CSCI 5525: Machine Learning (Fall'20)

Homework 0

(Due Tue, Sept 15, 11:59 PM central)

1. Have you read through the class syllabus, noted the important dates, and the class policies?

2. (i) Which of the following courses have you taken?
   - CSci 5512 - Artificial Intelligence II
   - CSci 5521 - Introduction to Machine Learning
   - CSci 5523 - Introduction to Data Mining

   (ii) Have you taken any course on Probability/Statistics? If yes, please write down the course department and course name.

   (iii) Have you taken any course on Linear Algebra? If yes, please write down the course department and course name.

   (iv) Have you taken any course on Optimization? If yes, please write down the course department and course name.

3. Let $X \in \mathbb{R}^{n \times p}$ and $y \in \mathbb{R}^n$ be given. The goal is to find a $w^* \in \mathbb{R}^p$ which solves the following problem:
$$\min_{w \in \mathbb{R}^p} \frac{1}{2}\|y - Xw\|^2 + \frac{c}{2}\|w\|^2 \ ,$$
where $c > 0$ is a constant. Give a closed form expression for w^* in terms of X, y and c. (Consult the *Matrix Cookbook* if you want to look up expressions for derivatives in matrix/vector form.)

4. Let A be a $n \times n$ positive definite matrix. The solutions to the following problems
$$\max_{w \in \mathbb{R}^n : w^T w \leq 1} w^T A w \quad \text{and} \quad \min_{w \in \mathbb{R}^n : w^T w \leq 1} w^T A w \tag{1}$$
have well known names—do you know what the solutions to these problems are called? (You can refer back to your Linear Algebra course if needed)

5. What is the probability density function $p(x; \mu, \Sigma)$ of a multivariate Gaussian distribution with mean μ and covariance Σ? Please provide an expression in terms of x, μ, Σ, and clearly define any special function you use in the expression.

   Let $\Theta = \Sigma^{-1}$ be the precision or inverse covariance matrix. What is expression of the probability density function $p(x; \mu, \Theta^{-1})$ of a multivariate Gaussian distribution in terms of the mean μ and precision matrix Θ?