choice_sim

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1 Simulation of reasonable choices

return choice

- 1.0.1 NOTE: We create a random matrix with zero diagonal.
- 1.0.2 Furthermore, let $r \in [0,1]$ be A's reasons to be chosen over B. Then r(A,B) = -r(B,A)

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In [13]: import numpy as np
         import string
         import pandas as pd
         from itertools import combinations, permutations, product, chain
         import warnings
         warnings.filterwarnings('ignore')
In [3]: # the following function creates a matrix with random entries in the upper diagonal,
        # the negative of the upper diagonal on the lower diagonal and zeros on the diagonal
        def init_reasons_matrix(size, num_reasons):
            init_array = np.random.uniform(-1*num_reasons,num_reasons,(size,size))
            X = np.triu(-init_array).T + np.triu(init_array)
            return pd.DataFrame(X, columns=[string.ascii_lowercase[i] for i in range(X[:,0].size
                                index=[string.ascii_lowercase[i] for i in range(X[:,0].size)])
In [10]: # Example: a 4*4 matrix, i.e. choice set X=a,b,c,d
         example = init_reasons_matrix(4, 1)
        print(example)
a 0.000000 -0.484926 -0.849135 0.737676
b 0.484926 0.000000 0.971414 0.179173
c 0.849135 -0.971414 0.000000 0.659471
d -0.737676 -0.179173 -0.659471 0.000000
In [11]: # the following function picks the alternative (i.e. letter) with the maximum
         # column sum
         def get_choice(reason_matrix):
             choice = np.argmax(np.sum(reason_matrix, axis=1))
```

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In [14]: # For example, with the reasons matrix above:
        the_choice = get_choice(example)
        print(the_choice)
b
In [16]: # Intermediary function to get all possible subsets of a choice problem
        def powerset(iterable):
            s = list(iterable)
            return chain.from_iterable(combinations(s, r) for r in range(len(s)+1))
In [23]: # Now, we generate a random 9*9 matrix as above and get the choices
         # for that and all possible "submatrices" (i.e. the choice from
         # the grand set and all possible subsets):
        11 = []
        ul = []
        X = init_reasons_matrix(9, 1)
        print(X.to_string())
        for sub in [list(i) for i in list(powerset(list(X))) if len(i)>1]:
            11.append(sub)
            ul.append(get_choice(X.loc[sub][sub]))
        the_pd = pd.DataFrame({'set': 11, 'choice':ul})
        print(the_pd.to_string())
                             С
                                       d
                                                           f
                                                                               h
a 0.000000 -0.705007 0.593389 0.018518 -0.085860 0.954000 0.911992 0.806396 -0.784077
b 0.705007 0.000000 0.645960 -0.816071 0.945257 0.671399 -0.470474 -0.556619 -0.154075
\texttt{c} \hspace{0.1cm} -0.593389 \hspace{0.1cm} -0.645960 \hspace{0.1cm} 0.000000 \hspace{0.1cm} 0.604809 \hspace{0.1cm} -0.874216 \hspace{0.1cm} -0.337586 \hspace{0.1cm} 0.924796 \hspace{0.1cm} -0.638917 \hspace{0.1cm} 0.570491
d -0.018518  0.816071 -0.604809  0.000000 -0.558400  0.651432  0.368800 -0.466420  0.878271
f -0.954000 -0.671399 0.337586 -0.651432 0.732254 0.000000 0.316948 -0.536624 0.513572
g -0.911992 0.470474 -0.924796 -0.368800 -0.020656 -0.316948 0.000000 -0.626545 0.140465
h -0.806396 0.556619 0.638917 0.466420 -0.007452 0.536624 0.626545 0.000000 -0.426278
set choice
                         [a, b]
0
                                     h
1
                         [a, c]
                                     a
2
                         [a, d]
                                     a
3
                         [a, e]
                                     е
4
                         [a, f]
                                     а
5
                         [a, g]
                                     a
                         [a, h]
6
7
                         [a, i]
                                     i
8
                         [b, c]
                                     b
                         [b, d]
9
                                     d
10
                         [b, e]
                                     b
11
                         [b, f]
                                     b
12
                         [b, g]
                                     g
```

13	[b, h]	h
14	[b, i]	i
15	[c, d]	С
16	[c, e]	е
17	[c, f]	f
18	[c, g]	С
19	[c, h]	h
20	[c, i]	С
21	[d, e]	е
22	[d, f]	d
23	[d, g]	d
24	[d, h]	h
25	[d, i]	d
	[e, f]	f
26		
27	[e, g]	е
28	[e, h]	e
29	[e, i]	i
30	[f, g]	f
31	[f, h]	h
32	[f, i]	f
33	[g, h]	h
34	[g, i]	g
35	[h, i]	i
36	[a, b, c]	Ъ
37	[a, b, d]	d
38	[a, b, e]	Ъ
39	[a, b, f]	b
40	[a, b, g]	b
41	[a, b, h]	b
42	[a, b, i]	i
43	[a, c, d]	a
44	[a, c, e]	е
45	[a, c, f]	a
46	[a, c, g]	a
47	[a, c, h]	a
48	[a, c, i]	i
49	[a, d, e]	е
50	[a, d, f]	a
51	[a, d, g]	a
52	[a, d, h]	a
53	[a, d, i]	d
54	[a, e, f]	a
55	[a, e, g]	a
56	[a, e, h]	
57	[a, e, n] [a, e, i]	a i
		i
58	[a, f, g]	a
59	[a, f, h]	a :
60	[a, f, i]	i

61	[a, g, h]	a
62	[a, g, i]	i
63	[a, h, i]	i
64	[b, c, d]	d
65	[b, c, e]	b
66	[b, c, f]	b
67	[b, c, g]	С
68	[b, c, h]	h
69	[b, c, i]	b
70	[b, d, e]	d
71	[b, d, f]	d
72	[b, d, g]	d
73	[b, d, h]	h
74	[b, d, i]	d
75	[b, e, f]	b
76	[b, e, g]	b
77	[b, e, h]	h
78	[b, e, i]	b
79	[b, f, g]	b
80	[b, f, h]	h
81	[b, f, i]	b
82	[b, g, h]	h
83	[b, g, i]	g
84	[b, h, i]	i
85	[c, d, e]	е
86	[c, d, f]	С
87	[c, d, g]	С
88	[c, d, h]	h
89	[c, d, i]	С
90	[c, e, f]	f
91	[c, e, g]	е
92	[c, e, h]	е
93	[c, e, i]	е
94	[c, f, g]	f
95	[c, f, h]	h
96	[c, f, i]	f
97	[c, g, h]	h
98	[c, g, i]	С
99	[c, h, i]	h
100	[d, e, f]	d
101	[d, e, g]	е
102	[d, e, h]	е
103	[d, e, i]	е
104	[d, f, g]	d
105	[d, f, h]	h
106	[d, f, i]	d
107	[d, g, h]	h
108	[d, g, i]	d

109	[d, h, i]	d
110	[e, f, g]	f
111	[e, f, h]	h
112	[e, f, i]	f
113	[e, g, h]	h
114	[e, g, i]	g
115	[e, h, i]	i
116	[f, g, h]	h
117	[f, g, i]	f
118	[f, h, i]	h
119	[g, h, i]	i
120	[a, b, c, d]	b
121	[a, b, c, e]	b
122	[a, b, c, f]	b
123	[a, b, c, g]	b
124	[a, b, c, h]	b
125	[a, b, c, i]	b
126	[a, b, d, e]	b
127	[a, b, d, f]	d
128	[a, b, d, g]	d
129	[a, b, d, h]	d
130	[a, b, d, i]	d
131	[a, b, e, f]	b
132	[a, b, e, g]	b
133	[a, b, e, h]	b
134	[a, b, e, i]	b
135	[a, b, f, g]	a
136	[a, b, f, h]	a
137	[a, b, f, i]	b
138	[a, b, g, h]	a
139	[a, b, g, i]	i
140	[a, b, h, i]	i
141	[a, c, d, e]	е
142	[a, c, d, f]	a
143	[a, c, d, g]	a
144	[a, c, d, h]	a
145	[a, c, d, i]	С
146	[a, c, e, f]	a
147	[a, c, e, g]	a
148	[a, c, e, h]	a
149	[a, c, e, i]	е
150	[a, c, f, g]	a
151	[a, c, f, h]	a
152	[a, c, f, i]	a
153	[a, c, g, h]	a
154	[a, c, g, i]	С
155	[a, c, h, i]	i
156	[a, d, e, f]	a

157	[a, d, e, g] a
158	[a, d, e, h] a
159	[a, d, e, i]
160	[a, d, f, g] a
161	[a, d, f, h] a
162	[a, d, f, i]
163	[a, d, g, h] a
164	[a, d, g, i]
165	[a, d, h, i]
166	[a, e, f, g] a
167	[a, e, f, h] a
168	[a, e, f, i] i
169	[a, e, g, h] a
170	[a, e, g, i] i
171	[a, e, h, i] i
172	[a, f, g, h] a
173	[a, f, g, i] a
174	[a, f, h, i] a
175	[a, g, h, i]
176	[b, c, d, e] k
177	[b, c, d, f]
178	[b, c, d, g]
179	[b, c, d, h] h
180	[b, c, d, i]
181	[b, c, e, f] k
182	[b, c, e, g] l
183	[b, c, e, h] h
184	[b, c, e, i] k
185	[b, c, f, g] h
186	[b, c, f, h] h
187	[b, c, f, i] k
188	[b, c, g, h] h
189	[b, c, g, i]
190	[b, c, h, i] h
191	[b, d, e, f]
192	[b, d, e, g]
193	[b, d, e, h] h
194	[b, d, e, i]
195	[b, d, f, g]
196	[b, d, f, h] h
197	[b, d, f, i]
198	[b, d, g, h] h
199	[b, d, g, i]
200	[b, d, h, i]
201	[b, e, f, g] l
202	[b, e, f, h] h
203	[b, e, f, i] k
204	[b, e, g, h] h

205	[b,	e,	g,	i]	g
206	[b,	e,	h,	i]	i
207	[b,	f,	g,	h]	h
208	[b,	f,	g,	i]	g
209	[b,	f,	h,	i]	h
210	[b,	g,	h,	i]	h
211	[c,	d,	e,	f]	е
212	[c,	d,	e,	g]	е
213	[c,	d,	e,	h]	е
214	[c,	d,	e,	i]	е
215	[c,	d,	f,	g]	С
216	[c,	d,	f,	h]	h
217	[c,	d,	f,	i]	d
218	[c,	d,	g,	h]	h
219	[c,	d,	g,	i]	С
220	[c,	d,	h,	i]	h
221	[c,	e,	f,	g]	f
222	[c,	e,	f,	h]	h
223	[c,	e,	f,	i]	f
224	[c,	e,	g,	h]	h
225	[c,	e,	g,	i]	е
226	[c,	e,	h,	i]	е
227	[c,	f,	g,	h]	h
228	[c,	f,	g,	i]	f
229	[c,	f,	h,	i]	h
230	[c,	g,	h,	i]	С
231	[d,	e,	f,	g]	d
232	[d,	e,	f,	h]	h
233	[d,	e,	f,	i]	d
234	[d,	e,	g,	h]	h
235	[d,	e,	g,	i]	d
236	[d,	e,	h,	i]	е
237	[d,	f,	g,	h]	h
238	[d,	f,	g,	i]	d
239	[d,	f,	h,	i]	d
240	[d,	g,	h,	i]	d
241	[e,	f,	g,	h]	h
242	[e,	f,	g,	i]	f
243	[e,	f,	h,	i]	f
244	[e,	g,	h,	i]	i
245	[f,	g,	h,	i]	h
246	[a, b,	С,	d,	e]	b
247	[a, b,	с,	d,	f]	b
248	[a, b,		d,		a
249	[a, b,		d,	h]	h
250	[a, b,	с,	d,	i]	d
251	[a, b,	с,	e,	f]	b
252	[a, b,	с,	e,	g]	b

[a,	b,	с,	e,	h]	b
Г					
[a,	b,	С,	e,	i]	b
[a,	b,	с,	f,	g]	a
[a,	b,	с,	f,	h]	a
[a,	b,	с,	f,	i]	b
		С,	g,	h]	a
			_		b
		с,	h,	i]	i
				f]	b
				_	d
			e,	h]	b
		d,	e,	i]	d
		d,	f,	g]	d
		d,		-	a
[a,	b,	d,	f,	i]	d
		d,	g,	h]	a
[a,	b,	d,	_		d
[a,	b,	d,	h,	i]	d
[a,	b,	e,	f,	g]	b
[a,	b,	e,	f,	h]	b
[a,	b,	e,	f,	i]	b
[a,	b,	e,	g,	h]	a
[a,	b,	e,	g,	i]	b
[a,	b,	e,	h,	i]	i
[a,	b,	f,	g,	h]	a
[a,	b,	f,	g,	i]	b
[a,	b,	f,	h,	i]	i
[a,	b,	g,	h,	i]	i
[a,	С,	d,	e,	f]	a
[a,	С,	d,	e,	g]	е
				h]	е
		d,			е
[a,	С,	d,	f,	g]	a
					a
					d
			0.		a
			_		С
	С,	d,			a
				_	a
					a
					a
			_		a
			_		е
					е
			_		a
			_		a
					a
Lα,	С,	g,	n,	1]	a
	[a,	[a, b, [a, b, [a, b, b, [a, c, c, c, [a, c, c, [a, c, c, c, c, c, c, [a, c, c, c, c, c, c, c, c, [a, c,	[a, b, c, [a, b, c, [a, b, c, [a, b, d, [a, b, d, [a, b, d, [a, b, d, [a, b, e, [a, b, e, [a, b, f, [a, c, d, [a, c, e, [a, c, f, [a, [a, c, f, [a, [a, c, f, [a, c, f, [a, c, f, [a, c, f, [a, [a, c, f, [a, [a, c, [a, [a, c, f, [a, [a, [a, [a, [a, [a, [a, [a, [a, [a	[a, b, c, f, [a, b, c, f, [a, b, c, g, [a, b, d, e, [a, b, d, f, [a, b, e, f, [a, b, f, g, [a, b, f, h, [a, b, f, g, [a, b, f, h, [a, b, g, h, [a, c, d, e, [a, c, d, e, [a, c, d, e, [a, c, d, f, [a, c, e, f, g, [a, c, f, g, [a, c, f, h, [a, c, f, g, [a, c, f, f, g, [a, c, f, g, [a, c, f, f, f, f]]])	[a, b, c, f, g] [a, b, c, f, i] [a, b, c, g, i] [a, b, c, g, i] [a, b, c, g, i] [a, b, c, h, i] [a, b, d, e, g] [a, b, d, e, i] [a, b, d, f, g] [a, b, d, f, i] [a, b, d, g, i] [a, b, d, g, i] [a, b, e, f, g] [a, b, e, f, i] [a, b, e, g, i] [a, b, f, g, i] [a, b, f, g, i] [a, b, f, g, i] [a, c, d, e, g] [a, c, d, e, g] [a, c, d, e, g] [a, c, d, f, i] [a, c, d, g, i] [a, c, d, f, i] [a, c, d, g, i] [a, c, d, f, i] [a, c, d, f, i] [a, c, d, f, i] [a, c, e, g, i] [a, c, e, f, i] [a, c, e, g, i] [a, c, e, f, i] [a, c, e, g, i] [a, c, e, g, i] [a, c, e, f, i] [a, c, e, g, i] [a, c, e, g, i] [a, c, f, g, i]

301	[a,	d,	e,	f,	g]	a
302	[a,				h]	a
303	[a,	d,	e,	f,	i]	d
304	[a,	d,	e,	g,	h]	a
305	[a,	d,	e,	g,	i]	d
306	[a,	d,	e,	h,	i]	е
307	[a,	d,	f,	g,	h]	a
308	[a,	d,	f,	g,	i]	d
309	[a,	d,	f,	h,	i]	d
310	[a,	d,	g,	h,	i]	a
311	[a,	e,	f,	g,	h]	a
312	[a,	e,	f,	g,	i]	a
313	[a,	e,	f,	h,	i]	a
314	[a,	e,	g,	h,	i]	i
315	[a,	f,	g,	h,	i]	a
316	[b,	С,	d,	e,	f]	b
317	[b,	С,	d,	e,	g]	е
318	[b,	С,	d,	e,	h]	h
319	[b,	С,	d,	e,	i]	b
320	[b,	С,	d,	f,	g]	d
321	[b,	С,	d,	f,	h]	h
322	[b,	С,	d,	f,	i]	d
323	[b,	С,	d,	g,	h]	h
324	[b,	С,	d,	g,	i]	d
325	[b,	с,	d,	h,	i]	h
326	[b,	с,	e,	f,	g]	b
327		с,		f,	h]	h
328	[b,	С,	e,	f,	i]	b
329	[b,	С,	e,	g,	h]	h
330	[b,	С,	e,	g,	i]	b
331	[b,	с,	e,	h,	i]	b
332	[b,	С,	f,	g,	h]	h
333	[b,	с,	f,			b
334	[b,	с,	f,	h,	i]	h
335	[b,	с,	g,	h,	i]	h
336	[b,	d,				d
337	[b,	d,	e,			h
338	[b,	d,	e,	f,	i]	d
339	[b,	d,	e,	g,	h]	h
340	[b,	d,		_	i]	d
341	[b,	d,			i]	d
342	[b,				h]	h
343	[b,			_	i]	d
344	[b,			_	i]	d
345	[b,				i]	d
346	[b,		_		h]	h
347	[b,			g,	i]	b
348	[b,			h,		b
-	,	- ,	- ,	-,		-

```
349
                   [b, e, g, h, i]
                                         h
                  [b, f, g, h, i]
350
                                         h
351
                   [c, d, e, f, g]
                                          f
                  [c, d, e, f, h]
352
                                         h
                   [c, d, e, f, i]
353
                                          f
                  [c, d, e, g, h]
354
                                         h
                   [c, d, e, g, i]
355
                                          е
356
                   [c, d, e, h, i]
                                          е
                  [c, d, f, g, h]
357
                                         h
358
                  [c, d, f, g, i]
                                          С
                  [c, d, f, h, i]
359
                                         h
                   [c, d, g, h, i]
360
                                          С
                  [c, e, f, g, h]
361
                                         h
                   [c, e, f, g, i]
362
                                          f
                  [c, e, f, h, i]
363
                                         f
                  [c, e, g, h, i]
364
                                         h
365
                   [c, f, g, h, i]
                                         h
                  [d, e, f, g, h]
366
                                         h
                   [d, e, f, g, i]
367
                                          d
368
                  [d, e, f, h, i]
                                         h
369
                  [d, e, g, h, i]
                                         h
                  [d, f, g, h, i]
370
                                          d
371
                  [e, f, g, h, i]
                                          f
372
               [a, b, c, d, e, f]
                                         b
373
               [a, b, c, d, e, g]
                                         b
               [a, b, c, d, e, h]
374
                                          b
               [a, b, c, d, e, i]
375
                                          b
376
               [a, b, c, d, f, g]
                                          a
377
               [a, b, c, d, f, h]
                                          a
378
               [a, b, c, d, f, i]
                                          d
379
               [a, b, c, d, g, h]
                                          a
380
               [a, b, c, d, g, i]
                                          d
381
               [a, b, c, d, h, i]
                                          d
               [a, b, c, e, f, g]
382
                                         b
               [a, b, c, e, f, h]
383
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384
               [a, b, c, e, f, i]
                                          b
385
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386
               [a, b, c, e, g, i]
                                         b
               [a, b, c, e, h, i]
387
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388
               [a, b, c, f, g, h]
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               [a, b, c, f, g, i]
389
                                         b
               [a, b, c, f, h, i]
390
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               [a, b, c, g, h, i]
391
                                          a
               [a, b, d, e, f, g]
392
                                          d
393
               [a, b, d, e, f, h]
                                          a
394
               [a, b, d, e, f, i]
                                          d
               [a, b, d, e, g, h]
395
                                          a
396
               [a, b, d, e, g, i]
                                          d
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               [a, b, d, f, g, h]
398
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399
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401
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404
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405
                                         i
406
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               [a, c, d, e, f, h]
408
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409
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421
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423
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424
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426
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427
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450
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452
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454
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457
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458
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466
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467
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468
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487
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488
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489
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491
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492
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[a, b, c, d, e, f, g, i]
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494
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495
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                                       h
        [a, b, c, d, f, g, h, i]
496
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        [a, b, c, e, f, g, h, i]
497
                                       b
        [a, b, d, e, f, g, h, i]
498
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        [a, c, d, e, f, g, h, i]
499
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        [b, c, d, e, f, g, h, i]
500
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501 [a, b, c, d, e, f, g, h, i]
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