Reasonable Choice vs Rational Choice*

Jesper Hansen, Christopher Kops†

Abstract

Many strands in philosophy and psychology emphasize that individuals expect to give reasons for their choices, indeed that reasons govern individual choices. In line with this idea, we propose a choice procedure by which the decision maker always chooses the alternative for which she can advance the most reasons to support and justify its choice. We interpret a reason as a consideration which counts in favor of choosing one alternative over another. Our conceptualization of this idea allows a reason to be overridden only by a more important fact which is a stronger reason for a contradictory action. We provide a behavioral foundation for identifying when choices are driven by such reasons and discuss the type of non-standard choices that our model can capture. Our analysis highlights important implications for decision making processes that curb detrimental phenomena such as discrimination and inertia.

JEL codes: D01, D81

Keywords: individual choice, justification, bounded rationality, choice by reasons, judgment aggregation, social choice, inertia, discrimination

*This draft: April 21, 2017

†Hansen: University of Cologne (e-mail: j.hansen@uni-koeln.de); Kops: University of Cologne (e-mail: ckops@uni-koeln.de). Correspondence address: Albertus-Magnus Platz, 50923 Cologne, Germany.

1 Introduction

Amid the tangle of violations of economic rationality documented in the literature, it seems to be a recurring theme that there is some alternative on offer whose choice is all too easily justified. Examples run from default options (Tversky and Kahneman 1991) over alternatives dominating a decoy (Huber, Payne, and Puto 1982) to compromise choices (Simonson 1989) and beyond. In this paper, we establish that the connection between appealing to readily accessible reasons in the process of choice and irrational choice behavior is indeed fundamental. Specifically, we propose and behaviorally characterize a choice procedure, the reasonable choice method (RCM), that, from our perspective, captures the spirit of reason-based choices (Shafir, Simonson, and Tversky 1993) and show that the RCM cannot only explain the most common irrational choices, but also serves to identify strategies that may curb detrimental phenomena such as discrimination and inertia.

Setting up a theory of reason-based choice requires that we formally pin down what a reason is. Given that the primitive concept in economics for modeling individual behavior, beliefs and attitudes is choice, we interpret a reason as a consideration which counts in favor of choosing one alternative over another. That is, we formally define a reason to be an ordered pair of two alternatives. Reasons in favor of choosing one alternative over another then translate into ordered pairs of these two alternatives listing the favored alternative as the first entry and the other one as the second. Admittedly, there are other ways to formally capture reasons, but a clear advantage of our conceptualization is that it allows reasons to differ in strength. Furthermore, it just so happens that ordered pairs of alternatives are also the atoms of preferences which makes for an easy comparison with the standard model of rational choice.

To elaborate on our conceptualization, the strength of a reason accounts for its power to override other reasons. In the context of our theory, a reason's strength conveys the relative importance that a decision maker ascribes to the underpinning consideration of alternatives. We should emphasize here that the notion of one reason overriding another has to be carefully distinguished from that of a reason being cancelled. A reason can be overridden only by a more important fact which is a stronger reason for a contradictory action. For instance, consider a DM faced with the need to take an injured man to hospital at the same time she promised to meet a friend at a restaurant. Helping a person in need is arguably more important than keeping an appointment and may, thus, override the promise. If, on the other hand, the DM's friend releases her from her promise to meet him at the restaurant, then this will cancel the DM's reason to go to the restaurant. Note that such cancellation conditions fall outside the scope of our analysis in this paper.

The above discussion lays out the operationalization of reasons that is at the heart of the RCM. But, how does a DM following our model actually go about making choices? Faced with the problem of choosing from a given set of alternatives, she turns to the set of all binary subsets of this set and considers, for each available alternative, her reasons (of potentially different strength) in favor of choosing this alternative over others. Then, to figure out which of the available alternatives is the most reasonable choice, she computes a score for each alternative by adding up all reasons in favor of it and correcting for differences in strengths that may arise. Whichever alternative maximizes this score in a given choice problem is the DM's choice.

To illustrate the RCM, suppose that a DM can choose, as in the example above, between different plans for the night which include staying home (h), meeting her friend (f), or taking the injured man to hospital (i). Being the nocturnal animal that she is, she craves for nightly entertainment and can find no reason to stay home. In line with this desire, it is conceivable that the ordered pair (f, h) is a reason of strength 2 and (i, h) one of strength 1. Confronted with the sight of the injured man, she can find no reason not to help him which results in (i, f) being a reason of strength 2. It follows that h has a score of 0, f a score of 2, and i a score of 3 which makes the latter option the most reasonable choice out of all three. So, faced with the problem of choosing her plan for the night, the DM reasons that she should take the injured man to hospital.

In the process of identifying the most reasonable choice, negative attributes of an alternative can, under the RCM, be compensated for by equal or higher value positive attributes provided that they translate into relatively stronger reasons. This makes the RCM a compensatory choice heuristic. It is this feature that, to the best of our knowledge, separates it from all other axiomatic approaches to boundedly rational choice in the recent literature. Such models rather propose non-compensatory choice procedures for decision-making that do not trade off the benefits of some attributes against the deficits of others. The most prominent examples among the surge of non-compensatory models in the recent literature are the rational shortlist method by Manzini and Mariotti (2007) and choice with limited attention by Masatlioglu, Nakajima, and Ozbay (2012).

The RCM and its concomitant conceptualization of reasons are theoretical constructs that cannot be observed. Rather, we explicitly assume that the domain of observables in this paper is constrained to the DM's choices. Based on this premise, our main result establishes that the RCM is falsifiable and renders it behaviorally testable. It does so by providing a list of consistency conditions on choices that hold if and only if the theoretical constructs of our model hold. We show that, under the assumption that reasons cannot differ in their strengths, the RCM permits the same consistency as rational choice does. If, on the other hand, any two reasons either share the same strength or one of them is half as strong as the other, we show that its consistency can be captured by four simple conditions. An expansion condition, weak WARP (Manzini and Mariotti 2007), a weak version of never chosen (Cherepanov, Feddersen, and Sandroni 2013) and a condition that we refer to as weak trinary independence which establishes that any violation of independence of irrelevant alternatives[1] can be attributed to the addition of one single alternative.

The rest of the paper is organized as follows. Section 2 introduces the general setup and Section 3 formally defines our choice procedure. In Section 4, we show what non-standard choices the RCM can accommodate and discuss policy implications that can be drawn. Section 5 provides the behavioral foundations of the RCM. Section 6 relates our theory to the literature and Section 7 concludes with remarks on the most general version of the RCM and alternative definitions of reasonable choice behavior.

2 Primitives

Let X be a finite set of alternatives with $|X| > 2$ and typical elements denoted by x, y, z etc. $\mathcal{P}(X)$ denotes the set of all nonempty subsets of X with typical elements S, T etc. A choice function, $\gamma : \mathcal{P}(X) \to X$ is a mapping that for any $S \in \mathcal{P}(X)$ selects an element $\gamma(S) \in S$. We abuse notation in a standard way by suppressing set delimiters such that we write $\gamma(x, y)$ instead of $\gamma(\{x, y\})$.

3 Reasonable Choice

We can now formally set up our theory under which we study the need to justify decisions with respect to its influence on individual choice. Consider, to this end, a DM faced with the task of deciding what to choose from some set $S \in \mathcal{P}(X)$. We posit that the influence of reasons on the DM's choice occurs through the following kind of cognitive process.

[1] Independence of irrelevant alternatives (IIA): If an alternative x is chosen from a set T, and x is also an element of a subset S of T, then x must be chosen from S.

First, we imagine that the DM has certain reasons in mind and that they may differ with respect to their strength. In our theory, a reason is a consideration which counts in favor of choosing one alternative over another. Formally, we define a reason to be an ordered pair of alternatives. The ordered pair (x, y), for instance, is a reason to choose x over y. To address differences in reasons' strengths, we introduce the concept of a reasonale, $\succ \subset X \times X$, which is a simple binary relation on X, and the set, $\mathcal{R}$, of all such reasonales. The number of all reasonales $\succ \in \mathcal{R}$ such that $(x, y) \in \succ$ then denotes the strength of the reason (x, y). Intuitively, the strength of a reason reflects the relative importance that the DM ascribes to the underpinning consideration of alternatives and the set of reasonales captures the potentially different criteria that she deems important for such considerations.

Next, given the set of reasonales, $\mathcal{R}$, for every set $S \in \mathcal{P}(X)$ we define a counting function $g_{\mathcal{R}|_S} : S \to \mathbb{N}$ by

$$g_{\mathcal{R}|_S}(x) = \sum_{\succ \in \mathcal{R}} \#\{y \in S \setminus \{x\} | (x, y) \in \succ\}$$

For a fixed choice problem $S \in \mathcal{P}(X)$, the function $g_{\mathcal{R}|_S}(.)$ assigns a number to each alternative $x \in S$ that scores how reasonable it is to choose this alternative from S. The way it does so is by taking all reasons into account for which the underlying consideration only involves alternatives that are both available from this set. We deem that this assumption best reflects the piling evidence (Grether and Plott 1979) that it is in the moment of choice that the DM feels the urge to advance reasons that support and justify her decision. And intuitively, considerations between available alternatives are more salient and more readily accessible than others. If, on the other hand, $g_{\mathcal{R}}(.)$ were to count all reasons irrespective of such availability issues, then each alternative's score would remain constant across different choice problems and choosing the most reasonable alternative would be indistinguishable from rational choice.

Having laid out our conceptualization of a reason and its strength, we now turn to the question of how these objects jointly govern what the DM chooses.

Our theory in this regard is the most obvious one that one can conceive of in the context introduced so far. We propose that the DMs choices are a result of the following choice procedure. For any set $S \in \mathcal{P}(X)$, the DMs reasoning process scores for each alternative how reasonable it is to choose this alternative from S. Finally, the DM chooses the most reasonable of the available alternatives, i.e., the alternative with the highest such score.

Definition 3.1. *A choice function γ is a reasonable choice method (RCM) whenever there exists a set of reasonales $\mathcal{R}$ such that for all $S \in \mathcal{P}(X)$,*

$$\gamma(S) = \{x \in S | g_{\mathcal{R}|_S}(x) > g_{\mathcal{R}|_S}(y), \forall y \in S \setminus \{x\}\}$$

In addition to being normatively appealing, our definition of reason-based choice also carries some descriptive appeal. Many heuristics that are used to determine the winner in sports competitions or elections rely processes similar to those underlying the RCM. Before we list some of them, note that according to the specification of the RCM provided above, a DM following this choice procedure engages, when faced with a choice problem, in a cognitive process that converts the set of available alternatives into the series of all possible binary comparisons between two such alternatives. It is this part of our conceptualization of reason-based choices that resurfaces in many other heuristics as well.

First, consider the group stages of the FIFA World Cup or the Cricket World Cup. These sports competitions are designed as round-robin tournaments in which each contestant meets all other contestants in turn and then proceeds to the next round based on his overall performance. Similarly, in social choice theory an election method that satisfies the Condorcet criterion elects the candidate that wins by majority rule in all pairings against the other candidates (if such a candidate exists). Indeed, in economics, any ranking of alternatives induced by complete ordinal preferences ultimately rests upon the series of all possible binary comparisons between two such alternatives because preference relations are defined as binary relations.

We maintain that this link between individual choice and concerns for justification is a fundamental one and substantiate this claim further by showing, in the next section, what behavioral anomalies it can accommodate. On the other hand, the setup of the RCM also applies to other choice problems such as judgment aggregation or social choice. Inspired by Arrow's impossibility theorem in social choice, Wilson (1975) posed the question whether Arrow's result extends to the aggregation of attributes other than preferences. This form of aggregation corresponds to the RCM's summation over individual reasons rather than over complete and transitive preferences. The caveat being that the RCM would still require that aggregated judgments always identify an alternative as the unique choice.

4 Non-Standard Choices

A natural question that arises is the following: if the DM's choices are an RCM, then are they rational; that is, does her choice function satisfy WARP? In this section we show that this need not be the case. To organize the discussion, we would first like to remind the reader of a result in Manzini and Mariotti (2007) that shows that all violations of WARP can be classified very simply in terms of violations of two weaker choice consistency conditions referred to as always chosen (AC) and no binary cycles (NBC). That is, the choice functions that violate WARP can be partitioned into three subclasses, the ones that violate exactly one of AC and NBC and the ones that violate both. We next formally define these two conditions and show that if the DM follows the RCM with two reasonales, then it can accommodate violations of NBC as well as of AC.

Definition 4.1. *A choice function $\gamma : \mathcal{P}(X) \to X$ satisfies no binary cycles (NBC) if for all $x_1, \ldots, x_{n+1} \in X$*

$$[\gamma(x_i, x_{i+1}) = x_i \textit{ for } i = 1, \ldots, n] \Rightarrow [\gamma(x_1, x_{n+1}) = x_1]$$

The example below shows that choices that are an RCM can violate NBC.

Example 4.1. *Let $X = \{x, y, z\}$ be the set of alternatives and $\mathcal{R} = \{\succ_1, \succ_2\}$ be the set of reasonales such that $\succ_1 = \{(x, y), (y, z)\}$ and $\succ_2 = \{(x, y), (z, x)\}$. Verify that for each alternative in X the reasonability score for choosing this alternative from some binary choice problem $S \in \mathcal{P}(X)$ is given by*

	$g_{\mathcal{R}\|S}(x)$	$g_{\mathcal{R}\|S}(y)$	$g_{\mathcal{R}\|S}(z)$
$S = \{x, y\}$	2	0	–
$S = \{x, z\}$	0	–	1
$S = \{y, z\}$	–	1	0

Accordingly, the DM's choices specified in Table 1 are an RCM, but violate NBC.

Table 1: Violation of NBC

	$\{x, y\}$	$\{x, z\}$	$\{y, z\}$
$\gamma(.)$	x	z	y

Definition 4.2. *A choice function $\gamma : \mathcal{P}(X) \to X$ satisfies always chosen (AC) if for all $S \subseteq X$,*

$$[x \in S \text{ and } \gamma(x, y) = x \text{ for all } y \in S\backslash\{x\}] \Rightarrow [\gamma(S) = x]$$

The example below shows that choices that are an RCM may also violate AC.

Example 4.2. *Let $X = \{x, y, z\}$ be the set of alternatives and $\mathcal{R} = \{\succ_1, \succ_2, \succ_3\}$ be the set of reasonales such that $\succ_1 = \{(x, y), (x, z), (y, z)\}$ and $\succ_2 = \succ_3 = \{(y, z)\}$. Verify that for each alternative in X the reasonability score for choosing this alternative from some choice problem $S \in \mathcal{P}(X)$ is given by*

Hence, the DM's choices specified in Table 2 are an RCM, but violate AC.

	$g_{\mathcal{R}\|S}(x)$	$g_{\mathcal{R}\|S}(y)$	$g_{\mathcal{R}\|S}(z)$
$S = \{x, y\}$	1	0	–
$S = \{x, z\}$	1	–	0
$S = \{y, z\}$	–	3	0
$S = \{x, y, z\}$	2	3	0

Table 2: Violation of AC

	$\{x, y\}$	$\{y, z\}$	$\{x, z\}$	$\{x, y, z\}$
$\gamma(.)$	x	y	x	y

4.1 Further Anomalies and Implications

This section takes a closer look at the various context effects that can be accommodated by the RCM. In particular, we show that the underlying process of choice may be responsible for as undesirable a phenomenon as inertia or discrimination. Motivated by this observation, we turn to the setup of the RCM to identify, in the context of our theory, the driving forces of such unwanted effects. As it turns out, our analysis generates insights that can be used to draw policy implications that may inhibit inert or discriminatory behavioral tendencies.

Attraction Effect The attraction effect refers to the ability of an asymmetrically dominated or relatively inferior alternative, when added to a set, to increase the choice frequency of the dominating alternative. To illustrate this effect, consider an experiment by Highhouse (1996) designed as an employee selection scenario. In the experiment, students were faced with the task of choosing a job candidate for recruitment of firm manager. To this end, a list was given to participants that outlined the scores on the two predictors "work sample rating" and "promotability ranking" for each eligible candidate, just as given in Table 5.

Table 3: Employee Selection

Candidate	Work sample rating	Promotability ranking
a	7	66
b	5	80
d_a	7	54
d_b	4	80

In the two decoy conditions of the experiment, subjects received information about the two predictors for each of the three job candidates a, b, and d_a (or, a, b, and d_b respectively) and were then asked to choose one of them. In each of the respective phantom conditions, the set of prospective candidates was reduced down to only contain candidates a and b. As is typical for instances of the attraction effect, choice data from the experiment reveals that choice frequencies significantly depend on the fact which one of the two decoy candidates, d_a or d_b, is available. That is, from the set $\{a, b, d_a\}$, 65% of subjects select candidate a and the remaining 35% choose candidate b. Conversely, merely 32% of subjects favor candidate a from the set $\{a, b, d_b\}$ and the remaining 68% vote for candidate b.

It is straightforward to verify that the RCM can accommodate the fact that a DM chooses a from $\{a, b, d_a\}$ and b from $\{a, b, d_b\}$. To this end, we simply have to define her set of reasonales to contain several reasonales of the kind $\succ = \{(b, d_b), (a, d_a)\}$. Intuitively, whereas the direct comparison between a and b can be hard, the ease of a comparison such as between a and d_a or between b and d_b may conveniently sway the decision in favor of the decoy-dominating alternative, i.e., a in $\{a, b, d_a\}$ and b in $\{a, b, d_b\}$.

Discrimination We surmise that there is a hidden form of discrimination which follows a similar pattern as the attraction effect does in the example above. It is based on the premise that if membership to the same group is shared by a strict majority of job candidates then it can be perceived as important and may, thus, enter the decision-making process. Research by Schwieren and Glunk (2008) on national composition of teams seems to back this claim. To illustrate, consider the following example.

Example 4.3. *Let $\{a, b, c, d\}$ be a set of prospective job candidates and let the DM receive the following information about each candidate*

Table 4: Candidate Information

Candidate	Group Membership	Score
a	g	80
b	g	90
c	f	100
d	f	70

Let the DM's set of rationales be $\mathcal{R} = \{\succ_{score}, \succ_{gr1}, \succ_{gr2}\}$, where $\succ_{score}$ provides a ranking of the eligible candidates with respect to their score, i.e., $\succ_{score} = \{(c, d), (c, b), (c, a), (b, a), (b, d), (a, d)\}$. The reasonales $\succ_{gr1}$ and $\succ_{gr2}$, on the other hand, rank candidates within the same group with respect to their score, i.e., $\succ_{gr1}, \succ_{gr2} = \{(b, a), (c, d)\}$. Clearly, by the RCM this DM reasons and chooses as follows

Table 5: Reasons and Choice by RCM

	{b,c}	{a,b,c}	{a,b,c,d}
$g_{\mathcal{R}\|.}(b)$	0	3	4
$g_{\mathcal{R}\|.}(c)$	1	2	5
$\gamma(.)$	c	b	c

Reasons that are based on two job candidates (or, more generally two alternatives) sharing a membership to the same group, class or category such as those subsumed under the reasonales $\succ_{\text{gr1}}$ or $\succ_{\text{gr2}}$ in the example above, naturally lead to choice behavior that is prone to discrimination. The way in which such discrimination unfolds, however, suggests an easy way to curb it. The solution we propose to this end, requires that a DM, when faced with the problem of choosing from some set of alternatives, restricts the impact of a reason's strength to its underpinning binary choice comparison. Definition 4.3 provides a formalization of this idea.

Definition 4.3. *A choice function γ is a sequentially reasonable choice method (SRCM) whenever there exists a set of reasonales $\mathcal{R}$ such that for all $S \in \mathcal{P}(X)$,*

$$\gamma(S) = \operatorname*{argmax}_{x \in S} \left\{ \# \left\{ y \in S \setminus \{x\} : g_{\mathcal{R}|_{\{x,y\}}}(x) > g_{\mathcal{R}|_{\{x,y\}}}(y) \right\} \right\}$$

Returning to Example 4.3, the fact that $g_{\mathcal{R}|_{\{b,c\}}}(c) = 1 > g_{\mathcal{R}|_{\{b,c\}}}(b) = 0$, $g_{\mathcal{R}|_{\{a,c\}}}(c) = 1 > g_{\mathcal{R}|_{\{a,c\}}}(a) = 0$ and $g_{\mathcal{R}|_{\{c,d\}}}(c) = 1 > g_{\mathcal{R}|_{\{c,d\}}}(d) = 0$ implies that under the SRCM the DM chooses $\gamma(b,c) = \gamma(a,b,c) = \gamma(a,b,c,d) = c$.

Never Chosen & Inertia A specific class of violations of AC that are particularly interesting are violations of a condition called never chosen. Violations of this condition constitute a behavioral anomaly often called a difficult choice (Cherepanov, Feddersen, and Sandroni 2013).

Definition 4.4. *A choice function $\gamma : \mathcal{P}(X) \to X$ satisfies never chosen (NC) if for all $S \in \mathcal{P}(X)$,*

$$[x \in S \text{ and } \gamma(xy) \neq x \text{ for all } y \in S \setminus \{x\}] \Rightarrow [\gamma(S) \neq x]$$

The example below shows that choices that are RCM can violate NC.

Example 4.4. *Let $X = \{x, y, z\}$ be the set of alternatives and $\mathcal{R} = \{\succ_1, \ldots, \succ_7\}$ be the set of reasonales such that $\succ_1 = \{(x,y), (x,z), (y,z)\}$, $\succ_i = \{(x,z), (y,z)\}$, for $i = 2,3,4$, and $\succ_j = \{(z,x), (z,y)\}$, for $j = 5,6,7$. Then, the choices specified in Table 6 show that $\gamma(.)$ is an RCM, but violates NC.*

Table 6: Violation of NC

	{x,y}	{x,z}	{y,z}	{x,y,z}
$g_{\mathcal{R}\vert .}(x)$	1	4	0	5
$g_{\mathcal{R}\vert .}(y)$	0	0	4	4
$g_{\mathcal{R}\vert .}(z)$	0	3	3	6
$\gamma(.)$	x	x	y	z

The choices in the example above are consistent with the following story about decision inertia as told by Shafir, Simonson, and Tversky (1993). Consider Thomas Schelling who wants to buy an encyclopedia. Let x and y be different encyclopdias and z the option of not buying anything. If only one of the encyclopdias is on offer in a bookstore, he will buy it, i.e., $\gamma(x, z) = x$ and $\gamma(y, z) = y$. If, on the other hand, more than one encyclopdia is available, he will struggle to justify choosing one over the other and, thus, buy none at all, i.e., $\gamma(x, y, z) = z$. As for the case of discrimination illustrated above, a variation of the RCM that restricts the impact of a reason's strength to its underpinning binary choice comparison, such as under the SRCM, cannot violate NC.

Compromise Effect The compromise effect refers to the ability of an extreme (but not inferior) alternative, when added to a set, to increase the choice frequency of a compromise alternative. Simonson and Tversky (1992) were the first to document this effect in a choice experiment. In one of the original settings, subjects were asked to choose between several microwave ovens, a cheap no-name device (e), an expensive brand oven (p) and another more expensive brand choice (p^*). The middle option, p, is usually referred to as the compromise alternative in such settings, because it is a compromise between the expensive brand option and the cheap no-name oven. Choice data from the experiment, listed in Table 7, shows that considerably more subjects choose p when p^* is also made available. Similarly to the case of the attraction effect, the RCM can explain such choices provided that the strength of reason (p, p^*) exceeds that of (e, p). Intuitively, the choice of p from $\{e, p, p^*\}$ is easier to justify than that of e from $\{e, p\}$, because as we alluded to above p can score on the two dimensions price and brand (=quality) when both e and p^* are also on offer.

Table 7: Compromise Effect

	$\{e, p\}$		$\{e, p, p^*\}$	
$\gamma_{agg}(.)$	e	(57%)	e	(27%)
	p	(43%)	p	(60%)
			p^*	(13%)

Choice Defaults & Inertia In an experiment about the effects of choice defaults on the willingness to donate organs, Johnson and Goldstein (2003) ask subjects to choose between donating (d) and not donating (nd) under different conditions. Conditions differ from each other as to whether and which of the two options is implemented as the automatic default. Choice data from the experiment for the three conditions, d as the default, nd as the default, and no default at all, are listed in Table 8.

Table 8: Choice Default (in bold)

	$\{d, \mathbf{nd}\}$	$\{\mathbf{d}, nd\}$	$\{d, nd\}$
$\gamma_{agg}(.)$	d (42%)	$\mathbf{d}$ (82%)	d (79%)
	$\mathbf{nd}$ (58%)	nd (18%)	nd (21%)

The share of subjects choosing option d from $\{d, nd\}$, when no default is implemented, reveals a clear tendency of the average subject to favor donating over not donating. Only when not donating is set as the automatic default does it become the majority choice. This strong impact of defaults on individual choices is well known and has, for instance, been successfully instrumented to significantly increase employee savings for retirement in the United States (Bhargava and Loewenstein 2015).

To see that the RCM can explain the choice default effect, consider a DM who sees merit in saving lives and is generally a faithful individual. Not sure whether her religious beliefs about the afterlife conform with the idea of giving away her organs, she reckons that defaults carry a signal as to which alternative she should choose. Formally, let this DM's set of reasonales be given by $\mathcal{R} = \{\succ_{ethics}, \succ_{faith1}, \succ_{faith2}\}$ such that $\succ_{ethics} = \{(d, \mathbf{nd}), (d, nd), (\mathbf{d}, nd)\}$ and $\succ_{faith1} = \succ_{faith2} = \{(\mathbf{nd}, d), (\mathbf{d}, nd)\}$. Then, if this DM's choices are an RCM, her choices will be in line with the majority choice in each of the three experimental conditions, i.e., $\gamma(d, \mathbf{nd}) = \mathbf{nd}$, $\gamma(\mathbf{d}, nd) = \mathbf{d}$, and $\gamma(d, nd) = d$.

The results from the study by Johnson and Goldstein (2003) point out another way to motivate people to register as organ donors rather than registering them as such by default. Note that the share of subjects choosing to donate remains almost the same whether no default option is set (79%) or whether donating is set as the default (82%). This suggests that the majority of subjects have a strict preference or reason to prefer donating over not donating. It is conceivable that what holds them back from registering when not donating is implemented as the default is just a lack of information. The rather weak strength of the reason (d, nd) in the example above may also express the DM's lack of insight into the process of donating organs and the circumstances under which he or she may become eligible to do so. If this explanation is valid, then the RCM suggests that providing a DM with more information will help her to build stronger support for her convictions that translate into stronger reasons. Indeed, the complexity of the issue itself may dissuade some individuals from actively taking a stance. One of the authors of this paper, for instance, decided to register for organ donation, after he received a letter with precise information on the issue together with a blank donor card from his health insurer.

The Disjunction Effect In an experiment by Tversky and Shafir (1992), students were told to imagine that they had just taken a tough exam and were then asked whether they want to purchase a vacation package. More specifically, subjects could choose between buying the vacation package (b), not buying it (n), and paying a fee to delay the offer (d). Subjects in the first condition were informed that they had successfully passed the exam, the second condition specified that subjects had failed the exam and in the third condition exam results were still pending. Delaying the offer in the last condition implied that subjects would receive notice of their results before they had to finalize their decision on the vacation offer. Table 9 summarizes the choice data from the experiment.

Table 9: Vacation Offers

	$\{b, n, d\}$ (pass)	$\{b, n, d\}$ (fail)	$\{b, n, d\}$ (pending)
	b (54%)	b (57%)	b (32%)
$\gamma_{agg}(.)$	n (16%)	n (12%)	n (7%)
	d (30%)	d (31%)	d (61%)

Intuitively, it is conceivable that learning the exam results provides a justification for choosing b independent of whether results are positive or negative. Specifically, if the student passes the exam, then the vacation may be seen as a reward, and if she fails the test, then buying the vacation may be seen as consolation. In the condition were exam results are still pending, however, information about these results is absent and so is any reason based on it. Hence, subjects in the experiment struggle to justify the choice of b, albeit being able to choose b given any possible outcome of the exam. Savage (1954) refers to the absence of such struggles as the sure-thing principle, i.e., if b is preferred to n knowing that some event A occurred, and if b is preferred to n knowing that this event did not occur, then b should be preferred to n even when it is not known whether A occurred. That is, majority choices in the experiment are in violation of this axiom.

Similarly to the choice default effect, it seems that what causes the disjunction effect is a dearth of reasons which provide a justification for choosing certain options. The dearth of such reasons is, in turn, the consequence of a lack of information. In view of the potential welfare loss that is associated with such a violation of the sure thing principle, it is advisable to provide useful information as early as possible. As to useful information of the kind such as exam results in the specific case of the experiment, compiling complete information in the form of final grades, for instance, isn't really necessary to improve welfare. Rather, it would be enough to issue preliminary information that indicates a tendency such as whether a student has passed or failed a test.

Uncertainty Aversion Fox and Tversky (1995) propose the comparative ignorance hypothesis, according to which a DM's attitude towards uncertainty is sensitive to the frame of comparison insofar as it only enters her choice process when she perceives the available alternatives to differ with respect to their degree of uncertainty. The setup of the RCM can easily capture the gist of this hypothesis if we define some reasonales to only capture differences in the degree of uncertainty between alternatives. Then, in line with the comparative ignorance hypothesis, attitude towards ambiguity will only enter the process of decision-making when the available alternatives differ with respect to their degree of uncertainty.

5 Behavioral Foundations

We now provide the behavioral foundations for our choice procedure. Since our characterization is based on observable choices, it allows any outside observer to verify whether data from individual or social choices is consistent with the RCM or not. Before we state our first main result, we want to remind the reader of the definition of the weak axiom of revealed preferences (WARP), a necessary and sufficient condition for rational choice. WARP demands that if an alternative x is chosen from some set where y is available, then y is never chosen from any set where x is available. Formally,

Definition 5.1. *A choice function $\gamma : \mathcal{P}(X) \to X$ satisfies WARP if for all $S, T \in \mathcal{P}(X)$,*

$$[\gamma(S) = x, y \in S, x \in T] \Rightarrow [\gamma(T) \neq y]$$

It is well known that if an individual's choice function satisfies WARP then her choices can be rationalized by a linear order.

Proposition 5.1. *A choice function $\gamma : \mathcal{P}(X) \to X$ satisfies WARP if and only if there exists a linear order*[2] *$\succ \subseteq X \times X$ such that for any $S \in \mathcal{P}(X)$,*

$$\gamma(S) = \{x \in S | x \succ y, \forall y \in S \backslash \{x\}\}$$

It turns out that if we require any two reasons to share the same strengths, then the corresponding version of the RCM is rational, i.e., satisfies WARP. The following characterization result establishes this equivalence between rational choice and this restricted version of the RCM.

Theorem 5.1. *A choice function γ on X is an RCM with the set of reasonales $\mathcal{R}$ on X such that for all $x, y, x', y' \in X$, it holds that if $(x, y) \in \succ$ and $(x', y') \in \succ'$ for some $\succ, \succ' \in \mathcal{R}$, then $\#\{\succ \in \mathcal{R} | (x, y) \in \succ\} = \#\{\succ \in \mathcal{R} | (x', y') \in \succ\}$ if and only if it satisfies WARP*

Proof: Please refer to Appendix A.

This characterization result establishes a strong connection between the traditional economic approach of rational choice and our conceptualization of the psychological interpretation that individual choice is governed by reasons.

[2] A linear order is a total, transitive and irreflexive binary relation

What drives the consistency of the RCM here is that we require all reasons to share the same strengths and, further, that a DM following the RCM is always able to identify a uniquely most reasonable alternative.

So far in our study of behavioral foundations, we have explicitly ignored the fact that reasons may differ in their strengths. We now extend our analysis of the RCM to examine this component of a DM's reasoning process. In this section, we restrict our attention to the case where any reason's strength cannot exceed two and defer the more general discussion of a reason's strength to Section 7.2. That is, the strength of a reason is either one or two if that reason exists, or zero if no such reason exists. Note that, under this assumption, the choice from a binary choice comparison does not reveal the number of reasons in favor of the chosen alternative, rather it only reveals that this number must be strictly positive. If we further assume that the reasonales in $\mathcal{R}$ are jointly asymmetric, then the choice from a binary choice comparison at least reveals whether the number of relevant reasons in favor of the available alternatives are zero (for the unchosen alternative) or strictly positive (for the chosen alternative).

We now address the question of identifying the underlying behavioral foundations of such an RCM. To that end, we introduce four axioms which all draw a connection between the choice from a binary choice comparison and that from larger sets. The first one is in the nature of an expansion condition, similar in spirit to other such conditions in the literature.

Axiom 5.1 (Gradual Expansion)**.** *For all $S \in \mathcal{P}(X)$ and all $y, z \in S$, $y \neq z$,* $[\gamma(S \setminus \{y\}) = \gamma(S \setminus \{z\}) = x$ *and* $\gamma(x, y) = \gamma(x, z) = x] \Rightarrow [\gamma(S) = x]$

Our next axiom was introduced by Manzini and Mariotti (2007) and conveys the idea of a confirmed revealed preference.

Axiom 5.2 (Weak WARP)**.** *For all $S, T \in \mathcal{P}(X)$ and all $x, y \in S$, $x \neq y$,* $[\gamma(x, y) = \gamma(T) = x] \Rightarrow [\gamma(S) \neq y]$

Weak WARP requires the revealed preference of one alternative over another to be revealed through the choice from a binary choice comparison and to be confirmed by the choice from a larger set. To elaborate on the idea of confirmed revealed preference, consider that in the axiom's precondition, x is chosen from the binary choice comparison with y, that is, without any other alternatives being available or without any "context effects". The precondition further specifies that this choice of x over y is confirmed by the choice of x from a superset T of $\{x, y\}$, i.e., with "context effects". We then say that x is confirmed revealed preferred for a set S, if S is sandwiched between the binary comparison of x and y from which x is chosen and the superset T from which x is chosen, i.e., $\{x, y\} \subset S \subset T$. Weak WARP then states that from any such set S for which we can say that x is confirmed revealed preferred over y, y cannot be chosen.

The third axiom requires that any revealed preference violation implies that the same such violation arises through the choices from a binary and a trinary choice comparison.

Axiom 5.3 (Weak Trinary Independence)**.** *For all $S \in \mathcal{P}(X)$ and all $x, y \in S$, $x \neq y$,*
$[\gamma(x, y) = y$ *and* $\gamma(S) = x] \Rightarrow [\gamma(x, y, z) = x$ *for some* $z \in S]$

In standard choice theory, when some alternative x is chosen from a set where y is also available, x is revealed to be preferred to y. The RCM does not allow to draw such general conclusions when reasons differ in their strengths. Rather, when x is chosen in the presence of y, the reasons that support this choice may depend on the choice context in the form of other alternatives being available and x faring better in a comparison with them than y does. In particular, it may even be the case that the DM's reasons from the direct comparison of x and y favor the latter over the former, but that, in the presence of other alternatives, she reasons that x is the most reasonable choice. Weak Trinary Independence then specifies that x can garner the reasons necessary to that end from the comparison to just one other alternative.

The fourth axiom is a weakening of a known consistency condition referred to as never chosen which we discussed in some detail in Section 4.

Axiom 5.4 (Almost Never Chosen)**.** *For all $S \in \mathcal{P}(X)$, $\#S \geq 4$,*
$[\gamma(x, y) \neq x$ for all $y \in S \setminus \{x, z\}] \Rightarrow [\gamma(S) \neq x]$

The main result of this section can now be stated as follows.

Theorem 5.2. *A choice function γ on X is an RCM with the set of reasonales $\mathcal{R}$ such that for all $x, y \in X$, it holds that if $(x, y) \in \succ$, for some $\succ \in \mathcal{R}$, then $(y, x) \notin \succ'$ for any $\succ' \in \mathcal{R}$ and $\#\{\succ \in \mathcal{R} | (x, y) \in \succ\} \leq 2$, if and only if it satisfies gradual expansion, weak WARP, weak trinary independence, and almost never chosen.*

This version of the RCM closely resembles the design of a double round-robin tournament which is the common basis on which most association football leagues in the world are organized. In such competitions, each contestant plays every other contestant twice. To determine a unique winner of home and away matches, if needed, other criteria such as superior goal difference or higher number of goals scored can be factored in.

6 Discussion and Literature

To establish a connection to the economic literature, consider an individual plunging her left hand in a bucket of ice water and her right one in a bucket of hot water. Then, if she simultaneously moves both of her hands into a pail of tap water having intermediate temperature, her left hand will feel the water as being warm while her right hand will feel it as cool. This mismatch in temperature sensation illustrates very vividly that it is much easier for an individual to make relative judgements, here by comparing the temperature of water in one bucket to that in another one, than it is for her to come to an absolute assessment, for instance, about the exact temperature of water in one bucket. How has economic theory accounted for this evidence? In economics, the departure from cardinal to ordinal preferences over alternatives allows to capture the phenomenon of temperature sensation as it renders absolute or cardinal assessments meaningless. In particular, ordinal preferences are usually implemented via a binary relation on the set of alternatives that collects ordered pairs of elements from that set. So, preferences are indeed a collection of relative judgements between alternatives.

Even if a DM's underlying preferences over alternatives rationalize her choices, it is far from clear whether they can be viewed as a genuine explanation of her choices. More specifically, the statement "because she prefers one alternative to another" is not very illuminating. Indeed, in the real world, individuals usually engage in practical reasoning by deliberating what to do and how to do it, and they often act in light of reasons which explain and justify their choices: Take a culprit in court, a firm announcing that layoffs are inevitable, or, choice in strategic interactions (List 2007). In each such scenario, an individual often provides reasons to justify her choices either to herself or to others. Since the alternatives from which she chooses or has chosen show numerous properties, so may there be different reasons or various strengths to justify her choice between them. This stands in stark contrast to the example above, where the alternatives (water in buckets) vary on one dimension (temperature) only.

The most prominent axiomatic approaches to individual choice behavior either define a sequential procedure by which the DM gradually narrows down the set of available alternatives until one is identified as her choice or they drop the assumption that the DM always considers all available alternatives. These two strands of behavioral choice theory in economics parallel the psychologic interpretation of individual choice behavior subsumed under the term of constructive preferences that breaks down in perceptual contrasting and effort minimization (Dhar, Nowlis, and Sherman 2000). In contrast, a DM following the RCM is neither sequentially rationalizing her choices nor is she in some way, consciously or unconsciously, restricting her attention to a subset of the available alternatives. Rather, such a DM always considers all available alternatives and chooses the most reasonable of the available alternatives in a one stage process.

In the most prominent of the sequential choice procedures, the rational shortlist method (RSM) by Manzini and Mariotti (2007), the DM has two criteria or aspects in mind that she deems relevant for her choice and that each translate into an asymmetric binary relation (rationale) on the set of alternatives. To arrive at her choice, the DM then narrows down the set of available alternatives by sequentially applying these two rationales and by dropping those alternatives that are inferior with respect to the current rationale in the sequence. In the basic version of the RSM, the number of rationales is fixed to two and also the order in which they are applied stays the same across all choice situations. Manzini and Mariotti (2012b) extend the RSM to sequences of more than two rationales. Kops (2016) presents a menu-dependent version of the RSM, referred to as the (f)lexicographic shortlist method (FSM), which retains the assumption that it is the same set of rationales that is applied to each choice situation, but drops the assumption of a fixed order of priority in which the two rationales are applied.

In the choice procedures described by Kalai, Rubinstein, and Spiegler (2002) and Apesteguia and Ballester (2005) multiple rationales are used to explain choice behavior. Their focus is not on a sequential application of several rationales, rather the authors are interested in identifying the minimum number of rationales necessary to explain choice data if the application of each relation can be restricted to specific subsets. By this procedure all choice functions can be rationalized such that the focus is shifted to identifying the minimum number of rationales necessary to explain choices. Houy and Tadenuma (2009) consider two distinct procedures to lexicographic compositions of two criteria for decision making. First, composing the separate relations into one and then selecting its maximal element, and second, the sequential approach of Manzini and Mariotti (2007).

Manzini and Mariotti (2012) propose a variation of the RSM in which the first stage involves a coarser form of maximization, using an asymmetric shading relation that is defined on sets of alternatives, rather than on alternatives themselves. This categorize then choose (CTC) procedure introduces into the elimination sequence the psychological process of categorization by which alternatives are clustered into sets. Complete categories may then eliminate others.

Other models describe a DM's choice procedure as following an ordered elimination. That is, choice is not the result of a maximization process, rather the DM selects an alternative following a series of binary comparisons. Choice by lists by Rubinstein and Salant (2006) and choice by sequential procedures by Apesteguia and Ballester (2013) are instances of such choice procedures.

The revealed preference argument of rational choice relies on the implicit assumption that a DM always considers all available alternatives. Masatlioglu, Nakajima, and Ozbay (2012) relax this full consideration assumption in their model of choice with limited attention (CLA) proposing the following two-stage procedure for choice. First, an attention filter determines which of the available alternatives are feasible and then the DM selects the unique feasible alternative that maximizes a complete and transitive binary relation. The attention filter is a conceivably unconscious process that identifies the alternatives to which the DM effectively pays attention. Once certain alternatives in a specific set are identified in this way by the attention filter they remain feasible in any smaller set as long as all of these feasible alternatives remain available in that set.

7 Remarks

In this section, we highlight different extensions and applications of the RCM. First, we provide an alternative definition of the choice procedure, the relatively reasonable choice method (RRCM), and establish that entirely ignoring the multiplicity of reasons leads again to the same internal consistency of WARP. Second, we discuss the consistency on choices implied by an RCM with unlimited multiplicity of reasons.

7.1 Relatively Reasonable Choices

Clearly, Definition 3.1 is not the only way how we can define a choice procedure by which a DM always chooses the alternative backed by the most context-dependent reasons. In this section, we introduce an alternative definition of such a choice procedure.

Given a set of reasonales $\mathcal{R}$, we now define a counting function $f_{\mathcal{R}|_S} : S \to \mathbb{N}$ for every set $S \subseteq X$, that is closely related to the counting function $g_{\mathcal{R}|_S}$, by

$$f_{\mathcal{R}|_S}(x) = \sum_{\succ \in \mathcal{R}} \#\{y \in S \setminus \{x\} | (x, y) \in \succ\} - \#\{y \in S \setminus \{x\} | (y, x) \in \succ\}$$

The intuition for this alternative definition is the following: For every $x \in S$, we still interpret an ordered pair $(x, y) \in \succ$ for some $y \in S \setminus \{x\}$ and $\succ \in \mathcal{R}$ as a reason in favor of x that is relevant in S. In contrast to the function $g_{\mathcal{R}|_S}$ in the definition of the RCM, here the function $f_{\mathcal{R}|_S}(x)$ not only counts all such reasons in S in favor of this alternative, but also subtracts all reasons in S against this alternative, i.e, it counts all the ordered pairs of alternatives that list x first and another alternative in $S \setminus \{x\}$ second and subtracts from this number the amount of all the ordered pairs of alternatives that list x second and some other alternative in $S \setminus \{x\}$ first. So, the value of $f_{\mathcal{R}|_S}(x)$ provides a measure of how reasonable the alternative x is relative to how unreasonable this alternative is in the context of all the other alternatives that are available in S.

With this alternative definition of the counting function, we can now also provide an alternative formalization of the psychologic interpretation of choice behavior according to which a DM chooses that alternative from the available ones for which she has the most context-dependent reasons to do so. The way we formalize this is captured by the following definition.

Definition 7.1. *A choice function γ is a relatively reasonable choice method (RRCM) whenever there exists a set of reasonales $\mathcal{R}$ such that for all $S \in \mathcal{P}(X)$,*

$$\gamma(S) = \{x \in S | f_{\mathcal{R}|_S}(x) > f_{\mathcal{R}|_S}(y), \forall y \in S \setminus \{x\}\}$$

In that case we say that $(\mathcal{R}, f_{\mathcal{R}})$ rationalizes γ.

There is a version of the Condorcet method in voting theory, referred to as Copeland's method, in which candidates are ordered by the number of pairwise victories, minus the number of pairwise defeats. The RRCM is the individual choice analogue of Copeland's election method that resolves ties by, for instance, the magnitudes of victories and defeats which is captured by the strength of a reason. If, on the other hand, we ignore the strengths of reasons entirely, as we did before in the simplest version of the RCM, the consistency on choices implied by the RRCM is the same internal consistency of choices that rationality in the economic sense implies. That is, the simplest version of the RRCM is characterized by WARP.

Theorem 7.1. *A choice function γ on X is an RRCM with the set of reasonales $\mathcal{R}$ on X such that $\succ_1 \cap \succ_2 = \emptyset$ for all $\succ_1, \succ_2 \in \mathcal{R}$, $\succ_1 \neq \succ_2$, if and only if it satisfies WARP.*

Proof: Please refer to Appendix C.

7.2 Generally Reasonable Choices

In the most general case possible under our definition of an RCM the size of the set of reasonales, $\#\mathcal{R}$, is given by some arbitrary natural number. This implies that even in binary choice comparisons between two alternatives the DM may have any number of reasons for choosing one of these alternatives over the other. We do not know what consistency conditions are sufficient for such a general RCM. What we know is that this general RCM cannot violate the following considerably weak consistency condition, referred to as intersecting expansion.

Axiom 7.1 (Intersecting Expansion)**.** *For all $S, T \in \mathcal{P}(X)$, $x \neq y$,*
$[\gamma(S) = \gamma(T) = x,\ \gamma(S \cap T) = y] \Rightarrow [\gamma(S \cup T) \neq y]$

Drawing the connection to voting systems, our definition of the RCM with arbitrary reason strengths would allow voters to also express the intensity of their feelings towards or against a certain candidate or attributes/aspects of that candidate.

Appendices

A Proof of Theorem 5.1

Proof. Necessity: Let γ be an RCM with the set of reasonales $\mathcal{R}$ on X such that for all $x, y, x', y' \in X$, it holds that if $(x, y) \in \succ$ and $(x', y') \in \succ'$ for some $\succ, \succ' \in \mathcal{R}$, then $\#\{\succ \in \mathcal{R} | (x, y) \in \succ\} = \#\{\succ \in \mathcal{R} | (x', y') \in \succ\}$.

First, we establish that $(x, y) \in \succ$ for some $\succ \in \mathcal{R}$ implies that $(y, x) \notin \succ$ for all $\succ \in \mathcal{R}$. Suppose otherwise, then $(x, y) \in \succ_1$ and $(y, x) \in \succ_2$ for some

$\succ_1, \succ_2 \in \mathcal{R}$. Since $\#\{\succ \in \mathcal{R} | (x,y) \in \succ\} = \#\{\succ \in \mathcal{R} | (x',y') \in \succ\}$, it holds that $g_{\mathcal{R}|_{\{x,y\}}}(x) = g_{\mathcal{R}|_{\{x,y\}}}(y)$. From the definition of the RCM it follows that $\gamma(x,y)$ is not an element of X which is a contradiction to $\gamma(.)$ being a function. A similar argument establishes that for all $x, y \in X$ either $(x,y) \in \succ$ for some $\succ \in \mathcal{R}$ or $(y,x) \in \succ$ for some $\succ \in \mathcal{R}$.

Second, we show that the set of reasonales is jointly transitive, i.e., if $(x,y) \in \succ_1$ and $(y,z) \in \succ_2$ for some $\succ_1, \succ_2 \in \mathcal{R}$, then $(x,z) \in \succ_3$ for some $\succ_3 \in \mathcal{R}$. Suppose, by contradiction that this is not true and that $(x,y) \in \succ_1$, $(y,z) \in \succ_2$ and $(z,x) \in \succ_3$ for some $\succ_1, \succ_2, \succ_3 \in \mathcal{R}$. Since then

$$\#\{\succ \in \mathcal{R} | (x,y) \in \succ\} = \#\{\succ \in \mathcal{R} | (y,z) \in \succ\} = \#\{\succ \in \mathcal{R} | (z,x) \in \succ\}$$

by the definition of the RCM it follows that $\gamma(x,y,z)$ is not an element of the codomain X which is a contradiction to $\gamma(.)$ being a function. Hence, the set of reasonales is jointly complete and transitive.

Now, we show that the RCM implies WARP. Consider $S, T \in \mathcal{P}(X)$ with $x, y \in S \cap T$ and let $\gamma(S) = x$. Since the set of reasonales is jointly complete and transitive, it follows that $(x,z) \in \succ_{\{x,z\}}$, for all $z \in S \backslash \{x\}$ and some $\succ_{\{x,z\}} \in \mathcal{R}$. To see this, suppose, by contradiction that $(x,y) \notin \succ$ for some $y \in S \backslash \{x\}$ and all $\succ \in \mathcal{R}$. By joint completeness of $\mathcal{R}$, it follows that $(y,x) \in \succ$, for some $\succ \in \mathcal{R}$. Joint transitivity of $\mathcal{R}$ then implies that for any $z \in S$ with $(x,z) \in \succ_{\{x,z\}}$ for some $\succ_{\{x,z\}} \in \mathcal{R}$ it also holds that $(y,z) \in \succ_{\{y,z\}}$ for some $\succ_{\{y,z\}} \in \mathcal{R}$. Furthermore, for any such z it holds that

$$\#\{\succ \in \mathcal{R} | (x,z) \in \succ\} = \#\{\succ \in \mathcal{R} | (y,z) \in \succ\}$$

Hence, $g_{\mathcal{R}|_S}(y) > g_{\mathcal{R}|_S}(x)$ and, by the definition of an RCM, this contradicts $\gamma(S) = x$. So, $(x,z) \in \succ_{\{x,z\}}$, for all $z \in S \backslash \{x\}$ and some $\succ_{\{x,z\}} \in \mathcal{R}$. Now, joint transitivity of $\mathcal{R}$ also implies that for any $z \in T$ with $(y,z) \in \succ_{\{y,z\}}$ for some $\succ_{\{y,z\}} \in \mathcal{R}$, it follows that $(x,z) \in \succ_{\{x,z\}}$ for some $\succ_{\{x,z\}} \in \mathcal{R}$. Hence, $g_{\mathcal{R}|_T}(x) > g_{\mathcal{R}|_T}(y)$ and, thus $\gamma(T) = x \neq y$ by the definition of the RCM.

<u>Sufficiency:</u> Suppose that γ satisfies WARP. We construct the set of reasonales

$\mathcal{R}$ explicitly. For any $\succ \in \mathcal{R}$, we define $(x, y) \in \succ$ if and only if $\gamma(x, y) = x$. By this definition, each $\succ$ is complete and transitive. To see that it is complete, suppose that both $(x, y) \notin \succ$ and $(y, x) \notin \succ$ for some $x, y \in X$. It follows that $\gamma(x, y) = \emptyset$, which, by the definition of a choice function, is not possible. To see that it is also transitive, suppose that $(x, y), (y, z), (z, x) \in \succ$, for some $x, y, z \in X$ and some $\succ \in \mathcal{R}$. By our definition of $\mathcal{R}$ above this implies that $\gamma(x, y) = x$, $\gamma(y, z) = y$ and $\gamma(x, z) = z$ such that WARP is violated. This establishes that $\succ$ is complete and transitive.

Next, we define the counting function $g_{\mathcal{R}|_S} : S \to \mathbb{N}$ for every set S by

$$g_{\mathcal{R}|_S}(x) = \#\{y \in S \setminus \{x\} | (x, y) \in \succ\}$$

and prove that $(\mathcal{R}, g_{\mathcal{R}})$ rationalizes γ. Consider $S \in \mathcal{P}(X)$ and let $\gamma(S) = x$. Suppose, by contradiction, that there exists $y \in S \backslash \{x\}$ such that $g_{\mathcal{R}|_S}(y) \geq g_{\mathcal{R}|_S}(x)$. By transitivity of $\succ$ this implies that $(x, y) \notin \succ$. Furthermore, completeness of $\succ$ ensures that $(y, x) \in \succ$. The definition of $\succ$ above then implies that $\gamma(x, y) = y$ such that WARP is violated. Hence, $\gamma(S) = x$ implies that $g_{\mathcal{R}|_S}(x) > g_{\mathcal{R}|_S}(y)$, for all $y \in S \backslash \{x\}$ such that $(\mathcal{R}, g_{\mathcal{R}})$ rationalizes γ. $\square$

B Proof of Theorem 5.2

Proof. Necessity: Let γ be an RCM on X and let $\mathcal{R}$ be the set of the corresponding two reasonales.

Gradual expansion. Let $\gamma(S \backslash \{y\}) = \gamma(S \backslash \{z\}) = x$ and $\gamma(x, y) = \gamma(x, z) = x$ for $S \in \mathcal{P}(X)$ and $x, y, z \in S$. $\gamma(S \setminus \{y\}) = x$ implies by the definition of RCM that $g_{\mathcal{R}|_{S \setminus \{y\}}}(x) > g_{\mathcal{R}|_{S \setminus \{y\}}}(w)$ for all $w \in S \setminus \{y, x\}$ and $\gamma(x, y) = x$ that $g_{\mathcal{R}|_{\{x,y\}}}(x) > 0$. Since $0 \leq g_{\mathcal{R}|_{\{w,y\}}}(w) \leq 2$ for all $w \in S \setminus \{y\}$, it follows that $g_{\mathcal{R}|_S}(x) \geq g_{\mathcal{R}|_S}(w)$ for all $w \in S \setminus \{y\}$. Analogously, $\gamma(S \setminus \{z\}) = x$ and $\gamma(x, z) = x$ imply that $g_{\mathcal{R}|_S}(x) \geq g_{\mathcal{R}|_S}(w)$ for all $w \in S \setminus \{z\}$. Since $y \neq z$, it follows that $g_{\mathcal{R}|_S}(x) \geq g_{\mathcal{R}|_S}(w)$ for all $w \in S$. Hence, the fact that γ is a function implies that $g_{\mathcal{R}|_S}(x) > g_{\mathcal{R}|_S}(w)$ for all $w \in S$ and, thus, it follows from the definition of RCM that $\gamma(S) = x$. So, the RCM implies gradual expansion.

Weak Trinary Independence (WTI). Let $\gamma(x,y) = x$ and $\gamma(S) = y$ for $S \in \mathcal{P}(X)$ and some $x, y \in S$. We prove via induction on the cardinality of S that this implies that there exists an element $z \in S$ such that $\gamma(x,y,z) = y$.

Base Case: Let $|S| = 3$, w.l.o.g., $S = \{x,y,z\}$. Further, let $\gamma(S) = y$ and $\gamma(x,y) = x$. Since $|S| = 3$, the desired conclusion follows immediately.

Induction hypothesis: If $\gamma(S) = y$ and $\gamma(x,y) = x$ for some $x \in S \setminus \{y\}$, then $\gamma(x,y,z) = y$ for some $z \in S$, for all $S \in \mathcal{P}(X)$ with $|S| = n \geq 3$.

Inductive step: $n \to n+1$. Let $|S| = n+1, \gamma(S) = y$ and $\gamma(x,y) = x$ for some $x \in S \setminus \{y\}$. Suppose, by contradiction, that $\gamma(S \setminus \{w\}) \neq y$ for all $w \in S \setminus \{x,y\}$. Since $\gamma(S) = y$ and, thus, by the RCM $g_{\mathcal{R}|_S}(y) > g_{\mathcal{R}|_S}(w)$ for all $w \in S \setminus \{x,y\}$, this implies that $2 = g_{\mathcal{R}|_{\{w,y\}}}(y) > 0$ and, thus, $\gamma(w,y) = y$ for all such w. Hence, $g_{\mathcal{R}|_{S\setminus\{w\}}}(y) = 2(|S|-3) \geq g_{\mathcal{R}|_{S\setminus\{w\}}}(z)$, for all $z \in S \setminus \{w\}, z \neq x$. So, $\gamma(S \setminus \{w\}) \neq y$ implies that $\gamma(S \setminus \{w\}) = x$ for all $w \in S \setminus \{x,y\}$. This can, however, only be the case if $g_{\mathcal{R}|_S}(x) = g_{\mathcal{R}|_S}(y) - 1$ and $g_{\mathcal{R}|_{S\setminus\{w\}}}(x) > g_{\mathcal{R}|_{S\setminus\{w\}}}(y)$ for all $w \in S \setminus \{x,y\}$. Hence, $g_{\mathcal{R}|_{\{x,w\}}}(x) = 0$ for all $w \in S \setminus \{x,y\}$. So, $g_{\mathcal{R}|_{S\setminus\{w\}}}(x) \leq 2$. By $g_{\mathcal{R}|_{\{y,z\}}}(y) = 2$ for all $z \in S \setminus \{x,y\}$ and $|S| \geq 4$, it follows that $g_{\mathcal{R}|_{S\setminus\{w\}}}(y) \geq g_{\mathcal{R}|_{S\setminus\{w\}}}(x)$. So, $\gamma(S \setminus \{w\}) \neq x$ and, thus, $\gamma(S \setminus \{w\}) = y$ for some $w \in S \setminus \{x,y\}$. Hence, the proof is complete by use of the induction hypothesis.

Almost Never Chosen (ANC). Let $|S| \geq 4$ and $\gamma(x,y) \neq x$ for all $y \in S \setminus \{x,z\}$ for some $z \in S$. We prove via induction on the cardinality of S that this implies that $\gamma(S) \neq x$.

Base Case: Let $|S| = 4$ and $S = \{x,y,z,w\}$. Suppose, by contradiction, that $\gamma(S) = x$ and $\gamma(x,y) \neq x, \gamma(x,w) \neq x$. By the definition of the RCM,

$g_{\mathcal{R}|_S}(x) = g_{\mathcal{R}|_{\{x,z\}}}(x) \leq 2$. On the other hand, there are five more binary comparisons, $\{x, y\}, \{x, w\}, \{w, y\}, \{w, z\}, \{y, z\}$, none of which is such that x is chosen from it. So, at least one alternative from $\{w, y, z\}$ is chosen from two of these comparisons, denote this alternative by v. Then, by the definition of the RCM, $g_{\mathcal{R}|_S}(v) \geq 2 \geq g_{\mathcal{R}|_S}(x)$ which implies that $\gamma(S) \neq x$ and we have arrived at our desired contradiction.

Induction hypothesis: For all $n \in \mathbb{N}$, $n \geq 4$, and for all $S \in \mathcal{P}(X)$ such that $|S| \geq n$ it holds that if $\gamma(x, y) \neq x$ for all $y \in S \setminus \{x, z\}$ and some $z \in S$, then $\gamma(S) \neq x$.

Inductive step: $n \to n + 1$. Let $|S| = n + 1$. Suppose, by contradiction, that $\gamma(S) = x$ and $\gamma(x, y) \neq x$ for all $y \in S \setminus \{x, z\}$ and some $z \in S$. Since $g_{\mathcal{R}|_{\{x,y\}}}(x) = 0$ for any such y and $g_{\mathcal{R}|_S}(x) > g_{\mathcal{R}|_S}(w)$ for all $w \in S \setminus \{x\}$, it follows that $g_{\mathcal{R}|_{S\setminus\{y\}}}(x) > g_{\mathcal{R}|_{S\setminus\{y\}}}(w)$ for all $w \in S \setminus \{x, y\}$. Hence, by the definition of the RCM, $\gamma(S \setminus \{y\}) = x$ and the induction hypothesis leads to the desired contradiction.

Weak WARP. Let $\gamma(x, y) = \gamma(S) = x$ for $S \in \mathcal{P}(X)$ and $x, y \in S$. We show that this implies that $\gamma(R) \neq y$ for all $R \in \mathcal{P}(X)$ with $\{x, y\} \subset R \subset S$. To this end, we first show that this is true for any such R, S with $|R| = 3$ and $R \subset S$ via induction on the cardinality of S.

Base case: $|S| = 4$. Let $S = \{w, x, y, z\}$ and $\gamma(S) = \gamma(x, y) = x$. Suppose, by contradiction, that for some R with $|R| = 3$ and $\{x, y\} \subset R \subset S$ it holds that $\gamma(R) = y$. WLOG, let $R = \{x, y, z\}$. By the definition of the RCM, this implies that $g_{\mathcal{R}|_R}(y) > g_{\mathcal{R}|_R}(x), g_{\mathcal{R}|_R}(z)$. Hence, $g_{\mathcal{R}|_R}(y) = g_{\mathcal{R}|_{\{y,z\}}}(y) = 2 > g_{\mathcal{R}|_R}(x) = g_{\mathcal{R}|_R}(z) = 1$, and, thus, $\gamma(y, z) = y$, $\gamma(x, z) = z$. By the RCM, $\gamma(S) = x$ then implies that $g_{\mathcal{R}|_S}(x) > g_{\mathcal{R}|_S}(v)$, for all $v \in S$, $v \neq x$. Since, $g_{\mathcal{R}|_{\{x,w\}}}(x) \leq 2$ it follows that $g_{\mathcal{R}|_{\{x,w\}}}(x) = 2$ and $g_{\mathcal{R}|_S}(x) = 3$. Since $g_{\mathcal{R}|_{\{y,z\}}}(y) = 2$, this implies that $g_{\mathcal{R}|_{\{w,y\}}}(y) = 0$. So, $g_{\mathcal{R}|_{\{y,z\}}}(w) > 0$. If, further, $g_{\mathcal{R}|_{\{z,w\}}}(w) = 2$, then $g_{\mathcal{R}|_S}(w) > 2$ which contradicts $\gamma(S) = x$. If, on the other hand, $g_{\mathcal{R}|_{\{w,z\}}}(w) = 1$, then either $g_{\mathcal{R}|_S}(w) > 2$ contradicting $\gamma(S) =$

x, or, $g_{\mathcal{R}|_{\{w,y,z\}}}(w) = 2$. But, then $g_{\mathcal{R}|_{\{w,y,z\}}}(w) = g_{\mathcal{R}|_{\{w,y,z\}}}(y) = 2$ and choice from $\{w, y, z\}$ is not unique which contradicts the definition of the RCM. Hence, $g_{\mathcal{R}|_{\{w,z\}}}(w) = 0$. If $g_{\mathcal{R}|_{\{z,w\}}}(z) = 2$, then $g_{\mathcal{R}|_S}(z) > 2$, a contradiction. If $g_{\mathcal{R}|_{\{z,w\}}}(z) = 1$, then $g_{\mathcal{R}|_{\{x,z,w\}}}(x) = g_{\mathcal{R}|_{\{x,z,w\}}}(z) = 2 > g_{\mathcal{R}|_{\{x,z,w\}}}(w) = 0$, contradicting uniqueness.

Lemma B.1. *A choice function γ on X satisfies gradual expansion only if it satisfies always chosen.*

Proof. Proof by induction on the cardinality of S. Base case: Let $|S| = 3$ and $S = \{x, y, z\}$. Further, w.l.o.g., let $\gamma(x, y) = \gamma(x, z) = x$. It follows immediately that $\gamma(S) = x$ by gradual expansion.

Induction hypothesis: $\forall S \in \mathcal{P}(X), |S| = n \geq 3$, if $\gamma(x, y) = x$ for all $y \in S \setminus \{x\}$, then $\gamma(S) = x$.

Inductive step: $n \to n+1$. Let $|S| = n+1$ and $\gamma(x, y) = x$ for all $y \in S \setminus \{x\}$. For any two $v, w \in S \setminus \{x\}, v \neq w$, we must have that $\gamma(S \setminus \{v\}) = \gamma(S \setminus \{w\}) = x$ by the induction hypothesis. Further, since $\gamma(x, v) = \gamma(x, w) = x$ by construction, gradual expansion implies that $\gamma(S) = x$. $\square$

Induction hypothesis: $\forall S \in \mathcal{P}(X), |S| = m \geq 4$, $\{x, y\} \subset R \subset S$, and $|R| = 3$, it holds that if $\gamma(x, y) = \gamma(S) = x$, then $\gamma(R) \neq y$.

Inductive step: $m \to m + 1$. Let S be such that $|S| = m + 1$ and let $\gamma(x, y) = \gamma(S) = x$ for some $x, y \in S$. Suppose, by contradiction, that $\gamma(R) = y$ for some $\{x, y\} \subset R \subset S$, $|R| = 3$. Let $R = \{x, y, z\}$ for some $z \in S$. By the induction hypothesis, we must have that $\gamma(S \setminus \{w\}) \neq x$ for all $w \in S \setminus \{x, y, z\}$. Since, by definition of the RCM, $\gamma(S) = x$ implies that $g_{\mathcal{R}|_S}(x) > g_{\mathcal{R}|_S}(z)$ for all $z \in S$, $z \neq x$, it follows that $g_{\mathcal{R}|_{\{x,w\}}}(x) > g_{\mathcal{R}|_{\{x,w\}}}(w)$ for all such w. If $\gamma(x, z) = x$, then together with $\gamma(x, y) = x$ it follows from Lemma B.1 that $\gamma(T) = x$ for all $T \subseteq S$, with $x \in T$. In particular, this is true for all such T with $|T| = m$ which together with $\gamma(R) = y$ contradicts the induction hypothesis. So, we have a arrived at our desired contradiction. If, on the other hand, $\gamma(x,z)=z$, then by WTI there exists $v \in S \setminus \{x,z\}$ such that $\gamma(v,x,z)=x$. If $v=y$, then this contradicts $\gamma(R)=y$. So, suppose $v \neq y$. Then, $\gamma(v,x,z)=x$ implies that $2=g_{\mathcal{R}|_{\{v,x,z\}}}(x) > g_{\mathcal{R}|_{\{v,x,z\}}}(z), g_{\mathcal{R}|_{\{v,x,z\}}}(v)$. By uniqueness and the fact that $\gamma(x,y)=x$ it follows that $\gamma(w,x,y,z)=x$. So, the induction hypothesis implies that $\gamma(R) \neq y$ and we have arrived at our desired contradiction.

Next, to establish the necessity of weak WARP by proving via induction on the cardinality of R that $\gamma(R) \neq y$ for all R such that $\{x,y\} \subset R \subset S$, if $\gamma(x,y)=\gamma(S)=x$. Note that we have already proven the base case where $|R|=3$ via the induction proof above.

Induction hypothesis: If $\gamma(x,y)=\gamma(S)=x$, then it holds for all R with $\{x,y\} \subset R \subset S$ and $|R|=n$ for some $n \in \mathbb{N}$ that $\gamma(R) \neq y$.

Inductive step: $n \to n+1$. Let R be such that $\{x,y\} \subset R \subset S$ and $|R| = n+1$. Furthermore, let $\gamma(x,y)=\gamma(S)=x$. Suppose, by contradiction, that $\gamma(R)=y$. Since $\gamma(x,y)=x$, $\exists z \in R \setminus \{x,y\}$ such that $\gamma(x,y,z)=y$, by WTI. However, notice that this contradicts the hitherto proved part of weak WARP for $|R|=3$. Hence, we have arrived at our desired contradiction.

Sufficiency: Suppose that γ satisfies the axioms. We construct the reasonales explicitly. Firstly, for every $x,y \in X$, we define $(x,y) \in \succ_2$ only if $(x,y) \in \succ_1$. Furthermore, we define $(x,y) \in \succ_i$ only if $(y,x) \notin \succ_i$ for $i=1,2$. By this definition, any $\succ \in \mathcal{R}$ as well as $\bigcup_{\succ \in \mathcal{R}}$ are asymmetric. In addition, we fix $(x,y) \in \succ_1$ if and only if $\gamma(x,y)=x$ such that the relation $\succ_1$ is complete by the definition of $\gamma(\cdot)$. Secondly, denote $(x,y) \in \succ_2$ if and only if either (i) $\gamma(x,y)=x, \gamma(y,z)=y, \gamma(x,z)=z$, and $\gamma(x,y,z)=x$, (ii) $\gamma(x,y)=x, \gamma(x,z)=x, \gamma(x,y,z)=x$ and (a) $\gamma(y,z)=y, (y,z) \in \succ_2$ or (b) $\gamma(y,z)=z, (z,y) \in \succ_2$, or (iii) $\gamma(x,y)=x, \gamma(x,w)=w, \gamma(y,w)=w, \gamma(x,z,w)=x$, and $(w,y) \in \succ_2$ for some $z,w \in X$. Next, we define the counting function

$g_{\mathcal{R}|S} : S \to \mathbb{N}$ for every set S by

$$g_{\mathcal{R}|_S}(x) = \#\{y \in S \setminus \{x\} | (x,y) \in \succ_1\} + \#\{y \in S \setminus \{x\} | (x,y) \in \succ_2\}$$

To check that this rationalizes the DM's choices, take any $S \in \mathcal{P}(X)$ with $|S| \geq 3$ and let $\gamma(S) = x$. We prove via induction that this implies that $g_{\mathcal{R}|_S}(x) > g_{\mathcal{R}|_S}(y)$ for all $y \in S \setminus \{x\}$.

Base case: $|S| = 3$. Let $S = \{x, y, z\}$ and $\gamma(S) = x$. First, suppose that $\gamma(x, y) = \gamma(x, z) = x$. By the assignment rule, we have that $(x, y), (x, z) \in \succ_1$ and $(y, x), (z, x) \notin \succ_1 \cup \succ_2$. W.l.o.g., let $\gamma(y, z) = y$. If $(y, z) \notin \succ_1 \cap \succ 2$, then $(y, z) \in \succ_1 \setminus \succ_2$ by the assignment rule, and we have $g_{\mathcal{R}|_S}(x) \geq 2 > g_{\mathcal{R}|_S}(y) = 1 > g_{\mathcal{R}|_S}(z) = 0$. If, on the other hand, $(y, z) \in \succ_1 \cap \succ_2$, then by (ii) of the assignment rule, we have that $(x, z) \in \succ_2$ such that $g_{\mathcal{R}|_S}(x) \geq 3 > g_{\mathcal{R}|_S}(y) = 2 > g_{\mathcal{R}|_S}(z) = 0$. Secondly, Suppose that $\gamma(x, y) \neq x$ and $\gamma(x, z) \neq x$. W.l.o.g., let let $\gamma(y, z) = y$. Hence, we have that $\gamma(x, y) = \gamma(y, z) = y$ and $\gamma(x, z) = z$. By gradual expansion, we have that $\gamma(S) = y$, a contradiction. Thirdly, suppose $\gamma(x, y) \neq x$ and $\gamma(x, z) = x$. If $\gamma(x, y) = \gamma(y, z) = y$, then $\gamma(S) = y$ by gradual expansion, a contradiction. If $\gamma(x, y) = y$ and $\gamma(y, z) = z$, then by part (i) of the assignment rule, $(x, z) \in \succ_1 \cap \succ_2$. By weak trinary independence, $(y, x) \in \succ_1 \setminus \succ_2$ and by weak WARP, $(z, y) \in \succ_1 \setminus \succ_2$ such that $g_{\mathcal{R}|_S}(x) = 2 > g_{\mathcal{R}|_S}(y) = g_{\mathcal{R}|_S}(z) = 1$. Finally, suppose $\gamma(x, y) = x$ and $\gamma(x, z) \neq x$. Notice that this case is analogous to the preceding one. That is, if $\gamma(y, z) = z$, we have that $\gamma(S) = z$ by gradual expansion. If $\gamma(y, z) = y$, we have that $(x, y) \in \succ_1 \cap \succ_2$ and $(z, x), (y, z) \in \succ_1 \setminus \succ_2$ by the assignment rule, weak trinary independence and weak WARP.

Base case: $|S| = 4$. Let $S = \{x, y, z, w\}$ and $\gamma(S) = x$. First, suppose that $\gamma(x, v) = x$ for all $v \in \{y, z, w\}$. By the assignment rule, $g_{\mathcal{R}|_{\{x,v\}}}(x) \geq 1$ for all $v \in \{y, z, w\}$. By the base case for $|S| = 3$, we have that $g_{\mathcal{R}|_{\{x,y,w\}}}(x) > g_{\mathcal{R}|_{\{x,y,w\}}}(v)$ for all $v \in \{y, w\}$, $g_{\mathcal{R}|_{\{x,z,w\}}}(x) > g_{\mathcal{R}|_{\{x,z,w\}}}(v)$ for all $v \in \{z, w\}$ and $g_{\mathcal{R}|_{\{x,y,z\}}}(x) > g_{\mathcal{R}|_{\{x,y,z\}}}(v)$ for all $v \in \{y, z\}$. Suppose, by contradiction, that $g_{\mathcal{R}|_S}(y) \geq g_{\mathcal{R}|_S}(x)$. Then $g_{\mathcal{R}|_{\{y,z\}}}(y) = 2$. By part (ii) of the assignment rule, it follows that $g_{\mathcal{R}|_{\{x,z\}}}(x) = 2$. Hence, $g_{\mathcal{R}|_S}(x) = g_{\mathcal{R}|_{\{x,y,w\}}}(x) + 2 > g_{\mathcal{R}|_{\{x,y,w\}}}(y) + 2 = g_{\mathcal{R}|_S}(y)$. So, we have arrived at our desired contradiction. Note that this holds analogously for the cases $g_{\mathcal{R}|_S}(z) \geq g_{\mathcal{R}|_S}(x)$ and $g_{\mathcal{R}|_S}(w) \geq g_{\mathcal{R}|_S}(x)$. Secondly, suppose that $\gamma(x, v) \neq x$ for all $v \in \{y, z, w\}$. Then, by almost never chosen, we have that $\gamma(S) \neq x$, a contradiction. Thirdly, suppose that $\exists! v \in \{y, z, w\}$ such that $\gamma(x, v) = x$. Once again, almost never chosen dictates that $\gamma(S) \neq x$. Finally, suppose that $\#\{v \in \{y, z, w\} | \gamma(x, v) = x\} = 2$. W.l.o.g., let $\gamma(x, y) = \gamma(x, z) = x$ and $\gamma(x, w) = w$. If $\gamma(y, w) = \gamma(z, w) = w$, we have that $\gamma(S) = w$ by lemma B.1, so this can not be the case. If $\gamma(y, w) = y$ and $\gamma(z, w) = w$, then weak WARP implies that $\gamma(x, y, z) \notin \{y, z\}$, i.e., $\gamma(x, y, z) = x$. Furthermore, it implies that $\gamma(x, y, w) \neq y$ and $\gamma(x, z, w) \neq z$. Since $\gamma(x, w) = w$, weak trinary independence implies that $\gamma(x, y, w) = x$ and/or $\gamma(x, z, w) = x$. Since $\gamma(x, z, w) = w$ by gradual expansion, we thus have that $\gamma(x, y, w) = x$. By the assignment rule, this implies that $(x, y) \in \succ_1 \cap \succ_2$, $(w, x), (y, w) \in \succ_1 \setminus \succ_2$. We have $g_{\mathcal{R}|_S}(z) = g_{\mathcal{R}|_{\{y,z\}}}(z) \leq 2$ and $g_{\mathcal{R}|_S}(x) = g_{\mathcal{R}|_{\{x,y\}}}(x) + g_{\mathcal{R}|_{\{x,z\}}}(x) \geq 3$. If $g_{\mathcal{R}|_S}(w) = 3$, then $(w, z) \in \succ_2$ and since $\gamma(x, z) = x, \gamma(x, w) = w, \gamma(z, w)$, and $\gamma(x, y, w) = x$, part (iii) of the assignment rule implies that $(x, z) \in \succ_2$. Hence, $g_{\mathcal{R}|_S}(x) = 4$. If $\gamma(y, w) = w$ and $\gamma(z, w) = z$, the argumentation is analogous to the preceding case. If $\gamma(y, w) = y, \gamma(z, w) = z$ and, w.l.o.g., $\gamma(y, z) = y$, then weak WARP implies that $\gamma(x, y, z) = x$, and, $\gamma(, y, w) \neq y$, so $\gamma(y, w) \notin \succ_2$. We have that $g_{\mathcal{R}|_S}(w) = g_{\mathcal{R}|_{\{x,w\}}}(w) \leq 2$ and $g_{\mathcal{R}|_S}(z) = g_{\mathcal{R}|_{\{z,w\}}}(z) \leq 2$. Furthermore, it implies that $\gamma(x, z, w) \neq z$. Since $\gamma(x, w) = w$, weak trinary independence implies that $\gamma(x, y, w) = x$ and/or $\gamma(x, z, w) = x$. By the assignment rule, this implies that either $(x, y) \in \succ_2$ or $(x, z) \in \succ_2$. Hence, $g_{\mathcal{R}|_S}(x) = g_{\mathcal{R}|_{\{x,y\}}}(x) + g_{\mathcal{R}|_{\{x,z\}}}(x) \geq 3$. If $g_{\mathcal{R}|_S}(y) = 3$, then $(y, z) \in \succ_2$. By part (ii) of the assignment rule , it follows that $(x, y), (x, z) \in \succ_2$.

Induction hypothesis: $\gamma(S) = x$ implies that $g_{\mathcal{R}|_S}(x) > g_{\mathcal{R}|_S}(z)$ for all $z \in S \setminus \{x\}$ and for all $S \in \mathcal{P}(X)$ with and $|S| = n$ for some $n \in \mathbb{N}$.

Inductive step: $n \to n + 1$. Let $|S| = n + 1$ and $\gamma(S) = x$. Suppose, by contradiction, that there exists $z \in S \setminus \{x\}$ such that $g_{\mathcal{R}|_S}(z) \geq g_{\mathcal{R}|_S}(y)$ for all $y \in S \setminus \{z\}$. First, note that this implies that $\gamma(S \setminus \{w\}) = z$ for some $w \in S \setminus \{z\}$. If not, then $g_{\mathcal{R}|_{S\setminus\{w\}}}(v_w) > g_{\mathcal{R}|_{S\setminus\{w\}}}(z)$ for all $w \in S \setminus \{z\}$ and for some $v_w \in S \setminus \{z, w)$. So, $g_{\mathcal{R}|_S}(z) > g_{\mathcal{R}|_S}(w)$ for all $w \in S \setminus \{z\}$ implies that $2 = g_{\mathcal{R}|_{\{z,w\}}}(z) > 0$, i.e., by our assignment rule, that $\gamma(z, w) = z$ for all $w \in S \setminus \{z\}$. By our assignment rule and gradual expansion, one can show easily via induction that then it follows that $\gamma(S) = z \neq x$, a contradiction. Hence, $\gamma(S \setminus \{w\}) = z$ for some $w \in S \setminus \{z\}$. In particular, $\gamma(S \setminus \{w\}) = z$ for some $w \neq x$. To see this, suppose that $\gamma(S \setminus \{w\}) = z$ only for $w = x$. Then, by our reasoning above it holds that $2 = g_{\mathcal{R}|_{\{z,w\}}}(z) > 0$ for all $w \neq z, x$ such that $g_{\mathcal{R}|_{S\setminus\{w\}}}(z) = 2(|S| - 3)$ for all $w \neq x$. So, $\gamma(S \setminus \{w\}) \neq z$ for some all $w \neq x$ implies that $\gamma(S \setminus \{w\}) = x$ for all such w. By the induction hypothesis, it follows that $g_{\mathcal{R}|_{S\setminus\{w\}}}(x) > g_{\mathcal{R}|_{S\setminus\{w\}}}(z)$. So, $g_{\mathcal{R}|_S}(z) \geq g_{\mathcal{R}|_S}(y)$ implies that also $2 = g_{\mathcal{R}|_{\{z,x\}}}(z) > 0$. Hence, by our assignment rule, it holds that $\gamma(w, z) = z$ for all $w \in S \setminus \{z\}$ and our assignment rule together with gradual expansion implies that $\gamma(S) = z \neq x$. Hence, $\gamma(S \setminus \{w\}) = z$ for some $w \neq x$.

Now, we show that z is unique in that $g_{\mathcal{R}|_S}(z) \geq g_{\mathcal{R}|_S}(x)$. To prove this, suppose otherwise, i.e., there exist $z_1, z_2, w_1, w_2 \in S$ such that $\gamma(S\setminus\{w_i\}) = z_i$ for $i = 1, 2$ by the reasoning above. Since neither can be chosen from all binary choice comparisons, $\exists w_i \in S \setminus \{z_i\})$ such that $\gamma(z_i, w_i) = w_i \neq z_i$ and by the induction hypothesis, it follows that $\gamma(S \setminus \{w_i\}) = z_i$. Following our reasoning above we can assume w.l.o.g. that $w_i \neq x$ for $i = 1, 2$. But, then by WTI and weak WARP $\gamma(x, z_i) = z_i, \gamma(w_i, x) = x, \gamma(w_i, z_i) = w_i$ and $\gamma(x, w_i, z_i) = x$ such that $(x, w_i) \in \succ_1 \cap \succ_2$ and $(z_i, x) \notin \succ_2$. Since $(z_i, x) \notin \succ_2$ is true for all such z_i, it follows that $g_{\mathcal{R}|_{S\setminus\{x\}}}(z_1) = g_{\mathcal{R}|_{S\setminus\{x\}}}(z_2) \geq g_{\mathcal{R}|_{S\setminus\{x\}}}(y)$ for all $y \in S \setminus \{x\}$ which is not possible by our induction hypothesis, since it establishes that the most reasonable alternative is unique. Hence, z is unique in that $g_{\mathcal{R}|_S}(z) \geq g_{\mathcal{R}|_S}(x)$.

Next, suppose $\exists y \in S \setminus \{x, z\}$ such that $\gamma(S \setminus \{v\}) = y$ for some $v \in S \setminus \{x\}$.

Then, by the induction hypothesis, $g_{\mathcal{R}|_{S\setminus\{v\}}}(y) > g_{\mathcal{R}|_{S\setminus\{v\}}}(x)$, and by axiom A and weak WARP $\gamma(x,y) = y, \gamma(x,v) = x, \gamma(v,y) = v$ and $\gamma(x,v,y) = x$. By our assignment rule it follows that $(x,v) \in \succ_1 \cap \succ_2$ and $(y,x),(v,y) \notin \succ_2$. By $g_{\mathcal{R}|_S}(z) \geq g_{\mathcal{R}|_S}(x)$, this implies that $g_{\mathcal{R}|_S}(z) = g_{\mathcal{R}|_S}(x)$ and $(z,v) \in \succ_1 \cap \succ_2$. Furthermore, $(y,x) \in \succ_1$ and $(x,y) \notin \succ_1 \cup \succ_2$, hence, $g_{\mathcal{R}|_S}(y) - 1 = g_{\mathcal{R}|_S}(z) = g_{\mathcal{R}|_S}(x)$. If $\gamma(y,z) = y$ then $g_{\mathcal{R}|_S}(z) = g_{\mathcal{R}|_S}(x) = g_{\mathcal{R}|_{S\setminus\{y\}}}(z) = g_{\mathcal{R}|_{S\setminus\{y\}}}(x) > g_{\mathcal{R}|_{S\setminus\{y\}}}(w)$ for all $w \in S\setminus\{y\}$, $w \neq x, z$, which is not possible by the induction hypothesis. If $\gamma(y,z) = z$ and $(z,y) \in \succ_1$ but $(z,y) \notin \succ_2$, then $g_{\mathcal{R}|_S}(x) - 1 = g_{\mathcal{R}|_{S\setminus\{y\}}}(x) = g_{\mathcal{R}|_{S\setminus\{y\}}}(z) \geq g_{\mathcal{R}|_{S\setminus\{y\}}}(w)$ for all $w \in S\setminus\{y\}$, which is not possible by the induction hypothesis. If $\gamma(y,z) = z$ and $(z,y) \succ_1 \cap \succ_2$, then for all $w \in S \setminus \{z\}$ and $\gamma(S \setminus \{w\}) \neq z$ it holds by the induction hypothesis that $\gamma(w,z) = z$ and $(z,w) \in \succ_1 \cap \succ_2$. Since also $\gamma(S \setminus \{v_z\}) = z$ only for one $v_z \neq x$, and $(z,y) \in \succ_1 \cap \succ_2$ it follows that $\gamma(S \setminus \{w\}) \neq x$ only for maximally two w. By $g_{\mathcal{R}|_S}(z) \geq g_{\mathcal{R}|_S}(x)$, z must be chosen from at least all binary comparisons with other elements from S, but one. This implies that $g_{\mathcal{R}|_{S\setminus\{w\}}}(z) \geq |S| - 2$ for all $w \in S\setminus\{z\}$ which in turn implies that $\gamma(x,w) = x$ for at least $\lceil \frac{|S|-2}{2} \rceil$ such $w \neq x$. Since $|S| \geq 5$, $\lceil \frac{|S|-2}{2} \rceil \geq \lceil \frac{5-2}{2} \rceil = \lceil \frac{3}{2} \rceil = 2$. By gradual expansion it follows that $g_{\mathcal{R}|_S}(z) = g_{\mathcal{R}|_S}(x)$ and that $\gamma(S) = x$.

Let $w \in S$ be such that $\gamma(S \setminus \{v\}) = x$ for all $v \in S \setminus \{w,z,x\}$. By $|S| = 5$, there exists at least two such v. Now, if $\gamma(S \setminus \{y\}) = z, y \in S \setminus \{x,z\}$, then by axiom A and weak WARP $\gamma(x,y) = x, \gamma(y,z) = y$, and $(x,y) \in \succ_1 \cap \succ_2, (z,y) \notin \succ_i$ for $i = 1,2$ and $(z,x) \in \succ_1$, but $(z,x) \notin \succ_2$. Hence, $g_{\mathcal{R}|_{\{x,y,z\}}}(x) = g_{\mathcal{R}|_{\{x,y,z\}}}(z) + 1$. By $g_{\mathcal{R}|_S}(z) \geq g_{\mathcal{R}|_S}(x)$, on the other hand, $g_{\mathcal{R}|_{\{z,v\}}}(z) > g_{\mathcal{R}|_{\{x,v\}}}(x)$. Hence, $g_{\mathcal{R}|_{S\setminus\{v\}}}(z) \geq g_{\mathcal{R}|_{S\setminus\{v\}}}(x)$ contradicting the induction hypothesis and $\gamma(S \setminus \{v\})$. $\square$

C Proof of Theorem 7.1

Proof. Necessity: Let γ be an RRCM with the set of reasonales $\mathcal{R}$ on X such that $\succ_1 \cap \succ_2 = \emptyset$ for all $\succ_1, \succ_2 \in \mathcal{R}$ with $\succ_1 \neq \succ_2$. First, we establish that $(x, y) \in \succ$ for some $\succ \in \mathcal{R}$ implies that $(y, x) \notin \succ$ for all $\succ \in \mathcal{R}$. Suppose otherwise, then $(x, y) \in \succ_1$ and $(y, x) \in \succ_2$ for some $\succ_1, \succ_2 \in \mathcal{R}$. Since $\succ_1 \cap \succ_2 = \emptyset$ for all $\succ_1, \succ_2 \in \mathcal{R}$, it holds that $(x, y) \notin \succ$ for all $\succ \in \mathcal{R} \setminus \{\succ_1\}$, and, analogously, that $(y, x) \notin \succ$ for all $\succ \in \mathcal{R} \setminus \{\succ_2\}$. So, $f_{\mathcal{R}|_{\{x,y\}}}(x) = f_{\mathcal{R}|_{\{x,y\}}}(y) = 0$ and from the definition of the RRCM it follows that $\gamma(x, y)$ is not an element of X which is a contradiction to $\gamma(.)$ being a function. A similar argument establishes that for all $x, y \in X$ either $(x, y) \in \succ$ for some $\succ \in \mathcal{R}$ or $(y, x) \in \succ$ for some $\succ \in \mathcal{R}$.

Second, we show that the set of reasonales is jointly transitive, i.e., if $(x, y) \in \succ_1$ and $(y, z) \in \succ_2$ for some $\succ_1, \succ_2 \in \mathcal{R}$, then $(x, z) \in \succ_3$ for some $\succ_3 \in \mathcal{R}$. Suppose, by contradiction that this is not true. Then by our reasoning above it follows that $(x, y) \in \succ_1$, $(y, z) \in \succ_2$ and $(z, x) \in \succ_3$ for some $\succ_1, \succ_2, \succ_3 \in \mathcal{R}$. This implies that $f_{\succ|_{\{x,y,z\}}}(w) = 0$, for all $w \in \{x, y, z\}$ and, thus, by the definition of the RCM it follows that $\gamma(\{x, y, z\})$ is not an element of the codomain X which is a contradiction to $\gamma(.)$ being a function. Hence, the set of reasonales is jointly complete and transitive.

Now, we show that RRCM implies WARP. Consider $S, T \in \mathcal{P}(X)$ with $x, y \in S \cap T$ and let $\gamma(S) = x$. Since the set of reasonales is jointly complete and transitive, it follows that $(x, z) \in \succ_{\{x,z\}}$, for all $z \in S \setminus \{x\}$ and some $\succ_{\{x,z\}} \in \mathcal{R}$. To see this, suppose, by contradiction that $(y, x) \in \succ$, for some $y \in S \setminus \{x\}$ and some $\succ \in \mathcal{R}$. Then the joint transitivity of $\mathcal{R}$ implies that for any $z \in S$ with $(x, z) \in \succ_{\{x,z\}}$ for some $\succ_{\{x,z\}} \in \mathcal{R}$ it also holds that $(y, z) \in \succ_{\{y,z\}}$ for some $\succ_{\{y,z\}} \in \mathcal{R}$. Hence, $g_{\mathcal{R}|_S}(y) > g_{\mathcal{R}|_S}(x)$ and, by the definition of an RCM, this contradicts $\gamma(S) = x$. So, $(x, z) \in \succ_{\{x,z\}}$, for all $z \in S \setminus \{x\}$ and some $\succ_{\{x,z\}} \in \mathcal{R}$. Now, joint transitivity of $\mathcal{R}$ also implies that for any $z \in T$ with $(y, z) \in \succ_{\{y,z\}}$ for some $\succ_{\{y,z\}} \in \mathcal{R}$, it follows that $(x, z) \in \succ_{\{x,z\}}$ for some $\succ_{\{x,z\}} \in \mathcal{R}$. Hence, $g_{\mathcal{R}|_T}(x) > g_{\mathcal{R}|_T}(y)$ and, thus $\gamma(T) = x \neq y$ by the definition of an RCM.

Now, we show that RRCM implies WARP. Consider $S, T \in \mathcal{P}(X)$ with $x, y \in S \cap T$ and let $\gamma(S) = x$. Since the reasonale is complete and transitive, it follows that $(x, z) \in \succ$, for all $z \in S \backslash \{x\}$. To see this, suppose, by contradiction that $(y, x) \in \succ$, for some $y \in S \backslash \{x\}$. Then transitivity of $\succ$ implies that for any $z \in S$ with $(x, z) \in \succ$ it also holds that $(y, z) \in \succ$. Hence, $f_{\succ|_S}(y) > f_{\succ|_S}(x)$ and, by the definition of an RRCM, this contradicts $\gamma(S) = x$. So, $(x, z) \in \succ$, for all $z \in S \backslash \{x\}$. Now, transitivity also implies that for any $z \in T$ with $(y, z) \in \succ$, it follows that $(x, z) \in \succ$. Hence, $f_{\succ|_T}(x) > f_{\succ|_T}(y)$ and, thus $\gamma(T) \neq y$ by the definition of an RRCM.

Sufficiency: Suppose that γ satisfies WARP. We construct the reasonale $\succ$ explicitly. Define $(x, y) \in \succ$ if and only if $\gamma(x, y) = x$. By this definition, $\succ$ is complete and transitive. To see that it is complete, suppose that both $(x, y) \notin \succ$ and $(y, x) \notin \succ$ for some $x, y \in X$. It follows that $\gamma(x, y) = \emptyset$, which, by the definition of a choice function, is not possible. To see that it is also transitive, suppose that $(x, y), (y, z), (z, x) \in \succ$, for some $x, y, z \in X$. By our definition of $\succ$ above this implies that $\gamma(x, y) = x$, $\gamma(y, z) = y$ and $\gamma(x, z) = z$ such that WARP is violated. This establishes that $\succ$ is complete and transitive.

Next, we define the counting function $f_{\succ|_S} : S \to \mathbb{N}$ for every set S by

$$f_{\succ|_S}(x) = \#\{y \in S \setminus \{x\} | (x, y) \in \succ\} - \#\{y \in S \setminus \{x\} | (y, x) \in \succ\}$$

and prove that $(\succ, f_\succ)$ rationalizes γ. Consider $S \in \mathcal{P}(X)$ and let $\gamma(S) = x$. Suppose, by contradiction that there exists $y \in S \backslash \{x\}$ such that $f_{\succ|_S}(y) \geq f_{\succ|_S}(x)$. This implies that $(x, y) \notin \succ$, by transitivity of $\succ$. Further, by completeness of $\succ$ it follows that $(y, x) \in \succ$. The definition of $\succ$ above then implies that $\gamma(x, y) = y$ such that WARP is violated. Hence, $\gamma(S) = x$ implies that $f_{\succ|_S}(x) > f_{\succ|_S}(y)$, for all $y \in S \backslash \{x\}$ such that $(\succ, f_\succ)$ rationalizes γ. $\square$

References

Apesteguia, J., and M. A. Ballester (2005): "Minimal Books of Rationales," ftp://ftp.econ.unavarra.es/pub/DocumentosTrab/DT0501.PDF.

——— (2013): "Choice by Sequential Procedures," *Games and Economic Behavior*, 77(1), 90–99.

Bhargava, S., and G. Loewenstein (2015): "Behavioral Economics and Public Policy 102: Beyond Nudging," *American Economic Review: Papers & Proceedings*, 105(5), 396–401.

Cherepanov, V., T. Feddersen, and A. Sandroni (2013): "Rationalization," *Theoretical Economics*, 8(3), 775–800.

Dhar, R., S. M. Nowlis, and S. J. Sherman (2000): "Trying Hard or Hardly Trying: An Analysis of Context Effects in Choice," *Journal of Consumer Psychology*, 9(4), 189–200.

Fox, C. R., and A. Tversky (1995): "Ambiguity Aversion and Comparative Ignorance," *The Quarterly Journal of Economics*, pp. 585–603.

Grether, D. M., and C. R. Plott (1979): "Economic Theory of Choice and the Preference Reversal Phenomenon," *American Economic Review*, 69(4), pp. 623–638.

Highhouse, S. (1996): "Context-Dependent Selection: The Effects of Decoy and Phantom Job Candidates," *Organizational Behavior and Human Decision Processes*, 65(1), 68–76.

Houy, N., and K. Tadenuma (2009): "Lexicographic Compositions of Multiple Criteria for Decision Making," *Journal of Economic Theory*, 144(4), 1770–1782.

Huber, J., J. W. Payne, and C. Puto (1982): "Adding Asymmetrically Dominated Alternatives: Violations of Regularity and the Similarity Hypothesis," *Journal of consumer research*, pp. 90–98.

JOHNSON, E. J., AND D. GOLDSTEIN (2003): "Do Defaults Save Lives?," *Science*, 302(5649), 1338–1339.

KALAI, G., A. RUBINSTEIN, AND R. SPIEGLER (2002): "Rationalizing Choice Functions by Multiple Rationales," *Econometrica*, 70(6), 2481–2488.

KOPS, C. (2016): "(F)Lexicographic Shortlist Method," Economic Theory, forthcoming.

LIST, J. A. (2007): "On the Interpretation of Giving in Dictator Games," *Journal of Political Economy*, 115(3), 482–493.

MANZINI, P., AND M. MARIOTTI (2007): "Sequentially Rationalizable Choice," *American Economic Review*, 97, 1824 – 1839.

——— (2012): "Categorize Then Choose: Boundedly Rational Choice and Welfare," *Journal of the European Economic Association*, 10(5), 1141–1165.

——— (2012b): "Choice by Lexicographic Semiorders," *Theoretical Economics*, 7(1), 1–23.

MASATLIOGLU, Y., D. NAKAJIMA, AND E. Y. OZBAY (2012): "Revealed Attention," *American Economic Review*, 102(5), 2183–2205.

RUBINSTEIN, A., AND Y. SALANT (2006): "A Model of Choice from Lists," *Theoretical Economics*, 1(1), 3–17.

SAVAGE, LEONARD, J. (1954): "The Foundations of Statistics," *NY, John Wiley*, pp. 188–190.

SCHWIEREN, C., AND U. GLUNK (2008): "Mechanisms Underlying Nationality-Based Discrimination in Teams," *Small Group Research*, 39(6), 643–672.

SHAFIR, E., I. SIMONSON, AND A. TVERSKY (1993): "Reason-based Choice," *Cognition*, 49(1), 11–36.

Simonson, I. (1989): "Choice Based on Reasons: The Case of Attraction and Compromise Effects," *Journal of consumer research*, pp. 158–174.

Simonson, I., and A. Tversky (1992): "Choice in Context: Tradeoff Contrast and Extremeness Aversion.," *Journal of marketing research*.

Tversky, A., and D. Kahneman (1991): "Loss Aversion in Riskless Choice: A Reference-dependent Model," *The Quarterly Journal of Economics*, 106(4), 1039–1061.

Tversky, A., and E. Shafir (1992): "The Disjunction Effect in Choice Under Uncertainty," *Psychological Science*, 3(5), 305–309.

Wilson, R. (1975): "On the Theory of Aggregation," *Journal of Economic Theory*, 10(1), 89–99.