Task Automation and Job Polarization*

(Preliminary draft - please do not cite/circulate)

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Abstract

I study the short-term and long-run effects of task automation when jobs consist of multiple tasks. Leveraging German survey data I show that task variety is ubiquitous in labor markets, and that computerization over the 1979-2018 period is associated with intra-occupational shifts away from lower-skill and routine task content. I explore the implications of task automation in a model that combines occupational assignment with a time allocation problem where workers must perform multiple tasks. The model predicts a reverse pattern of automation: low-skill tasks are automated first in high-skill occupations, where labor costs are higher. In the short-term this creates wage and employment polarization. In the long-run, low-skill automation has ambiguous implications for wage inequality and employment, with outcomes for low-skill workers generally improving as the costs of automation decrease. I test the model's short-run predictions against the historical time paths of computerization and occupational employment, and estimate a structural version of the model to obtain long-run predictions for German labor outcomes. Further declines in computer-related costs are expected to have little effect on wages, but to substantially increase employment in low- and middle-skill occupations.

1 Introduction

In both the popular and the academic literature, automation is associated with the wholesale replacement of human workers with machines. It is common to read predictions of robots "taking jobs".[1] This notion of one-to-one substitution has been formalized in theoretical

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[1] For example, "The Robots Are Coming For Phil in Accounting", The New York Times, March 6, 2021.

models of labor-substituting technology and in empirical studies that estimate occupation-level automation risk.[2] Historical examples of *job*-level automation are nevertheless difficult to find,[3] and case studies of technology adoption generally paint a far different picture, in which new technologies substitute for labor at particular tasks within jobs, changing but not eliminating the role of human labor.[4] Despite this, little is known about the macroeconomic implications of *task*-level automation when tasks are distinct from jobs.

In this paper, I study how labor markets are impacted by task automation when jobs consist of multiple tasks. I begin with an empirical analysis that draws on four decades of German survey data to show two motivating results. First, virtually all workers report performing a variety of tasks on the job, including both routine and non-routine work. Second, the most important technological shift over this time period - computerization - is associated with an intra-occupational increase in the proportion of time spent on non-routine tasks, consistent with the case study literature. To study the implications of task automation, I develop an assignment-based model of labor markets in which occupation-specific jobs involve multiple tasks, and workers solve a time allocation problem. Automation is formalized as the replacement of labor with capital at a single, low-skill task. In this environment, automation has multiple effects: a standard *labor-substitution* effect in which less labor is needed to produce a given amount of output, a *skill-complementarity* effect arising from the reallocation of workers' time towards more skilled task content, and a *demand* effect that depends on the responses of worker productivity and unit labor costs, together with the elasticity of substitution across occupations.

The model delivers several main predictions. In the short-run, when the automating technology is expensive, it will tend to be adopted only in skilled occupations for the reason that skilled labor has a comparative disadvantage at the unskilled task. The result is polarization of the wage and employment distributions: job losses from automation are concentrated around the "marginal" occupation where firms are indifferent between automating or not, which in turn drives polarization of wages. Post-adoption, employment in affected occupations recovers provided that occupations are not perfect complements, since declining capital rents tend to reduce unit labor costs and increase relative demand. In the long-run, after adoption is universal and as the cost of the automating technology approaches zero, employment in low-skill occupations rises and wage inequality falls; hence the effects of automation are less severe in the long-run than in the short-run.

In the final part of the paper I test the model's short-run predictions, and structurally

[2]Notable studies include Acemoglu and Autor (2011), Frey and Osborne (2017), and Acemoglu and Restrepo (2018).

[3]See Autor (2015) and Bessen (2016).

[4]Academic case studies include Levy and Murnane (1996), Fernandez (2001), and Autor, Levy, and Murnane (2002, 2003), while for an example from popular media see "How the World's Biggest Companies Are Fine-Tuning the Robot Revolution", The Wall Street Journal, May 14, 2018. Related are empirical studies on technological change and within-occupation task and skill requirements, such as Cappelli (1993), Spitz-Oener (2006), Bartel, Ichniowski, and Shaw (2007), Firpo, Fortin, and Lemieux (2011), and Atalay, Phongthiengtham, Sotelo, and Tannenbaum (2020).

estimate a continuous version of the model in order to obtain long-run forecasts. Consistent with model predictions, computerization in West Germany exhibited a top-down pattern, with significant PC adoption in low-skill occupations only occurring in the 2000's. PC adoption is associated with contemporaneous declines in occupational employment, while high rates of pre-existing PC use are predictive of occupational growth. The estimated structural model predicts that job losses in middle-skill occupations have peaked, as has the 90/50 wage ratio, with future computerization anticipated to drive wage inequality in the lower half of the wage distribution. In the very long-run, employment in high-skill occupations is anticipated to decline relative to current levels, and the 90/10 wage ratio to increase slightly. As a last exercise I use the quantitative model to explore implications for two commonly-studied elasticities. First, I show that the capital-labor elasticity of substitution is a complex function of rental rates and worker skill, and neither constant nor exogenous as is commonly assumed. Second, higher values of the labor-labor elasticity of substitution are associated with greater short-run (but not long-run) changes to occupational employment. This elasticity is therefore important for understanding the persistence of job loss from automation, and the efficacy of supply-side policies such as retraining programs.

The main contribution of this paper is to the macroeconomic literature on the effects of automation. Substitution of capital and labor is traditionally modeled in simple fashion as depending on the parameters of an aggregate production function, a tradition carried over to the case of automation by Zeira (1998). Subsequent work has also taken this approach, including Acemoglu and Autor (2011), Peretto and Seater (2013), Acemoglu and Restrepo (2018), Aghion, Jones, and Jones (2019), and Hemous and Olsen (2020). Theoretical models of capital-labor substitution yield a simple, negative relationship between technology adoption and employment, that has been used to generate forecasts for future disemployment as in Frey and Osborne (2017). Several strands of criticism exist. A number of authors have observed that history offers few examples of wholesale automation of jobs, which are rarely reducible to a single, automatable task.[5]. In addition, there have been significant within-occupation changes to task content that are difficult to explain through job-level automation.[6]. I reconcile these strands of literature with the notion of labor-substituting capital by relaxing the assumption of one-to-one substitution, and by providing both empirical evidence and theory on the automation of tasks at a sub-job level.

This paper also contributes to the literature on how occupational structure intermediates the effects of macroeconomic change. The framework developed here represents an intermediate point between models in which occupations consist of task bundles requiring task-specific skills, such as Gathmann and Schonberg (2010), Yamaguchi (2012), and Autor and Handel (2013), and models that abstract from occupational task structure and assign to each occu-

[5]For example Autor (2015), Bessen (2016), Arntz, Gregory, Zierahn (2017), and Dengler and Matthes (2018)

[6]See Levy and Murnane (1996), Fernandez (2001), Autor, Levy, and Murnane (2002, 2003), Spitz-Oener (2006), Bartel, Ichniowski, and Shaw (2007), Firpo, Fortin, and Lemieux (2011), and Atalay, Phongthiengtham, Sotelo, and Tannenbaum (2020)

pation an exogenous value (e.g. an index) describing the relationship between productivity and skill (or a skill composite), such as Costinot and Vogel (2010) and Acemoglu and Autor (2011). I am able to maintain the tractability in general equilibrium of the second class of models, while incorporating the occupational task structure studied by the first group. By explicitly modeling the allocation of workers' time across tasks, I am able to incorporate the time-reallocation effect often mentioned in anecdotal and case study accounts of automation, and to estimate task production shares from empirical task frequencies. This approach is more general than that taken in the task literature, where occupational output is usually constrained to be a linear function of task output.

The structure of this paper is as follows. In section 2 I show the main descriptive results: that virtually all workers report performing both routine and non-routine tasks, and computerization is associated with greater time spent on non-routine tasks. In section 3 I develop a model of task-based automation with multi-task occupations. I begin with a simple environment in which all jobs are homogeneous within an occupation, which in turn yields a set of precise analytical predictions. I then allow for intra-occupational heterogeneity, which weakens the analytical results but allows the model to be taken more directly to the data. Results on the qualitative model predictions are shown in section 5, and quantitative estimation and predictions are discussed in section 6.

2 Descriptive analysis

In this section I present motivating evidence on within-job task content. After describing the data and the task measures used in the analysis, I show two main facts. First, the vast majority of jobs involve both routine and non-routine tasks, indicating that "wholesale" automation of jobs is *ex ante* unlikely. Second, computerization of occupations is associated with a shift towards more non-routine task content, suggesting that *ex post*, technological automation has had an impact on the within-job distribution of task content. These facts are inconsistent with stylized models of automation that assume an equivalence between tasks and jobs, and they constitute evidence that technological change has interacted in a substantive way with the distribution of tasks within occupations, as well as between them.

2.1 Occupational routineness in the BIBB surveys

This analysis draws on the seven BIBB Employment Surveys, collected by the Federal Institute for Vocational Education and Training (BIBB) over the period 1979-2018 in partnership with the Institute for Employment Research (IAB, 1979-1999) and the Federal Institute for Occupational Health and Safety (2006-2018). Each survey draws on a random sample of the employed German labor force, and asks respondents a range of questions concerning job task content, the use of technology and tools, and other aspects of the job environment and the individual's work history. Also contained is information on monthly wage, which I convert to an hourly number using reported weekly hours.[7] The first two surveys do not contain data on East German workers and consequently, for comparability over time, I limit the analysis in this paper to West German workers between the ages of 18 and 65.

Table 2.1: Summary statistics for BIBB employment surveys, 1979-2018

	1979	1986	1992	1999	2006	2012	2018
Observations	28,595	26,090	23,940	27,371	15,905	16,330	16,699
with task data	27,709	25,859	23,830	27,229	15,893	16,257	16,617
with wage data*	26,474	22,631	21,198	20,840	13,607	13,230	13,952
PC use (%)	5.5	17.3	34.1	54.7	83.0	85.3	89.9
weighted	4.6	15.6	32.9	52.1	78.6	82.0	84.0
3-digit occupations	314	314	308	297/350	289/347	297	338

The detailed information contained in the BIBB surveys, together with the long time-frame over which they have been collected, makes them especially well-suited to research on the impact of technological change. Past examples of such research include Spitz-Oener (2006) and Bachmann, Cim, and Green (2019). There are nevertheless several challenges that must be addressed when using the BIBB surveys to study questions relating technology to labor markets. First, survey questions are often inconsistent across panels. For this project I rely on two sets of survey questions: those concerning PC use on the job, and those on task performance. Questions about PC use are broadly consistent over time, with one substantial change in format and wording occurring between 1992 and 1999. Questions regarding task content - for example, whether and how often workers "control machines", "advise others", and so forth - vary substantially across most survey years. For example, the 1979 survey considered more than 80 such tasks, while the 1999 survey contained only 13. Past research has relied on aggregation methods in order to obtain a smaller set of task categories that can be compared across time, but such methods are essentially *ad hoc* and Rohrbach-Shmidt and Tiemann (2013) show that different approaches can have a substantial impact on measures of occupational routineness.

A second issue is that job tasks do not map directly into conceptual frameworks like skillfulness or routineness, and researcher interpretation is therefore required when relating empirical results to hypotheses. This is especially problematic so when combined with the aggregation issues discussed above. Methods like factor analysis that provide a principled approach to dimensionality reduction are, by nature, unlikely to result in easily-interpreted task groups. Approaches based on intuition - combining tasks that "seem" similar - are perhaps more meaningful, but provide no guidance on precisely how tasks should be aggregated.

For both of these reasons I depart from past literature and limit attention to a set of survey questions about *task characteristics*. In all seven panels, and with minimal changes

[7]Wage data are highly aggregated for surveys prior to 2005/06, and consequently of limited usefulness in this study.

Table 2.2: Routine task content in the BIBB surveys

	Repeat tasks	Follow instructions	Adapt to new tasks	Improve procedures	Solve problems	Make decisions
Survey years						
1979-1992	.491	.654	.647	.455		
1999-2018	.504	.678	.694	.619	.799	.633
Education						
None	.530	.779	.585	.517	.583	.441
Vocational	.533	.767	.690	.619	.711	.587
University	.383	.546	.821	.752	.838	.766
Wage pct.						
1-25	.500	.780	.615	.554	.644	.490
26-50	.542	.749	.694	.624	.717	.586
51-75	.504	.703	.758	.680	.764	.655
76-100	.421	.586	.803	.732	.812	.750

Table notes. Survey answers re-coded as frequencies and means calculated using survey weights. Results by education and wage percentile are for the 2006 survey. See appendix for details.

in wording and response categories, respondents are asked how often they find themselves (1) repeating the same work process, (2) following detailed instructions, (3) adapting to new tasks, and (4) improving existing procedures or trying something new. These questions would seem to bear directly on the amount of routine task content present in the job, which is typically described in the literature as some combination of task repetition and the ability to codify task performance into a set of steps that a machine or computer might execute. I also consider questions from the 2006-2018 panels on whether respondents must (5) react to problems and solve them or (6) make difficult decisions on their own. These last three panels also feature a consistent set of questions regarding job tasks, for which I provide reference results in the appendix. Responses consist of between three and five verbal frequencies - e.g. "often" or "never" - which I interpret numerically and assign in even intervals to $[0, 1]$.

Table 2.2 shows conditional mean values after splitting the sample by year and, for the 2006 year survey, by educational attainment and wage quartile. Less education and a lower wage is associated as expected with greater task repetition and instruction, and less adaptive and cognitive content. Jobs do not appear less routine in later years, although there is a shift towards greater cognitive content that is likely the result of increased employment in professional and technical occupations. Comparisons with task content (see appendix) indicate that repetition and detailed instructions are associated with manual labor tasks, while non-routine characteristics are associated with tasks relating to analysis, instruction, advising, and sales and marketing. Notably, production-related tasks are associated with both routine and non-routine characteristics, suggesting a potentially complicated relationship between tasks and task characteristics like routineness.

2.2 Task variety within jobs

Are all tasks within a job equally susceptible to automation? In figure 2.3 I show, for each wage percentile, the distribution of survey responses for the 2006 survey[8]. Across all wage percentiles, the majority of respondents report performing both routine and non-routine tasks. Although higher-earning respondents perform non-routine content more frequently, and routine content less so, much of the variation in responses is within rather than between wage percentiles. In terms of task repetition, the 25th and 75th wage percentiles look similar; and even in the lowest quartile, three-quarters of respondents report that their job involves decision-making, problem-solving, and other task content that previous literature asserts is difficult to automate.

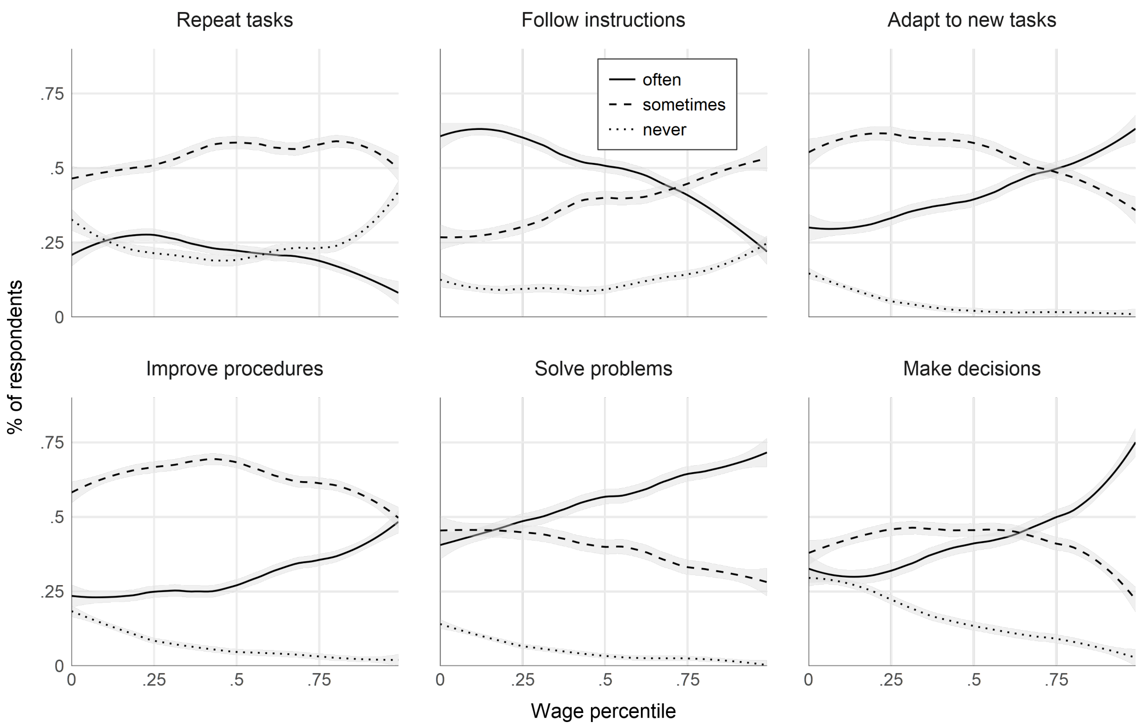

Figure 2.3: Routine and non-routine activity by wage percentile, 2006

Figure notes. LOESS interpolation of mean response by wage percentile, with gray regions indicating 95% confidence intervals.

Similar results can be shown for the *type* of tasks performed on the job, summarized in figure 2.4. Of the sixteen task categories present in the 2006 survey, four-fifths of respondents report performing at least 5. Of six broad task groups represented, four-fifths of respondents perform tasks in at least 3 of these groups. Similarly, when asked how often they must perform many different tasks at the same time, virtually all individuals report that this is at least sometimes true, and two-thirds that it is often the case. This and the previous figure

[8]This is the first year containing all of the measures discussed in the previous section, and also the first year in which wage data are not aggregated into coarse bins

offer a stark rebuttal to the assumption that jobs and tasks are equivalent. There exists a variety of tasks not just within a given occupation but within individual jobs, and in most cases this variety includes both routine and non-routine task content.

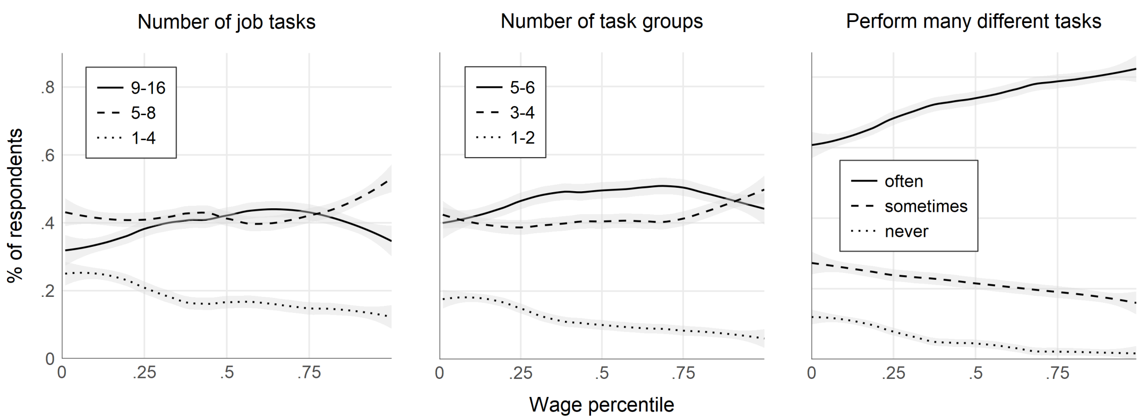

Figure 2.4: Task variety by wage percentile, 2006

FIGURE NOTES. LOESS interpolation of mean response by wage percentile, with gray regions indicating 95% confidence intervals. See appendix for description of tasks and task groups.

One notable aspect of the survey responses shown in figure 2.3 is the lack of any discernible discernible hump-shape in measures of routineness. Although Germany, like the United States, has experienced labor market polarization[9], and employment has generally declined in "middle-skill" jobs, there is little evidence in the BIBB surveys that jobs in the middle of the wage distribution involve greater routine task content. Measures of routineness peak in the first quartile, while measures of non-routine task content are generally increasing in wage. If routineness were the *only* predictor of employment effects from technology adoption, then it is difficult to avoid the conclusion that disemployment should be greatest at the bottom of the wage distribution. More broadly, these results would seem to be inconsistent with theories that pose a simple relationship between task content and automation.

2.3 Computerization and task content

If automation is task-specific and jobs vary in their task content, then we would expect automation to affect the within-job distribution of tasks. A simple test of this hypothesis is to see whether technology adoption is associated with changes in the overall routineness of job tasks. As my measure of technological automation I focus on the use of personal computers in the workplace. A large body of literature argues that computerization reduces time spent on routine tasks, and hence we should observe less frequent routine task content in this case. I begin with a cross-sectional analysis. In table 2.5 I show the marginal effect of computerization on task characteristics, estimated by performing fractional logit regressions of task content on computer use. All regressions control for 3-digit occupation, 1-digit

[9]See e.g. Dustmann, Ludsteck, and Schonberg (2007) and Bachmann, Cim, and Green (2019).

industry, and year. Because the wording and format of the survey question on computer use changes between 1992 and 1999, I split the sample into two subsamples covering the 1979-1992 and 1999-2018 periods. Conditional on the type of work being done, computerization is associated with relatively less routine task content, and relatively more non-routine content. This result holds within educational groups and wage quantiles, as well as across panels.

Table 2.5: Task characteristics and PC use, 1979-2018

Sample	*Dependent variable*					
	Repeat tasks	Follow instructions	Adapt to new tasks	Improve procedures	Solve problems	Make decisions
Survey years 1979 - 1992						
Full sample	**-.054** (.005)	**-.046** (.005)	**.103** (.004)	**.094** (.004)		
Education						
None	**-.043** (.021)	**-.043** (.016)	**.147** (.019)	**.107** (.017)		
Vocational	**-.038** (.006)	**-.033** (.006)	**.098** (.005)	**.093** (.005)		
University	**-.039** (.009)	**-.039** (.010)	**.051** (.007)	**.062** (.008)		
Wage pct.						
1-25	-.024 (.012)	-.016 (.011)	**.099** (.011)	**.072** (.010)		
26-50	**-.046** (.011)	-.011 (.010)	**.096** (.010)	**.071** (.009)		
51-75	**-.034** (.010)	**-.036** (.009)	**.080** (.009)	**.072** (.009)		
76-100	**-.044** (.009)	**-.056** (.010)	**.064** (.007)	**.084** (.008)		
Survey years 1999 - 2018						
Full sample	**-.013** (.006)	**-.045** (.006)	**.102** (.004)	**.116** (.005)	**.098** (.005)	**.145** (.007)
Education						
None	.009 (.017)	-.016 (.016)	**.119** (.014)	**.130** (.014)	**.128** (.018)	**.162** (.020)
Vocational	.003 (.007)	**-.039** (.006)	**.098** (.005)	**.102** (.006)	**.098** (.006)	**.133** (.008)
University	-.020 (.013)	**-.041** (.015)	**.090** (.008)	**.103** (.010)	**.082** (.009)	**.147** (.018)
Wage pct.						
1-25	.004 (.012)	**-.032** (.010)	**.107** (.009)	**.102** (.009)	**.089** (.011)	**.134** (.013)
26-50	-.005 (.012)	-.016 (.011)	**.079** (.009)	**.083** (.010)	**.79** (.011)	**.082** (.014)
51-75	-.025 (.013)	**-.058** (.013)	**.077** (.010)	**.088** (.010)	**.086** (.013)	**.095** (.018)
76-100	-.030 (.019)	**-.050** (.021)	**.100** (.012)	**.121** (.014)	**.068** (.017)	**.165** (.025)

TABLE NOTES. Marginal effects and robust standard errors from fractional logit regressions of task characteristics on PC use, aggregated by 3-digit occupation. All regressions include dummies for year, 3-digit occupation, 1-digit industry. Bold results indicate 95% significance. See appendix for details.

To better the answer the question of how *changes* in computer use and task content are related, I aggregate survey responses by 3-digit occupation and perform a series of difference-in-difference regressions, shown in figure 2.6. Regressions are performed for consecutive panels in order to minimize confounding from time-variation in other variables that may affect task content; results for 1992-1999 are shown, but indicated in italics due to the change in question format. The results are comparable to those shown previously: when significant, the coefficients on routine task content are negative and those on non-routine content are positive. Marginal effects are large, and indicate that a 10% increase in occupational computer use is associated with a 1-2% change in task frequencies. Note that even if the presumed direction of causality is correct - i.e. from computerization to task frequencies - there are two ways in which this effect might occur. Task content may change within individual jobs[10], or employment may substitute away from jobs in the same occupation but with less routine task content. Either story would nevetheless yield a similar implication: automation is unlikely to replace occupational labor wholesale, and employment outcomes would depend both on the degree to which labor is automated, and on any changes to occupational productivity.

Table 2.6: Occupation mean task characteristics and PC use (two-way FE)

Sample	*Dependent variable*					
	Repeat tasks	Follow instructions	Adapt to new tasks	Improve procedures	Solve problems	Make decisions
KLDB 1988						
1979-1986	-.049 (.034)	**-.077** (.035)	*-.022 (.052)*	**.073** (.036)		
1986-1992	**-.220** (.024)	**-.069** (.033)	**.072** (.027)	.004 (.027)		
1992-1999	*.046 (.030)*	*.028 (.028)*	*-.025 (.025)*	***.069** (.026)*		
1999-2006	**-.118** (.031)	**-.106** (.030)	**.111** (.019)	**.164** (.023)		
KLDB 1992						
1999-2006	**-.077** (.028)	**-.086** (.025)	**.114** (.018)	**.144** (.020)		
2006-2012	-.018 (.036)	-.037 (.033)	**.053** (.025)	**.078** (.026)	**.095** (.023)	**.176** (.033)
2012-2018	-.068 (.040)	-.051 (.042)	.053 (.030)	**.152** (.030)	**.064** (.029)	**.161** (.040)

TABLE NOTES. Marginal effects and robust standard errors from fractional logit regressions of task characteristics on PC use, aggregated by 3-digit occupation. All regressions include occupation and year dummies. Bold results indicate 95% significance, and italic results a change in change in survey wording. See appendix for details.

These findings, like those in the previous section, are difficult to reconcile with standard notions of automation as the wholesale substitution of capital for labor. Past authors have noted that it is difficult to find examples of jobs (i.e. occupations) eliminated by automation[11]. The results shown in this section indicate that it is difficult to find examples of jobs that *could* be entirely automated. The technological change most studied in the literature - widespread adoption of the PC - is associated with changes in occupational task content, indicating a reallocation of labor across tasks within occupations, and not just between occupations. These facts form the basis for the model developed in the next section.

[10] This would be consistent with case studies such as Autor, Levy, and Murnane (2002).
[11] Examples include Bessen (2016) and Autor (2015).

3 A model of partial automation

In this section I develop a model of automation where jobs consist of multiple tasks. As is common in the literature, automation is formalized as the substitution of capital for labor at a particular task. In contrast to one job-one task models like Acemoglu and Autor (2011), however, the effects of automation are not limited to a reduction in the demand for labor. Automation will also affect workers' time allocation across tasks, and hence occupational returns to skill. And because automation will tend to reduce unit costs within a given occupation, the overall effect on employment is ambiguous, and will depend on the relative magnitudes of countervailing substitution effects.

3.1 Environment

The environment is static and consists of workers and firms. Workers are heterogeneous in a continuous skill variable $s \in [0,1]$ with distribution $F(s)$. Intermediate good producers are heterogeneous in a continuous variable $j \in [0,1]$ indicating the type of labor output they produce. Here, j is interpreted as referring to an "occupation", with intermediate producers acting as aggregators of occupation-specific labor. The output of j-producers is in turn aggregated by final good producers that require no labor input.

Occupational output. Workers in a j-firm produce labor output by allocating a unit of time between a low-skill and a high-skill task, denoted l and h. Task output depends on worker skill, and is given by the functions $\gamma_l(s)$ and $\gamma_h(s)$. I assume that there are no agency problems: workers choose their time allocation so as to maximize firm output. The unskilled task may be partially automated, in which case a portion κ or less of task output may be performed by capital at a per-unit rental cost of r. Tasks are assumed to be perfect complements, with worker output given by the Leontief output function

$$y(j,s,K) = \min\left\{\frac{t\gamma_h(s)}{\alpha(j)}, \frac{(1-t)\gamma_l(s)+K_l}{1-\alpha(j)}\right\} .$$

In this equation t is the proportion of the worker's time allocated to the skilled task, and K_l the (per-worker) capital allocated to the unskilled task. The automation feasibility constraint implies that if K total capital is provided to the worker, then

$$K_l = \min\left\{K, \frac{\kappa}{1-\kappa}[1-t]\gamma_l(s)\right\} .$$

The task share function $\alpha(j) > 0$ is assumed to be continuously differentiable and to lie strictly between 0 and 1 for all j, implying that there are no jobs consisting of a single task. Wages are denoted $w(s)$ and intermediate output prices $p(j)$. Per-worker profits of intermediate good producers can be written as

$$\pi_i(j, s, K) = p(j)y(j, s, K) - w(s) - rK \ .$$

I assume there is free entry of producers, and for now I abstract from producer scale.

Aggregate technology. Final good producers aggregate j-output into a consumption good using the technology

$$Y = \left(\int \beta(j) Y(j)^{\frac{\rho-1}{\rho}} dj \right)^{\frac{\rho}{\rho-1}} \ .$$

I assume that production of the final good requires only j-inputs, and normalize the price of the final good to 1. Given these assumptions, the profit of the representative final good producer will be

$$\pi_f = Y - \int p(j)Y(j)dj \ .$$

Final good markets are assumed to be perfectly competitive.

Agents' problems. I now formalize the problems of workers and producers. Beginning with workers, the time spent on tasks is chosen to maximize output, given assigned capital K_l:

$$\max_{t \in [0,1]} \ \min \left\{ \frac{t\gamma_h(s)}{\alpha(j)}, \frac{(1-t)\gamma_l(s) + K_l}{1-\alpha(j)} \right\} \ , \tag{1}$$

where K_l is subject to the feasibility constraint above. Intermediate good producers choose worker skill s and per-worker capital K in order to maximize profits:

$$\max_{K \geq 0, s \in [\underline{s}, \overline{s}]} \ p(j)y^*(j, s, K) - w(s) - rK \tag{2}$$
$$\text{s.t. } y^*(j, s, K) \text{ solves (1)} \ .$$

Final good producers then choose the mix of j-goods that maximizes profits:

$$\max_{Y(j)} \left(\int \beta(j) Y(j)^{\frac{\rho-1}{\rho}} dj \right)^{\frac{\rho}{\rho-1}} - \int p(j)Y(j)dj \ . \tag{3}$$

Note that because capital costs are fixed when workers choose their time allocation, no inefficiency is introduced by separating the worker's time allocation problem from the intermediate producer's choice of capital.

3.2 Equilibrium

The worker's time allocation problem (1) yields two solutions, depending on whether the technological constrain $(1-\kappa)K \leq \kappa[1-t]\gamma_l(s)$ is binding:

$$t^*(j,s,K) = \begin{cases} \frac{\frac{\alpha(j)}{\gamma_h(s)}\left[1+\frac{K}{\gamma_l(s)}\right]}{\frac{\alpha(j)}{\gamma_h(s)}+\frac{1-\alpha(j)}{\gamma_l(s)}} & K \leq \frac{\kappa\left(1-\alpha(j)\right)}{\frac{\alpha(j)}{\gamma_h(s)}+\frac{1-\alpha(j)}{\gamma_l(s)}} \\ \frac{\frac{\alpha(j)}{\gamma_h(s)}}{\frac{\alpha(j)}{\gamma_h(s)}+(1-\kappa)\frac{1-\alpha(j)}{\gamma_l(s)}} & K > \frac{\kappa\left(1-\alpha(j)\right)}{\frac{\alpha(j)}{\gamma_h(s)}+\frac{1-\alpha(j)}{\gamma_l(s)}} \end{cases} .$$

Both costs and output are linear in capital, and so profit maximization by producers (2) will entail a corner solution: producers will choose either $K = 0$ or $(1-\kappa)K = \kappa[1-t]\gamma_l(s)$, depending on whether it is cheaper to hire additional labor or to automate the low-skill task. Formally,

$$K^*(j,s) = \begin{cases} 0 & w(s) \leq r\gamma_l(s) \\ \frac{\kappa\left(1-\alpha(j)\right)}{\frac{\alpha(j)}{\gamma_h(s)}+(1-\kappa)\frac{1-\alpha(j)}{\gamma_l(s)}} & w(s) > r\gamma_l(s) \end{cases} . \tag{4}$$

On the other hand the first-order condition for worker skill implies that in equilibrium we must have

$$w'(s) = \begin{cases} \frac{d}{ds}\left(\frac{\alpha(j)}{\gamma_h(s)}+\frac{1-\alpha(j)}{\gamma_l(s)}\right)^{-1} & w(s) \leq r\gamma_l(s) \\ \frac{d}{ds}\left(\frac{\alpha(j)}{\gamma_h(s)}+(1-\kappa)\frac{1-\alpha(j)}{\gamma_l(s)}\right)^{-1} & w(s) > r\gamma_l(s) \end{cases} .$$

The optimal choices of s and K will depend on the wage function. This is a potential problem, as it may not be possible to characterize the optimal assignment (and hence the wage function) without knowing producers' automation decisions. Nevertheless I show below that in this simple environment, K^* can be characterized *ex ante* to a degree sufficient for the optimal assignment and wage functions to be characterized and solved numerically without difficulty.

The final good producer's problem yields the first-order condition

$$Y(j) = \left(\frac{\beta(j)}{p(j)}\right)^{\rho} Y .$$

Free entry, on the other hand, has the implication that $p(j) = \frac{w(j)+rK(j)}{y(j)}$. Hence we can write total j-employment as

$$L(j,s) = \frac{y^*(j,s,K)^{\rho-1}}{\left(w(s)+rK^*(j,s)\right)^{\rho}}\beta(j)^{\rho}Y .$$

Fixing skill and wages, automation will increase labor demand whenever

$$\left(\frac{\frac{\alpha(j)}{\gamma_h(s)}+\frac{1-\alpha(j)}{\gamma_l(s)}}{\frac{\alpha(j)}{\gamma_h(s)}+(1-\kappa)\frac{1-\alpha(j)}{\gamma_l(s)}}\right)^{1-\rho} > \left(\frac{w(s)\left(\frac{\alpha(j)}{\gamma_h(s)}+(1-\kappa)\frac{1-\alpha(j)}{\gamma_l(s)}\right)+r(1-\alpha(j))\kappa}{w(s)\left(\frac{\alpha(j)}{\gamma_h(s)}+(1-\kappa)\frac{1-\alpha(j)}{\gamma_l(s)}\right)}\right)^{\rho}. \tag{5}$$

The right-hand side of (5) will always be greater than one, and hence for employment to increase it must be that automation decreases unit output costs, and that ρ is sufficiently large. Unit output costs will decrease by a greater amount when $\alpha(j)$ is large but automation has a large effect on labor productivity, e.g. when workers spend a large amount of time on the unskilled task because they have a comparative disadvantage at that task. Note, however, that $w(s)$ will be endogenous to r in equilibrium and so a critical factor in determining the employment effects of automation is the response of the wage function. On the other hand the capacity for automation κ will inform the overall magnitude of the employment effect. This is a departure from past, well-known models in that automation does not necessarily reduce employment, and the set of workers affected is an endogenous outcome depending on relative costs.

Optimal assignment. I now turn to the optimal assignment of workers to jobs, which will only be well-defined when different types of workers enjoy a comparative advantage at different types of jobs. I therefore impose the condition:

Assumption 1: $\frac{d}{ds}\log\gamma_h(s) > \frac{d}{ds}\log\gamma_l(s) > 0$ *and* $\alpha'(j) > 0$.

Task output is increasing in skill, and this increase is proportionally greater for skill-intensive tasks. That $\alpha(j)$ is increasing implies that jobs are ordered in terms of their return to skill, from which we can predict that in equilibrium, higher j will be associated with higher s. In order for the optimal assignment to be a smooth function, I further impose the conditions that $\beta(j)$ and $F(s)$ are continuously differentiable, and that $F(s)$ is a strictly increasing function. With these restrictions, the model can be shown to possess two properties that allow automation decisions (i.e. K^*) to be characterized separately from the wage function, allowing for a precise characterization of the equilibrium. Denoting $\overline{K}$ to be the optimal level of capital conditional on automation, the model satisfies the following conditions:

1. **rank-preserving**: for any two occupations j and j', if $\frac{d}{ds}\log y^*(j',s,0) > \frac{d}{ds}\log y^*(j,s,0)$ then $\frac{d}{ds}\log\left[p(j)y^*(j',s,\overline{K})-r\overline{K}\right] > \frac{d}{ds}\log\left[p(j)y^*(j,s,\overline{K})-r\overline{K}\right]$
2. **bias-consistent**: for all j and all s we have $\frac{d}{ds}\log y^*(j,s,0) > \frac{d}{ds}\log\gamma_l(s)$

Under the first property, the ordering of occupations from least to most skill-intensive is unchanged by automation, which greatly facilitates characterization of the optimal assignment. The second property implies that automation either everywhere increases the occupational return to skill, or everywhere decreases it, which simplifies the equilibrium pattern of automation across producers and allows it to be precisely characterized independent of the optimal assignment. These properties are satisfied in the present case due to the assumption that there are only two tasks; in an environment with three or more, they would require additional restrictions on worker productivity and task shares.

Now let $\lambda : s \to j$ indicate the set of jobs at which at least one s-worker is employed. Given assumption 1, λ will be a strictly increasing, piece-wise differentiable function. There will exist a unique s^* separating automated and non-automated labor types, where s^* may take an interior value or, in the cases where no or all employers automate, may be equal to $\underline{s}$ or $\overline{s}$. The wage and market tightness functions will satisfy the differential equations

$$\frac{w'(s)}{w(s)} = \begin{cases} \frac{d}{ds} \log y\big(\lambda(s), s, 0\big) & s < s^* \\ \frac{d}{ds} \log y\big(\lambda(s), s, \overline{K}\big) & s > s^* \end{cases} \tag{6}$$

$$\lambda'(s) = \begin{cases} \frac{y\left(\lambda(s),s,0\right)^{1-\rho} F'(s)}{\beta\left(\lambda(s)\right)^{\rho} Y} w(s)^{\rho} & s < s^* \\ \frac{y\left(\lambda(s),s,\overline{K}\right)^{1-\rho} F'(s)}{\beta\left(\lambda(s)\right)^{\rho} Y} \Big(w(s) + \kappa\big[1-\alpha(\lambda(s))\big] r y\big(\lambda(s), s, \overline{K}\big)\Big)^{\rho} & s > s^* \end{cases}. \tag{7}$$

where $\lambda(\underline{s}) = 0$ and $\lambda(\overline{s}) = 1$. If s^* lies in the interior of $[0, 1]$ then the wage and matching functions will be continuous but not differentiable at s^*, and continuously differentiable for all $s \neq s^*$.

All together, equilibrium in this environment is time and capital allocations t^* solving (1)-(2), intermediate good bundles $Y^*(j)$ satisfying the final good producer's problem (3), prices $p(j)$ such that zero-profit conditions hold and goods markets clear, and wage and matching functions satisfying the system of differential equations (6)-(7).

3.3 Automation, wages, and employment

In this section I characterize the short-run and long-run effects of automation, where (in this static setting) I interpret the short-run as describing scenarios where r is sufficiently large that $s^* > \underline{s}$, and the long-run corresponding to the case where $r \to 0$.

The first property of the model requires no additional proof and is shown in the system of differential equations (6)-(7): in the short-run, the automating technology is only adopted in high-skill jobs. Skilled labor has a comparative advantage at the skilled task, which in turn implies that it is more costly to have skilled workers performing the low-skill task. Automation therefore reduces unit costs by a greater amount when the worker is skilled. Declines in r will tend to lower the automation threshold s^* and, with $w(\underline{s}) > 0$, it is evident that for sufficiently low rental rates we will have $s^* = \underline{s}$ and all jobs will adopt the technology.

A second property is that in the short-run where $s^* > 0$, automation will tend to polarize the wage and employment distributions:

Theorem 1 (short-run polarization). *For $s^* \in (0, 1)$, low-skill employment $\int_0^{\lambda(s^*)} L(j)dj$ is greater under automation. If ρ is sufficiently small and whenever $\rho < 1$, there will ex-*

ist $j' > \lambda(s^*)$ such that $\int_{j'}^{\infty} L(j)dj / \int_{\lambda^{-1}(s^*)}^{j'} L(j)dj$ is greater under automation. Moreover for $s < s^*$ we will have $w'(s)/w(s)$ strictly smaller under automation, whereas for $s \in (s', \overline{s}]$ for s' sufficiently large we will have $w'(s')/w(s')$ greater under automation.

For low values of ρ disemployment effects will be strongest among automated jobs 'close' to the threshold skill level s^*, while a smaller share in the unskilled task and greater reductions in unit costs result in better employment outcomes for skilled jobs close to $\overline{s}$. As ρ becomes large, the short-run effects on employment are more difficult to characterize because changes to the wage function will have a larger impact on labor demand. With respect to wages, the immediate effect of automation is that low-skill workers shift towards lower-skill jobs, reducing $w'(s)/w(s)$ in the bottom part of the wage distribution. Workers at the upper end of the wage distribution see an increase in $w'(s)/w(s)$ as, even if $\lambda(s)$ shifts downwards (i.e. employment falls in skilled jobs), this will be more than compensated for by the increased return to skill at automated jobs. Assumption 1 and continuity then predict the existence of a region for which $w'(s)/w(s)$ is increasing, with the lower bound of this regional potentially equal to s^* but in all cases smaller than $\overline{s}$.

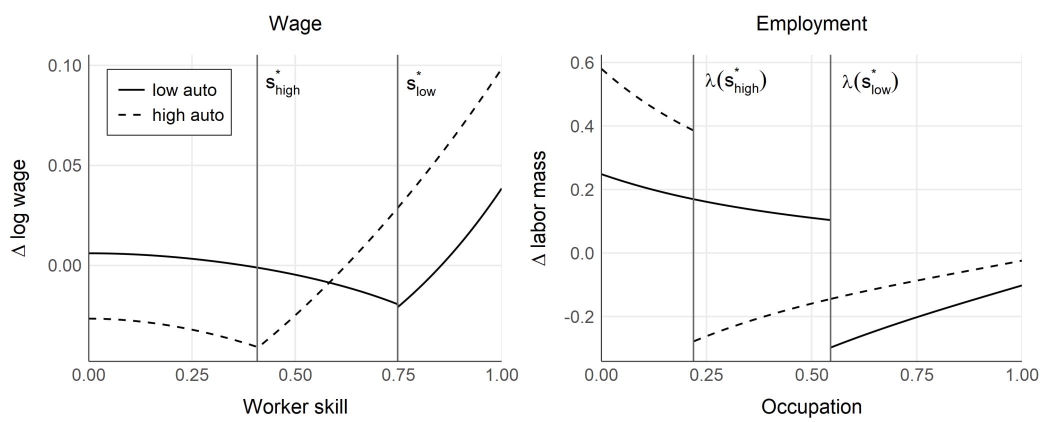

Figure 3.1: Short-run effects of automation

Figure notes. Linear task shares where $\gamma_l(s) = \exp(.5s)$, $\gamma_h(s) = \exp(s)$, $\kappa = .5$, and $\rho = .5$. Changes are relative to the case where $s^* = \overline{s}$; wages are demeaned prior to calculating change.

Long-run effects of automation will depend critically on the behavior of the rental rate r, the magnitude of the elasticity of substitution ρ, and on how $t^*(j, s, K)$ varies with s. I impose the condition, consistent with the empirical results shown previously, that t^* is increasing in s.

Assumption 2: *$t^*(j, s, K)$ is increasing in s.*

A general characterization is not possible due to the lack of a closed-form solution to (6) and (7), but results can be derived for the two cases $\rho \in \{0, 1\}$, which are helpful for developing intuition as to the implications of automation in this environment. First I define

$\overline{r} = \sup\{r|s^* = 0\}$ to be the highest rental rate consistent with 'full' automation. In the comparatively simple case where $\rho = 0$, declines in the rental rate below $\overline{r}$ will have no effect on labor demand or on wages and we can show that:

Theorem 2 (long-run effects of automation: $\rho = 0$). *If $r \le \overline{r}$ and $\rho = 0$, then relative to the case where $r \to +\infty$, $w'(s)/w(s)$ will be greater for all s and for any $j' \in (0,1)$ we will have $\int_0^{j'} L(j)ds$ smaller. In this case the wage and matching functions will be independent of $r \in [0, \overline{r}]$.*

The perfect complements case yields an intuitive outcome: low-skill automation reduces employment at low-skill jobs and increases wage inequality. This is consistent with standard notions of labor-substituting automation, and notwithstanding the distinction between short- and long-run effects of automation, this may be said to be the case where the model's predictions are closest to those of the one task-one job environment studied in the extant literature.

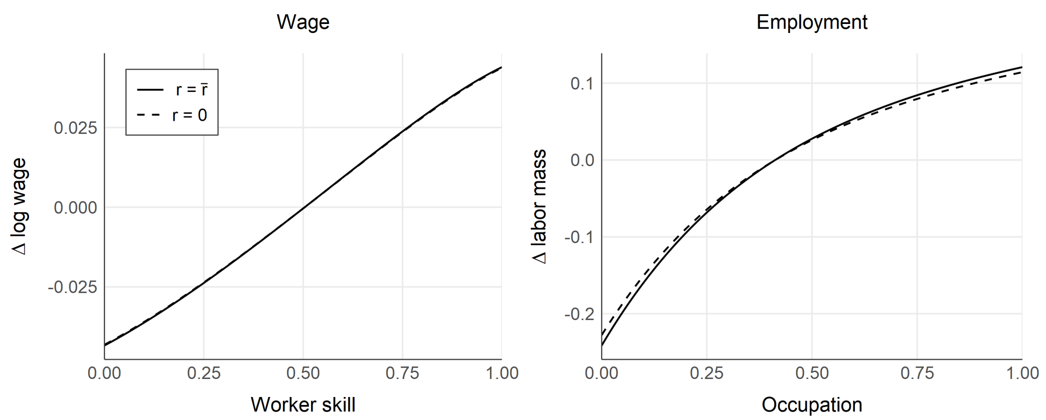

Figure 3.2: Long-run effects of automation, $\rho = 0$

FIGURE NOTES. Linear task shares where $\gamma_l(s) = \exp(.5s)$, $\gamma_h(s) = \exp(s)$, $\kappa = .5$, and $\rho \approx 0$. Changes are relative to the case where $s^* = \overline{s}$; wages are demeaned prior to calculating change.

Long-run effects of automation become considerably more complicated as we move away from the limiting case where $\rho = 0$, because there are two offsetting effects. On the one hand automation increases the intensity of the skilled task and therefore tends to raise wages of skilled workers, leading to substitution away from skilled occupations. On the other hand the cost of automation is smaller relative to wage costs for higher s, resulting in substitution in the opposite direction. The result is that while wage inequality will tend to increase overall, the implications for employment are ambiguous.

Theorem 3 (long-run effects of automation: $\rho = 1$). *If $r \le \overline{r}$ and $\rho = 1$, then for any s we will have $w(s)/w(\underline{s})$ greater under automation. If $r = 0$ then there will exist a $j' > 0$ such that $\int_0^{j'} L(j)ds$ is also greater under automation. Finally, comparing* *any two rental rates $r' < r''$, we will have $w'(s)/w(s)$ smaller under r' and, for any $j' \in (0,1)$ we will have $\int_0^{j'} L(j)ds$ larger.*

When $r = \overline{r}$ the effect of automation on employment is unclear, but further declines in r will tend to both reduce wage inequality and increase low-skill employment, and as r goes to zero the overall effect will be to raise employment in low-skill occupations. Hence technological disemployment will depend critically on the long-run behavior of r and the value of ρ.

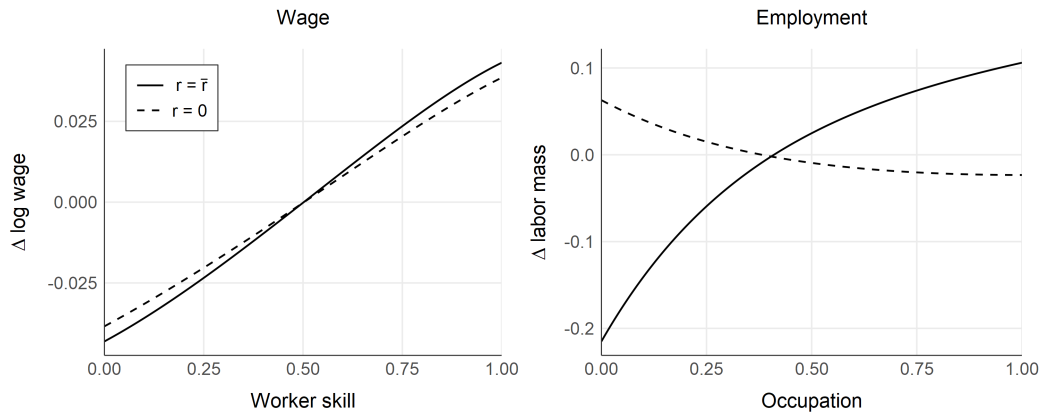

Figure 3.3: Long-run effects of automation, $\rho = 1$

FIGURE NOTES. Linear task shares where $\gamma_l(s) = \exp(.5s)$, $\gamma_h(s) = \exp(s)$, $\kappa = .5$, and $\rho = 1$. Changes are relative to the case where $s^* = \overline{s}$; wages are demeaned prior to calculating change.

For values of ρ greater than one characterization of the wage and matching functions becomes difficult. In general, larger values of ρ and smaller values of r will be associated with greater compression of the wage distribution and more employment in low-skill occupations. The extent to which automation affects wages rather than prices will also depend on the functional forms of γ_l and γ_h, which together with task shares α determine how easy it is to substitute one skill type for another at a given job, and hence the response of occupational labor supply.

3.4 Continuous model

The model developed thus far is of limited quantitative usefulness, as empirical adoption patterns are not binary at the occupation level, and consequently there is no empirical analogue of the automation threshold s^*. Intra-occupational heterogeneity must be taken into account if the model is to yield quantitatively meaningful predictions. Adding such heterogeneity will tend to weaken the theoretical results shown in the previous section, but they will hold *approximately* so long as the proportion of jobs automated is increasing in j. In this section I allow for a simple form of intra-occupation heterogeneity, and derive the equilibrium for a continuous model that will form the basis for the quantitative results shown below.

I assume that producers face idiosyncratic shocks ϵ to their rental cost, with total costs equal to ϵr per unit of capital. I further assume that these shocks are realized only after hiring labor, a simplification which has the implication that all j-firms will continue to hire the same worker type. I assume that ϵ is drawn from a continuously differentiable distribution $G(\epsilon)$, and that producers are constrained to a unit of output, so that $G(\epsilon)$ also gives the proportion of output attributable to producers with costs ϵ or lower[12]. I assume that workers are paid wages upon hiring and hence, firms have no incentive to leave the market upon discovering their rental costs as they can always choose not to automate and still receive positive revenue. Capital costs paid by automating firms will then be

$$\epsilon r K^*(j,s,z) = \kappa \frac{1-\alpha(j)}{z} y^*(j,s,\overline{K}) \ ,$$

and producers will automate whenever

$$w(s) > \epsilon r \gamma_l(s) \ .$$

I assume that G has full support over $\mathbb{R}^{++}$, in which case for each s there will exist a an $\epsilon^*(s)$ such that producers employing s automate whenever $\epsilon > \epsilon^*(s)$. The policy functions and optimal assignment are similar to before and so I provide them in the appendix but omit them here.

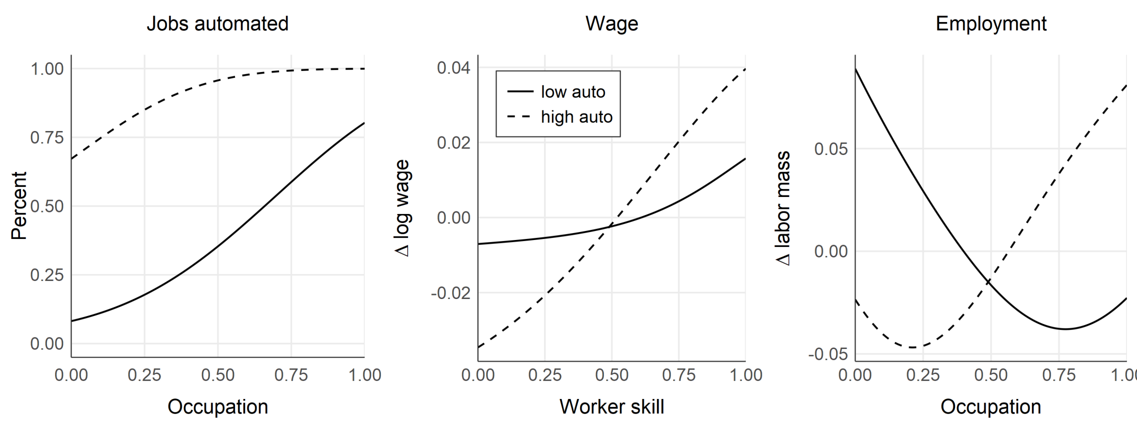

Figure 3.4: Short-run effects of automation, continuous model

Figure notes. Linear task shares where $\gamma_l(s) = \exp(.5s)$, $\gamma_h(s) = \exp(s)$, $\kappa = .5$, $\rho = .5$, and ϵ log-linear with mean 0 and standard deviation .1. Changes are relative to the case where $s^* = \overline{s}$; wages are demeaned prior to calculating change.

Although equilibrium in the continuous model is more difficult to characterize, two general predictions are preserved from static case. The first prediction is that $\epsilon^*(s)$ is increasing: a larger proportion of jobs will be automated in high-skill occupations. The reason for this

[12]Because firms will now enjoy greater or lesser rents depending on the value of ϵ, some restriction on scale is required for the equilibrium to be well-defined

is the same as before, that skilled workers have a comparative advantage at skilled tasks and therefore it is more costly when they must spend time on the unskilled task. The second prediction is that, holding r fixed, an increase in the automation threshold ϵ^* will tend to reduce occupational employment; whereas holding ϵ^* fixed, a decrease in r will tend to increase employment in occupations with a high ϵ^*. A lower cost of capital will tend to reduce expected costs in highly automated occupations and incentivize job creation, whereas an increase in the proportion of jobs being automated will result in less demand for labor. Hence as before, the effect of a change in r on j-employment will depend on the relative magnitude of these two effects.

4 Qualitative analysis

In this section I test the main qualitative predictions of model developed above, one concerning the cross-sectional distribution of technology adoption, and the other the effects of declining capital costs on employment. Although the model also delivers predictions regarding wages, these are difficult to test retrospectively, because in the first place they cannot be precisely characterized without restricting the model parameters, and in the second, wage data in the BIBB surveys are highly aggregated prior to the 2005/06 survey. I therefore defer consideration of the model's wage predictions until the quantitative section of this paper.

4.1 Time path of automation

The first qualitative prediction of the model is that automation of a low-skill task is "top-down", and at any given point in time is more likely to take place in high-skill occupations. In terms of model primitives:

Prediction #1: the automated proportion of jobs $G(\epsilon^*(s))$ is increasing in s.

This prediction is straightforward to test in the context of the BIBB surveys. Numerous case studies suggest that computers reduce the need for labor at low-skill tasks, for example by facilitating automation of manual tasks or allowing for faster execution of simple calculation and data manipulation tasks. To analyze computer adoption patterns, I calculate for each KLDB occupation the percentage of workers using a computer on the job and the mean log wage associated with the occupation. Taking a similar approach to Acemoglu and Autor (2011), I use wage data to construct a time-invariant ranking of occupations into percentiles, allowing for a more straightforward comparison across survey years. Results are shown in figure 4.1, and are consistent with model predictions. Between 1979 and 1999, PC adoption occurs primarily in high-paying and presumably high-skill occupations. By 1999 adoption rates are close to 100% in the highest-paying quartile of jobs, and only during the years after 1999 is there substantial PC adoption in the lower half of the occupation wage distribution.

One concern with these results is that, as a general purpose technology, PCs may substitute for labor at a variety of tasks, and do not necessarily reflect a distinct technological

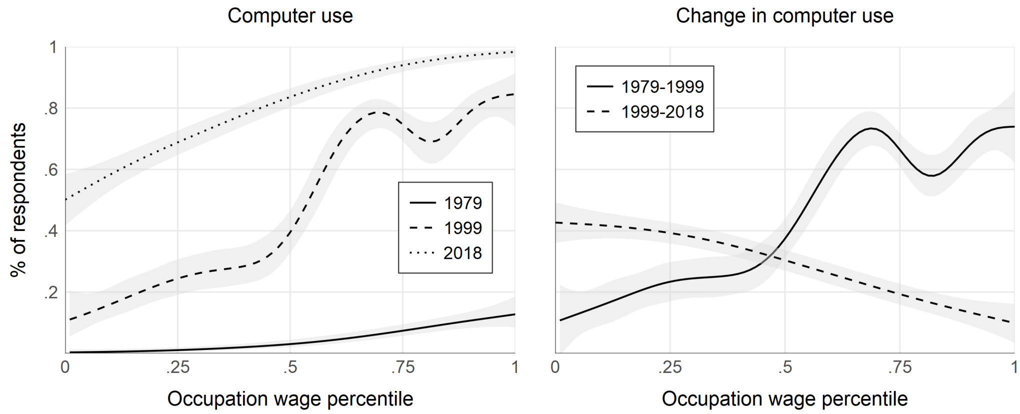

Figure 4.1: Computer use by occupation wage percentile

Figure notes. Log wage and PC use averaged by 1988 KLDB occupation for 1979-1999, and by 1992 KLDB occupation for 1999 (left panel) and 1999-2018 (right panel). Percentiles are time-invariant and reflect 1979 and 1999 wages. Shaded regions indicate 95% confidence intervals.

innovation as assumed in the model. I therefore consider two use cases for PCs - CNC machining in production jobs, and word processing for clerical tasks - for which data is available during the survey years 1986-1999. These applications are not relevant for all jobs, and so I therefore omit observations for which workers do not report using machinery (computer-controlled or not) and typing equipment (word processors or typewriters). Figure 4.2 shows similar adoption patterns to that for PCs as a whole, with early-stage use concentrated in high-paying jobs. A caveat is that most adoption of word processors occurs between two panels in the 1990's, and consequently I do not observe intermediate levels of adoption.

Figure 4.2: Technology penetration by wage percentile

Figure notes. First panel: use of CNC machines as percent of workers who report using heavy machinery. Second panel: use of word processors as percent of workers who report using typing equipment. Occupation-mean wages and weights calculated from equipment-using subsample. Shaded regions indicate 95% confidence intervals.

Summarizing these results, PC adoption over 1979-2018 exhibited a top-down pattern consistent with model predictions. While intuitive, this result is not explained by models of computerization based on capital-skill complementarity, which abstract from job-level interactions of labor and technology. It is also inconsistent with the simple labor-substitution framework of Acemoglu and Autor, in which technology adoption is associated with low-skill or routine jobs - which, given the empirical results shown in this paper, would suggest a "bottom-up" pattern of automation. The inability of extant models to predict adoption patterns is important, because these patterns can influence wage and employment trends *irrespective of the long-run effects of the technology.* In other words, the skill-bias of technological change at any point in time is likely to reflect both adoption patterns and the fixed characteristics of the technology, and disentangling these two factors is critical for predicting the long-run effects on labor markets.

4.2 Employment effects of automation

The second prediction of the model developed in this paper is that automation reduces contemporaneous employment, but will tend to increase employment as the cost of the technology declines further.

> **Prediction #2**: employment is decreasing in $G(\epsilon^*(s))$ given r, and decreasing in r for occupations with relatively high $G(\epsilon^*(s))$.

I test this prediction through a difference-in-difference approach relating occupational growth rates to levels of, and changes to, occupational PC use. A key limitation is that the BIBB surveys are not designed to measure occupational employment. Although the surveys are randomly sampled, non-response rates are not random and in addition, sample sizes are not large enough to ensure precise measures at the 3-digit occupational level. Survey weights correct for demographics but not occupation, and I find that in practice the weights introduce additional noise when calculating occupation growth rates. I therefore opt for the simplest approach and use raw counts to calculate occupational employment.[13] Results for the pooled panel regressions are shown in table C.5, and indicate that high levels of PC use are associated with employment growth, while increases in the rate of PC are generally accompanied by declining employment. This pattern is robust to controlling for occupation-mean wage. Inclusion of occupation fixed effects results in a somewhat stronger pattern for the years 1979-1999, but over the 1999-2018 period PC use contains no additional information over the occupational effects, which may reflect the fact that there is less cross-sectional and time variation in PC use after 2006.

To further assess the robustness of these results, in table 4.4 I show the estimated coefficients when considering only consecutive survey panels. The predicted employment patterns are present in four out of six regressions. Results are insignificant for the remaining two regressions, one of which (1992-1999) is compromised by changes to the wording and format

[13] Future plans for this project include the use of administrative employment data to calculate occupational wage and employment data.

Table 4.3: Regression: change in log occupational employment share

Independent variable	*Regression coefficients*							
	Years 1979-1999				Years 1999-2018			
PC use	**.268**	**.226**	**.497**	**.503**	**.472**	**.296**	-.043	-.117
	(.077)	(.085)	(.174)	(.172)	(.089)	(.116)	(.293)	(.300)
△ PC use	**-.462**	**-.472**	**-.520**	**-.519**	**-.378**	**-.296**	-.097	-.123
	(.186)	(.185)	(.192)	(.192)	(.133)	(.131)	(.177)	(.177)
Log(wage)		.079		-.064		**.223**		**.448**
		(.085)		(.272)		(.084)		(.215)
Occup. FE			X	X			X	X
Observations	830	830	830	830	874	874	874	874

Table notes. Difference-in-difference regression with occupational employment share as the dependent variable. Employment shares calculated from raw survey counts. All regressions include year fixed effects.

of the survey question concerning PC use. As in the previous table, the magnitude of the coefficients is large: a 10% point increase in PC use is associated initially with a 2-7% decline in occupational employment share, while a pre-existing PC use rate of 10% is associated with a 2-7% increase in share.

Table 4.4: Regression: change in log occupational employment share, by year

Indep. Var.	1979-86	1986-92	1992-99	1999-06	2006-12	2012-18
PC use	**.681**	**.205**	.128	**.704**	-.005	**.530**
	(.183)	(.099)	(.099)	(.135)	(.120)	(.170)
△ PC use	**-1.396**	**-.470**	.097	**-.380**	.135	**-.635**
	(.392)	(.195)	(.231)	(.174)	(.222)	(.244)
Observations	277	277	276	291	292	291

Table notes. Difference-in-difference regression with occupational employment share as the dependent variable. Employment shares calculated from raw survey counts.

These results suggest an intuitive explanation for why PCs have, in various cases, both complemented and substituted for occupational labor. Past studies such as Autor, Levy, and Murnane (2002) have made note of these disparate effects, and attributed them to fundamental (i.e. unexplained) differences in how technology interacts with different skill types, but the patterns shown in table C.5 are present even if one restricts attention to skilled occupations.[14] There is no mystery if only portions of jobs are automated, because this naturally introduces a distinction between the marginal (labor-substituting) and the long-run (labor-complementing) effects of automation.

[14] Tabulated results for high-wage occupations are provided in the appendix, and are qualitatively and quantitatively similar to those for the full sample.

5 Quantitative analysis

In this section I estimate the continuous model of partial automation developed above, and derive long-run predictions for wages and employment. I begin with an overview of the estimation procedure. Next I consider the model's long-run predictions regarding the wage and employment distributions. I close the section with results characterizing the capital-labor and labor-labor elasticities of substitution in this environment.

5.1 Estimation

Estimation is performed on the 2017/18 survey panel. As a first step, I reduce the dimensionality of the task space from 6 task variables (i.e. those used in the empirical analysis) to 2 variables, using a factor analysis approach.[15] Of the two composite variables, one loads principally on the routine task characteristics and the other on the non-routine characteristics. For each observation I divide factor scores by their sum and take the resulting values as measures of workers' time allocation t^* and $1 - t^*$.

I assume that the task production functions $\gamma_l(s)$ and $\gamma_h(s)$ are exponential functions taking the form $\gamma_k(s) = \exp(G(s)\gamma_k)$ with the normalization $\gamma_l = .5$. Once a value for γ_h is fixed, task shares can be estimated by solving the worker's time allocation policy function $t^*(j, s, K)$ for $\alpha(j)$, taking the expectation over all workers in j, substituting empirical values for t^* and the percentage of workers using PCs $G(\epsilon^*(s))$, and then numerically solving the resulting quadratic. The empirical values of t^* and $G(\epsilon^*(s))$ are averaged by occupation wage percentile and are generally increasing but, in order to ensure that equilibrium monotonicity conditions are met, I interpolate the empirical values subject to a non-decreasing constraint. The return to skill function $G(s)$ is then estimated by imposing the normalization $\lambda(s) = s$, solving the differential equation describing the wage function for $G'(s)$, and solving this system using empirical wages. The empirical and predicted distributions are shown for reference in figure 5.1.

The aggregate model parameters are estimated by using an indirect inference approach. The distribution $G(\epsilon)$ is assumed to be log-normal with standard deviation σ, and values of $\{r, \kappa, \sigma, \rho, \gamma_h/\gamma_l\}$ are obtained by minimizing the distance between empirical and model-predicted outcomes. The technological parameters r and σ are chosen to minimize the (squared) distance between the empirical and model-predicted distributions of PC use by wage percentile $G(\epsilon^*(s))$. To estimate κ, I match the empirical and model-predicted coefficients from a diff-in-diff regression of occupation-mean skilled task share $\mathbb{E}[t^*|s]$ on PC use $G(\epsilon^*(s))$. The elasticity of demand ρ can only be obtained by comparing outcomes over time, so I implement a second diff-in-diff regression of occupational employment shares on (1) PC use and (2) the time trend in PC use, and minimize the distance between the predicted

[15] Factor analysis is appropriate in this case because of the assumption of an underlying two-task structure; by preserving only the off-diagonal elements of the covariance matrix, factor analysis effectively ignores the idiosyncratic variance associated with individual survey questions.

Figure 5.1: Goodness of fit

and empirical coefficients on PC use. For each of the two regressions, I use data from the 2006-2018 surveys as they are identical in terms of questions and coding. Finally, the task skill differential γ_h is not separately identified from task shares $\alpha(j)$ in cross-sectional data, but will determine the wage response to automation. I therefore estimate γ_h by matching predicted and observed changes to the 90/10 wage ratio over 1979-2018.[16] Estimates are given in table 5.2.

Table 5.2: Aggregate parameter estimates

Parameter	Definition	Value	Obj. function	Survey years
r	rental rate	5.40	$(G^*(s) - \hat{G}^*(s))^2$	2018
σ	std. dev. $G(\epsilon)$	.157	$(G^*(s) - \hat{G}^*(s))^2$	2018
ρ	elasticity of subs.	1.72	$(\beta^L_{G^*} - \hat{\beta}^L_{G^*})^2$	2005-18
κ	automation feas.	.44	$(\beta^{t^*}_{G^*} - \hat{\beta}^{t^*}_{G^*})^2$	2005-18
γ_h/γ_l	task skill diff.	2.82	$\left(\triangle WR^{90}_{10} - \triangle \hat{WR}^{90}_{10}\right)^2$	1979, 2018

The most problematic aspect of model estimation is identification of ρ and γ_h. Changes over time to employment shares and wages are likely to reflect a number of factors, some of which may also be associated with PC use. The value of 1.72 is high relative to the "best guess" range of 1.4-1.5 proposed by Johnson (1997) for demand elasticity of substitution across skill types, but here the estimation procedure is influenced by other parameter values (in particular κ and γ) and interpretation of this parameter is therefore difficult. The value of γ_h determines the elasticity of labor supply, and is similarly unconstrained by past studies. I therefore consider alternative values of ρ and γ_h later in this section. For similar reasons the estimation of κ is also potentially problematic, but as κ is primarily a scaling parameter,

[16]Wage data for the 1979 survey are aggregated into bins, and so prior to calculations I perform similar aggregation on the wage data from the 2018 survey after adjusting for purchasing power using data on purchasing power comparisons of historical monetary amounts obtained from Deutsche Bundesbank. Wage ratios are calculated from occupation-mean log wage, and the model-predicted change is obtained by finding the rental rate for which aggregate PC adoption is equal to the observed 1979 value.

over- or under-estimation should not bias comparison between different points in time (i.e. different values of r) that will be the focus of the analysis in this section.

5.2 Employment, wages, and technology adoption

The model-predicted wage and employment distributions are shown in figure 5.3. The second panel shows how automation exerts a staggered effect on labor shares: employment declines initially in high-skill occupations, followed by middle-skill and finally low-skill occupations. Wages initially polarize, with the 90/50 ratio increasing at a faster rate than the 50/10 rate. As technology adoption approaches 100% for all j, polarization gives way to a general increase in wage dispersion. The main implication of partial automation, however, is the trajectory of wages and employment after the technology has been everywhere adopted. As r approaches zero, employment increases in low-skill jobs and wage inequality declines. The model predicts that employment in low-skill occupations ultimately increases, although automation tends also to increase the skill-intensity of these jobs in that workers spend a greater proportion of their time on the skilled task.

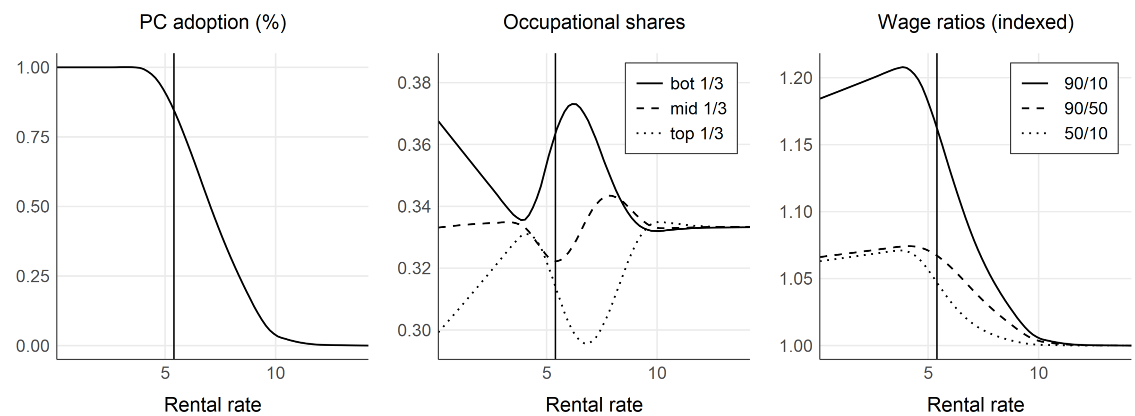

Figure 5.3: Technology adoption and labor outcome

FIGURE NOTES. Smoothed model output over a grid of rental rates r. Occupational groups correspond to the j-intervals $[0, 1/3)$, $[1/3, 2/3)$, and $[2/3, 1]$.

The distributional effects of automation are shown in greater detail in figure 5.4. Wage variance peaks as PC adoption approaches 100% (i.e. r approaches $\overline{r}$), and then declines slightly with further decreases in r. The small magnitude of this decline reflects the fact that the wage effects of automation are largely associated with the intensive task margin: automation increases the return to skill within a given automation because it results in workers spending a greater proportion of their time on the skilled task. The extensive margin - changes to the sorting of workers across jobs - is less important from a wage standpoint, with the caveat that job transitions in this environment are friction-less. If there are costs to switching jobs, for example due to non-transferable human capital or to search frictions, then the extensive margin is likely to exert a stronger effect on wages.

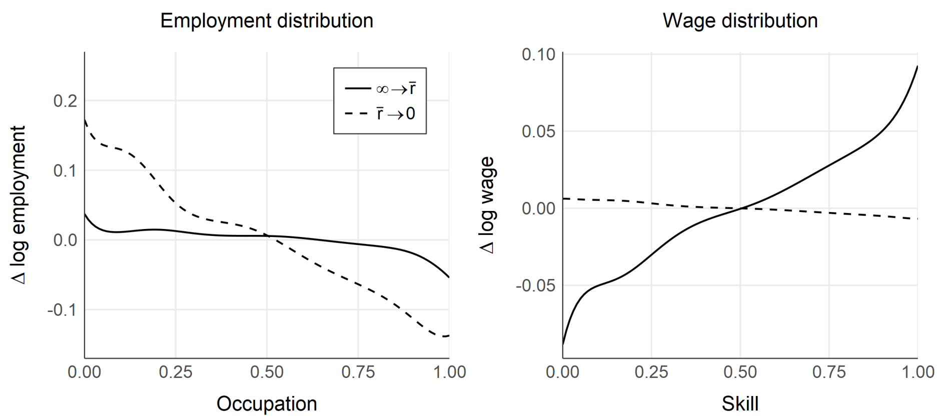

Figure 5.4: Short-run and long-run distributional effects

FIGURE NOTES. The value $\overline{r}$ is defined as the highest value of r for which $G(\epsilon^*(s)) \geq .99$ for all s.

Summarizing these results, the general prediction of the quantitative model is that automation of the low-skill task results in a persistent increase in wage inequality but non-persistent employment effects at the occupation level. In many respects this is a negative result. The undesirable outcome - wage inequality - is not expected to abate in the long-run, while the short-run nature of occupational disemployment suggests that policies facilitating job transitions may be counterproductive. Overall, figures 5.3 and 5.4 indicate that long-run effects of automation are important, but it is clear that the implications of this result will depend on the nature and magnitude of the labor market frictions affecting occupational choice.

5.3 Capital-labor substitution

The capital-labor elasticity of substitution $\partial \log L / \partial \log r$ is a common object of study in the skill-bias literature. A distinction is often made between technologies that are complementary with skill (i.e. "capital-skill complementarity") and those that substitute for low-skilled labor, while Autor, Levy, and Murnane (2003) make the case that both of characteristics are true of computers. A major departure of the model studied in this paper is that whether capital and labor are gross substitutes is not the result of assumptions imposed on technology, but is endogenous to occupational task structure and the rental rate. In figure 5.5 I plot cross-price elasticity labor demand over skill types and rental rates. In the early stages of computerization, capital *substitutes* for skilled labor, although it tends also to increase wage inequality as skilled workers devote a greater share of their time to the skilled task, and hence the return to skill rises. For intermediate values of the rental rate, capital complements labor in occupations at the extremes of the distribution, and substitutes for those in the middle. And in the final stages of adoption, capital substitutes for low-skill labor.

What determines this elasticity? It will generally be positive when changes to r have a large effect on $G(\epsilon^*(s))$ - when adoption is responsive to costs - and negative when $G(\epsilon^*(s))$ is

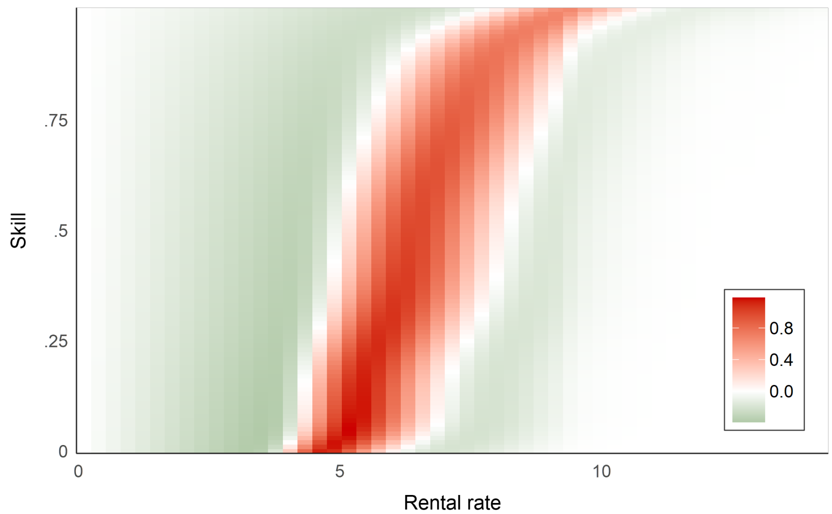

Figure 5.5: Cross-price elasticity of labor demand - rental rate

FIGURE NOTES. Partial derivative of $\log L(j, s, K)$ with respect to $\log r$, numerically calculated over rental rates and worker skill s.

large and relatively insensitive to the rental rate, which will be the case once the technology has been adopted in most jobs within an occupation. Note that for any given skill level, there exist rental rates for which capital is a gross substitute *and* rental rates for which capital complements labor. This result illustrates a main implication of task-level automation, which is that labor outcomes will depend critically on the time frame under consideration, and therefore the time-path of capital costs will be important for predicting labor outcomes in the long-run. These outcomes will also depend on the distribution of tasks across and within occupations - a point that is likely to hold with even more force under more general specifications of the model, in which more than two tasks are allowed.

5.4 Supply and demand elasticities

Two key elasticities in the model are the demand elasticity of substitution ρ, and the elasticity of occupational labor supply that, given the functional assumptions on match production, will depend on the ratio γ_h/γ_l. Together, these elasticities determine the response of wages and employment to automation, and it is for this reason that they are difficult to identify in practice. Historical changes to wages and employment are likely to reflect a multitude of other factors and therefore to be unreliable as measures of the effects of automation. In this section I consider the sensitivity of model predictions to alternative values of these parameters.

I begin with the demand elasticity ρ, which has some parallel to the labor-labor elasticity studied by previous authors, and typically formalized as the elasticity of substitution across skill types (typically education). Recent estimates of this elasticity include those by Ciccone and Peri (1.5, 2005) and Autor, Katz, and Kearney (1.57, 2008), and values in the neighborhood of 1.5 are common in this literature; but Gechert et al. (2021) suggest that published estimates are biased, and that the actual value of the labor-labor elasticity is as low as 1. In this paper, the elasticity of substitution across skill types is not a technological parameters but will depend on γ_h/γ_l as well as ρ, and so it is *ex ante* unclear what a plausible range for ρ would be. I therefore choose the somewhat arbitrary boundary points $\rho \in \{1, 2\}$, for which short-run and long-run employment predictions are shown in figure 5.6.[17] When ρ is

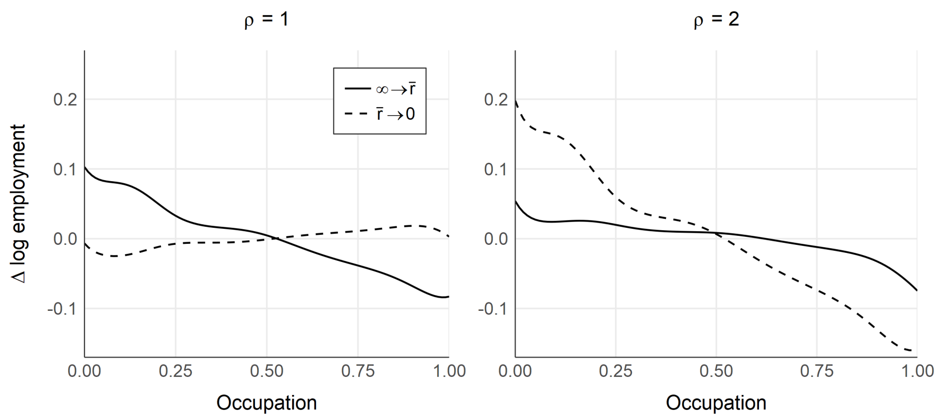

Figure 5.6: Varying the elasticity of labor demand

FIGURE NOTES. The value $\bar{r}$ is defined as the highest value of r for which $G(\epsilon^*(s)) \geq .99$ for all s. Occupational shares are re-estimated conditional on ρ.

larger, dis-employment from automation tends to be quickly reversed, as affected occupations enjoy greater benefits from further declines in r. For this reason, at the point $\bar{r}$ where adoption is complete, occupational labor shares are largely unchanged. As the rental rate continues to fall, however, low-skill occupations see growth, and this post-adoption effect is much stronger than in the case where ρ takes on a smaller value. Therefore a wide range of long-run employment outcomes are defensible, given uncertainty as to the true value of ρ. Wage predictions, on the other hand, are unaffected by this parameter and instead reflect the non-parametric skill productivity term $H(s)$ and the task skill differential γ_h/γ_l, discussed next.

Turning to the task skill ratio γ_h/γ_l, a larger value of this ratio will imply that skilled workers have a stronger comparative advantage at high-j occupations, and consequently occupational labor supply will be less elastic. For this reason wages will exhibit a stronger response to automation. To show this I plot results in figure 5.7 for two alternative parameter specifications where $\gamma_h/\gamma_l = 2$ and $\gamma_h/\gamma_l = 4$. The choice of these points is again somewhat arbitrary, but is sufficient for a demonstration of the sensitivity of model predictions. The first column of figure 5.7 shows occupational employment shares, from which we can see that a larger value of γ_h is associated with greater divergence over time in the share of high-skill and low-skill occupations. This in turn reflects the greater increase in the relative wages of skilled workers, indicated in the second column of the figure. Skilled workers experience a

[17] Wage predictions are unaffected by ρ and are therefore omitted from the plot.

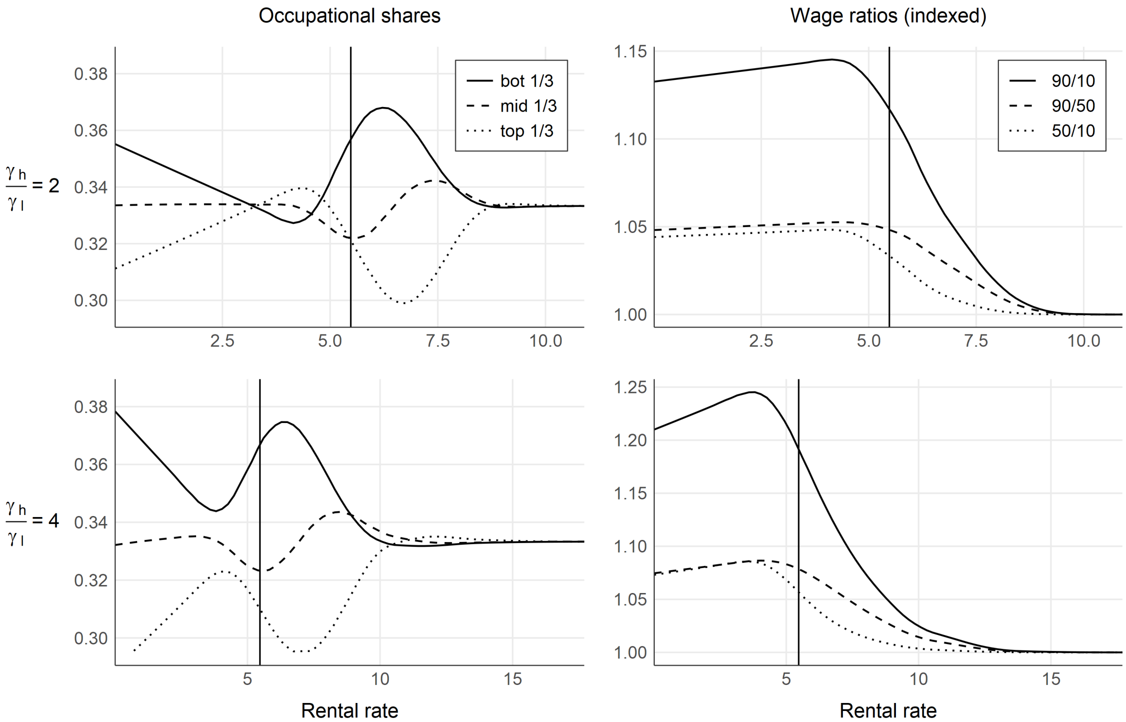

Figure 5.7: Varying the elasticity of labor supply

FIGURE NOTES. Smoothed model output over a grid of rental rates r. All model parameters other than ρ are re-estimated conditional on γ.

larger increase in wages in this case because the comparative advantage profile is steeper, and therefore as automation increases the portion of time spent on the skilled task, the return to skill increases by a greater amount. The long-run decline in wage inequality is somewhat greater in this case: from its peak, the 90/10 wage ratio falls by approximately 15% as capital costs fall towards zero. Uncertainty about the value of γ_h therefore suggests a wide range of plausible values for long-run employment shares, as well as some margin of error for wages, although in all cases the estimated model predicts only weak reversals of wage inequality in the long-run.

6 Conclusion

In this paper I have presented theoretical and quantitative results on the labor market effects of automation, when technology does not eliminate jobs but replaces labor at particular tasks within jobs. I show descriptively using German survey data that the vast majority of jobs contain both routine and non-routine task content, and that computerization over the 1979-2018 period was associated with intra-occupational changes in the frequency of routine content. The theoretical model developed in this paper predicts that in the short-run, automation of a low-skill task reduces employment as predicted by standard models; but in the long-run, as technology costs continue to fall, employment in low-skill occupations recovers and it is possible for long-run job growth to fully offset short-term losses. I show that the experience of West Germany is consistent with short-run model predictions concerning occupational computerization patterns and employment dynamics. A structurally estimated version of the model predicts that while increases in wage inequality associated with automation will be persistent, employment in middle-skill and low-skill jobs will recover as information technology costs continue to decline.

These results have two broad implications. First, the "traditional" effect of labor-substituting technology - dis-employment - is not an inevitable long-run outcome when automation is gradual, and jobs are not fully but only partially automated. Predictions of future occupational outcomes such as Frey and Osborne (2017) are generally based on the assumption of full automation, for which the empirical and historical support is weak. Second, because job outcomes in the immediate aftermath of automation are substantially different from job outcomes in the long-run, policies focused on worker retraining and occupational up-skilling are likely to be more costly over a long time-frame. The extent to which this is true will depend on how large and how fast are the declines in technology costs, and on the elasticity of labor supply and demand across occupations. On the other hand, policies intended to dampen job losses, such as employer subsidies, are likely to be more effective and less costly than they would be in the case of full automation.

A key shortcoming of this study is that I abstract from frictions affecting occupational labor supply, such as costs associated with retraining and re-schooling, barriers to entry such as licensure, and search frictions associated with finding a new job and, in many cases, a new employer. The effects of such frictions are unclear, as they will affect the response of both wages and employment to automation; but it is reasonable to expect that they would lead to greater variability of wages in the short-run. More broadly, the results in this paper reinforce the notion that factors influencing occupation transitions are key to understanding the quantitative effects of, and the policy trade-offs associated with automation.

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A Empirical appendix

The first set of variables use in this analysis are those concerning job routineness, and consist of answers to the following questions:

1. **repeat tasks**: in all survey years, how often that "one and the same operation is repeated down to the last detail"

2. **follow instructions**: in all survey years, how often the execution of work is "prescribed down to the last detail"

3. **adapt to new tasks**: the 1979 survey asks respondents how often they must adapt to "new situations." Beginning with the 1985/86 survey this changes to "new tasks that you must first think about and become familiar with."

4. **improve procedures**: in all survey years, how often respondents must "improve on previous procedures or try something new"

5. **solve problems**: for survey years 2006-2018, how often respondents must "react to and solve unforeseen problems"

6. **make decisions**: for survey years 2006-2018, how often respondents must "make difficult decisions independently and without guidance"

Response categories for 1979-1999 consist of five verbal frequencies: "practically always", "often", "every now and then", "seldom", and "practically never". For 2006-2018, response categories for variables 1-4 are changed to four frequencies: "often", "sometimes", "seldom", and "never". Variables 5-6, which are only present for 2006-2018, are coded in three frequencies: "often", "sometimes", and "never". I code the responses "practically always" and "often" as 1, "every now and then" and "sometimes" as 2/3 for variables 1-4 and 1/2 for variables 5-6, "seldom" as 1/3, and "never" and "practically never" as 0. Some discrepancies are observed between the 1989-99 and 2005-06 surveys, but they are inconsistent across variables and do not suggest a systematic bias due to the change in response frequencies.

The other principal variable used in this analysis concerns personal computers. In the 1979 survey respondents are asked whether they often work with computers, EDV equipment, terminals, or screened devices on the job. In 1985/86 and 1991/92 this question is split across manufacturing and office roles and by device type, and so I assign a value of "yes" (1) if the answer is affirmative for any of these roles or devices. There is a major change in 1998/99, with workers simply being asked whether they "work with computers and data processing equipment" in their professional activity. There is again a slight change for the 2006-2018 surveys, with respondents asked simply how often they "work with computers", the options being "often", "sometimes", and "never"; assign a value of "yes" to the first two responses.

With respect to other miscellaneous variables, occupation consists of 1988 KLDB 3-digit codes for the years 1979-2006, and 1992 KLDB 3-digit codes for 1999-2018. For consistency, the results in section two use 1988 KLDB codes for 1979-1992 results and 1992 KLDB codes for 1999-2018 results. Some occupations are not observed in all panels, and prior to performing difference-in-difference regressions I aggregate these codes into neighboring occupations. The same aggregated groupings are employed in the quantitative section. Occupational tasks from the 2006-2018 surveys are coded in the same frequencies as “solve problems” and “make decisions” above, and I assign them numerical values in identical fashion. The six aggregate groupings are composed as:

1. **analyze information**: “developing, researching, constructing [designing]”; “gathering information, investigating, documenting”
2. **advise others**: “organizing, planning, and preparing work processes [for others]”; “training, instructing, teaching, educating”; “providing advice and information”
3. **market goods**: “purchasing, procuring, selling”; “advertising, marketing, public relations”
4. **manual labor**: “measuring, testing, quality control”; “repairing, refurbishing”; “transporting, storing, shipping”; “cleaning, removing waste, recycling”
5. **produce goods**: “manufacturing, producing goods and commodities”; “monitoring, control of machines, plants, processes”
6. **care for others**: “entertaining, accommodating, preparing food”, “nursing, caring, healing”, “protecting, guarding, patrolling, directing traffic”

Numerical values are averaged by task group, and divided by the sum across task groups in order to remove variation due to individuals who report performing all tasks more frequently.

B Theoretical appendix

B.1 Assignment and wages

The environment is similar to that studied by Costinot and Vogel (CV, 2010), with the difference that unit output costs for automated firms are $\frac{w(s)+r\overline{K}}{y(j,s,\overline{K})}$ and not $\frac{w(s)}{y(j,s,0)}$. Following CV, I define $\omega(s)$ to be the set of jobs j for which at least one producer hires an s-worker, and $\sigma(j)$ the worker types assigned to j. I begin with a lemma establishing that non-automated jobs are always associated with lower-skill workers than automated job:

Lemma 1. *If $s' \geq s$ and $K(j) > 0$ for some firm employing s, then $K(j') > 0$ for any firm j' employing s'.*

Proof. This result is shown by establishing that $w(s)$ and $\gamma_u(s)$ satisfy the single-crossing: there exists at most one s such that $w(s) = r\gamma_u(s)$, and that for any $s' > s$ we will have $w(s') >$

Table A.1: Task characteristics and task frequencies, 2006-2018

Sample	Analyze information	Advise others	Market goods	Manual labor	Produce goods
	Year fixed effects				
Repetition	**-.469** (.023)	**-.179** (.024)	**-.255** (.025)	**.129** (.026)	.003 (.026)
Instructions	**-.267** (.024)	**-.139** (.025)	**-.370** (.026)	-.002 (.003)	**.155** (.027)
Adaptation	**.522** (.020)	**.205** (.020)	**.153** (.020)	**-.119** (.018)	**.176** (.019)
Improvement	**.281** (.022)	.039 (.023)	**.085** (.023)	**-.286** (.022)	**.053** (.023)
Solve problems	**.110** (.023)	.041 (.024)	**-.048** (.023)	**-.344** (.020)	**-.106** (.021)
Make decisions	-.013 (.026)	.040 (.028)	.048 (.028)	**-.509** (.026)	**-.203** (.026)
	Industry, occupation, and year fixed effects				
Repetition	**-.269** (.030)	**-.164** (.029)	**-.282** (.031)	.057 (.032)	.044 (.034)
Instructions	**-.179** (.031)	**-.171** (.031)	**-.306** (.032)	**-.089** (.032)	-.058 (.035)
Adaptation	**.263** (.023)	**.127** (.022)	**.135** (.023)	**-.083** (.022)	.043 (.024)
Improvement	**.208** (.025)	**.074** (.025)	**.135** (.026)	**-.167** (.025)	.029 (.027)
Solve problems	**.083** (.025)	**.110** (.024)	**.098** (.025)	**-.163** (.023)	-.017 (.025)
Make decisions	.010 (.030)	**.131** (.030)	**.187** (.032)	**-.332** (.030)	**-.128** (.032)

TABLE NOTES. Marginal effects and robust standard errors from fractional logit regressions of task characteristics on PC use, aggregated by 3-digit occupation. All regressions include dummies for year, 3-digit occupation, 1-digit industry. Bold results indicate 95% significance.

$r\gamma_u(s')$. Suppose the contrary: there exists an $s' > s$ such that $w(s') < r\gamma_u(s')$ but $w(s) > r\gamma_u(s)$. From free entry we have $p(j)y^*(j,s,K) \geq w(s)$ for any j employing worker s, and by assumption we have $y^*(j,s,K)$ increasing more quickly in s than $\gamma_u(s)$. But then $p(j)y^*(j,s',K) - w(s') > p(j)y^*(j,s,K) - w(s)$, violating the producer's profit-maximizing problem. The result then follows from (4), which states that $K > 0$ when $w > r\gamma_u$ and $K = 0$ otherwise. □

With this result it is possible to derive the main result:

Lemma 2. *There exists a continuous and strictly increasing function $\lambda : [\underline{s}, \overline{s}] \to [0,1]$ such that $L(j,s) > 0$ if and only if $\lambda(s) = j$, and where $\lambda(\underline{s}) = 0$ and $\lambda(\overline{s}) = 1$..*

Proof. In the proof below, I provide an abbreviated description whenever the steps follow closely those described by CV.

First, $\omega(s)$ and $\sigma(j)$ are non-empty. That $\omega(s)$ is non-empty follows from market-clearing, as labor is supplied inelastically and worker output is strictly positive. If there existed an s that was not assigned to any producer, it must be that $w(s)$ is sufficiently large that no firms wish to employ s-workers; but then market-clearing would imply that $w(s) \to 0$, a contradiction. Non-empty $\sigma(j)$ follows from the profit-maximizing condition for intermediate producers:

$$p(j) - \frac{w(s) + rK^*(j,s)}{y^*(j,s,K)} \leq 0$$

with equality for $s \in \sigma(j)$. If $\sigma(j)$ is empty for some j, then from $\omega(s)$ non-empty there must be

Table A.2: Task frequencies and PC use, 2006-2018

Sample	Analyze information	Advise others	Market goods	Manual labor	Produce goods	Care for others
Full sample	**.068** (.004)	**.031** (.003)	**.035** (.003)	**-.053** (.003)	**-.011** (.002)	**-.014** (.002)
Education						
None	**.048** (.009)	**.022** (.008)	**.030** (.007)	**-.065** (.010)	-.007 (.008)	**-.010** (.006)
Vocational	**.057** (.004)	**.027** (.004)	**.023** (.003)	**-.052** (.003)	**-.009** (.003)	**-.012** (.002)
University	**.072** (.015)	.012 (.15)	**.068** (.010)	**-.044** (.006)	**-.015** (.005)	**-.016** (.004)
Wage pct.						
1-25	**.058** (.006)	**.017** (.005)	**.020** (.005)	**-.048** (.006)	.000 (.005)	**-.021** (.004)
26-50	**.050** (.006)	**.029** (.006)	**.023** (.005)	**-.041** (.006)	**-.013** (.005)	**-.010** (.003)
51-75	**.050** (.009)	**.017** (.008)	**.059** (.007)	**-.045** (.006)	**-.017** (.007)	-.002 (.005)
76-100	.**.044** (.019)	**.042** (.019)	**.097** (.016)	**-.040** (.007)	**-.027** (.008)	.001 (.005)

Table notes. Marginal effects and robust standard errors from fractional logit regressions of task characteristics on PC use, aggregated by 3-digit occupation. All regressions include dummies for year, 3-digit occupation, 1-digit industry. Bold results indicate 95% significance.

Table A.3: Occupation mean task frequencies and PC use (D-in-D)

Years	Analyze information	Advise others	Market goods	Manual labor	Produce goods	Care for others
2006-2012	**.027** (.013)	**.033** (.011)	**.054** (.013)	**-.049** (.008)	**-.017** (.008)	.006 (.009)
2012-2018	**.060** (.012)	**.041** (.014)	.015 (.013)	**-.042** (.011)	-.012 (.010)	-.011 (.008)

Table notes. Marginal effects and robust standard errors from fractional logit regressions of task characteristics on PC use, aggregated by 3-digit occupation. All regressions include occupation and year dummies. Bold results indicate 95% significance.

a j' with $\sigma(j')$ non-empty and $p(j')/p(j) = \frac{A}{p(j)} = 0$ for any finite A. But then from the previous condition it must be that

$$\frac{p(j')}{p(j)} + r\frac{K^*(j',s) - K^*(j,s)}{p(j)y^*(j',s)} \geq \frac{y^*(j,s,K)}{y^*(j',s,K)} > 0$$

a contradiction since we must have the left-hand side equal to 0.

Second, $\sigma(j)$ is a non-empty interval of $[\underline{s}, \overline{s}]$, and if $j' > j$, then $s' > s$ for any $s' \in \sigma(j')$ and $s \in \sigma(j)$. Suppose instead that for some $j^+ > j^-$ and $s^+ > s^-$, we have $s^- \in \sigma(j^+)$ and

$s^+ \in \sigma(j^-)$. Then from intermediate producers' first-order condition we must have

$$\begin{aligned}
0 &\geq p(j^-) - \frac{w(s^-) + rK^*(j^-, s^-)}{y^*(j^-, s^-)} \\
&= \frac{w(s^+) + rK^*(j^+, s^+)}{y^*(j^-, s^+)} \left(1 - \frac{\frac{w(s^-)+rK^*(j^-,s^-)}{y^*(j^-,s^-)}}{\frac{w(s^+)+rK^*(j^-,s^+)}{y^*(j^-,s^+)}} \right) \\
&> \frac{w(s^+) + rK^*(j^+, s^+)}{y^*(j^-, s^+)} \left(1 - \frac{\frac{w(s^-)+rK^*(j^-,s^-)}{y^*(j^+,s^-)}}{\frac{w(s^+)+rK^*(j^-,s^+)}{y^*(j^+,s^+)}} \right) \\
&= \frac{w(s^+) + rK^*(j^+, s^+)}{y^*(j^-, s^+)} \left(1 - \frac{p(j^+)}{\frac{w(s^+)+rK^*(j^-,s^+)}{y^*(j^+,s^+)}} \right) \\
&= \frac{\frac{w(s^+)+rK^*(j^+,s^+)}{y^*(j^-,s^+)}}{\frac{w(s^+)+rK^*(j^-,s^+)}{y^*(j^+,s^+)}} \left(\frac{w(s^+) + rK^*(j^-, s^+)}{y^*(j^+, s^+)} - p(j^+) \right) \\
&\geq 0
\end{aligned}$$

a contradiction, with the strict inequality following from assumption 1 and lemma 1. The inequality holds trivially when either $K > 0$ for both s^- and s^+ or when $K = 0$ for both types. From lemma 1 the only other possibility is that $K > 0$ for s^+ but $K = 0$ for s^-, in which case $\frac{w(s^+)+rK^*(j^-,s^+)}{y^*(j^+,s^+)} = w(s^+)y(s^+, j^-, 0) - \kappa\big[1 - \alpha(j^-)\big]\gamma_u(s^+)\big[w(s^+) - r\gamma_u(s^+)\big]$. From $\alpha'(j) > 0$ we can then see that the inequality holds with even greater force than under the other two cases.

Fourth, ω and σ are single-valued almost everywhere. The proof is unchanged from CV and so I provide only the intuition: if ω (or σ) has positive measure over a domain with positive measure, then from the previous result the range of the correspondence will have measure greater than the measure of $[\underline{s}, \overline{s}]$, a contradiction.

Fifth, $\sigma(j)$ is single-valued. If this is not the case, then from step 3 there exists a non-degenerate interval $[s, s']$ for which all workers are assigned to job j. Step 4 implies that there exists another job j' that is assigned to a single worker type. But then $p(j)/p(j') = 0$, contradicting the free entry condition that $p(j)y(j, s'') \geq p(j')y(j', s'')$ for $s'' \in [s, s']$, a contradiction given that $y > 0$.

From the last step we have $\sigma(j)$ single-valued; from the third step, weakly increasing; from the first step, continuous and such that $\sigma(0) = \underline{s}$ and $\sigma(1) = \overline{s}$; and from the fourth step, σ is strictly increasing. Hence we have a continuous, strictly increasing bijection $\lambda(s) = \omega(s) = \{j \; | L(j, s) == 1\} = \sigma^{-1}(s)$. □

Lemma 3. *There exists a single threshold skill level s^*, which may be equal to $\underline{s}$ or $\overline{s}$], for which producers automate when $s > s^*$ and do not automate when $s > s^*$. The wage functions satisfies the differential equation*

$$\frac{w'(s)}{w(s)} = \begin{cases} \frac{d}{ds} \log y\big(\lambda(s), s, 0\big) & s < s^* \\ \frac{d}{ds} \log y\big(\lambda(s), s, \overline{K}\big) & s > s^* \end{cases}$$

where $w(s)$ is continuous but not differentiable at s^*.

Proof. Market-clearing, free entry, and continuity of $y(j,s)$ imply that $w(s)$ is continuous, while from lemma 1 we have that there exists at most one s^* satisfying $w(s^*) = r\gamma_u(s^*)$. Let $\mathbb{I}[s > s^*]$ be an indicator function taking the value 1 when $s > s^*$, and 0 otherwise.

From lemma 2, for any producer employing s-labor we must have

$$p\big(\lambda(s)\big) - \frac{w(s)}{y\big(\lambda(s), s, K\big)} + \mathbb{I}[s > s^*]\kappa\big[1 - \alpha(\lambda(s))\big]r \leq 0$$

For any $s \neq s^*$, there will exist a neighborhood around s such that $\mathbb{I}[s' > s^*]$ takes the same value for any $s' \in [s - ds, s + ds]$. In the case where $s > s^*$ the following inequalities must hold:

$$\begin{aligned}
&\Big[p(\lambda(s)) - \kappa\big[1 - \alpha(\lambda(s))\big]r\Big] - \frac{w(s)}{y\big(\lambda(s), s, \overline{K}\big)} \\
&\quad\geq \Big[p(\lambda(s)) - \kappa\big[1 - \alpha(\lambda(s))\big]r\Big] - \frac{w(s + ds)}{y\big(\lambda(s), s + ds, \overline{K}\big)} \\
&\Big[p(\lambda(s + ds)) - \kappa\big[1 - \alpha(\lambda(s + ds))\big]r\Big] - \frac{w(s + ds)}{y\big(\lambda(s + ds), s + ds, \overline{K}\big)} \\
&\quad\geq \Big[p(\lambda(s + ds)) - \kappa\big[1 - \alpha(\lambda(s + ds))\big]r\Big] - w(s)\frac{w(s)}{y\big(\lambda(s + ds), s, \overline{K}\big)}
\end{aligned}$$

For $s < s^*$ we must have the following inequalities hold:

$$\begin{aligned}
p(\lambda(s)) - \kappa\big[1 - \alpha(\lambda(s))\big]r\Big] - \frac{w(s)}{y\big(\lambda(s), s, 0\big)} &\geq p(\lambda(s)) - \frac{w(s + ds)}{y\big(\lambda(s), s + ds, 0\big)} \\
p(\lambda(s + ds)) - \frac{w(s + ds)}{y\big(\lambda(s + ds), s + ds, 0\big)} &\geq p(\lambda(s + ds)) - w(s)\frac{w(s)}{y\big(\lambda(s + ds), s, 0\big)}
\end{aligned}$$

It follows that in the first case we will have

$$\begin{aligned}
&\Big[p(\lambda(s)) - \kappa\big[1 - \alpha(\lambda(s))\big]r\Big]\Big[y\big(\lambda(s), s + ds, \overline{K}\big) - y\big(\lambda(s), s, \overline{K}\big)\big] \\
&\quad\leq w(s + ds) - w(s) \\
&\quad\leq \Big[p(\lambda(s + ds)) - \kappa\big[1 - \alpha(\lambda(s + ds))\big]r\Big]\Big[y\big(\lambda(s + ds), s + ds, \overline{K}\big) - y\big(\lambda(s + ds), s, \overline{K}\big)\Big]
\end{aligned}$$

and in the second

$$\begin{aligned}
&p(\lambda(s))\Big[y\big(\lambda(s), s + ds, 0\big) - y\big(\lambda(s), s, 0\big)\big] \\
&\quad\leq w(s + ds) - w(s) \\
&\quad\leq p(\lambda(s + ds))\Big[y\big(\lambda(s + ds), s + ds, 0\big) - y\big(\lambda(s + ds), s, 0\big)\Big]
\end{aligned}$$

We must have $p(j)$ continuous and therefore we can divide by ds and take the limit of the previous inequalities to show that

$$w'(s) = \begin{cases} p(\lambda(s))y\big(\lambda(s+ds), s, 0\big) & s < s^* \\ \big[p(\lambda(s)) - \kappa\big[1-\alpha(\lambda(s))\big]y\big(\lambda(s+ds), s, \overline{K}\big) & s > s^* \end{cases}$$

Substitution for $p(\lambda(s))$ and $p(\lambda(s)) - \kappa\big[1-\alpha(\lambda(s))\big]$ and division by $w(s)$ yields the final result. □

Lemma 4. *The matching function satisfies*

$$\lambda'(s) = \begin{cases} \frac{y\left(\lambda(s),s',0\right)^{1-\rho} F'(s)}{\beta\left(\lambda(s)\right)^{\rho} Y} w(s)^{\rho} & s < s^* \\ \frac{y\left(\lambda(s),s',\overline{K}\right) F'(s)}{\beta\left(\lambda(s)\right)^{\rho} Y}\Big(w(s) + \kappa\big[1-\alpha(\lambda(s))\big]ry\big(\lambda(s), s', \overline{K}\big)\Big)^{\rho} & s > s^* \end{cases}$$

where $\lambda(\underline{s}) = 0$ *and* $\lambda(\overline{s}) = 1$.

Proof. This portion of the proof follows closely Costinot and Vogel (2010). Total supply of j-labor is given by

$$L(j, s) = F'(s)\delta\big[j - \lambda(s)\big]$$

From market-clearing we have

$$Y\big(\lambda(s)\big) = \int_{s<s^*} y\big(\lambda(s), s', 0\big)L\big(\lambda(s), s'\big)ds' + \int_{s>s^*} y\big(\lambda(s), s', \overline{K}\big)L\big(\lambda(s), s'\big)ds'$$

and following CV we may derive the differential equation for the matching function, which is defined piece-wise:

$$\lambda'(s) = \begin{cases} \frac{y\left(\lambda(s),s',0\right)F'(s)}{Y\left(\lambda(s)\right)} & s < s^* \\ \frac{y\left(\lambda(s),s',\overline{K}\right)F'(s)}{Y\left(\lambda(s)\right)} & \mathbb{I}^s = 1 \end{cases}$$

From final good profit-maximization and free entry we have

$$Y\big(\lambda(s)\big) = \begin{cases} \beta\big(\lambda(s)\big)^{\rho} Y \left[\frac{y\left(\lambda(s),s',0\right)}{w(s)}\right]^{\rho} & s < s^* \\ \beta\big(\lambda(s)\big)^{\rho} Y \left[\frac{y\left(\lambda(s),s',\overline{K}\right)}{w(s)+\kappa\left[1-\alpha(\lambda(s))\right]ry\left(\lambda(s),s',\overline{K}\right)}\right]^{\rho} & s > s^* \end{cases}$$

Substitution of $Y\big(\lambda(s)\big)$ and $y\big(\lambda(s), s\big)$ in the previous equation then yields the result. □

B.2 Theorem 1

Proof. To establish the first part of the result I show that $\lambda(s^*)$ is strictly smaller under automation. Suppose otherwise. It can then be shown that for for all s, $\lambda(s')$ is greater under automation and hence from (6) $w'(s)/w(s)$ is everywhere larger. But then $\frac{w(s)^\rho}{Y} = \frac{w(s)^\rho}{\int \left[F'(s)w(s)+rK^*(\lambda(s),s)\right]ds}$ must be strictly smaller for $s < s^*$, implying that these workers are assigned to a smaller set of jobs and hence that $\lambda(s^*)$ is smaller, a contradiction.

For the second part of the proof, for any $j' > j$ and holding $\big(y(j',\lambda^{-1}(j'),0)/y(j,\lambda^{-1}(j),0)\big)^{\rho-1}$ we will have $L(j')/L(j)$ greater under automation both because $\alpha'(j) > 0$ and because $\lambda(s)$ and $w(s) - r\gamma_h(s)$ are both increasing in s. Now if $\rho < 1$ and employment decreases for all automated jobs then we must have $\big(y(1,\overline{s},0)/y(\lambda(s),s,0)\big)^{\rho-1}$ smaller under automation for any $s < \overline{s}$ and therefore $L(j')/L(j)$ greater. On the other hand if employment increases for some $j'' > \lambda^{-1}(s^*)$, then it must increase for all $j > j''$ and the result is shown. In both cases the result is shown for $\rho < 1$. Finally, from continuity the result must also hold for $\rho > 1$ but sufficiently small.

Finally, from above we will have $w'(s)/w(s)$ smaller for $s < s^*$, whereas $\lambda(\overline{s}) = 1$ and (6) imply that $w'(s)/w(s)$ will be greater around $\overline{s}$. Assumption 1 and continuity of all arguments is then sufficient to establish that either $w'(s)/w(s)$ is strictly greater for $s > s^*$ and smaller for $s < s^*$, in which case I define $s' = s^*$; or that there exists at least one value of s at which $w'(s)/w(s)$ is unchanged under automation, in which case I define s' equal to the largest such value and the proof is completed. □

B.3 Theorem 2

Proof. Both results may be shown by establishing that $\lambda(s)$ is strictly greater under automation for all s, which would in turn imply both that $w'(s)/w(s)$ is strictly greater and that $\int_0^j L(j,\lambda^{-1}(j))$ is strictly smaller. Now if t^* is increasing in s then holding λ fixed, $\frac{d}{ds}\log y^*(\lambda(s),s,K)$ will be smaller under automation and therefore $\frac{d}{ds}\log Y/y^*(\lambda(s),s,K)$ will be greater; and in particular, we must have $Y/y^*(\lambda(\underline{s}),\underline{s},K)$ smaller and $Y/y^*(\lambda(\overline{s}),\overline{s},K)$ bigger. It follows that $\lambda'(\underline{s})$ is greater under automation and $\lambda'(\overline{s})$ smaller, while labor market-clearing implies that we cannot have $\lambda(s)$ the same under automation for any s in $(\underline{s},\overline{s})$. Continuity of the matching function then establishes that $\lambda(s)$ is everywhere and the results follow. □

B.4 Theorem 3

Proof. I begin by noting that in the case where $\rho = 1$ we will have labor demand equal to

$$L(\lambda(s),s) = \frac{\beta(\lambda(s))Y}{\big(w(s) + r\kappa\big[1-\alpha(\lambda(s))\big]y^*(\lambda(s),s,K)\big)}$$

Making use of equation (6) it can be shown that

$$\begin{aligned}\frac{d}{ds}\log\big(w(s) + r\kappa\big[1-\alpha(\lambda(s))\big]y^*(\lambda(s),s,K)\big) &= \frac{y_s^*(\lambda(s),s,K)}{y^*(\lambda(s),s,K)} \\ &+\lambda'(s)r\kappa y^*(\lambda(s),s,K)\frac{\big[1-\alpha(\lambda(s))\big]\frac{y_j^*(\lambda(s),s,K)}{y^*(\lambda(s),s,K)} - \alpha'(\lambda(s))}{w(s) + r\kappa\big[1-\alpha(\lambda(s))\big]y^*(\lambda(s),s,K)}\end{aligned} \tag{8}$$

where the last term is strictly negative.

Suppose now that $w(\overline{s})/w(\underline{s})$ is smaller under automation. We must have $w'(\underline{s})/w(\underline{s})$ and $w'(\overline{s})/w(\overline{s})$ greater, which implies that there exists at least one s for which $w(s)/w(\underline{s})$ is unchanged under automation. For each of the smallest and largest such values of s, it must be that $w'(s)/w(s)$ is smaller, and hence that $\lambda(s)$ is strictly lower. But then between the lower of the two s values and $\underline{s}$ we must have average wage higher under automation, while between the higher value of s and $\overline{s}$ we must have average wage higher. Higher wages in the lower region imply a larger value of $\lambda'(s)$, and lower values of wages in the upper region imply a smaller value of $\lambda'(s)$, implying that at the points where $w(s)$ is unchanged we must have $\lambda(s)$ larger, a contradiction.

Now suppose that, for any $r \in (0, \overline{r})$, between any two points s' and s'' we have $\lambda(s)$ weakly greater relative to the case where $r = \overline{r}$. Then $w'(s)/w(s)$ must be strictly greater in this interval, and hence from (7) it must be that almost everywhere in $[s', s'']$ we have $\lambda(s)$ strictly greater. Suppose without loss of generality that $\lambda(s)$ is strictly greater in this interval. Then it must be that $\lambda'(s')$ is greater than before while $\lambda'(s'')$ is smaller. But since $w'(s)/w(s)$ is strictly larger between s' and s'', whereas rental costs decrease proportionally in r. it must be that the sum of wages and rental costs has increased for s'' relative to s', which from (7) implies that $\lambda'(s')$ has decreased relative to $\lambda'(s'')$, a contradiction. Hence over any interval we must have $\lambda(s)$ smaller almost everywhere, and the result follows.

Now when $r = 0$, the negative term in (8) drops out and so from the previous result we must have $\lambda'(\underline{s})$ smaller relative to $\lambda'(\overline{s})$. But then either the first of these terms is strictly smaller, or the second strictly larger, implying in either case that for some subset of $[\underline{s}, \overline{s}]$ we will have $\lambda(s)$ strictly smaller under automation, and the result follows. □

B.5 Continuous model

Free entry implies that

$$p(j,s) = \frac{w(s) + G(\epsilon^*(s)) r\kappa\big[1-\alpha(j)\big] y^*(j,s,\overline{K}) \int_0^{\epsilon^*(s)} G'(k)k\ dk}{y^*(j,s,0) + G(\epsilon^*(s))\Big[y^*(j,s,\overline{K}) - y^*(j,s,0)\Big]} .$$

while labor demand (i.e. firm entry) will be

$$\begin{aligned} L(j,s',s) &= \left(\frac{\beta(j)}{p(j,s',s)}\right)^{\rho} \frac{Y}{y^*(j,s,0) + G(\epsilon^*(s))\Big[y^*(j,s,\overline{K}) - y^*(j,s,0)\Big]} \\ &= \beta(j)^{\rho} Y \frac{\Big(y^*(j,s,0) + G(\epsilon^*(s))\Big[y^*(j,s,\overline{K}) - y^*(j,s,0)\Big]\Big)^{\rho-1}}{\Big(w(s) + G(\epsilon^*(s)) r\kappa\big[1-\alpha(j)\big] y^*(j,s,\overline{K}) \int_0^{\epsilon^*(s)} G'(k)k\ dk\Big)^{\rho}} \end{aligned}$$

Optimal choice of skill, on the other hand, implies that

$$w'(s) = \left[G(\epsilon^*(s))p(j) - r\kappa\big[1-\alpha(j)\big]\int_0^{\epsilon^*(s)} G'(k)k\ dk\right]\frac{\partial}{\partial s}y^*(j,s,\overline{K}) + \big[1-G(\epsilon^*(s))\big]p(j)\frac{\partial}{\partial s}y^*(j,s,0)$$
$$+\frac{d}{ds}\epsilon^*(s)\frac{\partial}{\partial \epsilon^*}\pi(j,s)$$

where profit maximization implies that the last term $\frac{\partial}{\partial \epsilon^*}\pi(j,s)$ is equal to zero. Hence we must have

$$\frac{w'(s)}{w(s)} = \frac{A(j,s)\frac{\partial}{\partial s}y^*(j,s,\overline{K}) + B(j,s)\frac{\partial}{\partial s}y^*(j,s,0)}{A(j,s)A(j,s)y^*(j,s,\overline{K}) + B(j,s)y^*(j,s,0) + B(j,s)A(j,s)y^*(j,s,0) + B(j,s)y^*(j,s,0)}$$
$$A(j,s) = G(\epsilon^*(s))p(j) - r\kappa\big[1-\alpha(j)\big]\int_0^{\epsilon^*(s)} G'(k)k\ dk$$
$$B(j,s) = \big[1-G(\epsilon^*(s))\big]p(j)\ ,$$

where since match output is log-supermodular and α is increasing, $\frac{A(j,s)\frac{\partial}{\partial s}y^*(j,s,\overline{K})+B(j,s)\frac{\partial}{\partial s}y^*(j,s,0)}{A(j,s)y^*(j,s,\overline{K})+B(j,s)y^*(j,s,0)}$ must be increasing in j and therefore $A(j,s)y^*(j,s,\overline{K}) + B(j,s)y^*(j,s,0)$ will also be log-supermodular. Optimal assignment in the continuous model will be characterized by the differential equations

$$\frac{w'(s)}{w(s)} = \frac{A(j,s)\frac{\partial}{\partial s}y^*(j,s,\overline{K}) + B(j,s)\frac{\partial}{\partial s}y^*(j,s,0)}{A(j,s)y^*(j,s,\overline{K}) + B(j,s)y^*(j,s,0)} \tag{9}$$

$$\lambda_m'(s) = \frac{\Big(y^*(\lambda(s),s,0) + G(\epsilon^*(s))\Big[y^*(\lambda(s),s,\overline{K}) - y^*(\lambda(s),s,0)\Big]\Big)^{1-\rho}}{\beta(\lambda(s))^\rho Y}$$
$$\times\left(w(s) + G(\epsilon^*(s))r\kappa\big[1-\alpha(\lambda(s))\big]y^*(\lambda(s),s,\overline{K})\int_0^{\epsilon^*(s)} G'(k)k\ dk\right)^\rho \tag{10}$$

where $\lambda(\underline{s}) = 0$ and $\lambda(\overline{s}) = 1$ as before, $\epsilon^*(s) = \frac{w(s)}{r\gamma_l(s)}$, and where

$$A(j,s) = G(\epsilon^*(s))p(j) - r\kappa\big[1-\alpha(j)\big]\int_0^{\epsilon^*(s)} G'(k)k\ dk$$
$$B(j,s) = \big[1-G(\epsilon^*(s))\big]p(j)\ .$$

In contrast to static model, (9) and (10) will be everywhere continuous.

C Qualitative results

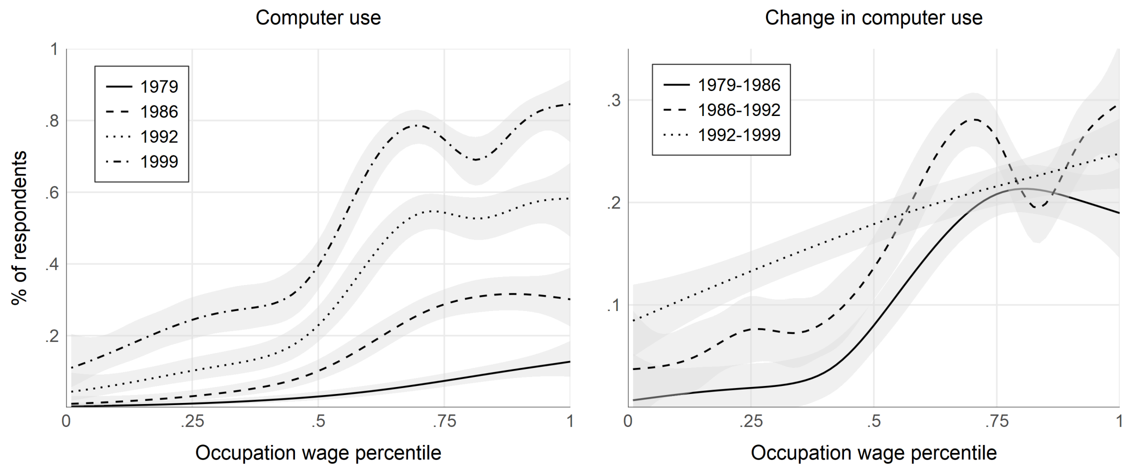

Figure C.2: Computer use by occupation wage percentile, 1979-1999

Figure notes. Log wage and PC use averaged by 1988 KLDB occupation. Percentiles are time-invariant and reflect 1979 wages. Shaded regions indicate 95% confidence intervals.

D Quantitative results

D.1 Estimation

To be completed.

D.2 Automation, wages, and employment

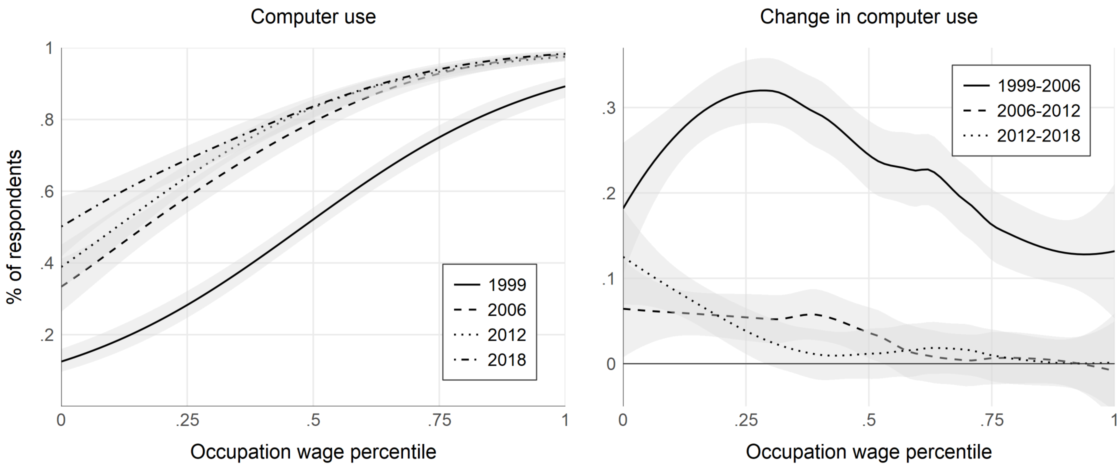

Figure C.3: Computer use by occupation wage percentile, 1999-2018

FIGURE NOTES. Log wage and PC use averaged by 1992 KLDB occupation. Percentiles are time-invariant and reflect 1999 wages. Shaded regions indicate 95% confidence intervals.

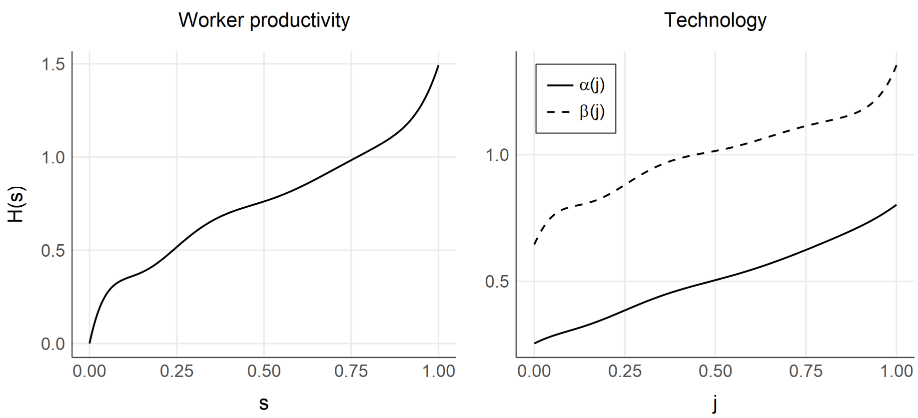

Figure C.1: Distributional parameter estimates

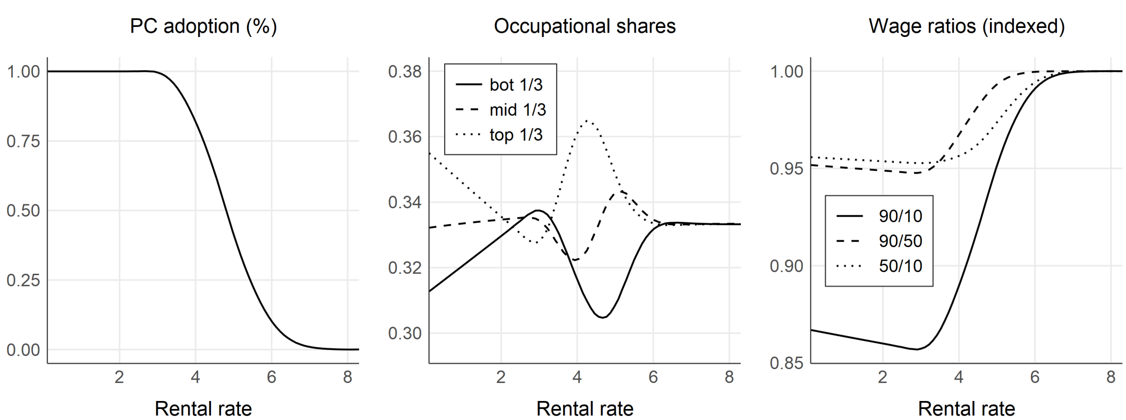

Figure C.6: Automation of skilled task

FIGURE NOTES. Smoothed model output over a grid of rental rates r_s. Model parameters are unchanged from above and reflect the limiting case where the skill-biased technology is cost-less ($r = 0$).

Table C.4: Regression: change in log occupational employment share, high-skill

Independent variable	*Regression coefficients*							
	Years 1979-1999				Years 1999-2018			
PC use	**.338**	**.284**	**.644**	**.659**	**.739**	**.707**	.565	.585
	(.108)	(.108)	(.268)	(.270)	(.115)	(.120)	(.449)	(.454)
△ PC use	**-.619**	**-.608**	**-.690**	**-.719**	**-.488**	**-.480**	-.305	-.291
	(.215)	(.207)	(.252)	(.257)	(.181)	(.184)	(.236)	(.233)
Log(wage)		**..268**		.380		.054		-.174
		(.128)		(.254)		(.123)		(.260)
Occup. FE			X	X			X	X
Observations	409	409	409	409	482	482	482	482

Table notes. Difference-in-difference regression with occupational employment share as the dependent variable, occupation wage percentiles between .5 and 1. Employment shares calculated from raw survey counts. All regressions include year fixed effects.

Table C.5: Regression: change in log occupational employment share, low-skill

Independent variable	*Regression coefficients*							
	Years 1979-1999				Years 1999-2018			
PC use	.102	.086	**.557**	**.585**	.209	.067	**-1.277**	**-1.402**
	(.165)	(.166)	(.266)	(.262)	(.122)	(.141)	(.426)	(.438)
△ PC use	-.042	-.044	-.571	-.575	.043	.066	**.696**	**.645**
	(.296)	(.296)	(.343)	(.344)	(.179)	(.174)	(.275)	(.279)
Log(wage)		.039		-.193		**.332**		**.790**
		(.142)		(.378)		(.137)		(.313)
Occup. FE			X	X			X	X
Observations	418	418	418	418	386	386	386	386

Table notes. Difference-in-difference regression with occupational employment share as the dependent variable, occupation wage percentiles between 0 and .5. Employment shares calculated from raw survey counts. All regressions include year fixed effects.