad (5)$$

$$X = \frac{F_0}{\left[(k-mp^2)^2 + c^2p^2\right]^{1/2}} \qquad \phi = \tan^{-1}\left(\frac{cp}{k-mp^2}\right)$$

In terms of frequency,

$$\frac{X}{\delta_{st}} = \frac{1}{\left[\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{2\zeta\frac{\omega}{\omega_n}\right\}^2\right]^{1/2}} \qquad \phi = \tan^{-1}\left\{\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right\}$$

10.2.3

$m = 3\ kg$

$K = 10\ N/m$

$F_0 = 5\ N$

$T = 1\ s$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1}\ rad/sec$$

The Eqn

$$m\ddot{x} + Kx = F_0 \cos\omega t \longrightarrow (1)$$

The homogeneous solution of this differential equation is given by.

$$x_h(t) = C_1 \cos\omega_n t + C_2 \sin\omega_n t$$

The Perticular solution is given by.

$$x_p(t) = X \cos\omega t$$

On substitution and solving (1) we obtain

$$X = \frac{F_0}{K - m\omega^2} \longrightarrow (2)$$

$$X = \frac{5}{10 - 3\times(2\pi)^2} =$$

$X = -0.1696$

The negative sign is because the ratio of $\left(\frac{\omega}{\omega_n}\right) > 1$.

$$X = \frac{F_0}{k - m\omega^2} = \frac{\delta_{st}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \longrightarrow (3)$$

In Equation (3) $\left(\frac{\omega}{\omega_n}\right) > 1$.

10.2.6

From logarithmic decrement

$$n = \frac{\sqrt{1-\xi^2}}{2\pi\xi} \ln\left[\frac{x_0}{x_n}\right]$$

$$3 = \frac{\sqrt{1-\xi^2}}{2\pi\xi} \ln[2]$$

$$740.52\xi^2 = 1$$

$$\xi^2 = 1.35\times10^{-3}$$

$$\xi = 0.036$$

No of cycles it takes for peak displacement to drop below 5% of the first peak.

$$n = \frac{\sqrt{1-\xi^2}}{2\pi\xi} \ln\left[\frac{x_0}{0.05x_0}\right]$$

$$n = 4.418 \times 2.995$$

$$n = 13.23$$

ie It should excute 14 cycles for its amplitude to fall below 5% of initial Amplitude

10.2.8

$m = 10\ kg$

$K = 50\ N/m$

$C = 5\ kg/s$

$$\xi = \frac{C}{\sqrt{4Km}} = \frac{5}{\sqrt{4 \times 10 \times 50}} = 0.11$$

$F_0 = 25\ N$

$P = \omega = 2.5\ rad/sec$

a) $$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{50}{10}} = \sqrt{5} = 2.23\ rad/sec$$

b) Steady state amplitude

$$X = \frac{(F_0/K)}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$$

$$X = 1.41\ m$$

Phase,

$$\phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$$

where $r = \frac{\omega}{\omega_n} = 1.12$

$$\phi = \tan^{-1}\left(\frac{2 \times 0.11 \times 1.12}{1 - (1.12)^2}\right)$$

$$\phi = -44.08^\circ$$

c) Displacement amplification is

$$\frac{X}{\left(\frac{F_0}{K}\right)} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\frac{X}{\delta_{st}} = 2.82$$

d)

$$W_F = \int_0^T F(t)\,\dot{x}(t)\,dt$$

$$x = X\cos(pt-\phi)$$

$$\dot{x} = -Xp\sin(pt-\phi)$$

$$W_F = \int_0^T F_0\cos(pt)\cdot -Xp\sin(pt-\phi)\,dt$$

On solving

$$W_F = F_0 \pi X \sin\phi$$

$$W_F = 25 \times \pi \times 1.41 \times 0.695$$

$W_F = 77.03$ Joules.

e) Energy lost in damper

$$W_d = \int_0^T c\dot{x}^2\,dt = \int_0^T X^2 c \cdot p^2 - \sin(pt-\phi) \cdot -\sin(pt-\phi) \cdot dt$$

$$= X^2 \int_0^T p^2 c \sin^2(pt-\phi)\,dt$$

$$= \pi c p X^2$$

$$W_d = \pi \times 5 \times 2.5 \times (1.41)^2$$

$$W_d = 78.072$$

10.2.9 $m = 2.05 \times 10^{-15} kg$ $k = 0.625 N/m$

$Q = 900$ $\Delta m = 1.36 \times 10^{-21} kg$

$Q = \frac{1}{2\zeta}$ $\Delta f = \frac{f_0}{Q}$ $Q = \frac{1}{2\zeta}$ $\Rightarrow \zeta = \frac{1}{2Q}$

$\Rightarrow \zeta = \frac{1}{1800}$

$\zeta = \frac{c_0}{\sqrt{4km}}$ $\omega_d = \omega\sqrt{1-\zeta^2}$

$= \sqrt{\frac{k}{m}}\sqrt{1-\frac{c_0^2}{4km}}$

$$\Rightarrow \Delta\omega_d = \sqrt{k}\,-\frac{1}{2}m^{-3/2}\Delta m\sqrt{1-\frac{c_0^2}{4km}} + \sqrt{\frac{k}{m}}\,\frac{1}{2}\left(1-\frac{c_0^2}{4km}\right) + \frac{c_0^2}{4km^2}\Delta m$$

$$\Rightarrow \Delta\omega_d = \sqrt{\frac{k}{m}}\left[\frac{-\Delta m}{2m}\sqrt{1-\frac{c_0^2}{4km}} + \frac{1}{2}\frac{(c_0^2/4km)}{(1-c_0^2/4km)^{3/2}}\Delta m\right]$$

$$\Delta\omega_d = \sqrt{\frac{k}{m}}\left[-\frac{\sqrt{1-\zeta^2}}{2m} + \frac{\zeta^2}{2m(1-\zeta^2)^{3/2}}\right]\Delta m$$

$$\Delta\omega_d = \sqrt{\frac{k}{m}}\left[-\sqrt{1-\zeta^2} + \frac{\zeta^2}{(1-\zeta^2)^{3/2}}\right]\frac{\Delta m}{2m}$$

$$= \sqrt{\frac{0.625}{2.05\times10^{-15}}}\left[-\sqrt{1-\frac{1}{(1800)^2}} + \frac{(1/1800)^2}{(1-(1/1800)^2)^{3/2}}\right]\frac{1.36\times10^{-21}}{2\times2.05\times10^{-15}}$$

$\Delta\omega_d = 5.792$ rad/sec.

~~$\Delta f = 2\pi\Delta\omega_d$~~ $\Delta\omega_d = 2\pi\Delta f$

$\therefore \Delta f = 0.922$ Hz

10.2.11

a) $x(t) = y(t) + z(t)$

$m\ddot{x}(t) = -kz - c\dot{z}$

b) $m\ddot{z} + kz + c\dot{z} = -m\ddot{y}$

c) $y(t) = y_0 \sin pt$

$\ddot{y}(t) = -p^2 y_0 \sin pt$

The magnitude of acceleration of the frame is $p^2 y_0$

d) From (b) we get

$m\ddot{z} + c\dot{z} + kz = -m\ddot{y}$

$\therefore \; m\ddot{z} + c\dot{z} + kz = mp^2 y_0 \sin pt$

Steady state amplitude is

$$= \frac{mp^2 y_0}{\sqrt{(k - mp^2)^2 + c^2 p^2}}$$

Phase ϕ is given by

$$\tan\phi = \frac{cp}{c - mp^2}$$

The steady state response is given by

$$\frac{mp^2 y_0}{\sqrt{(k-mp^2)^2 + c^2p^2}} \sin(pt - \phi)$$

e)

$$y = y_0 \sin pt$$

$$\ddot{y} = -p^2 y_0 \sin pt$$

$$\ddot{y} = p^2 y_0 \sin(pt + \pi)$$

$$\ddot{y} = A_y \sin(pt + \pi)$$

$$z = A_z \sin(pt - \phi)$$

$$A_z = \frac{m A_y}{\sqrt{(k-mp^2)^2 + c^2p^2}}$$

$$A_z = \frac{1}{\sqrt{(\omega_n^2 - p^2)^2 + \frac{c^2p^2}{m^2}}} A_y$$

$$A_z = \frac{1}{\sqrt{\omega_n^4\left(1-\frac{p^2}{\omega_n^2}\right)^2+\left(2\omega_n p\xi\right)^2}} \; A_y$$

$$A_z = \frac{1}{\omega_n^2} \times \frac{1}{\sqrt{\left(1-\frac{p^2}{\omega_n^2}\right)^2+\left(2\xi\frac{p}{\omega_n}\right)^2}} \; A_y$$

if $\frac{p}{\omega} \to 0$

$$A_z = \frac{A_y}{\omega_n^2}$$

$$A_z \propto A_y$$

f) $$z = \frac{m p^2 y_0}{\sqrt{(k-mp^2)^2+c^2p^2}}$$

$$z = \frac{r^2 y_0}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

where $r^2 = \left(\frac{p}{\omega_n}\right)^2$

$$z = \frac{r^2 y_0}{\sqrt{1+r^4+4\xi^2r^2-2r^2}}$$

$$z = \frac{y_0}{\sqrt{\frac{1}{r^4} + 1 + \frac{4\xi^2}{r^2} - \frac{2}{r^2}}}$$

$$r \to \infty$$

$$z = \frac{y_0}{\sqrt{0 + 1 + 0 - 0}}$$

$$z = y_0$$

11.1.1

$$\vec{r}(t) = A\sin\omega t\ \hat{i} + Bt\ \hat{j} + C\hat{k}$$

1) $\vec{V}(t) = \dot{\vec{r}}(t) = A\omega\cos\omega t\ \hat{i} + B\hat{j}$

2) $\vec{a}(t) = \ddot{\vec{r}}(t) = -A\omega^2\sin\omega t\ \hat{i}$

3) $\vec{r}(t)\times\vec{a}(t) = AB\omega^2 t\sin\omega t\ \hat{k} - AC\omega^2\sin\omega t\ \hat{j}$

$$= A\omega^2\sin\omega t\ [\,Bt\,\hat{k} - C\hat{j}\,]$$

11.1.2

$$\vec{r} = \sin\left(\frac{t}{s}\right)m\hat{i} + \cos\left(\frac{t}{s}\right)m\hat{j} + te^{t/s}\frac{m}{s}\hat{k}$$

a) $\vec{V} = \dot{\vec{r}}\Big|_{t=3s} = \cos\left(\frac{t}{s}\right)\frac{m}{s}\hat{i} - \sin\left(\frac{t}{s}\right)\frac{m}{s}\hat{j} + \left[e^{t/s}\frac{m}{s} + te^{t/s}\frac{m}{s^2}\right]\hat{k}$

$$= \left[-0.989\,\hat{i} - 0.141\,\hat{j} + 80.342\,\hat{k}\right]$$

b) $\vec{a} = \ddot{\vec{r}}\Big|_{t=3s} = -\sin\left(\frac{t}{s}\right)\frac{m}{s^2}\hat{i} - \cos\left(\frac{t}{s}\right)\frac{m}{s^2}\hat{j} + \left[2e^{t/s}\frac{m}{s^2} + te^{t/s}\frac{m}{s^3}\right]\hat{k}$

$$= \left[-0.141\,\hat{i} + 0.989\,\hat{j} + 100.42\,\hat{k}\right]$$

11.1.3

$$\vec{r} = 3t^2\,\hat{i}\,\frac{m}{s^2} + 4t^3\,\hat{j}\,\frac{m}{s^3}$$

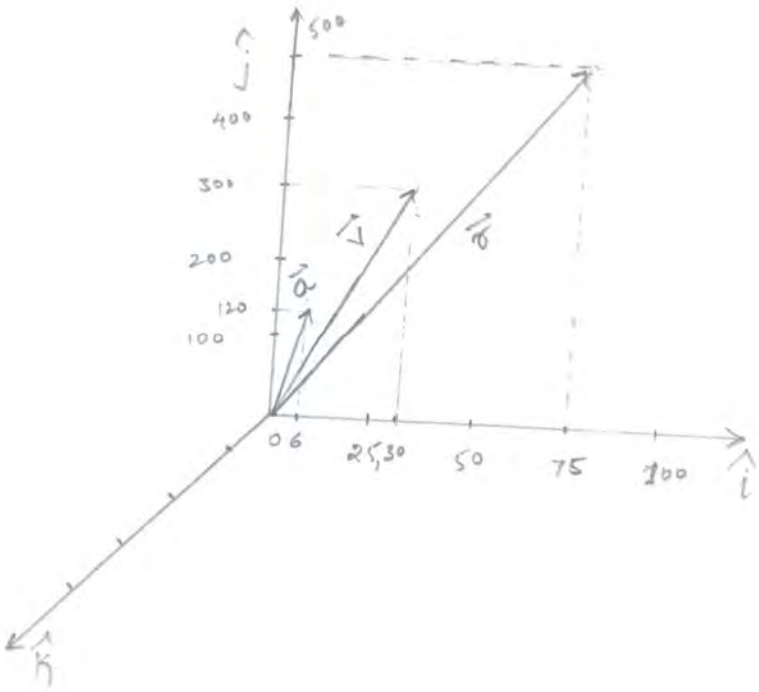
a) $\vec{V} = \dot{\vec{r}}\Big|_{t=5} = 6t\,\hat{i}\,\frac{m}{s} + 12t^2\,\hat{j}\,\frac{m}{s}$

$$= \left[30\,\hat{i} + 300\,\hat{j}\right]\frac{m}{s}$$

b) $\vec{a} = \ddot{\vec{r}}\Big|_{t=5} = \left[6\,\hat{i} + 24t\,\hat{j}\right]\frac{m}{s^2}$

$$= \left[6\,\hat{i} + 120\,\hat{j}\right]\frac{m}{s^2}$$

c)



11.1.4

a) $|\Delta\vec{r}| = |2(0.5\hat{i} - 1.5\hat{j})| = |\hat{i} - 3\hat{j}| = \sqrt{10}\,m$

b) Direction vector $= \dfrac{\hat{i}}{\sqrt{10}} - \dfrac{3\hat{j}}{\sqrt{10}}$

11.1.5

$$\dot{\vec{r}} = (u_0 \sin\Omega t)\hat{i} + v_0\hat{j}$$

$$\vec{r}(0) = x_0\hat{i} + y_0\hat{j}$$

integrate $\dot{\vec{r}}$

$$\int\dot{\vec{r}} = \vec{r} = \int(u_0 \sin\Omega t)\hat{i} + v_0\hat{j} + C_1$$

$$\vec{r} = -\frac{u_0}{\Omega}\cos\Omega t\,\hat{i} + v_0 t\,\hat{j} + C_1$$

$$\vec{r}(0) = x_0\hat{i} + y_0\hat{j} = -\frac{u_0}{\Omega}\hat{i} + C_1$$

$$C_1 = \left(x_0 + \frac{u_0}{\Omega}\right)\hat{i} + y_0\hat{j}$$

$$\vec{r} = \left[x_0 + \frac{u_0}{\Omega}(1 - \cos\Omega t)\right]\hat{i} + (y_0 + v_0 t)\hat{j}$$

11.1.6

$$\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$$

$$\vec{a} = 2\hat{i} - (3-t)\hat{j} - 5t^2\hat{k}$$

$$\vec{V}(t) = \int \vec{a}\, dt + C$$

$$\vec{V}(t) = \left[2t\,\hat{i} - \left(3t - \frac{t^2}{2}\right)\hat{j} - \frac{5t^3}{3}\hat{k}\right]$$

The scalar equations are.

$$V_x = 2t$$

$$V_y = -\left(3t - \frac{t^2}{2}\right)$$

$$V_z = -\frac{5t^3}{3}$$

11.1.7

$$\vec{r}(t) = \int \dot{\vec{r}}(t)\, dt + C_1$$

$$= \int (V_1 \sin ct\ \hat{i} + V_2\hat{j})\, dt + C_1$$

$$\vec{r}(t) = -\frac{V_1}{c}\cos ct\ \hat{i} + V_2 t\ \hat{j} + C$$

$$\vec{r}(t)\Big|_{t=0} = -\frac{V_1}{c}\hat{i} + C_1 = 2\hat{i} + 3\hat{j}$$

$$C_1 = \left(2 + \frac{V_1}{c}\right)\hat{i} + 3\hat{j}$$

$$\vec{r}(t)\Big|_{t=5} = -\frac{4}{4}\cos(20)\hat{i} + 5\times 5\,\hat{j} + \left(2+\frac{4}{4}\right)\hat{i} + 3\hat{j}$$

$$= -0.4080\hat{i} + 20\hat{j} + 3\hat{i} + 3\hat{j}$$

$$\vec{r}(t)\Big|_{5} = 2.5919\,\hat{i} + 23\,\hat{j}$$

11.1.8

$$\vec{r}(t) = \int \dot{\vec{r}}(t)\,dt + C$$

$$= \int \left[V_0\cos\alpha\,\hat{i} + V_0\sin\alpha\,\hat{j} + (V_0\tan\theta - gt)\hat{k}\right]dt + C_1$$

$$= t\left(V_0\cos\alpha\,\hat{i} + V_0\sin\alpha\,\hat{j} + V_0\tan\theta\,\hat{k}\right) - \frac{gt^2}{2}\hat{k} + C_1$$

$$\vec{r}(t)\Big|_{t=0} = 0 \quad \text{gives } C=0$$

$$\vec{r}(t) = t\left(V_0\cos\alpha\,\hat{i} + V_0\sin\alpha\,\hat{j} + V_0\tan\theta\,\hat{k}\right) - \frac{gt^2}{2}\hat{k}$$

11.1.9

$$\vec{r}(t) = \int\left(-R\sin t\,\hat{i} + R\cos t\,\hat{j} + gt\,\hat{k}\right)dt + C_1$$

$$= R\cos t\,\hat{i} + R\sin t\,\hat{j} + \frac{gt^2}{2}\hat{k} + C_1$$

$$\vec{r}(0) = R\hat{i} + C_1 = R\hat{i} \Rightarrow C_1 = 0$$

$$\vec{r}\left(\frac{\pi}{3}\right) = R\cos\frac{\pi}{3}\,\hat{i} + R\sin\frac{\pi}{3}\,\hat{j} + \frac{g\left(\frac{\pi}{3}\right)^2}{2}\hat{k}$$

$$= \frac{R}{2}\hat{i} + \frac{\sqrt{3}R}{2}\hat{j} + \frac{g\pi^2}{18}\hat{k}$$

11.1.10

$$y(x) = \sin^2(x)$$

$$x(t) = t^3$$

$$\vec{V}(t) = \frac{d}{dt}\left[t^3\,\hat{i} + \sin^2(t^3)\,\hat{j}\right]$$

$$\vec{V}(t) = \left[3t^2\,\hat{i} + 2\sin(t^3)\cos(t^3)\cdot 3\cdot t^2\;\hat{j}\right]$$

$$\vec{a}(t) = \left[6t\,\hat{i} + \left((3t^2)(6t^2\cos 2t^3) + 6t\sin 2t^3\right)\hat{j}\right]$$

$$\cancel{\vec{V}(t)\Big|_{t=(\pi)^{1/3}} = 3\pi^{2/3}\,\hat{i} \quad 6\pi^{1/3}\left(\hat{i} + 3\pi\,\hat{j}\right)}$$

$$\vec{V}(t)\Big|_{t=(\pi)^{1/3}} = 3\pi^{2/3}\,\hat{i}$$

$$\vec{a}(\pi^{1/3}) = 6\pi^{1/3}\left[\hat{i} + 3\pi\,\hat{j}\right]$$

11.1.11

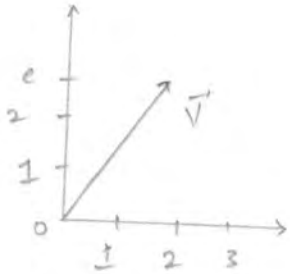
$$\vec{r}(t) = (t^2\,\hat{i} + e^{t/5}\,\hat{j})$$

$$\vec{v}(t) = (2t\,\hat{i} + e^{t}\,\hat{j})$$

$$\vec{a}(t) = 2\hat{i} + e^{t}\hat{j}$$

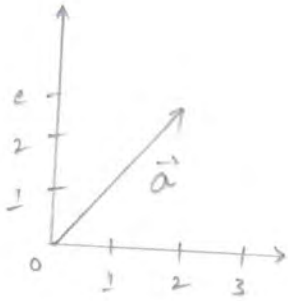
$$\vec{v}(1) = 2\hat{i} + e\hat{j}$$

$$\vec{a}(1) = 2\hat{i} + e\hat{j}$$



Velocity vector

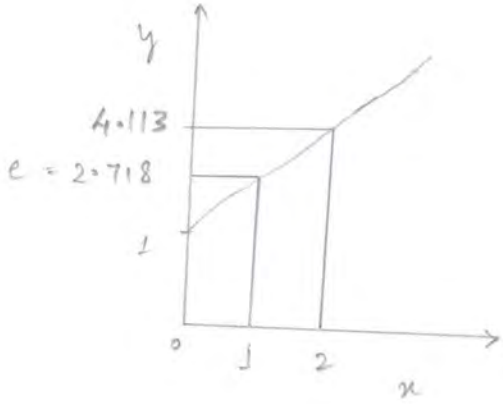
at $t = 1$



Acceleration vector

at $t = 1$

Trajectory



11.1.12

$$y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$$

$$x = \frac{a}{2} \Rightarrow y^2 = b^2\left(1 - \frac{1}{4}\right) = \frac{3b^2}{4}$$

$$y = \pm \frac{\sqrt{3}}{2} b \Rightarrow y = +\frac{\sqrt{3}}{2} b$$

$$\text{Speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = V \longrightarrow (1)$$

$$2y\frac{dy}{dt} = -b^2 \frac{2x}{a^2}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\left(\frac{b^2}{a^2}\frac{x}{y}\right)\frac{dx}{dt} \longrightarrow (2)$$

Putting in (1) we get

$$\frac{dx}{dt}\sqrt{1 + \left(\frac{b^2 x}{a^2 y}\right)^2} = V$$

$$V_x = \frac{dx}{dt} = \frac{V}{\sqrt{1 + \left(\frac{b^2}{a^2}\frac{x}{y}\right)^2}} = \frac{V}{\sqrt{1 + \frac{b^2}{3a^2}}}$$

similarly $V_y = \frac{dy}{dt} = \frac{V}{\sqrt{1 + \frac{3a^2}{b^2}}}$

$$\vec{V} = \frac{V}{\sqrt{1 + b^2/3a^2}}\hat{i} + \frac{V}{\sqrt{1 + 3a^2/b^2}}\hat{j}$$

11.1.14

$V(t) = \sqrt{1+4t^2}$ $\qquad x > 0$

$x(t) = t$ $\qquad y > 0$

$V_x = \dot{x}(t) = 1$

$V(t) = \sqrt{V_x^2 + V_y^2} = \sqrt{1 + V_y^2} = \sqrt{1+4t^2}$

$V_y = \pm 2t \quad \Rightarrow V_y = 2t$

$\vec{r}(t) = \int \vec{V}(t)\,dt = \int (\hat{i} + 2t\hat{j})\,dt + C_1$

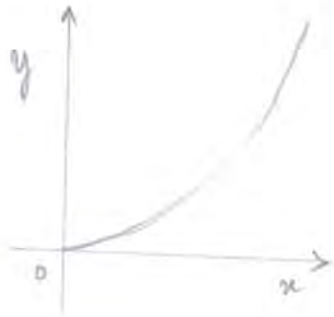
$\vec{r}(t) = t\hat{i} + \frac{2t^2}{2}\hat{j} + C_1$

$C_1 = 0$

$\vec{r}(t) = t\hat{i} + t^2\hat{j}$

$\vec{V}(t) = \hat{i} + 2t\hat{j}$

$\vec{a}(t) = 2\hat{j}$



Rate of change of speed

$$\dot{V} = \frac{1}{2}\frac{8t}{\sqrt{1+4t^2}} = \frac{4t}{\sqrt{1+4t^2}}$$

Path of the particle is parabolic upwards constant acceleration.

At $x = 3$, $\quad t = 3$;

$y = t^2 = 9$

distance from origin $= \sqrt{9^2 + 3^2} = 3\sqrt{10}$

velocity $= (\hat{i} + 6\hat{j})$ m/s

acceleration $= 2\hat{j}$ m/s^2

11.1.15

$$\vec{a} = -2\hat{i} + 5\hat{j}$$

$$m = 2$$

a) $\sum \vec{F} = m\vec{a}$

$$\vec{F}_1 + \vec{F}_2 = m(-2\hat{i} + 5\hat{j})$$

$$= -2m\hat{i} + 5m\hat{j}$$

$$\vec{F}_2 = |\vec{F}_2| \left(-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right), \cancel{|\vec{F}|} =$$

$$\vec{F}_1 = |\vec{F}_1| \left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$$

$$\left(\frac{\sqrt{3}}{2}|\vec{F}_1| - \frac{|\vec{F}_2|}{\sqrt{2}}\right)\hat{i} + \left(\frac{|\vec{F}_1|}{2} + \frac{|\vec{F}_2|}{\sqrt{2}}\right)\hat{j} = -2m\hat{i} + 5m\hat{j}$$

b) Scalar form

$$\frac{\sqrt{3}}{2}|\vec{F}_1| - \frac{|\vec{F}_2|}{\sqrt{2}} = -2m \quad \text{— (i)}$$

$$\frac{|\vec{F}_1|}{2} + \frac{|\vec{F}_2|}{\sqrt{2}} = 5m \quad \text{— (ii)}$$

c) Matrix form

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} |\vec{F}_1| \\ |\vec{F}_2| \end{bmatrix} = \begin{bmatrix} -2m \\ 5m \end{bmatrix}$$

d) Solution : Hand Algebra

$$\frac{\sqrt{3}+1}{2}|\vec{F_1}| = 3m \Rightarrow |\vec{F_1}| = \frac{6m}{\sqrt{3}+1}$$

$$|\vec{F_2}| = \frac{\sqrt{2}\,(5\sqrt{3}+2)m}{\sqrt{3}+1}$$

Matrix Method

$$|\vec{F_1}| = \frac{\begin{vmatrix} -2m & -\frac{1}{\sqrt{2}} \\ 5m & \frac{1}{\sqrt{2}} \end{vmatrix}}{\begin{vmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{\sqrt{2}} \end{vmatrix}}$$

$$|\vec{F_2}| = \frac{\begin{vmatrix} \sqrt{3}/2 & -2m \\ 1/2 & 5m \end{vmatrix}}{\begin{vmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{\sqrt{2}} \end{vmatrix}}$$

Solution from vector method.

unit vector along $\vec{F_1} \Rightarrow \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j} = \hat{e}_1$

Along $\vec{F_2} = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} = \hat{e}_2$

$$\vec{F_1} + \vec{F_2} = m(-2\hat{i} + 5\hat{j})$$

$$\hat{e}_2 \times (\vec{F}_1 + \vec{F}_2) = \hat{e}_2 \times [m(-2\hat{i} + 5\hat{j})]$$

$$|\vec{F}_1| \left(-\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} \right) \hat{k} = m \left(\frac{-5}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \hat{k}$$

$$|\vec{F}_1| = \frac{6m}{1+\sqrt{3}}$$

lly

$$\hat{e}_1 \times (\vec{F}_1 + \vec{F}_2) = \hat{e}_1 \times m(-2\hat{i} + 5\hat{j})$$

$$|\vec{F}_2| \left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \right) = \left(\frac{5\sqrt{3}}{2} + 1 \right) m$$

$$|\vec{F}_2| = \frac{\sqrt{2}\,(5\sqrt{3}+2)\,m}{1+\sqrt{3}}$$

11. 1.16

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = m\vec{a}$$

$$\Rightarrow [20\hat{i} - 5\hat{j}] + [F_{2x}\hat{i} + F_{2z}\hat{k}] + [\frac{F_3}{2}\hat{i} + F_3\frac{\sqrt{3}}{2}\hat{j}] = 2[-0.2\hat{i} + 2.2\hat{j} + 1.7\hat{k}]$$

$$[20 + F_{2x} + \frac{F_3}{2}]\hat{i} + [-5 + \frac{F_3\sqrt{2}}{2}]\hat{j} + F_{2z}\hat{k} = -0.4\hat{i} + 4.4\hat{j} + 3.4\hat{k}$$

$$20 + F_{2x} + \frac{F_3}{2} = -0.4 \rightarrow (i)$$

$$-5 + \frac{F_3\sqrt{3}}{2} = 4.4 \rightarrow (ii)$$

$$F_{2z} = 3.4 \rightarrow (iii)$$

$F_{2z} = 3.4$

$F_3 = 10.85$

$F_{2x} = -25.827$

11.1.17

Say the tension be T_1, T_2, T_3

then

$$T_1 \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} + T_2 \frac{\vec{r}_{AC}}{|\vec{r}_{AC}|} + T_3 \frac{\vec{r}_{AD}}{|\vec{r}_{AD}|} = m\vec{a}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 3.4675 \\ 0.3366 \\ 0.1083 \end{bmatrix}$$

11.1.18

$$\sum \vec{F} = \dot{\vec{L}}$$

$$\begin{bmatrix} 25 \\ 32 \\ 75 \end{bmatrix} + \begin{bmatrix} F_{2x} \\ F_{2y} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -F_3 \end{bmatrix} = \begin{bmatrix} 20 \\ -18 \\ i_z \end{bmatrix}$$

This gives.

$$F_{2x} = -5N$$

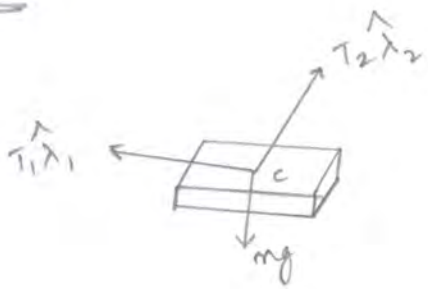
$$F_{2y} = -50N$$

$$[F_3 = 75 - i_z]$$

Insufficient data to determine F_3

11.1.19

FBD



$$\hat{\lambda}_1 = \frac{\vec{r}_{CA}}{|\vec{r}_{CA}|}$$

$$\hat{\lambda}_1 = \frac{(\hat{i}+\hat{j}) - (1.2\hat{i} + 1.2\hat{k})}{|\vec{r}_{CA}|}$$

$$= -\frac{1}{\sqrt{1.08}}\left[-0.2\hat{i} + \hat{j} - 0.2\hat{k}\right]$$

lly.

$$\hat{\lambda}_2 = -\frac{[\hat{i} - 0.2\hat{j} - 0.2\hat{k}]}{\sqrt{1.08}}$$

using $\sum \vec{F} = m\vec{a}$

we get $m\vec{a} = T_1\hat{\lambda}_1 + T_2\hat{\lambda}_2 - mg\hat{k}$

$$m\vec{a} = \frac{T_1}{\sqrt{1.08}}\begin{bmatrix}0.2\\-1\\0.2\end{bmatrix} + \frac{T_2}{\sqrt{1.08}}\begin{bmatrix}-1\\0.2\\0.2\end{bmatrix} - mg\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\vec{a} = 5.2\hat{i} - 25.98\hat{j} - 4.8038\hat{k}$$

11.1.20

$$\vec{F} = m\vec{a}$$

$$3g\,\frac{[3\hat{i}+4\hat{j}+12\hat{k}]}{13} = m\vec{a}$$

$$\cancel{\vec{a} = \frac{3}{32}\times(32)}$$

$$\vec{a} = \frac{3}{2.5}\times 32\left[\frac{3\hat{i}+4\hat{j}+12\hat{k}}{13}\right]$$

$$\vec{a} = 5.9\hat{i} + 7.87\hat{j} + 23.63\hat{k}$$

11.1.21

$$\sum \vec{F} = m\vec{a}$$

a) $-T_1\hat{k} + T_2\hat{j} + T_3\frac{1}{13}(3\hat{i}+12\hat{j}+4\hat{k}) = \left[-0.5\hat{i}+2.5\hat{j}+\frac{1}{3}\hat{k}\right]6$

$$\Rightarrow \quad \frac{3T_3\hat{i}}{13} = -3\hat{i}$$

$$T_3 = -13N$$

11.1.23

$$\vec{r} = 5\cos^2(t^2)\,\hat{i} + 5\sin(t^2)\cos(t^2)\,\hat{j}$$

1. Plot trajectory

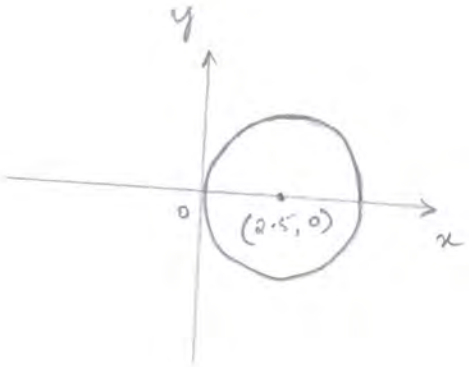
$x = 5\cos^2(t^2)$, $y = 5\sin(t^2)\cdot\cos(t^2)$

$x = 2.5(\cos 2t^2 + 1)$, $y = 2.5\sin(2t^2)$

$\frac{x-2.5}{2.5} = \cos 2t^2$, $\frac{y}{2.5} = \sin 2t^2$

$$(x-2.5)^2 + y^2 = (2.5)^2$$

Trajectory is a circle with radius 2.5 and the centre $x = 2.5$, $y = 0$



11.1.27

a) Equation of motion

$$m\vec{a} = -mg\hat{j}$$

$$\ddot{y} = -g\hat{j}$$

$$\ddot{x} = 0$$

$$\vec{V}_y = \dot{y} = (V_0 \sin\theta_0 - gt)\hat{j}$$

$$\vec{V}_x = V_0 \cos\theta_0 \hat{i}$$

$$\vec{V} = V_0 \cos\theta_0 \hat{i} + (V_0 \sin\theta_0 - gt)\hat{j}$$

b) acceleration

$$\vec{a} = -g\hat{j}$$

Velocity

$$\cancel{\vec{V} = V_0 t \cos}$$

$$\vec{V} = V_0 \cos\theta_0 \hat{i} + (V_0 \sin\theta_0 - gt)\hat{j}$$

Position

$$\vec{r} = V_0 t \cos\theta_0 \hat{i} + (V_0 t \sin\theta_0 - \tfrac{1}{2} g t^2)\hat{j}$$

c) maximum height

$$\frac{d}{dt}(v_y) = 0 \Rightarrow v_0 \sin\theta_0 - gt = 0 \Rightarrow t = \frac{v_0 \sin\theta_0}{g}$$

$$h_{max} = \left(\frac{v_0 \sin\theta_0}{g}\right) v_0 \sin\theta_0 - \frac{1}{2} g \left(\frac{v_0 \sin\theta_0}{g}\right)^2$$

$$h_{max} = \frac{v_0^2 \sin^2\theta_0}{2g}$$

d) $y = 0 \Rightarrow v_0 t \sin\theta_0 - \frac{1}{2} g t^2 = 0$

$$t = \frac{2v_0 \sin\theta_0}{g} = t_0$$

$$r_x \Big|_{t=t_0} = v_0 \cos\theta_0 \left(\frac{2v_0 \sin\theta_0}{g}\right) = \frac{v_0^2 \cos 2\theta_0}{g}$$

e) $y = v_0 t \sin\theta_0 - \frac{1}{2} g t^2$

$x = v_0 t \cos\theta_0$

eliminating 't' we get

$$y = v_0 \sin\theta_0 \left(\frac{x}{v_0 \cos\theta_0}\right) - \frac{1}{2} g \left(\frac{x}{v_0 \cos\theta_0}\right)^2$$

$$y = \frac{x}{V_0 \cos\theta_0}\left[V_0 \sin\theta_0 - \frac{gx}{2V_0\cos\theta_0}\right]$$

$$-y = \frac{gx^2}{2V_0^2\cos^2\theta_0} - x\,\mathrm{Tan}\,\theta_0$$

$$-\frac{2V_0^2\cos^2\theta_0}{g}\,y = x^2 - 2x\left(\frac{V_0^2\cos^2\theta_0\,\mathrm{Tan}\,\theta_0}{g}\right)$$

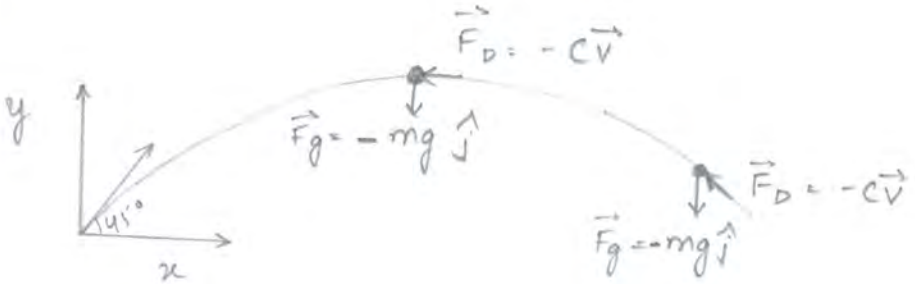
$$\left(x - \frac{V_0^2\sin 2\theta}{2g}\right)^2 = -\frac{2V_0^2\cos^2\theta}{g}\,y + \left(\frac{V_0^2\sin^2\theta}{2g}\right)^2$$

$$\left(x - \frac{V_0^2\sin 2\theta}{2g}\right)^2 = -\frac{2V_0^2\cos^2\theta}{g}\left[y - \frac{V_0^2\sin^2\theta}{2g}\right]$$

This equation represents a parabola

11.1.28

a)



b) Linear momentum balance equation

$$ma_x = -cV_x$$

$$ma_y = -mg - cV_y$$

Where $\vec{V} = v_x\hat{i} + v_y\hat{j}$

$$m\ddot{x} = -c\dot{x}$$

$$m\ddot{y} = -mg - c\dot{y}$$

c) We can put in the state space form as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{c}{m} & 0 \\ 0 & 0 & 0 & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -g \end{bmatrix}$$

d) Matlab

e) $m\ddot{x} = -c\dot{x} \Rightarrow m\frac{dv_x}{dt} = -cv_x$

$$\int \frac{dv_x}{v_x} = -\frac{c}{m}\int dt$$

$$\Rightarrow \log v_x = -\frac{ct}{m} + c_1$$

$$v_x = Ae^{-ct/m}$$

at $t=0$, $v_x|_{t=0} = u\cos\theta \Rightarrow v_x = u\cos\theta\, e^{-\frac{ct}{m}} \rightarrow (1)$

Similarly

$$\int \frac{dv_y}{v_y + \frac{mg}{c}} = \frac{-c}{m}\int dt$$

$$\log\left(v_y + \frac{mg}{c}\right) = -\frac{ct}{m} + c_1$$

$$v_y = Ae^{-\frac{ct}{m}} - \frac{mg}{c}$$

$$A - \frac{mg}{c} = u\sin\theta$$

$$A = u\sin\theta + \frac{mg}{c}$$

$$v_y = u\sin\theta\, e^{-ct/m} + \frac{mg}{c}\left(e^{-ct/m} - 1\right)$$

Further Integration

$$\frac{dx}{dt} = u\cos\theta\, e^{-ct/m}$$

$$x = u\cos\theta\left(-\frac{m}{c}\right)e^{-ct/m} + B$$

$$x\Big|_{t=0} = 0 \Rightarrow \quad B = u\cos\theta\left(\frac{m}{c}\right)$$

$$x = u\cos\theta\left(\frac{m}{c}\right)\left[1 - e^{-ct/m}\right]$$

Similarly

$$\dot{y} = (u\sin\theta + mg/c)\,e^{-ct/m} - \frac{mg}{c}$$

$$y = (u\sin\theta + mg/c)\left(-\frac{m}{c}\right)e^{-ct/m} - \frac{mgt}{c} + B$$

$$y\Big|_{t=0} = 0 \Rightarrow \quad B = \left(u\sin\theta + \frac{mg}{c}\right)\left(\frac{m}{c}\right)$$

$$y = \frac{m}{c}\left(u\sin\theta + \frac{mg}{c}\right)\left(1 - e^{-ct/m}\right) - \frac{mgt}{c}$$

11.1.29

Equation of motion

$$m\ddot{x} = \frac{-b(\dot{x}^2+\dot{y}^2)\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} = -b\dot{x}\sqrt{\dot{x}^2+\dot{y}^2}$$

$$m\ddot{y} = -mg - \frac{b(\dot{x}^2+\dot{y}^2)\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}} = -mg - b\dot{y}\sqrt{\dot{x}^2+\dot{y}^2}$$

$$\ddot{x} = -\frac{b\dot{x}}{m}\sqrt{\dot{x}^2+\dot{y}^2}$$

$$\ddot{y} = -g - \frac{b\dot{y}}{m}\sqrt{\dot{x}^2+\dot{y}^2}$$

11.1.30

a) System of first order differential Equation

$$\dot{x} = u$$
$$\dot{y} = v$$
$$\dot{u} = -\frac{b}{m} u \sqrt{u^2+v^2}$$
$$\dot{v} = -g - \frac{b}{m} v \sqrt{u^2+v^2}$$

b) using $g = 10\ m/sec^2$, $m = 10\ kg$

$b = 5\ N/(m/sec)^2$, $v_0 = 100\ m/sec$

c) Matlab

d) From Matlab we get initial angle that maximizes range is

Theta = 15.2432°

Without Air resistance (b=0)

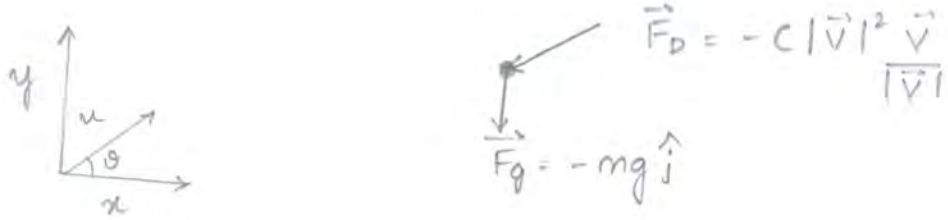
Theta = 45°

11.1.32

$m = 1\ kg$, $\theta = 30^\circ$, $u = 172\ m/sec$

$F_D \propto c v^2$, $c = 0.01\ kg/m$, $g = 10\ m/sec^2$

FBD



Linear momentum balance

$$m\vec{a} = \vec{F}_D + \vec{F}_G$$

$$= -mg\hat{j} - cv_x\sqrt{v_x^2+v_y^2}\,\hat{i} - cv_y\sqrt{v_x^2+v_y^2}\,\hat{j}$$

$$m\ddot{x} = -c\dot{x}\sqrt{\dot{x}^2+\dot{y}^2}$$

$$m\ddot{y} = -mg - c\dot{y}\sqrt{\dot{x}^2+\dot{y}^2}$$

$u = \dot{x}$, $v = \dot{y}$ $\longrightarrow$ (i), (ii)

$$\dot{u} = -\frac{c}{m}u\sqrt{u^2+v^2} \longrightarrow \text{(iii)}$$

$$\dot{v} = -g - \frac{c}{m}v\sqrt{u^2+v^2} \longrightarrow \text{(iv)}$$

Matlab :- height at $t = 1$ sec

from matlab solution height at $t = 1$.

is $h = 46.5047\ m$

b) Assume 'g' is negligible

If 'g' is assumed to be negligible drag force is always in opposite direction of velocity. Hence no force component $\perp$ to the current trajectory acts on it.

ie it is a straight line motion, with $\theta = 30°$ along the line

$$ma = -cv^2$$

$$m\frac{dv}{dt} = -cv^2$$

$$\int \frac{dv}{v^2} = -\int \frac{c}{m}\,dt$$

$$-\frac{1}{v} = -\frac{ct}{m} + A$$

$$v\Big|_{t=0} = u_0 \Rightarrow A = -\frac{1}{u_0}$$

$$\frac{1}{v} = \frac{ct}{m} + \frac{1}{u_0}$$

$$v = \frac{mu_0}{cu_0t+1} = \frac{ds}{dt}$$

$$S = \int \frac{mu_0}{cu_0t+1}\,dt = \frac{mu_0}{cu_0}\log(cu_0t+1) = \frac{m}{c}\log(cu_0t+1)$$

for $t = 1$ $\quad S = 100\log(2-72)$

$$h = 100\sin 30° = 50\,m$$

11.2.2

No,

$\vec{L} = \frac{d}{dt}(m\vec{v})$, is the same with respect to any point in the same Newtonian refrence frame in which $\vec{v}$ is measured.

11.2.3

No.

11.2.4

$$\vec{F}_{net} = \sum \vec{F} = \frac{d}{dt}(\vec{L}) = \dot{\vec{L}}$$

11.2.5

$$\vec{M}_{net/c} = \frac{d}{dt}(\vec{H}_{/c}) = \dot{\vec{H}}_{/c}$$

11.2.6

$m = 0.1$ Kg ;

$h = 10$ m

$\vec{V} = 120 \frac{km}{hr} \hat{i} - 120 \, Km/hr \, \hat{j}$

$\vec{V} = \frac{100}{3} [\hat{i} - \hat{j}]$ m/sec

$KE = \frac{1}{2} m \vec{V} \cdot \vec{V} = \frac{0.1}{2} \left[\left(\frac{100}{3}\right)^2 + \left(\frac{100}{3}\right)^2 \right]$ Joules

$K.E = 111.11$ Joules

11.2.7

$m = 0.2$ kg , $h = 20$ m

$\vec{V} = (120\hat{i} - 120\hat{j} - 10\hat{k})$ Km/hr

$KE = \frac{1}{2} m \vec{V} \cdot \vec{V} = 0.1 [V_x^2 + V_y^2 + V_z^2]$

$= 0.1 \left[\left(\frac{100}{3}\right)^2 + \left(\frac{100}{3}\right)^2 + (2.77)^2 \right]$

$K.E = 222.989$ Joules.

11.2.8

$$\text{Power} = \sum_{i=1}^{n} \frac{d}{dt} (\vec{F}_i \, d\vec{x}) = \sum_{i=1}^{n} \vec{F}_i \cdot \vec{V}$$

11.2.9

$$m_B = m_A \ , \ \vec{V}_B = 2\vec{V}_A$$

a) $\vec{L}_B = m_B\vec{V}_B = 2m_A\vec{V}_A = 2\vec{L}_A$

$$\vec{L}_B = 2\vec{L}_A$$

b) $\vec{H}_{B/C} = \vec{r}_{B/C} \times \vec{L}_B = 2\vec{r}_{B/C} \times \vec{L}_A$

$$\vec{H}_{A/C} = \vec{r}_{A/C} \times \vec{L}_A$$

If $\vec{r}_{B/C} = \vec{r}_{A/C}$

then,

$$\vec{H}_{B/C} = 2\,\vec{H}_{A/C}$$

c) $E_{KA} = \frac{1}{2} m_A \vec{V}_A \cdot \vec{V}_A \ , \ E_{KB} = \frac{1}{2} m_B \vec{V}_B \cdot \vec{V}_B$

$$E_{KB} = \frac{1}{2} m_B \vec{V}_B \cdot \vec{V}_B = \frac{1}{2} m_A (2\vec{V}_A)\cdot(2\vec{V}_A) = 4E_{KA}$$

$$E_{KB} = 4E_{KA}$$

11.2.10

$m = 50$ grams

$\vec{V} = [0.8\hat{i} + 0.6\hat{j}]$ km/sec

Linear momentum of the Bullet

$$\vec{L} = m\vec{V} = \frac{50}{1000}[0.8\hat{i} + 0.6\hat{j}] \text{ km/sec}$$

$$\vec{L} = [40\hat{i} + 30\hat{j}] \text{ kg m/sec}$$

11.2.11

$\vec{r} = (4\hat{i} + 7\hat{j})\ m$

$\vec{V} = (6\hat{i} - 3\hat{j})\ m/sec$

$\vec{a} = (-2\hat{i} + 9\hat{j})\ m/sec^2$

say mass = m

$\vec{H}_{/P} = \vec{r}_{/P} \times \vec{L} = m\,\vec{r}_{/P} \times \vec{V}$

a) $\vec{r}_P = (4\hat{i} + 7\hat{j})m$

$\vec{H}_{/P} = m(4\hat{i} + 7\hat{j}) \times (6\hat{i} - 3\hat{j})\ kg\,m^2/sec$

$= m(-12 - 42)\hat{k}\ kg\,m^2/sec$

$\vec{H}_{/P} = -54m\,\hat{k}\ kg\,m^2/sec$

b) $\vec{r}_P = (-2\hat{i} + 7\hat{j})\ m$

$\vec{H}_{/P} = m(-2\hat{i} + 7\hat{j}) \times (6\hat{i} - 3\hat{j})\ kg\,m^2/sec$

$= m(6 - 42)\hat{k}\ kg\,m^2/sec$

$\vec{H}_{/P} = -36m\,\hat{k}\ kg\,m^2/sec$

c) $\vec{H}_{/P} = m(7\hat{j}) \times (6\hat{i} - 3\hat{j})\ kg\,m^2/sec$

$\vec{H}_{/P} = -42m\,\hat{k}\ kg\,m^2/sec$

d) $\vec{H}_{/P} = m(\vec{0}) \times (6\hat{i} - 3\hat{j})$

$\vec{H}_{/P} = 0\ kg\,m^2/sec$

11.2.12

$m = 1\,kg$, $\vec{r} = (2\hat{i} - 0.5\hat{j})$

$\vec{V} = -4\hat{i} + 3\hat{j}$

a) Linear Momentum,

$\vec{L} = m\vec{V} = (-4\hat{i} + 3\hat{j})$ kg m/sec

b) $\vec{H}_{/o} = m\vec{r} \times \vec{V}$

$= (2\hat{i} - 0.5\hat{j}) \times (-4\hat{i} + 3\hat{j})$ kg m²/sec²

$= (6-2)\,\hat{k}$ kg m²/sec

$\vec{H}_{/o} = 4\hat{k}$ kg m²/sec

11.2.13

$m = 0.5\,kg$

$\vec{r}(t) = l \sin\omega t\,\hat{i} + h\,\hat{j}$

$\vec{v}(t) = \frac{d\vec{r}}{dt} = \omega l \cos\omega t\,\hat{i}$

$\omega = 2\,rad/sec$

$h = 2m$

$l = 2m$

a) $E_K = \frac{1}{2} m\,\vec{v}\cdot\vec{v}$

$= \frac{1}{2} m(\omega l \cos\omega t)^2 = \frac{1}{2} m\omega^2 l^2 \cos^2\omega t = 4\cos^2\omega t$

at $t = 0$

$E_K = 4$ Joules

at $t = 5$ sec

$E_K = 4\left[\cos(2\times 5)\right]^2$

$E_K = 2.8161$ Joules

b) $\dot{E}_K = \frac{d}{dt}\left[\frac{1}{2} m\omega^2 l^2 \cos^2\omega t\right]$

$= 4\frac{d}{dt}\cos^2\omega t = -8\cos\omega t \sin\omega t = -4\sin 2\omega t$

at $t = 0$

$\dot{E}_K = 0$ Joules/sec

at $t = 5$ sec

$\dot{E}_K = -4\sin(20)$

$\dot{E}_K = -3.6517$ Joules/sec

11.2.14

$$E_K = \frac{1}{2} m \vec{v} \cdot \vec{v}$$

$$\dot{E}_K = \frac{d}{dt}\left(\frac{1}{2} m \vec{v} \cdot \vec{v}\right)$$

$$= \frac{1}{2} m \left[\vec{v} \cdot \frac{d\vec{v}}{dt} + \frac{d\vec{v}}{dt} \cdot \vec{v} \right]$$

$$= m \left[\vec{v} \cdot \frac{d\vec{v}}{dt} \right]$$

$$\dot{E}_K = m \vec{v} \cdot \vec{a}$$

11.2.15

$\vec{F}_D = -bv^2 \hat{e}_t$ $\quad$ $\vec{F}_g = -mg\hat{j}$

Say $\vec{v} = v_x \hat{i} + v_y \hat{j}$

$$F_D = -bv^2 \hat{e}_t = -bv^2 \frac{\vec{v}}{v} = -bv\vec{v} \quad , \quad v = \sqrt{v_x^2 + v_y^2}$$

Net Power of force

$$P = \sum \vec{F} \cdot \vec{v}$$

$$= -(bv)\vec{v} \cdot \vec{v} - mg\hat{j} \cdot \vec{v}$$

$$= -bv^3 - mgv_y$$

$$= -bv^3 - mg(v \sin\theta)$$

$$P = -mgv\sin\theta - bv^3$$

11.2.16

$m = 10$ gms

$$E_k = \frac{1}{2} m \vec{V} \cdot \vec{V} = \frac{1}{2}\left(\frac{10}{1000}\right)\left(9^2 + 24^2 + 30^2\right) \text{ Joules}$$

$$E_k = 7.785 \text{ Joules}$$

11.2.17

$m = 2$ kg

$$\vec{r} = [5 \sin at\, \hat{i} + bt^2 \hat{j} + ct\hat{k}]$$

$$\vec{V} = 5a\cos at\, \hat{i} + 2bt\, \hat{j} + c\hat{k}$$

$$\vec{a} = -5a^2 \sin at\, \hat{i} + 2b\, \hat{j}$$

a) $$\vec{L}\Big|_{t=1} = m\vec{V}\Big|_{t=1} = 2[5\pi\cos\pi + \cancel{2\cos}\ 2(0.25)\hat{j} + 2\hat{k}]$$

$$= 2[-15.708\,\hat{i} + 0.5\,\hat{j} + 2\hat{k}]$$

$$\vec{L}\Big|_{t=1} = -10\pi\hat{i} + \hat{j} + 4\hat{k}$$

b) $$\vec{F}\Big|_{t=1} = m\vec{a}\Big|_{t=1}$$

$$= 2[-5\pi^2 \sin\pi\, \hat{i} + 0.5\hat{j}]$$

$$\vec{F} = 1\hat{j} \text{ N}$$

11.2.18

$$m_B = 2m_a$$
$$\vec{V_B} = \vec{V_A}$$
$$\vec{r_B} = \vec{r_A}$$

(i) $\vec{L_B} = m_B \vec{V_B} = 2m_A \vec{V_A} = 2\vec{L_A}$

$$\vec{L_B} = 2\vec{L_A}$$

(ii) $\vec{H}_{B/C} = \vec{r}_{B/C} \times \vec{L_B} = \vec{r}_{A/C} \times 2\vec{L_A}$

$$\vec{H}_{B/C} = 2\vec{H}_{A/C}$$

(iii) $E_{KB} = \frac{1}{2} m_B \vec{V_B} \cdot \vec{V_B} = 2\left(\frac{1}{2} m_A \cdot \vec{V_A} \cdot \vec{V_A}\right) = 2E_{KA}$

$$E_{KB} = 2E_{KA}$$

11.2.19

$m = 3\,kg$

a) $\vec{L} = m\vec{v}$

$= 3[-3\hat{i} + 8\hat{j} + 10\hat{k}]$ kg m/sec

$\vec{L} = [-9\hat{i} + 24\hat{j} + 30\hat{k}]$ kg m/sec

b) $\dot{\vec{L}} = m\dfrac{d\vec{v}}{dt} = m\vec{a}$

$= 3[-5\hat{i} + 12\hat{j} + 16\hat{k}]$ kg m/sec^2

$\dot{\vec{L}} = [-15\hat{i} + 36\hat{j} + 48\hat{k}]$ kg m/sec^2

c) $\vec{H}_{/o} = \vec{r}_{/o} \times m\vec{v}$

$= [2\hat{i} + 3\hat{j} + 5\hat{k}] \times [-9\hat{i} + 24\hat{j} + 30\hat{k}]$ kg m^2/sec

$= [(90-120)\hat{i} + (-60-45)\hat{j} + (48+27)\hat{k}]$ kg m^2/sec

$\vec{H}_{/o} = [-30\hat{i} - 105\hat{j} + 75\hat{k}]$ kg m^2/sec

d) $\dot{\vec{H}}_{/o} = \vec{r}_{/o} \times m\vec{a}$

$= [2\hat{i} + 3\hat{j} + 5\hat{k}] \times [-15\hat{i} + 36\hat{j} + 48\hat{k}]$ kg m^2/sec^2

$\dot{\vec{H}}_{/o} = [-36\hat{i} - 171\hat{j} + 117\hat{k}]$ kg m^2/sec^2

e) $E_k = \frac{1}{2} m\vec{v}\cdot\vec{v} = \frac{3}{2}[3^2 + 8^2 + 10^2] = 256.6$ Joules

f) $\dot{E}_k = m\vec{a}\cdot\vec{v} = [-15\hat{i} + 36\hat{j} + 48\hat{k}]\cdot[3\hat{i} + 8\hat{k} + 10\hat{j}]$

$= 45 + 288 + 480 = 813$ Joules/sec

11.2.20

$$\vec{H}_{/P} = \vec{r}_{/P} \times m\vec{a}$$

$$= (\vec{r} - \vec{r}_P) \times m\vec{a}$$

a) $m\{(3\hat{i} - 2\hat{j} + 4\hat{k}) - (3\hat{i} - 2\hat{j} + 4\hat{k})\} \times [\hat{i} - 8\hat{j} + 3\hat{k}]$

$= m[0 \times (\hat{i} - 8\hat{j} + 3\hat{k})]$

$\dot{\vec{H}}_{/P} = 0$

b) $\dot{\vec{H}}_{/P} = m[(3\hat{i} - 2\hat{j} + 4\hat{k}) - (6\hat{i} - 4\hat{j} + 8\hat{k})] \times [\hat{i} - 8\hat{j} + 3\hat{k}]$

$= -m[(3\hat{i} - 2\hat{j} + 4\hat{k}) \times (\hat{i} - 8\hat{j} + 3\hat{k})]$

$\dot{\vec{H}}_{/P} = -m[26\hat{i} - 5\hat{j} - 22\hat{k}]$

c) $\dot{\vec{H}}_{/P} = m[(3\hat{i} - 2\hat{j} + 4\hat{k}) - (-9\hat{i} + 6\hat{j} - 12\hat{k})] \times [\hat{i} - 8\hat{j} + 3\hat{k}]$

$= m[12\hat{i} - 8\hat{j} + 16\hat{k}] \times [\hat{i} - 8\hat{j} + 3\hat{k}]$

$\dot{\vec{H}}_{/P} = m[104\hat{i} - 20\hat{j} - 92\hat{k}]$

11.2.21

$m_1 = 5\,kg$	$m_2 = 5\,kg$
$\vec{r}_1 = 3\hat{i}\ m$	$\vec{r}_2 = -6\hat{i}$
$\vec{v}_1 = -4\hat{j}\ m/s$	$\vec{v}_2 = 5\hat{j}$
$\vec{a}_1 = 6\hat{j}\ m/s^2$	$\vec{a}_2 = -4\hat{j}$

a) $\vec{L} = \sum m_i \vec{v}_i$

$= 5(-4\hat{j}) + 5(5\hat{j})$

$\vec{L} = 5\hat{j}\ kg\,m/sec$

b) $\dot{\vec{L}} = \sum m_i \vec{a}_i$

$= 5(6\hat{j}) + 5(-4\hat{j})$

$\dot{\vec{L}} = 10\hat{j}\ kg\,m/sec^2$

c) $\vec{H}_{/o} = \sum \vec{r}_{/o} \times m\vec{v}$

$= 5[3\hat{i} \times (-4\hat{j}) + (-6\hat{i}) \times (5\hat{j})]$

$\vec{H}_{/o} = -210\hat{k}\ kg\,m^2/sec$

d) $\dot{\vec{H}}_{/o} = \sum \vec{r}_{/o} \times m\vec{a}$

$= 5[3\hat{i} \times 6\hat{j} + (-6\hat{i}) \times (-4\hat{j})]$

$= 5[18\hat{k} + 24\hat{k}]$

$\dot{\vec{H}}_{/o} = 210\hat{k}\ kg\,m^2/sec^2$

e) $E_K = \sum \frac{1}{2} m_i \vec{v}_i = \frac{5}{2}[(4)^2 + (5)^2] = 102.5$ Joules.

$E_K = 102.5$ Joules

f) $\dot{E}_K = \sum m\vec{a}\cdot\vec{v}$

$= 5\left[(6j)\cdot(-4\hat{j}) + (-4\hat{j})\cdot(5\hat{j})\right]$

$= 5\left[-24 - 20\right]$

$\dot{E}_K = -220$ Joules/sec

<u>11.2.22</u>

a) $\sum \vec{F} = \dot{\vec{L}} = m\vec{a}$

$= 6\left[5\hat{i} + 11\hat{j} - 9\hat{k}\right]$

$\vec{F} = 30\hat{i} + 66\hat{j} - 54\hat{k}$ N

b) $\sum \vec{M}_{/o} = \dot{\vec{H}}_{/o}$

$= m\,\vec{r}_{/o} \times \vec{a}_{/o}$

$= m\left[\hat{i} - 2\hat{j} + 4\hat{k}\right] \times \left[5\hat{i} + 11\hat{j} - 9\hat{k}\right]$

$= 6\left[-26\hat{i} + 29\hat{j} + 21\hat{k}\right]$

$\sum \vec{M}_{/o} = -156\hat{i} + 174\hat{j} + 126\hat{k}$ N·m

c) Power,

$P = \sum m\vec{a}\cdot\vec{v}$

$= 6\left[5\hat{i} + 11\hat{j} - 9\hat{k}\right]\cdot\left[3\hat{i} + 4\hat{j} - 7\hat{k}\right]$

$= 6\left[15 + 44 - 63\right]$

$P = -24$ Joules/sec

$m = 250 \text{ grams} = \frac{1}{4} \text{ kg}$

$\vec{r}_0 = 2\hat{i}$

$\vec{V} = \sqrt{V_0^2 - 2ah}\,\hat{j}$

$V_0 = 100 \text{ m/sec}$

$a = 10 \text{ m/sec}^2$

$h = y$ if $r_{P/O} = x\hat{i} + y\hat{j}$

a) $\vec{L}\Big|_{t=0} = m\vec{V}\Big|_{t=0}$

$= \frac{1}{4}\left[\sqrt{V_0^2 - 2a(0)}\;\hat{j}\right] \text{kg m/sec}$

$= \frac{V_0}{4}\hat{j} \text{ kgm/sec}$

$\vec{L}\Big|_{t=0} = 25\hat{j} \text{ kg m/sec}$

b) $\vec{H}\Big|_{t=0} = \vec{r}_{/0} \times m\vec{V}\Big|_{t=0}$

$= 2\hat{i} \times 25\hat{j}$

$\vec{H}\Big|_{t=0} = 50\hat{k} \text{ kg m}^2\text{/sec}$

c) for $h = 20\text{m}$

$\vec{L} = m\vec{V}$

$= \frac{1}{4}\left[\sqrt{V_0^2 - 2ah}\right]\hat{j}$

$\vec{L} = 24.5\,\hat{j} \text{ kgm/sec}$

d) for $h = 20\text{m}$

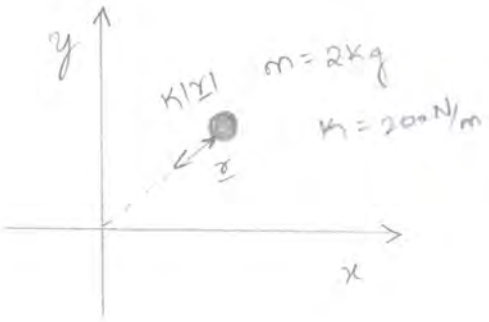
$\vec{r}_{/0} = \hat{i} + 20\hat{j}$

$\vec{H}_{/0} = \vec{r}_{/0} \times m\vec{V}$

$= (2\hat{i} + 20\hat{j}) \times (24.5\,\hat{j})$

$\vec{H}_{/0} = 99\hat{k} \text{ kgm}^2\text{/sec}$

11.3.6



$$\sum F = m\underline{a}$$

$$-K\underline{r} = m\ddot{\underline{r}}$$

Let $\underline{r} = x\hat{i} + y\hat{j}$

$$\ddot{r} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

$$-K(x\hat{i} + y\hat{j}) = m(\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

$$m\ddot{x} = -Kx \quad \& \quad m\ddot{y} = -Ky$$

If $|\underline{r}|$ = constant Let $r = a\cos\omega t$

$$\dot{\underline{r}} = -\underline{a}\,\omega\sin\omega t$$

$$\ddot{\underline{r}} = -\underline{a}\,\omega^2\cos\omega t = -\omega^2 r$$

$$\ddot{\underline{r}} = -\left(\frac{K}{m}\right)\underline{r}$$

Let $x = A\sin\lambda t + B\cos\lambda t = C_1\sin(\lambda t + C_2)$

$y = C\sin\lambda t + D\cos\lambda t = C_3\sin(\lambda t + C_4)$

If initial conditions are choosen such that $C_2 = 0$, $C_4 = \frac{\pi}{2}$ & $C_1 = \pm C_3$

Then x, y trace a circle.

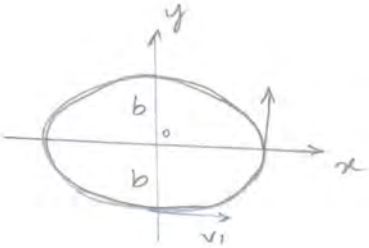
$x = r_0\sin\lambda t$ & $y = r_0\cos\lambda t$

$$\dot{x} = r_0\lambda\cos\lambda t$$

$$r_0 = \sqrt{x^2+y^2} = r \qquad \dot{y} = -r_0\lambda\sin\lambda t$$

speed, $v = \sqrt{\dot{x}^2+\dot{y}^2} = r_0\lambda \Rightarrow v = r_0\sqrt{K/m}$

b) $a = 1m$, $b = 0.8m$



$$x = A\sin\lambda t + B\cos\lambda t$$

$v|_{x=0} = V_1$

At $t=0$ say $x=0$ & $y=-b$

$B=0$ & $y = C\sin\lambda t + D\cos\lambda t \Rightarrow D = -b$

$\dot{x} = A\lambda\cos\lambda t - B\lambda\sin\lambda t$, $\dot{y} = C\lambda\cos\lambda t - D\lambda\sin\lambda t$

$t=0$

$\dot{x} = V_1$ $\quad V_1 = A\lambda$ & $0 = C\lambda \Rightarrow C=0,\ D=-b$; $A = \frac{V_1}{\lambda}$ & $B=0$

$\therefore\ x = \frac{V_1}{\lambda}\sin\lambda t$ & $y = -b\cos\lambda t$ $\Big\}$ $\dot{x} = V_1\cos\lambda t$ & $\dot{y} = b\lambda\sin\lambda t$

$\lambda t = \frac{\pi}{2} \Rightarrow V = \dot{y} = b\lambda$

ΔV required $= (a-b)\lambda$ or $\Delta V = (a-b)\sqrt{\frac{K}{m}}$

11.3.9

$$V_1 = \sqrt{\frac{GM}{R_1}} \quad , \quad V_2 = \sqrt{\frac{GM}{R_2}}$$

$$TE_1 = \frac{1}{2} m \frac{GM}{R_1} - \frac{GMm}{R_1} = -\frac{GMm}{2R_1}$$

$$TE_2 = -\frac{GMm}{2R_2}$$

$$\frac{TE_2}{TE_1} = \frac{R_1}{R_2}$$

$$\therefore \quad TE_1 = \frac{R_2}{R_1} TE_2$$

$$TE_1 > TE_2$$

11.3.1

$$\left.\begin{aligned} \underline{r} &= R[\cos\lambda t\ \hat{i} + \sin\lambda t\ \hat{j}] \\ \dot{\underline{r}} &= R\lambda[-\sin\lambda t\ \hat{i} + \cos\lambda t\ \hat{j}] \\ \ddot{\underline{r}} &= R\lambda^2[-\cos\lambda t\ \hat{i} - \sin\lambda t\ \hat{j}] \end{aligned}\right\} \ddot{\underline{r}} = -\lambda^2 \underline{r}$$

$$\underline{F} = m\ddot{\underline{r}}$$

a) $\underline{F} = \dfrac{-\underline{r}}{|\underline{r}|} \cdot F(r) = \dfrac{-\underline{r}}{|\underline{r}|} \cdot Kr = -K\underline{r} = m\ddot{\underline{r}} = -m\lambda^2 \underline{r}$

$\Rightarrow \lambda^2 = \dfrac{k}{m}$ $\quad \therefore$ velocity $v = r\sqrt{\dfrac{K}{m}}$

b) $F(r) = \dfrac{GMm}{r^2} \Rightarrow \underline{F} = \dfrac{-\underline{r}}{|\underline{r}|} \cdot \dfrac{GMm}{r^2} = -\dfrac{GMm}{r^3} \cdot \underline{r} = -m\lambda^2 \underline{r}$

As $r = R$

$\Rightarrow \lambda^2 = \dfrac{GM}{r^3}$ $\quad \therefore V = r\sqrt{\dfrac{GM}{r^3}}$

$V = \sqrt{\dfrac{GM}{r}}$ $\quad [\because GM = gR^2]$

$V = \sqrt{\dfrac{gR^2}{r}}$

c) $F(r) = r^n \Rightarrow \underline{F} = \dfrac{-\underline{r}}{|\underline{r}|} = -\underline{r}\, r^{n-1} = -m\lambda^2 \underline{r} \Rightarrow \lambda^2 = \dfrac{r^{n-1}}{m}$

$\therefore V = r\sqrt{\dfrac{r^{n-1}}{m}} \Rightarrow V = \sqrt{\dfrac{r^{n+1}}{m}}$

d) $F(r) = \frac{-r}{|r|} \cdot F(r) = -m\lambda^2 r \Rightarrow \lambda^2 = \frac{F(r)}{mR}$

$$\therefore V = R\sqrt{\frac{F(r)}{mR}} \Rightarrow V = \sqrt{\frac{F(r) \cdot r}{m}}$$

e) $F(r) = \frac{1}{R}$ If $F(r) = \frac{1}{r}$ $V = \sqrt{\frac{1}{m}}$

11.3.8

a) $\underline{F} = m\underline{a}$

$$-\frac{\underline{r}}{|\underline{r}|} \cdot F(r) = m\ddot{\underline{r}}$$

$$m\ddot{\underline{r}} = -K\underline{r}$$

$$m\ddot{x} = -Kx \quad \& \quad m\ddot{y} = -Ky$$

Let

$$z_1 = x$$
$$z_2 = \dot{x}$$
$$z_3 = y$$
$$z_4 = \dot{y}$$

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = -\frac{K}{m} z$$
$$\dot{z}_3 = z_4$$
$$\dot{z}_4 = -\frac{K}{m} z_3$$

b) $\underline{F} = m\underline{a}$

$$-\frac{\underline{r}}{|\underline{r}|} \cdot F(r) = m\ddot{\underline{r}}$$

$$-\frac{\underline{r}}{r^3} \cdot GMm = m\ddot{r}$$

$$m\ddot{x} = -\frac{GMm}{r^3} x$$

$$m\ddot{y} = -\frac{GMm}{r^3} y$$

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = -\frac{GM}{\left(\sqrt{z_1^2 + z_3^2}\right)^3} \cdot z_1$$

$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = -\frac{GM}{\left(\sqrt{z_1^2 + z_3^2}\right)^3} \cdot z_3$$

c) $F(r) = r^n$

$$-\underline{r}\, r^{n-1} = m\ddot{\underline{r}}$$

$$-(x\hat{i} + y\hat{j})\left(\sqrt{x^2+y^2}\right)^{n-1} = m(\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

$$m\ddot{x} = -\left(\sqrt{x^2+y^2}\right)^{n-1} x$$

$$m\ddot{y} = -\left(\sqrt{x^2+y^2}\right)^{n-1} y$$

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = -\left(\sqrt{z_1^2 + z_3^2}\right)^{n-1} \frac{z_1}{m}$$

$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = -\left(\sqrt{z_1^2 + z_3^2}\right)^{n-1} \frac{z_3}{m}$$

d) $F(r) = \text{Arbitrary}$

Let $F(r) = e^{-r}$

$$\frac{-\underline{r}}{r} \cdot e^{-r} = m\ddot{r} = -m\lambda^2 r \Big\} \Rightarrow \lambda^2 = \frac{e^{-r}}{mr}$$

$$V = r\lambda \Rightarrow V = r\sqrt{\frac{e^{-r}}{mr}} \Rightarrow V = \sqrt{\frac{r^2 e^{-r}}{m}}$$

$$-(x\hat{i} + y\hat{j}) \cdot \frac{e^{-(\sqrt{x^2+y^2})}}{\sqrt{x^2+y^2}} = m\ddot{x}\,\hat{i} + m\ddot{y}\,\hat{j}$$

$$\ddot{x} = -\frac{e^{-\sqrt{x^2+y^2}}}{m\sqrt{x^2+y^2}} \cdot x \quad \Bigg| \quad \ddot{y} = -\frac{e^{-\sqrt{x^2+y^2}}}{m\sqrt{x^2+y^2}} \cdot y$$

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = -\frac{e^{-\sqrt{z_1^2+z_3^2}}}{m\sqrt{z_1^2+z_3^2}} \cdot z_1$$

$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = -\frac{e^{-\sqrt{z_1^2+z_3^2}}}{m\sqrt{z_1^2+z_3^2}} \cdot z_3$$

e) $F(r) = 1/r$

$$-\frac{\underline{r}}{r} \cdot \frac{1}{r} = m\ddot{r} \Rightarrow -m\lambda^2 \underline{r} \Rightarrow \lambda^2 = \frac{1}{mr^2}$$

$$V = r\sqrt{\frac{1}{mr^2}} = \frac{1}{\sqrt{m}}$$

$$-\frac{(x\hat{i} + y\hat{j})}{x^2+y^2} = m\ddot{x}\,\hat{i} + m\ddot{y}\,\hat{j}$$

$$\ddot{x} = -\frac{x}{(x^2+y^2)m}, \quad \ddot{y} = -\frac{y}{(x^2+y^2)m}$$

11.3.10

Let $F(r) = Kr$

$$\sum \underline{F} = m\underline{a}$$

$$-\frac{\underline{r}}{|\underline{r}|} \cdot K|\underline{r}| - c\underline{v} = m\underline{a}$$

$$-K(x\hat{i} + y\hat{j}) - c(\dot{x}\hat{i} + \dot{y}\hat{j}) = m(\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

$$\ddot{x} = -\frac{c}{m}\dot{x} - \frac{K}{m}x \quad ; \quad \ddot{y} = -\frac{c}{m}\dot{y} - \frac{K}{m}y$$

$$\dot{z}_1 = z_2 \quad , \quad \dot{z}_3 = z_4$$

$$\dot{z}_2 = -\frac{c}{m}z_2 - \frac{K}{m}z_1 \quad ; \quad \dot{z}_4 = -\frac{c}{m}z_4 - \frac{K}{m}z_3$$

12.1.1

$$\vec{F} = m\vec{a}$$

$\vec{a}$ → Acceleration of the C.M in the inertial frame of reference

m → Total mass of the system

$\vec{F}$ → External force on the system

12.1.2

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix} \text{kg}$$

$$\underline{r}_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \text{m}, \quad \underline{r}_2 = \begin{pmatrix} -6 \\ -4 \end{pmatrix} \text{m}$$

$$\dot{\underline{r}}_1 = \begin{pmatrix} -16 \\ 6 \end{pmatrix} \text{m/s}, \quad \dot{\underline{r}}_2 = \begin{pmatrix} 8 \\ 4 \end{pmatrix} \text{m/s}$$

$$\ddot{\underline{r}}_1 = \begin{pmatrix} 10 \\ -24 \end{pmatrix} \text{m/s}^2, \quad \ddot{\underline{r}}_2 = \begin{pmatrix} 5 \\ -16 \end{pmatrix} \text{m/s}^2$$

a) $\vec{L}_1 = m_1 \dot{\underline{r}}_1 = 6 \begin{pmatrix} 3 \\ 2 \end{pmatrix} \text{kg m/s}$

$\dot{\vec{L}}_1 = m_1 \ddot{\underline{r}}_1 = 6 \begin{pmatrix} 10 \\ -24 \end{pmatrix} \text{kg m/s}^2$

$$\vec{L}_2 = m_2 \dot{\underline{r}}_2 = 10 \begin{pmatrix} 8 \\ 4 \end{pmatrix} \text{ kg m/s}$$

$$\dot{\vec{L}}_2 = m_2 \ddot{\underline{r}}_2 = 10 \begin{pmatrix} 5 \\ -16 \end{pmatrix} \text{ kg m/s}^2$$

b) $$\vec{L} = \vec{L}_1 + \vec{L}_2 = \begin{pmatrix} 98 \\ 52 \end{pmatrix} \text{ kg m/s}$$

$$\dot{\vec{L}} = \dot{\vec{L}}_1 + \dot{\vec{L}}_2 = \begin{pmatrix} 110 \\ -304 \end{pmatrix} \text{ kg m/s}$$

c) $$\vec{r}_c = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$$\vec{r}_c = \frac{6 \begin{pmatrix} 3 \\ 2 \end{pmatrix} + 10 \begin{pmatrix} -6 \\ -4 \end{pmatrix}}{16} = \frac{1}{16} \begin{pmatrix} -42 \\ -28 \end{pmatrix} = -\frac{7}{8} \begin{pmatrix} 3 \\ 2 \end{pmatrix} \text{ m}$$

$$\dot{\vec{r}}_c = \frac{m_1 \dot{\vec{r}}_1 + m_2 \dot{\vec{r}}_2}{m_1 + m_2}$$

$$= \frac{6 \begin{pmatrix} -16 \\ 6 \end{pmatrix} + 10 \begin{pmatrix} 8 \\ 4 \end{pmatrix}}{16} = \frac{1}{16} \begin{pmatrix} -16 \\ 76 \end{pmatrix} = \begin{pmatrix} -1 \\ 4.75 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

12.1.3

$m_1 = 5\ kg$

$\vec{r}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} m$

$\vec{v}_1 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} m/s$

$\vec{a}_1 = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} m/s^2$

$m_2 = 10\ kg$

$\vec{r}_2 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} m$

$\vec{v}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} m/s$

$\vec{a}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} m/s^2$

a) $\vec{L} = m_1\vec{v}_1 + m_2\vec{v}_2$

$= 5\begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + 10\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ 10 \end{pmatrix} kg\,m/s$

b) $\vec{L} = 5\begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} + 10\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ 15 \end{pmatrix} kg\,m/s^2$

c) Angular momentum about origin

$\vec{H}_{/0} = m_1\vec{r}_1 \times \vec{v}_1 + m_2\vec{r}_2 \times \vec{v}_2$

$= 5\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + 10\begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -20 \\ 10 \end{pmatrix} kg\,m^2/s$

d) $\dot{\vec{H}}_{/0} = m_1 \vec{r}_1 \times \vec{a}_1 + m_2 \vec{r}_2 \times \vec{a}_2$

$$= 5 \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} + 10 \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 15 \\ -15 \\ 20 \end{pmatrix} \text{ kg m}^2/\text{s}^2$$

e) $E_K = \frac{1}{2} m_1 \dot{r}_1^2 + \frac{1}{2} m_2 \dot{r}_2^2$

$$= \frac{1}{2} \times 5 \times 4 + \frac{1}{2} \times 10 \times 1$$

$$= 15 \text{ kg m}^2/\text{s}^2$$

f) $\dot{E}_K = m_1 \dot{r}_1 \ddot{r}_1 + m_2 \dot{r}_2 \cdot \ddot{r}_2$

$$= 10 \times 3 + 10 \times 1 \times 1$$

$$= 40 \text{ kg m}^2/\text{s}^3$$

12.1.4

$$\vec{H}_{/o} = \int \vec{r} \times dm\, \vec{v}$$

$$= \int (\vec{r} \times \vec{v})\, dm$$

$$= \int (\vec{r}_{co} + \vec{\rho}_c) \times \vec{v}\, dm$$

$$\vec{H}_{/o} = \int \vec{r}_{co} \times \vec{v}\, dm + \int \vec{\rho}_c \times \vec{v}\, dm$$

$$= \vec{r}_{co} \times m\vec{v}_c + \int \vec{\rho}_c \times \vec{v}\, dm$$

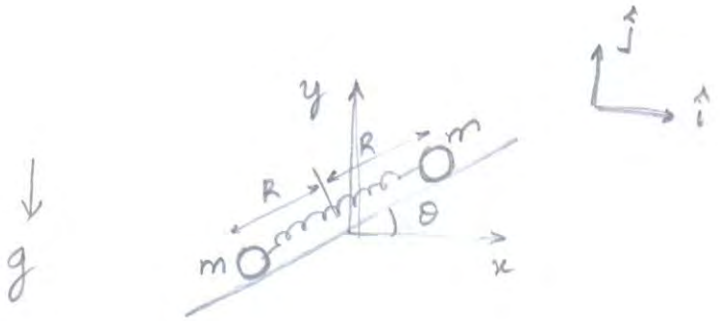
if $\vec{\rho}_c \times \vec{v} = 0$, it is possible, particles are moving towards centre of mass

12.1.5

a) $\vec{F}_{ij} = -\vec{F}_{ji}$

b) Total force on 5 $= \sum_{j=1}^{17} \vec{F}_{5j}$

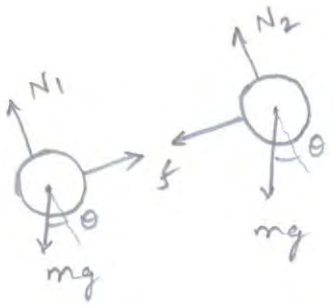
12.1.6



a) Total linear momentum is not conserved. Gravity is acting on the system.

b) Yes, due to gravity

c)



$N_1 = mg\cos\theta = N_2$

$\sin\theta\, mg - f = ma$

both masses fall together so $f = 0$

$a = g\sin\theta$ along the plane

d)

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = -g \sin\theta \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$\ddot{x} = -g \sin\theta \cos\theta \ \hat{i}$$

$$\ddot{y} = -g \sin^2\theta \ \hat{j}$$

e) The total Kinetic Energy + Potential Energy = constant

12.2.1

$$\begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix} \begin{pmatrix} v'_{Ax} \\ v'_{Ay} \end{pmatrix} = v_0 \begin{pmatrix} e\cos\theta \\ \sin\theta \end{pmatrix}$$

$$\begin{pmatrix} v'_{Ax} \\ v'_{Ay} \end{pmatrix} = v_0 \begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}^{-1} \begin{pmatrix} e\cos\theta \\ \sin\theta \end{pmatrix}$$

$$= \frac{v_0}{-1} \begin{bmatrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} \begin{pmatrix} e\cos\theta \\ \sin\theta \end{pmatrix}$$

$$\begin{pmatrix} v'_{Ax} \\ v'_{Ay} \end{pmatrix} = -v_0 \begin{bmatrix} (1-e)\sin\theta\cos\theta \\ e\cos^2\theta + \sin^2\theta \end{bmatrix}$$

12.2.2

$$(\vec{v}_1' - \vec{v}_2') \cdot \hat{n} = e(\vec{v}_2 - \vec{v}_1) \cdot \hat{n}$$

$$(v_{1x}\,\hat{i} + v_{1y}\,\hat{j} + v_0\,\hat{i}) \frac{1}{\sqrt{2}}(\hat{i} + \hat{j}) = e(-2\hat{i} + 5\hat{j}) \cdot \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$$

$$(v_{1x} + v_0)\hat{i} + v_{1y} = e \times (-2 + 5)$$

$$v_{1x} + v_{1y} + v_0 = 0.5 \times (3)$$

$$v_{1x} + v_{1y} + v_0 = 1.5$$

12.2.3

v_0 , v_B , v_c are unknown

$$\begin{bmatrix} 0 & -.67m_B & -0.58m_c \\ 0 & 0.33m_B & .03m_c \\ -m_0 & -0.67m_B & -.58m_c \end{bmatrix} \begin{bmatrix} v_0 \\ v_B \\ v_c \end{bmatrix} = \begin{pmatrix} -23m_A \\ -36m_A \\ -24m_A \end{pmatrix}$$

12.2.4

$$\begin{bmatrix} -\cos\theta & -\sin\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \\ m_A & 0 & m_B \end{bmatrix} \begin{pmatrix} v'_{Ax} \\ v'_{Ay} \\ v'_{Bx} \end{pmatrix} = \begin{bmatrix} -10 \\ -36 \\ -60m_A \end{bmatrix}$$

12.2.6

$\theta = 20^\circ$ and $v_0 = 5$ ft/s

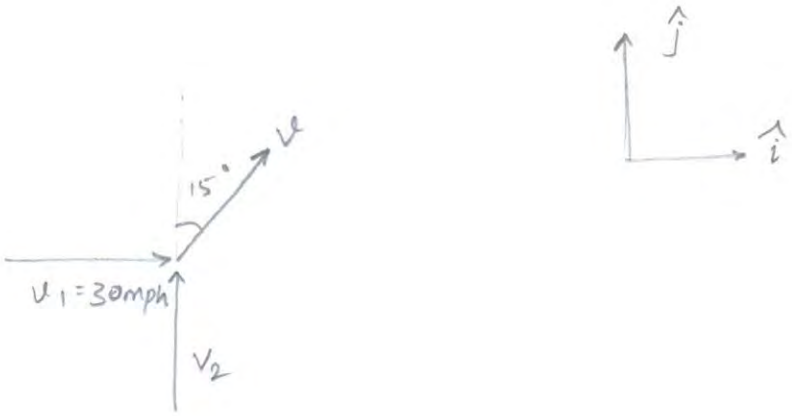
From 12.2.1

$$\begin{pmatrix} V'_{Ax} \\ V'_{Ay} \end{pmatrix} = -v_0 \begin{bmatrix} (1-e)\sin\theta\cos\theta \\ e\cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$\begin{pmatrix} V'_{Ax} \\ V'_{Ay} \end{pmatrix} = -5 \begin{bmatrix} (1-e)\sin 20^\circ \cos 20^\circ \\ e\cos^2 20^\circ + \sin^2 20^\circ \end{bmatrix}$$

$$\begin{pmatrix} V'_{Ax} \\ V'_{Ay} \end{pmatrix} = \begin{pmatrix} (e-1)\times 1.6 \\ -4.4e - 0.58 \end{pmatrix}$$

12.2.12



$m_1 = m_2 = m$

From conservation of Linear momentum

$$m_1\vec{v_1} + m_2\vec{v_2} = (m_1+m_2)\vec{v}$$

$$m \cdot 30\,\hat{i} + m \cdot v_2\,\hat{j} = 2mv(\sin 15^\circ\,\hat{i} + \cos 15^\circ\,\hat{j})$$

$$30\,\hat{i} + v_2\,\hat{j} = 2v\sin 15^\circ\,\hat{i} + 2v\cos 15^\circ\,\hat{j}$$

$$v_2\,\hat{j} = 2v\cos 15^\circ\,\hat{j}$$

$$30\,\hat{i} = 2v\sin 15\,\hat{i}$$

$$v = \frac{30}{2\sin 15}$$

$$\therefore\ v_2 = 30 \cdot \cot 15^\circ$$

$$v_2 = 112 \text{ mph}$$

12.2.14

$h = 30{,}000\ ft \equiv 5.5\ mi$

$g = 9.8\ m/s^2 \equiv 79380\ mi/hr^2$

From conservation of linear momentum,

Let mass of the Airplane be 2m.

$$2mv\hat{i} = mv_1\hat{i} + mv_2\hat{i}$$

$$\therefore \quad 2v = v_1 + v_2 \longrightarrow (1)$$

Now, we know that the first half of the aircraft was found at 8 mi from the point of explosion then from the Equation of Projectiles we obtain

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5.5}{79380}} = 0.012\ hr$$

Range = 8 mi

$$\therefore \quad R_1 = v_1 t$$

$8 = v_1 \times 0.012$

$v_1 = 666.6 \ mi/hr$

We know that $v = 500 \ mi/hr$

Substituting in (1) we obtain

$2 \times 500 = 666.6 + v_2$

$v_2 = 333.4 \ mi/hr$

The range of the second half of the airplane

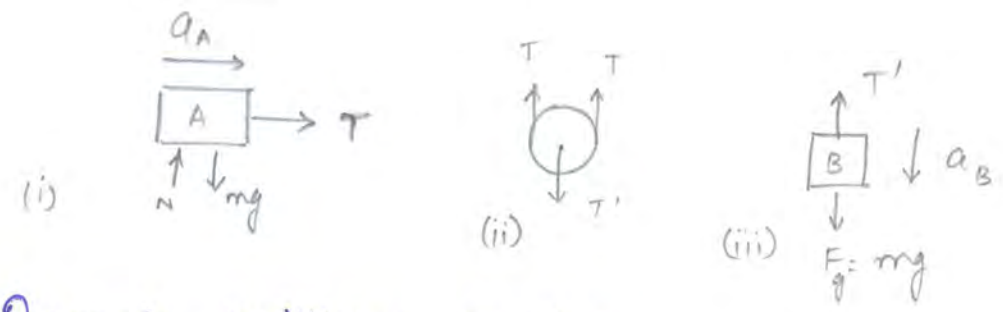
$R_2 = v_2 \times t$

$R_2 = 333.4 \times 0.012 \ hr$

$R_2 = 4 \ mi$

13.1.3

FBD



a) Dynamic equilibrium equation

$$m a_A = T \longrightarrow (i)$$

$$2T = T'$$

$$m a_B = mg - T' = mg - 2T \longrightarrow (ii)$$

Kinematic constraint ($\Delta x_A = 2\Delta y_B$)

$$a_A = 2a_B \longrightarrow (iii)$$

Solving (i), (ii), (iii) ; ~~According of~~

Acc. of Block 'A' Just after release

$$\frac{m a_A}{2} = mg - 2 m a_A$$

$$a_A = \frac{2g}{5} = 4 \text{ m/sec}^2$$

b) $a_B = \frac{a_A}{2} = \frac{g}{5}$

Speed of block B after it has fallen 2m.

$u_B = 0$, $V_B = ?$, $a_B = g/5$, $s = 2m$

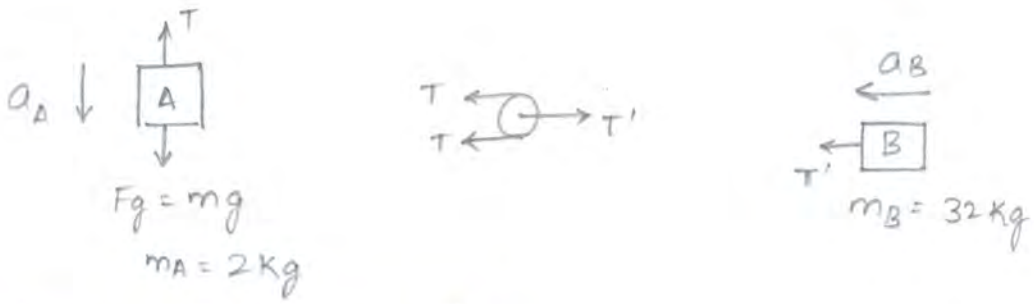
$$V_B^2 - u_B^2 = 2a_B s$$

$$V_B = \sqrt{2 \times \frac{g}{5} \times 2}$$

$$V_B = \sqrt{4 \times 4} = 4 \, m/sec$$

13.1.4

FBD



Dynamic equilibrium equation

$$T' = 2T$$

$$m_B a_B = T' = 2T \longrightarrow (i)$$

$$m_A a_A = m_A g - T \longrightarrow (ii)$$

Kinematic constraint

$$\Delta y_A = 2 \Delta x_B$$

$$a_A = 2a_B \longrightarrow (iii)$$

Solving (i), (ii) and (iii) we get

$$m_A a_A = m_A g - \frac{m_B a_B}{2} \Rightarrow 2a_A + \frac{32}{2}\left(\frac{a_A}{2}\right) = 20$$

$$2a_A + 8a_A = 20$$

$$a_A = 2 \, m/sec^2$$

13.1.5

a_B using Energy Balance

$a_A = 2a_B$, $V_A = 2V_B$

$P_{in} = m_A g V_A$

$\dot{E}_K = m_A a_A V_A + m_B a_B V_B$

Power balance requires

$m_A g V_A = m_A a_A V_A + m_B a_B V_B$

$\Rightarrow m_A g V_A = m_A (2a_B) V_A + m_B a_B \left(\frac{V_A}{2}\right)$

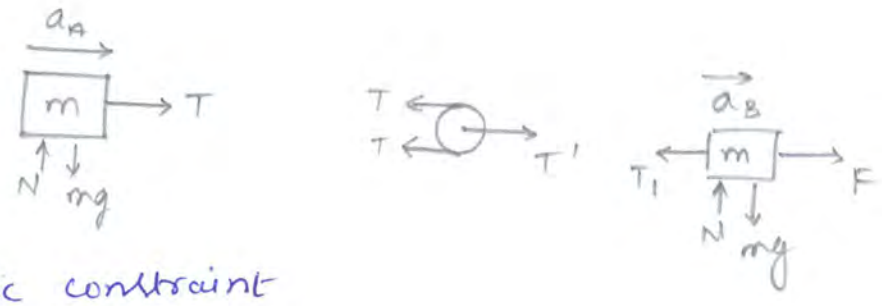
$$a_B = \frac{m_A g}{\left(2m_A + \frac{m_B}{2}\right)}$$

$$a_B = \frac{2 \times 10}{2 \times 2 + \frac{32}{2}} = \frac{20}{4+16}$$

$a_B = 1 \, m/sec^2$

13.1-9

F.B.D



Kinematic constraint

$$\Delta x_A = 2\Delta x_B$$

$$a_A = 2a_B \longrightarrow (i)$$

Dynamic Equilibrium

$$T' = 2T$$

$$ma_A = T \longrightarrow (ii)$$

$$ma_B = F - T' = F - 2T \longrightarrow (iii)$$

solving (i) (ii) and (iii)

$$ma_B = F - 2m(2a_B)$$

$$a_B = \frac{F}{5m} \Rightarrow a_A = \frac{2F}{5m}$$

13.1.10

Power input by force

$$P_{in} = F V_B$$

Kinematic requires:

$$\Delta x_A = 2 \Delta x_B$$

$$V_A = 2 V_B$$

$$a_A = 2 a_B$$

$$\dot{E}_K = m \vec{V}_B \cdot \vec{a}_B + m \vec{V}_A \cdot \vec{a}_A$$

$$F V_B = m V_B a_B + m (2V_B)(2a_B)$$

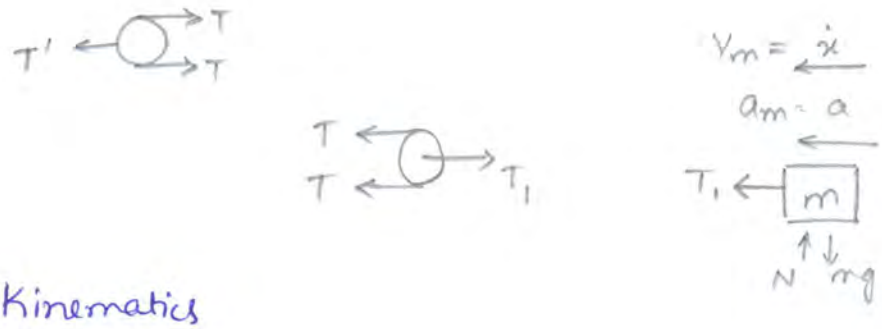
$$F = 5 m a_B$$

$$a_B = \frac{F}{5m}$$

$$a_A = \frac{2F}{5m}$$

13.1.11

FBD



Kinematics

$\Delta x_A = 2\Delta x_m$

$V_A = 2V_m$

$a_A = 2a_m$

Input power to the system at any instant

$P_{in} = FV_A$

$= 2FV_m$

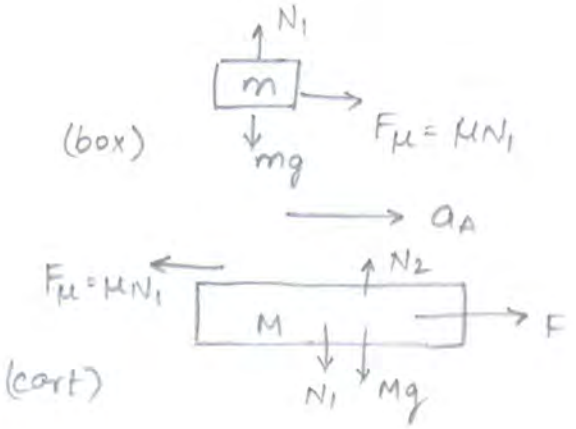
$V_m = \dot{x}$

$P_{in} = 2F\dot{x}$

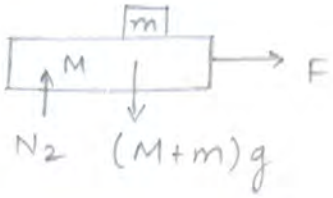
$P_{in} = 2F\dot{x}$

13·1·13

a)



Together



b) Linear momentum Balance

$$N_1 = mg$$

box $\quad m a_B = \mu N_1 = \mu m g \Rightarrow a_B = \mu g \longrightarrow$ (i)

cart $\quad m a_A = F - \mu N_1 = F - \mu m g$

$$a_A = \frac{F - \mu m g}{M} \longrightarrow \text{(ii)}$$

combined

$$(M+m)\, a_{cm} = F$$

$$a_{cm} = \frac{F}{M+m} \longrightarrow \text{(iii)}$$

c) Equation of motion of centre of mass

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1+m_2} = \frac{mx_1 + Mx_2}{m+M}$$

$$\Rightarrow \quad a_{cm} = \frac{ma_B + Ma_A}{m+M}$$

$$a_{cm} = \frac{m(\mu g) + M\left(\frac{F-\mu mg}{M}\right)}{M+m}$$

$$a_{cm} = \frac{(\mu mg + F - \mu mg)}{M+m} = \frac{F}{M+m}$$

This is same as (iii) Hence proved

d) At $t=0$, $u_A = u_B = 0$

Acc of the Box when seen from cart frame

$$a_{B/A} = a_B - a_A = \mu g - \frac{F-\mu mg}{M} = \frac{\mu g(m+M) - F}{M}$$

Displacement 's' $= -l$

initial velocity $u_{B/A} = 0$

$$-l = -\frac{1}{2}\left(\frac{F-\mu(M+m)}{M}\right)t^2$$

$$t = \sqrt{\frac{2Ml}{F-\mu(M+m)}}$$

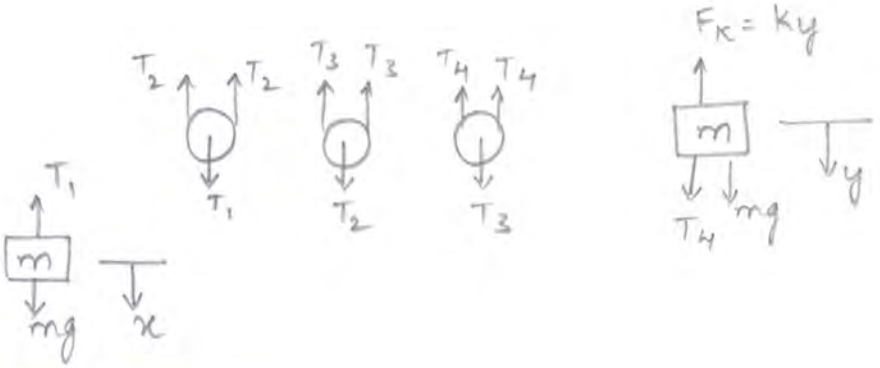
Displacement of Cart in this time

$$x_A = \frac{1}{2}\left(\frac{F-\mu mg}{M}\right)t^2$$

$$x_A = \frac{1}{2}\left(\frac{F-\mu mg}{M}\right)\left(\frac{2Ml}{F-\mu(M+m)}\right)$$

$$x_A = \frac{l(F-\mu mg)}{F-\mu(M+m)}$$

13.1.18



Kinematic constraint

$$\Delta x = \frac{\Delta y}{8}$$

$$\dot{x} = \frac{\dot{y}}{8} \quad \& \quad \ddot{x} = \frac{\ddot{y}}{8} \longrightarrow (i)$$

Dynamical equilibrium

$$m\ddot{x} = mg - T_1 \longrightarrow (ii)$$

$$m\ddot{y} = mg + T_4 - Ky' \longrightarrow (iii)$$

$$T_1 = 2T_2 \; ; \; T_2 = 2T_3 \; ; \; T_3 = 2T_4 \; ; \; T_1 = 8T_4$$

$$\Rightarrow m\ddot{y} = mg + \frac{1}{8}(mg - m\ddot{x}) - Ky'$$

$$\Rightarrow m\ddot{y} = mg + \frac{1}{8}\left(mg - \frac{m\ddot{y}}{8}\right) - Ky'$$

Now Assume $y' = y_0 + y$; y = Position from equilibrium

$$\ddot{y}' = \ddot{y}$$

Such that

$$Ky_0 = \frac{9mg}{8}$$

$$m\ddot{y}'\left(1+\frac{1}{64}\right) = mg\left(\frac{9}{8}\right) - k(y_0+y')$$

$$m\ddot{y}'\left(\frac{65}{64}\right) = -ky'$$

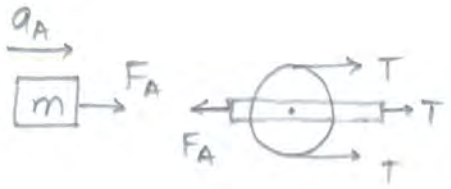
$$\ddot{y}' = -\frac{64k}{65m}y' = -\omega^2 y'$$

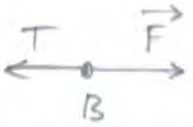
Natural frequency of vibration

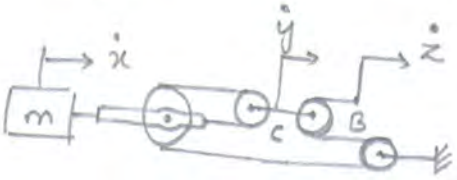
$$\omega = \sqrt{\frac{64k}{65m}}$$

13.1.19

FBD







The rate of change of length of rope is

$$\dot{L} = -\dot{x} + 2(\dot{y} - \dot{x}) - \dot{y} + (\dot{z} - \dot{y}) = 0$$

$$\Rightarrow \quad -3\dot{x} + \dot{z} = 0$$

$$\dot{z} = 3\dot{x} \quad \& \quad \ddot{z} = 3\ddot{x} \longrightarrow (i)$$

Dynamic equilibrium

$$m a_A = F_A = 3T = 3F$$

$$a_A = \frac{3F}{m} \longrightarrow (ii)$$

using relation (i) we get

$$a_B = 3a_A = \frac{9F}{m} \longrightarrow (iii)$$

13.1.20

using Energy methods

as derived already

$$\dot{z} = 3\dot{x}$$

$$P_{in} = F\dot{z}$$

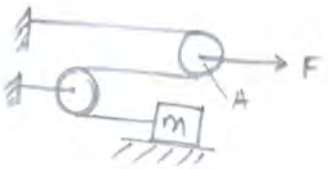
$$\dot{E}_K = m a_A \dot{x}$$

$$m a_A \dot{x} = F\dot{z}$$

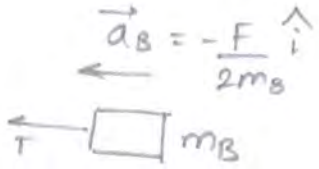
$$m a_A = 3F$$

$$a_A = \frac{3F}{m}$$

13.1.21



FBD



Kinematics

$F\dot{x}_A = TV_B$

$\dot{x}_A = \frac{V_B}{2} \Rightarrow a_A = \frac{a_B}{2}$

$F = 2T \longrightarrow (i)$

$m_B a_B = T$

$\Rightarrow a_B = \frac{T}{m_B} = \frac{F}{2m_B}$

$\vec{a}_B = -\frac{\vec{F}}{2m_B}$

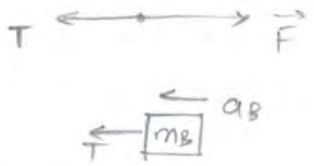
$\vec{a}_A = -\frac{\vec{a}_B}{2} = -\frac{\vec{F}}{4m_B}$

13 · 1 · 22

$$\vec{F} = F\hat{i}$$

a) $\vec{a}_B = -\dfrac{F}{m_B}\hat{i}$

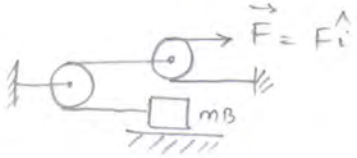
FBD



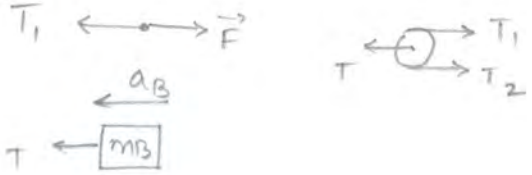
$$\vec{T} = -\vec{F}$$

$$\vec{a}_B = \frac{\vec{T}}{m_B} = -\frac{\vec{F}}{m_B}$$

b) $\vec{a}_B = -\dfrac{2F\hat{i}}{m_B}$



F.B.D



equation

$$F = T_1$$

$$T = 2T_1$$

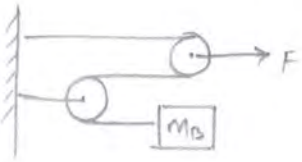
$$m_B a_B = T = 2T_1 = 2F$$

$$a_B = \frac{2F}{m_B}$$

Taking direction

$$\vec{a}_B = -\frac{2\vec{F}}{m_B}$$

c) $\vec{a}_B = -\dfrac{F\hat{i}}{2m_B}$



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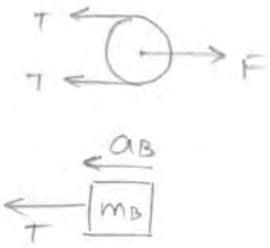


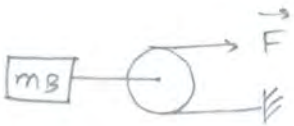
Equation

$m_B a_B = T = \dfrac{F}{2}$

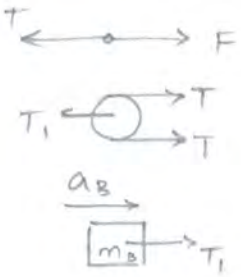
$a_B = \dfrac{F}{2m}$

$\vec{a}_B = \dfrac{-\vec{F}}{2m}$

d) $\vec{a}_B = \dfrac{2\vec{F}}{m_B}$



FBD



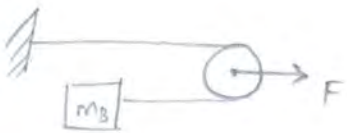
Equation

$m_B a_B = T_1 = 2T = 2F$

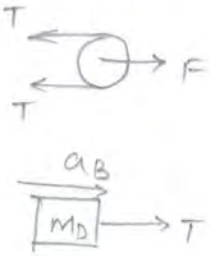
$a_B = \dfrac{2F}{m_B}$

$\vec{a}_B = \dfrac{2\vec{F}}{m_B}$

e) $\vec{a}_B = \dfrac{\vec{F}}{2m_B}$



FBD



Equation

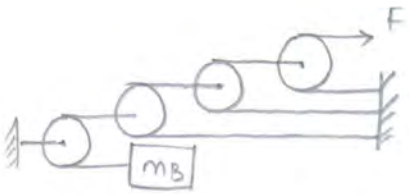
$F = 2T$

$m_B a_B = T = \dfrac{F}{2}$

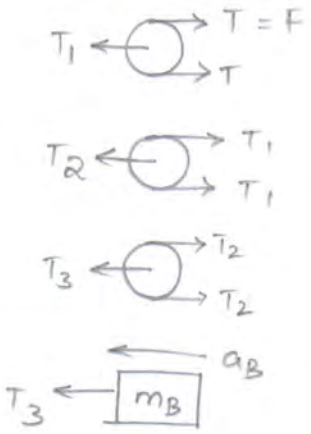
$a_B = \dfrac{F}{2m_B}$

$a_B = \dfrac{\vec{F}}{2m_B}$

f) $\vec{a}_B = \dfrac{-8\vec{F}}{m_B}$



F.B.D



Equation

$T_1 = 2T$; $T_2 = 2T_1$; $T_3 = 2T_2$

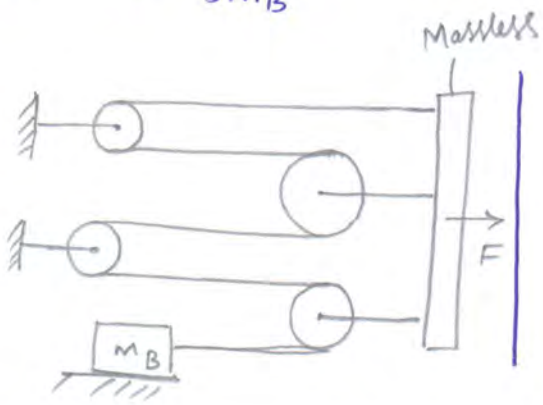
$T_3 = 8T = 8F$

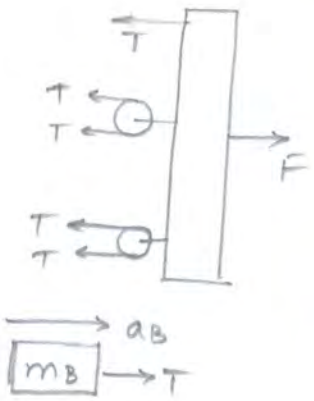
$m_B a_B = T_3$

$a_B = \dfrac{T_3}{m_B} = \dfrac{8F}{m_B}$

$\vec{a}_B = -\dfrac{8\vec{F}}{m_B}.$

g) $\vec{a}_B = \dfrac{\vec{F}}{5m_B}$





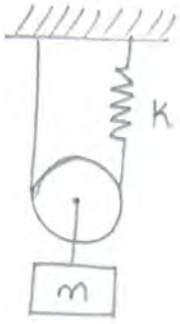
Equation

$5T = F$

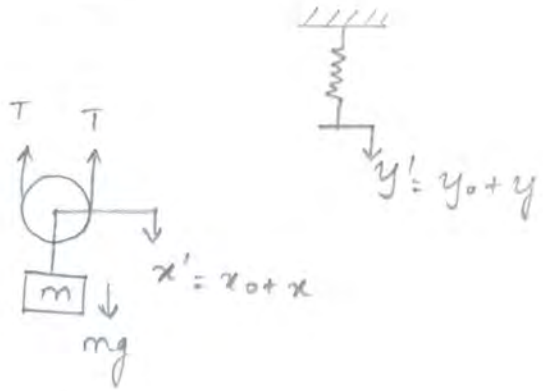
$m_B a_B = T = \dfrac{F}{5}$

$\vec{a}_B = \dfrac{\vec{F}}{5m_B}$

13.1.23



FBD



Equations;

Kinematics

$$\dot{y}' = 2\dot{x}'$$
$$\dot{y} = 2\dot{x}$$
$$\ddot{y} = 2\ddot{x}$$

$$\left.\begin{aligned} m\ddot{x} &= mg - 2T \\ T &= Ky' \end{aligned}\right| \Rightarrow m\ddot{x} = mg - 2Ky'$$

At equilibrium $\ddot{x} = 0 \Rightarrow mg = 2Ky_0 \Rightarrow y_0 = \frac{mg}{2K}$

$$\Rightarrow m\ddot{x} = mg - K\left[\frac{mg}{2K} + y\right] \Rightarrow m\ddot{x} = -2Ky$$

$$m\ddot{y} = -4Ky$$

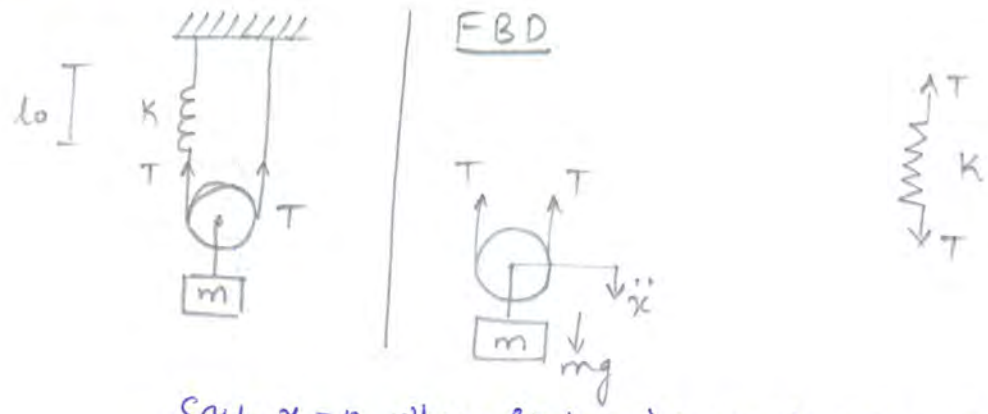
and

$$m\ddot{x} = -4Kx$$

Angular frequency $\omega = \sqrt{\frac{4K}{m}} = 2\sqrt{\frac{K}{m}}$

$$T = \pi\sqrt{\frac{m}{K}}$$

13.1.27



Say $x = 0$ when spring is just relaxed

$$m\ddot{x} = mg - 2T$$

where

$$T = K\delta$$

for equilibrium $\ddot{x} = 0$

$$mg - 2K\delta = 0$$

$$\delta = \frac{mg}{2K}$$

For deformation δ in spring mass deflects by

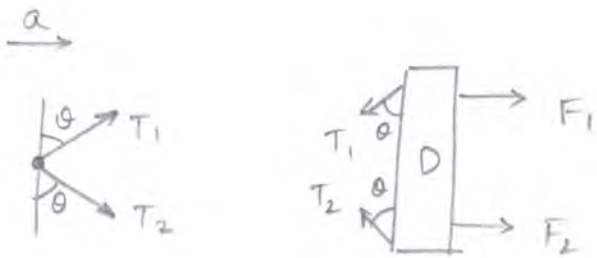
$$x = \frac{\delta}{2}$$

deflection in mass is

$$x = \frac{mg}{4K}$$

13·2·1

$a = 5\,m/sec^2\ \hat{i}$



$T_1 = T_2 = T$ because of Balance in vertical direction

$T_1 \cos\theta = T_2 \cos\theta \Rightarrow T_1 = T_2$

$ma = 2T \sin\theta$

$$T = \frac{ma}{2\sin\theta}$$

$$= \frac{5 \times m}{2 \times \sin\theta}$$

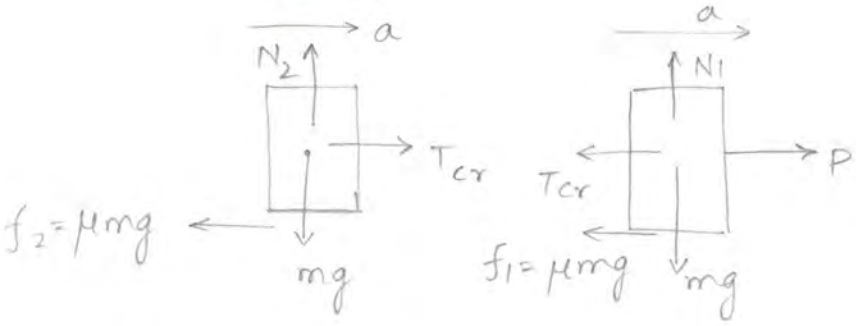
$$\sin\theta = \frac{4}{5}$$

$$T = \frac{5m \times 5}{2 \times 4}$$

$$T = \frac{25m}{8}\ N$$

13.2.2

FBD



Equation

$$ma = P - [T_{cr} + \mu mg]$$

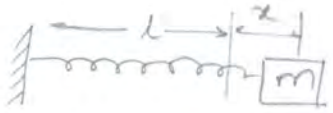
$$ma = T_{cr} - \mu mg$$

solving these we get

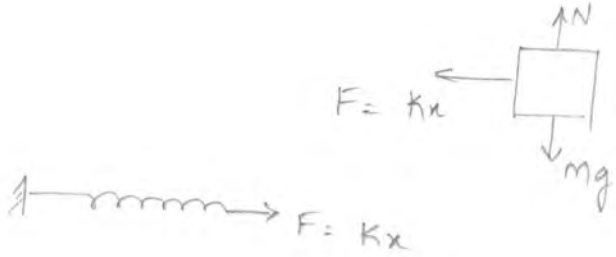
$$0 = P - 2T_{cr}$$

$$P = 2T_{cr}$$

13.2.4



a) F.B.D



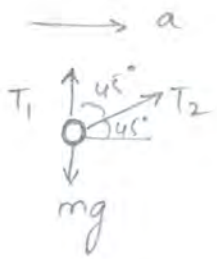
b) $\vec{a} = \ddot{x}\,\hat{i}$

linear momentum Balance

$$m\vec{a} = \vec{F} = -Kx\,\hat{i}$$

$$m\ddot{x} = -Kx$$

13.2.5



By linear momentum balance

$$ma\hat{i} = -mg\hat{j} + T_1\hat{j} + T_2\cos\theta\,\hat{i} + T_2\sin\theta\,\hat{j}$$

$$ma = T_2\cos\theta$$

$$0 = -mg + T_1 + T_2\sin\theta$$

$$\Rightarrow \quad T_1 = mg - T_2\sin\theta$$

$$T_1 = mg - \frac{ma}{\cos\theta}\cdot\sin\theta$$

$$T_1 = m(g-a)$$

$$T_1 = 2(10-3)\ N$$

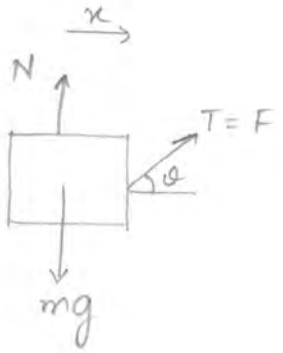
$$T_1 = 14\ N$$

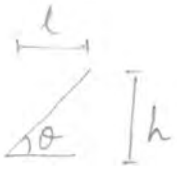
$\theta = 45°$

$\sin 45 = \cos 45$

$m = 2\ kg.$

13.2.6





Rate of work done on mass

$$W = \vec{F} \cdot \vec{v}$$

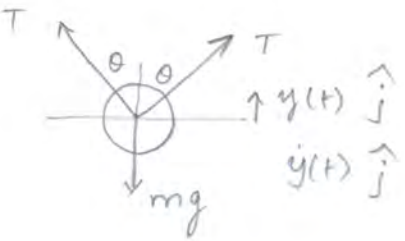
$$= TV \cos\theta$$

$$= 50 \times 3 \times \cos\theta$$

$$W = \frac{150\, l}{\sqrt{l^2 + h^2}}$$

13.2.7

FBD



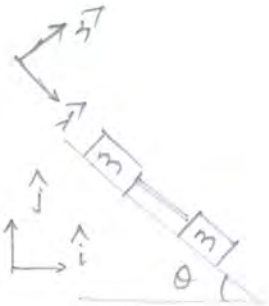
Power input to the mass

$$P = \Sigma(\vec{F} \cdot \vec{v})$$

$$= T\dot{y}\cos\theta + T\dot{y}\cos\theta$$

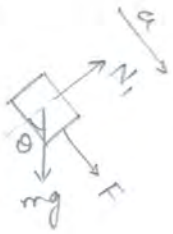
$$P = 2T\dot{y}\cos\theta$$

13.2.8

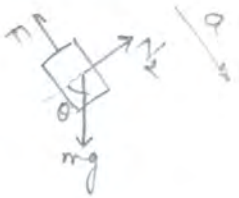


FBD

Block 1



Block 2



Rod

Blocks and rod

b) Linear momentum Balance

i) $ma_1 = F + mg\sin\theta$

ii) $F = F$

iii) $ma_2 = mg\sin\theta - F$

$a_1 = a_2 = a$

solving we get

$F = 0$

$a = g\sin\theta$

$\vec{a} = g\sin\theta\,\hat{\lambda}$

c) $2ma_{cm} = 2mg\sin\theta$

$\vec{a}_{cm} = g\sin\theta\,\hat{\lambda}$

d) Force on the rod

$F = 0$

e) $V_{cm} = g \sin\theta \cdot t$

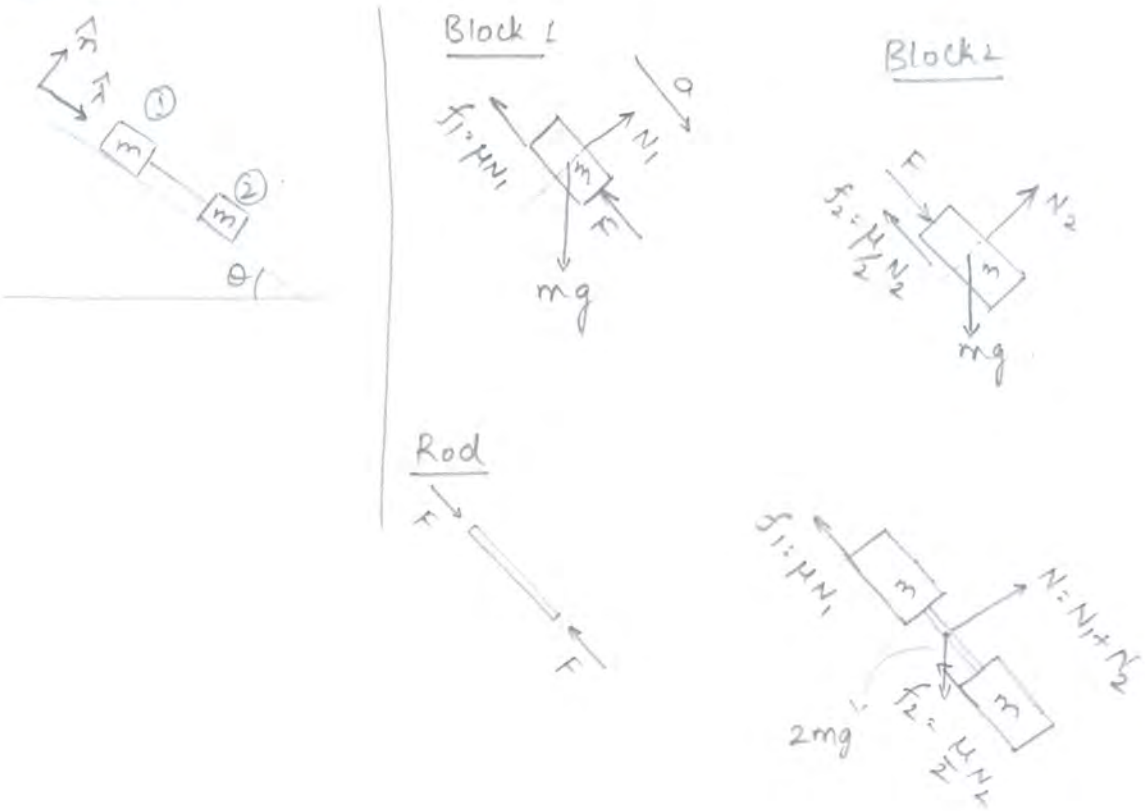
where

$$t = \sqrt{\frac{2d}{g \sin\theta}}$$

$$V_{cm} = \sqrt{2dg \sin\theta}$$

$$V_{cm} = \sqrt{2dg \sin\theta}$$

13.2.9



Equations

$m\vec{g}$ can be written in $\hat{\lambda}, \hat{n}$ co-ordinates as

$$m\vec{g} = mg \sin\theta \, \hat{\lambda} - mg \cos\theta \, \hat{n}$$

Linear Momentum balance

1) $$ma\hat{\lambda} = mg \sin\theta \, \hat{\lambda} - mg \cos\theta \, \hat{n} + N_1 \hat{n} - \mu N_1 \hat{\lambda} - F\hat{\lambda}$$

$$N_1 = mg \cos\theta$$

$$ma = mg \sin\theta - \mu mg \cos\theta - F \quad \text{——} \quad (i)$$

2) $$ma\hat{\lambda} = mg \sin\theta \, \hat{\lambda} - mg \cos\theta \, \hat{n} + N_2 \hat{n} - \frac{\mu}{2} N_2 \hat{\lambda} + F\hat{\lambda}$$

$$N_2 = mg \cos\theta$$

$$ma = mg \sin\theta - \frac{\mu}{2} mg \cos\theta + F \quad \text{——} \quad (ii)$$

c) Momentum balance for centre of mass

$$2ma_{cm} = 2mg\sin\theta - f_1 - f_2$$

$$a_{cm} = \frac{2mg\sin\theta - \mu mg\cos\theta - \frac{\mu}{2}mg\cos\theta}{2m}$$

$$a_{cm} = a = g\left[\frac{2\sin\theta - (3/2)\cos\theta}{2}\right]$$

d) Force in rod eqn (ii) - eqn (i) gives

$$0 = -\frac{\mu}{2}mg\cos\theta + \mu mg\cos\theta + 2F$$

$$F = -\frac{\mu}{4}mg\cos\theta$$

Force in the rod is tensile, $\frac{\mu}{4}mg\cos\theta$

e) $V_{cm} = \sqrt{2a_{cm}d}$

$$V_{cm} = \sqrt{dg(2\sin\theta - \frac{3}{2}\mu\cos\theta}$$

f) If slippery mass was on top

$$f_2 = \mu mg\cos\theta, \quad f_1 = \frac{\mu}{2}mg\cos\theta$$

$$a_{cm} = \frac{2mg\sin\theta - f_1 - f_2}{2m}$$

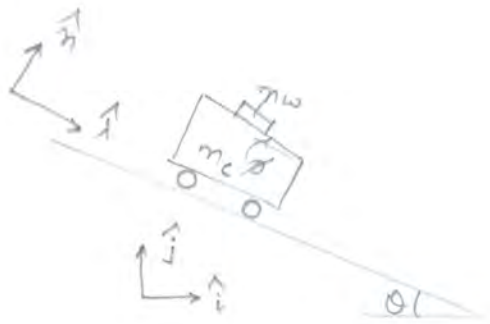
$$a_{cm} = g\left[\frac{2\sin\theta - (3/2)\mu\cos\theta}{2}\right]$$

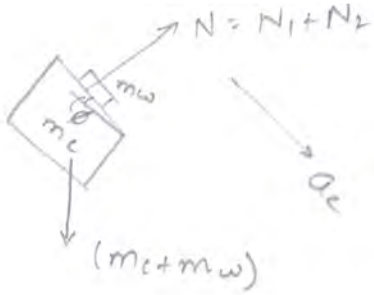
Now; $0 = -\mu mg\cos\theta + \frac{\mu}{2}mg\cos\theta + 2F$

$$F = \frac{\mu}{4}mg\cos\theta$$

Force in the rod is compressive instead of tensile.

13-2-10





a) $\hat{n} = \sin\theta\,\hat{i} + \cos\,\hat{j}$

$\hat{\lambda} = \cos\theta\,\hat{i} - \sin\theta\,\hat{j}$

$$(m_c + m_w)\,\vec{a}_c = -g(m_c + m_w)\,\hat{j} + N\hat{n}$$

now, $\vec{a}_c = a_c\hat{\lambda}$

$$\Rightarrow (m_c + m_w)\,a_c\,\hat{\lambda} = -g(m_c + m_w)\,\hat{j} + N\hat{n}$$

dot product both sides with $\hat{\lambda}$ we get

$$(m_c + m_w)\,a_c = -g(m_c + m_w)\,\hat{j}\cdot(\cos\theta\,\hat{i} - \sin\theta\,\hat{j}) + 0$$

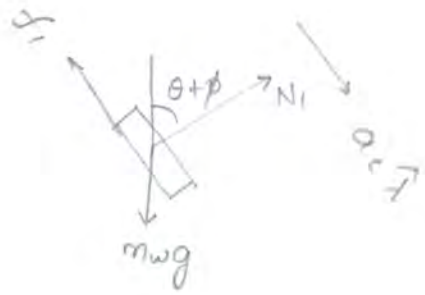
$$(m_c + m_w)\,a_c = -g(m_c + m_w)(-\sin\theta)$$

$$a_c = g\sin\theta$$

$$\vec{a}_c = a_c\hat{\lambda} = g\sin\theta\,[\cos\theta\,\hat{i} - \sin\theta\,\hat{j}]$$

$$\vec{a}_c = g\sin\theta\cos\theta\,\hat{i} - g\sin^2\theta\,\hat{j}$$

b) FBD of mw



$$\vec{N}_1 = N_1 [\sin(\theta+\phi)\,\hat{i} + \cos(\theta+\phi)\,\hat{j}]$$

$$\vec{f}_1 = \mu N_1 [-\cos(\theta+\phi)\,\hat{i} + \sin(\theta+\phi)\,\hat{j}]$$

$$\vec{w} = -m_w g\,\hat{j}$$

$$\vec{a}_w = \vec{a}_c = g\sin\theta\,[\cos\theta\,\hat{i} - \sin\theta\,\hat{j}]$$

By linear momentum balance

$$m_w g \sin\theta [\cos\theta\,\hat{i} - \sin\theta\,\hat{j}] = N_1[\sin(\theta+\phi) - \mu\cos(\theta+\phi)]\hat{i} + N_1[\cos(\theta+\phi) + \mu\sin(\theta+\phi)] - m_w g\,\hat{j}$$

we get 2 eqns by doing dot product with $\hat{i}$ and $\hat{j}$

$$\Rightarrow m_w g \sin\theta.\cos\theta = N_1[\sin(\theta+\phi) - \mu\cos(\theta+\phi)]$$

$$\Rightarrow N_1 = \frac{m_w g \sin\theta.\cos\theta}{\sin(\theta+\phi) - \mu\cos(\theta+\phi)}$$

$$-m_w g \sin^2\theta = -m_w g + \frac{m_w g \sin\theta\cos\theta}{\sin(\theta+\phi) - \mu\cos(\theta+\phi)}\left[\cos(\theta+\phi) + \mu\sin(\theta+\phi)\right]$$

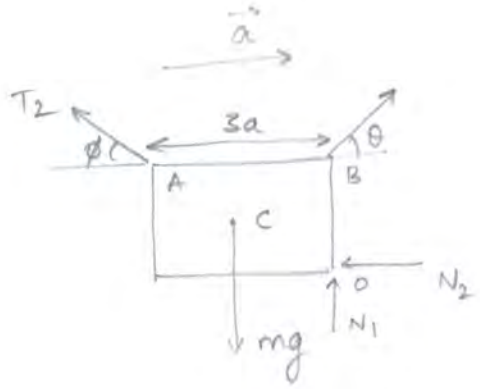
$$m_w g \cos^2\theta = m_w g \sin\theta\cos\theta \left[\frac{\cos(\theta+\phi) + \mu\sin(\theta+\phi)}{\sin(\theta+\phi) - \mu\cos(\theta+\phi)}\right]$$

$$\Rightarrow \tan\theta = \frac{\tan(\theta+\phi) - \mu}{1 + \mu\tan(\theta+\phi)}, \quad \text{Put } \mu = \tan\alpha$$

$$\Rightarrow \tan\theta = \tan(\theta+\phi-\alpha) \Rightarrow \alpha = \phi$$

$$\mu = \tan^{-1}(\phi)$$

13.2.12



b) Linear momentum balance equation

$$m a_{cm}\hat{i} = -mg\hat{j} + T_1(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) + T_2(-\cos\phi\,\hat{i} + \sin\phi\,\hat{j}) + N_1\hat{j} - N_2\hat{i}$$

by dot product with $\hat{i}$ and $\hat{j}$ we get.

$$0 = -mg + T_1\sin\theta + T_2\sin\phi + N_1 \quad \text{——} \quad (i)$$

$$m a_{cm} = T_1\cos\theta - T_2\cos\phi - N_2 \quad \text{——} \quad (ii)$$

c) Angular momentum balance about 'O'.

$$\vec{r}_{c/o} \times m a_{cm}\hat{i} = -\frac{3a}{2}\hat{i} \times -mg\hat{j} + 2a\hat{j} \times T_1(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$(-\frac{3}{2}a\hat{i} + a\hat{j})$

$$+(-3a\hat{i} + 2a\hat{j})(-T_2\cos\phi\,\hat{i} + T_2\sin\phi\,\hat{j})$$

by dot product with $\hat{k}$ on both sides we get

$$-m a_{cm} = \frac{3}{2}mg - T_1 2\cos\theta - 3T_2\sin\phi + 2T_2\cos\phi$$

$$m a_{cm} = 2T_1\cos\theta - \frac{3}{2}mg + 2T_2\cos\phi - 3T_2\sin\phi$$

$$\longrightarrow (iii)$$

Eqn (i) (ii) & (iii) have four variables

$\tan\theta = 2$, $\tan\phi = \frac{2}{3}$ => $2T_2\cos\phi = 3T_2\sin\phi$

=> eqn (iii) => $2T_1\cos\theta = ma_{cm} + \frac{3}{2}mg$

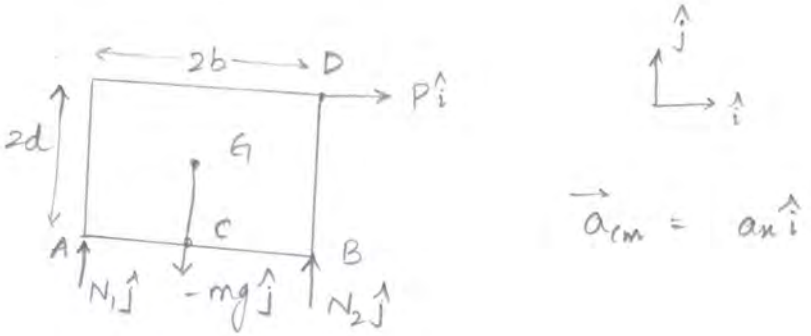
$$T_1 = \frac{m\left[a_{cm} + \frac{3}{2}g\right]}{2\cos\theta}$$

$$\cos\theta = \frac{1}{\sqrt{5}}$$

therefore

$$T_1 = \frac{\sqrt{5}\,m}{2}\left[a_{cm} + \frac{3}{2}g\right]$$

13.2.16



$$\vec{a}_{cm} = a_x\hat{i}$$

$$\sum \vec{F} = m\vec{a}_{cm}$$

$$P\hat{i} + (N_1 + N_2)\hat{j} - mg\hat{j} = m\vec{a}_{cm} = ma_x\hat{i}$$

$$a_x = \frac{P}{m} \Rightarrow \vec{a}_{cm} = \frac{P}{m}\hat{i}$$

$$N_1 + N_2 = mg \quad \text{—— (i)}$$

Moment balance about point C

$$(N_2 b - N_1 b)\hat{k} - 2Pd\hat{k} = 0$$

$$N_2 - N_1 = \frac{2Pd}{b} \quad \text{—— (ii)}$$

Solving (i) and (ii) we get

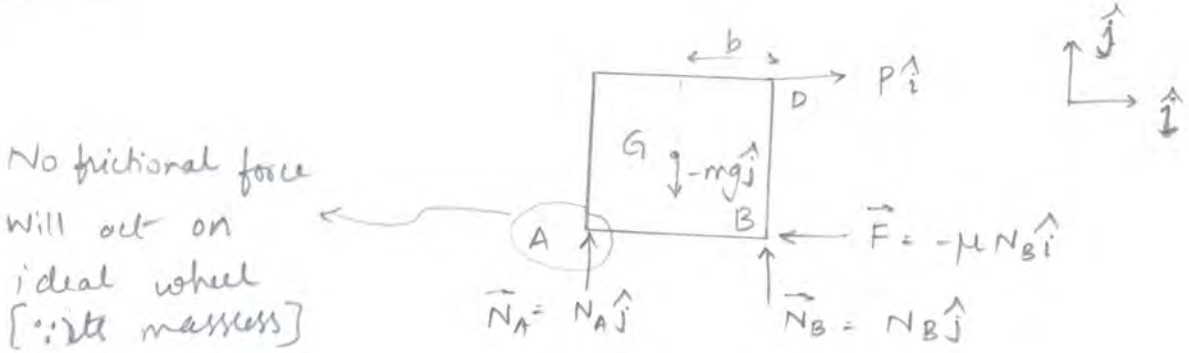
$$N_2 = \frac{mg}{2} + \frac{Pd}{b}$$

$$N_1 = \frac{mg}{2} - \frac{Pd}{b}$$

$$\vec{N}_A = \left(\frac{mg}{2} - \frac{Pd}{b}\right)\hat{k}$$

$$\vec{N}_B = \left(\frac{mg}{2} + \frac{Pd}{b}\right)\hat{k}$$

13·2·17



Force balance

$\vec{a}_{cm} = a_x \hat{i}$

$$\sum \vec{F} = m\vec{a}_{cm}$$

$$P\hat{i} - \mu N_B \hat{i} + (N_A + N_B - mg)\hat{j} = m a_x \hat{i}$$

$$a_x = \frac{P - \mu N_B}{m} \quad \text{——— (i)}$$

$$N_A + N_B = mg \quad \text{——— (ii)}$$

moment balance about point 'c' gives.

$$N_B b \hat{k} - N_A b \hat{k} - P(2d)\hat{k} = 0 \quad [\text{ block is not rotating }]$$

$$\Rightarrow \quad N_B - N_A = \frac{2Pd}{b} \quad \text{——— (iii)}$$

solving (ii) and (iii) for N_A and N_B gives

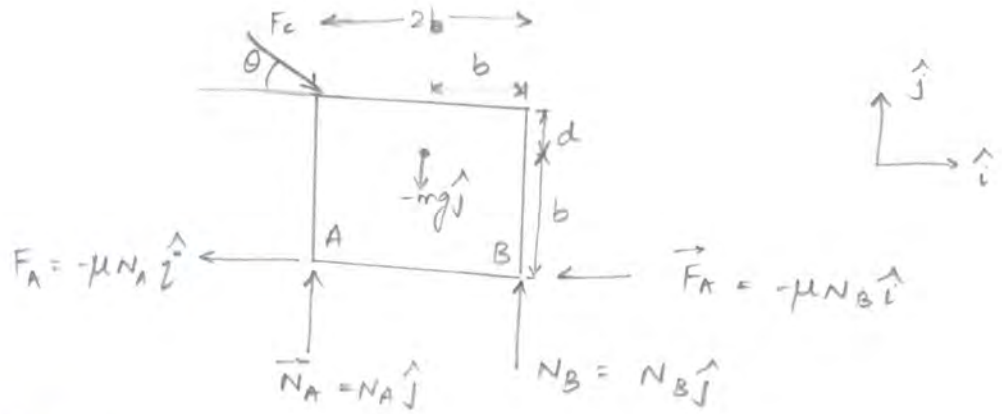
$$N_B = \frac{mg}{2} + \frac{Pd}{2} \quad ; \quad N_A = \frac{mg}{2} - \frac{Pd}{b}$$

$$a_x = \frac{P}{m}\left(1 - \frac{\mu d}{b}\right) - \frac{\mu g}{2}$$

$$\vec{N}_A = \left(\frac{mg}{2} - \frac{Pd}{b}\right)\hat{j} \quad ; \quad \vec{N}_B = \left(\frac{mg}{2} + \frac{Pd}{b}\right)\hat{j}$$

$$\vec{a}_{cm} = \left[\frac{P}{m}\left(1 - \frac{\mu d}{b}\right) - \frac{\mu g}{2}\right]\hat{i}$$

13.2.18



Force balance

$$\sum \vec{F} = m\vec{a}_{cm}$$

$$F_c \cos\theta \hat{i} - F_c \sin\theta \hat{j} - \mu(N_A + N_B)\hat{i} + (N_A + N_B - W)\hat{j} = ma_x \hat{i}$$

$$a_x = \frac{F_c \cos\theta - \mu(N_A + N_B)}{m} \quad \text{—— (i)}$$

$$N_A + N_B = W + F_c \sin\theta \quad \text{—— (ii)}$$

Moment balance about point A gives

$$-F_c \cos\theta\,(b+d)\,\hat{k} + N_B(2b)\,\hat{k} - W(b)\,\hat{k} = 0$$

$$N_B = \frac{Wb + F\cos\theta\,(b+d)}{2b}$$

$$N_B = \frac{W}{2} + \frac{F_c \cos\theta\,(b+d)}{2b} \quad \text{—— (iii)}$$

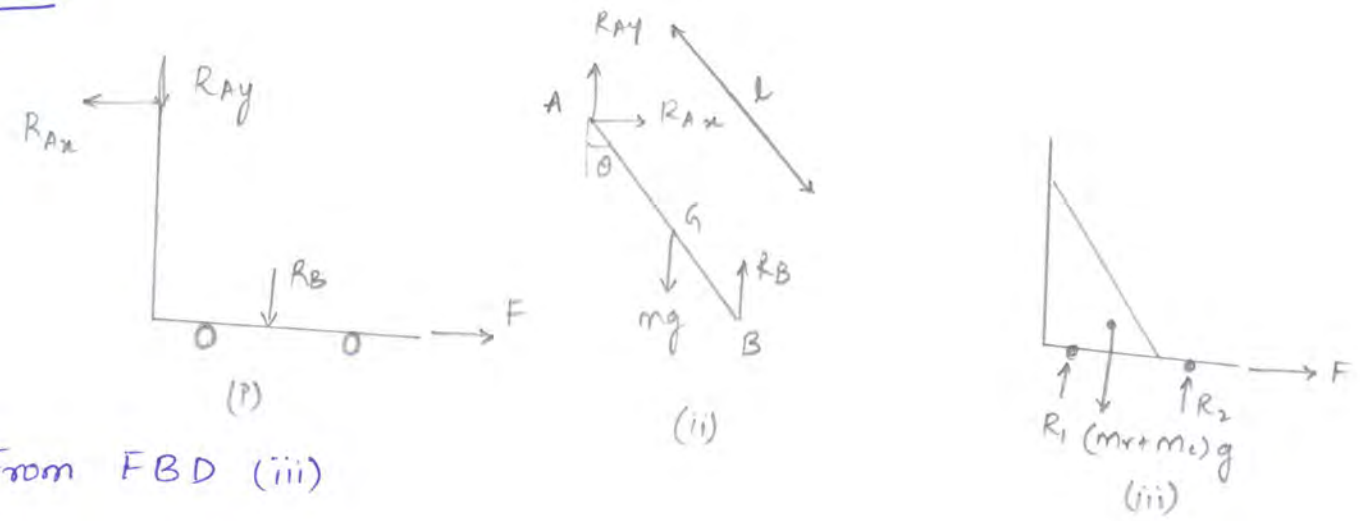
Solving (ii) and (iii) we get

$$N_A = \frac{W}{2} + F_c\left[\sin\theta - \left(\frac{b+d}{2b}\right)\cos\theta\right]$$

$$a_x = \frac{F_c \cos\theta - \mu\left[W + F_c \sin\theta\right]}{m}$$

$$a_x = \frac{F_c}{m}(\cos\theta - \mu\sin\theta) - \frac{\mu W}{m}$$

13.2.19



From FBD (iii)

$$F\hat{i} + (R_1+R_2)\hat{j} - (m_r+m_c)g\hat{j} = (m_r+m_c)\,a_x\hat{i}$$

$$a_x = \frac{F}{m_r+m_c} \quad \text{——} \quad (i)$$

From FBD (ii)

$$R_{Ay}\hat{j} + R_{Ax}\hat{i} + R_B\hat{j} - m_r g\hat{j} = m_r a_x \hat{i}$$

$$R_{Ax} = \frac{m_r F}{m_r+m_c}$$

Moment balance about point A gives $\vec{M}_{/A} = \dot{\vec{H}}_{/A} = \vec{r}_{G/A} \times m_r \vec{a}_{cm}$

$$-m_r g \frac{l}{2}\sin\theta\,\hat{k} + R_B l \sin\theta\,\hat{k} = \left(-\frac{l}{2}\cos\theta\,\hat{j} + \frac{l}{2}\sin\theta\,\hat{i}\right) \times \left(m_r a_x \hat{i}\right)$$

$$= \frac{m_r a_x l}{2}\cos\theta\,\hat{k}$$

$$R_B = \left[\frac{m_r a_x l\cos\theta}{2} + \frac{m_r g}{2} l\sin\theta\right] \cdot \frac{1}{l\sin\theta}$$

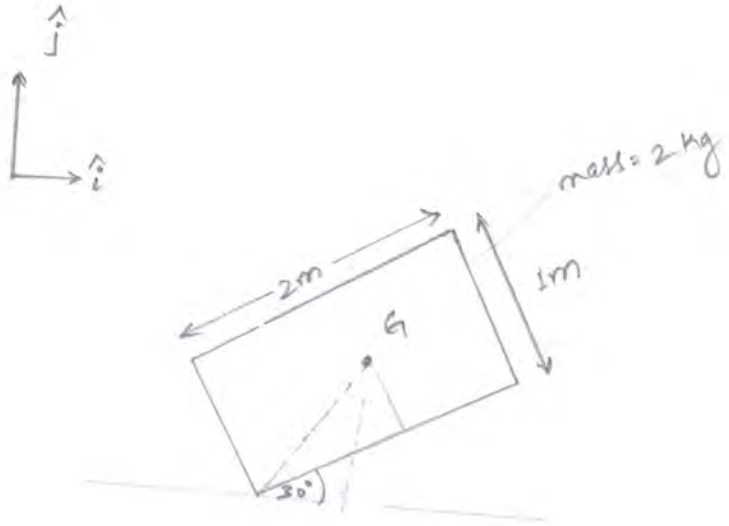
$$R_B = \frac{m_r g}{2} + \frac{m_r}{2\tan\theta} \cdot \frac{F}{(m_r+m_c)}$$

$$\vec{R}_B = \left[\frac{m_r g}{2} + \frac{F}{2\tan\theta}\left(\frac{m_r}{m_r+m_c}\right)\right]\hat{j}$$

$$R_{Ay} = m_r g - R_B = \frac{m_r g}{2} - \frac{F}{2\tan\theta}\left(\frac{m_r}{m_r+m_c}\right)$$

$$\vec{R}_A = \left(\frac{F m_r}{m_r+m_c}\right)\hat{i} + \left[\frac{m_r g}{2} - \frac{F}{2\tan\theta}\left(\frac{m_r}{m_r+m_c}\right)\right]\hat{j}$$

13.2.20



$\vec{a} = 0.5\ m/s^2\ \hat{i}$

$\vec{v} = 0.8\ m/s\ \hat{i}$

a) Linear momentum

$$\vec{p} = m\vec{v}_{cm} = 2 \times 0.8 = 1.6\ kg\ m/sec$$

b) Rate of change of linear momentum

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}_{cm}}{dt} = m\vec{a}_{cm}$$

$$\dot{\vec{p}} = 2 \times 0.5 = 1\ kg\ m/sec$$

c) Angular momentum about point 'O'.

$$\vec{H}_{/o} = \vec{r}_{cm/o} \times m\vec{v}_{cm} \qquad [\ \vec{\omega}\ \text{is zero}\ ]$$

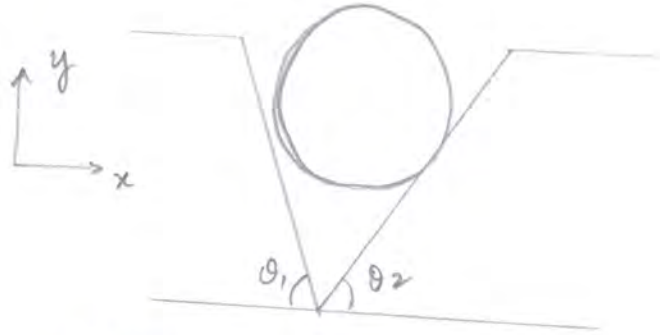
$$= -OG \sin(30° + \tan^{-1} 0.5)\, m\, |\vec{v}_{cm}|\, \hat{k}$$

$$= -\frac{\sqrt{5}}{2} \times 0.8345 \times 2 \times 0.8\, \hat{k} = -1.49282\, \hat{k}\ kg\ m^2/sec$$

d) $$\dot{\vec{H}}_{/o} = \vec{r}_{cm/o} \times m\vec{a}_{cm}$$

$$= -\frac{\sqrt{5}}{2} \times 0.8345 \times 0.5 \times 2\, \hat{k} = -0.933\, \hat{k}\ kg\ m^2/sec^2$$

13.2.21



FBD

Force balance

$$N_1 \cos\theta_1 \hat{j} + N_2 \sin\theta_1 \hat{i} + N_2 \cos\theta_2 \hat{j} - N_2 \sin\theta_2 \hat{i} - mg\,\hat{j} = ma\hat{j}$$

$$N_1 \sin\theta_1 = N_2 \sin\theta_2$$

$$N_1 \cos\theta_1 + N_2 \cos\theta_2 = mg + ma$$

$$N_1 = \frac{m(g+a)}{\cos\theta_1 + \cos\theta_2 \left(\frac{\sin\theta_1}{\sin\theta_2}\right)}$$

$$N_1 = \frac{m(g+a)\sin\theta_2}{\cos\theta_1 \sin\theta_2 + \cos\theta_2 \sin\theta_1} = \frac{m(g+a)\sin\theta_2}{\sin(\theta_1+\theta_2)}$$

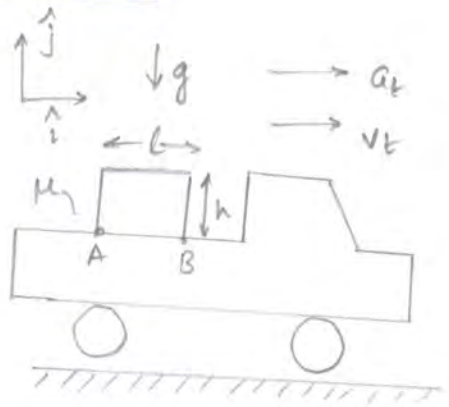
$$N_2 = \frac{m(g+a)\sin\theta_1}{\sin(\theta_1+\theta_2)}$$

So, forces on disk due to groove are

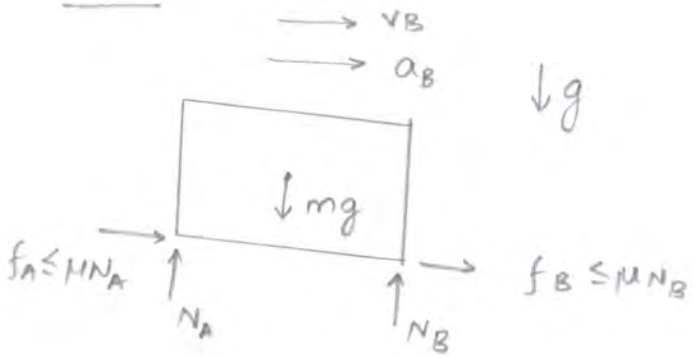
$$\vec{N_1} = \frac{m(g+a)\sin\theta_2}{\sin(\theta_1+\theta_2)} \left[\sin\theta_1 \hat{i} + \cos\theta_1 \hat{j}\right]$$

$$\vec{N_2} = \frac{m(g+a)\sin\theta_1}{\sin(\theta_1+\theta_2)} \left[-\sin\theta_2 \hat{i} + \cos\theta_2 \hat{j}\right]$$

13.2.22



FBD



a) Total force applied by truck on the box

$$\vec{F} = (N_A + N_B)\,\hat{j} + (f_A + f_B)\,\hat{i}$$

From FBD

$$-mg\,\hat{j} + (N_A + N_B)\,\hat{j} + (f_A + f_B)\,\hat{i} = m a_t\,\hat{i}$$

$$N_A + N_B = mg$$

$$f_A + f_B = m a_t$$

$$\vec{F} = mg\,\hat{j} + m a_t\,\hat{i}$$

b) Moment balance about point 'A' gives

$$\vec{M}_{/A} = \dot{\vec{H}}_{/A}$$

$$\frac{l}{2}\hat{i} \times mg(-\hat{j}) + l\hat{i} \times N_B\hat{j} = \left(\frac{l}{2}\hat{i} + \frac{h}{2}\hat{j}\right) \times m a_t\,\hat{i}$$

$$-\frac{mgl}{2} + N_B l = -m a_t \frac{h}{2}$$

$$N_B = \frac{m}{l}\left(\frac{gl}{2} - \frac{a_t h}{2}\right) \Rightarrow N_B = \frac{m}{2}\left(g - \frac{a_t h}{l}\right)$$

$$N_A = mg - N_B$$

$$= \frac{m}{2}\left[g + \frac{a_t h}{l}\right]$$

Now we need to find f_A and f_B

$$f_A + f_B = ma_t$$

we assume that

$$f_A \propto N_A$$

and

$$f_B \propto N_B$$

$$f_A\left[1 + \frac{g - \frac{a_t h}{l}}{g + \frac{a_t h}{l}}\right] = ma_t$$

$$f_A = (ma_t)\left[\frac{lg + a_t h}{2gl}\right]$$

$$f_B = ma_t\left[\frac{gl - a_t h}{2gl}\right]$$

Reaction at A

$$\vec{N}_A = \frac{m}{2}\left[g + \frac{a_t h}{l}\right]\hat{j} + \frac{ma_t}{2gl}\left[gl + a_t h\right]\hat{i}$$

Reaction at B

$$\vec{N}_B = \frac{m}{2}\left[g - \frac{a_t h}{l}\right]\hat{j} + \frac{ma_t}{2gl}\left[gl - a_t h\right]\hat{i}$$

c) Total force exerted by the truck on the box

$$N_A + N_B = mg$$

$$f_A + f_B = \mu[N_A + N_B] = \mu mg$$

$$\vec{F}_{net} = mg\,\hat{j} + \mu mg\,\hat{i} = mg\,[\hat{j} + \mu\hat{i}]$$

d) If the box slides

$$a_B \neq a_t$$

$$f_A = \mu N_A \;,\; f_B = \mu N_B$$

Force balance

$$-mg\,\hat{j} + N_A\hat{j} + \mu N_A\hat{i} + N_B\hat{j} + \mu N_B\hat{i} = m a_B\hat{i}$$

$$N_A + N_B = mg$$

$$a_B = \frac{\mu[N_A + N_B]}{m} = \mu g$$

Moment balance about 'A' gives

$$-\frac{mgl}{2} + N_B l = -\frac{m a_B h}{2}$$

$$\Rightarrow N_B = \frac{m}{2}\left[g - \frac{a_B h}{l}\right] = \frac{mg}{2}\left[1 - \frac{\mu h}{l}\right]$$

$$N_A = \frac{mg}{2}\left[1 + \frac{\mu h}{l}\right]$$

Reaction forces

$$\vec{N}_A = \frac{mg}{2}\left[1 + \frac{\mu h}{l}\right]\left[\hat{j} + \mu\hat{i}\right]$$

$$\vec{N}_B = \frac{mg}{2}\left[1 - \frac{\mu h}{l}\right]\left[\hat{j} + \mu\hat{i}\right]$$

e) At critical acceleration $N_B = 0$

using N_B we get

$$\frac{m}{2}\left[g - \frac{a_t h}{l}\right] = 0$$

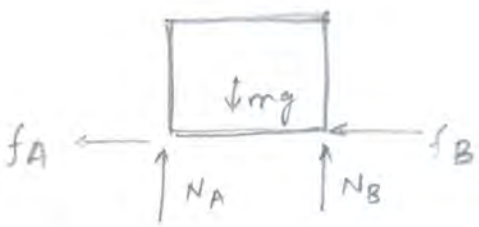
$$a_t = \frac{gl}{h}$$

f) Maximum acceleration for block to not slide;

$$a_{t\,max} = a_{s\,sliding}$$

$$a_{t\,max} = \mu g$$

g) If the truck stops suddenly; the block will start deaccelerati



Moment balance about point B gives

$$-N_A l + \frac{mgl}{2} = m a_B \frac{h}{2}$$

$$N_A = \frac{m}{2}\left[g - \frac{a_B h}{l}\right]$$

$$N_A = \frac{m}{2}\left[g - \frac{\mu g h}{l}\right]$$

$$N_A = \frac{mg}{2}\left[1 - \frac{\mu h}{l}\right]$$

For block to tip over $N_s = 0$

$$\frac{\mu h}{l} \geq 1$$

$$h \geq \frac{l}{\mu}$$

If the block does not tip over

$$\vec{a}_B = -\mu g \hat{i} \qquad \vec{V}_{Bi} = v_t \hat{i}$$

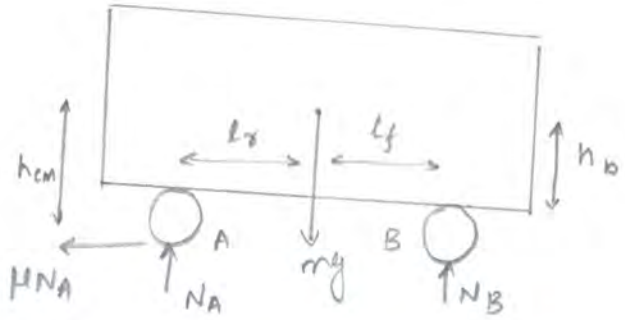
distance covered by the block before stopping is

$$S = \frac{v_f^2 - v_i^2}{2a_B}$$

$$S = \frac{v_t^2}{2\mu g}$$

13.2.24

FBD of car



$$-m a_c \hat{i} = -k[l_0 - l_s]\hat{i} - \mu N_A \hat{i} + (N_A + N_B)\hat{j} - mg\hat{j}$$

$$a_c = \frac{\mu N_A + k[l_0 - l_s]}{m}$$

$$N_A + N_B = mg$$

Moment balance about point B gives

$$mg l_f \hat{k} - N_A(l_r + l_f)\hat{k} + h_b k[l_0 - l_s]\hat{k} = m h_{cm} a_c \hat{k}$$

$$N_A = \frac{-m h_{cm} a_c + mg l_f + h_b k[l_0 - l_s]}{l_r + l_f}$$

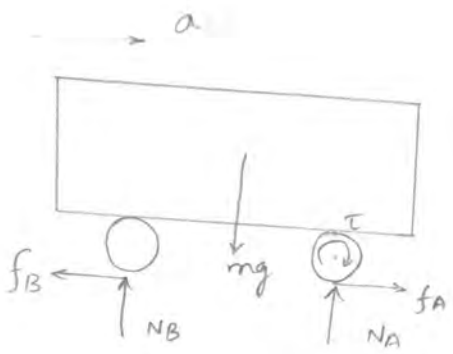
$$m a_c = \mu\left[\frac{-m h_{cm} a_c + mg l_f + h_b k(l_0 - l_s)}{(l_r + l_f)}\right] + k[l_0 - l_s]$$

$$m\left(1 + \frac{\mu h_{cm}}{l_r + l_f}\right) a_c = k(l_0 - l_s)\left[1 + \frac{\mu h_b}{l_r + l_f}\right] + \frac{\mu mg l_f}{l_r + l_f}$$

$$a_c = \frac{k(l_0 - l_s)(l_r + l_f + \mu h_b) + \mu mg l_f}{l_r + l_f + \mu h_{cm}}$$

13.2.26

FBD



a) $f_B = 0$ [Wheel is massless]

$f_{A\,max} = \mu N_A$

Force balance

$$m a = f_A = \mu N_A$$

$$N_A + N_B = mg$$

moment balance about A gives

$$-mgl_r + N_A(l_r + l_f) = -mah$$

$$N_A = \frac{m(gl_r - ah)}{l_r + l_f}$$

$$a_{max} = \frac{\mu N_A}{m} = \frac{\mu(gl_r - a_{max}h)}{l_r + l_f}$$

$$a_{max} = \frac{\mu g l_r}{l_r + l_f + \mu h}$$

b) $N_B = mg - N_A = mg - \dfrac{m(g\,l_r - ah)}{l_r + l_f}$

$$N_B = \frac{m(g l_f + ah)}{(l_r + l_f)} = \frac{m\left(g l_f + \dfrac{\mu g l_r h}{l_r + l_f + \mu h}\right)}{l_r + l_f}$$

$$N_B = \frac{mg}{l_r + l_f}\left[l_f + \frac{\mu l_r h}{l_r + l_f + \mu h}\right]$$

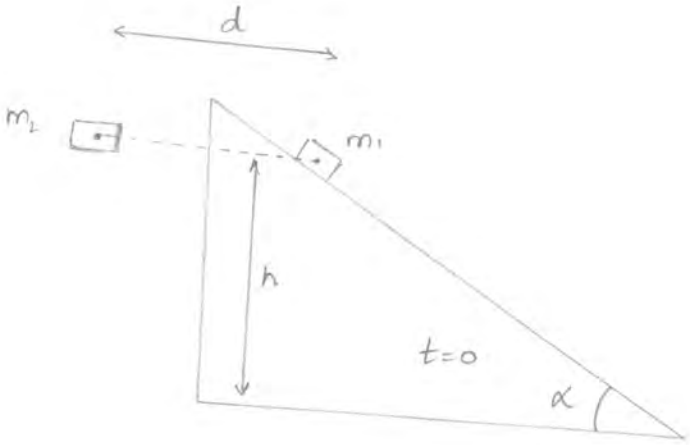
$$N_A = \frac{m(g l_f - ah)}{l_r + l_f}$$

$$N_A = \frac{mg}{l_r + l_f}\left[l_f - \frac{\mu l_r h}{l_r + l_f + \mu h}\right]$$

13.228

FBD

(i)



Force balance gives

$$a_2 = -g\hat{j} \quad , \quad a_1 = g\sin\alpha\,[\hat{i}\cos\alpha - \sin\alpha\,\hat{j}]$$

After t seconds

$$\vec{V}_2 = -gt\,\hat{j} \;;\; \vec{V}_1 = gt\sin\alpha\,[\cos\alpha\,\hat{i} - \sin\alpha\,\hat{j}]$$

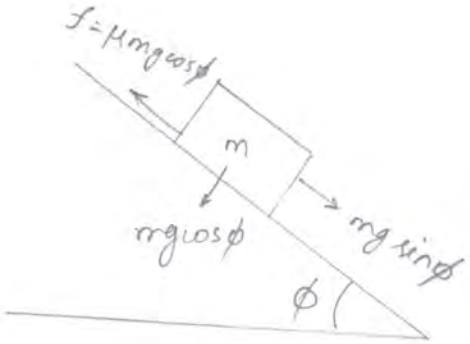
$$\vec{V}_{1/2} = \vec{V}_{1/G} - \vec{V}_{2/G}$$

$$= -gt\,[-\hat{j} + \sin^2\alpha\,\hat{j} - \sin\alpha\cos\alpha\,\hat{i}]$$

$$= -gt\,[-\cos^2\alpha\,\hat{j} - \sin\alpha\cos\alpha\,\hat{i}]$$

$$\vec{V}_{1/2} = gt\cos\alpha\,[\sin\alpha\,\hat{i} + \cos\alpha\,\hat{j}]$$

13.2.30



While going up

$$ma = -mg\sin\phi - \mu mg\cos\phi$$

$$a_1 = -g[\sin\phi + \mu\cos\phi]$$

while sliding down acceleration is

$$ma_2 = mg\sin\phi - \mu mg\cos\phi$$

$$a_2 = g[\sin\phi - \mu\cos\phi]$$

If the initial velocity is V.

time taken to reach top is

$$t_{up} = \frac{-V}{a_{up}} = \frac{V}{g(\sin\phi + \mu\cos\phi)}$$

distance covered

$$d = \frac{V^2}{2g(\sin\phi + \mu\cos\phi)}$$

Time taken to slide down

$$\frac{1}{2} a_{down} t_{down}^2 = d$$

$$t^2_{down} = \frac{2d}{a} = \frac{2v^2}{g(\sin\phi - \mu\cos\phi)} \times \frac{1}{2g(\sin\phi + \mu\cos\phi)}$$

$$t_d = \frac{v}{g} \frac{1}{\sqrt{\sin^2\phi - \mu^2\cos^2\phi}}$$

$$t_d = 2t_u$$

$$\frac{1}{\sqrt{\sin^2\phi - \mu^2\cos^2\phi}} = \frac{2}{\sin\phi + \mu\cos\phi}$$

$$\Rightarrow \quad \sin^2\phi + \mu^2\cos^2\phi + 2\mu\sin\phi\cos\phi = 2\sin^2\phi - 2\mu^2\cos^2\phi$$

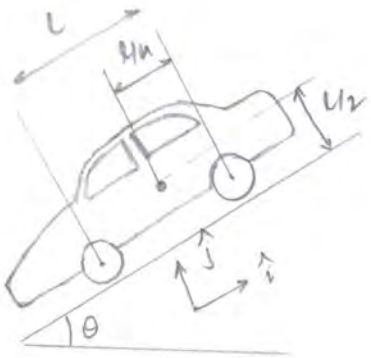
$$3\mu^2\cos^2\phi + 2\mu\cos\phi(\sin\phi) - \sin^2\phi = 0$$

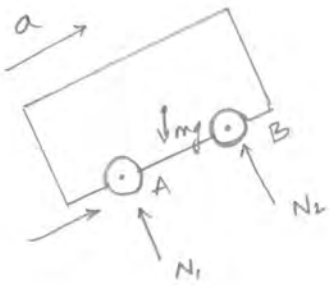
This is a quadratic equation in μ

$$\mu = \frac{-2\cos\phi\sin\phi \pm 4\sin\phi\cos\phi}{6\cos^2\phi}$$

$$\mu = \frac{\tan\phi}{3}$$

13.2.31





Assuming breaking acceleration $= a\hat{i}$

$$f_1 = \mu N_1$$

Force balance

$$-mg\cos\theta\,\hat{j} - mg\sin\theta\,\hat{i} + \mu N_1\hat{i} + (N_1+N_2)\hat{j} = ma\hat{i}$$

$$ma = -(mg\sin\theta - \mu N_1) \quad \text{——— (i)} \quad ; \quad N_1 + N_2 = mg\cos\theta \;;$$

Moment Balance about point B gives

$$-N_1 l\,\hat{k} + (mg\cos\theta)\left(\frac{l}{4}\right)\hat{k} + mg\sin\theta\left(\frac{l}{2}\right)\hat{k} = \left(\frac{l}{2}\hat{j} - \frac{l}{4}\hat{i}\right)\times ma\hat{i}$$

$$\left[-N_1 l + \frac{mgl}{4}\cos\theta + \frac{mgl}{2}\sin\theta\right]\hat{k} = -\frac{mal}{2}\hat{k}$$

$$N_1 = \frac{mg}{4}(2\sin\theta + \cos\theta) + \frac{mal}{2}$$

$$N_1 = \frac{mg}{4}(2\sin\theta + \cos\theta) + \frac{ma}{2}$$

Substituting in (i) we get, for $\mu = 1$

$$\frac{a}{2} = -g\sin\theta + \left[\frac{g}{4}(2\sin\theta + \cos\theta)\right] = -\frac{g}{2}\sin\theta + \frac{g}{4}\cos\theta$$

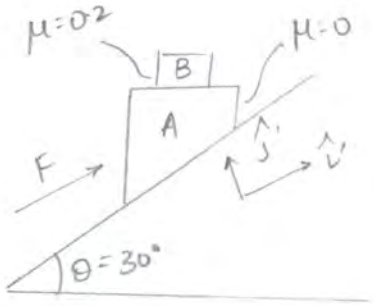
$$a = -g\sin\theta + \frac{g}{2}\cos\theta$$

$$a = g\left[\frac{\cos\theta}{2} - \sin\theta\right]\hat{i}$$

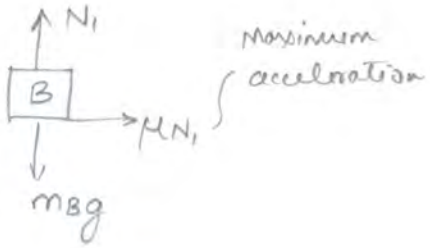
13.2.32

$\hat{i}' = \hat{i}\cos\theta + \hat{j}\sin\theta$

$\hat{j}' = -\hat{i}\sin\theta + \hat{j}\cos\theta$



FBD(I)



Say acceleration of blocks is $a\hat{i}'$

then, Force balance gives

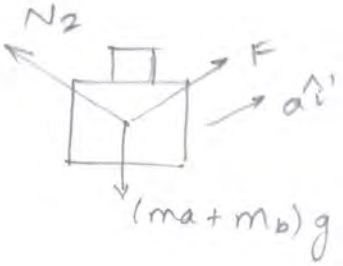
$$-m_B g\hat{j} + \mu N_1\hat{i} + N_1\hat{j} = m_B a\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{\hat{j}}{2}\right]$$

$$-m_B g + N_1 = \frac{m_B a\hat{j}}{2}$$

$$\mu N_1 = \frac{m_B a\sqrt{3}}{2}$$

$$a = \frac{g}{\frac{\sqrt{3}}{2\mu} - \frac{1}{2}}$$

FBD (II)



$$(m_B + m_A)a = F - (m_a + m_B)\sin\theta\, g$$

$$F = (m_B + m_A)\left[a + \frac{g}{2}\right]$$

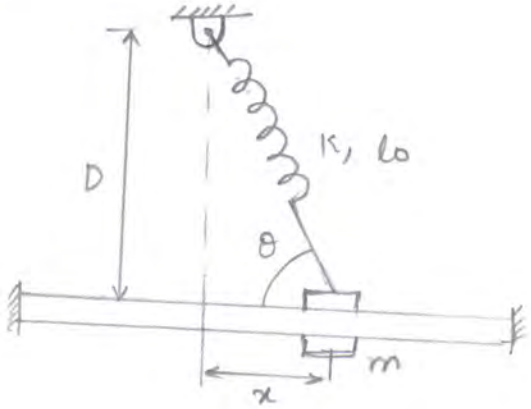
$$F = (m_A + m_B)\left[\frac{g}{\frac{\sqrt{3}}{2\mu} - \frac{1}{2}} + \frac{g}{2}\right]$$

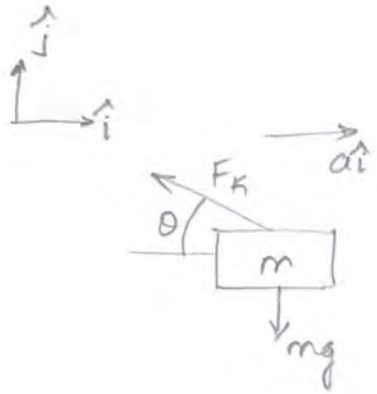
$$F_{max} = (5+2)\left[\frac{10}{\frac{\sqrt{3}}{2\times 0.2} - \frac{1}{2}} + \frac{10}{2}\right] \qquad \because g = 10\ m/sec^2$$

$$= 7\left[5 + \frac{20}{17.32 - 1}\right] = 7\left[5 + \frac{20}{16.32}\right]$$

$$F_{max} = 43.578\ N$$

13.2.33





a) Force balancing along $\hat{i}$ gives

$$ma\hat{i} = -F_x \cos\theta \, \hat{i}$$

$$a = \frac{-F_x \cos\theta}{m}$$

$$a = \frac{-k\left[\sqrt{x^2+D^2} - l_0\right]\cos\theta}{m}$$

$$\vec{a} = a\hat{i}$$

$$a = -\frac{k}{m}\left[\sqrt{x^2+D^2} - l_0\right]\frac{x}{\sqrt{x^2+D^2}} = -\frac{kx}{m}\left[1 - \frac{l_0}{\sqrt{x^2+D^2}}\right]$$

b) $a = \ddot{x}$

$$\ddot{x} = -\frac{kx}{m}\left[1 - \frac{l_0}{\sqrt{x^2+D^2}}\right]$$

c) If $l_0 = 0$

$$\ddot{x} = -\frac{Kx}{m}$$

Integrating we get

$$x = x_0 \cos\left(\sqrt{\frac{K}{m}}\, t\right)$$

Time taken to return to starting position

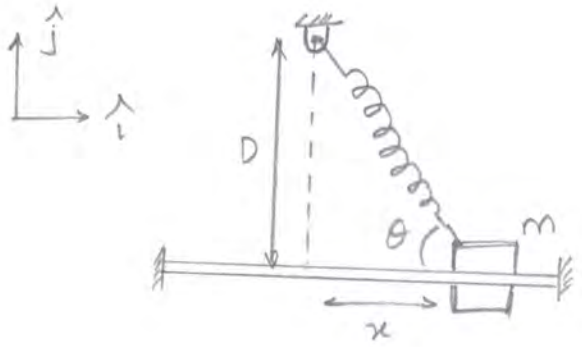
$$\sqrt{\frac{K}{m}}\, t_f = \frac{\pi}{2} \times 4$$

$$t_f = \frac{2\pi}{\sqrt{\frac{K}{m}}}$$

$$t_f = 2\pi \sqrt{\frac{m}{K}}$$

13.2.34

$F = K(l - l_0)$



a)

Same as 13.2.33

Force balance along $\hat{i}$ gives

$$m\ddot{x} = -K(l - l_0)\cos\theta$$

$$m\ddot{x} = -K\left[\sqrt{D^2 + x^2} - l_0\right]\frac{x}{\sqrt{D^2+x^2}}$$

$$\ddot{x} = -\frac{Kx}{m}\left[1 - \frac{l_0}{\sqrt{D^2+x^2}}\right]$$

b) at $t=0$, $x = x_0$, $\dot{x} = 0$

at $x = 0$ By energy balance

$$\frac{1}{2}m\dot{x}^2 = \int_{l \text{ at } x = x_0}^{D} K(l - l_0)\,dl = \frac{1}{2}Ky^2 \Big|_{l - l_0 \text{ at } x = x_0}^{l - l_0 \text{ at } x = 0}$$

$$= -\frac{1}{2}K\left[\left(\sqrt{D^2 + x_0^2} - l_0\right)^2 - (D - l_0)^2\right]$$

$$\frac{1}{2}m\dot{x}^2 = \frac{1}{2}K\left[x_0^2 + 2Dl_0 - 2l_0\sqrt{D^2+x_0^2}\right]$$

$$\dot{x} = \sqrt{\frac{K}{m}} \cdot \sqrt{\left(x_0^2 + 2Dl_0 - 2l_0\sqrt{D^2+x_0^2}\right)}$$

c) For $l_0 = 0$ eqn of motion

$$\ddot{x} = -\frac{Kx}{m} \Rightarrow x = x_0 \cos\left(\sqrt{\frac{K}{m}}\, t\right)$$

$$\dot{x}\Big|_{x=0} = x_0\sqrt{\frac{K}{m}} \sin\left(\frac{\pi}{2}\right) = x_0\sqrt{\frac{K}{m}}$$

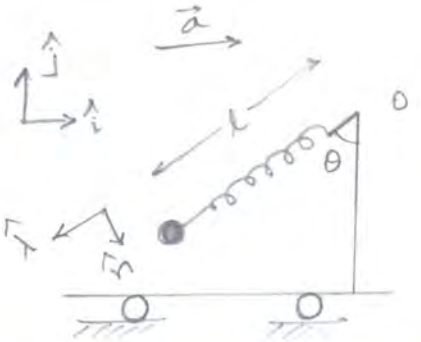
d) Put $l_0 = 0$ in $\dot{x}$ gives

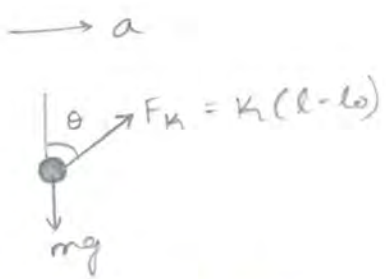
$$\dot{x} = \sqrt{\frac{K}{m}x_0^2} = x_0\sqrt{\frac{K}{m}}$$

The solution agrees.

13.2.36

$\hat{\lambda} = -\cos\theta\,\hat{j} - \sin\theta\,\hat{i}$





Solving force balance for ball along $\hat{i}$ we get

$$F_K \sin\theta = ma \longrightarrow \text{(i)}$$

Moment balance about 'O' gives

$$l\hat{\lambda} \times mg(-\hat{j}) = l\hat{\lambda} \times ma\hat{i}$$

$$\Rightarrow mgl\sin\theta = mal\cos\theta$$

$$\tan\theta = \frac{a}{g}$$

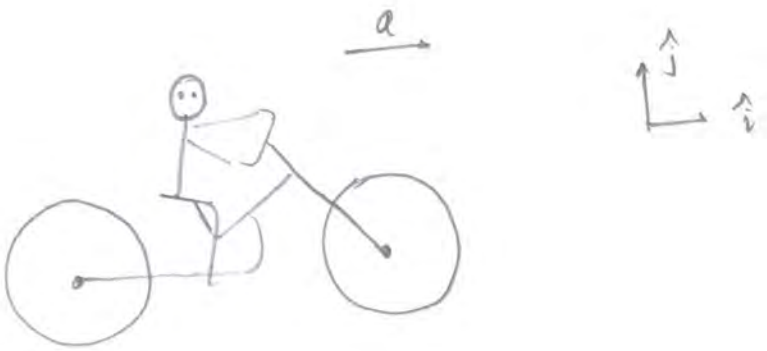
Putting in (i) we get

$$K(l - l_0)\frac{a}{\sqrt{a^2 + g^2}} = ma$$

$$l = \frac{m\sqrt{a^2 + g^2}}{K} + l_0$$

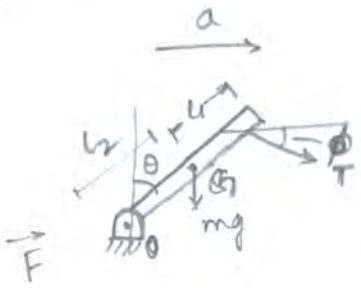
13.2.37

a)



b)

F.B.D



c) Linear Momentum Balance

$$ma\hat{i} = -mg\hat{j} + T\cos\phi\,\hat{i} - T\sin\phi\,\hat{j} + \vec{F} \longrightarrow \text{(i)}$$

Angular momentum about O ; $\vec{\tau}_{/o} = \dot{\vec{H}}_{/o}$

Assuming that $\dot{\theta}$ and $\dot{\phi} = 0$ we get

$$\vec{\tau}_{/o} = -mg\,l_2 \sin\theta\,\hat{k} - l_1(\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) \times T(\cos\phi\,\hat{i} - \sin\phi\,\hat{j}]$$

$$\dot{\vec{H}}_{/o} = \vec{r}_{cm/o} \times m\vec{a}$$

$$= l_2(\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) \times (ma\hat{i})$$

$$\Rightarrow -mgl_2\sin\theta\,\hat{k} + (Tl_1\sin\theta\sin\phi + Tl_1\cos\theta\cos\phi)\,\hat{k} = -mal_2\cos\theta\,\hat{k} \longrightarrow \text{(ii)}$$

d)

$$T = \frac{ml_2[g\sin\theta - a\cos\theta]}{l_1[\sin\theta\sin\phi + \cos\theta\cos\phi]}$$

From (i) $\vec{F} = (ma - T\cos\phi)\hat{i} + (mg + T\sin\phi)\hat{j}$

All Forces on rider

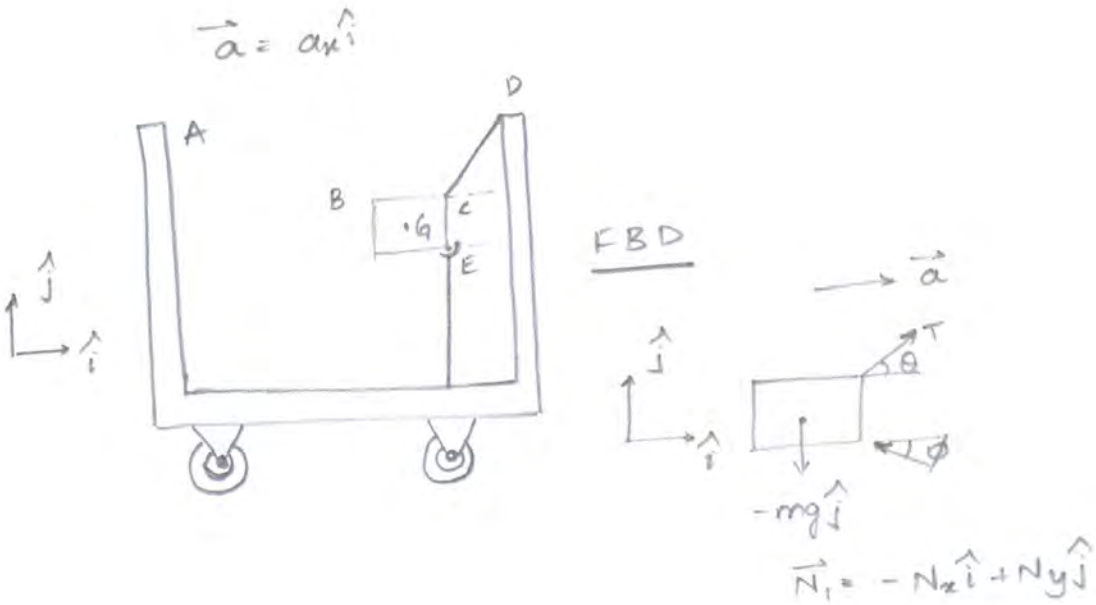
$$\vec{F} = (ma - T\cos\phi)\,\hat{i} + (mg + T\sin\phi)\,\hat{j}$$

$$T = \frac{ml_2[g\sin\theta - a\cos\theta]}{l_1[\sin\theta\sin\phi + \cos\theta\cos\phi]}\left[\cos\phi\,\hat{i} - \sin\phi\,\hat{j}\right]$$

e) All forces on motor cycle from rider

$$\vec{F}_1 = -\vec{F} \;;\quad \vec{T}_1 = -\vec{T}$$

13.2.42



Force Balance

$$m a_x \hat{i} = -mg\hat{j} + T[\cos\theta\,\hat{i} - \sin\theta\,\hat{j}] - N_x\hat{i} + N_y\hat{j}$$

Moment balance about E

$$-T\cos\theta\,(2l)\,\hat{k} + mg\left(\frac{3l}{2}\right)\hat{k} = l\hat{j} \times [m a_x \hat{i}]$$

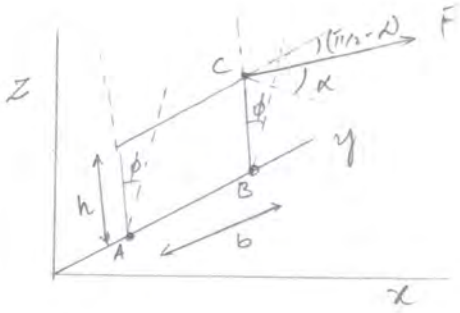
$$= -m a_x l \hat{k}$$

$$T = \frac{m a_x l + 3\frac{m}{2} g l}{2 l \cos\theta}$$

$$\left[\cos\theta = \frac{l}{\sqrt{l^2 + (2l)^2}}\right] = 1/\sqrt{5}$$

$$T = \frac{\sqrt{5}\, m \left[a_x + \frac{3g}{2}\right]}{2}$$

13.2.45



Moment balance about y-axis

$$\dot{\vec{H}} = 0$$

$$\vec{\tau}_y = (F\cos\alpha)(h\cos\phi)\,\hat{j} - mg\,\frac{h}{2}\sin\phi\,\hat{j} = 0$$

$$F = \frac{mg\sin\phi}{2\cos\alpha\cos\phi}$$

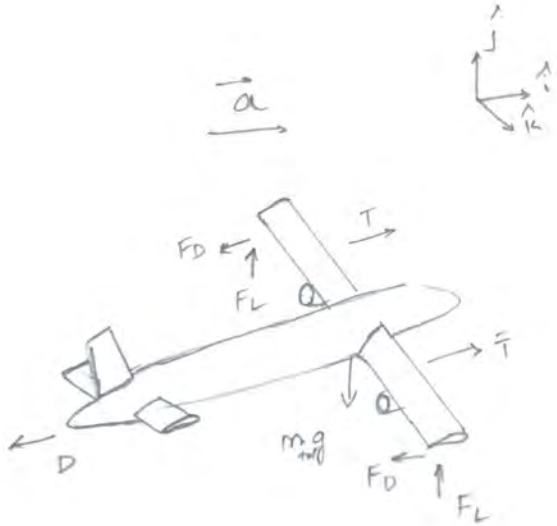
Force Balance along $\hat{j}$

$$ma_y\,\hat{j} = F\sin\alpha\,\hat{j}$$

$$a_y = \frac{g\sin\phi\sin\alpha}{2\cos\alpha\cos\phi}$$

$$a_y = \frac{g\tan\phi\tan\alpha}{2}$$

13·2·46

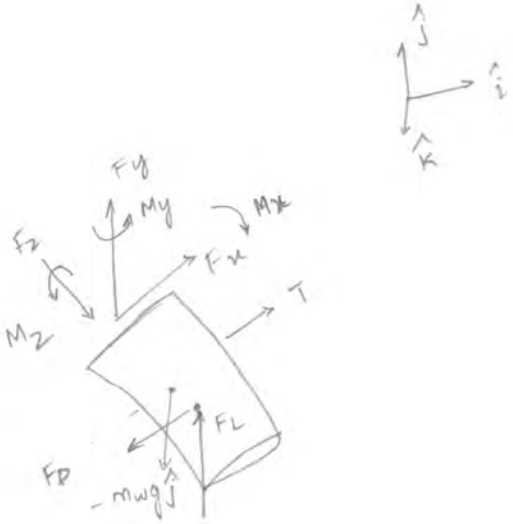


a) Force balance gives

$$m_{tot}\, a_p\, \hat{i} = 2T\,\hat{i} - D\,\hat{i} - 2F_o\,\hat{i} - m_{tot}\, g\,\hat{j} + 2F_L\,\hat{j} \longrightarrow \text{(i)}$$

eqn (i) $\cdot\, \hat{j}$ => $0 = 2F_L - m_{tot}\, g$

$$F_L = \frac{m_{tot}\, g}{2}$$



b) Eqn (i) $\hat{i}$ gives

$$m_{tot}\, a_p = 2T - D$$

$$a_p = \frac{2T - D}{m_{tot}}$$

c) F.B.D of wing

Force Balance gives

$$m_w a_p \hat{i} = (T - F_o)\hat{i} + F_x\,\hat{i} + (F_L - m_w g + F_y)\hat{j} + F_z\hat{k} \longrightarrow \text{(ii)}$$

(ii) $\cdot\, \hat{k}$ => $F_z = 0$

(ii) $\cdot\, \hat{i}$ => $m_w a_p = (T - F_o + F_x)$ => $F_x = m_w\left[\left(\frac{2T - D}{m_{tot}}\right)\right] + F_o - T$

(ii) $\cdot\, \hat{j}$ => $F_y = m_w g - F_L$

Moment balance about base of wing

$$(T_c - F_D b + M_y)\hat{j} + (m_w g a - F_L b + M_x)\hat{i} + M_z \hat{k} = \dot{\vec{H}}/_O$$

$$= a\hat{k} \times m_w a_p \hat{i}$$

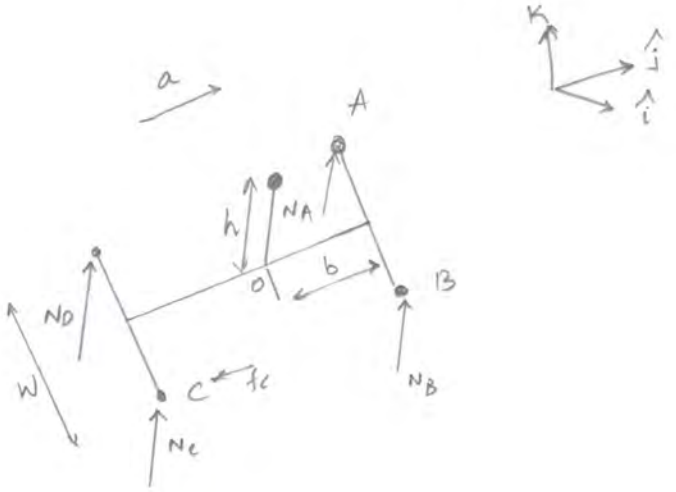
$$= m_w a \left[\frac{2T - D}{m_{tot}}\right]\hat{j} \longrightarrow \text{(iii)}$$

(iii) $\cdot\,\hat{i}$ gives : $H_x = F_L b - m_w g a$

(iii) $\cdot\,\hat{j}$ gives : $H_y = F_D b - T_c + \dfrac{m_w(2T - D)}{m_{tot}} \cdot a$

(iii) $\cdot\,\hat{k}$ gives: $H_z = 0$

13.2.48



Ignoring mass of wheels in comparision to the mass of the car

Force Balance

$$mg\hat{k} + ma\hat{i} = -\mu N_c \hat{i} + (N_A + N_B + N_c + N_D)\hat{k} \longrightarrow \quad (i)$$

$$a = -\frac{\mu N_c}{m}$$

Moment balance about point 'O' gives

$$\left(b\hat{j} + \frac{w}{2}\hat{i}\right) \times N_B\hat{k} + \left(b\hat{j} - \frac{w}{2}\hat{i}\right) \times N_A\hat{k} + \left(-(l-b)\hat{j} + \frac{w}{2}\hat{i}\right) \times \left(N_c\hat{j} - f_c\hat{j}\right)$$

$$+ \left[-(l-b)\hat{j} - \frac{w}{2}\hat{i}\right] \times N_D\hat{j} = h\hat{k} \times ma\hat{j} + I_{car/cm}\alpha$$

$$\Rightarrow \left(b\hat{j} + \frac{w}{2}\hat{i}\right) \times N_B\hat{k} + \left(b\hat{j} - \frac{w}{2}\hat{i}\right) \times N_A\hat{k} + \left[(b-l)\hat{j} - \frac{w}{2}\hat{i}\right] \times N_D\hat{k} + \left[(b-l)\hat{j} + \frac{w}{2}\hat{i}\right]$$

$$\times \left(N_c\hat{k} - f_c\hat{j}\right) = -hma\hat{i} + I_{car/cm}\alpha\hat{k}$$

$$\left[N_B b\hat{i} - \frac{N_B w}{2}\hat{j}\right] + \left[N_A b\hat{i} + \frac{N_A w}{2}\hat{i}\right] + \left[(b-l)N_D\hat{i} + \frac{wN_D}{2}\hat{j}\right] + \left[(b-l)N_c\hat{i} - \frac{N_c w}{2}\hat{j}\right]$$

$$- f_c\frac{w}{2}\hat{k} = -hma\hat{i} + I\alpha\hat{k} \longrightarrow \quad (ii)$$

(i) $\hat{k} \Rightarrow \quad N_A + N_B + N_C + N_D = mg$ —— (a)

(ii). $\hat{k} \Rightarrow \quad \alpha = -\frac{f c w}{2}$ $\longrightarrow$ The car rotates

(ii). $\hat{i} \Rightarrow \quad -hma = (N_A + N_B + N_C + N_D)b - l(N_C + N_D)$ —— (b)

(ii). $\hat{j} \Rightarrow \quad 0 = (N_A + N_D - N_B - N_C)\frac{w}{2}$ —— (c)

(a), (b), (c) can be solved for N_A, N_B, N_C

this gives

$$N_A + N_D = N_B + N_C \longrightarrow \text{(iii)}$$

$$N_C + N_D = \frac{mgb + mah}{l}$$

$$\Rightarrow N_C = \left(\frac{mgb}{l} - N_D\right)\left[\frac{1}{1 + \mu h/l}\right]$$

$$N_C = \frac{mgb - N_D l}{l + \mu h}$$

$$a = -\frac{\mu h}{m}\left[\frac{mgb - N_D l}{l + \mu h}\right] = -\frac{\left(gb - \frac{N_D l}{m}\right)}{\left(\frac{l}{\mu h} + 1\right)}$$

$$\Rightarrow \quad N_A + N_B = mg - N_D - \left[\frac{mgb - N_D l}{l + \mu h}\right]$$

and,

$$N_A - N_B = N_C - N_D = \frac{mgb - N_D l}{l + \mu h} - N_D$$

$$N_A = \frac{mg - 2N_D}{2} \Rightarrow N_A = \frac{mg}{2} - N_D$$

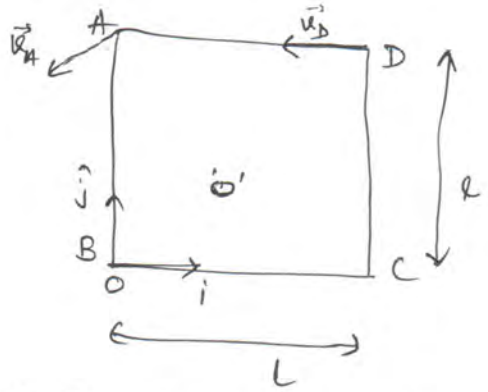
$$N_B = \frac{mg}{2} - \frac{mgb - N_D l}{l + \mu h}$$

We could find all the reactions because rections at A, B, C and D are required to balance roll about $\hat{i}$ and $\hat{j}$ and balance the weight. There are three equations and we have only three reactions

CHAPTER 16

2

16.1.3



Co ordinates of A $(0, l)$; $l\hat{j}$

" " D (l, l) ; $l(\hat{i} + \hat{j})$

Let the " " O' [center] ; $(a\hat{i} + b\hat{j})$

Center O' has zero velocity.

Position vectors of points A and D w.r.t. O' are $\vec{O'A}$ & $\vec{O'D}$ respectively.

$\vec{O'A} \perp \vec{v}_A$

$\vec{O'D} \perp \vec{v}_D$

$\vec{O'A} = \vec{OA} - \vec{OO'}$

$= (-a)\hat{i} + (l-b)\hat{j}$

$\vec{O'D} = \vec{OD} - \vec{OO'}$

$= (l-a)\hat{i} + (l-b)\hat{j}$

$\vec{O'A} \cdot \vec{v}_A = 0 \Rightarrow [-a\hat{i} + (l-b)\hat{j}] \cdot [-2i - 2j] = 0$

$\Rightarrow 2a - 2l + 2b = 0$

$\Rightarrow \boxed{a - l + b = 0}$

$\vec{O'D} \cdot \vec{v}_D = 0 \Rightarrow [(l-a)\hat{i} + (l-b)\hat{j}] \cdot [-2\hat{i}] = 0$

$\Rightarrow -2l + 2a = 0$

$\Rightarrow \boxed{l = a}$

$\therefore b = 0$

(a) $\therefore$ Center O' is at $l\hat{i}$.

(b) Angular velocity of the plate? Lets $\omega = \omega_0 \hat{k}$ (rotating in XY plane)

$$\vec{V}_D = \vec{V}_A + \vec{\omega} \times \vec{r}_{D/A}$$

$$\Rightarrow -2\hat{i} = -2(\hat{i}+\hat{j}) + \omega_0\hat{k} \times (l\hat{i})$$

$$\Rightarrow 0 = -2\hat{j} + (-\omega_0 l)\hat{j}$$

$$\Rightarrow 2 + \omega_0 l = 0$$

$$\Rightarrow \omega_0 = -\frac{2}{l} \text{ rad/sec}$$

velocity of center of mass?

$$\vec{V}_G = \vec{V}_A + \vec{\omega} \times \vec{r}_{G/A}$$

$$= (-2\hat{i} - 2\hat{j}) + \left(-\frac{2}{l}\right)\hat{k} \times \left(\frac{l}{2}\hat{i} - \frac{l}{2}\hat{j}\right)$$

$$= (-2\hat{i} - 2\hat{j} - \hat{j} - \hat{i}$$

$$= -3(\hat{i}+\hat{j})$$

Acceleration of the center of mass?

$$\vec{a}_G = \vec{a}_{O'} + \vec{a}_{G/O'}$$

$$= \underbrace{\vec{a}_{O'}}_{0} + \underbrace{\vec{\omega} \times \vec{r}_{G/O'}}_{0} - \omega^2 \vec{r}_{G/O'}$$

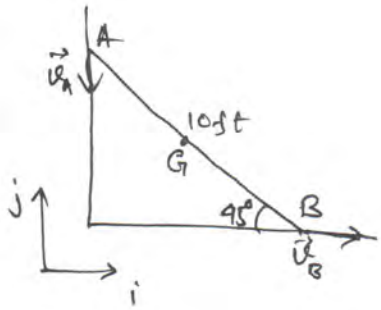
[const. angular velocity; given]

$$\Rightarrow \vec{a}_G = -\omega^2 \vec{r}_{G/O'}$$

$$\Rightarrow \vec{a}_G = -\left(\frac{2}{l}\right)^2 \left[-\frac{l}{2}\hat{i} + \frac{l}{2}\hat{j}\right]$$

$$= \frac{2}{l}\left[\hat{i} - \hat{j}\right]$$

Q
16.1.4



$\vec{v}_A = (-1 \text{ ft/sec})\hat{j}$

$\vec{v}_B = v\hat{i}$

Angular velocity of ladder?

$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$

$\Rightarrow v\hat{i} = -1\hat{j} + \omega_0\hat{k} \times \left[\frac{10}{\sqrt{2}}\hat{i} - \frac{10}{\sqrt{2}}\hat{j}\right]$

$\Rightarrow v\hat{i} = -1\hat{j} + \omega_0 \cdot \frac{10}{\sqrt{2}}\left[\hat{j} + \hat{i}\right]$

$(\omega_0)\frac{10}{\sqrt{2}} - 1 = 0$

$\Rightarrow \omega_0 = \frac{\sqrt{2}}{10}$ rad/sec

<u>Velocity of midpoint G</u>:

$\vec{V}_G = \vec{V}_A + \vec{r}_{G/A}$

$= \vec{V}_A + \vec{\omega} \times \vec{r}_{G/A}$ $\qquad \vec{r}_{G/A} = \frac{10}{\sqrt{2}}(i - j)$

$= -1\hat{j} + \frac{\sqrt{2}}{10}\hat{k} \times \frac{10}{\sqrt{2}}(i - j)$

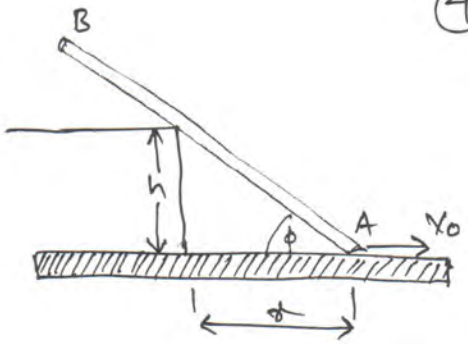
$= -\hat{j} + \hat{k} \times (\hat{i} - \hat{j})$

$= -\hat{j} + (\hat{j} + \hat{i})$

$= \hat{i}$

$\Rightarrow \vec{V}_G = 1$ ft/sec along +ve x direction.

Q
16.1.5



$$\tan\phi = \frac{h}{x}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{h}{x}\right)$$

$$\Rightarrow \frac{d\phi}{dt} = \frac{1}{1+\left(\frac{h}{x}\right)^2} \times \frac{x\cdot\cancel{\frac{dh}{dt}}^{\,0} - h\frac{dx}{dt}}{x^2}$$

$$= \frac{1}{x^2+h^2} \times -h\frac{dx}{dt}$$

$$\Rightarrow \frac{d\phi}{dt} = \frac{-hv_0}{x^2+h^2}$$

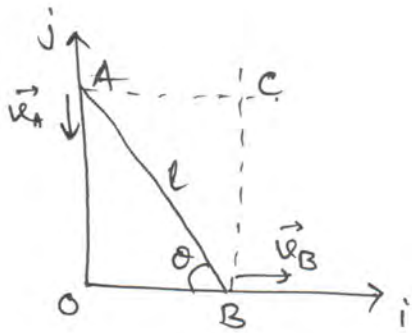
Denominator $\rightarrow$ +ve
v_0 $\rightarrow$ +ve
h $\rightarrow$ +ve

$\therefore \dot{\phi} \rightarrow$ -ve.

$$\frac{d^2\phi}{dt^2} = \frac{-hv_0\left[-2x\frac{dx}{dt}\right]}{(x^2+h^2)^2} = \frac{2hxv_0^2}{(x^2+h^2)^2}$$

$\Rightarrow \ddot{\phi} \Rightarrow$ +ve.

Q
16.1.6



Length of ladder l.

$\vec{v}_A \Rightarrow$ along $-\hat{j}$
$\vec{v}_B \Rightarrow$ along $\hat{i}$

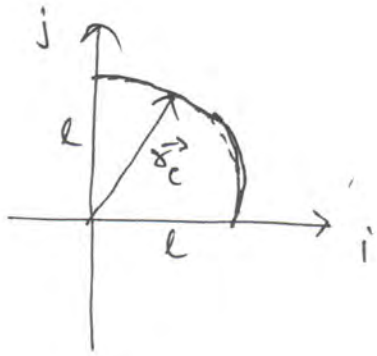
Lines perpendicular to the velocities $\vec{v}_A$ & $\vec{v}_B$ intersect at C.
Hence C is center of rotation [COR]

co-ordinates of C w.r.t. origin O are $(l\cos\theta, l\sin\theta)$

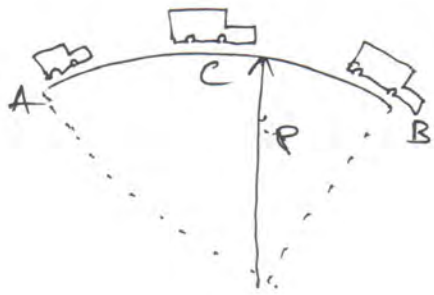
(a) $\therefore \vec{r}_c = l\left[\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right]$

As the ladder falls from straight up to lying on the floor θ varies from 90° to 0°.

(b) The locus of center C will be quarter of a circle present in the first quadrant of co-ordinate axes.



Q
16.1.7



Acceleration in path coordinates:

$$\bar{a} = a_t \hat{e}_t + a_n \hat{e}_n$$

$$a_t = \ddot{s} \qquad a_n = \frac{\dot{s}^2}{\rho}$$

$$\dot{s}_A = 50 \text{ mph}$$

$$\dot{s}_B = 60 \text{ mph}$$

$$\dot{s}_B = \dot{s}_A + a_t T$$

$$\Rightarrow a_t = \frac{60-50}{1} = 10 \text{ miles/hour}^2$$

Total distance travelled

$$s_{total} = \dot{s}_A T + \frac{1}{2} a_t T^2$$

$$= 50 \times 1 + \frac{1}{2} \times 10 \times 1^2$$

$$= 55 \text{ miles.}$$

Velocity at half way:

$$\dot{s}_C^2 = \dot{s}_A^2 + 2 . a_t s_{1/2}$$

$$= 50^2 + 2 . 10 . \frac{55}{2}$$

$$= 2500 + 550$$

$$= 3050$$

Acceleration at C:

$$\bar{a} = a_t \hat{e}_t + a_n \hat{e}_n$$

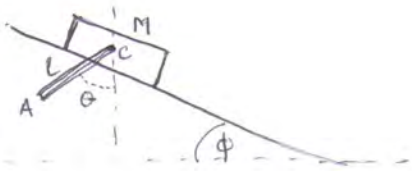
$$= 10 \hat{e}_t + \frac{3050}{25} \hat{e}_n$$

$$= 10 \hat{e}_t + 122 \hat{e}_n \text{ miles/hr}^2$$

Q

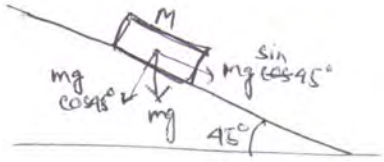
16.1.10

Acceleration of the cart $= g/\sqrt{2}$

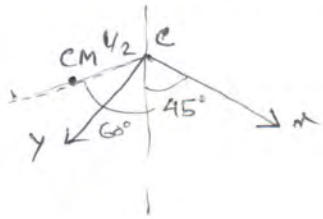


velocity of the cart at the position shown

$$v_c = u + at$$
$$= 0 + \frac{g}{\sqrt{2}} \cdot 2$$
$$\Rightarrow v_c = \sqrt{2}g \ m/s$$



The co-ordinate axes are choosen as shown.



Velocity of the center of mass of the rod :

$$\vec{v}_{cm} = \vec{v}_c + \vec{\omega}_{rod} \times \vec{r}_{cm/c}$$

Here $\vec{v}_c = \sqrt{2}g\,\hat{i}$ m/s

$\vec{\omega}_{rod} = 0.5\,\hat{k}$ rad/s

$$\vec{r}_{cm/c} = \frac{l}{2}\left[-\cos 75^\circ \hat{i} + \sin 75^\circ \hat{j}\right]$$
$$= \frac{l}{2}\left[-0.258\,\hat{i} + 0.966\,\hat{j}\right]$$
$$= -0.258\,\hat{i} + 0.966\,\hat{j} \ m \quad [\because l = 2\,m]$$

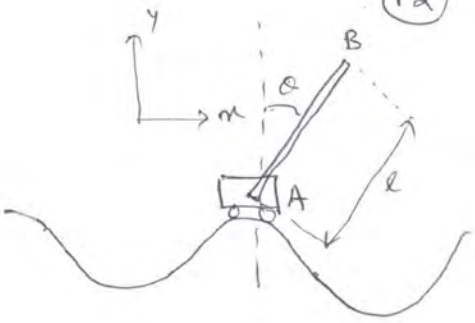
$$\therefore \vec{v}_{cm} = \sqrt{2}g\,\hat{i} + 0.5\hat{k} \times \left[-0.258\,\hat{i} + 0.966\,\hat{j}\right]$$
$$= 13.87\,\hat{i} + 0.129(-\hat{j}) + 0.483(-i)$$
$$= \left[13.387\,\hat{i} - 0.129\,\hat{j}\right] m/s$$

Acceleration of the center of mass of the rod:

$$\vec{a}_{cm} = \vec{a}_c \times \vec{r}_{cm/c} + \vec{\omega}_{rod} \times (\vec{\omega}_{rod} \times \vec{r}_{cm/c})$$
$$= 1\hat{k} \times \left[-0.258\,\hat{i} + 0.966\,\hat{j}\right] + 0.5\hat{k} \times \left[0.5\hat{k} \times (-0.258\,\hat{i} + 0.966\,\hat{j})\right]$$
$$= -0.258\,\hat{j} - 0.966\,\hat{i} + 0.5\hat{k} \times \left[-0.129\,\hat{j} - .483\,\hat{i}\right]$$
$$= -0.258\,\hat{j} - 0.966\,\hat{i} + 0.0648\,\hat{i} - 0.2415\,\hat{j}$$
$$= -0.9012\,\hat{i} - 0.4995\,\hat{j} \ m/s^2$$

Q

16.1.11



$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{AB}$$

$$\cancel{= 2\hat{i} + \vec{\omega}_{AB} \times \vec{r}_{AB}}$$

$$\theta(t) = \theta_0 \sin \omega t$$

$$\Rightarrow \dot{\theta}(t) = (\theta_0 \omega) \cos \omega t$$

$$\Rightarrow \omega_{AB} = \frac{\pi^2}{6} \cos \pi t$$

Here at $t=0$ $\theta(t) = -\theta_0 = -\frac{\pi}{6}$

$$\therefore \theta(t) = \theta_0 \sin\left(\omega t - \frac{\pi}{2}\right) = \frac{\pi}{6} \sin\left(\pi t - \frac{\pi}{2}\right)$$

Now ω_{AB} modifies to

$$\vec{\omega}_{AB} = \frac{\pi^2}{6} \cos\left(\pi t - \frac{\pi}{2}\right) \hat{k}$$

(a) velocity of point B when $\theta = 0$?

time at $\theta = 0$?

$$0 = \frac{\pi}{6} \sin\left(\pi t - \frac{\pi}{2}\right)$$

$$\Rightarrow \pi t - \frac{\pi}{2} = 0 \Rightarrow t = \frac{1}{2} \text{ sec.}$$

$$\vec{\omega}_{AB}\Big|_{t=\frac{1}{2}} = \frac{\pi^2}{6} \text{ rad/s}$$

$$\vec{r}_{AB}\Big|_{\theta=0} = 4\,j \text{ ft}$$

At $t = \frac{1}{2}$, horizontal distance covered by the cart

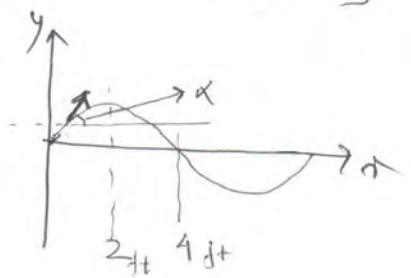
$$= v_0 \times \frac{1}{2} = 2 \times \frac{1}{2} = 1 \text{ ft.}$$

y-co-ordinate of the sinusodial track at $x = 1$ ft.

$$y(x) = y_0 \sin\left(\lambda x + \frac{\pi}{2}\right) \qquad \left[\because \text{the cart starts from a crest}\right]$$

$$= (0.5) \sin\left(\frac{\pi}{4} + \frac{\pi}{2}\right) = \frac{1}{2\sqrt{2}} \text{ ft}$$

At point $\left(1, \frac{1}{2\sqrt{2}}\right)$ slope (direction of $\vec{v}_A$):



$$y'(x)\Big|_{x=1} = y_0 \lambda \cos\left(\lambda x + \frac{\pi}{2}\right)$$

$$= \left(0.5 \times \frac{\pi}{4}\right) \cos\left(\frac{\pi}{4} \times 1 + \frac{\pi}{2}\right)$$

$$= -0.2776$$

Q

16.1.11 contd...

$\tan^{-1}(\alpha) = -0.2776$

$\Rightarrow \alpha = -15.51°$

$v_A \cos\alpha = 2$ ft/sec

$\Rightarrow v_A \cos(15.51) = 2$

$\Rightarrow v_A = 2.076$ ft/sec.

$\therefore \vec{v}_A = (v_A \cos\alpha)\hat{i} + (v_A \sin\alpha)\hat{j}$

$= 2\hat{i} - 0.55\hat{j}$

$\vec{v}_B$?

$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{AB}$

$= (2\hat{i} - 0.55\hat{j}) + \frac{\pi^2}{6} \cdot \cos\left(\pi \cdot \frac{1}{2} - \frac{\pi}{2}\right)\hat{k} \times (4\hat{j})$

$= 2\hat{i} - 0.55\hat{j} - \frac{\pi^2}{6} \times 4\,\hat{i}$

$= (-4.58\,\hat{i} - 0.55\,\hat{j})$ ft/s

(b) Locus of point B:

$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

$\vec{r}_A = 2t\,\hat{i} + 0.5 \sin\left[\frac{\pi}{4}(2t) + \frac{\pi}{2}\right]\hat{j}$

$= 2t\,\hat{i} + \frac{1}{2}\sin\frac{\pi}{2}(t+1)\,\hat{j}$

$\vec{r}_{B/A} = AB \sin\theta\,\hat{i} + AB\cos\theta\,\hat{j}$

$\cancel{= 4\left[\frac{\pi}{6}\sin\left(\pi t - \frac{\pi}{2}\right)\right.}$

$= 4\sin\left[\frac{\pi}{6}\sin\left(\pi t - \frac{\pi}{2}\right)\right]\hat{i} + 4\cos\left[\frac{\pi}{6}\cos\left(\pi t - \frac{\pi}{2}\right)\right]\hat{j}$

$= 4\sin\left[-\frac{\pi}{6}\cos(\pi t)\right]\hat{i} + 4\cos\left[\frac{\pi}{6}\sin(\pi t)\right]\hat{j}$

Chapter - 16

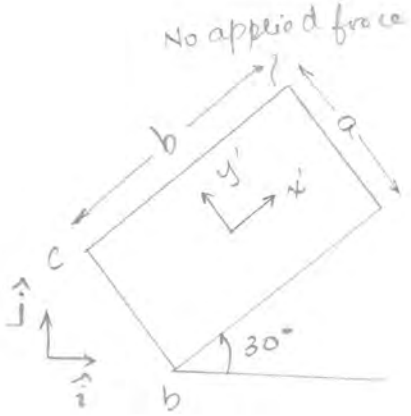
16.2.1

$a = 1m;\ b = 2m;\ m = 1kg$

$(x_c, y_c) \equiv (3m, 2m)$

$\vec{L} = 4\hat{i} + 3\hat{j}\ [kg \cdot m/sec]$

$\vec{H}_{cm} = 5\hat{k}\ [kg\, m/sec^2]$



$\Rightarrow \sum \vec{F} = m\vec{a}_{cm} = 0 \Rightarrow \vec{a}_{cm} = 0$

$\vec{H}_{cm} = [I]^{cm}\vec{\omega} \Rightarrow 5\hat{k} = \frac{m}{2}(a^2+b^2)\vec{\omega}$

$\vec{\omega} = \frac{10}{4+1}\hat{k} = 2\hat{k}\ rad/sec$

$\dot{\vec{\omega}} = 0$ [No net torque on body]

Acceleration of an arbitrary point on plate

$\vec{a}_p = \vec{a}_{cm} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{p/cm})$

$\vec{a}_p = -\omega^2 \vec{r}_{p/cm}$

For point B

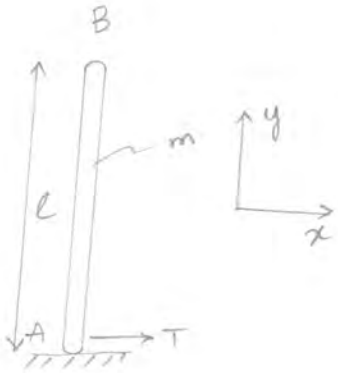
$$\vec{a}_b = -4 \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} -b/2 \\ -a/2 \end{bmatrix}$$

$$= -4 \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} -1 \\ -1/2 \end{bmatrix}$$

$$\vec{a}_b = -4 \left[\left(-\sqrt{3}/2 + 1/4 \right) \hat{i} + \left(-1/2 - \sqrt{3}/4 \right) \hat{j} \right] \quad m/sec^2$$

<u>16.2.2</u>

a) $\vec{F} = m\vec{a}_{cm}$



$$\vec{a}_{cm} = \left(\frac{T}{m}\right)\hat{i} = \ddot{x}_{cm}\hat{i} + \ddot{y}_{cm}\hat{j}$$

$$\ddot{x}_{cm} = \frac{T}{m}$$

b) $\vec{M}_{/A} = 0 = \vec{r}_{cm/A} \times m\vec{a}_{cm} + \underline{I}_{cm}\dot{\vec{\omega}} + (\vec{\omega} \times \underline{I}\vec{\omega})$

$$0 = \frac{l}{2}\hat{j} \times m\left(\frac{T}{m}\right)\hat{i} + \frac{ml^2}{12}\dot{\vec{\omega}}$$

$$\dot{\vec{\omega}} = \frac{6T}{ml}\hat{k}$$

$$\ddot{\theta}_{AB} = \frac{6T}{ml} \quad rad/sec^2$$

$$\vec{a}_B = \vec{a}_{cm} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/cm}) + \dot{\vec{\omega}} \times \vec{r}_{B/cm}$$

$$= \left(\frac{T}{m}\right)\hat{i} + \left(\frac{6T}{ml}\right)\hat{k} \times \frac{l}{2}\hat{j}$$

$$= \frac{T}{m}\hat{i} + \left(\frac{-3T}{m}\right)\hat{i}$$

$$\vec{a}_B = -\frac{2T}{m}\hat{i}$$

$$\ddot{x}_B = -\frac{2T}{m} \quad m/sec^2$$

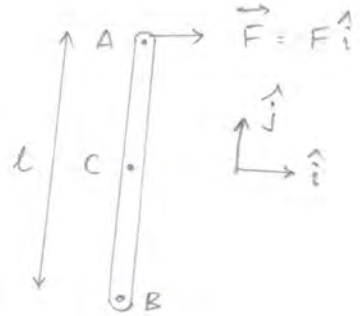
<u>16.2.3</u>

(i) Force Balance

$$\vec{F} = m\vec{a}_c$$

$$\vec{a}_c = \frac{F}{m}\hat{i}$$

$$\vec{a}_c = \frac{F}{m}\hat{i}$$



(ii) Momentum balance about point 'A' gives

$$\vec{\tau}_{/A} = \vec{M}_{/A} = 0 = \dot{\vec{H}}_{/A}$$

$$0 = \vec{r}_{cm/A} \times m\vec{a}_c + \bar{I}_{cm}\dot{\vec{\omega}}$$

$$-\frac{l}{2}\hat{j} \times m\left(\frac{F}{m}\hat{i}\right) + \frac{ml^2}{12}\vec{\alpha} = 0$$

$$\vec{\alpha} = -\frac{6F}{ml}\hat{k} \quad rad/sec^2$$

(iii) $^{f}\vec{a}_A = {}^{f}\vec{a}_c + {}^{f}\vec{a}_{A/c} = \vec{a}_c + \dot{\vec{\omega}} \times \vec{r}_{A/c}$

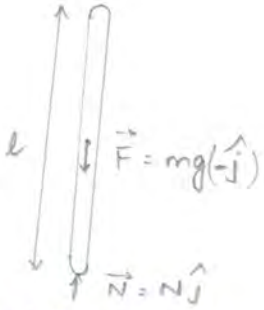
$$= \frac{F}{m}\hat{i} - \frac{6F}{ml} \times \frac{l}{2}(-\hat{i})$$

$$\vec{a}_A = \frac{4F}{m}\hat{i} \quad m/sec^2$$

16.2.4

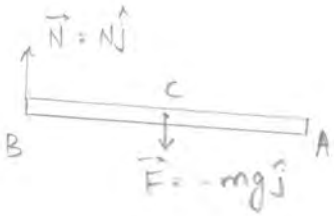
a) Path of centre of Mass

$$\vec{a}_{cm} = -g\hat{j} + \frac{N}{m}\hat{j}$$



So, centre of mass moves on straight line along $-\hat{j}$ ie falls straight down.

b) Normal force will act such that $(\vec{a}_B)_v = 0$, momentum balance just before rod becomes horizontal.



$$\vec{M}_{/c} = -\frac{l}{2}\hat{i} \times N\hat{j} = -\frac{Nl}{2}\hat{k}$$

$$= \dot{\vec{H}}_{/c} = \frac{ml^2}{12}\dot{\vec{\omega}}$$

$$\dot{\vec{\omega}} = -\frac{6N}{ml}\hat{k}$$

$$\vec{a}_B = 0 = \vec{a}_{cm} + \dot{\vec{\omega}} \times \vec{r}_{B/cm} = -g\hat{j} + \frac{N}{m}\hat{j} + \left(-\frac{6N}{ml}\hat{k}\right) \times \left(-\frac{l}{2}\hat{i}\right)$$

$$-g\hat{j} + \frac{N}{m}\hat{j} + \frac{3N}{m}\hat{j} = 0$$

Normal force $\vec{N} = \frac{mg}{4}\hat{j}$

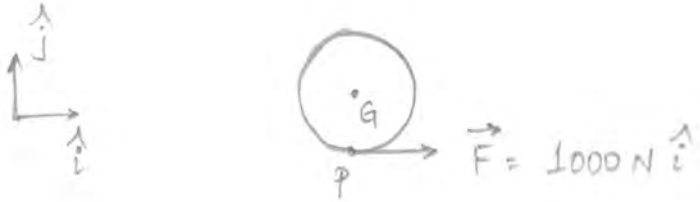
16-2-5

$r = 1\,m$

$m = 10\,kg$

$\vec{F} = 1000\,N\,\hat{i}$

$\Delta t = 0.0001\,sec$



a) $\vec{a}_{cm} = \dfrac{F}{m} = 100\,\hat{i}\ m/sec^2$

$$\frac{\Delta \vec{V}_{cm}}{\Delta t} = 100\,\hat{i}\ m/sec^2 \Rightarrow \vec{V}_{cm} = 100 \times 10^{-4}\,\hat{i}\ m/sec$$

$$\vec{V}_{cm} = 10^{-2}\,\hat{i}\ m/sec$$

b) exact

c) moment balance about G:

$$\vec{r}_{P/G} \times \vec{F} = I\,\dot{\vec{\omega}}$$

Say $\vec{r}_{P/G} = -r\hat{j}$

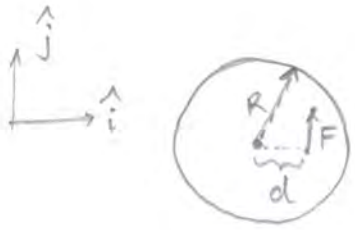
$$\dot{\vec{\omega}} = \frac{-r\hat{j} \times 1000\hat{i}}{mr^2/2} = 200\hat{k}\ rad/sec^2$$

$$\frac{\Delta \vec{\omega}}{\Delta t} = 200\hat{k}\ ; \quad \vec{\omega}_{final} = 2 \times 10^{-2}\,\hat{k}\ rad/sec$$

d) Approximate:

Since $F = 1000\,\hat{i}$; it always acts on point 'p' during that time interval, the torque changes with the change in orientation and the Angular acceleration is not constant.

16.2.6



a) $\vec{a}_c = ?$

$$\sum \vec{F} = m\vec{a}_c$$

$$\vec{a}_c = \frac{F}{m}\hat{j}$$

b) Moment balance about 'c':

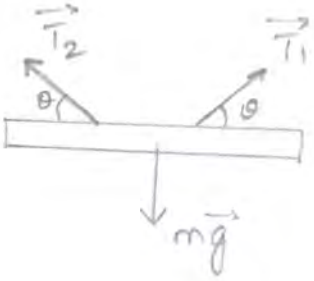
$$\vec{M}_{/c} = \vec{r}_{P/c} \times \vec{F} = I\dot{\vec{\omega}}$$

$$d\hat{i} \times F\hat{j} = \frac{mr^2}{2}\dot{\vec{\omega}}$$

$$\dot{\vec{\omega}} = \frac{2Fd}{mr^2}\hat{k}$$

16.2.8

a) F B D



$$\vec{T}_1 = T\cos\theta\,\hat{i} + T\sin\theta\,\hat{j}$$

$$\vec{T}_2 = -T\cos\theta\,\hat{i} + T\sin\theta\,\hat{j}$$

$$\vec{F}_G = -mg\,\hat{j}$$

$$\Rightarrow \sum \vec{F} = (2T\sin\theta - mg)\hat{j}$$

b) After the string is cut

$$\sum \vec{F} = T\cos\theta\,\hat{i} + T\sin\theta\,\hat{j} - mg\,\hat{j}$$

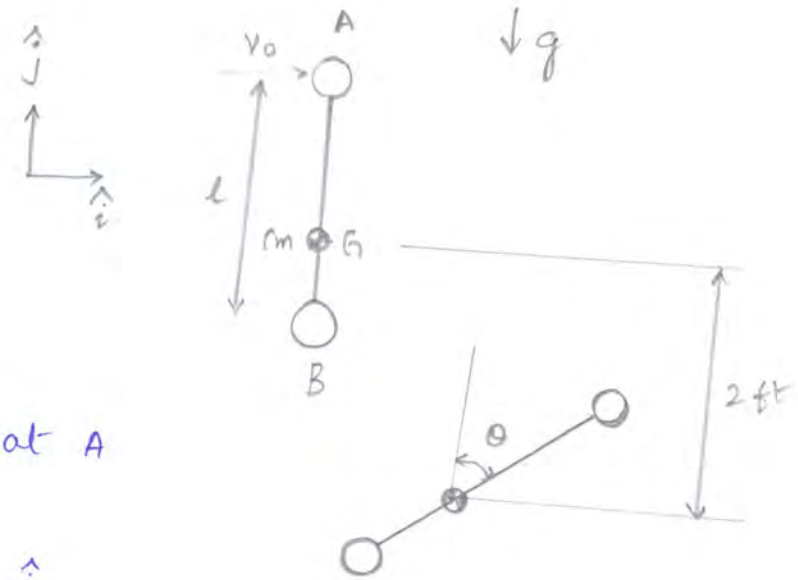
$$\sum \vec{F} = T\cos\theta\,\hat{i} + (T\sin\theta - mg)\hat{j}$$

16.2.10

$w = 5\,lb$

$l = 1\,ft$

$V_0 = 3.0\,ft/sec$



If a force is applied at A

$$\vec{a}_{cm} = \frac{\vec{F}}{m} \; ; \; \frac{F}{m}\hat{i}$$

$$\vec{M}_{/cm} = \frac{2}{3} l \hat{j} \times F \hat{i}$$

$$= \left[m\left(\frac{2}{3} l\right)^2 + 2m\left(\frac{l}{3}\right)^2 \right] \alpha \, (-\hat{k})$$

$$\Rightarrow \alpha = -\left(\frac{2lF}{3} \times \frac{g}{6ml^2} \right)\hat{k} = \frac{-F}{ml}\hat{k}$$

$$\vec{a}_A = \frac{-F}{ml}\hat{k} \times \left(\frac{2}{3} l \hat{j}\right) + \vec{a}_{cm}$$

$$= \left(\frac{2F}{3m} + \frac{F}{m} \right)\hat{i} = \frac{5}{3} \cdot \frac{F}{m}\hat{i}$$

$$\vec{a}_A = \frac{5}{3}\vec{a}_{cm}$$

At the end of the impulse

$$\vec{V}_A = \frac{3\,ft}{sec} \quad ; \quad \vec{V}_{cm} = \frac{9}{5}\ ft/sec$$

$$\vec{V}_A - \vec{V}_{cm} = \vec{\omega} \times \vec{r}_{A/cm}$$

$$\frac{6}{5}\hat{i} = -\omega\hat{k} \times \left(\frac{2}{3}l\hat{j}\right)$$

$$\omega = 1.8\ rad/sec$$

$$\vec{\omega} = -1.8\hat{k}\ rad/sec$$

Angular momentum

$$\vec{H}/cm = I\vec{\omega} = \left(\frac{2}{3}ml^2 \times 1.8\right)(-\hat{k}) = -1.2\ ml^2\hat{k}$$

$$\vec{H}/cm = -12\hat{k}\ \frac{lb\ ft^2}{sec}$$

$$g = 32.56\ ft/sec^2$$

$$2 = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{4}{32.56}}\ sec$$

$$\theta = \omega t = 1.8\sqrt{\frac{4}{32.56}}\ radians.$$

$$\theta = 0.63\ rad$$

b) Velocity of sphere A

$$\vec{V}_{cm} = \left(\frac{9}{5}\right)\hat{i} - \sqrt{2 \times g \times 2}\ \hat{j}$$

$$\vec{V}_A = \vec{V}_{cm} + \vec{\omega} \times \vec{r}_{A/cm}$$

$$= \frac{9}{5}\hat{i} - \sqrt{4g}\ \hat{j} + (-1.8\,\hat{k}) \times \left(\frac{2}{3} l \sin\theta\,\hat{i} + \frac{2}{3} l \cos\theta\,\hat{j}\right)$$

$$\vec{V}_A = 9/5\,\hat{i} - 2\sqrt{g}\ \hat{j} - 1.2 \sin\theta\,\hat{j} + 1.2 \cos\,\hat{i}$$

c) Total Kinetic Energy

$$= \frac{1}{2}(3m)V_{cm}^2 + (3m)g(h) + \frac{1}{2} I_{cm}(\omega)^2$$

$$= \left(\frac{15}{2}\right)\left(\frac{9}{5}\right)^2 + \left(\frac{1}{2}\right)\left(\frac{2}{3} m l^2\right)(1.8)^2 + 3mg(h)$$

$$= \frac{243}{10} + \frac{5}{3} \times (1.8)^2 + 15 \cdot 32.66 \times 2$$

$$KE = 24.3 + 5.4 + 30 \times 32.66$$

$$K.E = 1009.5 \quad \frac{lb\ ft^2}{sec^2}$$

16.2.11

(i) $W = F\Delta S$

$$LHS = \left[\frac{Kg\ M^2}{sec^2}\right]$$

$$RHS = \left[\frac{Kg\ M}{sec^2}\right] \cdot \left[M\right] = \left[\frac{Kg \cdot M^2}{sec^2}\right]$$

(ii) $W = M\Delta\theta$

$$LHS = \left[\frac{Kg\ M^2}{sec^2}\right]$$

$$\left[\begin{array}{l} \because [M][rad] = [M] \\ \because r\theta^{c} = s \end{array}\right]$$

$$RHS = \left[\frac{Kg\ M^2}{sec^2}\right][rad] = \left[\frac{Kg\ M^2}{sec^2}\right]$$

16.2.12

$$K.E = \frac{1}{2} m \vec{v} \cdot \vec{v} + \frac{1}{2} I \vec{\omega} \cdot \vec{\omega}$$

$$K.E = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} I \omega^2$$

16.2.13

$$E = \frac{1}{2} K \delta^2 = \frac{1}{2} \cdot 100 \cdot \left(\frac{100}{1000}\right)^2$$

$$= 0.5 \text{ Joules}$$

16-2.14

Input power

$$= \vec{F} \cdot \vec{V}$$

$$= 50\,\hat{j} \cdot 2\,\hat{j}$$

$$= 100 \text{ Joules}$$

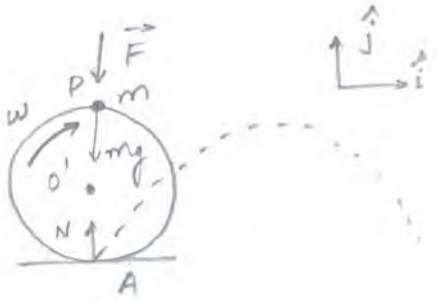
16-2.15

a) Input power $= \vec{M} \cdot \vec{\omega} = M_{in}\,\omega_{in}$

b) output torque $\vec{M}_{out} \cdot \vec{\omega}_{out} = \vec{M}_{in} \cdot \vec{\omega}_{in}$

$$M_{out} = \frac{M_{in}\,\omega_{in}}{\omega_{out}}$$

16.3.2



$$\vec{a}_A = -\omega^2 r \hat{j}$$

$$\vec{F} = m\vec{a}$$

$$\vec{a}_{P/O} = \vec{a}_{A/O} + \vec{a}_{P/A}$$

$$= \omega^2 r \hat{j} - 2\omega^2 r \hat{j}$$

$$= -\omega^2 r \hat{j}$$

Force required to hold the disk

$$m\vec{g} + \vec{N} + \vec{F} = m\vec{a}$$

$\vec{N}$ can only be positive

$$\vec{F} + \vec{N} = -m\omega^2 r \hat{j} + mg\hat{j}$$

so, if $g > \omega^2 r$

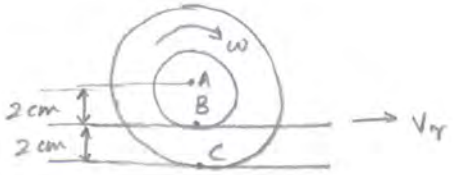
$\vec{N} = (mg - m\omega^2 r)\hat{j}$ and $\vec{F}_{required} = 0$

if $g < \omega^2 r$

then,

$$\vec{F}_{min} = -(m\omega^2 r - mg)\hat{j}$$

1b · 3.4



$V_r = 1\,m/sec$

$\vec{V}_{B/A} = -\omega r \hat{i}$

$\vec{V}_{C/A} = -2\omega r \hat{i}$

$\vec{V}_{A/G} = 2\omega r \hat{i}$

$\vec{V}_{B/G} = \vec{V}_{B/A} + \vec{V}_{A/G} = -\omega r \hat{i} + 2\omega r \hat{i} = 20 \hat{i}$

$\omega = \frac{20}{r} = -\frac{20}{2} \times 100$

$\omega = -1000\ rad/sec^2$

16.4.1

Rolling rate = constant = ω

$$\vec{V}_c = \vec{\omega} \times \vec{r} \Rightarrow \vec{V}_c = -\omega \hat{k} \times r\hat{j} \Rightarrow V_c = r\omega \hat{i}$$

$$K.E = \frac{1}{2} m \vec{V}_c \cdot \vec{V}_c + \frac{1}{2} I (\vec{\omega} \cdot \vec{\omega})$$

$$= \frac{1}{2} m (r\omega)^2 + \frac{1}{2} \frac{mr^2}{2} \omega^2$$

$$K.E = \frac{3}{4} m r^2 \omega^2$$

16.4.2

$$I = m \times \frac{r^2}{4}$$

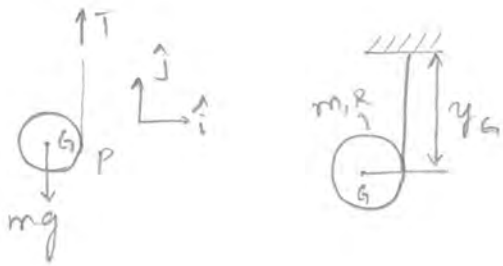
$$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} = -\omega \hat{k} \times \frac{3r}{2} \hat{j} = \frac{3\omega r}{2} \hat{i}$$

$$T_B = \frac{1}{2} m (\vec{V}_{B/A} \cdot \vec{V}_{B/A}) + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} m \left(\frac{3r}{2} \omega\right)^2 + \frac{1}{2} m \left(\frac{r}{2}\right)^2 \times \omega^2$$

$$T_B = \frac{5}{4} m r^2 \omega^2$$

16.4.3



$$\sum F = m\vec{a}_G$$

$$mg(-\hat{j}) + T\hat{j} = m\vec{a}_G \} \cdot \hat{i}$$

$$\Rightarrow \quad m\vec{a}_G \cdot \hat{i} = 0 \quad \Rightarrow \quad a_{Gx} = 0 \qquad \vec{a}_G = a\hat{j}$$

$$\vec{a}_{G/P} = \dot{\vec{\omega}} \times \vec{r}_{G/P}$$

$$= \dot{\vec{\omega}} \times R(-\hat{i})$$

$$= -\dot{\omega} R \hat{j}$$

$$\vec{a}_G = -\dot{\omega} R \hat{j}$$

$$\therefore a_G = -\dot{\omega} R$$

$$\sum \underline{M}_{/P} = \dot{\underline{H}}_{/P}$$

$$R(-\hat{i}) \times mg(-\hat{j}) = R(-\hat{i}) \times m a_G + \underline{\underline{I}}^{cm} \cdot \dot{\omega} \hat{k}$$

$$= -R\hat{i} \times m a \hat{j} + mR^2 \cdot \dot{\omega} \hat{k}$$

$$mgR\hat{k} = -maR\hat{k} + mR^2\left(-\frac{a}{R}\right)\hat{k}$$

$$mgR\hat{k} = -2maR\hat{k}$$

$$a = -\frac{g}{2}$$

$$V = \int a\,dt$$

$$V = \frac{-gt}{2} + A$$

$$y = \int v\,dt = -\frac{g}{2}\frac{t^2}{2} + At + B$$

At $t=0$, $y = y_G(-\hat{j})$ & $\dot{y} = 0$

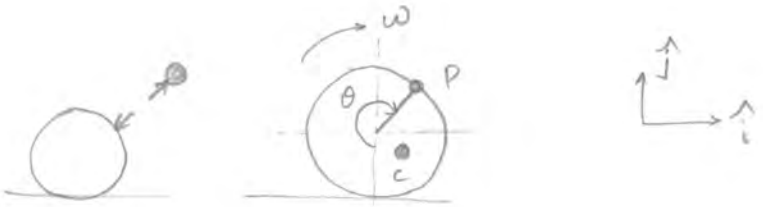
$\Rightarrow$ $A=0$, $B = -y_G\hat{j}$

$$\vec{y} = \left(-\frac{g}{4}t^2 - y_G\right)\hat{j}$$

$$\vec{a}_G = a\hat{j} = -\frac{g}{2}\hat{j}$$ Horizontal acceleration is 0.

$\therefore$ The hoop doesnot moves sideways

16.4.5



$$\vec{r}_c = R\theta\,\hat{i} + R\,\hat{j}$$

$$\vec{V}_c = R\dot{\theta}\,\hat{i} \qquad \vec{\omega} = -\dot{\theta}\,\hat{k}$$

$$\dot{\vec{\omega}} = -\ddot{\theta}\,\hat{k}$$

$$\vec{r}_P = \vec{r}_c + \vec{r}_{P/c} = R\theta\,\hat{i} + R\,\hat{j} + (-R\cos\theta\,\hat{j} - R\sin\theta\,\hat{i})$$

$$= R(\theta - \sin\theta)\,\hat{i} + R(1-\cos\theta)\,\hat{j}$$

$$\vec{V}_P = R\left[(\dot{\theta} - \cos\theta\,\dot{\theta})\,\hat{i} + \sin\theta\,\dot{\theta}\,\hat{j}\right]$$

$$V_P = R\dot{\theta}\left[(1-\cos\theta)\,\hat{i} + \sin\theta\,\hat{j}\right]$$

if $\theta = 90^\circ \Rightarrow V_P = R\dot{\theta}\,(\hat{i} + \hat{j})$

$\theta = 180^\circ \Rightarrow V_P = 2R\dot{\theta}\,\hat{i}$

$$|\vec{V}_P|_{max} = 2R\dot{\theta}$$

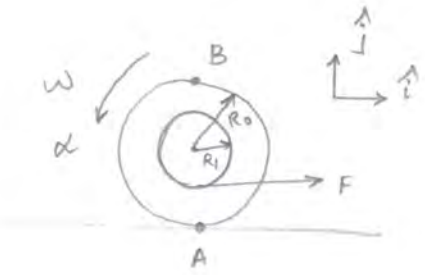
$$\vec{a}_P = \left.\frac{dV_P}{dt}\right|_{\ddot{\theta}=0} = R\dot{\theta}^2(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$

$$|a_P| = R\dot{\theta}^2\sqrt{\sin^2\theta + \cos^2\theta} = R\dot{\theta}^2$$

when $\theta = \pi$

$$\vec{a}_P = -R\dot{\theta}^2\cos\theta\,\hat{j} - g\,\hat{j}$$

16.4.7



$$\sum F = m\vec{a}_{cm}$$

$$\vec{a}_c = \vec{a}_A + \vec{a}_{c/A}$$

LMB

$$F\hat{i} = m\vec{a}_{cm}$$

$$= m\ddot{x}\hat{i} + m\ddot{y}\hat{j}$$

$$\ddot{x} = \frac{F}{m} \qquad \therefore \vec{a}_{cm} = \frac{F}{m}\hat{i}$$

AMB/c

$$\sum \vec{M}_{/c} = \dot{\vec{H}}_{/c}$$

$$-R_i\hat{j} \times F\hat{i} = \vec{r}_{cm} \times m a_{cm} + I^{cm}\dot{\vec{\omega}}\hat{k} + \vec{\omega} \times I\vec{\omega}$$

$$R_i F\hat{k} = I^{cm}\dot{\omega}\hat{k} = \frac{mR_i^2}{2}\cdot\dot{\omega}\hat{k}$$

$$\dot{\vec{\omega}} = \frac{2F}{mR_i}\hat{k}$$

$$F = \frac{mR_i\dot{\omega}}{2}$$

$$\Rightarrow \quad \vec{a}_{cm} = \frac{mR_i\dot{\vec{\omega}}}{2m} = \frac{R_i\dot{\vec{\omega}}}{2}$$

$$\vec{a}_B = \vec{a}_c + \vec{a}_{B/c} = \vec{a}_{cm} + \left[\dot{\vec{\omega}} \times \vec{R}_o\right] + \vec{\omega} \times (\vec{\omega} \times \vec{R}_o)$$

$$= \vec{a}_{cm} + \frac{2}{mR_i} F\hat{k} \times R_o\hat{j} + \omega\hat{k} \times (\omega\hat{k} \times R_o\hat{j})$$

$$= \frac{F}{m}\hat{i} + \frac{2R_oF}{mR_i}(-\hat{i}) - \omega^2 R_o\hat{j}$$

$$\vec{a}_B = \frac{F}{m}\left(1 - \frac{2R_o}{R_i}\right)\hat{i} - \omega^2 R_o\hat{j}$$

$$\vec{a}_B = \frac{F}{m}\left(1 - \frac{2R_o}{R_i}\right)\hat{i}$$

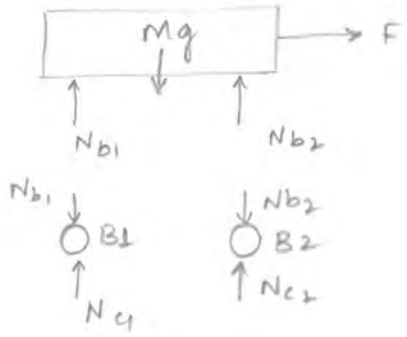
$$\dot{\omega} = \frac{2F}{mR_i}$$

$$\omega = \frac{2F}{mR_i}t + C_1$$

$$\omega = \frac{2F}{mR_i}t$$

$$t = 0,\ \omega = 0$$

16.4.10



$$\sum \vec{F} = m\vec{a}_b$$

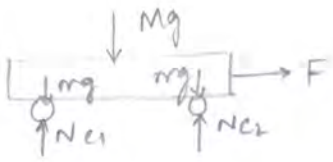
$$F\hat{i} - Mg\hat{j} + N_{b_1}\hat{j} + N_{b_2}\hat{j} = M\vec{a}_b$$

$$\vec{a}_b = a_x\hat{i} + a_y\hat{j}$$

a) $F = Ma_x \therefore a_x = \frac{F}{M}$

$$\therefore \vec{a}_b = \frac{F}{M}\hat{i}$$

b)

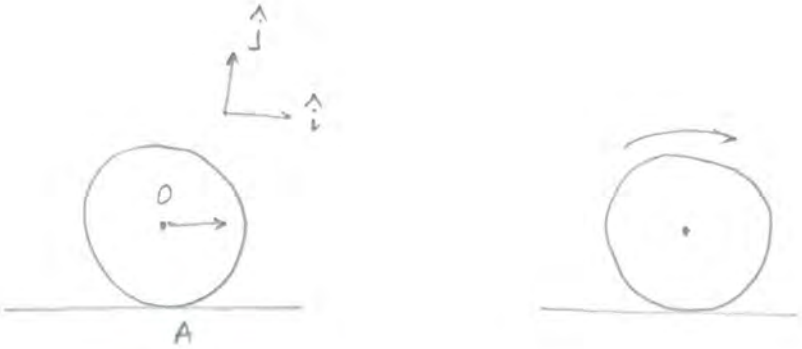


$$\sum \vec{F} = m\vec{a}_{cm}$$

$$F\hat{i} + N_{c_1}\hat{j} + N_{c_2}\hat{j} - \underbrace{Mg\hat{j} - 2mg\hat{j}}_{N_{c1}+N_{c2} = Mg + 2mg} = (M+2m)\,\vec{a}_{cm}$$

$$\vec{a}_{cm} = \frac{F}{M+2m}\hat{i}$$

16.4.12



Just before rolling

$$^{1}\underline{H}_{/A} = \underline{r}_{O/A} \times m\,\underline{v}_{cm} \longrightarrow (1)$$

During rolling

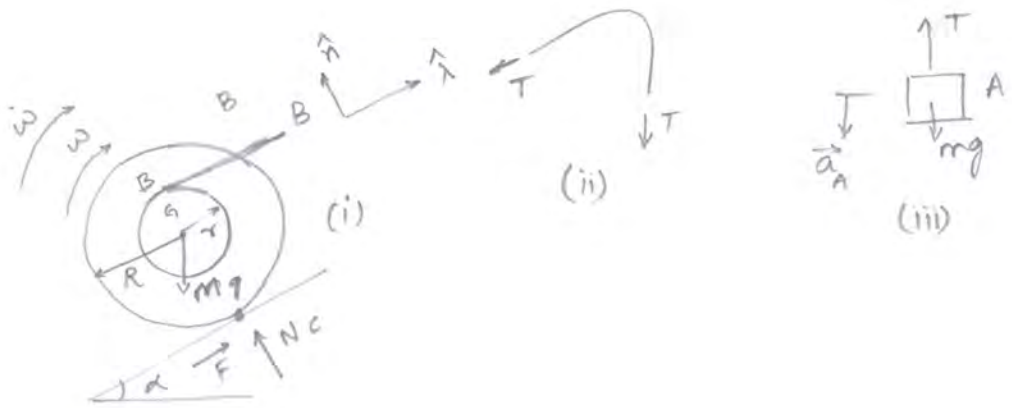
$$^{2}\underline{H}_{/A} = \underline{r}_{O/A} \times m\,\underline{v}_{f} + \underline{\underline{I}}^{cm} \cdot \underline{\omega}_{f} \longrightarrow (2)$$

Equating (1) and (2)

$$-mRV_0 = -RmV_f - \frac{mr^2}{2} \times \frac{V_f}{r}$$

$$-V_0 = -V_f - \frac{V_f}{2} \Rightarrow V_0 = \frac{3V_f}{2} \Rightarrow V_f = 2\,m/s$$

16.4.15



$$\sum \underline{M}_{/c} = \dot{H}_{/c}$$

$$R\hat{n} \times mg(-\hat{j}) + (R+r)\hat{n} \times T\hat{\lambda} = \underline{R} \times m\underline{a}_G + \underline{I}^{cm}\dot{\omega}\hat{k}$$

$$mgR\sin\alpha\,\hat{k} - (R+r)T\hat{k} = R\hat{n} \times m(-\dot{\omega}R)\hat{\lambda} + \underline{I}\dot{\omega}\hat{k}$$

$$(mgR\sin\alpha - (R+r)T)\hat{k} = (mR^2\dot{\omega} + \underline{I}\dot{\omega})\hat{k} \longrightarrow (1)$$

Now, From the FBD of the mass 'm'.

$$T\hat{j} - mg\hat{j} = m\underline{a}_A$$

$$|a_A| = |a_B|$$

$$\therefore \quad (T - mg) = m(\underline{a}_G + a_{B/G})$$

$$(T - mg) = m(\dot{\omega}R + \dot{\omega}r)$$

$$T = mg - m\dot{\omega}(R+r)$$

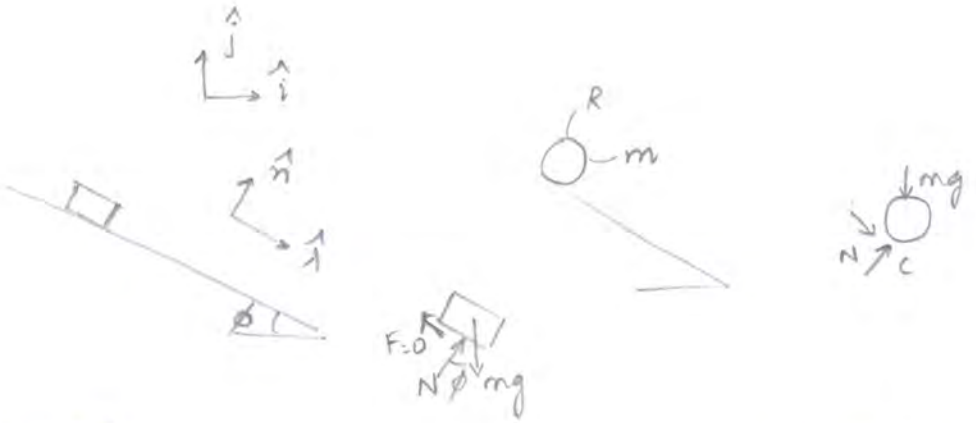
on substituting the value in (I).

$$mgR\sin\alpha - mg(R+r) + m\dot{\omega}(R+r)^2 = (mR^2 + I)\dot{\omega}$$

$$\dot{\omega} = \frac{mg(R\sin\alpha - (R+r))}{(mR^2 + I - m(R+r)^2)}$$

$$\underline{a}_{cm} = \frac{mgR[R\sin\alpha - (R+r)]}{(mR^2 + I - m(R+r)^2)}\hat{\lambda}$$

16.4.16



$$\sum \vec{F} = m\vec{a}$$

$$N\hat{n} - mg\hat{j} = ma\hat{\lambda}$$

$$N\hat{n} + mg\sin\phi\,\hat{\lambda} - mg\cos\phi\,\hat{n} = ma\hat{\lambda}$$

$$\hat{j} = -\sin\phi\,\hat{\lambda} + \cos\phi\,\hat{n}$$

$\{\ \}\cdot\hat{\lambda} \Rightarrow \quad a = g\sin\phi$

$\{\ \}\cdot\hat{n} \Rightarrow \quad N = mg\cos\phi$

$$s = \frac{1}{2}at^2 \Rightarrow L = \frac{1}{2}g\sin\phi$$

$$\vec{a}_G = \dot{\vec{\omega}} \times R\hat{n}$$

$$= -\dot{\omega}\hat{k} \times R\hat{n}$$

$$\vec{a}_G = a_G\hat{\lambda}$$

$$\sum M_{/c} = \dot{\underline{H}}_{/c}$$

$$R\hat{n} \times mg(-\hat{j}) = R\hat{n} \times m\vec{a}_G + I^{cm}(-\dot{\omega})\hat{k}$$

$$-mgR\sin\phi\,\hat{k} = -ma_G R\hat{k} - mRa_G\hat{k}$$

$$a_G = \frac{g}{2}\sin\phi$$

$$t^2 \propto \frac{1}{a}$$

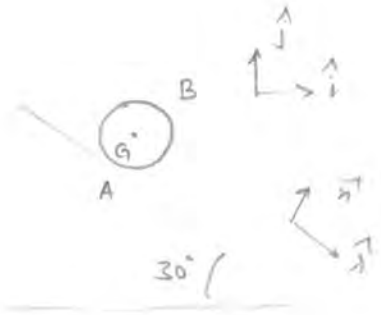
$$\frac{t_h^2}{t_b^2} = \frac{a_b}{a_h} \quad \times t_h^2 = \frac{a_b}{a_h}\cdot t_b^2$$

$$= \frac{g\sin\phi}{\frac{g\sin\phi}{2}}$$

$$t_h = \sqrt{2}\,t_b$$

The hoop takes $\sqrt{2}$ times longer than the mass

16.4.17



a) $a_G = ?$ (from previous problem)

$$a_G = \frac{g}{2}\sin\phi = \frac{g}{4}\hat{\lambda}$$

or

$$a_G = \dot{\vec{\omega}} R \hat{\lambda}$$

b) $a_A = ?$

$$\vec{a}_A = \vec{a}_G + \vec{a}_{A/G}$$

$$\vec{a}_G = \dot{\omega} R \hat{\lambda}$$

$$\vec{a}_A = \vec{a}_G + \vec{a}_{A/G} = \omega^2 R \hat{n}$$

$$\vec{a}_{A/G} = \dot{\vec{\omega}} \times R(-\hat{n}) + \vec{\omega} \times (\vec{\omega} \times R(-\hat{n}))$$

$$= -\dot{\vec{\omega}}\hat{k} \times R(-\hat{n}) + (-\omega\hat{k}) \times (-\omega\hat{k} \times -R\hat{n})$$

$$= \dot{\vec{\omega}} R(\hat{k} \times \hat{n}) - \omega\hat{k} \times \omega R(-\hat{\lambda})$$

$$\vec{a}_{A/G} = -\dot{\omega} R \hat{\lambda} + \omega^2 R \hat{n}$$

c) $\vec{a}_B = ?$

$$\vec{a}_B = \vec{a}_G + \vec{a}_{B/G}$$

$$\vec{a}_{B/G} = -\dot{\omega}\hat{k} \times R(\hat{n}) + \left[-\omega\hat{k} \times (-\omega\hat{k} \times R\hat{n})\right]$$

$$= \dot{\omega} R \hat{\lambda} - \omega^2 R \hat{n}$$

Now,

$$\vec{a}_B = 2\dot{\omega} R \hat{\lambda} - \omega^2 R \hat{n}$$

d)

As $a_G = \dot{\omega} R = \frac{g}{2} \sin 30^\circ = \frac{g}{4}$

$\dot{\omega} = \frac{g}{4R} \Rightarrow \omega = \frac{gt}{4R} + C_1$ at $t=0$, $\omega = 0$

$C_1 = 0$

$\Rightarrow \omega = \frac{gt}{4R} \Rightarrow \theta = \frac{1}{2} \frac{g}{4R} t^2$

No slip $\Rightarrow$

$x = R\theta \Rightarrow L = R \cdot \frac{1}{2} \frac{g}{4R} t^2$

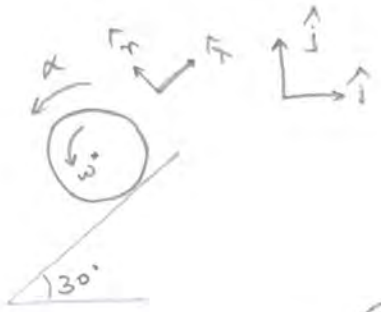
$t = \sqrt{\frac{8L}{g}} = 4\sqrt{\frac{y}{g}}$

$\omega = \frac{g4}{4R}\sqrt{\frac{y}{g}} = \sqrt{\frac{yg}{R^2}} = \frac{\sqrt{2g}}{R}$

$\therefore \vec{a}_B = 2 \cdot \frac{g}{4} \hat{\lambda} - \frac{2g}{R^2} \cdot R \cdot \hat{n}$

$\vec{a}_B = \frac{g}{2} \hat{\lambda} - \frac{2g}{R} \hat{n}$

16.4.18



a) $\sum \underline{M}_{/c} = \dot{\underline{H}}_{/c}$

$$r\hat{n} \times mg(-\hat{j}) = r\hat{n} \times m\vec{a}_G + I^{cm}\dot{\omega}\hat{k}$$

$$\sin\alpha \, mgr\hat{k} = mr^2\dot{\omega}\hat{k} + \frac{mr^2}{2}\dot{\omega}\hat{k}$$

$$\dot{\omega} = \frac{2g\sin\alpha}{3r} \Rightarrow \dot{\omega} = \frac{g}{3r}$$

b) Minimum value of co-efficient of friction to ensure no slip

$$\sum \vec{F} = m\vec{a}_G$$

$$N\hat{n} - F\hat{\lambda} - mg\hat{j} = ma_G(-\hat{\lambda})$$

$$\Rightarrow \left\{ N\hat{n} - F\hat{\lambda} - mg\sin\hat{\lambda} - mg\cos\alpha\,\hat{n} = -m\dot{\omega}r\hat{\lambda} \right\}$$

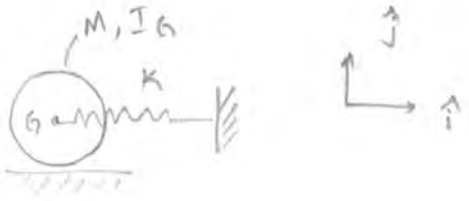
$$\{\}\cdot\hat{n} \Rightarrow N = mg\cos\alpha \;, \quad \{\}\hat{\lambda} \Rightarrow -F - mg\sin\alpha = -m\dot{\omega}r = -\frac{mg}{3}$$

$$F = \frac{mg}{3} - \frac{mg}{2} \Rightarrow F = -\frac{mg}{6} \Rightarrow F = \mu N \;, \quad \mu = \frac{F}{N}$$

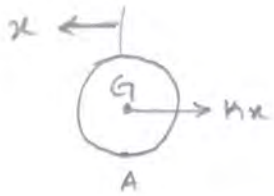
$$\mu = \frac{-\frac{mg}{6}}{mg\cos\alpha} = \frac{1}{3\sqrt{3}}$$

Any $\mu < \frac{1}{3\sqrt{3}}$, at $\theta > 30°$ results in cylinder slip

16.4.20



a) No friction



$$\sum \underline{M}_{/A} = \dot{\underline{H}}_{/A}$$

$$r\hat{j} \times Kx\,\hat{i} = r\hat{j} \times m\vec{a}_G$$

$$Krx(-\hat{k}) = -r\hat{j} \times m\ddot{x}\,\hat{i} = m\ddot{x}r\hat{k}$$

$$m\ddot{x} + Kx = 0 \Rightarrow \quad \omega = \sqrt{\frac{K}{m}}, \quad T = 2\pi\sqrt{\frac{m}{K}}$$

b) Perfect rolling

$$\sum \underline{M}_{/A} = \dot{\underline{H}}_{/A}$$

$$r\hat{j} \times Kx\,\hat{i} = r\hat{j} \times m\ddot{x}(-\hat{i}) + I\dot{\omega}\hat{k}$$

$$Kxr(-\hat{k}) = m\ddot{x}r\hat{k} + I\frac{\ddot{x}}{r}\hat{k}$$

$$\left(m + \frac{I}{r^2}\right)\ddot{x} + Kx = 0$$

$$\omega = \sqrt{\frac{K}{m + \frac{I}{r^2}}}$$

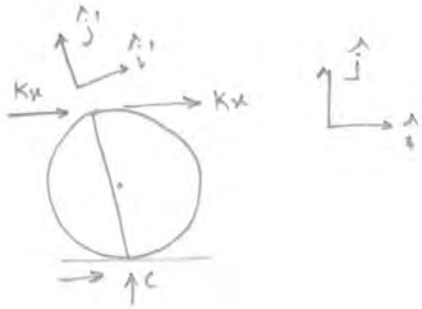
$$T = 2\pi\sqrt{\frac{m + \frac{I}{r^2}}{K}}, \quad I = \frac{mr^2}{2}$$

$$T = \left(\sqrt{\frac{3}{2}}\right) 2\pi\sqrt{\frac{m}{K}}$$

16.4.21

$$\sum \underline{M}_{/c} = \dot{\underline{H}}_{/c}$$

a) $\cos\alpha\, 2R\hat{j} \times 2Kx\hat{i} = R\hat{j} \times \underbrace{m\underline{a}_{cm}}_{-m\dot{\omega} r \hat{i}} + \underline{I}^{cm}\dot{\omega}\hat{k}$



$$\cos\alpha(-4KxR)\hat{k} = m\dot{\omega}R^2\hat{k} + \frac{mR^2}{2}\dot{\omega}\hat{k}$$

$$= mR\ddot{x}\hat{k} + mR\frac{\ddot{x}}{2}\hat{k}$$

$$(\cos\alpha)(-4KxR\hat{k}) = \frac{3}{2}mR\ddot{x}\hat{k}$$

$$\frac{3}{2}m\ddot{x} + 4Kx\cos\alpha = 0$$

$$m\ddot{x} + \frac{8}{3}Kx\cos\alpha = 0$$

$$\omega = \sqrt{\frac{8K\cos\alpha}{3m}}$$

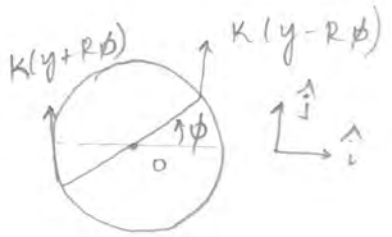
Differential Eqn

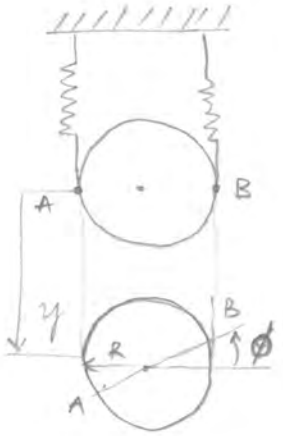
$$3m\ddot{x} + 8K\cos\alpha = 0$$

b) For small oscillations $\cos\alpha = 1$

$$f = \frac{1}{2\pi}\sqrt{\frac{8K}{3m}}$$

16.4.22





$$\sum \underline{M}_{/o} = \dot{H}_{/o}$$

Let $\phi =$ small

$$I\ddot{\phi} = KR(y - R\phi) - KR(y + R\phi) \Rightarrow I\ddot{\phi} = -KR^2\phi - KR^2\phi$$

$$m\ddot{\phi} + 4K\phi = 0$$

$$\omega_1 = \sqrt{\frac{4K}{m}}$$

Now,

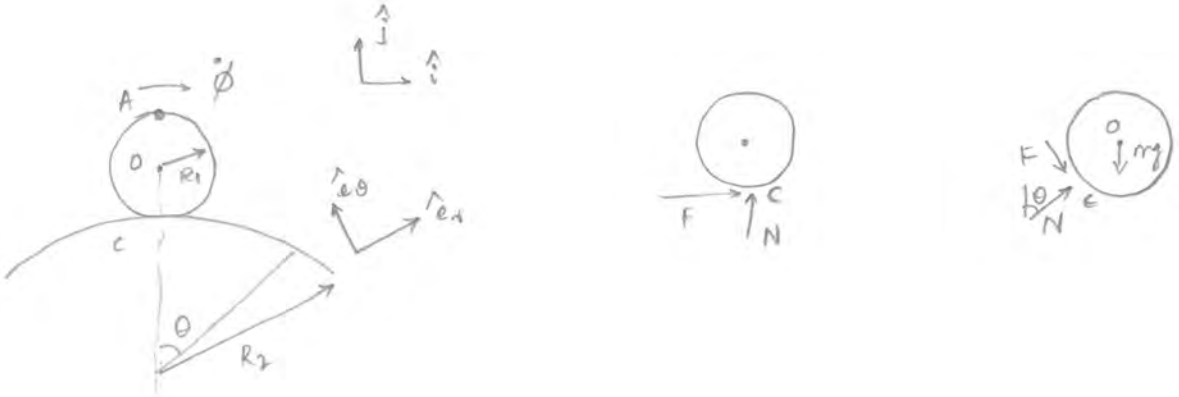
$$\sum F = m\vec{a}$$

$$\left[K(y - R\phi) + K(y + R\phi)\right]\hat{j} = m\ddot{y}(-\hat{j})$$

$$m\ddot{y} + 2Ky = 0$$

$$\omega_2 = \sqrt{\frac{2K}{m}}$$

16.4.24



'O' - goes in circles with radius (R_1+R_2)

$$\underline{V}_O = \underline{\omega} \times (R_1+R_2)\hat{e}_r$$

$$= \dot{\theta}(-\hat{k}) \times (R_1+R_2)\hat{e}_r$$

$$\underline{V}_O = \dot{\theta}(R_1+R_2)(-\hat{e}_\theta)$$

$$\underline{a}_O = \underline{\omega} \times \underline{\omega} \times (R_1+R_2)\hat{e}_r + \dot{\underline{\omega}} \times (R_1+R_2)\hat{e}_r$$

$$= -\dot{\theta}\hat{k} \times (-\dot{\theta}\hat{k} \times (R_1+R_2)\hat{e}_r) - \ddot{\theta}\hat{k} \times (R_1+R_2)\hat{e}_r$$

$$= \dot{\theta}\hat{k} \times (R_1+R_2)\dot{\theta}\hat{e}_\theta - (R_1+R_2)\ddot{\theta}\hat{e}_\theta$$

$$= -(R_1+R_2)\dot{\theta}^2\hat{e}_r - (R_1+R_2)\ddot{\theta}\hat{e}_\theta$$

$$\hat{j} = \cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta$$

$$\sum \underline{F} = m\underline{a}$$

$$\left\{ -F\hat{e}_\theta + N\hat{e}_r - mg\hat{j} = m\left[-(R_1+R_2)(\dot{\theta}^2\hat{e}_r + \ddot{\theta}\hat{e}_\theta)\right]\right\}$$

$$\{\ \}\cdot\hat{e}_r \Rightarrow \quad N - mg\cos\theta = -m(R_1+R_2)\dot{\theta}^2$$

Hoop looses contact $\Rightarrow N = 0$

$$\Rightarrow \quad \dot{\theta}^2 = \frac{g\cos\theta}{(R_1+R_2)}$$

Energy Balance

$$E_1 = E_2$$

$$E_1 = P.E = mg(R_1+R_2)$$

$$E_2 = K.E + P.E = \frac{1}{2}mV_0^2 + mg(R_1+R_2)\cos\theta + \frac{1}{2}I\omega^2$$

No slip

$$V_0 = \underline{\omega} \times R_1\hat{j}$$

$$= -\dot{\phi}\hat{k} \times R\hat{e}_r$$

$$V_0 = \dot{\phi}R_1(-\hat{e}_\theta)$$

$$\Rightarrow \dot{\phi} = \left(1 + \frac{R_2}{R_1}\right)\dot{\theta}$$

$$\Rightarrow mg(R_1+R_2) = \frac{1}{2}m(\dot{\phi}R_1)^2 + mg(R_1+R_2)\cos\theta + \frac{1}{2}mR_1^2\dot{\phi}^2$$

$$= \frac{2}{2}m\left(1 + \frac{R_2}{R_1}\right)^2\dot{\theta}^2R_1^2 + mg(R_1+R_2)\cos\theta$$

$$\Rightarrow mg(R_1+R_2)(1-\cos\theta) = \frac{2}{2}m(R_1+R_2)^2\dot{\theta}^2$$

$$g(1-\cos\theta) = (R_1+R_2)\frac{g\cos\theta}{(R_1+R_2)}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 60^\circ$$

Angular velocity at that instant,

$$\dot{\phi} = \frac{R_1 + R_2}{R_1} \cdot \sqrt{\frac{g\cos\theta}{R_1+R_2}}$$

$$\underline{\dot{\phi}} = \underline{\omega} = \frac{1}{R_1}\sqrt{\frac{g(R_1+R_2)}{2}}\ (-\hat{k})$$

$$\underline{V} = \sqrt{\frac{g(R_1+R_2)}{2}}\ (-\hat{e}_\theta)$$

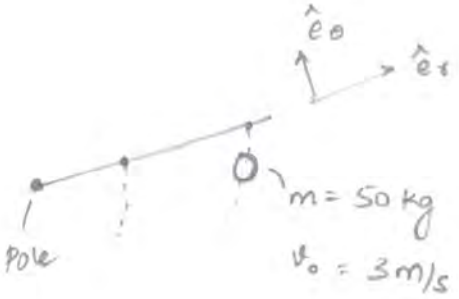
16.5.3

Angular momentum about 'O' is conserved

$$H_o = \vec{r} \times m\vec{v}$$

$$= r\hat{e}_r \times m v_o \hat{e}_\theta$$

$$= m r v_o \hat{k}$$



$$H_o^+ = \vec{r} \times m\vec{v} = m r v_o$$

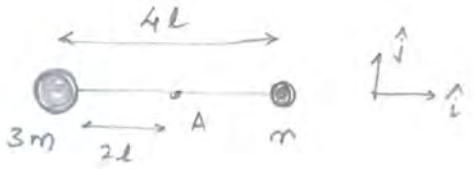
$$\frac{mr}{2} v \hat{k} = r m v_o \hat{k}$$

$$V = 2v_o$$

Speed of skater at 5 m distance is 6 m/s.

16.5.4

Before collision



$L = 4mV_0$

$\vec{H}^- = \vec{H}^+$

$\vec{H}^+ = \vec{r} \times mV^+ + I^{cm}\omega^+$

$\vec{H}^+ = 2l\hat{i} \times mv\hat{j} + 2l(-\hat{i}) \times (3m)V^+\hat{j}$

$= 2l\hat{i} \times m(2l\omega)\hat{j} + 2l(-\hat{i}) \times 3m(-2l\omega\hat{j})$

$= 4l^2m\omega\hat{k} + 12ml^2\omega\hat{k}$

$\vec{H}^- = 16ml^2\omega\hat{k}$

$V_m = \omega\hat{k} \times 2l\hat{i}$

$V_M = \omega\hat{k} \times 2l(-\hat{i})$

$V_m = 2l\omega\hat{j}$

$V_M = -2l\omega\hat{j}$

$\underline{H}^-_{/A} = \underline{I}^{cm}\omega^-\hat{k} + r \times mv^-$

$= 2l\hat{i} \times m v_0(-\hat{j}) + (-2l)\hat{i} \times 3m(-v_0\hat{j})$

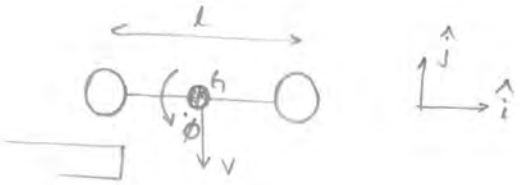
$= -2mlv_0\hat{k} + 6mlv_0\hat{k}$

$\therefore \quad 4mlv_0\hat{k} = 16ml^2\omega\hat{k}$

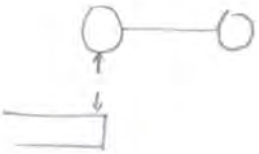
$\omega = \dfrac{v_0}{4l}$

$\vec{\omega} = \dfrac{v_0}{4l}\hat{k}$

16.5.5



$$\vec{H}^- = \vec{H}^+$$



$$I^{cm}\dot{\phi}\hat{k} + \frac{l}{2}\hat{i} \times mv.-\hat{j} + \frac{l}{2}(-\hat{i}) \times mv(-\hat{j})$$

$$\vec{H}^- = I^{cm}\dot{\phi}\hat{k}$$

$$m\left(\underline{v}_1^+ + \left(v + \frac{\dot{\phi}l}{2}\right)\hat{j}\right) = P\hat{j}$$

$$I^{cm}(\omega^+ - \omega^-) = \underline{r}_{/G} \times P\hat{j}$$

$$2m\left(\frac{l}{2}\right)^2\left[\underline{\omega}^+ - \dot{\phi}\hat{k}\right] = \frac{l}{2}(-\hat{i}) \times P\hat{j}$$

$$\frac{ml^2}{2}(\underline{\omega}^+ - \dot{\phi}\hat{k}) = \frac{Pl}{2}-\hat{k} = -2m\left(v + \frac{\dot{\phi}l}{2}\right)\frac{l}{2}\hat{k}$$

$$\underline{\omega}^+ - \dot{\phi}\hat{k} = -\frac{(2v + \dot{\phi}l)}{l}\hat{k}$$

$$\underline{\omega}^+ = -\frac{2v}{l}\hat{k}$$

$$m(\underline{v}_1^+ - \underline{v}_1^-) = P_j$$

$$v_1^- = v_{cm} + \dot{\phi}\hat{k} \times \frac{l}{2}(-\hat{i})$$

$$= v(-\hat{i}) - \frac{\dot{\phi}l}{2}\hat{j} = -\left(v + \frac{\dot{\phi}l}{2}\right)\hat{j}$$

$$\left\{(\underline{v}_t^+ - \underline{v}_1^+) = -e(\underline{v}_t^- - \underline{v}_1^-)\right\}\cdot\hat{j}$$

$$(0 - v_1^+\cdot\hat{j}) = -\left[0 - v_1^-\cdot\hat{j}\right]$$

$$-v_1^+\hat{j} = v_1^-\cdot\hat{j} = -\left(v + \frac{\dot{\phi}l}{2}\right)\hat{j}$$

$$\therefore \underline{v}_1^+ = \left(v + \frac{\dot{\phi}l}{2}\right)\hat{j}$$

$$KE^{\text{rotation}+} = \frac{1}{2}I\omega^{+2}$$

$$= \frac{1}{2}\frac{ml^2}{2}\cdot\frac{4v^2}{l^2} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \text{K.E Translation}^-$$

$$KE^- = \frac{1}{2}m\left(v + \frac{\dot{\phi}l}{2}\right)^2 + \frac{1}{2}m\left(v - \frac{\dot{\phi}l}{2}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}m\left(\frac{\dot{\phi}l}{2}\right)^2 + \frac{1}{2}m\left(\frac{\dot{\phi}l}{2}\right)^2$$

$$= \text{K.E Trans}^- + \frac{ml^2}{4}\dot{\phi}^2$$

$$= K.E^{Trans^-} + \frac{1}{2}\left(\frac{ml^2}{2}\right)\dot{\phi}^2$$

$$= K.E^{Trans^-} + K.E^{rot^-}$$

$K.E^{Trans^+}$

$$V_2^+ = V_1^+ + (\underline{\omega} \times l\hat{i})$$

$$V_2^+ = \left(V + \frac{\dot{\phi} l}{2}\right)\hat{j} + \left[\frac{-2V}{l}\hat{k} \times l\hat{i}\right]$$

$$= \left(V + \dot{\phi}\frac{l}{2}\right)\hat{j} - 2V\hat{j}$$

$$\underline{V}_2^+ = \left(-V + \frac{\dot{\phi} l}{2}\right)\hat{j}$$

$$K.E^+ = \frac{1}{2}m\left[\left(V + \frac{\dot{\phi} l}{2}\right)\hat{j}\right]^2 + \frac{1}{2}m\left[\left(-V + \dot{\phi}\frac{l}{2}\right)\hat{j}\right]^2$$

$$= \frac{1}{2}mV^2 + \frac{1}{2}mV^2 + \frac{1}{2}m\dot{\phi}^2\frac{l^2}{2}$$

16.5.6

$$\underline{H}^- = \underline{H}^+$$

$$\underline{r} \times m\underline{v}_{cm} + \underline{I}^{cm}\,\underline{\omega}^-\hat{k} = \underline{I}^{cm}\,\underline{\omega}^+\hat{k} + \underline{r}_{cm} \times m\underline{v}_{cm}$$

$$d\hat{e}_r \times m v_0(-\hat{e}_\theta) + \frac{ml^2}{12}\omega_0\hat{k} = \frac{ml^2}{12}\omega^+\hat{k} + d\hat{e}_r \times m(\underline{\omega}^+ \times d\hat{e}_r)$$

$$\Rightarrow \{\ \}\cdot\hat{k}$$

$$-mdv_0\hat{k} + \frac{ml^2}{12}\omega_0\hat{k} = \frac{ml^2}{12}\omega^+\hat{k} + d\hat{e}_r \times m(d\omega^+\hat{e}_\theta) = \frac{ml^2}{12}\omega^+\hat{k} + md^2\omega^+\hat{k}$$

$$\frac{ml^2}{12}\omega_0 - mdv_0 = \left(\frac{ml^2}{12} + \frac{ml^2}{4}\right)\omega^+ = \frac{1}{2}ml^2\omega^+$$

$$\omega^+ = \frac{3}{l^2}\left[\frac{l^2}{12}\omega_0 - \frac{l}{2}v_0\right]$$

$$\omega^+ = \frac{1}{4}\left(\omega_0 - \frac{3v_0}{d}\right)$$

$$\omega^+_{los} = \omega^+ = \text{constant}$$

$$KE^+ = \frac{1}{2}I\omega_f^2 = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\left[\frac{1}{4}\left(\omega_0 - \frac{3v_0}{d}\right)\right]^2$$

$$KE^- = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{1}{12}ml^2\right)\omega_0^2$$

$$E_{lost} = KE^- - KE^+$$

$$= \frac{1}{2}mv_0^2 + \frac{ml^2\omega_0^2}{32} - \frac{3mv_0^2}{8} + \frac{m}{8}l\omega_0 v_0$$

$$= \frac{1}{8}mv_0^2 + \frac{ml^2}{32}\omega_0^2 + \frac{ml\omega_0 v_0}{8} = \frac{m}{8}\left(v_0^2 + \frac{(l\omega_0)^2}{4} + 2\left(\frac{l\omega_0}{2}\right)v_0\right)$$

$$= \frac{m}{8}\left(v_0 + \frac{l\omega_0}{2}\right)^2$$

16.5.7

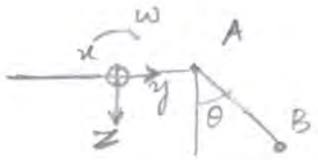
Angular momentum is conserved during impact.

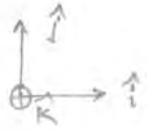
$\therefore \underline{H}^-_{/A} = \underline{H}^+_{/A}$

$\underline{H}^-_{/A} = \frac{L}{2}(-\hat{i}) \times m\left[-\omega\hat{k} \times \frac{L}{2}(-\hat{i})\right] + I^{cm}(-\omega)\hat{k}$

$= \frac{ml^2}{4}\omega(-\hat{i}) \times (\hat{j}) - mk_x^2\omega\hat{k}$

$= -\left(\frac{ml^2}{4}\omega + mk_x^2\omega\right)\hat{k}$





$\underline{H}^+_A = I^A_{zz}\,\omega^+\hat{k} = \left[mk_x^2 + m\left(\frac{L}{2}\right)^2\right]\omega^+\hat{k}$

$\underline{\omega}^+ = -\omega\hat{k} \Rightarrow \omega^+ = -\omega$

b) $\underline{V}_{cm} = (\underline{\omega} \times \underline{r})$

$= (\omega^+\hat{k} \times \frac{L}{2}\hat{\lambda})$

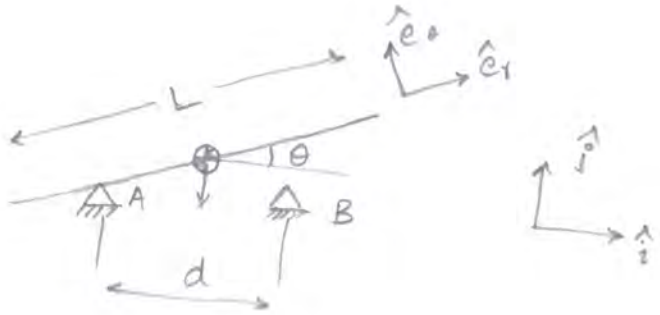
$= \left(\frac{\omega^+ l}{2}\right)\hat{n}$

$\hat{n} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$V_{cm} = \left(\frac{\omega^+ l}{2}\right)\cos\theta\,\hat{i} + \left(\frac{\omega^+ l}{2}\right)\sin\theta\,\hat{j}$

c) If the hands are released prior to the impact between the centre of mass and the lower bar, gymnast swings about her C.M as she initially has ω^+ rotation rate.

16.5.9



$$\sum \underline{M}_{/A} = \dot{\underline{H}}_{/A}$$

$$\sum \underline{M}_{/A} = \frac{d}{2}\hat{e}_r \times mg(-\hat{j})$$

$$\hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\sum \underline{M}_{/A} = -\frac{mgd}{2}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) \times \hat{j}$$

$$\sum \underline{M}_{/A} = -\frac{mgd}{2}\cos\theta\,\hat{k}$$

$$\dot{\underline{H}}_{/A} = \underline{I}^{cm}\ddot{\theta}\hat{k} + \underline{r} \times m\,\underline{a}^{cm}$$

$$= \frac{ml^2}{12}\ddot{\theta}\hat{k} + \frac{d}{2}\hat{e}_r \times \left[\ddot{\theta}\frac{d}{2}\hat{e}_\theta - \dot{\theta}^2\frac{d}{2}\hat{e}_r\right]\cdot m$$

$$= \frac{ml^2}{12}\ddot{\theta}\hat{k} + \frac{md^2}{4}\hat{k}\ddot{\theta}$$

$$\underline{a}^{cm} = (\dot{\underline{\omega}} \times \underline{r}) + [\underline{\omega} \times (\underline{\omega} \times \underline{r})]$$

$$= \ddot{\theta}\hat{k} \times \frac{d}{2}\hat{e}_r + \left(-\dot{\theta}^2\frac{d}{2}\hat{e}_r\right)$$

$$= \ddot{\theta}\frac{d}{2}\hat{e}_\theta - \dot{\theta}^2\frac{d}{2}\hat{e}_r$$

$$m\left(\frac{l^2}{12} + \frac{d^2}{4}\right)\ddot{\theta} = -\frac{mgd}{2}\cos\theta$$

$$\ddot{\theta} = -\frac{gd}{2\left(\frac{l^2}{12} + \frac{d^2}{4}\right)}\cos\theta$$

$$\ddot{\theta} = -\frac{6gd}{L^2 + 3d^2}\cos\theta$$

on integration

$$\frac{1}{2}\dot{\theta}^2\Big|_0^{\dot{\theta}_0} = -\frac{6gd}{L^2+3d^2}\sin\theta\Big|_\theta^0$$

$$\dot{\theta} = \frac{\sqrt{2 \times 6 g d \sin\theta}}{\sqrt{L^2 + 3d^2}}$$

$$\dot{\theta} = \sqrt{\frac{12 g d}{L^2 + 3d^2} \sin\theta}$$

Using Principle of work and energy

$$U = mg\left[\frac{d}{2}\sin\theta\right]$$

$$\cancel{T_1}^{0} + U = T_2$$

$$U = T_2$$

$$T_2 = \frac{1}{2} m v^2 + \frac{1}{2} I^{cm} \omega^2$$

$$T_2 = \frac{1}{2} m (\underline{\omega} \times \underline{r})^2 + \frac{1}{2} \frac{m l^2}{12} \omega^2$$

$$T_2 = \frac{1}{2} m \left(\dot{\theta}^2 \frac{d^2}{4}\right) + \frac{1}{2} \frac{m l^2}{12} \omega^2$$

$$T_2 = \frac{m \omega^2 d^2}{8} + \frac{m l^2 \omega^2}{24}$$

since $U = T_2$

$$mgd\sin\theta = \frac{m\dot{\theta}^2 d^2}{4} + \frac{m\dot{\theta}^2 l^2}{12}$$

$$\dot{\theta}^2 = \left(\frac{12 g d}{3d^2 + L^2}\right) \sin\theta$$