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The purpose of this chapter is to provide general information and guidance on lamina constituent materials, properties, manufacturing, and structural analysis techniques. This information is for guidance only. Refer to your program for specific guidance on composite materials and laminae.

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[1] In 2002, administration of MIL-HDBK-17 was transferred to Materials Sciences Corporation. Future releases will be released as Composite Materials Handbook 17, Materials Sciences Corporation, Secretariat.

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Table 3.1-1 Symbols and Nomenclature

Symbol	Description	Units
BMI	Bismaleimide	
[C]	Stiffness matrix	
C_{ij}	Stiffness matrix components	psi
CLPT	Classical Laminated Plate Theory	
E_1	Axial stiffness in the 1 direction. (Young's Modulus)	psi
E_2	Axial stiffness in the 2 direction. (Young's Modulus)	psi
E_3	Axial stiffness in the 3 direction (Young's Modulus)	psi
F_{11}^{tu}	Ultimate tensile stress allowable in the fiber direction	psi
F_{11}^{cu}	Ultimate compressive stress allowable in the fiber direction	psi
F_{22}^{tu}	Ultimate tensile stress allowable in the transverse direction	psi
F_{22}^{cu}	Ultimate compressive stress allowable in the transverse direction	psi
F_{33}^{tu}	Ultimate tensile stress allowable in the out-of-plane direction	psi
F_{33}^{cu}	Ultimate compressive stress allowable in the out-of-plane direction	psi
F_{12}^{su}	Ultimate engineering shear stress allowable in the 1-2 plane	psi
F_{13}^{su}	Ultimate engineering shear stress allowable in the 1-3 plane	psi
G_{12}	Shear stiffness in the 1-2 material plane.	psi
G_{12s}	Secant shear stiffness in the 1-2 material plane.	psi
G_{13}	Shear stiffness in the 1-3 material plane.	psi
G_{23}	Shear stiffness in the 2-3 material plane.	psi
IDAT	Integrated Detailed Analysis Toolset	
Kevlar	Du Pont Company trademark name for aramid material	
MATUTL	IDAT – Material Editing and Viewing Utility	
N	Load Cycles	
PE	Polyethylene	
PMR	Polyimide	

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$[Q]$	Reduced stiffness matrix	
Q_{ij}	Reduced stiffness components	psi
$[\overline{Q}]$	Reduced transformed stiffness matrix	
$[R]$	Reuter Matrix	
S	Load Spectrum	
$[S]$	Compliance matrix	
SiC	Silicon Carbide	
S_{ij}	Compliance matrix components	1/psi
T	Operating temperature	˚F
T_g	Glass Transition Temperature	˚F
T_{sf}	Stress Free Temperature	˚F
$[T]$	Transformation Matrix	
α_1	Coefficient of thermal expansion in material direction 1.	
α_2	Coefficient of thermal expansion in material direction 2.	
α_{12} or α_6	Coefficient of thermal expansion in material direction 1-2.	
α_3	Coefficient of thermal expansion in material direction 3.	
β_1	Coefficient of moisture expansion in material direction 1.	
β_2	Coefficient of moisture expansion in material direction 2.	
β_{12} or β_6	Coefficient of moisture expansion in material direction 1-2.	
β_3	Coefficient of moisture expansion in material direction 3.	
ΔC	% Moisture content by weight	
ΔT	Change from stress free temperature	˚F
ε_{ij}	Axial strain components (Tensor notation)	in/in
ε_{11}^{tu}	Ultimate tensile strain allowable in the fiber direction	
ε_{11}^{cu}	Ultimate compressive strain allowable in the fiber direction	
ε_{22}^{tu}	Ultimate tensile strain allowable in the transverse direction	
ε_{22}^{cu}	Ultimate compressive strain allowable in the transverse direction	
ε_{33}^{tu}	Ultimate tensile strain allowable in the out-of-plane direction	

ε_{33}^{cu}	Ultimate compressive strain allowable in the out-of-plane direction	
γ_{12}^{su}	Ultimate engineering shear strain allowable in the 1-2 plane	
γ_{13}^{su}	Ultimate engineering shear strain allowable in the 1-3 plane	
γ_{ij}	Engineering shear strain components (Tensor notation)	psi
$\mu_{ij,.kl}$	Chentsov coefficients	
ν_{12}	Poisson's ratio for the 1-2 plane	
ν_{13}	Poisson's ratio for the 1-3 plane	
ν_{23}	Poisson's ratio for the 2-3 plane	
$\nu_{i,ij}$	Coefficients of mutual influence of the first kind	
$\nu_{ij,i}$	Coefficients of mutual influence of the second kind	
σ_i	Axial stress components	psi
τ_{ij}	Shear stress components (Tensor notation)	psi

3.2   Introduction

The term "Composite" indicates that two or more materials have been combined on a macroscopic scale to form a more useful material. Constituent materials in a composite consist of a matrix material with embedded reinforcement fibers. The matrix material binds the reinforcement fibers together and transfers load to and between the fibers. The fibers provide stiffness and carry loads oriented in the fiber direction. The fiber material form is crucial to the structural advantage of composite materials. Fibers are significantly stronger and stiffer when compared to the same material in bulk form. The improved structural properties occur because the small fiber cross section has fewer material defects and dislocations. This combination of a reinforcement material in fiber form embedded within a matrix material can result in specific strength and specific stiffness properties (in the fiber direction) that are difficult to achieve with conventional homogeneous materials. Specific strength and stiffness values for commonly used aerospace materials are presented in Table 3.2-1.

Table 3.2-1 Specific Strength and Stiffness Comparison of Common Aerospace Structural Materials

	Graphite/Epoxy (unidirectional)		Kevlar Epoxy (cloth)	Glass Epoxy (cloth)	Boron Epoxy	Aluminum	Beryllium	Titanium	Steel (4130)
	High Strength	High Modulus							
Specific Strength 10^6 in	5.4	2.1	1	0.7	3.3	0.7	1.1	.8	0.34
Specific Stiffness 10^6 in	400	700	80	45	457	100	700	100	102
Density lb/in^3	0.056	0.063	0.05	0.065	0.07	.10	0.07	.16	0.28

A "composite lamina" is a single ply (or layer) of composite material. Unidirectional tape and woven fabric are the most common forms for lamina materials used in aerospace structural applications. Reinforcement fibers are bundled together as tows, and the tows are aligned side by side for unidirectional tape, or woven together for fabric. Laminae can also be created by direct placement of fiber tows using either fiber placement or filament winding techniques. A unidirectional lamina is strong and stiff in the fiber direction and relatively weak and flexible in the transverse directions.

When viewed from the macroscopic level, composite laminae can be treated analytically as homogenous orthotropic materials. The relevant homogenous orthotropic material properties of a composite lamina can be determined by a series of structural tests. Knowledge of detailed interactions between fiber and matrix materials at the micromechanics level is usually not required for structural analysis purposes. When combined with classical laminated plate theory, the macroscopic homogenous orthotropic lamina properties are sufficient to predict the strength and stiffness of a laminated composite part.

A composite material is "designed" and manufactured. Reinforcement fibers and matrix materials are combined to achieve a desired specific strength, specific stiffness, and toughness. Thermal response, moisture response, electromagnetic response and resistance to environmental and chemical exposure can be other important design considerations affecting the selection of fiber and matrix materials for an application. Material processing and storage requirements, lay-up options, tooling options, cure options, and total cost are also affected by the choice of constituent materials for a specific application. Matrix and fiber reinforcement materials are chosen and combined to create a composite material that is optimal across structural design, manufacturing, and business criteria for the intended application.

3.3   Lamina Constituent Materials

Constituent materials in a composite lamina consist of a matrix material with embedded reinforcement fibers.

3.3.1   Matrix

The matrix is an essentially homogeneous material in which the fiber system of a composite is embedded. The matrix binds the reinforcement fibers together and transfers load to and between fibers. The matrix protects and supports the fibers. The matrix is usually an organic resin, but metal, carbon, ceramic, and other materials are also used. The matrix properties influence: compressive strength, transverse strength

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and stiffness, interlaminar shear strength, service operating temperature, fabrication process and tool design, fracture toughness, and more.

Resin is an organic polymer or prepolymer[1] used as a matrix to contain the fibrous reinforcement in a composite material. Resin is also used as an adhesive. This organic form of matrix material may be a thermoset or a thermoplastic, and may contain a wide variety of components or additives to influence: handleability, toughness, processing, and ultimate properties.

3.3.2   Thermoset Resin

Thermoset resins are a class of polymers that when cured using heat, chemical, or other means, change into a substantially infusible and insoluble material. Thermoset resins dominate the composite industry. Low viscosity allows ready impregnation of fibers and malleability permits the manufacture of complex forms. Thermosets provide a means of achieving high strength, high stiffness crosslinked networks in a cured part.

Thermoset resins undergo an irreversible chemical change when cured. The curing process transforms a thermoset resin into a plastic through a chemical cross-linking process. Energy and/or catalysts cause molecular chains to react at chemically active sites, linking into a rigid 3-D structure. Thermoset resin cure is an irreversible exothermic chemical reaction. A thermoset material cannot be heated and re-shaped or recycled after it is cured. The cross-linking process increases the resin material's molecular weight and melting point. Thermoset materials are generally stronger than thermoplastic materials due to the 3-D network of molecular bonds that is created during cure.

The cure process for thermoset resin materials affects the quality, strength, and temperature resistance of the finished part. Thermoset resin materials are often cured in an autoclave using a combination of heat, vacuum, and pressure. Moderately elevated temperatures lower viscosity allowing the resin to flow and thoroughly wet the embedded fibers. A "thermal soak" in the vacuum bag at moderate temperatures consolidates the laminate and draws out moisture and volatile[2] substances. Elimination of gaseous substances is essential to prevent or minimize the formation of porosity, voids and other defects in the finished part. After moisture and volatiles have been eliminated, the autoclave is pressurized, and the vacuum bag is vented. The autoclave pressure prevents any remaining volatiles from becoming gaseous as the temperature is increased to cure level. The cure process is completed by "soaking" the part at cure temperature and pressure while the chemical cross links form and the resin solidifies. Thermoset cure times are long with relatively low processing temperatures. Several resins commonly used for structural parts are described in the following paragraphs.

3.3.2.1   Epoxy

Epoxies are polymerizable thermosetting resins containing one or more epoxide groups curable by reaction with amines, acids, amides, alcohols, phenols, acid anhydrides, or mercaptans. These polymers are available in a variety of viscosities from liquid to solid. Epoxies are widely used for prepregs[3] and structural adhesives.

Advantages of epoxies include high strength and modulus, low levels of volatiles, excellent adhesion, low shrinkage, good chemical resistance, and ease of processing. Disadvantages include brittleness of finished parts and degradation in properties in presence of moisture.

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[1] A low-molecular-weight macromolecule, capable of further polymerization.

[2] A volatile is a liquid with a vapor pressure. At any given temperature, for a particular substance, there is a specific pressure at which the gas of that substance is in dynamic equilibrium with its liquid form. This is the vapor pressure of that substance at that temperature.

[3] Prepreg materials are two dimensional sheets of fiber material pre-impregnated with a staged resin.

Processing techniques for epoxies include autoclave molding, filament winding, press molding, vacuum bag molding, resin transfer molding, and pultrusion. Cure temperatures vary from room temperature to 350˚F, and commonly range from 250˚F to 350˚F. Higher temperature cures produce higher temperature resistance. Cure pressures range from vacuum bag[1] to 100 psi.

3.3.2.2 Bismaleimides (BMI)

Bismaleimides are the maleimide formed from the reaction of a diamine and maleic anhydride. A wide range of materials can be added to tailor the properties of a bismaleimide resin. The form can vary from a solid to a liquid pourable at room temperature. Bismaleimides are now commercially available as pregreg tapes, fabrics, rovings, and sheet molding compounds. Aerospace applications require sticky prepregs which typically require proprietary resin formulations.

Bismaleimide resins deliver higher temperature capability and higher toughness when compared to epoxies. Bismaleimides have a relatively high glass transition temperature. Glass transition temperatures of 500-600˚F are typical, with high-temperature epoxies being less than 500˚F. Bismaleimides also have high elongation compared to epoxies. Elongation for bismaleimides is 2-3% vs. 1% for epoxies. The primary disadvantage of bismaleimide resin over epoxy is higher cost.

Bismaleimides are suitable for standard autoclave processing, injection molding, resin transfer molding, sheet molding compounds and other methods. The processing time of bismaleimide resin is similar to epoxies except that higher service temperatures require a free-standing post cure process. Bismaleimide resins that cure at room temperature are not available at this time.

3.3.2.3 Polyimides (PMR)

Polyimide resins are a group of polymers characterized by an aromatic heterocyclic ring structure. Bismaleimide resin is a subset within this group. The polyimide resin group contains both thermoset and thermoplastic resins, with some polyimide materials exhibiting characteristics of both groups.

Polyimide resins excel in high temperature environments. Thermal resistance, oxidative stability, low coefficient of thermal expansion, and high solvent resistance are the primary benefits of polyimide resins. Their primary uses are in circuit boards and high temperature aerospace structures. The primary disadvantage of polyimides is that they are expensive and difficult to process.

Polyimides require high cure temperatures, usually in excess of 550˚F. Cure materials commonly used for vacuum bags, breather plies, and release films cannot be used because of the high temperatures. High cure temperature also requires the use of steel tooling.

3.3.2.4 Polyester

Thermosetting polyester resin is generally an orthophthalic polyester resin, or an isophthalic polyester resin. Polyester resins are relatively inexpensive and fast processing resins used for low cost applications.

Polyester cure processing is primarily dependent on temperature. Processing temperatures range from room temperature to 350˚F depending on the formulation. A quick cure will occur at the temperature

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[1] Vacuum bag pressure will depend on the vacuum applied and the local atmospheric pressure. Pressures up to 14.7 psi are achievable.

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specified for the specific polyester formulation. Without sufficient heat, the product will remain plasticized.

Processing methods for polyester resins include matched metal molding, wet lay-up, vacuum bag molding, injection molding, filament winding, pultrusion, and autoclaving. Compared to epoxies, polyesters are easier to process, and are tougher[1], but have lower strength and temperature resistance.

3.3.2.5   Phenolic

Phenolic polymers are condensation polymers based either on a reaction of excess formaldehyde with a base catalyst and phenol (resoles) or a reaction of excess phenol with an acidic catalyst and formaldehyde (novolacs). Resoles are cured to a solid of high crosslink density. Ten to twelve percent of the resin by weight is lost as water during cure. Novolacs are mixtures of low molecular weight polymers soluble in acids. Novolac materials will form an infusible resin unless an excess amount of phenol is present.

3.3.2.6   Cyanate Ester

Cyanate ester resins are generally based on a bisphenol or novolac derivative. Cyanate esters have higher temperature performance compared to epoxies, with glass transition temperatures in the range of 440˚ to 520˚ F. Other advantages include improved toughness (with elongation at failure in the range of 2.5% to 6%), low moisture absorption, low dielectric loss, and good peel strength. Cyanate esters can be cured by heating at elevated temperatures or at lower temperatures in the presence of a suitable catalyst. Cyanate esters are easily processed and toughened. Cost is competitive with bismaleimide.

Table 3.3-1 Characteristics of Common Aerospace Thermoset Resins

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[1] In a composite material, toughness represents the energy absorbed per unit crack extension and thus represents the part's ability to withstand impact damage.

Property	Polyester	Epoxy	Phenolic	Bismaleimide	Polyimide
Processibility	Good	Good	Fair	Good	Fair to difficult
Mechanical Properties	Fair	Excellent	Fair	Excellent	Good
Temperature Resistance	180 F	250 F	350 F	350 F	500-600 F
Price Range	Low - Medium	Low - Medium	Low - Medium	Medium	High
Delamination Resistance	Fair	Good	Good	Good	Good
Toughness	Good	Fair - Good	Poor	Good	Fair - Good
Remarks	Secondary structures, primarily with fiberglass	Most widely used, with best properties for primary structures	Secondary structures, primarily with fiberglass	Good structural properties, intermediate temperature resistant alternative	Specialty use for high temperature applications

Table 3.3-2 Manufacturing Attributes of Common Aerospace Thermoset Resins

Resin	Tack	Drape	Thermal Stability	Cure Temp	Cure Pressure	Void Content
Epoxy	Excellent	Excellent	205 Dry 180 Wet	350 F	100 psi	Low
Tough Epoxy	Very Good	Excellent	180 Dry 160 Wet	350 F	100 psi	Low
BMI	Good Warm	Good Warm	400 Dry 350 Wet	350 F 450 F Post Cure	100 psi	Low
PMR	Good	Good	400-500	Complex To 650 F	500 psi	Moderate

3.3.3   Thermoplastic Resin

Thermoplastic resins are high-molecular-weight polymers that can be softened by heating and hardened by cooling through a temperature range characteristic of the material. When in the softened stage, thermoplastic resins can be molded or extruded into shaped articles. Once formed, thermoplastic parts can be joined with adhesives or welded with a variety of techniques including heat and ultrasonic vibration. Thermoplastic parts are remoldable and the resin material is recyclable.

Ease of processing, high temperature capability and retention of properties after impact are the primary advantages that make thermoplastic resins attractive to the aerospace industry. Service temperatures over 350°F and toughness two to three times that of thermoset resin is typical. Moisture resistance is high and strength is generally low when compared to cured thermoset resins

Several physical forms of thermoplastic material are available including films, powders, and filaments. Thermoplastic resins are often combined with reinforcing fibers and supplied in compression-moldable random orientation sheets, unidirectional tapes, towpregs, and woven prepregs. Thermoplastic material forms are often boardy and do not possess the tack and drape qualities of thermoset materials. Some thermoplastic materials have unlimited shelf life and do not require refrigeration.

Thermoplastic resins are non-reacting. No cure process is required and processing cycles are short. Parts may be produced by stamp molding, thermoforming, autoclave molding, diaphragm forming, roll forming, filament winding, pultrusion, and other methods. Processing temperatures are typically high. Tooling and material costs are generally high, but total production cost may be reduced due to shorter processing times and the potential for reprocessing parts and material.

The majority of thermoplastic resins have an amorphous form which is characterized by a random molecular orientation. Amorphous thermoplastic resins are noted for their processing ease and speed, high temperature capability, good mechanical properties, excellent toughness, impact resistance, and chemical stability. Primary concerns are poor solvent resistance and reduced 0° compression properties when compared to thermoset resins.

- Examples of amorphous thermoplastics include; polysulfone, polyamide-imide(PAI), polyphenylsulphone, polyphenylene sulfide sulfone, polyether sulfone(PES), polystyrene, polyetherimide(PEI), and polyarylate

The semi-crystalline form of thermoplastic resin has a percentage of molecules that are able to form three-dimensionally-ordered crystalline arrays. The remaining volume has the random amorphous molecular orientation. Semi-crystalline thermoplastics have superior creep resistance and solvent resistance when compared to amorphous thermoplastics. The degree of crystallinity is affected by the cooling rate during processing.

- Examples of semi–crystalline thermoplastics include; polyethylene, polypropylene, polyamides, polyphenylene sulfide, polyetheretherketone (PEEK).

Thermoplastic resins were developed with the intent to rapidly stamp or compression mold structural composite parts at low cost. This potential has yet to be realized due to low production volumes, high capital equipment and tooling costs, and excessive fiber distortion in the formed parts. Thermoplastics are mainly used in interiors and other non-structural parts. The lack of an extensive database of performance properties over service time is a factor limiting use in aircraft structures.

3.3.4   Glass Transition Temperature

Resin materials have a glass transition temperature (T_g) that is a function of the resin material, moisture content, and for thermoset resins, the degree of cure. Glass transition is the change in an amorphous polymer from a hard and relatively brittle condition to a viscous and relatively rubbery condition. Glass transition is a reversible process. The resin material will start to soften and deform under load at temperatures above the glass transition. The general effect of glass transition temperature on lamina strength and stiffness is illustrated in Figure 3.3-1. Wet Glass Transition Temperature should be a minimum of 50° F higher than maximum operating temperature. For thermoset resins, a post cure is sometimes used to drive glass transition temperature (T) higher and increase temperature performance.

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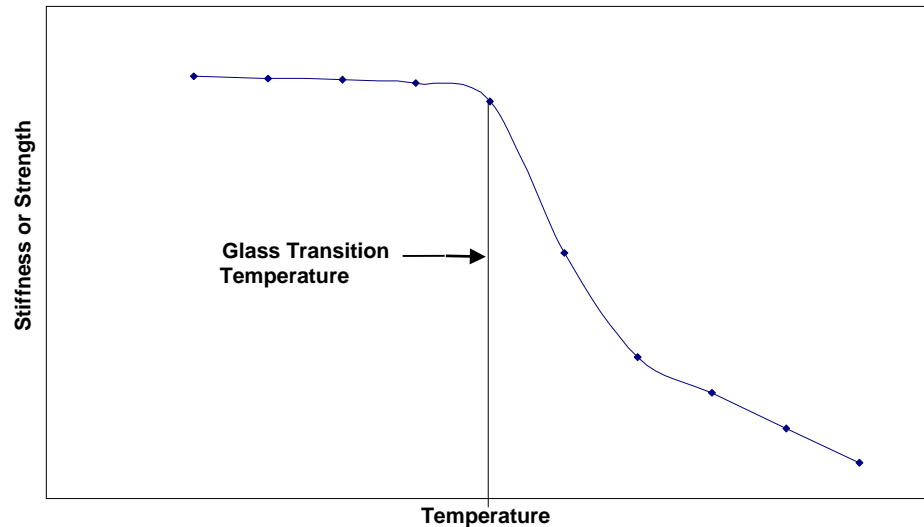| Page 3-12 | **Revision A** |
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Figure 3.3-1 Glass Transition Temperature

3.3.5   Matrix Toughness

Matrix toughness is the measure of a matrix material's ability to absorb work and is proportional to the area under the load-elongation curve from the origin to the breaking point. Matrix Toughness governs several important structural properties of a composite material:

- Retention of compressive strength and strain allowables after impact
- Damage containment and fracture toughness.

Matrix toughness can be increased through both chemical and mechanical means. Chemical toughening methods include: increasing molecular weight to reduce cross linking, fluorine substitution with 6F groups[1], and copolymerization with elastomer segments. Mechanical toughening is accomplished by adding rubber or thermoplastic particles to a thermoset resin.

3.3.6   Reinforcement Fiber

Fiber is the general term used to refer to the smallest unit in a filamentary material. Fibers are characterized by extreme length and a small diameter. Commonly used structural reinforcement fibers have diameters ranging from 7 to 200 microns, with 7 to 10 microns being most common. A 7 micron carbon fiber is shown in comparison to a human hair in Figure 3.3-2.

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[1] A 6F group is a hexagonal arrangement of 6 fluorine molecules.
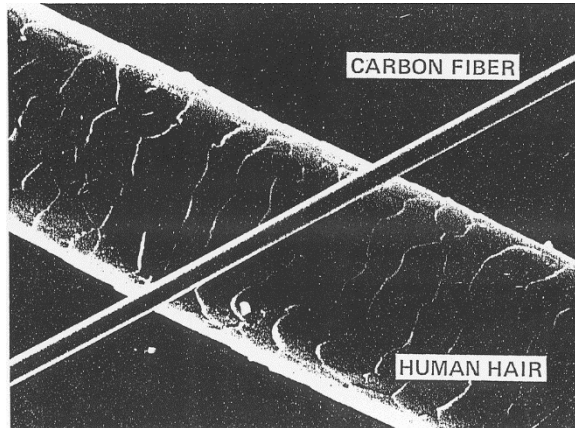
Figure 3.3-2 Size of Carbon Reinforcement Fiber Compared to Human Hair.

Reinforcement fibers are the principal load-carrying members in a composite material. Load is transferred to and between the fibers by the matrix. Fiber properties determine the tensile strength, compressive strength, flexural strength, and stiffness of a composite lamina in the fiber direction(s). Fiber properties also determine the composite electrical conductivity and the coefficient of thermal expansion in the fiber direction.

Structural reinforcement fibers have much higher strength and stiffness properties when compared to the same material in bulk form. For example, plate glass fractures at only a few thousand psi, while structural glass fibers have strengths of 400,000 to 700,000 psi. The increased strength for a fiber is explained by the fiber's more perfect nature. The small cross section diameter of the fiber has fewer internal defects and dislocations that can slip or grow to result in permanent deformation and/or failure of the material. Some fiber materials such as graphite have crystals that are aligned parallel to the fiber's long axis. The alignment of crystals along the fiber axis creates stronger molecular bonds and further increases strength of the fiber. A comparison of properties for some commonly used structural fibers[1] is shown in Table 3.3-3.

Table 3.3-3 Comparison of Properties for Common Structural Fibers

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[1] Relative strength properties of fibers can be misleading. The strength of a composite material will depend on the combination of fiber material, matrix material, resin content, cure, sizing, and other factors.

Fiber	Diameter (microns)	Density (lb/in^3)	Tensile Strength (ksi)	Modulus (msi)	Service Temp (F)
S-Glass	7	0.09	500-650	13	600-700
Aramid (Kevlar)	12	0.052	400	10-25	500
Polyethelene	27-38	0.035	375-430	17-25	230
Graphite	7	0.06	350-450	33-55	1000
Quartz	9	0.079	500	10	2000
Silicon Carbide (SiC)	10-20	0.083-0.094	400	28	2400
Alumina	20	0.141	200-300	55	1800
Boron	50-200	0.09	500	58	3500

3.3.6.1   Glass

Glass fibers are derived from sand. Molten material is pulled through a die or bushing to create continuous fibers. High strength "S-2" glass formulation is a low-alkali magnesium-alumina-silicate composition. Surface treatments such as binders or sizing can be applied directly to the fibers during the pulling process. Structural grade glass is higher strength, stiffness, and lower density when compared to other glass fiber grades[1].

Advantages of structural glass fibers are: low cost on a weight or volume basis, high chemical resistance, high resistance to galvanic corrosion, and good electrical properties. Structural glass fibers have a low coefficient of thermal expansion compared to graphite. A large variety of glass fiber product forms are readily available.

Limitations for some applications may include a lower melting temperature, low thermal conductivity and low electrical conductivity. Density is relatively high compared to other structural reinforcement fiber materials.

3.3.6.2   Aramid (Kevlar)

Aramid is an organic fiber with high specific tensile modulus, high specific tensile strength, and exceptional toughness. Aramid laminates have excellent vibration damping characteristics, are resistant to shattering upon impact, are resistant to fatigue, and have relatively low density compared to glass and graphite. Aramid fibers do not melt, but will decompose at temperatures above 900˚F. Typical temperature range of use is -33 to 390˚F. Aramid fibers are non-conductive and exhibit no galvanic reaction with metals.

Aramid fibers are manufactured by extruding a polymer solution through a spinneret. Fibers are available in two forms: Kevlar 29 with the lowest modulus and highest toughness is used mostly in ballistics and soft composite systems, and Kevlar 49 for laminate structural reinforcement and other applications.

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[1] "E" Glass – Electrical grade, "C" Glass – Chemical grade

The primary disadvantage of aramid laminates is low compression strength and stiffness. Aramid laminates are relatively flexible and ductile under compression and flexure. The compressive load-displacement relation becomes non-linear at about 20% of the ultimate tensile load allowable. Ultimate strength under compression is lower than for glass or carbon. Processing aramid laminates can be more expensive since inherent fiber toughness necessitates special tools for cutting fabric and machining cured parts.

3.3.6.3 Polyethylene (PE)

Ultrahigh molecular weight polyethylene (UHMWPE) is a high performance fiber more widely known by the trade name Spectra®. Polyethylene fibers have a highly crystalline structure that is oriented axially in fully-extended polymer chains. Polyethylene fibers are formed by a gel spinning process where the polymer is dissolved in order to disentangle the polymer chains, and then the dissolved chains are drawn from the solution to axially align the molecules to a high degree.

Polyethylene fiber use is limited by a service temperature of 230°F and a melting point of 300°F. These relatively low temperatures create a concern with resins requiring higher cure or processing temperatures. Polyethylene bonds poorly to most resins due to its chemical inertness and poor wetability. Fiber surface treatments to improve bonding are available.

Ballistic protection items such as lightweight body armor and riot shields are among this fiber's primary uses. Polyethylene has outstanding impact strength and a 33% weight reduction over Kevlar. Other applications include cut-resistant fabrics, heavy-lift cables, snow skies, sporting goods, and high wear applications. Polyethylene fiber is also used in radomes due to its low dielectric constant and loss tangent.

3.3.6.4 Graphite

Graphite is the most widely used fiber with the best balance of properties. High specific strength and specific stiffness make graphite an attractive fiber for high performance structures. With proper fiber orientation, graphite composites can be stronger and stiffer than steel at less than half the weight. In addition, graphite composite laminates have fatigue limits far in excess of aluminum or steel and provide superior vibration damping.

The coefficient of thermal expansion for graphite is very low, allowing the design of thermally stable structures. The high thermal stability of graphite can be a problem when paired with a material like aluminum which has a coefficient of thermal expansion an order of magnitude higher. A large mismatch in thermal expansion coefficients can result in significant thermally induced stresses.

Although the terms carbon and graphite are often used interchangeably, graphite fibers are more "ordered" internally when compared to carbon fibers. Carbon and graphite fibers are both based on the graphene layer networks present in carbon. If the graphene layers stack with three dimensional order the material is defined as graphite. Graphite is a crystalline form of carbon.

Graphite fibers are typically made by processing polyacrlyonitrile (PAN) fibers. Pitch and Rayon are also used as graphite/carbon fiber precursors. White precursor fibers are made by polymerization, spinning, drawing and washing the precursor material. The familiar finished carbon "Black fiber" is made by oxidation, pyrolysis, and surface treatment of the precursor fiber. Strength and stiffness are extremely process and precursor-dependent. Fibers created from the same precursor but with different processing can have dramatically different properties.

Typically, limitations on the use of graphite fibers in composite structure are more dependent on the resin than the fiber. Extended time and temperature processing are required to produce the crystalline structure of graphite, making the product more expensive than carbon.

3.3.6.5 Ceramic Fibers

When the matrix material is metal or ceramic, temperatures associated with fabrication can be extremely high. For these applications, ceramic fibers are the material of choice.

3.3.6.5.1 Quartz

Quartz fibers are a very pure form of fused silica glass. With a tensile strength of 500 ksi and a density of .079 lb/in^3, quartz fibers have the highest specific strength of current high-temperature fiber materials. Continuous service temperatures up to 1920˚F are possible. Quartz fibers do not melt or vaporize until temperatures exceed 3000˚F. Quartz fibers are chemically stable, have high electrical resistivity, high radar transparency, and are relatively expensive.

3.3.6.5.2 Silicon Carbide (SiC)

Silicon Carbide fibers have high strength, high modulus, and relatively high density. The majority of silicon carbide fibers are produced for use as reinforcement in metal matrix composites. Aluminum, titanium, and molybdenum alloys are produced with Silicon Carbide reinforcement. Silicon Carbide fibers are also used in epoxy, bismaleimide, polyimide, and ceramic matrix composites. Processing and cure of organic matrix silicon carbide composites uses methods similar to those used for graphite epoxy.

Silicon Carbide/Ceramic composites have been engineered to provide service temperatures in excess of 2640˚F, and are unique in several thermal properties. In addition to the high temperature capability, Silicon Carbide/Ceramic composites have very low thermal conductivity, thermal expansion, and thermal creep. Due to the ceramic matrix, Silicon Carbide/Ceramic composites are not easily joined or machined and must be manufactured to near net shape.

3.3.6.5.3 Alumina

Continuous polycrystalline alumina fiber is suitable for reinforcement of plastics, metals, and ceramics. Alumina is an oxide of aluminum and is produced from the ore bauxite. Alumina is primarily used in the production of aluminum metal and abrasives. The most common form of crystalline alumina is α-aluminum oxide also known as corundum. Crystalline α-alumina is very strong and exceptionally hard. The most common use of alumina fiber is in the reinforcement of metal matrix and ceramic composites.

Alumina fibers are good electrical insulators, have high modulus, high temperature capability (3713˚F melting point), and good compressive strength. Fibers more than 99% α-alumina have excellent chemical resistance and have higher modulus and temperature capabilities than ceramic fibers containing silica. The α-alumina fiber modulus of 55 msi is comparable to boron and graphite.

Alumina fibers are of interest for high temperature applications, radar transparent structures, circuit boards, and antenna supports.

3.3.6.6 Boron

Boron fiber is unmatched for its combination of strength, stiffness, and density. Boron fibers are formed by a deposition reaction on a hot tungsten wire that is continuously drawn through a reactor. Boron fibers are stronger and stiffer than graphite as shown in Table 3.3-3. Thermal conductivity and coefficient of thermal expansion are low. Boron fibers are used with both organic and metal matrices to form strong lightweight composite materials.

Difficulty of handling is one major impediment to the use of boron fibers. Boron fibers are extremely stiff, brittle, and difficult to work with. Brittleness limits the bend radius to greater than three inches. The large diameter of the fibers (up to 200 microns) also creates a problem. Large fiber diameters increase ply thickness for boron fiber laminae. For thin aerospace structures, increased ply thickness limits the ability to tailor ply orientations and achieve a structurally optimized part. Cost is another factor limiting use. Cost of boron fiber is approximately an order of magnitude higher than standard carbon fiber. Cured boron composites are extremely difficult to machine and drill.

3.3.7   Fiber Tows

Reinforcement fibers bundled together in groups of continuous untwisted parallel fibers are referred to as tows. The number of fibers in a tow depends on the fiber material and the application. For example, graphite fibers are available in bundles of 1000, 3000, 6000, and 12000 fibers. The larger fiber material bundles are usually less expensive on a per pound basis. The smaller tow sizes are used in weaving, filament winding, and braiding, while the larger tow sizes are used for unidirectional tapes. Yarn is a generic term for tows that have been twisted. Yarn is typically used for making fabric.

Figure 3.3-3 contains two micrographs of a tow cross section. Note that the tow cross sections for this application are flattened and have tapered ends. The tapered tow shape is typically used for fiber placement and allows the tows to be placed side by side with a slight overlap. Tows produced with a more rectangular cross section are used for uni-axial slit tape prepregs.
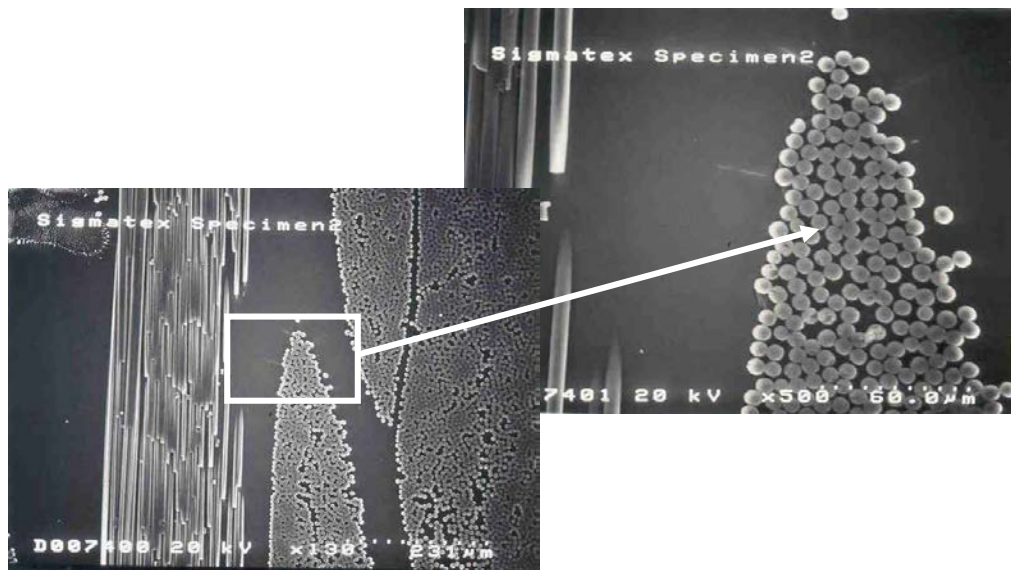
Figure 3.3-3 Micrograph of Fiber Cross Sections in a Tow

3.4   Lamina Material

Fiber and matrix materials are available in a variety of forms which allow for great flexibility in the design and manufacture of composite structure. Continuous fiber tows, two dimensional unidirectional plies, and two dimensional textile weaves are the common continuous fiber material forms used to create laminae. Continuous fiber tows can be machine placed or filament wound to create a unidirectional lamina. Unidirectional tape and woven fabric are materials in lamina form that can be laid either by hand or by machine. Most lamina materials are available either dry or pre-impregnated with a specified resin ("prepreg").

In additional to the continuous fiber lamina forms, short fiber, whisker, and particulate composite reinforcements can be impregnated with resin and formed into two-dimensional ply materials. Highly aligned discontinuous fiber tapes have been developed mainly for use with thermoplastic resin. Post-cure formability allows discontinuous thermoplastic tapes to be molded into complex shapes without significantly affecting material strength. Three dimensional braided reinforcements are also produced.

Properties of unidirectional and textile continuous fiber laminae are highly dependent on fiber direction. By convention, the material coordinate system of a two-dimensional continuous-fiber lamina is aligned with the

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primary fiber direction. The coordinate system axes for the lamina material system are designated 1, 2, and 3 as shown in Figure 3.4-1. The "1" axis is aligned in the fiber direction for unidirectional forms, and in the warp direction for textile forms.
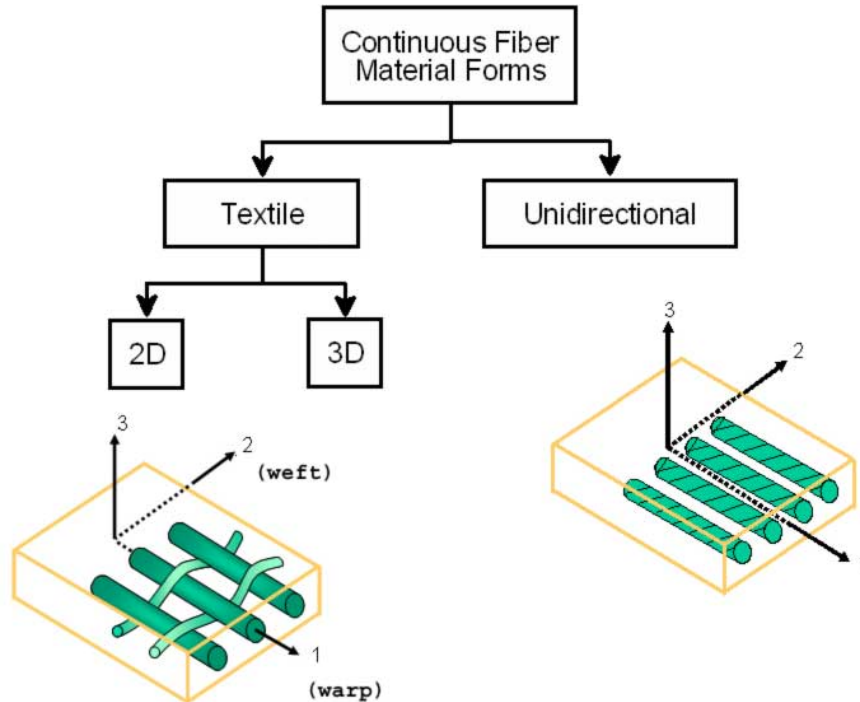

Figure 3.4-1 Continuous Fiber Lamina Forms and Coordinate Systems

3.4.1   Prepreg

The most common lamina materials used in advanced aerospace applications are pre-impregnated unidirectional tape and pre-impregnated woven fabric. These material forms are commonly referred to as "prepreg tape" and "prepreg fabric". Prepreg designates that the fiber reinforcement materials have been pre-impregnated with a staged thermoset resin. Prepreg is ready to mold or cure and does not require resin application during processing. Ninety to ninety-five percent of the advanced composite material sold today is pre-impregnated.

Prepreg materials are created by applying a staged thermoset resin to a reinforcement material using either a hot melt, or a solution process. For the hot melt process, a resin film of specified weight is cast upon a release film and stored. Dry fiber in unidirectional or fabric form is then fed through two heated rollers along with the cast resin film. The heated rollers cause the resin film to melt and saturate the fibers creating a pre-impregnated sheet. New release films are applied and the material is frozen and stored for later use. The solution process involves dipping dry fiber material into a trough containing a solution of resin and solvent as shown in Figure 3.4-2. The solvents are flashed off in a drying oven, a release film is applied, and the material is frozen and stored for later use.
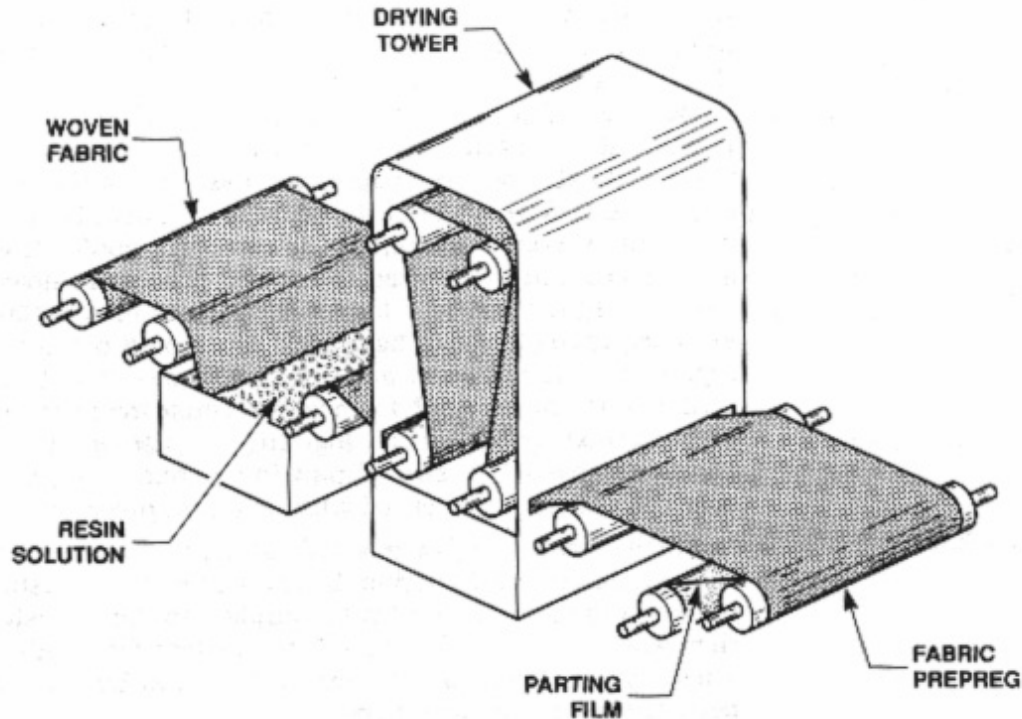
Figure 3.4-2 Prepreg Fabrication Process

There are several stages of prepreg resin cure. A-stage is an early stage reaction of certain thermosetting resins in which the material is fusible and still soluble in certain liquids. A-stage is sometimes referred to as resol. B-stage is an intermediate stage reaction in certain thermosetting resins in which the material softens when heated and swells when in contact with certain liquids, but may not entirely fuse or dissolve. B-stage is sometimes referred to as resitol. C-stage is the final stage reaction of certain thermosetting resins in which the material is relatively insoluble and infusible. C-stage is sometimes referred to as resite.

Prepreg materials require frozen storage to inhibit curing of the staged resin. Working time at room temperature must be monitored and recorded. Shelf life from date of manufacture to cure is limited. Prepreg materials are not used if shelf life or working time exceeds established limits.

3.4.2   Unidirectional Lamina

A unidirectional lamina is a two-dimensional layer of straight[1] continuous unidirectional fiber reinforcement material combined with resin. Continuous reinforcement fibers are bundled together as rectangular or tapered cross section tows. The untwisted tows are aligned side by side to create a thin two-dimensional layer of unidirectional reinforcement material. An illustration of a unidirectional lamina is shown in Figure 3.4-3.

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[1] Fibers are not curved or kinked within the thickness dimension of the lamina.
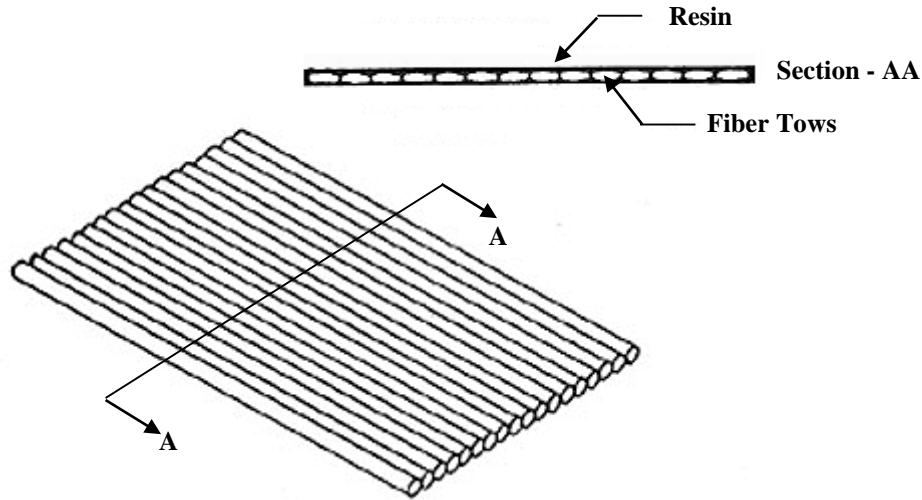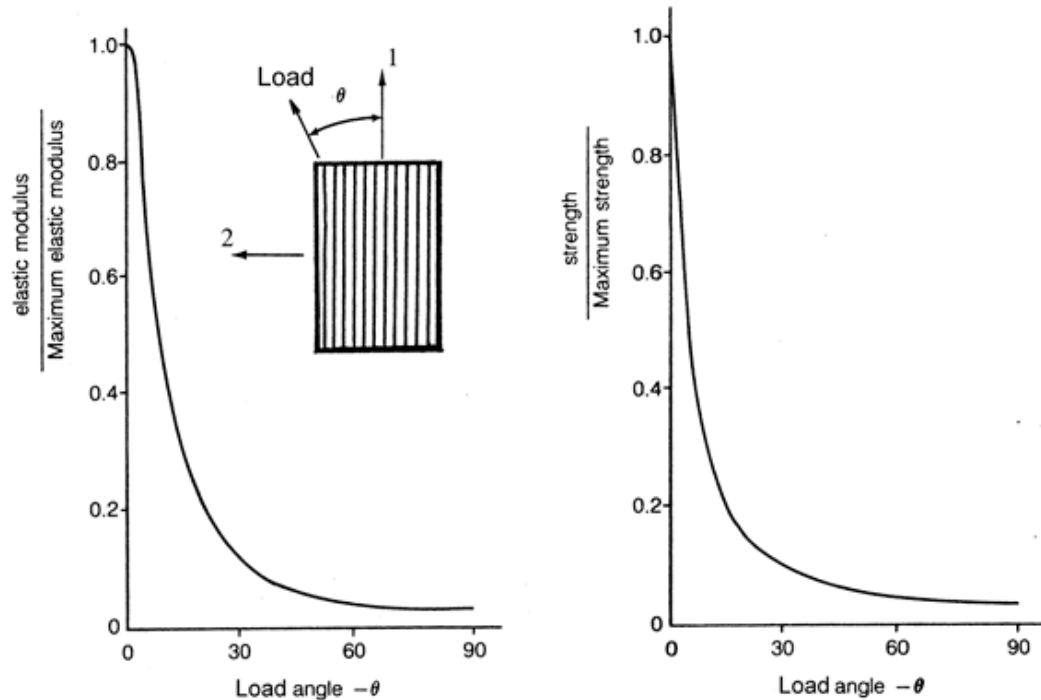
Figure 3.4-3 Unidirectional Lamina

Unidirectional laminae can be created using unidirectional prepreg tape, by direct placement of fiber tows using fiber placement, or by filament winding techniques. Prepreg unidirectional tape has become the industry standard for unidirectional laminae. Resin impregnated fiber tows are aligned side by side on a removable backing film to create tape. The number of fibers or tows dictates the areal weight or fiber density of the tape material. Unidirectional tapes are available in widths from 0.125 to 60-inches wide. Slit tape is manufactured in wide rolls and cut or "slit" into smaller width strips. Fiber tows used for slit tape are more rectangular versus tapered in cross section.

Unidirectional lamina materials provide the highest material direction mechanical properties of all forms of reinforcement materials. Maximum properties are achieved because reinforcement fibers are most efficient when they are straight. Woven fibers are usually crimped and will lose about 30% to 40% of their load-carrying capabilities.

Strength and stiffness properties of unidirectional materials are maximized in the fiber direction and drop rapidly as the angle from fiber direction increases. Relatively small angular deviations from the fiber direction will result in large reductions in strength and stiffness. The fiber direction for a unidirectional lamina is aligned with material axis "1". Transverse[1] properties of unidirectional laminae are dependent on matrix material and are therefore usually very weak compared to longitudinal properties. Figure 3.4-4 shows the normalized stiffness and strength of a typical unidirectional lamina as a function of load orientation to the material axis.

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[1] Transverse indicates the material axis "2" direction, which is at a 90° angle from the fiber direction.

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Figure 3.4-4 Normalized Stiffness and Strength as a Function of Load Angle

Unidirectional lamina materials allow designers the most flexibility to stack and orient material layers such that strength and stiffness properties are optimized to carry the applied loads with a minimum structural weight. Unidirectional lamina thickness is controlled by the fiber tow thickness and is typically lower than woven product forms. Typical thickness values range from .005" to .010". Because unidirectional material is thinner, more laminae can be placed and oriented within a given thickness. Increased ply count for a specified thickness increases the probability of achieving a tailored optimized laminate that meets strength, stiffness, design, and weight requirements for a given design.

The primary disadvantages for unidirectional tape are poor drapability and a high sensitivity to fiber misalignment.

3.4.3   Woven Fabric

Woven fabric is a generic material form based on the ancient art of weaving. A two dimensional planar fabric is woven by interlacing two orthogonal sets of fiber tows referred to as the warp and weft. The warp runs parallel to the length of the fabric and the weft runs perpendicular to the warp. By convention, material direction "1" is aligned with the warp direction, and material direction "2" is aligned with the weft.

The interlocking orthogonal weave pattern increases ply thickness for woven fabrics when compared to unidirectional material forms. Smaller more expensive fiber tows are often utilized to mitigate the thickness doubling effect of fiber tow crossover. Thicker plies limit flexibility to optimize stacking sequence and can result in increased weight of finished parts. This potential weight disadvantage of woven fabric is offset by at least two advantages. First, increased ply thickness improves fracture toughness with

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respect to delamination by reducing the number of interlaminar boundaries in a specified laminate thickness, and second, lay-up costs are reduced because fewer plies must be applied, oriented, and inspected to achieve the same thickness laminate.

Strength and stiffness in the material direction are reduced for woven fabric lamina forms. The primary cause of reduced properties is fiber crimp. Reinforcement fibers become "crimped" as the woven fiber tows pass over and under each other. Fiber crimp can reduce specific strength and stiffness by as much as 30% to 40% when compared to the same material in unidirectional lamina form. Although strength and stiffness are reduced, fiber crimp increases fracture toughness with respect to delamination. Interlaminar cracks are deflected by the crimped fiber tows. Another cause of reduced properties for woven fabric is lower fiber volume fraction. Surface mismatch at adjacent tow boundaries and crossings create irregular high and low areas on the woven reinforcement fabric surfaces. Low spots on each surface fill with excess resin which lowers fiber volume fraction. Fiber volume fractions average between 50% and 60% for fabric versus 70% for unidirectional tape. Lower fiber volume fraction is associated at the micromechanics level with reduced strength and stiffness.

Sensitivity of strength and stiffness to load angle is reduced for woven fabrics. The weak transverse direction associated with unidirectional tape is eliminated because woven fiber tows provide strength and stiffness in both the warp and weft directions. Strength and stiffness in direction "1" is approximately equal to strength and stiffness in direction "2". An example comparison of stiffness versus load angle for tape and fabric is shown in Figure 3.4-5.
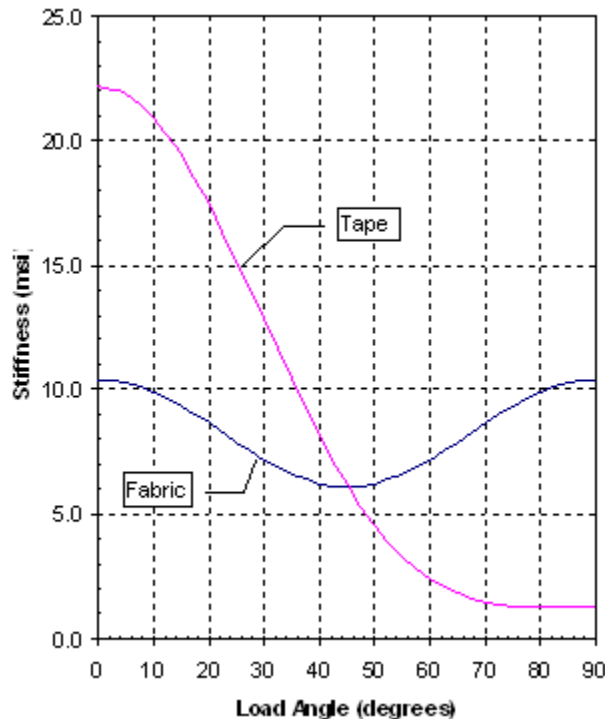
Figure 3.4-5 Stiffness versus Load Angle for Tape and Fabric

Woven fabric materials are usually more expensive than unidirectional materials because they require an additional manufacturing process and typically use smaller more expensive fiber tows. Although woven fabric material is more expensive, improved handleability and reduced ply count can lower manufacturing costs and lead to overall cost savings for woven fabric parts.

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The primary benefits of woven fabric compared to unidirectional tape are improved drapability for certain weaves, increased toughness with respect to delamination, improved damage tolerance, and reduced lay-up and inspection costs. The primary disadvantages are increased material costs, reduced properties, width limitations on availability of goods, higher scrap rates, and reduced design flexibility due to increased ply thickness.

3.4.3.1   2D Woven Fabric Forms

Woven fabrics are available in a variety of weave patterns in both dry and prepreg forms. Woven fabric is described by the over-and-under pattern of the tows. The over-and-under tow pattern can refer to the warp or the weft tows. Three important weave patterns are described in this section: plain weave, satin weave, and twill.

3.4.3.1.1   Plain Weave

Plain weave is characterized by an over-one, under-one weave pattern. Plain weave is the firmest, most stable woven fabric construction. Plain weave fabric has the highest toughness and the lowest in-plane properties. Plain weave fabrics are suitable for flat or slightly contoured parts. Strength is uniform in both directions. The plain weave pattern is shown in Figure 3.4-6.
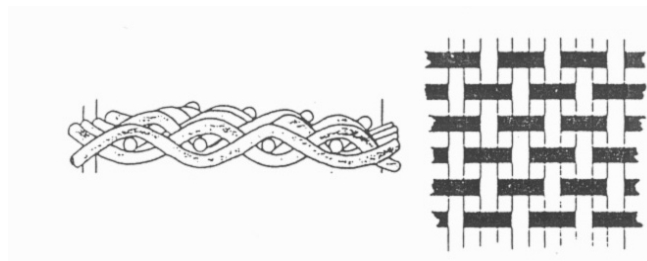

Figure 3.4-6 Plain Weave

3.4.3.1.2   Satin Weave

Satin weave construction is characterized by one warp tow passing over several weft tows and then under one and repeating. The pattern is offset by one weft tow at each new row. A satin pattern designated 4x1 would indicate a warp tow passing over four weft tows and under one. A 4x1 pattern would also be referred to as five harness satin or 5HS. Satin weaves are more easily draped and strength is higher in both directions. The commonly used 8HS retains most of the strength of unidirectional tape and can be easily draped over complex mold shapes. A 5HS satin weave is shown in Figure 3.4-7.
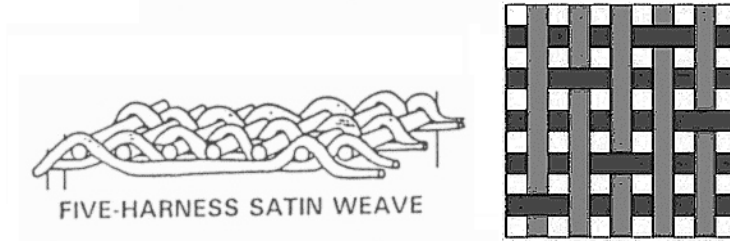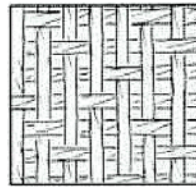
Figure 3.4-7 5HS Satin Weave

The 4HS crowfoot satin weave pattern is shown in Figure 3.4-8. For the crowfoot pattern, one warp tow passes over three fill tows and under one fill tow. Crowfoot satin conforms easily to contoured surfaces.

Figure 3.4-8 Crowfoot Satin Weave

3.4.3.1.3   Twill Weave

The twill weave is characterized by an over-X, under-X diagonal pattern, where X indicates the number of fiber tows being crossed. The pattern is offset one tow per row which creates a diagonal pattern on the fabric. Twill weaves are easier to drape over complex shapes than plain weaves. The higher the number X, the more easily the fabric will drape. An example twill weave fabric pattern is shown in Figure 3.4-9.
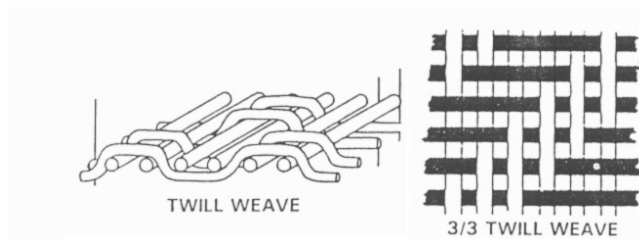
Figure 3.4-9 Twill Weave

3.4.3.2   Knitting

Fiber tows are knitted together with non-reinforcing binder fibers which go over and under the reinforcement fibers. The reinforcement fibers remain straight and retain strength approaching that of unidirectional tape. Knitted fabric is available preplied in +45/-45 or 0/90 ply combinations. Knitted fabrics save labor and reduce trim scrappage.

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3.4.3.3   Braiding

Braiding refers to interweaving two or three separate tow orientations in a diagonally overlapping pattern to create a tubular or conical reinforcement weave on a shaped mandrel. Most braids are biaxial, having interlocking tows wound in two orientations. The braiding angle can be varied between approximately 10 and 85 degrees. In triaxial braiding, a third fiber tow is added longitudinally along the braiding axis. Small braiding angles are best for tensile strength and large angles are best for hoop strength.

Braiding is particularly suited to tubular and contoured parts such as tubes of various cross sections, nose cones, radomes, rocket nozzles, and other specialized applications. Unlike filament winding, the fiber tows in a braided part are interlocked. The interlocked fiber tows improve torsion, shear, and impact resistance. The net result is lower in-plane properties due to fiber crimp and higher out-of-plane properties.

3.5   Lamina Behavior

Understanding the macromechanical behavior of a lamina is essential to understanding the behavior of laminated reinforced composites. Macromechanical behavior considers the averaged apparent mechanical properties rather than detailed interactions of constituent materials. A lamina is considered homogeneous and orthotropic when viewed from the macromechanical level. Homogeneous indicates that properties do not vary from point to point within the material. Orthotropic indicates that properties at a point are dependent on the direction, with three perpendicular directions of material property symmetry. A lamina is the basic building block in a laminated reinforced composite.

3.5.1   Lamina Coordinate System

The principal material coordinate system axes of composite laminae are designated 1, 2, and 3. The integer axis designators 1, 2, and 3 are used to differentiate the lamina material system from the laminate coordinate system(x, y, and z). For a unidirectional lamina, the "1" axis is aligned in the fiber direction and the "3" axis is normal to the plane of the lamina. For a fabric lamina, the "1" axis is aligned in the warp direction and the "3" axis is normal to the plane of the lamina. Alignment of the material coordinate systems for a unidirectional lamina and a woven fabric is shown in Figure 3.5-1.
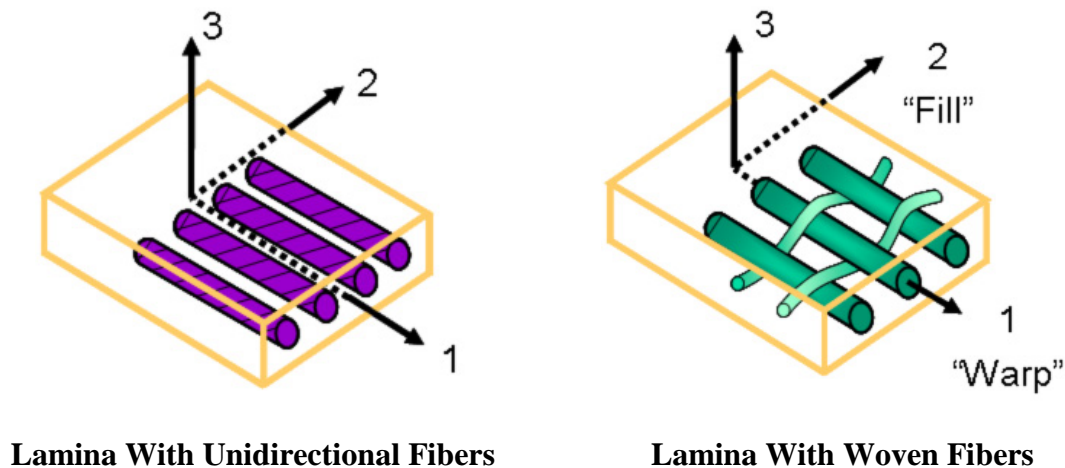

Lamina With Unidirectional Fibers   **Lamina With Woven Fibers**

Figure 3.5-1 Lamina Material Coordinate System

3.5.2   Hooke's Law

In structural analysis, constitutive equations relate stress to strain. The constitutive relation for linear elastic material behavior is known as Hooke's law. Hooke's law relates stress to strain via the stiffness matrix [C], and is defined relative to a specified material coordinate system. Hooke's law is written in contracted notation as shown in Equation 3.5-1.

$$\sigma_i = C_{ij}\varepsilon_j \quad i,j = 1,...,6$$

<div align="right">**Equation 3.5-1**</div>

Where:

σ_i are the stress components

C_{ij} are components of the stiffness matrix

ε_j are the strain components

Stress and strain are second order tensors. In the contracted form of the constitutive relation shown in Equation 3.5-1, these second order tensors are written as vectors. The contracted notation stress and strain vectors are defined by their tensor notation equivalents in Equation 3.5-2. It is common practice to write stress and strain as vectors using tensor notation. The contracted notation is used to simplify mathematical equations.

$$\{\sigma\} = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}, \{\varepsilon\} = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix}$$

(*Contracted Notation*, *Tensor Notation*)

<div align="right">**Equation 3.5-2**</div>

Note that engineering shear strain is implied by the contracted notation for strain in Equation 3.5-2. Engineering shear strain is defined as follows in Equation 3.5-3.

$$\gamma_{ij} = \varepsilon_{ij} + \varepsilon_{ji} = 2\varepsilon_{ij}$$

<div align="right">**Equation 3.5-3**</div>

Components of the three dimensional state of stress given by the tensor notation stress vector in Equation 3.5-2 are shown in Figure 3.5-2. Note that each component of stress is a vector.
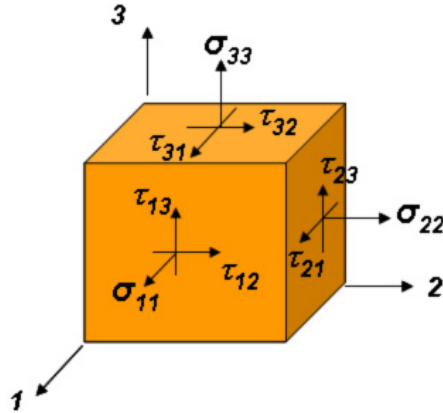
Figure 3.5-2 The 3 Dimensional State of Stress at a Material Point

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3.5.2.1   Anisotropic Materials

An anisotropic material has no planes of material property symmetry. An alternative name for an anisotropic material is a triclinic material. The stiffness matrix [C] for an anisotropic material is shown expanded in Equation 3.5-4. Guaranteed symmetry[1] along the main diagonal reduces the possible number of independent constants in the stiffness matrix from 36 to 21. The fully populated symmetric stiffness matrix with 21 independent constants represents the most general form of Hooke's law in the framework of linear elasticity. When viewed from any coordinate system, anisotropic materials have non-zero shear-extension and shear-shear coupling terms. This general form of the stiffness matrix characterizes an anisotropic material.

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix}$$

Equation 3.5-4

Using Equation 3.5-4 with Equation 3.5-2, the expanded form of Hooke's law for an anisotropic material is given by Equation 3.5-5.

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix}$$

Equation 3.5-5

Hooke's law can be manipulated to yield an expression for strain in terms of stress via the compliance matrix [S]. The compliance matrix relates material strain to stress and is the inverse of the stiffness matrix [C]. The contracted form of the strain-stress relation is given by Equation 3.5-6.

$$\{\varepsilon_i\} = [S_{ij}]\{\sigma_j\}, \ i,j=1,\ldots,6 \ \text{where} \ [S] = [C]^{-1}$$

Equation 3.5-6

The compliance matrix for an anisotropic material is symmetric, with 21 independent constant terms. The expanded form of the strain-stress relation is shown in Equation 3.5-7.

---

[1] Symmetry of the linear elastic stiffness matrix ($C_{ij} = C_{ji}$) can be proven using energy methods.

$$
\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\ S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}
$$

Equation 3.5-7

There are 21 independent engineering constants that characterize Hooke's law for an anisotropic material. The compliance matrix for an anisotropic material can be written in terms of the engineering constants as follows in Equation 3.5-8.

$$
[S_{ij}] = \begin{bmatrix} \dfrac{1}{E_1} & \dfrac{-v_{21}}{E_2} & \dfrac{-v_{31}}{E_3} & \dfrac{\eta_{1,23}}{G_{23}} & \dfrac{\eta_{1,13}}{G_{13}} & \dfrac{\eta_{1,12}}{G_{12}} \\ \dfrac{-v_{12}}{E_1} & \dfrac{1}{E_2} & \dfrac{-v_{32}}{E_3} & \dfrac{\eta_{2,23}}{G_{23}} & \dfrac{\eta_{2,13}}{G_{13}} & \dfrac{\eta_{2,12}}{G_{12}} \\ \dfrac{-v_{13}}{E_1} & \dfrac{-v_{23}}{E_2} & \dfrac{1}{E_3} & \dfrac{\eta_{3,23}}{G_{23}} & \dfrac{\eta_{3,13}}{G_{13}} & \dfrac{\eta_{3,12}}{G_{12}} \\ \dfrac{\eta_{23,1}}{E_1} & \dfrac{\eta_{23,2}}{E_2} & \dfrac{\eta_{23,3}}{E_3} & \dfrac{1}{G_{23}} & \dfrac{\mu_{23,13}}{G_{13}} & \dfrac{\mu_{23,12}}{G_{12}} \\ \dfrac{\eta_{13,1}}{E_1} & \dfrac{\eta_{13,2}}{E_2} & \dfrac{\eta_{13,3}}{E_3} & \dfrac{\mu_{13,23}}{G_{23}} & \dfrac{1}{G_{13}} & \dfrac{\mu_{13,12}}{G_{12}} \\ \dfrac{\eta_{12,1}}{E_1} & \dfrac{\eta_{12,2}}{E_2} & \dfrac{\eta_{12,3}}{E_3} & \dfrac{\mu_{12,23}}{G_{23}} & \dfrac{\mu_{12,13}}{G_{13}} & \dfrac{1}{G_{12}} \end{bmatrix}
$$

Equation 3.5-8

Where:

E_i is the axial stiffness (Young's moduli) of the material (psi)

G_{ij} is the shear stiffness of the material (psi)

v_{ij} is the Poisson's ratio of the material

$\eta_{i,ij}$ are coefficients of mutual influence of the first kind

$\eta_{ij,i}$ are coefficients of mutual influence of the second kind

$\mu_{ij,kl}$ are Chentsov coefficients

Each independent engineering constant term in the compliance matrix must be determined by material testing. The engineering constants are derived from test data using the relations in Equation 3.5-8. Coefficients of mutual influence and Chentsov coefficients are additional measured physical engineering constants that are non-zero for anisotropic materials. These coefficients are similar to Poisson's ratio in that they relate one type of material deformation to another. Chentsov coefficients relate shear deformation with shear. Coefficients of mutual influence relate shear deformation to extension and vice versa. Coefficients of mutual influence can be expressed in terms of strain as shown in Equation 3.5-9.

The first kind is given by

$$\eta_{i,ij} = \frac{\varepsilon_i}{\gamma_{ij}}$$

Equation 3.5-9

and the second kind is given by

$$\eta_{ij,i} = \frac{\gamma_{ij}}{\varepsilon_i}$$

3.5.2.2   Monoclinic Materials

A monoclinic material is characterized by one plane of material property symmetry. For a monoclinic material, a material coordinate system exists for which the material properties are symmetric about one plane. Extensional properties measured perpendicular to this material plane are constant throughout the material. Shear properties measured in planes parallel to the plane of symmetry are constant throughout the material. The monoclinic stiffness matrix is symmetric along the main diagonal and has at most thirteen independent elastic constants. The expanded form of Hooke's law for a monoclinic material with symmetry about z=0 is shown in Equation 3.5-10.

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{54} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix}$$

Equation 3.5-10

Symmetric composite laminates can often be characterized as monoclinic materials with material property symmetry about z=0. Matrix properties dominate in the z-direction and are approximated as constant in the z=0 plane, throughout the thickness of the laminate.

3.5.2.3   Orthotropic Materials

An orthotropic material is characterized by three perpendicular planes of material property symmetry. For an orthotropic material, a material coordinate system exists for which the shear-extension and shear-shear coupling terms in the stiffness matrix[C] are zero. Extensional properties measured perpendicular to each of these material planes are constant throughout the material. Shear properties measured in planes parallel to each of these material planes are constant throughout the material. The orthotropic stiffness matrix is symmetric along the main diagonal and has at most nine independent constants. The expanded form of Hooke's law for an orthotropic material is shown in Equation 3.5-11.

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix}$$

Equation 3.5-11

Components of the compliance matrix for an orthotropic material can be written in terms of the engineering material constants as shown in Equation 3.5-12.

$$[S_{ij}] = \begin{bmatrix} \dfrac{1}{E_1} & \dfrac{-v_{21}}{E_2} & \dfrac{-v_{31}}{E_3} & 0 & 0 & 0 \\ \dfrac{-v_{12}}{E_1} & \dfrac{1}{E_2} & \dfrac{-v_{32}}{E_3} & 0 & 0 & 0 \\ \dfrac{-v_{13}}{E_1} & \dfrac{-v_{23}}{E_2} & \dfrac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{1}{G_{12}} \end{bmatrix}$$

Equation 3.5-12

Where:

E_i is the axial stiffness (Young's moduli) of the material (psi)

G_{ij} is the shear stiffness of the material (psi)

v_{ij} is the Poisson's ratio of the material (psi)

Nine independent engineering constants characterize Hooke's law for an orthotropic material; Young's moduli in the 1, 2, and 3 material directions (E_1, E_2, E_3), shear moduli in the 2-3, 1-3, and 1-2 planes (G_{23}, G_{13}, G_{12}), and three independent Poisson's ratio(v_{ij}), for transverse strain in the j-direction when stressed in the i-direction. Symmetry of the orthotropic stiffness matrix leads to the set of reciprocal relations for Poisson's ratio shown in Equation 3.5-13. The reciprocal relations for Poisson's ratio leave us with three independent Poisson's ratio terms for a total of nine independent engineering constants.

$$\frac{v_{ij}}{E_i} = \frac{v_{ji}}{E_j} \quad i, j = 1,2,3$$

Equation 3.5-13

The nine engineering constants for an orthotropic material are calculated using the compliance matrix of Equation 3.5-12, combined with measured data collected from standardized coupon specimen tests. Normally these coupon tests are performed with a known load or stress and the resulting displacement or strain is measured. The coordinate system for which the material is orthotropic must be known in advance

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of testing. Hooke's law is defined relative to a specific material coordinate system and an orthotropic material will appear different if examined from another system.

Measured values of the engineering constants can be checked against limiting values determined by thermodynamic constraints. The product of a stress component and the corresponding strain component represents the work done by the stress. The sum of work done by all the stress components must be positive in order to avoid the creation of energy. This condition leads to the thermodynamic constraints shown in Equation 3.5-14.

$$
\begin{aligned}
&S_{11}, S_{22}, S_{33}, S_{44}, S_{55}, S_{66} > 0 \\
&C_{11}, C_{22}, C_{33}, C_{44}, C_{55}, C_{66} > 0 \\
&E_1, E_2, E_3, G_{23}, G_{13}, G_{12} > 0 \\
&\left| v_{ji} \right| < \left(\frac{E_j}{E_i} \right)^{\frac{1}{2}} \quad i, j = 1,2,3
\end{aligned}
$$

Equation 3.5-14

3.5.2.4 Hooke's Law for Continuous Fiber Laminae

Our macroscopic view of composite laminae as homogeneous orthotropic materials limits the maximum number of independent engineering constants to nine. Continuous fiber lamina material forms have fiber and matrix dominated material directions. For these common material forms, it is sometimes possible to equate properties in two material directions based on fiber or matrix dominance. Equal properties in two material directions can equate to equal properties for all directions in that plane. A material which has equal properties for all directions in only one plane is referred to as transversely isotropic. If the composite lamina can be approximated as transversely isotropic the number of independent material constants will be reduced from nine to five

For a unidirectional lamina, material properties in directions 2, and 3 are matrix dominated therefore we can assert the following equalities: $E_{33} = E_{22}$, $v_{13} = v_{12}$, $G_{13} = G_{12}$, and $v_{23} = (E_{33}/2G_{23})-1$. These asserted equalities establish the 2-3 material plane as a plane of isotropy, and leaves the following five independent engineering constants for a unidirectional lamina: *E_{11}, E_{22}, v_{12}, G_{12}, and G_{23}*. The symmetric compliance matrix for a transversely isotropic unidirectional lamina is shown in Equation 3.5-15.

$$
\left[S_{ij} \right] =
\begin{bmatrix}
S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\
 & S_{22} & S_{23} & 0 & 0 & 0 \\
 & & S_{22} & 0 & 0 & 0 \\
 & & & S_{44} & 0 & 0 \\
 & \text{Symmetric} & & & S_{66} & 0 \\
 & & & & & S_{66}
\end{bmatrix}
$$

Equation 3.5-15

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Where:

$$S_{11} = 1/E_1$$
$$S_{12} = -\nu_{21}/E_2$$
$$S_{22} = 1/E_2$$
$$S_{23} = -(E_2/2G_{23}-1)/E_2$$
$$S_{44} = 1/G_{23} = 2(S_{23}-S_{22})$$
$$S_{66} = 1/G_{12}$$

For a balanced woven fabric, material properties in directions 1 and 2 are fiber dominated and equal, therefore we can assert the following equalities: $E_{22} = E_{11}$, $\nu_{23} = \nu_{13}$, $G_{23} = G_{13}$. For a balanced woven fabric lamina, empirical data shows that $\nu_{12} \neq (E_{22}/2G_{12})-1$, therefore this material form is not considered transversely isotropic. The asserted equalities leave the following six independent engineering constants for a balanced woven fabric: *E11, E33, ν12, ν13, G12, G13*. The symmetric compliance matrix for a balanced woven fabric is shown in Equation 3.5-16

$$[S_{ij}] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{11} & S_{13} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & \textit{Symmetric} & & & S_{44} & 0 \\ & & & & & S_{66} \end{bmatrix}$$

Equation 3.5-16

Where:

$$S_{11} = 1/E_1$$
$$S_{12} = -\nu_{21}/E_1$$
$$S_{33} = 1/E_3$$
$$S_{13} = -\nu_{13}/E_3$$
$$S_{44} = 1/G_{23}$$
$$S_{66} = 1/G_{12}$$

3.5.3 Reduced Hooke's Law for Composite Laminae

Composite laminates are generally suited for use as thin shell structures, with stress at a point determined using "Classical Laminated Plate Theory"(*CLPT*). Development of "Classical Laminated Plate Theory" begins with development of the transformed reduced constitutive relation for a composite lamina. This relation is a reduced two-dimensional thin shell form of Hooke's law for orthotropic materials. The reduced constitutive relation can

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be transformed by an in-plane rotation to accommodate laminae placed off-axis to the laminate material system. Development of the transformed reduced constitutive relation is presented in this section.

3.5.3.1   Plane Stress Assumption

A basic building block of the transformed reduced constitutive relation is the plane stress assumption for laminae. The plane stress assumption is made by setting the out-of-plane stress components σ_{33} , τ_{13}, and τ_{23} equal to zero. With the plane stress assumption, the three-dimensional state of stress at a point is reduced to non-zero stresses in the 1-2 coordinate plane. The effect of this assumption on the stress state is illustrated in Figure 3.5-3.
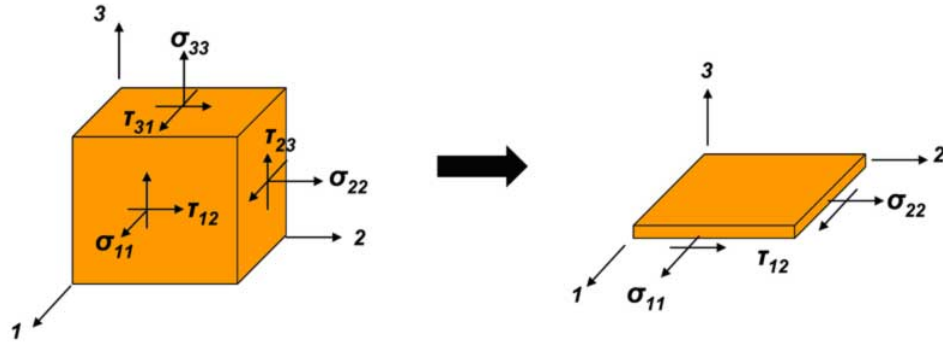

Figure 3.5-3 The Effect of the Plane Stress Assumption

By assuming a state of plane stress, columns 3, 4, and 5 of the 3-D orthotropic compliance matrix[S] are effectively eliminated. Terms in columns 3, 4, and 5 are now multiplied by zero and do not contribute to strain calculation. With columns 3, 4, and 5 eliminated, the following values for out-of-plane strains are implied: $\varepsilon_{33} = S_{13}\sigma_{11} + S_{23}\sigma_{22}$, $\gamma_{23} = \gamma_{31} = 0$. If the out-of-plane strain ε_{33} is assumed to be negligible, rows 3, 4, and 5 of the orthotropic compliance matrix are also eliminated. The net effects of these assumptions on the compliance matrix[S] are illustrated in Figure 3.5-4.

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

The plane stress assumption eliminates columns 3, 4, and 5.

Rows 3, 4, and 5 are eliminated when γ_{13}, γ_{23}, and ε_{33} are assumed to be negligible.

Figure 3.5-4 Effect of the Plane Stress Assumption on the Compliance Matrix[S]

A reduced strain-stress relation for a plane stress orthotropic material can now be written as shown in Equation 3.5-17. With the plane stress assumption, the strain-stress relation for a lamina has been reduced from three dimensions to two dimensions.

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix}$$

Equation 3.5-17

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Where:

$$S_{11} = \frac{1}{E_{11}}$$

$$S_{12} = -\frac{v_{12}}{E_{11}} = -\frac{v_{21}}{E_{22}}$$

$$S_{22} = \frac{1}{E_{22}}$$

$$S_{66} = \frac{1}{G_{12}}$$

The number of independent engineering constants required to characterize the state of stress in a lamina has also been reduced from the five - or six for the three-dimensional problem of Equation 3.5-12 - to the four required for the two-dimensional plane stress problem of Equation 3.5-17. Engineering constants E_{33}, v_{13}, and v_{23} are eliminated by the plane stress assumption.

3.5.3.2 Reduced Stiffness Matrix

The reduced compliance matrix [S] of Equation 3.5-17, is inverted to obtain the ply-level reduced stiffness matrix [Q]. Using the reduced stiffness matrix, the reduced constitutive relation for a plane stress orthotropic lamina is given by Equation 3.5-18.

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix}$$

Equation 3.5-18

Components of the reduced stiffness matrix [Q] in Equation 3.5-18 can be expressed in terms of the remaining four independent engineering constants as follows in Equation 3.5-19.

$Q_{11} = \quad E_{11}/ (1-v_{12}\,v_{21})$,

$Q_{22} = \quad E_{22}/ (1-v_{12}\,v_{21})$,

$Q_{12} = \quad v_{21}\,E_{11}/ (1-v_{12}\,v_{21}) =$
$\qquad\qquad v_{12}\,E_{22}/ (1-v_{12}\,v_{21})$,

Equation 3.5-19

$Q_{66} = \quad G_{12}$,

$Q_{16} = \quad Q_{26} = 0$,

$v_{21} = \quad E_{22}\,v_{12}/E_{11}$

The in-plane stress components given by the ply-level constitutive relation in Equation 3.5-18 are shown in Figure 3.5-5

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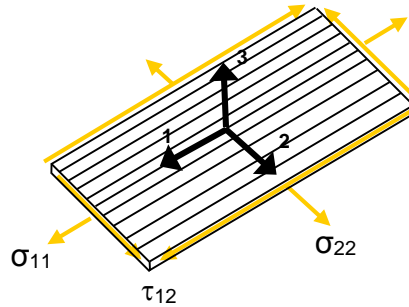| Page 3-37		**Revision A**
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Figure 3.5-5 In-plane Stress Components

3.5.3.3 Lamina Orientation

Laminates are created from stacks of oriented laminae. In practical structures, some laminae must be oriented off axis with respect to the laminate coordinate system. An orientation variable "θ" is used to designate the angle between the laminate material direction "x", and the lamina material direction "1" as shown in Figure 3.5-6. The lamina orientation angle θ designates a positive[1] rotation in degrees about the laminate z axis, from the laminate "x" axis, to the lamina "1" axis.
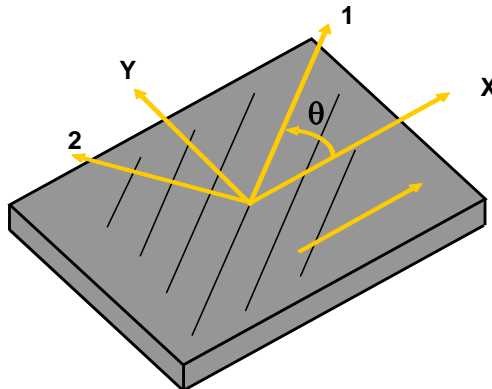

Figure 3.5-6 Convention for Lamina Orientation Angle θ

3.5.3.4 Stress and Strain Transformation

Structural analysis of composites laminates is conducted in the laminate x-y coordinate system. For laminae oriented off-axis ($\theta \neq 0$), the reduced ply level constitutive relation in Equation 3.5-18 must be transformed through the ply angle θ and expressed in the laminate x-y coordinate system. The first step in accomplishing this transformation of the constitutive relation is to develop transformations for lamina stress and strain. The state of stress in each coordinate system is illustrated in Figure 3.5-7.

---

[1] right hand rule

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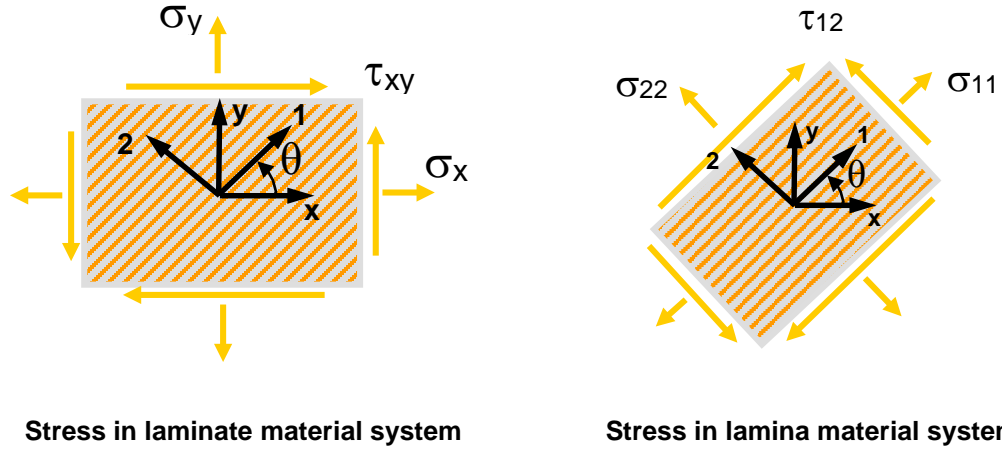| Page 3-38	***PM-4056 Composite Structural Analysis Manual***	**Revision A**
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Figure 3.5-7 Transformation of Lamina Stress

Understanding that stress and strain are actually 2nd order tensors is important when transformations from one coordinate system to another are developed. The symmetric second order tensor forms for stress$\{\sigma\}$ and strain$\{\varepsilon\}$ are shown in Equation 3.5-20 and Equation 3.5-21. Note that engineering shear strain does not appear in the tensor form of Equation 3.5-21.

$$[\sigma] = \begin{bmatrix} \sigma_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \sigma_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \sigma_{33} \end{bmatrix} \text{ where, } \tau_{12} = \tau_{21}, \tau_{31} = \tau_{13}, \tau_{23} = \tau_{32} \qquad \textbf{Equation 3.5-20}$$

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \text{ where, } \varepsilon_{12} = \varepsilon_{21}, \varepsilon_{31} = \varepsilon_{13}, \varepsilon_{23} = \varepsilon_{32}, \qquad \textbf{Equation 3.5-21}$$

The tensor form for strain can be used to develop a transformation equation for a ply rotated by an angle θ, from the x-y system to the 1-2 system. The unexpanded form of this tensor transformation is given by Equation 3.5-22. The out-of-plane strains have been dropped in keeping with the plane stress assumptions.

$$\begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{12} & \varepsilon_{22} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \varepsilon_x & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_y \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \qquad \textbf{Equation 3.5-22}$$

Where:

θ is the angle from the material x axis to the lamina "1" axis as shown in Figure 3.5-7

After multiplying and collecting terms a transformation matrix [T] can be written such that:

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$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} = [T] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}$$

Equation 3.5-23

Where:

ε_{11}, ε_{22}, ε_{12} are strains in the lamina 1, 2, coordinate system

ε_x, ε_y, and ε_{xy} are strains in the laminate x, y coordinate system

[T] is the matrix given by Equation 3.5-24

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

Equation 3.5-24

By substituting the stress tensor for the strain tensor in Equation 3.5-22, it can be shown that the plane stress vector transforms as follows:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = [T] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

Equation 3.5-25

Where:

σ_{11}, σ_{22}, σ_{12} are strains in the lamina 1, 2, coordinate system

σ_x, σ_y, and σ_{xy} are strains in the laminate x, y coordinate system

[T] is the matrix given by Equation 3.5-24

The strain transformation given by Equation 3.5-23 must be modified to support the use of engineering shear strain. This modification is necessary because engineering shear strain appears in the strain vector of the constitutive relation given by Equation 3.5-18. Engineering shear strain γ_{xy} is equal to $2\varepsilon_{xy}$.

Substituting engineering shear strain into Equation 3.5-23 yields the transformation shown in Equation 3.5-26.

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 1/2\gamma_{12} \end{Bmatrix} = [T] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ 1/2\gamma_{xy} \end{Bmatrix}$$

Equation 3.5-26

The ½ factor on engineering shear strain in the strain vectors of Equation 3.5-26 can be eliminated by introduction of the "Reuter" matrix [R] given by Equation 3.5-27.

$$[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

<div align="right">**Equation 3.5-27**</div>

The Reuter matrix transforms a vector as follows in Equation 3.5-28.

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = [R] \begin{Bmatrix} x \\ y \\ \dfrac{z}{2} \end{Bmatrix}, \text{ or } [R]^{-1} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} x \\ y \\ \dfrac{z}{2} \end{Bmatrix}$$

<div align="right">**Equation 3.5-28**</div>

Inserting the Reuter matrix into the strain transformation given by Equation 3.5-26 yields Equation 3.5-29.

$$[R]^{-1} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = [T][R]^{-1} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

<div align="right">**Equation 3.5-29**</div>

After rearranging terms to get a strain transformation in terms of the vector forms used in the constitutive relation of Equation 3.5-18, the final form of the strain transformation is given by Equation 3.5-30.

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = [R][T][R]^{-1} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

<div align="right">**Equation 3.5-30**</div>

The Reuter matrix [R] allows use of engineering shear strain with one common transformation matrix [T(θ)] for both stress and strain. It can be shown that $[R][T][R]^{-1} = [T]^{-T}$, where the -T superscript designates the transpose of the inverse of the [T] matrix. This equality can be used to eliminate direct use of the Reuter matrix in the strain transformation.

So that:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = [T]^{-T} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

<div align="right">**Equation 3.5-31**</div>

3.5.3.5 Transformed Reduced Stiffness

The constitutive relation in lamina coordinates was defined by Equation 3.5-18 in section 3.5.3.2. By substituting the stress transformation of Equation 3.5-25 and the strain transformation of Equation 3.5-31 into the constitutive relation of Equation 3.5-18, the constitutive relation in the laminate x-y system can be written as given by Equation 3.5-32.

$$[T]\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} [T]^{-T} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Equation 3.5-32

After rearranging terms, the constitutive relation in laminate x-y coordinates is given by Equation 3.5-33.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1}[Q][T]^{-T} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Equation 3.5-33

The transformed relation for lamina stiffness $[T]^{-1}[Q][T]^{-T}$ in the constitutive relation of Equation 3.5-33 is referred to as the "Transformed Reduced Stiffness Matrix", and is designated $[\overline{Q}]$. $[\overline{Q}]$ is a function of reduced lamina stiffness [Q], and ply orientation angle θ.

$$[\overline{Q}] = [T]^{-1}[Q][R][T][R]^{-1} = [T]^{-1}[Q][T]^{-T} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}$$

Equation 3.5-34

The six independent transformed reduced stiffness terms can be expressed in terms of the four independent reduced stiffness terms and the ply orientation angle θ as given by Equation 3.5-35.

$$\begin{aligned}
\overline{Q}_{11} &= Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta \\
\overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta) \\
\overline{Q}_{22} &= Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta \\
\overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^3\theta\cos\theta \\
\overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})\sin^3\theta\cos\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta\cos^3\theta \\
\overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta)
\end{aligned}$$

Equation 3.5-35

The transformed reduced stiffness matrix $[\overline{Q}]$ relates lamina stress to strain in the laminate x-y coordinate system. The transformed reduced stiffness matrix allows off-axis laminae to be included in the formulation of laminate stiffness. Off-axis orthotropic laminae appear anisotropic; therefore $[\overline{Q}]$ will normally have non-zero terms in all nine locations. When the ply orientation angle equals 0˚, or 90˚, the $\overline{Q}_{16}$, and $\overline{Q}_{26}$ terms will reduce to zero.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Equation 3.5-36

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3.5.4   Static Material Response

For routine design and analysis purposes, the response of a composite lamina to static loading is linearly elastic to failure. By definition, static loads are applied slowly and do not change rapidly with time. Dynamic loads from impact events, gusts, or vibration are usually analyzed as statically equivalent loads.

3.5.4.1   Fiber Direction

Lamina properties in the fiber direction are governed by the fiber material. The fiber direction of a unidirectional lamina is typically very strong, and very stiff. Fiber direction strength and stiffness properties for a unidirectional graphite epoxy material are shown in Figure 3.5-8. Fiber direction stress-strain response is linear to failure. For the representative example shown, the ultimate tensile strength in the fiber direction (F_{11}^{tu}) is 343 ksi, stiffness in the fiber direction (E_{11}) is 24.5x10^6 psi, and the ultimate strain to failure is 0.014 in/in. The lamina fails catastrophically at the ultimate strain level and does not continue to carry load.
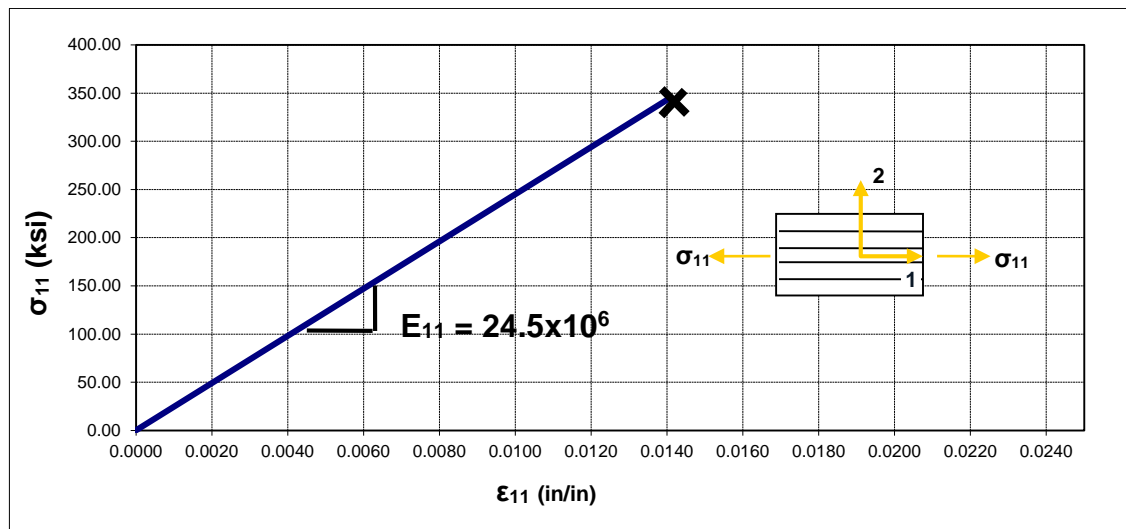
Figure 3.5-8 Fiber Direction Stress-Strain Response Is Linear to Failure
(Tension Shown, Compression Similar)

3.5.4.2   Transverse Direction

Properties in the transverse direction of a unidirectional lamina are governed by the matrix material. For unidirectional laminae, the transverse direction is relatively weak and flexible. Transverse stress-strain response is linear to failure. Example transverse strength and stiffness properties for a unidirectional graphite epoxy material are shown in Figure 3.5-9. For the representative example shown, the ultimate tensile strength in the transverse direction (F_{22}^{tu}) is 10.7 ksi, stiffness in the transverse direction (E_{22}) is 1.26x10^6 psi, and the ultimate strain to failure is 0.0085 in/in. Transverse failure of a unidirectional lamina is characterized by matrix cracking that occurs between the longitudinal fibers. Transverse lamina failure is typically ignored if transversely oriented laminae are present in the laminate stack. Transverse failure does not significantly impact a lamina's ability to carry load in the fiber direction.
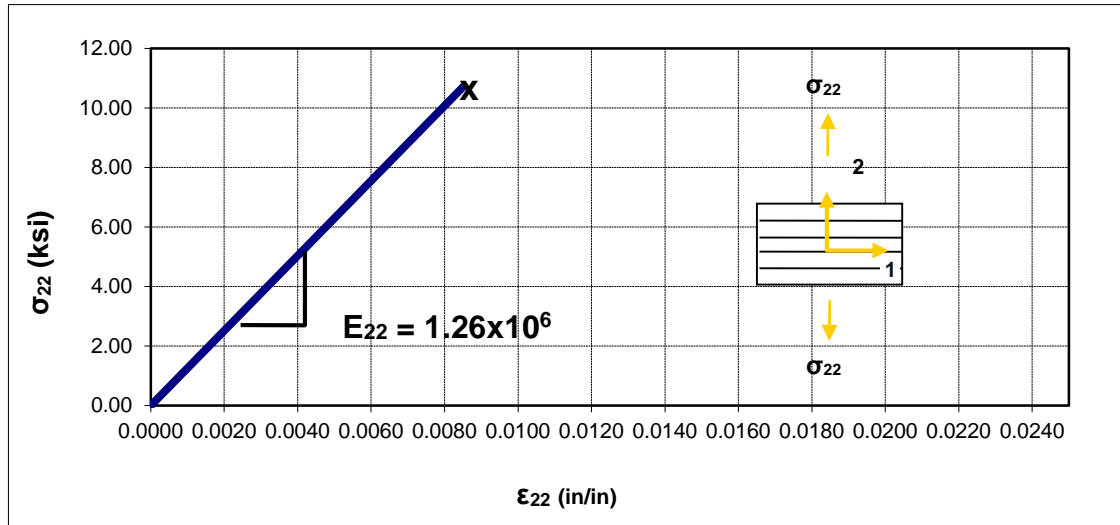
Figure 3.5-9 Transverse Direction Stress-Strain Response is Linear

3.5.4.3 Shear

Lamina properties in the transverse shear direction of a unidirectional lamina are governed by the matrix material. For unidirectional laminae, the transverse shear direction is relatively weak, and flexible. The transverse shear properties of a unidirectional lamina are non-linear to failure. Representative transverse shear strength and stiffness properties for a unidirectional graphite epoxy material are shown in Figure 3.5-10. For the representative example shown, the ultimate transverse shear strength (τ_{12}^{su}) is 12.28 ksi, transverse shear stiffness (G_{12}) is 0.69×10^5 psi, and the ultimate strain to failure is 0.0328 in/in. Transverse shear failure of a unidirectional lamina is usually prevented by guidelines that require ±45 degree plies in the stacking sequence. The ±45 degree plies will carry laminate shear loads as tension and compression in the fiber direction.
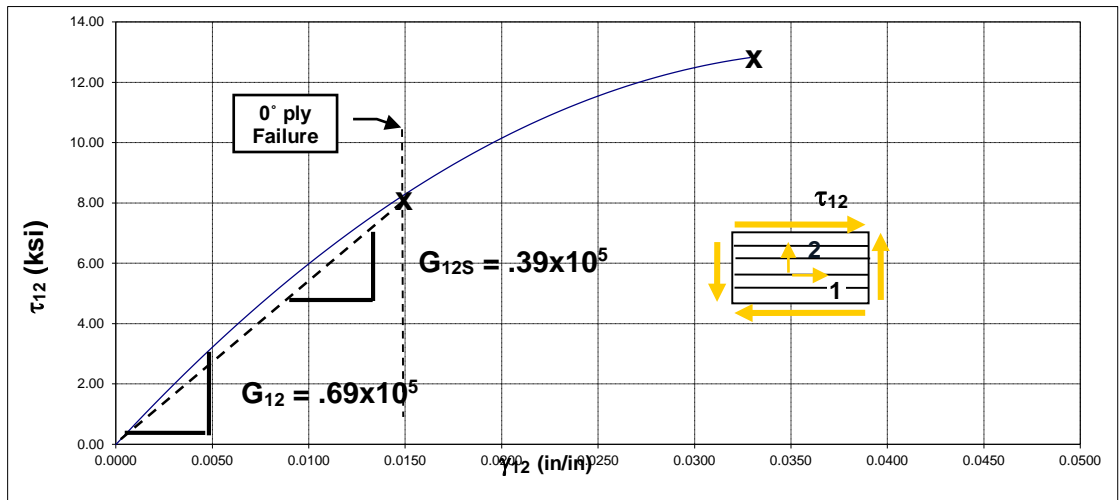

Figure 3.5-10 Shear Direction Stress-Strain Response Is Non-Linear

The secant shear modulus (G_{12S}) is used when calculating the laminate stiffness matrices used for failure predictions. The secant modulus accounts for non-linearity of the shear stress-strain response, and provides

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better correlation with failure test data, especially for soft laminates. G_{12S} is calculated as the slope from the origin to the location on the shear stress-strain curve corresponding to the tensile strain failure of a zero degree ply.
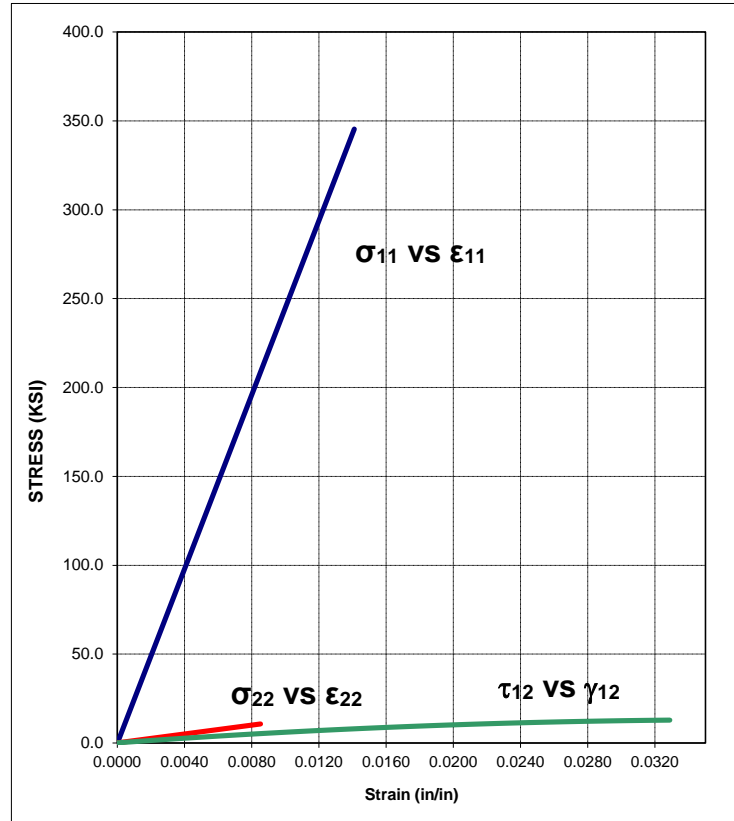
3.5.4.4 Relative Strength and Stiffness Comparison

Figure 3.5-11 Comparison of uni-tape Lamina Stress-Strain Curves

Figure 3.5-11 shows a comparison of in-plane strength and stiffness for a typical unidirectional graphite epoxy material. For unidirectional materials, the fiber direction is significantly stiffer and stronger when compared to transverse and shear directions.

3.5.5 Repeated Loading

Composite materials exhibit higher fatigue threshold stresses than metals. In addition, design criteria such as damage tolerance often limit the stress levels in composite structures to such low values that fatigue does not generally represent a design constraint. As shown in Figure 3.5-12, the max spectrum stress is typically below the runout stress for most composite properties. Exceptions can include thermoplastics, interlaminar stresses,

certain joint configurations, and high-cycle fatigue (n $>10^7$). In these cases an appropriate fatigue knockdown is incorporated into the static analysis or allowables.
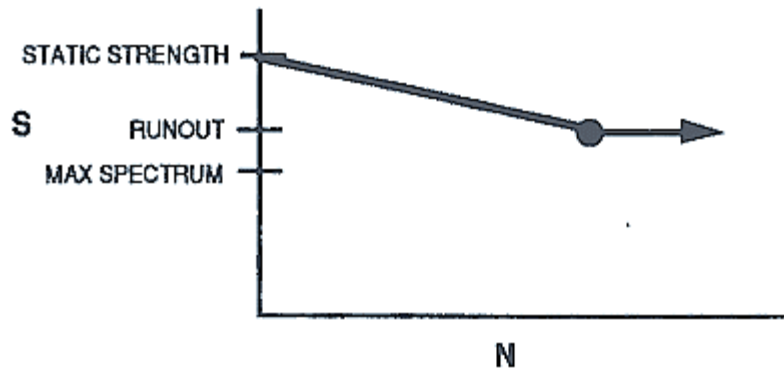

Figure 3.5-12 Typical Composite Material Fatigue S-N Behavior

3.5.6   Creep/Rupture

 -This section is reserved for future use-

3.5.7   Lamina Failure Modes and Criteria

There are five failure modes for an orthotropic plane-stress lamina. The allowable load level for a lamina is dependent on the type of loading (tension, compression, or shear), and on load orientation (fiber direction, or transverse direction). These dependencies result in a minimum of five separate allowable strength values for an orthotropic lamina. The five allowables correspond to the five failure modes: tension in the fiber direction, compression in the fiber direction, tension in the transverse direction, compression in the transverse direction, and shear. Because laminae are assumed to be linear elastic to failure, the five allowable values are commonly expressed in terms of both stress and strain. Table 3.5-1 shows the ultimate material allowable notation for a composite lamina.

Table 3.5-1 Material Allowable Notation for a Composite Lamina

Stress   Strain

F_{11}^{tu} ε_{11}^{tu} Ultimate tensile allowable in the fiber direction

F_{11}^{cu} ε_{11}^{cu} Ultimate compressive allowable in the fiber direction

F_{22}^{tu} ε_{22}^{tu} Ultimate tensile allowable in the transverse direction

F_{22}^{cu} ε_{22}^{cu} Ultimate compressive allowable in the transverse direction

F_{12}^{su} γ_{12}^{su} Ultimate engineering shear allowable in the 1-2 plane

Tensile failure of a lamina in the fiber direction is primarily a function of fiber tensile strength. As the first fibers begin to fail, stress in the remaining fibers will increase. Matrix cracks may develop parallel to the fibers due to shear transfer of load from the broken fibers to the intact fibers. Increased loading in the intact fibers results in additional fiber failures and initiates a rapid failure sequence that continues until total failure of the lamina has occurred. Once initiated, tensile failure is typically rapid and catastrophic.

Compressive failure of laminae in the fiber direction is primarily a function of matrix properties and fiber stiffness. Small wave-length micro-instability of the fibers occurs in a manner analogous to the buckling of a beam on an elastic foundation. For fiber volume fractions greater than approximately 0.2, the dominant microbuckling mode is "shear". In shear mode, fibers buckle in unison, with the same pattern and direction. Microbuckling effects are more dependent on straightness and uniformity of fibers than on the diameter of fibers. The dominant property controlling compressive failure in the fiber direction is matrix shear modulus.

The remaining failure modes are transverse tension, transverse compression and transverse shear. These are considered matrix failure modes and are primarily dependent on matrix properties. For failure in the plane of the lamina, fibers provide very little reinforcement in transverse tension, transverse compression, and shear. Matrix mode lamina failures occur without fracture of the fibers. For the transverse tension mode, failure occurs by sudden formation of a matrix crack parallel to the fiber direction. Lamina transverse tensile strength is governed by tensile strength of the matrix material. Matrix compression and shear properties dominate the transverse compression allowable. Transverse shear strength of a lamina is dominated by shear strength of the matrix material. Matrix dominated lamina failure modes are delayed or in some cases ignored if ply stacking sequence guidelines are followed. Stacking sequence guidelines insure that there are sufficient laminae oriented in each direction to allow the laminate to carry the majority of transverse loads and shear loads in the fiber dominated directions of other laminae.

Numerous criteria exist to predict the allowable loads or combination of loads that will result in first fiber failure of a lamina at a point. At Lockheed Martin Aeronautics, failure criteria and material property testing are interrelated. Coupon specimen tests are conducted to establish allowable values of un-notched strength and stiffness. Tests are conducted on a family of coupon test specimens to ensure that results are applicable to a range of laminate stacking sequences. Analytical failure predictions are correlated with test data by adjusting the strength and stiffness values to achieve a best conservative fit. **Lamina strength and stiffness values developed using this methodology are correlated to a specific failure criterion and may not be accurate or conservative if used with an alternate failure criterion. Specified failure criteria may not be accurate or conservative if used with allowables developed using a different methodology.** Always refer to program guidance for the proper failure criteria and allowables to use in composite analysis.

Lockheed Martin Aeronautics program specific design, mechanical, and physical properties for approved composite material can be found in the IDAT tool MATUTIL.
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3.5.7.1　　Maximum Strain Failure Criteria

The lamina failure criterion used most frequently at Lockheed Martin Aeronautics is "Maximum Strain". Lamina strains are bounded by maximum and minimum failure values. Failure at a point occurs if strains in the material directions exceed the allowable values. Interaction between strains is not considered. The maximum strain failure criteria envelope is illustrated in Figure 3.5-13.
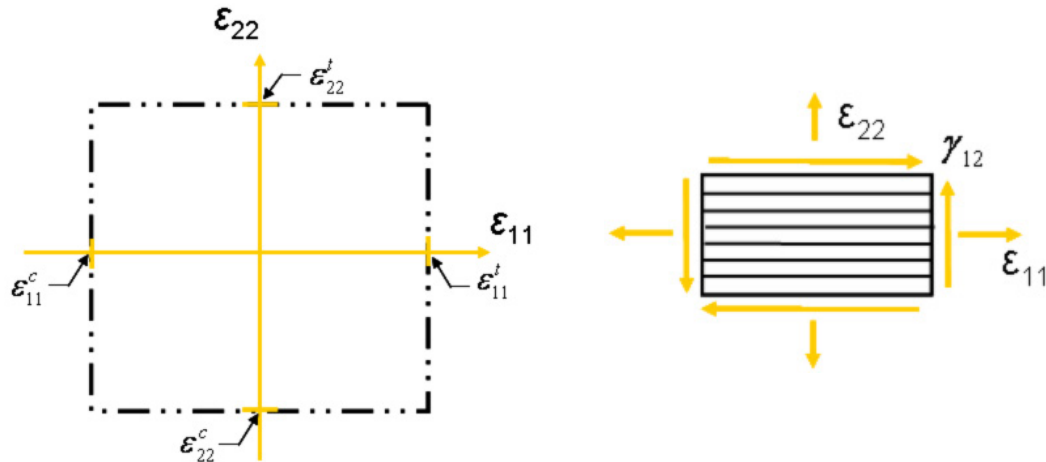

Figure 3.5-13 Maximum Strain Failure Criteria

The maximum strain failure envelope is given mathematically by Equation 3.5-37.

$$\varepsilon_{11}^{c} < \varepsilon_{11} < \varepsilon_{11}^{t}$$
$$\varepsilon_{22}^{c} < \varepsilon_{22} < \varepsilon_{22}^{t} \qquad\qquad\qquad \textbf{Equation 3.5-37}$$
$$\left| \gamma_{12} \right| < \gamma_{12}^{s}$$

3.6　　Environmental Effects

Composite materials and parts must be capable of surviving the environmental conditions expected to be encountered in service. Common environmental conditions that must be considered are: temperature, humidity, chemical exposure, corrosive environments, lightning, erosion, impact, ultraviolet rays, and fire.

Changes in temperature and moisture content are the primary environmental factors which influence the analysis of composite laminae. Hail, tool drop, ballistic damage, and other impact scenarios are addressed from a damage tolerance and durability perspective with material toughness and strain energy release rates as key factors in the analysis. Chemical resistance and flammability are primarily functions of material selection and otherwise do not impact analysis and sizing. Lightning, erosion, corrosion, ultraviolet rays and other environmental effects impact laminate and part design with requirements for barrier plies, specialty fasteners, specialty coatings and other means, but do not otherwise influence analysis at the lamina level.

Changes in temperature and moisture content affect analysis of composite materials in two principal ways:

- Changes in material properties including: stiffness, strength, glass transition temperature (T_g), strain energy release rates, and density
- Material expansion and contraction primarily in the matrix-dominated material directions.

3.6.1   Material Property Changes

The matrix, and therefore the laminae, will lose strength and stiffness when the temperature and humidity rise. The general trend of temperature and moisture effects on stiffness is shown below in Figure 3.6-1. The temperature and moisture effects on strength follow a similar trend but can be more complex. The trends will vary depending on material and load orientation.
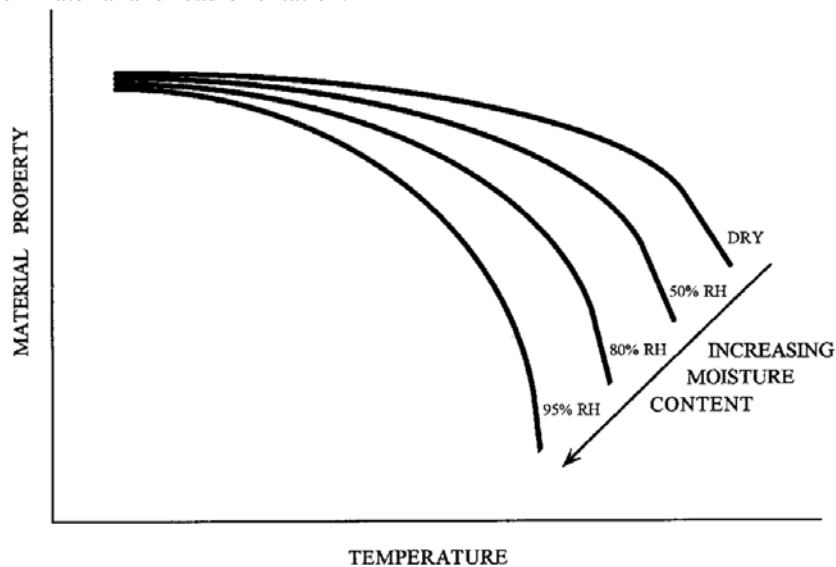
Figure 3.6-1 Effects of Temperature and Moisture on Material Properties

Material properties associated with the load condition's critical environment are used for analysis. The normal practice for composite allowables development at Lockheed Martin Aeronautics is to provide material properties at a range of different temperatures (-65°F, 75°F, 180°F, 275°F, and 350°F). Material testing at each temperature level is conducted with specimens containing critical moisture content. Properties associated with the higher temperature are provided "wet" (moisture content (ΔC) $\approx$ 1.2%) and properties associated with the lower temperatures are provided "dry" (moisture content (ΔC) $\approx$ 0%). These temperature and moisture combinations are commonly referred to as "Elevated Temperature Wet" (ETW), and "Cold Temperature Dry" (CTD). Thermal effects are accounted for explicitly by use of material properties at the load condition temperature, and the moisture effects are accounted for implicitly as "worst case" impact to properties for the specified temperature.

3.6.1.1   Thermal Effects on Properties

Composite material's resistance to heat is primarily dependent on the resin system and the cure. With the exception of polyethelene, service temperature of fiber is not an issue when used with organic resin matrix materials. The glass transition temperature usually determines the heat resistance of thermoset resin laminae. A comparison of heat resistance values for common resin systems is presented in Table 3.6-1.

Table 3.6-1 Comparison of Heat Resistance for Common Organic Resin Systems

Property	Polyester	Epoxy	Phenolic	Bismaleimide	Polyimide
Heat Resistance	180° F	250° F	350° F	350° F	500-600° F

A comparison of heat resistance values for common fiber materials is presented in Table 3.6-2

Table 3.6-2 Comparison of Heat Resistance for Common Fiber Materials

Property	S-Glass	Aramid (Kevlar)	Polyethelene	Carbon	Quartz	Silicon Carbide (SiC)	Alumina	Boron
Service Temperature	600-700° F	500° F	230° F	1000° F	2000° F	2400° F	1800° F	3500° F

3.6.1.2 Moisture Absorption Effects on Properties

When a composite lamina with a polymeric matrix is placed in a humid environment, the matrix material will absorb moisture. The rate of moisture absorption or desorption is dependent on temperature, humidity, and current moisture concentration in the material. The moisture absorption/desorption process can be modeled with Fick's 2nd law of diffusion. The matrix absorbs moisture until an equilibrium saturation point is reached. Typical equilibrium moisture content for severe humidity exposure of common epoxy materials is represented by a 1.1 to 1.3 percent weight gain. Representative moisture absorption rates are shown for room temperature average conditions in Figure 3.6-2, and for hot wet conditions in Figure 3.6-3.

With the exception of aramid fibers, the moisture absorption of most fiber materials is negligible. Moisture absorption for aramid fiber laminae can exceed 2% weight gain. Moisture absorption can also occur along the fiber/matrix interface of glass and quartz fibers and is a well-known degrading phenomenon.

Composite material properties are dependent upon the amount of moisture in the matrix. Absorbed moisture generally lowers resin glass transition temperature (T_g) and causes a decrease in matrix dominated properties at elevated temperatures. Research indicates that these moisture effects on heat resistance are reversible. When moisture content of the material is reduced, the glass transition temperature and other material properties return to their original values. Elevated Temperature Wet (ETW) is generally a critical condition for the analysis composite structures.
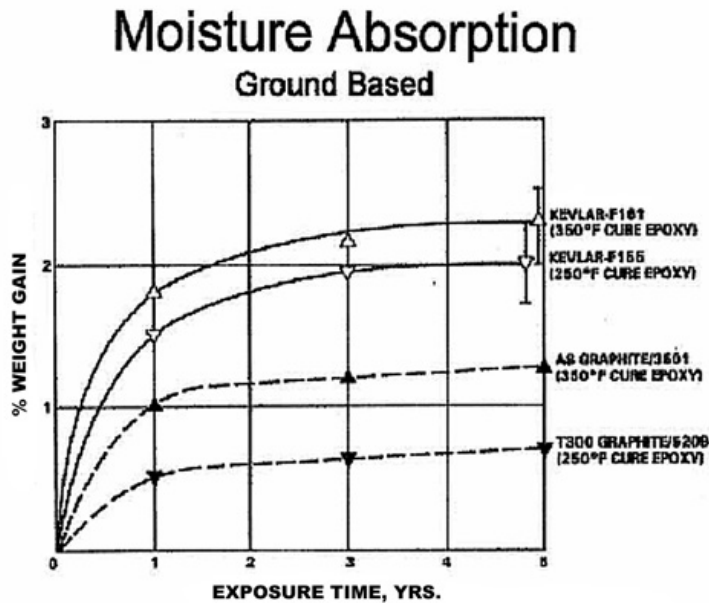
Figure 3.6-2 Example moisture absorption at Room Temperature Average (RTA)
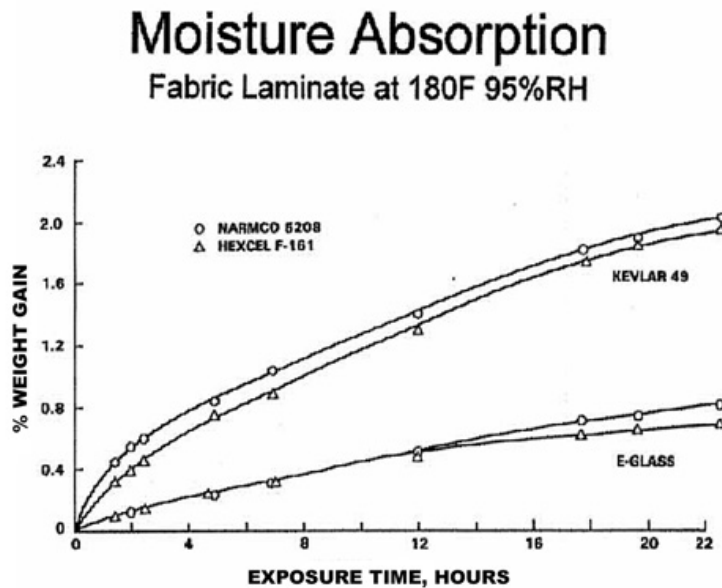

Figure 3.6-3 Example Moisture absorption at Elevated Temperature Wet (ETW)

3.6.2 Material Expansion and Contraction

Changes in temperature and humidity result in expansion or contraction of composite materials. In the general case for a single layer of anisotropic material, the dimensional changes can include shear deformations along

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with expansion or contraction in each of the three orthogonal coordinate planes. As with mechanical load effects, the three-dimensional effects of environmental loads are reduced to two dimensions with the plane stress assumptions of Classical Laminated Plate Theory (CLPT). Unidirectional and woven fiber laminae are assumed to expand or contract orthotropically in plane with no shear deformation. Classical Laminated Plate Theory assumptions neglect the effects of out of plane lamina expansion or contraction. For thick curved laminates, the out-of-plane environmental deformations of laminae may cause significant distortion of a laminate and can not always be neglected.

Environmental expansion and contraction effects cause residual strain in laminae. Individual laminae attempt to expand or contract due to changes in temperature and moisture but are constrained by the overall hygrothermal[1] response of the laminate. The overall hygrothermal response is determined by equilibrium of internal forces resulting from differential hygrothermal strains in the individual composite laminae. The difference between the overall hygrothermal response of the laminate and the free expansion of a specific ply determines the residual strain in that specific ply. The first step in calculating the residual strain in a lamina is to understand free expansion effects of temperature and moisture. The discussion of free expansion effects that follows forms a basis for the residual strain calculations presented in chapter 4.

3.6.2.1   Thermal Expansion

Temperature change causes a volumetric change in a lamina. Laminae expand orthotropically when temperature is increased and contract orthotropically when temperature is decreased. Thermal expansion or contraction in each orthotropic material direction is linearly related to temperature change (ΔT) by a constant coefficient of thermal expansion (α_i). Unrestrained (free) thermal expansion of a lamina is expressed mathematically as follows in Equation 3.6-1.

$$\{\varepsilon\}_{\Delta t} = \{\alpha\}\Delta T \, , \quad \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix}_{\Delta t} = \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix}\Delta T \qquad \text{\textbf{Equation 3.6-1}}$$

Where:

ΔT is the change in temperature from cure

α_i are the coefficients of thermal expansion (CTE)

The temperature change (ΔT) of Equation 3.6-1 is referenced to the stress-free temperature of a lamina. The stress-free temperature (T_{sf}) is a material property that is defined as the temperature at which the residual cure stresses of Equation 3.6-1 are zero. T_{sf} is a function of use temperature and is not a material constant. For most Lockheed Martin structural prepreg materials the temperature-dependent values for T_{sf} are determined empirically, as described in Reference 3-9 "CSE-101" section 4. Values for T_{sf} are typically between the cure temperature and the dry glass transition temperature (T_g) of the material. Stress-free temperatures (T_{sf}) are normally provided as temperature dependent material data in IDAT MATUTL.

Unidirectional laminae are thermally stable in the fiber direction ($\alpha_1 \approx 0$). Balanced fabric laminae have relatively low and equal coefficients of thermal expansion in the 1 and 2 directions. Representative coefficients of thermal expansion for example graphite/bismaleimide lamina materials are presented in Table 3.6-3.

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[1] Combined effects of temperature and moisture

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Table 3.6-3 Coefficients of Thermal Expansion for a Graphite/Bismaleimide Material

B_IM7/5250, 75˚ F Dry		
	Unidirectional Tape	Fabric
α_1	0.00 $\mu\varepsilon$/˚F	1.75 $\mu\varepsilon$ /˚F
α_2	16.10 $\mu\varepsilon$ /˚F	1.75 $\mu\varepsilon$ /˚F
α_6	0.00 $\mu\varepsilon$ /˚F	0.00 $\mu\varepsilon$ /˚F
α_3	16.10 $\mu\varepsilon$ /˚F	16.10 $\mu\varepsilon$ /˚F

The thermal strains of Equation 3.6-1 are zero at the stress-free temperature ($\Delta T = 0$), and the laminae will be in a stress-free state if no other loads are present. At any temperature below the stress-free temperature (ΔT is negative), each unidirectional ply in a laminate wants to contract primarily in the 2-direction. Figure 3.6-4 illustrates the unrestrained thermal contraction of a unidirectional lamina as temperature is decreased from cure (350ºF) to room temperature (75ºF).
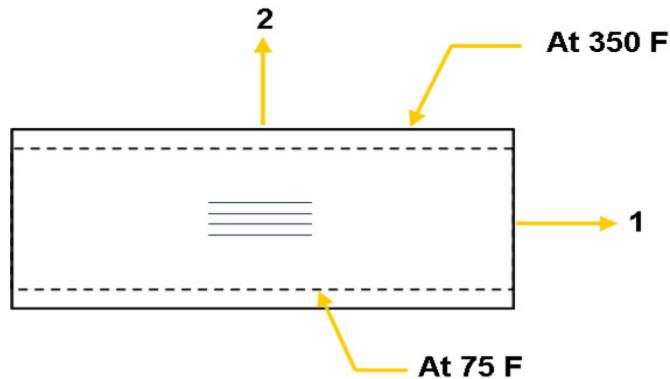
Figure 3.6-4 Unrestrained Thermal Expansion of a Lamina

For temperature ranges typically seen on aircraft, thermal coefficients of expansion for composite materials are assumed to be constant with respect to temperature and are in general much lower in the fiber direction(s) when compared to aluminum. The temperature independence of composite thermal coefficients is assumed for analytical simplicity in IDAT. This constant α approximation is reasonable in the operating environment of most aircraft, but is generally not physically accurate across all temperature ranges. Figure 3.6-5 shows α as a function of temperature for the 2024 series aluminum alloys compared to the assumed constant values for graphite/bismaleimide tape and fabric materials.

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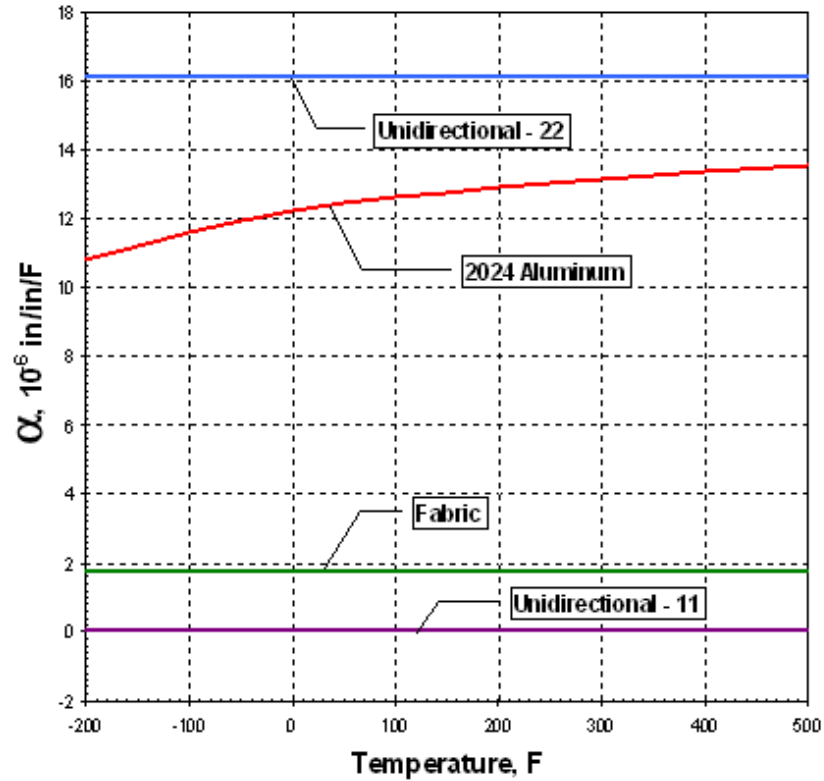| Page 3-53	**PM-4056 *Composite Structural Analysis Manual***	**Revision A**
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Figure 3.6-5 Comparison of Thermal Expansion Coefficients

3.6.2.2   Moisture Swelling

Moisture absorption causes a volumetric change in a lamina. Laminae expand orthotropically when moisture is absorbed and contract orthotropically when moisture is desorbed. Moisture expansion or contraction in each orthotropic material direction is linearly related to a change in specific moisture content (ΔC) by a constant coefficient of moisture expansion (β_i). Unrestrained moisture expansion of a lamina is expressed mathematically as follows in Equation 3.6-2.

$$\{\varepsilon\}_{\Delta t} = \beta \Delta C \, , \quad \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix}_{\Delta t} = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{Bmatrix} \Delta C \qquad\qquad \textbf{Equation 3.6-2}$$

Where:

ΔC is the percent change in specific moisture content

β_l are the coefficients of moisture expansion

Unidirectional laminae are moisture-stable in the fiber direction ($\beta_l \approx 0$). Balanced fabric laminae have relatively low and equal coefficients of moisture expansion in the 1 and 2 directions. Coefficients of moisture expansion for example graphite/bismaleimide lamina materials are presented in Table 3.6-4.

Table 3.6-4 Coefficients of Moisture Expansion for a Graphite/Bismaleimide Material

B_IM7/5250, 300˚ F		
	Unidirectional Tape	Fabric
β_1	0.00 με/ΔC	0.00 με/ΔC
β_2	0.0024 με/ΔC	0.00 με/ΔC
β_6	0.00 με/ΔC	0.00 με/ΔC
β_3	0.0024 με/ΔC	0.0024 με/ΔC

Moisture change (ΔC) is a percentage weight gain referenced to the "Dry" state of a lamina. Moisture absorbed into the resin causes each ply to swell primarily in the 2-direction. Figure 3.6-4 illustrates the unrestrained moisture swelling of a unidirectional lamina as moisture content is increased from dry (ΔC = 0) to wet (ΔC ≈ 1.2).
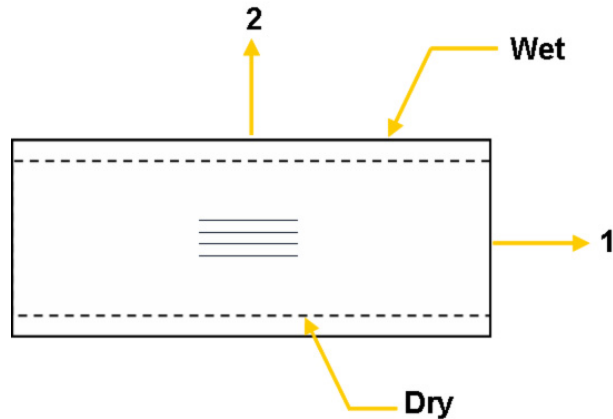
Figure 3.6-6 Unrestrained Moisture Expansion of a Lamina

Exposed laminate surfaces pick up and lose moisture rapidly, leaving moisture gradients through the laminate thickness. Moisture desorption gradients may put the surface of a laminate in tension as the surface shrinks relative to the interior of the laminate. If the residual tension stress at the surface is beyond the strength of the matrix, microcracking will occur. When the laminate has been exposed to the humid environment for a sufficient period of time, moisture concentration throughout the matrix will be uniform.

3.6.2.3　Combined Mechanical and Environmental Stress

Strains caused by temperature and moisture changes are added to mechanical strains of Equation 3.5-17 as follows in Equation 3.6-3.

$$\varepsilon_i = S_{ij}\sigma_j + \alpha_i\Delta T + \beta_i\Delta C \qquad\qquad \textbf{Equation 3.6-3}$$

Where:

α_i are the coefficients of thermal expansion

ΔT is the change in temperature(°F) relative to the cure temperature or "stress free temperature"

β_i are the coefficients of moisture expansion

ΔC is the change in moisture content(%)

S_{ij} is the lamina reduced compliance matrix

Solving the strain equation for the combined stress yields Equation 3.6-4.

$$\sigma_i = C_{ij}(\varepsilon_j - \alpha_j \Delta T - \beta_j \Delta C)$$

Equation 3.6-4

Where:

C_{ij} is the lamina reduced stiffness matrix

The combined mechanical and environmental stress in lamina "k" is written in matrix form as follows in Equation 3.6-5.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \left(\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}_k + \begin{Bmatrix} -\alpha_1 \Delta T \\ -\alpha_2 \Delta T \\ 0 \end{Bmatrix}_k + \begin{Bmatrix} -\beta_1 \Delta C \\ -\beta_2 \Delta C \\ 0 \end{Bmatrix}_k \right)$$

Equation 3.6-5

Coefficients of thermal and moisture expansion for lamina k represent strain tensors and are transformed to laminate coordinates using the transformation matrix for lamina strain as presented earlier in Equation 3.5-24.

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k = [T]_k^T \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix}_k \qquad \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k = [T]_k^T \begin{Bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{Bmatrix}_k$$

Equation 3.6-6

The transformed equation for combined mechanical and environmental stress in lamina "k" is written in matrix form as given by Equation 3.6-7.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left(\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k + \begin{Bmatrix} -\alpha_x \Delta T \\ -\alpha_y \Delta T \\ -\alpha_{xy} \Delta T \end{Bmatrix}_k + \begin{Bmatrix} -\beta_x \Delta C \\ -\beta_y \Delta C \\ -\beta_{xy} \Delta C \end{Bmatrix}_k \right)$$

Equation 3.6-7

Equation 3.6-7 is an extended form of the transformed reduced constitutive relation given by Equation 3.5-36. This extended form includes the transformed relations for environmental strains caused by changes in temperature and moisture content. The extended transformed reduced constitutive relation given by Equation 3.6-7 is used to calculate lamina mechanical and environmental stress in the laminate x-y coordinate system.

3.7 Material Specification and Qualification Process

Material specifications and process controls are necessary to ensure that the laminate structural properties of the finished part consistently meet specified strength, quality, and durability design requirements.

Unlike metallic parts, where material is formed and/or machined from a homogenous base to achieve a desired shape and thickness profile, advanced composite parts are "Laminates" assembled from stacked and oriented layers of unidirectional tape or fabric. The laminated nature of the composite material is being built as part of the part fabrication process. Design factors such as material selection and stacking sequence are combined with design requirements for a host of features such as ply terminations and ramp angles to determine the maximum theoretical strength, stiffness, and durability of the laminate. The actual strength, quality, and durability of the finished laminate are dependent on the material properties and the fabrication process. The actual material properties of a lamina are dependent on the material processes. This section addresses key considerations of the materials side of the process.

3.7.1 Material Processes

The processes and requirements for a specific composite material form are specified in detail to ensure consistent properties of finished parts manufactured from that material. Each material form will have a unique process specification which controls the material, qualification, handling and storage, but not the parts built.

3.7.1.1 Material Specification

A material specification is a document that controls the constituent elements of a composite material system. The specified items might include: fiber, sizing, fabric style, resin system, areal weight, traceability, uniformity and workmanship, splicing methods, material form, marking requirements, tolerances, physical and chemical properties. A detailed material specification is developed for each material system that will be used for production parts. The material specification will identify acceptable values for properties and will specify the test methods and controlling documents that are used to verify that specified properties are consistent and within the acceptable range. Material specifications will normally be referenced as part of a manufacturing process specification for composite parts. Material specifications will also be referenced in ply tables of laminated component drawings.

3.7.1.2 Material Qualification and Acceptance

Each material property or requirement in a material specification must be verifiable by test or measurement. The material qualification tests and acceptance criteria are governed by program documents that establish the test matrix necessary for verification of a specified physical or chemical property. Verification of a material system's specified properties usually requires testing of at least three material batches, which contain at least two different resin batches and two different fiber lots. Following successful qualification of a material system, each lot of material received is subjected to acceptance testing as specified in the material qualification and acceptance document. Vendors are contractually required to report any changes in their fabrication process to a specific contact within Lockheed Martin. Reported changes will be reviewed by the Lockheed Martin Material and Process group to determine if re-qualification of the material is required.

3.7.1.3   Material Storage and Handling

The storage and handling of composite materials must be monitored and controlled. Prepreg materials require time, temperature, and humidity controls to prevent premature curing of the staged resin. The cumulative elapsed time in each storage and handling condition from manufacture of material to cure of the finished part is specified and monitored. Storage and handling conditions include: inventory life, storage life, working life, and open mold life. Material storage and handling requirements are normally defined in the material specification document.