

**PROBLEM SET #4**  
(The last one for the semester! ☺)  
**Due 11:59 P.M., December 10, 2019**

## Problem 1

A rather common problem that appears regularly in signal processing is that of estimating the total phase  $\varphi_k = 2\pi f t_k + \theta$ . The frequency of the signal  $f$  is known and, thus, the dynamic model is  $\dot{\varphi} = 2\pi f$ . The measurement model relates a measurement of its amplitude  $y_k$  to  $\varphi_k$  as follows:

$$y_k = \sin \varphi_k + v_k = \widehat{y}_k + v_k, \quad v_k \sim N(0, R_k)$$

This is a nonlinear measurement model. What we want to show in this problem is how a nonlinear measurement model results in a predicted measurement  $\widehat{y}_k$  with a distribution that is very different from that of *a priori* one or that of the measurement  $y$ . For what follows, ignore the measurement noise and just focus on the random variable  $\varphi_k$ .

- Derive an analytical expression for  $f_{\widehat{Y}}(y)$  when  $\varphi \sim U(0, \pi)$ ?
- Using the pdf you derived in (a), determine  $E(\widehat{Y})$  and  $\sigma_{\widehat{Y}}^2$  when  $\varphi \sim U(0, \pi)$ .
- Using MATLAB draw 10,000 samples of  $\varphi$  and run a simple Monte Carlo experiment to generate samples of  $\widehat{y}$ . Using the generated data approximate  $E(\widehat{Y})$  and  $\sigma_{\widehat{Y}}^2$  (you can use the MATLAB commands `mean` and `var`). Also plot a histogram-based approximation of  $f_Y(y)$ . To do this, use the `histogram` command in MATLAB (use at least 50 bins with `histogram` and the option 'Normalization' set to 'pdf'). How well do the Monte Carlo results for  $E(\widehat{Y})$ ,  $\sigma_{\widehat{Y}}^2$  and  $f_{\widehat{Y}}(y)$  agree to the analytical results?
- Use the analytical approach or a Monte Carlo simulation (your choice) to determine  $E(\widehat{Y})$ ,  $\sigma_{\widehat{Y}}^2$  and  $f_{\widehat{Y}}(y)$  when  $\varphi \sim N(0, 1)$ . Which approach did you use (analytical or Monte Carlo)? Why?

## Problem 2

Do Problem 13.3 in your text.

## Problem 3

Do Problem 14.4 in your text.

## Problem 4

Do Problem 15.2 in your text.

## **Problem 5**

Do Problem 15.1 in your text.

## **Problem 6**

Do 15.15 in your text.