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Program Structures & Algorithms

Fall 2021 Assignment No. 1

Tasks

* Relationship Conclusion:

The relationship conclusion by mathematical deduction is: $d \approx \sqrt{n}$

However, the program simulation result is: $d \approx 0.89\sqrt{n}$

Here the n is the step, and the d, specifically is the root-mean-square of the distance, because of the d actually is a probability density function, but we are no need to calculate it.

* Evidence to support the conclusion:

1. Mathematical deduction

First come to my mind is to divide the calculation of x and y, which they are independent. The distance of the x and y are probability distributions, however, I found that the average distance is enough, which are also in the code.

Assume that there are totally n steps, and x moved m steps and y moved l steps.
The sum of the x and y, let's say x_s and y_s, which is the final coordinates of the drunken man.

The two equations show the sum of the x and y:
(1) $x_s = x_1 + x_2 + \dots x_m$
(2) $y_s = y_1 + y_2 + \dots y_l$

And the distance is:

$$d = \sqrt{x_s^2 + y_s^2}$$

Then replace the x_s and y_s with the (1) and (2) respectively:

(3) $d^2 = x_s^2 + y_s^2$

$$= x_1^2 + x_2^2 + \ldots + x_m^2 + x_1x_2 + x_1x_3 + \ldots + x_1x_m$$

$$+y_1^2 + y_2^2 + \ldots + y_l^2 + y_1y_2 + y_1y_3 + \ldots + y_1y_l$$

Assume that n is a really big number, and the x_n and y_n are probably become positive or negative, the terms in the (3) $x_1x_2 + x_1x_3 + \ldots + x_1x_m$ and $y_1y_2 + y_1y_3 + \ldots + y_1y_l$ can be ignored.

Then an approximation can be showed:

$$d^2 \approx \underbrace{[x_1^2 + y_1^2] + [x_2^2 + y_2^2] \ldots + [x_m^2 + y_l^2]}_{\text{n steps}}$$

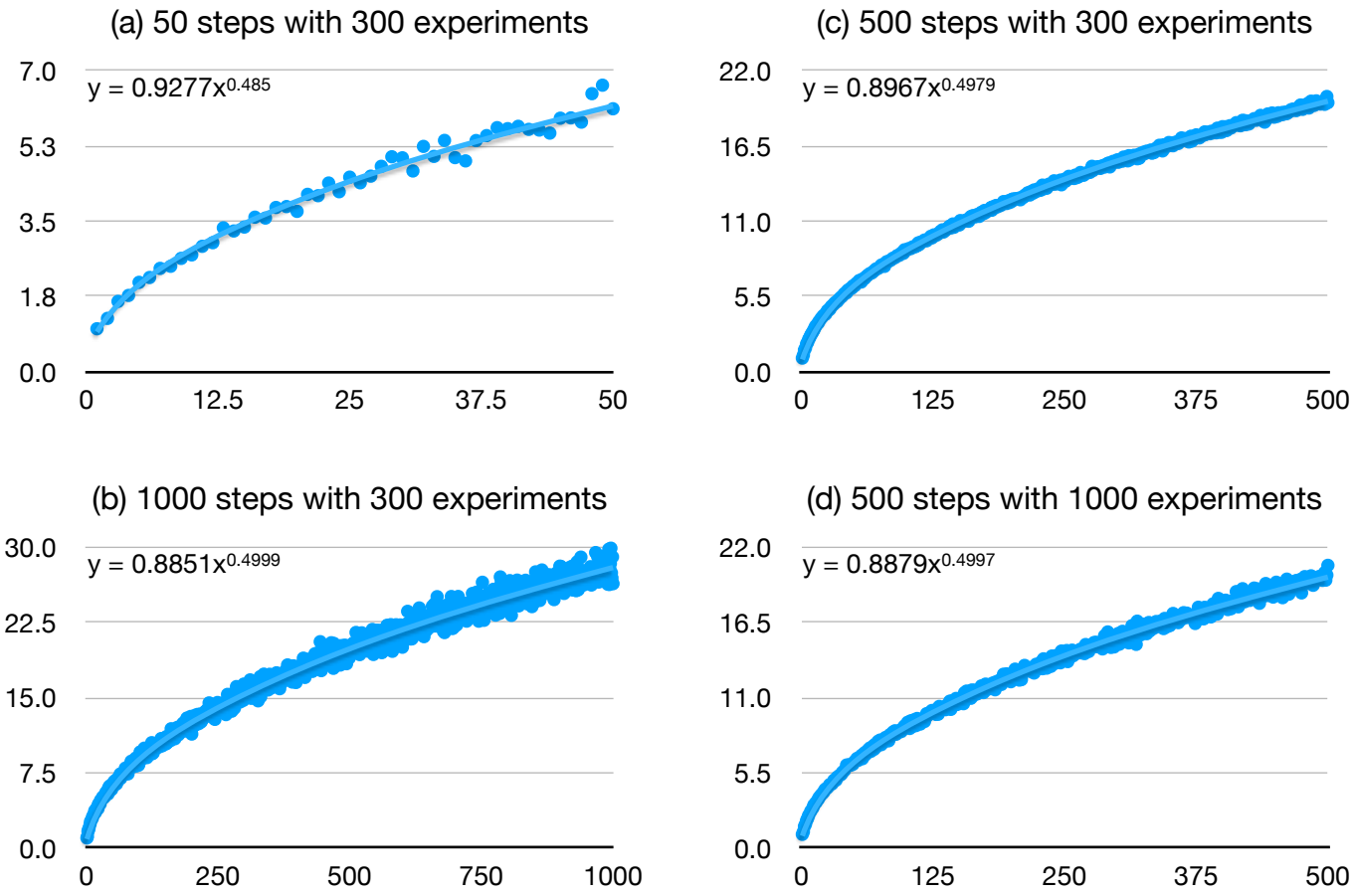
The m and l shared the same probability, 50% and the total steps are n steps.
The result is

$$d^2 \approx n$$

Then

$$d \approx \sqrt{n}$$

2. Graphical Representation from program simulation



The y axis represent the output of the program, which is the mean distance of the experiments.
The x axis means the steps of the drunk man moved.

Some observations:

* Under the same experiments(plot a, b, and c), the increase of the steps can improve the precision of the power of x, but not the coefficient of the x.
* Under the same steps, the increase of the experiments can also improve the precision of the power of x, but not the coefficient of the x.
* The average of the coefficient of the x is approximately 8.99.

3. Reproduction the experiments

The Bash script I ran to generate the statistics, which located with RandomWalk.java:

run_experiments.sh

A file “results.csv” will be generated, then it can be plotted.

* Unit tests result: