CHAPTER

9

INTEREST RATE RISK: GAP ANALYSIS

Learning Objectives

- The causes and consequences of interest rate risk.
- The link between interest rate risk and mismatching of assets and liabilities.
- The strengths and weaknesses of gap analysis.

Introduction

Repricing
The resetting of the interest rate on that loan (e.g., a term loan whose interest rate is reset every 90 days is repriced every 90 days).

In Chapter 6 we noted that, while performing its asset transformation function, an FI often mismatches its assets and liabilities. In Chapter 7 you learned that an FI with maturity mismatches between its assets and liabilities confronts liquidity risk. This chapter shows that if the **repricing** of the assets and liabilities is mismatched, the FI also faces interest rate risk. The seriousness of interest rate risk is a function of two factors: the volatility of interest rates and the degree of the FI's repricing mismatch. An FI's interest rate risk would be inconsequential if interest rates were entirely predictable or if the FI totally matched the repricing o its assets and liabilities. Unfortunately, neither condition holds in practice. Interest rates have been particularly volatile over the last 20 years. An FI could perfectly match the repricing of its assets and liabilities only at considerable cost; it would be foregoing much of its asset transformation role an the income accruing to that role. We will first briefly discuss the macroeconomic environment that leads to interest rate volatility before returning to the problem of measuring interest rate risk in the FI.

The Central Bank and Interest Rate Risk

An interest rate is a price for the use of money. Among the major factors behind its determination[1] are

- The real (i.e., inflation-free) riskless rate of interest. Historically, it has been in the range of 2 to 3 percent.

[1]A full discussion of interest rate determination is beyond the scope of this book. We assume that the reader has a basic understanding of macroeconomics. If not, the reader should review a text such as Dornbursch, Fischer, Startz, Atkins and Sparks, *Macroeconomics* (Toronto: McGraw Hill Ryerson, 2000), Chs. 9, 12, 16 and 17 to trace from the theoretical determination of interest rates through actual monetary policy in Canada. Stephen Kellision's *The Theory of Interest* (Boston: Irwin, 1991) is a good interest rate theory handbook.

Fisher Effect
A nominal rate of interest is made up of two components: the real rate of interest and the expected rate of inflation.

Riding the Yield Curve
Taking interest rate exposure to earn profits, typically by borrowing at short-term rates and lending at long-term rates of interest.

- Inflation. The **Fisher effect**[2] notes that the observed (nominal) rate of interest is really made up of two components, the real rate of interest and the inflation rate.

  $$R = (1 + rr)(1 + \pi - 1 \approx rr + \pi$$

  where R = the nominal rate of interest, rr = the real rate of interest (in a zero inflation environment), and π = the inflation rate *expected* to prevail over the interest period. The Fisher effect says that any expectation that inflation will rise over the relevant interest period will cause interest rates to rise.
- The length of investment. Investors may require a liquidity premium to induce them to place funds for differing periods of time. the liquidity premium is traditionally positive, giving rise to a normally shaped upward-sloping yield curve (see Appendix 9A for a review of the term structure of interest rates). FIs typically **ride the yield curve** by borrowing at shorter maturities and lending at longer maturities.
- Credit risk. As we discuss in Chapter 12, if there is a positive probability that less than 100 percent of principal and accrued interest will be repaid, then the FI will adjust the interest rate upwards so that the expected value of payments equals *or exceeds* the expected value of comparable risk-free loans.
- Government, corporate, and private demand for credit. Interest is simply the price for credit. The higher the demand for credit, the higher its price (all other factors being equal).
- Central bank monetary policy. By influencing the cost and availability of short-term funding, the central bank indirectly controls the supply of credit. All other things being equal, the higher the money supply, the lower interest rates are. The problem is that all other things do not remain equal. Particularly, increasing the money supply will probably increase expected inflation—which increases interest rates.

Figure 9–1 shows the yields of Canadian three-month T-bills from 1950 to 2000. Interest rates are volatile; moreover that volatility is directly linked to the Bank of Canada's monetary policy. Three recent peaks in interest rates followed directly from the Bank of Canada's action to tighten the supply of money: mid-year 1981, when interest rates peaked at over 20 percent; early 1990, when they plateaued at over 13 percent; and from the early January 1993 trough of 3.6 percent to the March 1995 peak of 8 percent.

When discussing liquidity management in Chapter 8, we described the mechanics by which the Bank of Canada greatly affects short-term interest rates. Because it exercises this control over interest rates, the Bank of Canada is frequently blamed for the recession caused by the first two peaks and the dampening of the recovery caused by the last peak. As the former governor of the Bank of Canada, Gordon Thiessen, comments in Professional Perspectives on page 000 however, the Bank of Canada is constrained by its mandate to maintain inflation at targeted low rates. Even if the bank wished to maintain higher levels of inflation, international capital flight would quickly discipline such a lax Canadian monetary policy. Clearly, our monetary policy cannot be made in isolation from that of the Bank of Japan, the European Central Bank in Frankfurt, Germany, and, especially, the Federal Reserve Bank of the United States.[3]

Volatility of interest rates in our increasingly global credit markets puts the measurement and management of interest rate risk among the major problems facing modern FI managers. If the interest rate risk an FI takes is sufficiently large—i.e., the mismatch is great enough in

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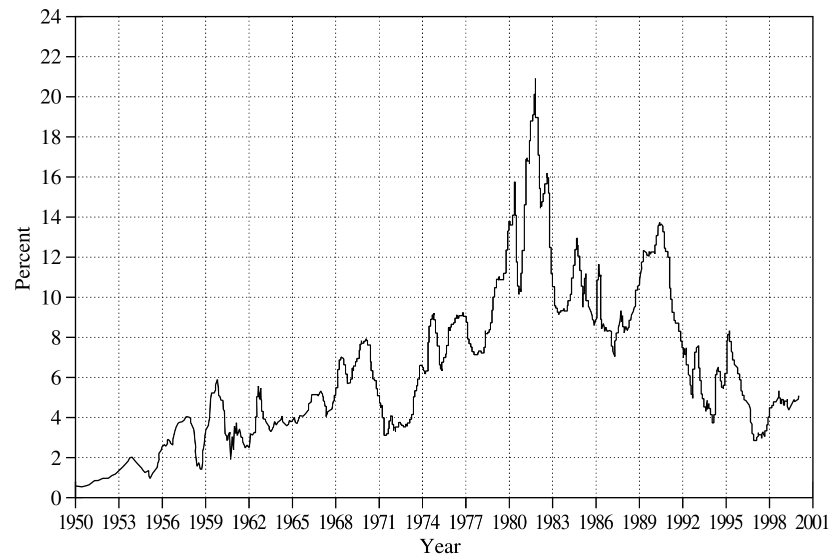
[2]Named after I. Fisher, who first observed the effect in his classic work *The Theory of Interest,* 1930 (reprint New York: Augustus M. Kelley, 1986). We discuss the Fisher effect in Chapter 12.

[3]For more details on the volatility of interest rates over different Federal Reserve regimes, see A. Saunders and T. Urich, "The Effects of Shifts in Monetary Policy and Reserve Accounting Regimes on Bank Reserve Management Behaviour in the Federal Funds Market," *Journal of Banking and Finance* 12, 1988, pp. 523–35.

FIGURE 9–1

Yields of Canadian 3-month T-bills, January 1950–2000.



Source: CanSim.

a volatile interest rate environment—and the FI loses its bet, it will be driven to insolvency. As the Professional Perspectives on page 000 describes, interest rate risk precipitated the crisis that developed into one of the largest government bailouts of FIs in world history.

In this chapter and the next, we analyze the different ways FIs measure the exposure they face in running a mismatched book of assets and liabilities in a world of interest rate volatility.

In particular, we concentrate on two models of measuring the interest rate exposure of an FI:

- Gap[4] analysis (Chapter 9).
- Duration analysis (Chapter 10).

The Effect of Interest Changes on FI Portfolios

Book Value Accounting Accounting in which the assets and liabilities of the FI are reported according to their historic values and thus are insensitive to changes in market rates.

FIs record assets on their balance sheets using a mixture of accounting methods. Loans and debt securities for investment are shown according to **book value accounting.** This shows the historic value of the loan booked (less a provision for credit risk, as we discuss in Chapter 12), regardless of the interest rate on the loan and the change in the interest rate environment.[5]

[4]Note that the gap analysis discussed in this chapter concerns the interest rate gap or, equivalently, the repricing gap. It differs materially from the *financing gap* discussed in Chapter 7 under liquidity management.

[5]Bonds are actually carried at *amortized cost.* the cost is adjusted by a straight-line amortization of the discount or premium from the purchase date to the maturity (or first call). Insurance companies typically carry their stock and real estate portfolios using a moving average market method, an accounting method that recognizes changes in the market value of assets gradually and systematically over a period of time. (See Canadian Institute of Chartered Accountants, *Accounting Recommendations,* May 1988, pp. 3201–02.) For instance, the insurance company may have an accounting policy that, if the market price differs from the carrying value, the carrying value will be adjusted by 15 percent of the difference each year. Use of a moving average method can be justified by the observation that equity and real estate markets tend to overreact, so value smoothing may be appropriate for a long-term institutional investor such as a life insurance company.

Professional Perspectives

The Role of the Bank of Canada in Setting Interest Rates

Gordon G. Thiessen
Governor of the Bank of Canada
February 1994–January 2001

There is a commonly held view that the Bank of Canada has the capacity to set interest rates in Canada at whatever level it wishes. So why has the bank not used this capacity to counter these unwanted pressures on our interest rates in financial markets?

The reality is, however, that the Bank of Canada cannot arbitrarily set interest rates. We have an important influence on very short-term money market interest rates, but our influence beyond that on other short-term rates and out to longer-term rates is indirect. It depends on how savers and investors see our actions affecting inflation and the external value of the Canadian dollar. If the bank is seen as encouraging inflation and an associated downward trend in the value of the Canadian dollar, the result will be higher interest rates.

That is why the proposals that you sometimes hear for the bank to push down interest rates and stimulate the economy still further so as to help government solve its budget problems would not work. Actions by the bank to force interest rates lower would require us to pump more liquidity into the financial system. Such actions would raise worries about inflation and a declining trend in the Canadian dollar. This is a recipe, not for low interest rates, but for higher rates and for more pressure on government debt-service costs and deficits.

However, by strongly promoting price stability, the Bank of Canada provides an important underpinning to the expected future value of the dollar and thus to lower interest rates than would otherwise be possible.

Gordon G. Thiessen made these remarks to the Board of Trade of Metropolitan Montreal on January 19, 1995. For the full text, see "Financial Markets and the Canadian Economy," *Bank of Canada Review,* Spring 1995, pp. 79–84.

Fair Value Accounting
Accounting in which the assets and liabilities of the FI are revalued according to the current level of interest rates.

Banks and investment dealers, however, show their trading inventories of marketable securities according to **fair value accounting.** The recording of market values means that assets or liabilities are revalued to reflect current market conditions. Thus, if a fixed-coupon bond was purchased at \$100 per \$100 of face value in a low-interest rate environment, a rise in current market rates reduces the present value of the cash flows from the bond to the investor. Such a rise also reduces the price—say, to \$97—at which it could be sold in the secondary market today. That is, marking to market, implied by the fair value accounting method, reflects economic reality or the true values of assets and liabilities of the FI's portfolio were to be liquidated at today's securities prices rather than the prices at which the assets and liabilities were originally purchased or sold.

An Example

Consider the value of a bond held by an FI that has one year to maturity, one single annual coupon of 10 percent (C) plus a face value of 100 (F) to be paid on maturity, and a current yield (R) to maturity (reflecting current interest rates) of 10 percent. the price of the one-year bond, P_1^{B}, is

$$P_1^{B} = \frac{F + C}{(1 + R)} = \frac{100 + 10}{(1.1)} = 100$$

Professional Perspectives

The Savings and Loans Crisis in the United States: A Massive Interest Rate Mismatch

From the 1930s to the 1970s, savings and loan companies (S&Ls) in the United States had a simple, protected, and profitable business. The Federal Home Loan Bank (FHLB) encouraged the S&Ls to specialize in mortgage lending by restricting each institution's proportion of nonmortgage loan assets to 20 percent of total loan assets. The Federal Savings and Loan Insurance Corporation (FSLIC) insured their deposits. Retail deposits were their major source of funds, so they effectively funded massive maturity mismatches. They took short-term deposits and lent long-term (in 25-year mortgages) at fixed rates of interest. As long as rates of interest remained low and stable, and depositors could find no higher-yielding, safe, liquid investment, S&Ls took advantage of upward-sloping yield curves to earn good profits.

In the 1970s, things changed. The S&Ls saw their average cost of funds creep from 5.38 percent in 1971 to 7.47 percent in 1979, while their average return on mortgages grew from 6.81 percent to 8.83 percent. although they preserved a profitable spread, by the end of the decade inflation—and interest rates—were increasingly rising. At the same time, the S&Ls' natural deposit base was being eroded by higher-yielding money market funds. The S&Ls countered by raising deposit rates and looking elsewhere for funds. In 1979 they began issuing negotiable orders of withdrawal (NOW) accounts, which functioned as interest-bearing chequing accounts. Some also began issuing large denomination CDs, which (because they were issued in amounts of greater than $100,000) were not protected by FSLIC insurance.

The whole S&L industry was overtaken by the events of October 1979. The Federal Reserve Board, moving to control inflation by restricting money supply growth, forced interest rates up sharply. In the subsequent recession, with fewer high-rate mortgages being booked, the S&Ls' existing portfolios of old mortgage loans held down earnings. By 1981, the average cost of funds for S&Ls had risen to 10.92 percent while their mortgage portfolio return remained at 9.87 percent. the entire industry, operating at a loss, was saved from oblivion by FSLIC and the FHLB Board, which provided security to depositors and lines of credit to the S&Ls themselves. The U.S. Congress released the S&Ls from the restrictions governing their assets and liabilities, increasing the array of services the S&Ls could offer. The crisis abated in late 1982 because of rapid declines in interest rates and the end of the recession.

The equity capital of the industry had been so eroded, however, that many S&Ls would have been insolvent if the true value of their assets had been assessed, yet continued government support allowed them to remain in business. Faced with increased competition from deregulation, new powers to expand their operations, and nothing to lose from increasing their risk, many tried to grow their way out of their insolvency. The failure of the regulatory authorities through the 1980s to close insolvent S&Ls led many to plough themselves deeper into insolvency. Only in 1989 did Congress pass the Financial Institutions Reform, Recovery and Enforcement Act to truly address the problems. By 1995, with the cleanup nearly completed, the final Bill to the U.S. taxpayers for the S&L debacle is estimated to be some U.S. $180 billion.

For an introduction to the vast literature on the S&L crisis, see Alva W. Stewart, *The Savings and Loan Crisis: A Bibliography* (Vance Bibliographies, 1991).

Suppose that the Bank of Canada tightens monetary policy so that the required yield on the bond rises instantaneously to 11 percent. the market value of the bond falls to:

$$P_1^B = \frac{100 + 10}{(1.11)} = 99.10$$

Thus, the market value of the bond is now only $99.10 per $100 of face value instead of its original book value, $100. The FI has suffered an realized capital loss (ΔP_1) of $0.90 per $100 of face value in holding this bond, or

$$\Delta P_1 = 99.10 - 100 = -0.90\%$$

This example simply demonstrates the fact that

$$\frac{\Delta P}{\Delta R} < 0$$

A rise in the required yield to maturity reduces the price of fixed-income securities held in FI portfolios. Note that if the bond under consideration is issued as a liability by the FI (e.g., a fixed-interest deposit such as a CD) rather than being held as an asset, the effect is the same: the market value of the FI's deposits will fall. However, the economic interpretation is different. Although rising interest rates that reduce the market value of its assets are bad news, a reduction in the market value of its liabilities is good news for the FI. The economic intuition is straightforward. Suppose the FI issued a one-year deposit with a promised interest rate of 10 percent and principal or face value of $100.[6] When the current level of interest rates is 10 percent, the market value of the liability is 100:

$$P_1^D = \frac{100 + 10}{(1.1)} = 100$$

If interest rates on new one-year deposits rise instantaneously to 11 percent, the FI has gained by locking in a promised interest payment to depositors of only 10 percent. the market value of the FI's liability to its depositors would fall to $99.10; alternatively, this would be the price the FI would need to pay the depositor if it repurchased the deposit in the secondary market:

$$P_1^D = \frac{100 + 10}{(1.11)} = 99.10$$

That is, the FI gained from paying only 10 percent on its deposits rather than 11 percent if they were newly issued after the rise in interest rates.

You can see that in a fair value accounting framework, rising interest rates generally lower the fair values of both assets and liabilities on an FI's balance sheet. Clearly, falling interest rates have the reverse effect; they increase fair values of both assets and liabilities. In the preceding example, both the bond and the deposit were of one-year maturity. We can easily show that if the bond or deposit had a two-year maturity with the same annual coupon rate, the same increase in market interest rates from 10 to 11 percent would have had a more *negative* effect on the fair value of the bond's price. That is, before the rise in required yield:

$$P_2^B = \frac{10}{1.1} + \frac{100 + 10}{(1.1)^2} = 100$$

After the rise in market interest rates yields from 10 to 11 percent,

$$P_2^B = \frac{10}{1.11} + \frac{100 + 10}{(1.11)^2} = 98.29$$

and

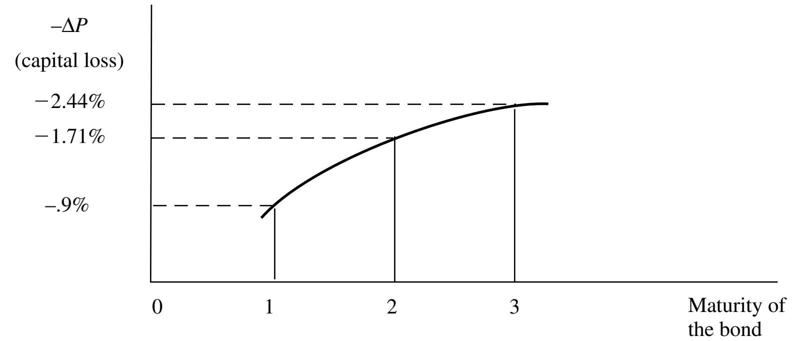
$$\Delta P_2 = 98.29 - 100 = -1.71\%$$

This example demonstrates another general rule of portfolio management for FIs: the *longer* the maturity of a fixed income asset or liability, the greater is its fall in price and fair

[6]In this example, we assume for simplicity that the promised rate on the CD is 10 percent. in reality, for positive returns to intermediation to prevail, the promised rate on the CD would be less than the received rate (coupon) on assets.

FIGURE 9–2

The relationship among ΔR, maturity and ΔP.



value capital (i.e., the greater the absolute value of the change in price) for any given increase in the level of market interest rates:

$$\underline{|\Delta P_1|} < \underline{|\Delta P_2|} < \ldots < \underline{|\Delta P_{30}|}$$

Note, however, that while a two-year bond's fall in price is greater than the one-year bond's, the difference between the two price falls $\Delta P_2 - \Delta P_1$ is $-1.71\% - (-0.9\%) = -0.81\%$. The fall in a 3-year, 10-percent-coupon bond's price when yield increases to 11 percent is $-$ 2.44 percent. Thus, $\Delta P_3 - \Delta P_2 = -2.44\% - (-1.71\%) = -0.73\%$. This establishes an important result: While P_3 falls more than P_2 and P_2 falls more than P_1, the size of the capital loss increases at a diminishing rate as we move into the higher maturity ranges. This effect is graphed in Figure 9–2.

So far, we have shown that for an FI's fixed-income assets and liabilities:

1. A rise (fall) in interest rates generally leads to a fall (rise) in the market value of an asset or liability.
2. The longer the maturity of a fixed-income asset or liability, the greater the fall (rise) in fair value for any given interest rate increase (decrease).
3. The value of longer-term securities falls at a diminishing rate for any given change in interest rates.

Gap Analysis

We used the preceding example to remind you of the relationship between bond prices and market interest rates that you studied in your introductory finance or economics courses. The effect of that relationship on an FI will depend on whether the bond—or similar instrument—is held as an asset or issued as a liability. As we discussed in Chapters 1 through 4, the majority of the assets (deposits, money market securities, loans, mortgages, bonds, etc.) and liabilities (deposits, GICs, actuarial liabilities, debentures, etc.) of FIs are instrument priced wholly or partly by prevailing market interest rates. The analyst must cancel out the offsetting interest rate risks posed by assets and liabilities[7] in order to understand the net interest rate risk.

[7]The off-balance-sheet interest rate risk position must also be added to obtain the full interest rate risk position of a modern FI. We will return to this aspect in Chapters 14 and 20–22.

Table 9–1 Gap Analysis ($ billions)

Interest Repricing Interval	*Assets*	*Liabilities*	*Gap*	*Cumulative Gap*
One day	$ 2.4	$ 2.3	$0.1	$0.1
One day–three months	3.0	4.0	−1.0	−0.9
Three–six months	6.9	8.5	−1.6	−2.5
Six–12 months	9.1	6.8	2.3	−0.2
One-five years	4.0	0.9	3.1	+2.9
Over five years	1.3	0.2	1.1	+4.0
Not interest rate sensitive	1.4	5.4	−4.0	0.0
Total	$28.1	$28.1	$0.0	0.0

Repricing Gap
The difference between those assets whose interest rates will be repriced or changed over some future period (rate-sensitive assets) and those liabilities whose interest rates will be repriced or changed over some future period (rate-sensitive liabilities).

Repricing Bucket
A grouping of assets (or liabilities) according to the time until their interest rates are reset.

Cumulative Gap
A measure of the aggregate gap over any given planning horizon.

Gap analysis, also known as the repricing model, is essentially a book value accounting cash flow analysis of the **repricing gap** between the interest revenue earned on an FI's assets and the interest paid on its liabilities over some particular period.

Look at Table 9–1. An analyst has divided the FI's entire balance sheet into seven **repricing buckets** (or bins) by looking at the rate sensitivity of each asset and each liability on its balance sheet. *Rate sensitivity* here means the time to repricing of the asset or liability. More simply, it means how long the FI manager has to wait to change the posted rates on any asset or liability. In Table 9–1, we show how the assets and liabilities of an FI are categorized into each of the six previously defined buckets according to their time to repricing. Note that $1.4 billion in assets (including property, plant, and equipment) and $5.4 billion in liabilities (mostly demand deposits, common equity, and retained earnings) are not interest rate-sensitive.

While the **cumulative gap** shown in Table 7–1 over the whole balance sheet must by definition be zero, the advantage of the repricing model lies in its information value and its simplicity in pointing to an FI's *net interest income exposure* (or earnings exposure) to interest rate changes at different repricing buckets.

For example, the one-day gap indicates a positive difference between assets and liabilities being repriced in one day. As we discussed in Chapters 7 and 8, assets and liabilities that are repriced each day are likely to be Bank of Canada advances, interbank borrowings, and repos (repurchase agreements). This gap suggests that a rise in the overnight interest rates would raise the bank's net interest income because the bank has more rate-sensitive assets than liabilities in this bucket. In other words, it has sold more short-term funds (such as interbank deposits) than it has purchased. Specifically, let

RSA = Rate-sensitive assets

RSL = Rate-sensitive liabilities

ΔNII_i = Change in net interest income in the ith bucket

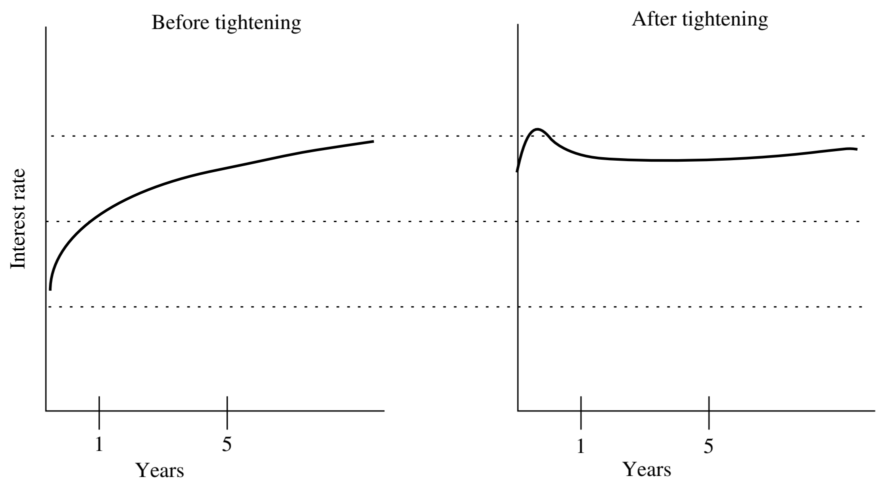
GAP_i = The dollar size of the gap between the book value of assets and liabilities in maturity bucket i

ΔR_i = The change in the level of interest rates affecting assets and liabilities in the ith bucket: Then

$$\Delta NII_i = (GAP_i)\,\Delta R_i = (RSA_i - RSL_i)\,\Delta R_i$$

FIGURE 9–3

Possible effect of tightening monetary policy



In the first bucket, if the gap is positive $0.1 billion and overnight interest rates rise by 1 percent, the annualized[8] change in the bank's future net interest income is

$$\Delta NII_{\text{i}} = \$100 \text{ million} \times 0.01 = \$1 \text{ million}$$

As you can see, this approach is very simple and intuitive. Moreover, it is flexible. The analyst can define the repricing buckets as finely (e.g., daily buckets) or as coarsely (buckets of several years) as she wishes, depending on the FI's portfolio. And the effect of various changes in the term structure of interest rates can be seen easily. In the example, the analyst may consider that, although short- and medium-term rates will rise from the tightening of the Bank of Canada's monetary policy, the interest rates on maturities in excess of two years will fall as markets revise their inflation expectations downwards. The assumptions in this shift in the yield curve, illustrated in Figure 9–3, can be used to determine the expected changes in FI income.

Remember from our example at the beginning of this chapter, however, that capital or fair value losses occur when rates rise. The capital loss effect is not evident here. The reason is that in the book value accounting world of the repricing model, assets and liabilities are reported at their *historic* values or costs. Thus, interest rate changes affect only future interest income or interest costs—that is, net interest income.[9]

[8]By *annualized* change, we mean the change in income that would result if this change in daily income caused by the shift in interest rates were replicated for 365 days and no other effects were present. Clearly, this is highly unlikely, as maturity buckets of greater than one day but less than one year would also be repriced as a calendar year progressed.

[9]For example, if an FI bought the 8-percent-coupon Government of Canada bond maturity on June 1, 2023, on January 12, 1994, it would have paid $111.25 per $100 face value. A year later, the yield on the bond had risen from 7.083 percent to 9.50 percent, driving the bond's price down to $85.25, a 23-percent capital loss that gap analysis would ignore! In a fair value world, this loss to asset and liability values would be reflected in the balance sheet as rates changed. If the FI had bought this long bond for trading purposes, it would consider its loss a realization of market risk (see Chapter 11).

TABLE 9–2 Bank Balance Sheet for Gap Analysis ($ billions)

Assets		*Liabilities*	
1. Deposits with other banks	$ 2.6	1. Demand deposits	$ 1.7
2. Treasury bills and maturing bonds	5.7	2. Notice deposits	8.5
3. Short-term, maturing, and floating-rate loans	7.5	3. Term deposits less than one year	12.0
4. Customer liability under BAs	1.1	4. BAs	1.1
5. Treasury, provincial, and municipal bonds	0.8	5. Term deposits more than one year	1.0
6. Floating-rate mortgages (six-month rate adjustment)	4.5	6. Debentures	0.9
7. Fixed-rate mortgages (rate adjustments in more than one year)	4.5	7. Preferred shares	0.6
8. Buildings and equipment	1.4	8. Common equity	2.3
	$28.1		$28.1

The FI manager can also estimate cumulative gaps (CGAP) over various repricing buckets. A common cumulative gap of interest is the one-year repricing gap, estimated from Table 9–1 as:

$$\text{CGAP} = (+0.1) + (-1.0) + (-1.6) + (+2.3) = -0.2$$

If ΔR_i is the average rate change affecting assets and liabilities that can be repriced within a year, the cumulative effect on the bank's net interest income is

$$\begin{aligned} \Delta NII_i &= (CGAP)\,\Delta R_i \\ &= (-200 \text{ million})\,(.01) = -2 \text{ million} \end{aligned}$$

We can now look at how an FI manager would calculate the cumulative one-year gap from a balance sheet. Remember that the manager asks: Will or can this asset or liability have its interest rate changed within the next year? If the answer is yes, it is a rate-sensitive asset or liability; if the answer is no, it is not rate sensitive within the one-year bucket.

Consider the simplified balance sheet facing the FI manager in Table 9–2. Rather than the original maturities, the maturities are those remaining on different assets and liabilities at the time the repricing gap is estimated.

Rate-sensitive Assets

As we look down the asset side of the balance sheet, these five items are one-year rate-sensitive assets (RSA):

1. Deposits with other banks: $2.6 billion. These include overnight loans, repos, and term deposits under one year.
2. Treasury bills and maturing bonds: $5.7 billion. These are money market instruments and bonds with less than one year before final maturity. Note that Treasury, provincial and municipal bonds maturing in more than one year are not RSA within one year.
3. Short-term, maturing, and floating-rate loans: $7.5 billion. These include loans of up to one year remaining until maturity as well as term loans and lines of credit that are priced over **prime rate, LIBOR,** or other floating rates.
4. Customer liability under BAs: $1.1 billion. These are obligations of the payees (the bank clients on whose behalf the BAs were created) to reimburse the bank on the payment date.

Prime Rate
A variable annual rate of interest that can be changed at any time without notice, charged to a bank's most creditworthy borrowers.

LIBOR
The London Interbank Offer Rate is the interest rate at which banks active in the interbank market will place deposits of a set maturity with other banks.

5. Floating-rate mortgages: $4.5 billion. These are repriced (i.e., the mortgage rate is reset) every six months. Thus, these long-term assets are rate-sensitive assets in the context of repricing with a one-year horizon.

Summing these five items produces a one-year RSA total of $21.4 billion.

Rate-sensitive Liabilities

As we look down the liability side of the balance sheet, these three liability items clearly fit the one-year repricing sensitivity test:

1. Notice deposits: $8.5 billion. The interest dates on these deposits can be varied daily at the FI's discretion.
2. BAs: $1.1 billion. This is the bank's obligation to make payment on BAs that mature within the year.
3. Term deposits less than 1 year: $12 billion. These mature within the year and are repriced on their **rollover date.**

Rollover Date
The date on which a term deposit that is expected to be renewed matures. Instead of withdrawing the interest and principal, the depositor *rolls* the total into a new deposit.

Summing these three items produces one-year RSL of $21.6 billion.

Note that demand deposits were not included here. We can make strong arguments for and against their inclusion as rate-sensitive liabilities.

Against inclusion: The explicit interest rate on demand deposits is zero. Moreover, as we discussed in the last chapter, demand deposits act as *core deposits* for banks, meaning they are a long-term source of funds. This is the reason for not including demand deposits in the one-year repricing bucket.

For inclusion: Even if they pay no explicit interest rates, demand deposits do pay implicit interest in the form of the bank not charging fully for chequing services through fees. Further, if interest rates rise, individuals draw down (or run off) their demand deposits, forcing the bank to replace them with higher-yielding, interest-bearing, rate-sensitive funds. This is most likely to occur when the interest rates on alternative instruments are high. Then the opportunity cost of holding funds in demand deposit accounts is likely to be larger than in a low-interest-rate environment.

The three repriced liabilities of $\$8.5 + \$1.1 + \$12.0$ sum to $21.6 billion, and the five repriced assets of $\$2.6 + \$5.7 + \$7.5 + \$1.1 + \$4.5$ sum to $21.4 billion. Given this, the cumulative one-year repricing gap for the bank is

$CGAP =$ One-year rate-sensitive assets $-$ one-year rate-sensitive liabilities
$CGAP = RSA - \text{RSL}$
$CGAP = \$21.4 - \$21.6 = -\$0.2$ billion

This can also be expressed as a percentage of assets:

$$\frac{CGAP}{A} = \frac{-0.2}{28.1} = -0.0071 = 0.71\%$$

Expressing the repricing gap in this way is useful since it tells us: (1) the direction of the interest rate exposure (positive or negative CGAP) and (2) the scale of that exposure (by dividing the gap by the asset size of the institution).

In our example, the bank has fewer rate-sensitive assets than liabilities in the one-year and under bucket, and the gap is −0.71 percent of its balance sheet. This is a small, tactical gap. If the one-year cumulative gap as a percent of assets were in excess of 10 percent, it would reflect a strategic, speculative positioning.

TABLE 9–3 One-Year Rate-Sensitive Gap as Percentage of Total Assets October 31, 1993

Bank	
Bank of Montreal	0.17
Bank of Nova Scotia	3.47
Canadian Imperial Bank of Commerce	1.34
National Bank of Canada	−0.63
Royal Bank of Canada	−4.91
Toronto-Dominion Bank	−5.88

Note: Shows cumulative gap, including off-balance-sheet exposure.
Source: Banks' annual reports.

Look at the one-year percentage gaps of the Big Six banks in Table 9–3 in October 1993, prior to the last large interest-rate spike shown in Figure 9–1. Notice that some banks were taking quite large interest-rate gambles relative to their asset size and that different banks held different opinions as to the direction interest rates would go. For example, the average one-year repricing gap of the Toronto-Dominion was −5.88 percent, while that of the Bank of Nova Scotia was +3.47 percent in October 1993. If interest rates had risen and TD had not adjusted its position, it would have been exposed to significant net interest income losses due to the cost of refinancing its large amount of rate-sensitive liabilities.[10] As it turned out, interest rates first dropped, then rose to finish the year well above their October 1993 levels. BNS was more correct than TD.

Clearly, the rate sensitivity gap can be a useful tool for managers and regulators in identifying interest rate risk-taking or exposure. Nevertheless, the repricing gap model has a number of serious weaknesses.[11]

Concept Questions

1. Why do some banks in Table 9–3 have positive gaps while others have negative gaps?
2. How can FIs quickly change the size and direction of their gaps?

Weaknesses of the Repricing Model

Market Value Effects

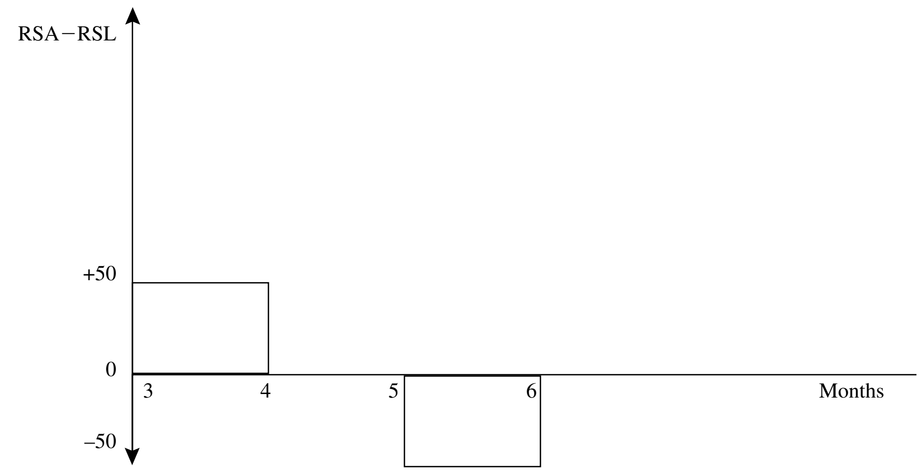
As we discussed earlier, interest rate changes have a fair value effect in addition to an income effect on asset and liability values. The repricing model ignores the former (implicitly assuming a book value accounting approach). Thus, the repricing gap is only a *partial* measure of the true interest rate exposure of an FI.

[10]As we discuss in Chapter 20, positions can be adjusted quickly using off-balance-sheet instruments such as FRAs and swaps. The gaps in Table 9–3 reflect both on- and off-balance-sheet interest rate exposure.

[11]See Elijah Brewer, "Bank Gap Management and the Use of Financial Futures," *Federal Reserve Bank of Chicago Economic Perspectives,* March–April 1985, pp. 12–22, for an excellent analysis of the repricing gap model and its strengths and weaknesses.

FIGURE 9–4

The overaggregation problem: The three- to six-month bucket



Overaggregation

The problem of defining buckets over a range of maturities ignores information regarding the distribution of assets and liabilities within that bucket. For example, the dollar values of rate-sensitive assets and liabilities within any repricing bucket range may be equal; however, on average, liabilities may be repriced towards the end of the bucket's range while assets may be repriced towards the beginning.

Look at the simple example for the three- to six-month bucket in Figure 9–4. Note that \$50 million more rate-sensitive assets than liabilities are repriced between months 3 and 4, while \$50 million more liabilities than assets are repriced between months 5 and 6. The FI shows a zero repricing gap for the three- to six-month bucket $(+50 + (-50) = 0$, but, as you can easily see, the FI's assets and liabilities are *mismatched* within the bucket. Clearly, the shorter the range over which bucket gaps are calculated, the smaller this problem is. If an FI manager calculated one-day bucket gaps out into the future, this would give a very good idea of the net interest income exposure to rate changes. Many large banks reportedly have internal systems that show their repricing gaps on any given day in the future (252 days hence, 1,329 days, etc.). This suggests that although regulators require only the reporting of repricing gaps over relatively wide maturity bucket ranges, FI managers could set up internal information systems to report the daily future patterns of such gaps.[12]

The Problem of Runoffs

In the simple repricing model in the first section, we assumed that all consumer loans matured in one year or that all conventional mortgages matured in 20 years' time. in reality,

[12]Another way to deal with the overaggregation problem is to adjust the buckets to interest rate repricing within the bucket. Let RSA and RSL be rate-sensitive assets and liabilities in a bucket, R denote initial interest rates on an asset or liability, K be new interest rates after repricing, A be assets, and L be liabilities. Let t be the proportion of the bucket period for which the asset's (liability's) old interest rate (R) is in effect; thus, $1 - t$ is the proportion of the bucket period in which the new interest rate (K) is in operation:

$$\Delta NII = \text{RSA}\,[(1 + R_A)^{tA} \bullet (1 + K_A)^{1-tA}] - \text{RSL}\,[(1 + R_L)^{tL} \bullet (1 + K_L)^{t-tL}]$$

See Brewer, "Bank Gap Management," for details.

TABLE 9–4 Runoffs of Different Assets and Liabilities ($ millions)

Assets			Liabilities		
Item	*Amount Run Off in Less Than One Year*	*Amount Run Off in More Than One Year*	*Item*	*Amount Run Off in Less Than One Year*	*Amount Run Off in More Than One Year*
Short-term consumer loans	$ 50	$ 0	Equity	$ 0	$20
Long-term consumer loans	5	20	Demand deposits	30	10
Three-month T-bills	30	0	Notice deposits	15	15
Six-month T-bills	35	0	Three-month term deposits	40	0
Three-year notes	10	60	Six-month term deposits	20	0
10-year notes	2	18	Nine-month term deposits	60	0
20-year mortgages	4	36	One-year term deposits	20	0
			Two-year term deposits	20	20
Total	$136	$134	Total	$205	$65

Runoff
Periodic cash flow of interest and principal amortization payments on long-term assets, such as conventional mortgages, that can be reinvested at market rates.

the bank continuously originates and retires consumer and mortgage loans as it creates and retires deposits. For example, today some 20-year original maturity mortgages may have only one year left before they mature; that is, they are in their 19th year. And virtually all long-term mortgages pay at least some principal back to the FI each month. As a result, the FI receives a **runoff** cash flow from its conventional mortgage portfolio that can be reinvested at current market rates; that is, this runoff component is rate sensitive. The FI manager can easily deal with this in the repricing model by identifying for each asset and liability item the proportion that will run off, reprice, or mature within the next year. For example, consider Table 9–4.

Notice in Table 9–4 that while the original maturity of an asset or liability may be long term, it still generates some cash flows that can be reinvested at market rates. This table is a more sophisticated measure of the one-year repricing gap that takes into account the cash flows received on each asset and liability item during that year. Adjusted for runoffs, the repricing gap is

$$GAP = \$136 - \$205 = -\$69$$

Note that the runoffs themselves are not independent of interest rate changes. Specifically, when interest rates rise, many people may delay repaying their mortgages (and the principal on those mortgages) causing the runoff amount of $4 million in Table 9–4 to be overly optimistic. Similarly, when interest rates fall, people may prepay their fixed-rate mortgages to refinance at a lower interest rate. Then runoffs could balloon to much more than $4 million. This sensitivity of runoffs to interest rate changes is a further weakness of the repricing model.

Concept Questions

1. What is meant by a runoff?
2. What are three major problems with the repricing model?

Summary

- Interest rate risk arises from interest rate volatility and asset and liability repricing mismatching.
  - The central bank's inflation rate and foreign exchange stability targets remove its ability to set interest rates independently.
  - International and domestic macroeconomic shocks are transmitted into interest rates, leading to high volatility.
  - Some mismatching is an inescapable part of financial intermediation.
- Bond price and interest rates are related.
  - Price is an inverse function of the market interest rate.
  - The longer the bond maturity, the greater its sensitivity to the interest rate.
  - The increase in sensitivity decreases with increasing maturity.
- Gap analysis is a useful and intuitive tool for analyzing interest rate risk.
  - Rate-sensitive liabilities are subtracted from assets for each maturity bucket.
  - The cumulative annual change in noninterest income attributable to each bucket can be estimated.
- Gap analysis presents problems with:
  - Ignoring immediate capital losses and gains through fair valuation.
  - Overaggregation.
  - Runoffs.

Questions

1. How did interest rate risk lead to the S&L crisis in the United States?
2. Why is it important to use fair (as opposed to book) values in financial decision-making?
3. What are some advantages of using book values as opposed to fair values?
4. List three possible explanations for a reduction in the fair value of a purchased financial security below book value.
5. Gap analysis requires specification of the length of the repricing period. Why must a time period be specified? How does the choice of the repricing period affect the delineation between rate-sensitive and fixed-rate assets and liabilities?
6. What determines the optimal length of the repricing period?
7. What are the shortcomings of very long repricing periods?
8. Which of the following assets or liabilities fit the one-year rate or repricing sensitivity test?
   a. 91-day treasury bills.
   b. One-year treasury notes.
   c. 20-year Canada bonds.
   d. 20-year floating-rate corporate bonds with annual repricing.
   e. 20-year floating-rate mortgages with annual repricing.
   f. 20-year floating-rate mortgages with biannual repricing.
   g. Overnight interbank loans.
   h. Nine-month fixed-rate CDs.
   i. One-year fixed-rate CDs.
   j. Five-year floating-rate CDs with annual repricing.
   k. Common stock.

Problems

1. Evaluate the prices of the following pure discount (zero-coupon) bonds:
   a. \$1,000 face value received in five years, yielding an annual rate of 8 percent.
   b. \$10,000 face value received in three years, yielding an annual rate of 6 percent.
   c. \$100,000 face value received in 10 years, yielding an annual rate of 13 percent.
   d. \$1,000,000 face value received in two years, yielding an annual rate of 7 percent.
   e. \$1,000,000 face value received in six months, yielding an annual rate of 7 percent.
2. Calculate the value of each bond in Question 1 if all yields increased by 1 percent.
3. Calculate the percentage price changes for each bond in Question 1 if all yields increased by 1 percent (as in Question 2).
4. What can you conclude about bond price volatility from your answer to Question 3?
5. If the bonds in Question 1 were coupon instruments selling at par, calculate the annual coupon payment for each bond.
6. Calculate the prices of each coupon bond in Question 5 if all yields increased by 1 percent.
7. Calculate the percentage price changes for each bond in Question 6 if all yields increased by 1 percent. (Recall that the coupon bonds were originally priced at par.)

8. Compare your answers to Questions 3 and 7. What can you conclude about bond price volatility?
9. Consider a five-year coupon bond with a face value of $1,000 paying an annual coupon of 15 percent.
   a. If the current market yield is 8 percent, what is the bond's price?
   b. If the current market yield increases by 1 percent, what is the bond's new price?
   c. Using your answers to Parts a and b, what is the result of the 1 percent increase in interest rates?
10. Compare your answers to Questions 3, 7, and 9. What can you conclude about bond price volatility?
11. Calculate the repricing gap and impact on net interest income of a 1-percent increase in interest rates for the following positions:
   a. Rate-sensitive assets = $100 million, rate-sensitive liabilities = $50 million.
   b. RSA = $50 million, RSL = $150 million.
   c. RSA = $75 million, RSL = $70 million.
   d. Compare the interest rate risk exposure of the institutions in Parts *a, b,* and *c.*
12. Use the following data to answer Parts *a* through *c.*

Givebucks Bank, Inc.
($ millions)

Assets		*Liabilities*	
Rate-sensitive	$50	Rate-sensitive	$70
Fixed rate	50	Fixed rate	20
		Equity	10

Notes: All RSAs currently earn 10 percent interest per annum. All fixed-rate assets earn 7 percent per annum. RSLs currently pay 6 percent per annum, while fixed-rate liabilities offer 6 percent.

   a. What is Givebucks Bank's current net interest income?
   b. What will the net interest income be if interest rates increase by 2 percent?
   c. What is Givebucks' repricing or interest rate gap? Use it to check your answer to Part *b.*
   d. Why might Givebucks' change in NII differ from that predicted by gapping?
13. Use the following information about a hypothetical government security dealer named J. P. Mersal Citover to answer Parts *a* and *b.* (Market yields are in parentheses.)

J. P. Mersal Citover
($ millions)

Assets		*Liabilities*	
Cash	$ 10	Overnight repos (7.00%)	$170
T-bills: 30-day (7.05%)	75	Subordinated debt: Seven-year fixed at (8.55%)	150
T-bills: 91-day (7.25%)	75		
Government notes: Two-year (7.50%)	50	Equity	15
Canada bonds: 10-year (8.96%)	100		
Municipal notes: Five-year quarterly floating rate (8.20%)	25		

   a. What is the repricing or interest rate gap if the planning period is 30 days? 91 days? Two years? (Recall that cash is a noninterest-earning asset.)
   b. Use gapping to estimate the impact over the next 30 days on net interest income if all interest rates rise by 50 basis points.
   c. If the 50-basis-point increase in *b* were the only change in interest rates during the year, would you expect the actual change in annual NII to be as calculated in *b*? Why not?
14. Assume a planning period of 120 days when answering Parts *a* through *e.*

Assets	*Rate*
30-year, fixed-rate mortgages: $11 million	10%
90-day, fixed-rate loans: $35 million	9%
Property: $4 million	

Liabilities and Equity	*Rate*
Demand deposits: $12 million	0%
Interbank borrowings (with maturities less than 90 days): $30 million	7%
Equity: $8 million	

   a. Calculate this bank's repricing gap.
   b. What is the bank's annual net interest income, assuming that all rates stay constant and the principals of all assets and liabilities are rolled over on maturity?
   c. Suppose that all interest rates decrease by 50 basis points over the planning period. What will be the impact on net interest income?
   d. Suppose that all interest rates increase 1 percent over the planning period. What will be the impact on net interest income?
   e. What is the bank's interest rate risk exposure? How can the bank protect itself from unanticipated reductions in net interest income?
15. *Challenge Question*
   Spot rates are
   One-year CD: 7.80%
   Two-year CD: 7.95%
   One-year municipal note: 7.95%
   Two-year municipal note: 8.15%
   Overnight interbank rates: 8.075%

a. Using the preceding term structure, describe the leveraged transaction with the highest interest spread. (Recall that a typical transaction for an FI consists of the simultaneous purchase of an interest-earning asset financed with the issuance of a financial liability.)

b. What is the interest rate risk exposure of the transaction in Part *a*?

c. If all interest rates increase 50 basis points at the end of one year, what are the cash flows at the end of each of the first and second years? (Hint: Use implied forward rates to form expectations about future spot rates for one-year CDs.)

d. List two transactions that have no interest rate risk exposure. What are the cash flows over the life of the investment?

APPENDIX 9–A
TERM STRUCTURE OF INTEREST RATES

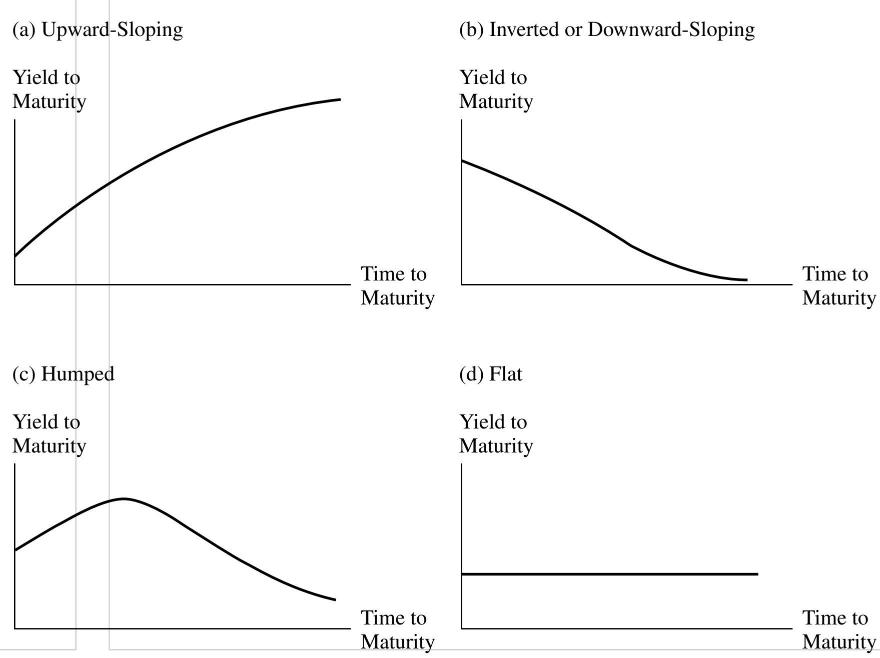
To explain the process of estimating the impact of an unexpected shock in short-term interest rates on the entire term structure of interest rates, we can use the theory of the term structure of interest rates of the yield curve. The term *structure of interest rates* compares the market yields on securities assuming that all characteristics (default risk, coupon rate, etc.) except maturity are the same. The yield curve for Treasury securities is the most commonly reported and analyzed yield curve. The shape of the yield curve on Treasury securities has taken many forms over the years, but the four most common shapes are shown in Figure 9A–1. In graph (a), yields rise steadily with maturity when the yield curve is upward sloping. This is the most commonly seen yield curve. Graph (b) shows an inverted or downward-sloping yield curve where yields decline as maturity increases. Inverted yield curves were prevalent just before interest rates dropped in early 1992. Graph (c) shows a humped yield curve, one most recently seen in mid-1991 to late 1991. Finally, graph (d) shows a flat yield curve in which the yield to maturity is not affected by the term to maturity. This shape of the yield curve was last seen in the fall of 1989. Explanations for the shape of the yield curve fall predominantly into three theories: the unbiased expectations theory, the liquidity premium theory, and the market segmentation theory.

Unbiased Expectations Theory

According to the unbiased expectations theory for the term structure of interest rates, at a given point in time the yield curve reflects the market's current expectations of future short-term rates. Thus, an upward-sloping yield curve reflects the market's expectation that short-term rates will rise throughout the relevant time period (e.g., the Bank of Canada is expected to tighten monetary policy in the future). Similarly, a flat yield curve reflects the expectation that short-term rates will remain constant over the relevant time period. The unbiased expectations theory posits that long-term rates are a geometric average of current and expected short-term interest rates. That is, the interest rate that equates the return on a series of short-term security investments with the return on a long-term security with an equivalent maturity reflects the market's forecast of future interest rates. The mathematical equation representing this relationship is

FIGURE 9A–1

Common Shapes for Yield Curves on Treasury Securities



$$\bar{R}_N = [(1 + \bar{R}_1)\,(1 + E(\bar{r}_2))\ldots(1 + E(\bar{r}_N)]^{1/N} - 1$$

where

$\bar{R}$ = actual N-period rate
N = term to maturity
$\bar{R}_1$ = current one-year rate
$E(\bar{r}_i)$ = expected one-year yield during period I.

For example, suppose one-year Treasury Bill rates for the next four years are expected to be as follows:

$$\bar{R}_1 = 6\%,\ E(\bar{r}_2) = 7\%,\ E(\bar{r}_3 = 7.5\%,\ E(\bar{r}_4) = 8.5\%$$

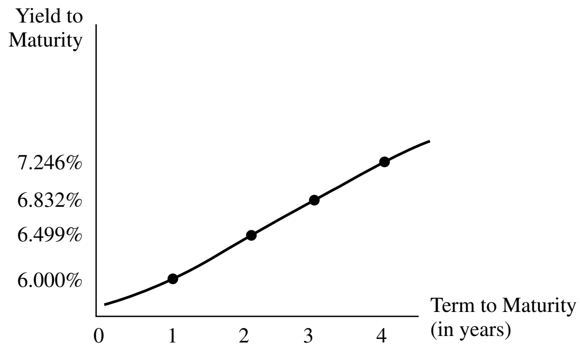
This would be consistent with the market expecting the Bank of Canada to increasingly tighten monetary policy. Using the unbiased expectations theory, current long-term rates for one-, two-, three-, and four-year maturity Treasury securities should be:

$\bar{R}_1 = 6\%$

$\bar{R}_2 = [(1 + .06)(1 + .07)]^{1/2} - 1 = 6.499\%$

$\bar{R}_3 = [(1 + .06)(1 + .07)(1 + .075)]^{1/3} - 1 = 6.832\%$
$\bar{R}_4 = [(1 + .06)(1 + 0.07)(1 + .075)(1 + .085)]^{1/4} - 1 = 7.246\%$

The yield curve should look like the following:



Thus, the current yield curve reflects the market's expectation of consistently rising short-term interest rates in the future.

Although we have discussed unbiased expectations here, the perceptive student will note that forward contracts can be written on deposits and, by arbitrage, these forward rates should equal the rates we have referred to here as the expected rates under the Unbiased Expectations Theory. We will return to a discussion of such forward rates in Chapter 20, Forward Contracts and Swaps.

Liquidity Premium Theory

The unbiased expectations theory has the shortcoming that it neglects to recognize that forward rates are not unbiased predictors of future interest rates. With uncertainty about future interest rates (and future monetary policy actions) and hence about future security prices, these instruments become risky in the sense that the return over a future investment period is unknown. In other words, because of future uncertainty of return, there is a risk in holding long-term securities and that risk increases with the security's maturity.

The liquidity premium theory of the term structure of interest rates allows for this future uncertainty to be priced. It is based on the idea that investors will hold long-term maturities if they are offered a premium to compensate for the future uncertainty associated with the long term. In other words, the liquidity premium theory states that long-term rates are the geometric average of current and expected short-term rates plus a "liquidity" or risk premium that increases with the maturity of the security. Thus, according to the liquidity premium theory, an upward-sloping yield curve may reflect the market's expectation that future short-term rates will rise, be flat, or fall, while the liquidity premium increases such that overall the yield to maturity on securities increases with the term to maturity. The liquidity premium theory may be mathematically represented as

$$\bar{R}_N = [(1 + \bar{R}_1)(1 + E(\bar{r}_2) + L_2)\ldots(1 + E(\bar{r}_N) + L_N)]^{1/N} - 1$$

where

L_t = liquidity premium for a period t and $L_2 < L_3 < \ldots < L_N$.

Market Segmentation Theory

Market segmentation theory rejects the assumption that risk premiums must rise uniformly with maturity but instead recognizes that investors have specific maturity needs or preferences. Accordingly, securities with different maturities are not seen as perfect substitutes under market segmentation theory. Instead, investors have holding periods dictated by the nature of the assets and liabilities they hold and/or by regulation, internal policy constraints, etc. As a result, interest rates are determined by distinct supply and demand conditions within a particular maturity bucket or market segment (e.g., the short end and the long end of the market). Market segmentation theory then assumes that neither investors nor borrowers are willing to shift from one maturity sector to another to take advantage of opportunities arising from changes in yields (e.g., insurance companies generally prefer long-term securities and banks generally prefer short-term securities). Figure 9A–2 demonstrates how changes in the supply curve for short- versus long-term bonds result in changes in the shape of the yield curve. Such a change may occur if the Treasury decides to issue fewer short-term bonds and more long-term bonds (i.e., to lengthen the average maturity of government debt outstanding). Specifically in Figure 9A–2, the higher the yield on securities, the higher the demand for them. Thus, as the supply of securities decreases in the short-term market and increases in the long-term market, the shape of the yield curve becomes steeper. If the supply of short-term securities increased while the supply of long-term securities had decreased, the yield curve would have become flatter (and may even have sloped downward).

FIGURE 9A–2

Caption appears here.