

Program Structures & Algorithms

Fall 2021 Assignment No. 1

Tasks

* Relationship Conclusion:

The relationship conclusion is: $d \approx 0.89\sqrt{n}$

However, the result of the mathematical deduction is: $d \approx \sqrt{n}$

Here the n is the step, and the d is the of the approximately average distance, because of the distance actually is a probability density function, but there is no need to calculate it.

I believed that the program simulation will be more correct and there are some factor in the mathematical deduction I didn't considered.

* Evidence to support the conclusion:

1. Mathematical deduction

First come to my mind is to divide the calculation of x and y , which they are independent. The distance of the x and y are probability distributions, however, I found that the average distance is enough, which are also in the code.

Assume that there are totally n steps, and x moved m steps and y moved l steps.

The sum of the x and y , let's say x_s and y_s , which is the final coordinates of the drunken man.

The two equations show the sum of the x and y :

$$(1) x_s = x_1 + x_2 + \dots x_m$$

$$(2) y_s = y_1 + y_2 + \dots y_l$$

And the distance is:

$$d = \sqrt{x_s^2 + y_s^2}$$

Then replace the x_s and y_s with the (1) and (2) respectively:

$$\begin{aligned}
 (3) \quad d^2 &= x_s^2 + y_s^2 \\
 &= x_1^2 + x_2^2 + \dots + x_m^2 + x_1x_2 + x_1x_3 + \dots + x_1x_m \\
 &\quad + y_1^2 + y_2^2 + \dots + y_l^2 + y_1y_2 + y_1y_3 + \dots + y_1y_l
 \end{aligned}$$

Assume that n is a really big number, and the x_n and y_n are probably become positive or negative, the terms in the (3) $x_1x_2 + x_1x_3 + \dots + x_1x_m$ and $y_1y_2 + y_1y_3 + \dots + y_1y_l$ can be ignored.

Then an approximation can be showed:

$$d^2 \approx \underbrace{[x_1^2 + y_1^2] + [x_2^2 + y_2^2] \dots + [x_m^2 + y_l^2]}_{n \text{ steps}}$$

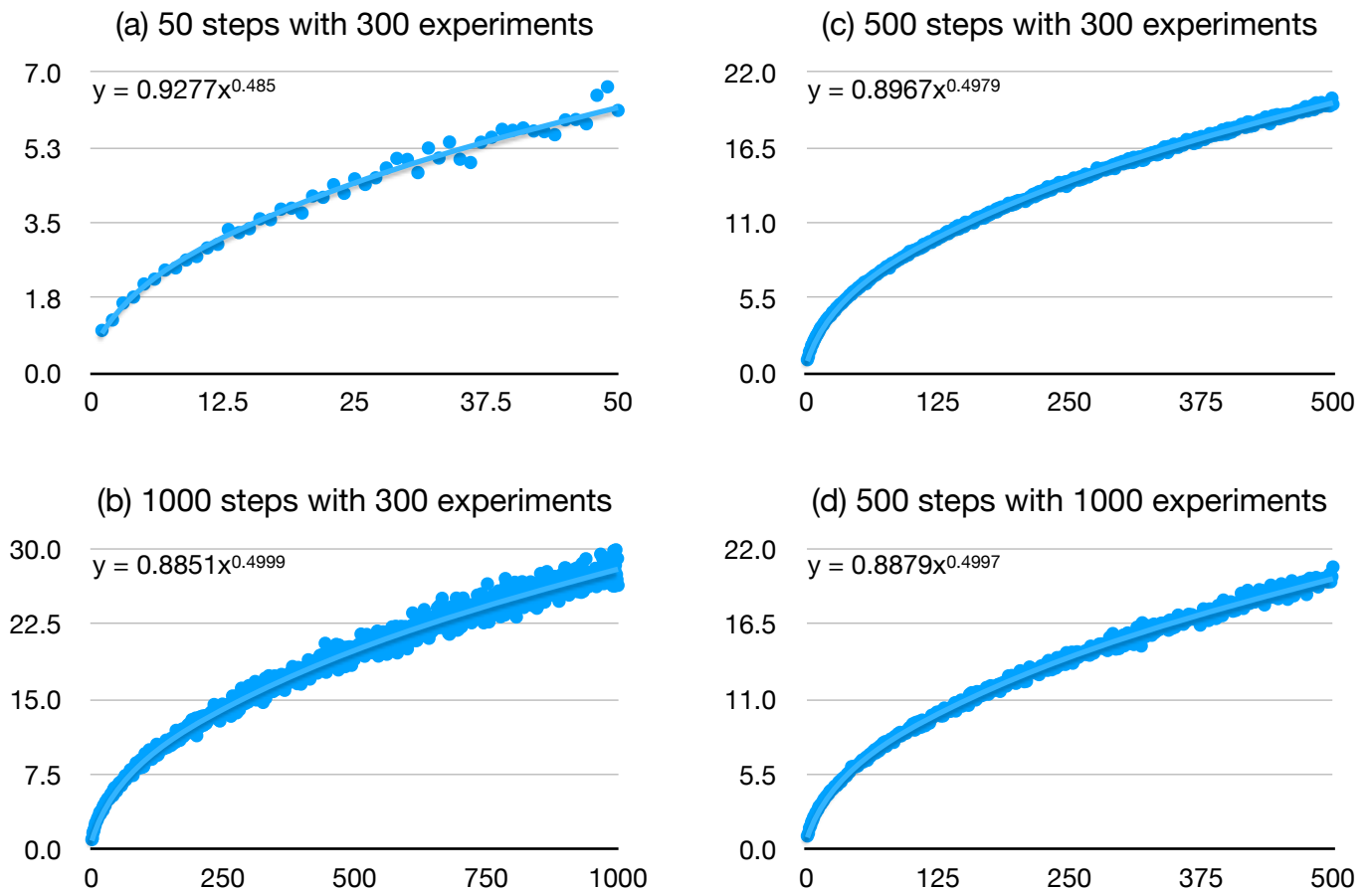
The m and l shared the same probability, 50% and the total steps are n steps.
The result is

$$d^2 \approx n$$

Then

$$d \approx \sqrt{n}$$

2. Graphical Representation from program simulation



The y axis represent the output of the program, which is the mean distance of the experiments.

The x axis means the steps of the drunk man moved.

Some observations:

- * Under the same experiments(plot a, b, and c), the increase of the steps can improve the precision of the power of x, but not the coefficient of the x.
- * Under the same steps, the increase of the experiments can also improve the precision of the power of x, but not the coefficient of the x.
- * The average of the coefficient of the x is approximately 8.99.

3. Reproduction the experiments

The Bash script I ran to generate the statistics, which located with RandomWalk.java:

```
run_experiments.sh
```

A file “results.csv” will be generated, then it can be plotted.

* Unit tests result:

The screenshot shows an IDE window titled "INFO6205_Assignments - RandomWalk.java". The main editor displays the `RandomWalk` class with the `main` method. The `main` method calls `randomWalkMulti(m, n)` and prints the mean distance. The `randomWalkMulti` method is partially visible, showing a loop that accumulates distances.

```
76: return totalDistance / n;
77: }
78:
79: public static void main(String[] args) {
80:     if (args.length == 0)
81:         throw new RuntimeException("Syntax: RandomWalk steps [experiments]");
82:     int m = Integer.parseInt(args[0]);
83:     int n = 10000;
84:     if (args.length > 1) n = Integer.parseInt(args[1]);
85:     double meanDistance = randomWalkMulti(m, n);
86:     System.out.println(m + " steps: " + meanDistance + " over " + n + " experiments");
87: }
88:
89:
90: }
```

The Run window at the bottom shows the test results for `RandomWalkTest`. The tests passed are:

- testRandomWalk2: 23 ms
- testMove0: 15 ms
- testMove1: 8 ms
- testMove2: 10 ms
- testMove3: 2 ms
- testRandomWalk: 452 ms

The total test time is 510 ms, and all 6 tests passed.